Is the secrecy of the parametric configuration of slot machines rationally justified? The exposure of the mathematical facts of games of chance as an ethical obligation

Barboianu, Catalin

Is the secrecy of the parametric configuration of slot machines rationally justified? The exposure of the mathematical facts of games of chance as an ethical obligation

Catalin Barboianu

Abstract

Slot machines gained a high popularity despite a specific element that could limit their appeal: non-transparency with respect to mathematical parameters. The PAR sheets, exposing the parameters of the design of slot machines and probabilities associated with the winning combinations are kept secret by game producers, and the lack of data regarding the configuration of a machine prevents people from computing probabilities and other mathematical indicators. In this article, I argue that there is no rational justification for this secrecy by giving two reasons, one psychological and the other mathematical. For the latter, I show that mathematics provides us with some statistical methods of retrieving the missing data, which are essential for the numerical probability computations in slots. The slots case raises the problem of the exposure of the parametric configuration and mathematical facts of any game of chance as an ethical obligation.

Keywords: gambling mathematics; statistical recording; outcome recording; slot machines; slots probabilities; slots mathematics; gambling ethics

Popularity of slot machines and their particularities in regard to exposure of the parametric configuration

There is a relation between the popularity of a game and excessive gambling in it, which is also visible on any pathway model to problem gambling in its first phase, namely *ecological factors* involving *availability* and *accessibility*. For decades, the slots game has remained one of the most popular games of chance and the main elements that contribute to its position – from the perspective of both a problem gambler and a non-problem gambler – are these:

a) variety; Players may choose among a wide palette of games with respect to rules, parameters, design, according to their profile, goals, or even hobbies (Griffiths, M., 1999; Wood & al., 2004).

b) privacy: Slots games – played either in a public place or in front of the computer – assume a private space in which there is only the player and the machine (Parke, J. & Griffiths, M., 2002), with minimal exposure, unlike a roulette table, for example, where you place your bets together with other players.

c) attractive design: From the graphics of the symbols to the design of the interface and even case, all is sparkling and brightly colored (Wood & al., 2004).

d) brevity: A game timeline is short, lasting few seconds from credit insertion to the stop of the spin, and a high number of games played within a time unit is preferred by players. (Griffiths, M., 1999).

e) illusions and sensations: There are multiple design and configuration features that distort player's perceptions (e.g., just missing the jackpot by one symbol can cause players to think a big win is imminent; use of a "stop" button can foster the illusion of control over machine outcomes**.** (Griffiths, M., 1999).

Slot machines gained and maintained this popularity despite some specific elements that could limit their appeal:

f) non-transparency: Players do not know the configurations of the machines they play at, as this information is not exposed. Blackjack players know the composition of the decks in play, roulette players know the numbers on the wheel, lottery players know the numbers from which the winning line is drawn, and so on. Slots remains the only game in which players are not aware of the essential parameters of the game, such as number of stops of the reels, number of symbols and their distribution on the reels.

g) prevention from odds estimation: Obviously, the lack of data regarding the configuration of a machine prevents people from computing the odds of winning and other mathematical indicators.

The so-called PAR sheets (Probability Accounting Reports), exposing the weighting of the reels, some of the probabilities associated with the winning combinations, and other statistical indicators, are kept secret by game producers.

Practical questions arise on whether this popularity would decrease and the slot player's behavior would be influenced if these hidden probabilities were exposed and whether there exists a rigorous method through which qualified persons studying slot games (statisticians, applied mathematicians, programmers, etc.) can retrieve the parameters of the configurations of slot machines in order to generate their own PAR sheets for the players. I answer the first question in the section *The psychological argument* and the second in the section *Statistical methods for estimating the parameters of the configuration of a slot machine*.

The parametric configuration of slot machines as a base for the probabilistic models for the slot games

Although less difficult than other games with respect to the ease of probability calculus (compared to card games, for example), slot games still fall into that category of games of chance for which the probability computations cannot be conducted and performed by the average player, as such computations require medium to advanced probability theory knowledge and skills. Therefore, the final probability results for these games can be delivered to gamblers only by qualified persons, as numerical probabilities or formulas (or software programs /applets using those formulas) ready to be computed by inserting the parameters of the specific game design and of the event to be measured.

For the applied mathematician, the hardest task in establishing the mathematical model for the probability calculus in slots is the optimal categorization of slot games, so as to be able to obtain general probability formulas with variables describing all possible parametric designs of the machines and all winning events. This difficulty is due to the wide variety of existing and possibly forthcoming slot games with respect to their parametric configuration, which consists of configuration of the reels and configuration of the display.

Configuration of a reel

The configuration of a reel refers to the distribution of the symbols over the stops of that reel and the arrangement of the symbols:

Denoting by *t* the number of stops and by *p* the number of distinct symbols $S_1, S_2, \ldots, S_p$ on the reel, and denoting by c_i the number of symbol S_i on the reel ($1 \le i \le p \le t$), then the vector $\left(c_1, c_2, \ldots, c_p\right)$ is called the *distribution of the symbols* $S_1, S_2, \ldots, S_p$ on the reel, also known as the *weighting* of a reel. Each reel has its own distribution of symbols.

Given the number of stops *t*, the number of distinct symbols *p*, and a distribution $\left(c_1, c_2, \ldots, c_p\right)$ of the symbols on a reel, there are several ways of arranging those symbols on the stops of that reel. Any function *a* from the set of stops to the set of distinct symbols, such that $\left|\left\{x \middle| a(x) = S_i\right\}\right| = c_i$ for any *i* from 1 to *p* (that is, the number of stops having assigned symbol S_i by function *a* is c_i) is called *arrangement of the symbols* on the reel.

The distribution and arrangement of the symbols on each reel identify a slot game and are determined by the game producer.

The configuration parameters are essential for the probability computations. The numbers of stops of the reels and the symbol distributions on the reels stand as variables for any general formula for the probability of a winning event defined on a payline made of independent stops (stops belonging to independent reels – that is, a payline that crosses over the reels without overlapping them). The symbol arrangements on the reels count toward the probability computations of winning events defined on paylines holding stops of the same reel, assuming the arrangement is known.

Consequently, the parameters of the configuration of a slot machine should be present in any technical sheet describing that machine – for either internal or external use – and in its associated PAR sheet, along with the computed probabilities of the winning combinations and other statistical indicators, owing to ethical reasons for which I advocate in a further section.

Configuration of the display

The configuration of the display refers to the shape and structure of the set of windows showing the visible symbols of the reels, which produce the outcome of the game, and the shape, length, and position of the paylines. All these properties are described mathematically through geometrical and topological properties of those sets (a rigorous model for the configuration of the display is a rectangular grid in which the lines are defined as discrete paths linking neighboring points).

From the whole configuration of a display, the length of a payline is the only parameter that counts toward the probability computations for events related to that line, regardless of its shape or other properties; however, for more complex events defined on several paylines (for instance, of the type '*a specific winning combination of symbols on any payline from a given group of paylines'*), particular properties and parameters of that group, such as related to *intersection* and *independence* do also count.

The mathematical model of the configuration and also the probabilistic models based on it are idealized models not representing *all* types of slot machines on the market.

Variables for the general formulas of probability and expected value

Under the assumption that the reels spin independently (either physically or virtually, in the sense of probabilistic independence), we can obtain general formulas for the probability of the various winning events related to one or several paylines, having as variables the parameters of the configuration of the slot machine described in previous sections.

Most slot machines do not have the same number of stops on their reels, nor the same distribution of symbols on them (unfortunately, for the ease of computations!) Yet we can assume the same number of distinct symbols on each reel (denoted by *p*) through a convention: if a symbol does not appear on a reel, we could simply take its distribution on that reel as being zero. A blank is considered as a distinct symbol within the mathematical model.

We distinguish two possible types of slot machines with regard to the parametric equality of their reels:

Type A – All reels have the same number of stops and the same distribution of symbols; each symbol *S* has the same distribution (number of instances) c_S on the *t* stops of each reel; denote by c_i the distribution of symbol S_i on each reel ($1 \le i \le p$);

Type B – The reels have different numbers of stops t_j and each symbol has different distributions on the stops of the reels, denoted by c_i^j (c_i^j is the distribution of symbol S_i on reel number *j* ($1 \le i \le p$ and $1 \le j \le n$), where *n* is the number of reels).

Call the two situations case A and case B.

Given a specific symbol S_i, the probability of S_i occurring on a reel after a spin is $q_i = \frac{c_i}{t}$ in case A and $q_i^j = \frac{c_i^j}{t_j}$ in case B, where *j* is the number of that reel.

Numbers q_i, respectively q_i^j are the *basic probabilities* in slots.

For an event *E* related to an independent-stops line of length *n*, the general formula of the probability of *E* is:

$$P(E) = \frac{F(E)}{t^n} \text{ in case A and } P(E) = \frac{F(E)}{\prod_{j=1}^{n} t_j} \text{ in case B,} \quad (1)$$

where *F*(*E*) is the number of combinations of stops favorable for the event *E* to occur.

For winning events *E* defined in a cumulative manner (that is, through numbers (quantities) of specific symbols necessary for the payline to hold, regardless of their position on the payline), which is the case for most slot games, *F*(*E*) has a polynomial expression, being a function of *t* and c_i in case A or of t_j and c_i^j in case B ($1 \le i \le p$ and $1 \le j \le n$). For instance, if the event *E* is in particular *Exactly one instance of a specific symbol S*, formula (1) is written as:

$$P(E) = \frac{nc_s\left(t - c_s\right)^{n-1}}{t^n} \text{ in case A and } P(E) = \frac{\sum_{i=1}^{n} c_S^i \prod_{\substack{1 \le j \le n \\ j \ne i}} \left(t_j - c_S^j\right)}{\prod_{j=1}^{n} t_j} \text{ in case B} \quad (2)$$

Still in particular, for an event *E* expressed through the number of instances of *each* symbol on a payline in case A, formula (1) becomes the classical formula of probability in a polynomial field:

$$P(E) = \frac{n!}{a_1!a_2!\dots a_p!}\left(q_1\right)^{a_1}\left(q_2\right)^{a_2}\dots\left(q_p\right)^{a_p} \quad (3)$$

where a_1 is the number of instances of S_1, and so on, a_p is the number of instances of S_p ($a_1 + a_2 + \cdots + a_p = n$). Parameters a_1 to a_p characterize the winning combination, while q_1 to q_p characterize the configuration of the reels.

Formula (3) is used when the winning event is defined through an exact distribution of *all* symbols on the payline (even if some of them do not appear in the winning combination, having the distribution zero).

Particularizing in case B, consider the event *E* as *Exactly m instances of a specific symbol S* ($m \le n$). Formula (1) becomes in this particular case:

$$P(E) = \sum_{1 \le i_1 < i_2 < \cdots < i_m \le n} \prod_{j \in \{i_1, i_2, \dots, i_m\}} q_S^j \prod_{\substack{1 \le k \le n \\ k \ne i_1, i_2, \dots, i_m}} \left(1 - q_S^k\right) \quad (4)$$

where q_S^i is the basic probability of occurrence of symbol *S* on reel no. *i*.

The general formula (1) holds for simple events related to *one* payline. For more complex events like unions of winning events on one or several paylines, formula (1) is used along with other properties of probability and also methods of approximations for obtaining applicable (although overloaded) formulas for the probability of those events (Bărboianu, 2013a).

From the expression of the probability formulas presented, one can see that these are functions of *n* (the length of the payline), *t*, and c_i in case A, or of t_j and c_i^j in case B (*t* or t_j are the numbers of stops of the reels; c_i or c_i^j are the distributions of the symbols on the reels) and other variables describing the event to be measured. For events related to several paylines, other variables describing the event also appear in the formulas (the number of lines of the group, cardinalities of the intersections of those lines, etc.). The formulas can also be written in terms of basic probabilities q_i or q_i^j, which can replace the *c* and *t* variables as ratios between them.

The same variables will also appear in the general formulas of the expected value of the slots bets, along with the payout rates from the payout schedule of the game.

I present this overview of the base of the mathematical model necessary for probability calculus in slots and also few general results to emphasize the necessity of having the data describing the configuration of a slot machine as inputs for the probability computations. By applying mathematics within a general model, we can obtain only general formulas for probability and expected value. However, these formulas or even associated tables of values are useless for the player without final numerical computations for a given game, and these can be performed only if we are provided in advance with the numerical parameters of the configuration of that game.

The secrecy of slots PAR sheets – facts, justifications, and implications

The secrecy of game producers on PAR sheets is a certitude, verified by the lack of these files. One can see unsuccessful PAR sheet requests from slot players by browsing their forums, while game researchers can obtain them only through legal intervention.

Since 2007, game researchers obtained the PAR sheets of some slot games through FIPPA (Freedom of Information and Protection of Privacy Act) in Canada (Harrigan & Dixon, 2009). Most of these PAR sheets became public after researchers studied them; others were also available before 2007 through other channels (see Wilson 2004a, 2004b, 2004c, 2004d, 2004e). Still, these are the PAR sheets of a miniscule part of all slot games on the market. *Slot Tech* magazine has published articles dedicated to PAR sheets; however, they are limited in the number of games covered and in that the description is adapted to the audience of this magazine, who are technicians serving the machines (Harrigan & Dixon, 2009).

Browsing the appeal decisions of IPC (Information and Privacy Commissioner) in Ontario, Canada, with respect to PAR sheets requests, one can see that game producers who declined the requests invoked the exemption set forth for scientific and technical information, through one or more of the facts that PAR sheets contain information routinely considered to be trade secrets in the gaming industry and consist of mathematical formulas and equations developed by their engineers. They further claim that information provided on PAR sheets significantly prejudices their competitive position and interferes significantly with the contractual obligations of the company (Information and Privacy Commissioner [IPC], 2009, 2010).

Major slot companies have in circulation for players brochures presenting their games, in which mathematical aspects of the games are barely touched upon, thus being more promotional than informational materials. For instance, International Game Technology (IGT)'s brochure, called *Introduction to Slots and Video Gaming*, has a section titled *Slot Math*, which presents three examples of games, each one from a large category (3-reel, 4-reel, and 5-reel slot machines), for which the following configuration parameters and mathematical facts are shown: number of stops of each reel, the distribution of the top-award symbol (that symbol triggering the jackpot in a line-up combination) on each reel, hit frequency of the top-award combination, and the overall hit frequency of any winning combination. Summarizing, the math section provides the player with the numbers of stops of the reels, distributions of *one* symbol on the reels and *two* probabilities (one for the top-award combination and the other for any winning combination), for three specific games; there are no distributions of the other symbols, no probabilities for other winning combinations, and no expected value, which is a relevant indicator of the practical risk over the long run. Also, the use of the term *symbol combination* in probability context is confusing, as with this term the winning combinations shown on the payout panel are defined; the combinations involved in the probability computations are *combinations of stops* (holding those symbols), and not combinations of symbols. To "explain" the brevity of their math section, the editor wrote in its introduction (International Game Technology, 2009):

> [...] One such tool, par sheets, can be complicated to understand. However, investing the time learning to read them is time well spent. They offer important information for optimizing the revenue for each machine, as well as offering data for technicians. In this section, we provide examples of simple slot math that is found on par sheets for three types of games – spinning reel and video reel slots, video poker, and bonus games. These equations represent the most basic operations only. For more detailed information, please ask a gaming representative or attend training classes. Par sheets for all IGT games [...] are available online at [...]

Going to the webpage indicated as providing PAR sheets "for all IGT games," I found a link labeled *PAR sheets* hidden on a third-level page, for which the browser returned a 310 error when accessed. The problem of accessing that section might have been temporary; however, there is a disclaimer on that page that certain areas of the site are secured and require an active member account. I made no further investigation of whether opening such an account leads unconditionally to a positive answer regarding a PAR sheet request.

Given all the facts presented above, it is clear that slot producers are strongly reticent in exposing the parametric configuration of their games.

The psychological argument related to competition

Coming now to the justification of this reticence, the slot producers' reasons for declining PAR sheet requests, shown in the IPC's appeal decisions, seem to be judicially formal rather than factual. I present the following arguments for this claim:

- The trade secret and intellectual ownership reasons fail against the generality of the math formulas and equations. Although the parametric details vary from game to game, the mathematical results concerning probability, expected value, and other statistical indicators are just *applications* of general formulas that are publicly available in mathematics, thus common across all slot machines, and no individual or corporate body can claim ownership of such a pattern or formula. The argument also holds if talking only about the protection of the parametric configuration. Protecting a certain finite sequence (combination or arrangement) of symbols reverts to protecting the sequence of numbers that can be put in bijection with the former, since others may use it by just replacing the symbols with new ones through a new bijection on that sequence. But protecting a sequence of numbers falls within the same argument that mathematics is freely available.
- There are three possible reasons for competitive prejudice against competition, two of them coming from the situation in which another producer copies and uses the revealed parametric configuration: a) the possibility of losing a share of the market to the infringer; b) the unethical development of the infringing company; c) the exposure to bad publicity from a competitor or neutral entities. The arguments as to why these reasons fail are the following:

  a) The infringing company would develop a slot machine different in its external (physical) design from the machine having the original parametric configuration and having this configuration in common with the original (because the entire machine of the original producer is patented, even though its parametric configuration in and of itself may not be). With the parametric configuration invisible, if the new machine is successful at and after launch, there may be possibly three types of elements responsible for this success, alone or in combination: its physical design, the marketing, and its statistical indicators.

The first two types of elements have nothing to do with parametric configuration, so they do not apply to the original producer's competitive-prejudice argument. If statistical indicators like frequency of wins and payback percentage prove in time to be responsible for the new machine's success – which is very likely to happen – this is not a consequence of the use of that specific parametric configuration alone, but together with a given payout schedule. First, a producer can manipulate the game parameters, including the payout schedule, in unlimited ways so as to obtain the desired statistical indicators for the house; that is, if the goal is to have the payback percentage of another machine, a better alternative is to use a new parametric configuration yielding the same percentage instead of replicating one; besides, within the same goal, if copying the parametric configuration of the reels, the infringing company would need to copy the payout schedule also; while the parametric configuration itself may not fall within patent restrictions, parametric configuration *plus* payout schedule is likely to do so, therefore the infringing company should expect the original producer to recognize his own payout schedule on the new machine and take a legal action the potential (at that time) infringers.

b) The infringing company would get a ready-to-use parametric configuration, possibly non-patented, which he can use in two ways: keeping also the original statistical indicators, including payback percentage, or adding a new payout schedule and getting different statistical indicators. The former alternative assumes copying the entire game parameters (and avoiding the expense of the mathematical work, which compounds the unethical behavior), which may be protected as a whole. If this happens, the infringing company exposes himself to lawsuit and bad publicity, since the parametric configuration used can be discovered through statistical observation (see the next section) by the original producer. The latter alternative gives no rational motive for the infringing company to use a copied parametric configuration instead of a new one, since he already must do the math work for finding a payout schedule which will yield the desired statistical indicators. In addition, using only the copied parametric configuration would again expose him to bad publicity when revealed through statistical observation.

c) In the situation of bad publicity as result of revealing a PAR sheet (which would have as issue the fact that probabilities and/or statistical indicators are not favorable for the players), the producer would have the option of defending himself with an acceptable answer: the fact that all existent slot games have similar figures attached, not much different from those exposed, while he at least showed them to the public ("I am not the bad guy here, but those who keep the PAR sheets secret"). Such an answer may turn the negative publicity into a good marketing strategy.

The hypothetical situations related to reasons a) and b) would actually apply to a start-up company in the role of the infringing company, as it is very unlikely that an established producer would risk his position just to avoid the expense of the math work.

Thus, the slot producers' justification for the secrecy of PAR sheets related to competition is insubstantial, as I argue above, even though some of them claim the opposite in their judicial litigations. The previous arguments (related to competition) are part of what I called at large *the psychological argument*. The possible justification exclusively related to their players remains to be considered. That would mean that they are afraid of losing players who face the PAR sheets of their games, and so the popularity of slots would decrease. I argue in the next section that such a claim is likewise insubstantial and propose further research to confirm what it is hypothesized.

The psychological argument related to players

The facts are:

1. PAR sheets are kept secret by slot producers, with isolated and rare exceptions.

2. Slot players continue to play slots in the absence of information regarding parametric configuration, probabilities, and statistical indicators of the games, maintaining the popularity of these games.

When we talk about hiding a data sheet, obviously it is about the *content* that is hidden, and for a PAR sheet this means parametric configuration (numbers of stops and symbol weighting of the reels), probabilities for the prize-award combinations, other probabilities, frequencies, and other statistical indicators.

With respect to the content of a PAR sheet: by putting myself in both the producer's and the player's position, I see two possible reasons for the secrecy,– one related to competition (hiding the parametric configuration and the statistical indicators), and the other related to players (hiding probabilities and statistical indicators). Further empirical study on slot players can confirm the existence of these two reasons and perhaps find others, which may be treated thereafter.

The former reason cannot change players' behavior with regard to the willingness to play slots, as that is not related to any *new* unethical or fraudulent strategy of the producer against them – everything is the same as it was.

In treating the latter reason, I will focus on the PAR sheet's content that is the most accessible (as knowledge) to and has the greatest impact on gamblers. Among all the mathematical data related to a game, the *probabilities* of the various winning events are the most important for a player with respect to the objective evaluation of winning/losing possibilities and general gaming knowledge, even though their influence might not lead to gaming decisions and changes in gaming behavior. This top status of probabilities is due to the fact that all players, regardless of their level of mathematical knowledge, have a basic understanding (although many times distorted or misconceived) of the notion of probability according to its common classical definition (as the ratio between the situations favorable for an event to occur and the number of all equally possible situations) and a basic interpretation of it as a degree of belief in the occurrence of an event. Moving then to *expected value*, this notion already requires a new level of mathematical education (as a mean of a discrete random variable), not available to the majority of slot players. Thus, it can be hypothesized that players perceive that the main reason for secrecy of PAR sheets (related to themselves) is to hide the odds/probabilities of winning. If further empirical research confirms this hypothesis, we shall arrive at the conclusion that slot players *expect* low to very low odds of winning without acquiring this information[1], since there is no other reason for keeping them secret if they were high compared to other types of games on the market.

Now, fact 2 along with the last conclusion leads to the prediction that slot gamblers will not stop playing in the event of exposure of the PAR sheets of the games they play, since they already expect low to very low odds of winning. There are other visible elements of the games that keep them attractive and popular (Griffiths, M., 1999; Wood & al., 2004) despite their special status among games of chance (in respect to the exposure of their parametric configuration, which I talked about in the first section of this article). Obviously, there is also an addictive component of the other elements of attractiveness, and slots addiction should also be studied as a particular type of addiction, given the missing-parametric-configuration feature of the slot games.

Further empirical study of slot players[2] can confirm this theoretical prediction of not quitting slots under the condition of low to very low odds of winning exposed through PAR sheets. Until then, we have an example, provided by lottery players, whose behavior seems to confirm my prediction in the slots case.

---

[1] I have excluded the statistical indicators from this analysis and focused on probabilities, also given the possibility of their being misinterpreted by non-math gamblers. A proper interpretation of the mathematical data of a game with respect to favoring the player takes into account both probabilities and statistical indicators based on expected value, not one category or the other individually. For instance, a payback percentage of over 90% can be seen as high and therefore favorable by (and for) a player. However, there is mainly the probability of the biggest win (along with the payout of that win) that yields this high payback percentage and the calculation is a mean over the long run (infinity in mathematical terms). Since probability translates to frequency in the gaming experience, it won't be the same for the player if that winning event (balancing the computed payback percentage) occurs on average in a lifetime or less. Besides, payback percentage is an indicator usually exposed outside the PAR sheet.

[2] possibly conducted together with those proposed earlier as one stand-alone research

The lottery example. From all games of chance, lottery offers by far the lowest odds of winning for the top prizes, on the order of one to millions or tens of millions for the first prize, and one to tens or hundreds of thousands for the second prize, for the common lottery designs.

With a history traced back to B.C. antiquity for its birth and to 15^{th}-16^{th} century for expansion, lottery remained the most stable and respected game of chance; contemporaneous studies of this game recognize its popularity and the fact that no decrease of this popularity has been reported (National Impact Gambling Study Commission [NIGSC], 2004). Most lottery players play regularly and are aware of the very low odds of winning. Even though most of them might not know the exact figures, all have a clue about the size order of these odds, knowing that they are very close to zero, because this information has spread widely enough in common communication between lottery players and through the media as well to become a proven well-known characteristic of lottery. Even knowing that the winning odds are very low, lottery players still continue to buy tickets on a regular basis and the lottery has never lacked for business. Why, then, should slot game producers worry about slots doing otherwise, since the odds of winning at slots are generally higher than those of lottery?

Some will be quick to point out the following elements in which the two types of games differ at least with respect to financial expectations and player's options to estimate and manage these expectations:

- Prize amount – Lottery offers first- to third- category prizes in amounts higher than the similar prizes in slots games, and the high prizes somehow compensate for the low probabilities of winning with respect to the decision of quitting the game.
- Enhancing the probability of winning – The lottery games allow an increase in the overall winning probability[3] through buying several tickets or playing

[3] "Overall" has the sense of a disjunction of winning events that is measured in probability, that is "winning with line no.1" or "winning with line no. 2" and so on in our lottery example. The increase in probability refers to moving from a single event to a disjunction

systems with several lines in unlimited numbers for the same draw, while in slots the player may enable paylines only in limited number for such an increase.

- Game frequency – Slot players can spin the reels of a slot machine thousands times in a day, while in lottery players must wait several days for a new game.

In response, I would argue as follows to ignore these non-equivalences:

- Prize amount – The objection argument assumes that the high prize amount is the dominant factor in the lottery player's behavior of playing against the minute odds of winning. My argument: If this factor is merely addictive, in parallel the slot games also have their specific addictive and entertaining components, and the existent balance between lottery and slots is inclined toward the latter due to its generally higher odds of winning. If the high-prizes factor has a practical side in lottery player's mind, like "someday these [high prizes] will make me rich" given the assumed basic knowledge of the probabilities involved, the player can estimate and face the overall probability of this fortunate event happening in a lifetime as very low and the invested money as serious, which eliminates the practical expectation, contradicts the assumption, and thus reduces the factor to its addictive component, which has been already addressed. (Such probability estimation is at hand for everyone, by taking an average playing lifetime, assuming a weekly play, and multiplying the winning probability by the total number of plays.)
- Enhancing the probability of winning – My argument: This increase is actually limited by the player's available funds for the tickets' total price, and therefore, increasing the probability of winning cannot change the size order of this probability from *very low* or *low* (for an one-line ticket) to *medium* (for a multi-line ticket or several tickets). For example, assume a player buys 500 one-line tickets at $2 each (a serious investment for one draw), or an

---

of events that includes that single event. This convention applies in every instance where these terms were used in the article.

equivalent line-system ticket, at a 6/49 lottery. The overall winning probability will increase a maximum of 500 times, leading to a maximum of 0.00003575 (from 0.0000000715) for the first-category prize and 0.0092 (from 0.0000184) for the second-category prize, which are very low and low respectively (these maximums apply if the played lines are independent, in the sense of a restriction on the number of the common numbers of those lines) (Bărboianu, 2009). There is another limitation in playing large numbers of lines once, coming from the distribution of the prize fund – the prizes are not given by a fixed payout schedule, but the lottery company assigns a certain prize amount for each prize category as a fixed percentage from the ticket sales of that draw. Therefore, investing large amounts of money for increasing the chances for a draw might result at some point in winning less than invested. On the other hand, in slots the payout schedule of a machine is fixed for all games (spins), and the increase in the overall winning probability can be acquired not necessarily by enabling more paylines, but by running more spins – by the technical features of the slot game, a player can run hundreds of independent spins in a reasonable time, and so we get the equivalent effect as in the multi-line play in the lottery case with respect to probability increasing.

- Game frequency – My argument: This conclusion was drawn based on the criterion of probability size order for *one* game, under the assumption that the basic probability knowledge is accessible to most gamblers. Game frequency is not a criterion for and does not change the argument of the conclusion, as it does not change the probabilities attached to a game; game frequency can be a criterion for other comparative analyses of the two types of games – for instance, regarding how deviation from the expected value is directly perceived and accounted by the math-inclined player.

In my analysis, when talking about probabilities of *winning*, I considered only the slots prizes as given by the winning combinations of a single machine and ignored the progressive-jackpot prizes, because including them would just incline the balance more toward slots in the lottery-slots comparison, with respect to the analyzed prediction of not quitting slots under the condition of PAR sheets exposure.

With the above counter-arguments I conclude the lottery example and declare it as *relevant* within the psychological argument for arguing for the prediction of not quitting slots under the condition of PAR sheets exposure. Of course, there are other characteristic features (except those related to financial-expectation) that distinguish the two types of games from eachother and might be responsible for a different behaviour of the slot players than the lottery players. Further research is needed to confirm or refute that idea.

I will also present another argument, this time merely mathematical, for the insubstantiality of the secrecy of slots PAR sheets, in the next section.

Statistical methods for estimating the parameters of the configuration of a slot machine

The methods I briefly describe here can be applied in an organized professional environment in order to estimate and expose the parametric configuration of any slot game whose PAR sheet is missing.

In the next sections I use the same denotations used in section *The parametric configuration of slot machines as a base for the probabilistic models for the slot games*.

The raw approximation. This method is based on the well-known result from probability theory called the Bernoulli's Theorem, which states that in a sequence of independent experiments performed under identical conditions, the sequence of the relative frequencies of the occurrence of an event is convergent toward the probability of that event.

Applied to slots, that principle says that if N is the number of spins of a reel with t stops where we observe as an outcome a specific symbol S that is placed on c stops and $n(N)$ is the number of occurrences of S after the N spins, then the sequence $\left(n / N\right)_N$ is convergent toward the probability of occurrence of S, namely $P(S) = c/t$.

The ratio n / N is the relative frequency of occurrence of S. It follows that for large values of N, the relative frequency of occurrence of S approximates the probability of S occurring. The higher N, the more accurate this approximation.

Obviously, the number of spins *N* must be large enough for obtaining good approximations of the ratios c_i / t, and this is the main issue of this method. As theory does not provide us with tools for choosing *N* for a given error range, all we have is the principle "the larger *N*, the better."

As one can notice, this method of approximation based on statistical observation is subject to errors coming from idealizations and various assumptions, and the error ranges are not even quantifiable. Given these issues, the best way to use this method is not for individual records, but cumulating progressively the records coming from several sources and refining the estimations in correlation with the increase in total number of spins *N*. This principle is also common for the odds calculators based on partial simulations, used for various games.

Note that the described method provides us with approximations of the ratios c_i / t (the basic probabilities) for each reel and not the parameters of the configuration individually (c_i and *t*). However, knowing the basic probabilities is enough for any probability computation for a slot game, as seen in section *The parametric configuration of slot machines as a base for the probabilistic models for the slot games*.

A more accurate approximation of the ratios c_i / t and even of c_i and *t* individually is still possible through statistical observation, using a method which can refine the raw estimations obtained through the previously described method. Such a method is briefly described in the next section.

Denominator-match method. Denote by $n_1(N), n_2(N), \ldots, n_p(N)$ the number of occurrences of symbols S_1 to S_p respectively after *N* spins of a reel. There is a slight correlation between the recorded values $n_1, n_2, \ldots, n_p$ for various large numbers of spins *N*. Based upon this correlation, we can refine the estimation of the ratios c_i / t obtained through the previous method and also find estimations for $\left(c_1, c_2, \ldots, c_p\right)$ and *t*, by recognizing a numerical pattern across some sequences of fractions representing the ratios between possible values for c_i and *t*.

The denominator-match method is based on the numerical analysis of the fractions n_i / N and on a five-step algorithm briefly explained below:

We write each fraction n_i / N as a chain of equal fractions, having numerators from 1 upward and denominators not necessarily integers, for every *i* from 1 to *p*. Across the *p* chains of equal fractions obtained, we choose that of the minimal length (let *m* be the minimal length). Then, across the *p* chains of equal fractions, we extract *m* sequences of fractions (one fraction from each equality chain), having the denominators the nearest to the denominators from the minimal equality chain respectively. From the *m* sequences of fractions obtained, we choose one sequence of *p* fractions by applying progressively the following filtering criteria: having denominators as close to each other as possible, having the highest number of instances of the same denominator, and the repeating denominator with the largest share being an integer. As final step, we adjust the numerators of the final sequence of fractions, as follows: If the sum of the numerators lies between the minimum and maximum of the denominators, then we take the numerators as the symbol distribution on the reel (c_i) and their sum as the number of stops of the reel (*t*); if their sum does not lie in that interval, then through addition or subtraction, we distribute, proportionally with their values, the difference between their sum and the integer nearest to the mean of the minimal and maximal denominator, rounding the added/subtracted quantities to integers. For our resulting estimation, we take the adjusted numerators as the symbol distribution on the reel (c_i), and the integer nearest to the mean of the minimal and maximal denominators as the number of stops of the reel (*t*).

This method provides us with the most probable number of stops *t* and associated symbol distribution $(c_1, c_2, \ldots, c_p)$ of a reel in a certain probability field; the error range of this approximation is quantifiable in terms of probability (Bărboianu, 2013b).

Regarding the practical application of the methods through statistical observation, it is obviously an arduous task, since we have to watch and record spins in numbers of thousands. For online games, software can be developed to help in such an endeavour. For physical machines, it is far more difficult to watch and note down thousands of outcomes just for one reel of a machine, not to mention that the slots operator might not allow this action. Of course, technology based on video capturing might help with such a task, but that is not the concern of the current study.

Physical measurements.

Any information acquired on t besides the presented statistical methods of estimation is useful with respect to the accuracy of the approximations because it can give a clue as to how high we should choose N for avoiding irrelevant results (for example, if $t = 100$, we intuit that choosing $N = 1{,}000$ or lower would not be high enough for relevant results).

Besides the methods based on statistical observation, there exists a method of estimating t through physical measurements, applicable to some particular types of slot machines. This method exploits the information given by the appearance of the reel on the display. As we know, only a small part of the reel (either physical or virtual) is visible on the display and this part can be seen as one or several adjacent symbols (usually 3, up to 5). So we can view from 1 up to 5 consecutive symbols of the reel. If the appearance of this part of the reel is three-dimensional (which is possible for both physical and virtual reels), by measuring some parameters of this image, we can deduce an estimation for the number of stops of that reel (t). Basically, the apparent lengths of the visible symbols give full information on the curvature of the reel, which then leads to an estimation of the entire number of stops, since the number of visible symbols per the circular length of the visible reel is proportional to the total number of stops per the circular length of the entire reel (Bărboianu, 2013c). This method can be applied only to reels showing at least two consecutive symbols on the display in three-dimensional view. The method cannot be applied to virtual reels showing several consecutive symbols in flat image. As in the case of the previous method through statistical observation, there are issues with the practical application of the method through physical measurements. There might be technical issues regarding acquiring the proper position for measurement or placing the measurement tool on the surface of the machine. Also for this method, an alternative would be for the observer to take photos and make the measurements on the photos. Of course, the slot machine operator might not allow the direct measurement and/or taking photos.

With this incursion into the mathematical methods of approximating the parameters of the configuration of a slot machine through statistical observation, I conclude the analysis of the possible reasons for slot producers keeping secret their PAR sheets. Summarizing below the arguments against the justifications for these possible reasons, I draw the conclusion that the secrecy of slot producers on the parametric configuration of their slot machines is not rationally justified:

- Protection against competition fails against the generality of the math formulas and equations and the open possibility for all slot producers to configure any parametric design for their slot machines, also manipulating the game parameters and the payout schedule in unlimited ways, so as to obtain the desired statistical indicators for the house.
- The fear of losing players who face the real odds of winning attached to their games, thereby affecting the popularity of slot games, fails against the *a priori* expectation of the players for low and very low odds of winning induced by the experienced secrecy of PAR sheets and against the lottery example, in which lottery players continue to play against the lowest odds of winning due to other addictive elements that slots also hold.
- The secrecy itself fails against the statistical methods that mathematics provides us with for retrieving the missing data through statistical observation, even tough as approximated results.

The exposure of the parametric configuration and the mathematical facts of a game as an ethical obligation

The requirement for the exposure of the parametric configuration applies only to slots, since it is the only existent game of chance for which such data is hidden. Indeed, while a slot machine displays only a part of each reel in stop position, for the other games all the configuration from which the outcome is produced is visible for the players – the roulette numbers are shown on the wheel and table, the deck composition is known for every card game, dice faces and number of dice are visible for every dice game, lottery numbers are known for each lottery design, etc.

The information to be exposed would be in the form of a technical/mathematical sheet specific to each slot game, either consisting only of the parametric configuration, or of the parametric configuration plus basic mathematical results such as the probabilities of the winning combinations shown on the payout schedule, probability of any win, and expected value. For the former variant, which is merely informative and provided either by the slot producer or retrieved through the methods I described in the previous section, it would remain for the player to inquire further for the mathematical results as an optional action. For the latter variant, the probability/statistics part that comes along with the parametric configuration would be completed by an assigned mathematical authority.

The goal of exposing parametric configurations for slot games is not so much to place slots in the same status as other games of chance as it is a matter of ethics.

The exposure of the parametric configuration of a game to the player prior to playing is an ethical obligation in two ways – one commercial and the other humanitarian.

The commercial way treats the game as any commercial service, for which full technical specifications are required from the producer to the customer – as a public coffee machine must show the coffee brand name and the coffee volume returned for the unit price, so the slot machine must show the reels' numbers of stops and symbol weighting; a bet is still a purchased service once the players inserts a non-returnable coin in the machine. A relevant example of unethical procedures would be having identical-looking machines yielding different payback percentages due to different (missing) parametric configurations or even the same machine changing its payback percentage with the replacement of a single chip.

The humanitarian way is related first to the free will of thought and second, to the limitation of addiction risk. Being informed on all parameters of a game one plays is a condition for unconstrained (constrained through omission) personal thinking leading to personal actions. It is as if someone asks you to bet you can jump from a high place and land on your feet; of course, if you know in advance the height from which you will jump or measure it before you bet, you might decline the bet or propose another one for a certain measurement, and this means *free decision*. Such comparison can also be an argument for the sceptical slot player who could ask: "Given that slots is not a strategic game played against opponents (as with poker for instance, where the odds are essential in evaluating the advantage in a given situation), why do we need 'all the math stuff' associated with slots?" The answer is simple: information and strategy. The argument for information is expressed through the above comparison. Regarding strategy, in slots there is a trivial strategy, namely the strategy of choosing: choosing one game or another, choosing how many paylines to enable, choosing the parameters of time and money management, and of course, choosing to quit one game for another or choosing not to play at all. The only objective criteria for such a strategy are probabilities and expected value. Similar comparisons that also hold in some ethical aspects are the illness-danger warnings on cigarette packs and the "possible adverse effects" statistics in drug leaflets.

Regarding the limitation of addiction risk, past and ongoing studies debating the issue of whether mathematical knowledge (as either provided pre-calculated results like winning odds and other statistical indicators or theoretical and applied probability theory basics learning) causes a decrease in gambling behavior have not yet reached a clear conclusion. Several empirical studies found no significant changes in college students' gambling activity after they received a scholastic intervention on gambling mathematics (Hertwig et al., 2004; Steenbergh et al., 2004; Williams & Connolly, 2006). On the other hand, more theoretical studies proved that post-secondary statistics education developed critical thinking, which also applies to gambling, and the gamblers who get such education tend to have significantly lower rates of problem gambling (Abbot & Volberg, 2000; Gerstein et al., 1999; Gray &

Mill, 1991). I am personally inclined to think that such a decrease follows an optimal mathematical learning, which can be devised and developed according to its scope, and this will be the focus of forthcoming research.

Given the ethical obligation to expose the parametric configuration of the slot games, the question arises as to how this information can be technically exposed. Due to its relation with the risk factors, the exposure on a website would not be enough, because in a physical casino or slot room, there are specific physical addictive elements that might distract a player's mind from the mathematical facts seen earlier on the internet, not to mention that the player might encounter a slot machine for which he/she had not studied its technical/mathematical sheet beforehand. It follows that the technical/mathematical sheet must accompany each slot machine in a printed form or at least be available upon request from the slot operator. Instead, online sheets are applicable to the online slot games. Since slot operators, like slot producers, might consider that it is not to their advantage to provide the technical/mathematical sheets to their customers, such an action is imposable only by law, which can also certify as official the authority providing the mathematical facts of the games.

The debate remains open as to whether the technical/mathematical sheets must contain only the parametric configuration (as sufficient information for someone to compute further mathematical results optionally) or in addition, basic mathematical results concerning probabilities and other statistical indicators, with respect to the ethical requirement. The former alternative raises the question of the usefulness of mathematical didactical intervention undergone to gamblers, and both alternatives raise the question of *understanding* and *interpretation* of the exposed or learned mathematical concepts and facts related to games of chance, since the simple acquisition of numerical probabilities and statistical indicators as mere quantities might not be enough toward the decisions made based upon them. These issues are treated in a forthcoming article, as conditions for an optimal mathematical didactical intervention in gambling.

Acknowledgements

I want to thank to the reviewer who suggested relevant examples for the hypothetical situations presented in the paper: the start-up company example within the competitive-prejudice counterargument and the example of two identical-looking slot machines having different parametric configurations within the ethical argument.

References

Abbott, M.W. & Volberg, R.A. (2000). Taking the Pulse on Gambling and Problem Gambling in New Zealand: A Report on Phase One of the 1999 National Prevalence Survey. Department of Internal Affairs, Government of New Zealand.

Bărboianu, C. (2009). Enhancing the Winning Probability. In Infarom (Ed.), *The Mathematics of Lottery: Odds, Combinations, Systems* (pp.75-91). Craiova: Infarom

Bărboianu, C. (2013a). The probabilities of the winning events related to several lines. In Infarom (Ed.), *The Mathematics of Slots: Configurations, Combinations, Probabilities* (pp.107-125). Craiova: Infarom

Bărboianu, C. (2013b). How to estimate the number of stops and the symbol distribution on a reel: Denominator-match method. In Infarom (Ed.), *The Mathematics of Slots: Configurations, Combinations, Probabilities* (pp.50-58). Craiova: Infarom

Bărboianu, C. (2013c). How to estimate the number of stops and the symbol distribution on a reel: Physical measurements. In Infarom (Ed.), *The Mathematics of Slots: Configurations, Combinations, Probabilities* (pp.52-63). Craiova: Infarom

Blaszczynski, A., & Nower, L. (2002). A pathways model of problem and pathological gambling. *Addiction*, *97*(5), pp. 487-499.

Gerstein, D., Volberg, R.A., Murphy, S., Toce, M., et al. (1999). *Gambling impact and behavior study*. Report to the National Gambling Impact Study Commission. Chicago: National Opinion Research Center at the University of Chicago.

Gray, T. & Mill, D. (1991). Critical abilities, graduate education, and belief in unsubstantiated phenomena. *Canadian Journal of Behavioral Science,* 22, pp. 162-172.

Griffiths, M. (1999). Gambling Technologies: Prospects for Problem Gambling. *Journal of Gambling Studies*, *15*(3), pp. 265-283, doi: 10.1023/A:1023053630588

Harrigan, K.A., & Dixon, M. (2009). PAR Sheets, probabilities, and slot machine play: Implications for problem and non-problem gambling. *Journal of Gambling Issues*, 23, pp. 81-110. doi: 10.4309/jgi.2009.23.5

Hertwig, R., Barron, G., Weber, E.U., Erev, I. (2004). Decisions from experience and the effect of rare events in risky choice. *Psychological Science, 15*(8), 534-539.

Information and Privacy Commissioner, Ontario, Canada (2009). Decisions and resolutions: PO-2744. Retrieved from http://www.ipc.on.ca/images/Findings/PO-2774.pdf

Information and Privacy Commissioner, Ontario, Canada (2010). Decisions and resolutions: PO-2744. Retrieved from http://www.ipc.on.ca/images/Findings/PO-2903.pdf

International Game Technology (2009). *Introduction to Slots and Video Gaming*. Retrieved from http://media.igt.com/Marketing/PromotionalLiterature/IntroductionToGaming.pdf

National Impact Gambling Study Commission (2004). *Lotteries*. Retrieved from http://govinfo.library.unt.edu/ngisc/research/lotteries.html

Parke, J. & Griffiths, M. (2002). Slot Machine Gamblers — Why Are They So Hard to Study? *Journal of Gambling Issues*, 6, pp. 1-11. doi: 10.4309/jgi.2002.6.7

Steenbergh, T.A., Whelan, J.P, Meyers, A.W., May, R.K., & Floyd, K. (2004). Impact of warning and brief intervention messages on knowledge of gambling risk, irrational beliefs and behaviour. *International Gambling Studies, 4* (1), 3-16.

Williams, R.J., Connolly, D. (2006). Does learning about the mathematics of gambling change gambling behavior? *Psychology of Addictive Behaviors, 20* (1), 62-68.

Wilson, J. (2004a). PAR excellence: Improve your edge. Slot Tech Magazine, February 2004, pp.16–23.

Wilson, J. (2004b). PAR excellance: Part 2. Slot Tech Magazine, march 2004, pp.16–21.

Wilson, J. (2004c). PAR excellance: Part 3. Slot Tech Magazine, April 2004, pp. 20–26.

Wilson, J. (2004d). PAR excellence - Improving you game - Part IV. Slot Tech Magazine, May 2004, pp. 21–24.

Wilson, J. (2004e). PAR excellance: Part V: The end is here! Slot Tech Magazine, June 2004, pp. 24–29.

Wood, R.T.A., Griffiths, M.D., Chappell, D., & Davies, M.N. O. (2004). The Structural Characteristics of Video Games: A Psycho-Structural Analysis. *CyberPsychology & Behavior*, *7*(1), pp 1-10. doi:10.1089/109493104322820057

http://probability.infarom.ro/slots.html