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## Empfohlene Zitierung / Suggested Citation:

Allanson, P. (2022). Ordinal health disparities between population subgroups: measurement and multivariate analysis with an application to the North-South divide in England. The Journal of Economic Inequality, 20(4), 841-860. https:// doi.org/10.1007/s10888-021-09511-9

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# Ordinal health disparities between population subgroups: measurement and multivariate analysis with an application to the North-South divide in England 

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Received: 7 February 2020 / Accepted: 30 August 2021 /Published online: 23 February 2022
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#### Abstract

Health disparities between population subgroups classified on the basis of nominal characteristics such as sex, caste, race or region are of major academic and policy concern. The paper develops a novel analytical framework to not only measure differences in ordinal health outcomes between population subgroups but also account for such disparities in terms of the individual-level socioeconomic and demographic characteristics of their members. The measurement approach is directly applicable to the ordinal health and well-being data commonly available from general social surveys, building on the concept of statistical preference to motivate the definition of summary indices of comparative subgroup health and between-group variation in health. The analysis employs indirect standardisation techniques based on the estimation of a health distribution regression model for the population to identify the effects of compositional and conditional health differences on subgroup health outcomes. An illustrative empirical study finds that about half of the regional variation in self-reported health within England in 2016/17 can be accounted for by sociodemographic factors, with age and educational qualifications both more important predictors than income.


Keywords Ordinal health • Subgroup disparities • Multivariate decomposition • English regions
JEL classifications D63•I14•I18

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## 1 Introduction

Allison and Foster (2004, p.505) identify "evaluation of the inequality in the distribution of health status across individuals in a population" as the main focus of health inequalities research. However health disparities between groups classified on the basis of some nominal characteristic - such as sex, caste, race or region - are also of immediate academic and policy concern from a public or population health perspective (McCartney et al. 2019). In particular, systematic differences in health outcomes between groups may be deemed socially inequitable to the extent that they are avoidable by reasonable action to tackle the circumstances that limit the chances of people in disadvantaged groups to live longer, healthier lives.

The evaluation of health disparities between population subgroups is a routine exercise for quantitative outcome measures, such as longevity and disease incidence, given that many popular measures of inequality are decomposable into between-group and within-group components, where the former is usually defined in terms of inequality among subgroup means and the latter as a weighted sum of subgroup inequality levels. However, the mean is not well defined for the qualitative or ordinal measures of self-reported health status and subjective well-being that are routinely collected in surveys: for example, respondents are commonly asked whether their health is very good, good, fair, bad, or very bad. One way round this problem might be to use an inequality index based on the spread of outcomes about the median health category (see e.g. Apouey 2007; Abul Naga and Yalcin 2008), but only some of these indices are groupwise decomposable and only then under the highly restrictive condition that all subgroups share a common median (Kobus and Miłos 2012). In practice, the overwhelming majority of studies first convert ordinal health measures into binary, interval or ratio scale variables in order to make use of the standard tools of inequality analysis.

The main contribution of this paper is to propose a novel analytical framework to not only measure differences in ordinal health outcomes between population subgroups but also account for such disparities in terms of the individual-level socioeconomic and demographic characteristics of their members. The basis of our measurement approach is the comparative evaluation of pairs of population subgroup health distributions by means of the statistical preference criterion (De Schuymer et al. 2003; Montes et al. 2015), whereby the health of one group is judged to be better than that of another if the (strictly) healthier of any randomly chosen pair of individuals from the two groups is more likely to be from the first rather than the second group. Statistical preference offers two main advantages over first-order stochastic or rank dominance, which is commonly used within economics to compare pairs of ordinal distributions (see e.g. Zheng 2011). First, it will always be possible to say whether one group health profile is better, worse or equivalent to another, whereas this is not guaranteed with rank dominance as some pairs of profiles may be noncomparable according to this criterion. Rank dominance implies statistical preference, but not vice versa, if the two groups are independent of each other, which will be the case if each individual is a member of one and only one group as in the current study. The use of statistical preference as a normative criterion has been derived axiomatically by Dubois et al. (2003) within a decision-making framework in which individual (health) outcomes are assumed to be only ordinally measurable. Second, statistical preference is able to not only rank any pair of group health profiles but also provide a readily intelligible measure of the degree to which one profile is better or worse than the other (De Baets and De Meyer 2007).

We proceed to define two summary measures of ordinal health differences between population subgroups. First, the comparative health of a subgroup is given by the difference
in the probabilities that a randomly chosen subgroup member will be in better rather than worse health than a randomly chosen individual from the population as a whole. Second, the total variation in health across subgroups is captured by an overall or 'pure' headcount stratification index that measures the average absolute difference in the chances of being in better rather than worse health as a result of being a member of one subgroup rather than another. Importantly, calculation of neither measure is dependent on all subgroups sharing a common median health state, nor on any other such restrictive condition.

Following the lead of Allison and Foster (2004) the bulk of the literature on the measurement of ordinal health inequalities has been restricted to evaluating overall inequality in health, without focusing on any particular cause or justification (see Josa and Aguado (2020) for a recent review). Notable exceptions are Zheng (2011), Makdissi and Yazbeck (2014) and Allanson (2017), which all offer new bivariate measures of the strength of the association between income and health that are designed specifically for use with ordinal health data, while Makdissi and Yazbeck (2017) establish conditions for the robust ordering of joint distributions of income and ordinal health status in terms of socioeconomic health inequality. However, the bivariate analysis of (income-related) health inequalities may be misleading from a policy perspective because it fails to control for the confounding effects of other socioeconomic and demographic characteristics - such as age, sex, ethnicity and education - that affect health and may be correlated with income (Gravelle 2003). In this paper we take a different approach that makes use of indirect standardisation techniques to provide the basis for a multivariate decomposition of our measure of overall health stratification.

The first step of our procedure uses the results from a distribution regression model for the whole population (Chernozhukov et al. 2013; Silbersdorff et al. 2018) to generate a set of indirectly standardised group health profiles. Specifically, we estimate a sequence of binary choice models for the whole population where the dummy dependent variable in each model takes a value of one if the health of an individual is no better than $h(h=1, \ldots . C-1$ of $C$ distinct health states) and zero otherwise, and use the predicted values from this set of models to calculate the group health profiles that would be expected if health outcomes conditional upon sociodemographic characteristics were the same for each group as in the whole population. The second step uses these counterfactual health profiles to perform an aggregate decomposition (cf. Fortin et al. 2011) of the overall headcount stratification index into compositional and conditional health effects, where the former reflects differences between the sociodemographic composition of the groups and the latter differences in individual health outcomes conditional upon sociodemographic characteristics. We further use the distribution regression model results to obtain a detailed breakdown of the aggregate composition effect in terms of the separate contributions of between-group differences in individual sociodemographic factors to stratification.

Application of the new analytical framework is illustrated by an empirical investigation of differences in adult population health between the regions of England, making use of the responses to the self-reported health question included in the Family Resources Survey 201617. The link between regional health outcomes and deprivation levels in England has long been an object of official concern (Black et al. 1980; Acheson 1998; Marmot et al. 2010), with Public Health England recently commissioning an independent inquiry (Whitehead 2014) specifically to look at the health divide between the poorer North and more prosperous South of the country. Our analysis shows that about half of the regional variation in self-reported health can be accounted for by sociodemographic factors, with age and educational qualifications both more important predictors than income.

The structure of the paper is as follows. The next section provides a discussion of the specification and properties of the comparative health and headcount stratification indices. Section 3 outlines the indirect standardisation and decomposition procedures. Section 4 presents the empirical study. The final section discusses the contribution and offers some suggestions for further applications of the methodology.

## 2 Measurement of disparities in ordinal health between population subgroups

The basis of the measurement approach is the pairwise comparison of health profiles. We therefore start with the specification and properties of a pairwise index that provides a measure of how much better or worse one group health profile is than another based on the statistical preference criterion. We go on to define the comparative health and headcount stratification indices as summary statistics of the set of all possible pairwise indices for some population of interest composed of two or more groups.

Specifically, we consider some population $\Omega$ consisting of $K \geq 2$ mutually exclusive and exhaustive groups. The population size and share of group $k(k=1, . K)$ are given as $n_{k}$ and $p_{k}=n_{k} / N$ respectively, where $N=\sum_{k} n_{k}$ is the total population size. The health profile or distribution of group $k$ is a vector of the form $H_{k}=\left(h_{1 k}, \cdots, h_{n_{k} k}\right)$, where the health status $h_{i k}$ of the $i$ 'th person is defined as an ordinal variable with $C$ distinct categories. The population health profile $H_{\Omega}=\left(H_{1}, H_{2}, \ldots H_{K}\right)$ is obtained as the concatenation of the $K$ independent subgroup health distributions. The probability that the health of a randomly chosen individual from group $k$ is at least as good as - i.e. strictly better than or the same as - that of a randomly chosen individual from group $k^{\prime}$ is given as

$$
\mathrm{P}\left(H_{k} \geq H_{k^{\prime}}\right)=\mathrm{P}\left(H_{k}>H_{k^{\prime}}\right)+\mathrm{P}\left(H_{k}=H_{k^{\prime}}\right)
$$

and from the whole population (including group $k$ itself) as

$$
\mathrm{P}\left(H_{k} \geq H_{\Omega}\right)=\sum_{k^{\prime}} \cdot \mathrm{p}_{k^{\prime}} \mathrm{P}\left(H_{k} \geq H_{k^{\prime}}\right) .
$$

### 2.1 Measurement of health differences between pairs of groups

Following Allanson (2017) let the difference in health between any two groups $k$ and $k^{\prime}$ be defined by the pairwise index:

$$
\begin{equation*}
\Delta_{k k^{\prime}}=P\left(H_{k^{\prime}} \geq H_{k}\right)-P\left(H_{k} \geq H_{k^{\prime}}\right)=P\left(H_{k^{\prime}}>H_{k}\right)-P\left(H_{k}>H_{k^{\prime}}\right) ; \quad \forall k, k^{\prime} \in K \tag{1}
\end{equation*}
$$

where the second equality follows by definition. $\Delta_{k k^{\prime}}$ is thus equal to the difference in the chances that a randomly chosen individual from group $k^{\prime}$ will have (strictly) better rather than worse health than a randomly chosen individual from group $k . \Delta_{k k^{\prime}}$ is defined for both continuous and discrete health distributions, with the latter being the norm for self-reported health data from surveys in which individuals are typically asked to choose between a finite number of descriptive categories (e.g. very bad, bad, fair, good, very good). Thus, importantly, $\Delta_{k k^{\prime}}$ is well defined even if only ordinal health data are available. For example, if health is given by a binary $0 / 1$ variable with $20 \%$ of group $k$ and $60 \%$ of group $k^{\prime}$ individuals in good health then $P\left(H_{k^{\prime}}>H_{k}\right)=P\left(H_{k}=0, H_{k^{\prime}}=1\right)=(1-0.2) * 0.6=0.48$ and $P\left(H_{k}>H_{k^{\prime}}\right)$
$=P\left(H_{k}=1, H_{k^{\prime}}=0\right)=0.2 *(1-0.6)=0.08$ given independent groups, so $\Delta_{k k^{\prime}}=(0.48$ $-0.08)=0.4$ using (1). For binary indicators $\Delta_{k k^{\prime}}$ is simply the difference in the proportions in good health, but it can also be calculated for ordinal indicators with three or more health states without the need for dichotomisation. A more computationally efficient approach if there are more than 3 health states (i.e. $C>3$ ) is to use the relation $\Delta_{k k^{\prime}}=\left(1-2 \bar{F}_{k k^{\prime}}\right)$, where $\bar{F}_{k k^{\prime}}$ $=\mathrm{P}\left(H_{k}>H_{k^{\prime}}\right)+0.5 \mathrm{P}\left(H_{k}=H_{k^{\prime}}\right)$ is the mean fractional rank of group $k$ health outcomes in the group $k^{\prime}$ health distribution if ties are assigned their average rank and $\bar{F}_{k k}=0.5$ by definition.

The interpretation of $\Delta_{k k^{\prime}}$ as a measure of subgroup health differences is motivated by the notion of statistical preference (De Schuymer et al. 2003). Specifically, $H_{k^{\prime}}$ may be said to be statistically preferred to $H_{k}$ if $\Delta_{k k^{\prime}}$ is greater than zero, since there is a greater than an even chance that a randomly chosen individual from group $k^{\prime}$ will be in better rather than worse health than someone from group $k . \Delta_{k k^{\prime}}$ will take a value of zero if the health of the two groups is statistically indifferent, although this does not necessarily imply that the health distributions of the two groups are identical; a maximum value of one when the least healthy individual in group $k^{\prime}$ is strictly healthier than the healthiest individual in group $k$; and a minimum value of minus one when the opposite is the case. Thus $\Delta_{k k^{\prime}}$ provides a directional measure in the sense of Dagum (1997), which may be further interpreted as the signed 'distance' between the health distributions of the two groups given that $\Delta_{k k}=0$ and $\Delta_{k k^{\prime}}=-\Delta_{k^{\prime} k}$ by construction. Unlike symmetric measures of distributional differences, such as the Permanyer and D'Ambrosio (2015) overlap index, $\Delta_{k k^{\prime}}$ indicates which group has the better health as well as the degree of separation of their health profiles.

### 2.2 Comparative health index

We define the comparative health of a subgroup as the difference in the probabilities that a randomly chosen subgroup member will be in better rather than worse health than a randomly chosen individual from the population as a whole. The comparative health index for group $k$ may be written as:

$$
\begin{equation*}
\Delta_{\Omega k}=P\left(H_{k}>H_{\Omega}\right)-P\left(H_{\Omega}>H_{k}\right)=\sum_{k^{\prime}=1}^{K} p_{k^{\prime}}\left(P\left(H_{k}>H_{k^{\prime}}\right)-P\left(H_{k^{\prime}}>H_{k}\right)\right)=\sum_{k^{\prime}=1}^{K} p_{k^{\prime}} \Delta_{k^{\prime} k} ; \forall k \in K \tag{2}
\end{equation*}
$$

providing a summary measure of the comparative health of the group as a weighted average of its pairwise index values (including with itself). $\Delta_{\Omega k}$ can take values in the closed interval from $-\left(1-p_{k}\right)$ to $+\left(1-p_{k}\right)$, since $\Delta_{k k}=0$ by definition, with the sign of the index indicating whether the health profile of group $k$ is better or worse than that of the population and its magnitude indicating the extent of any distributional difference. By definition, the population-weighted average of the set of comparative health indices is zero, i.e. $\sum_{k} p_{k} \Delta_{\Omega k}=\Delta_{\Omega \Omega}=0$.

### 2.3 Headcount stratification index

We measure the total variation in health across subgroups as the average difference in the chances that the healthier of two randomly chosen individuals from the entire population will be a member of the subgroup with the better rather than worse health. Specifically, the headcount stratification index S is defined as:

$$
\begin{equation*}
S=\sum_{k=1}^{K} \sum_{k^{\prime}=1}^{K} p_{k} p_{k^{\prime}}\left|\Delta_{k k^{\prime}}\right|=\sum_{k=1}^{K} \sum_{k^{\prime}=1}^{K} p_{k} p_{k^{\prime}}\left|P\left(H_{k^{\prime}}>H_{k}\right)-P\left(H_{k}>H_{k^{\prime}}\right)\right| \tag{3}
\end{equation*}
$$

which is simply the population-weighted mean absolute value of the full set of pairwise indices. Following Allanson (2018), $S$ may also be interpreted as a measure of the extent to which the group health profiles form distinct strata or layers in the overall health distribution. $S$ will take a minimum value of zero only if all groups have the same comparative health and a maximum value of $\left(1-\sum_{k} p_{k}^{2}\right)$ if the ranges of the group health distributions are completely disjoint. Given symmetry, the pairwise indices $\left|\Delta_{k k^{\prime}}\right|$ may be meaningfully aggregated to yield estimates $S_{k}$ of the contribution of each group to the total variation or stratification between population subgroups.
$S$ is a unit free measure that is invariant to rank-preserving transformations of individual health outcomes. If the health outcome measure is given by a binary indicator variable, taking values of zero and one, then $S=2 \mu G_{B}=\sum_{k} \sum_{k^{\prime}} p_{k} p_{k^{\prime}}\left|\mu_{k}-\mu_{k^{\prime}}\right|$, i.e. twice the conventional between-group absolute Gini index where $\mu=\sum_{k} p_{k} \mu_{k}=\sum_{k} p_{k} P\left(H_{k}=1\right)$ may be interpreted as a measure of population mean health and $G_{B}$ is the between-group health Gini index. But unlike the between-group (absolute) Gini index, $S$ is also defined for polytomous categorical variables without the need to either dichotomise or cardinalise the health measure.
$S$ will be sensitive to any change in individual health status within the population unless the change is over some health range occupied exclusively by members of the same group as the individual. Following Allanson (2017) it is possible to show that $S$ satisfies a between-group exchange condition which holds that any exchange in group identity (or health status) between an individual from one group and an individual in no better (i.e. the same or worse) health from a group with worse health will not lead to an increase in headcount stratification provided that the ordering of groups is transitive and not affected by the exchange. This is a 'global' exchange condition unlike the 'local' version of the principle identified by Reardon (2009) as a characteristic of a class of vertical segregation indices that measure the extent to which ordinal variation within subgroups is less than the total ordinal variation in the population. A so-called 'Hammond' transfer that merely reduces the spread of health between the two individuals (cf. Gravel et al. 2020) may in contrast increase $S$ if, for example, the stratification of the first group does not change as a result while that of the second increases in relation to groups with even worse health.

## 3 Decomposition of the comparative health and headcount stratification indices

Health differences between subgroups may reflect differences both in sociodemographic composition and in individual health outcomes conditional upon sociodemographic characteristics, where the former may call for policies to address disparities in the social determinants of health whereas the latter may in part reflect differences in health system performance. The aggregate decomposition of our summary measures of comparative health and headcount stratification serve to identify these compositional and conditional health effects. For this purpose, we employ an indirect standardisation technique based on the construction of the counterfactual health distribution that would be expected if health outcomes conditional upon sociodemographic characteristics were the same in each subgroup as in the population as a whole.

Let $F_{H_{\Omega}}(h)=P\left(H_{\Omega} \leq h\right)=\sum_{k} p_{k} F_{H_{k}}(h)$ be the cumulative distribution function (cdf) of health outcomes for the whole population, where $F_{H_{k}}(h)=P\left(H_{k} \leq h\right)$ is the cdf of health in
group $k$. Defining the (joint) distributions $F_{X_{\Omega}}(x)$ and $F_{X_{k}}(x)$ analogously for some vector of sociodemographic characteristics $x=\left(x_{1}, x_{2}, \ldots x_{V}\right)$, it may be noted that $F_{H_{k}}(h)=\int$ $F_{H_{k} \mid X_{k}}(h \mid X=x) d F_{X_{k}}(x)$ where $F_{H_{k} \mid X_{k}}(h \mid X=x)$ is the conditional distribution of health in group $k$. Thus the indirect standardisation counterfactual may be written as:

$$
\begin{equation*}
F_{H_{\Omega}^{I}}(h)=\sum_{k=1}^{K} p_{k} F_{H_{k}^{I}}(h)=\sum_{k=1}^{K} p_{k}\left[\int F_{H_{\Omega} \mid X_{\Omega}}(h \mid X=x) d F_{X_{k}}(x)\right] \tag{4}
\end{equation*}
$$

where $F_{H_{\Omega}^{I}}(h)$ and $F_{H_{k}^{I}}(h)$ respectively denote the population and subgroup counterfactual unconditional health distributions, and $F_{H_{\Omega} \mid X_{\Omega}}(h \mid X=x)$ is the population conditional health distribution. We use the distribution regression approach of Chernozhukov et al. (2013) to estimate $F_{H_{\Omega} \mid X_{\Omega}}(h \mid X=x)=\Lambda\left(X \beta_{\Omega}(h)\right)=\Lambda(z)$, where $\Lambda\left(X \beta_{\Omega}(h)\right)$ is a chosen link function and $\beta_{\Omega}(h)$ is a vector of parameters. Specifically, the link function is estimated for each distinct health state $h(h=1, \ldots C-1)$ observed in the population by creating a dummy dependent variable that takes a value of one if the observation on $H_{\Omega}$ is no better than $h$ and zero otherwise. The estimate of the counterfactual $F_{H_{k}^{I}}(h)$ for each group is then obtained by averaging the predicted probabilities over all observations in that group. Our preferred link is the identity function since the derivation of the detailed decomposition from the resultant linear probability distribution regression model (LPDRM) is straightforward. However, we also obtain results for the standard normal cdf link $\Phi(z)$ to examine the robustness of the findings to the choice of functional specification.

The next step is to construct a set of indirectly standardised pairwise indices

$$
\Delta_{k k^{\prime}}^{I}=\mathrm{P}\left(H_{k^{\prime}}^{I}>H_{k}^{I}\right)-\mathrm{P}\left(H_{k}^{I}>H_{k^{\prime}}^{I}\right)
$$

using the counterfactual health distributions, from which it follows immediately that $\Delta_{\Omega k}^{I}$ $=\sum_{k^{\prime}} p_{k^{\prime}} \Delta_{k^{\prime} k}^{I}$ measures what the comparative health of group $k$ would be if health differences between groups were due to compositional differences alone. The aggregate decomposition of the headcount stratification index is defined as:

$$
\begin{equation*}
S=\sum_{k=1}^{K} \sum_{k^{\prime}=1}^{K} p_{k} p_{k^{\prime}}\left|\Delta_{r k^{\prime}}\right|=\sum_{k=1}^{K} \sum_{k^{\prime}=1}^{K} p_{k} p_{k^{\prime}}\left\{\left|\Delta_{k k^{\prime}}^{I}\right|+\left(\left|\Delta_{k k^{\prime}}\right|-\left|\Delta_{k k^{\prime}}^{I}\right|\right)\right\}=S^{I}+\left(S-S^{I}\right) \tag{5}
\end{equation*}
$$

where the indirectly standardised index $S^{I}$ provides a measure of the 'explained' component of total health stratification that is due to compositional differences. The 'unexplained' component due to conditional health differences is captured by the difference ( $S-S^{I}$ ), which may be either positive or negative. Specifically, if a separate model $F_{H_{k} \mid X_{k}}(h \mid X)=\Lambda\left(X \beta_{k}(h)\right)$ was estimated for each group then $\left(S-S^{I}\right)$ would reflect the differences between $\beta_{\Omega}(h)$ and the set of parameter vectors $\beta_{k}(h)$, and also, if the link function was non-linear, non-zero average prediction errors by group and health state. If the distribution regression model has no explanatory power then $S^{I}=0$, implying that all stratification is due to conditional health differences between groups.

The 'explained' component $S^{I}$ may be further decomposed to yield estimates of the individual contribution of differences between the groupwise distributions of each sociodemographic characteristic. Assigning average ranks to ties,

$$
\Delta_{k k^{\prime}}^{I}=\bar{F}_{k^{\prime} k}^{I}-\bar{F}_{k k^{\prime}}^{I}=1-2 \bar{F}_{k k^{\prime}}^{I}
$$

where $\bar{F}_{k k^{\prime}}^{I}=1-\bar{F}_{k^{\prime} k}^{I}$ is the average rank of an individual from group $k$ in the counterfactual health distribution of group $k^{\prime}$. Specifically,

$$
\bar{F}_{k k^{\prime}}^{I}=\sum_{h=1}^{C} P\left(H_{k}^{I}=h\right) \bar{F}\left(H_{k^{\prime}}^{I}=h\right)
$$

where $P\left(H_{k}^{I}=h\right)$ is the counterfactual probability that an individual from group $k$ is in heath state $h$ and $\bar{F}\left(H_{k^{\prime}}^{I}=h\right)=P\left(H_{k^{\prime}}^{I}<h\right)+0.5 P\left(H_{k^{\prime}}^{I}=h\right)$ is the counterfactual average rank of individuals from group $k^{\prime}$ in health state $h$. It follows that

$$
\Delta_{k k^{\prime}}^{I}=2 \sum_{h=1}^{C} P\left(H_{k}^{I}=h\right)\left(\bar{F}\left(H_{k}^{I}=h\right)-\bar{F}\left(H_{k^{\prime}}^{I}=h\right)\right),
$$

since $\bar{F}_{k k}^{I}=0.5$ by definition, which shows that the counterfactual pairwise index is equal to twice the weighted average difference in ranks. Hence if $F_{H_{\Omega} \mid X_{\Omega}}(h \mid X=x)$ is given by a LPDRM then $\Delta_{k k^{\prime}}^{I}=2 \sum_{h=1}^{C} P\left(H_{k}^{I}=h\right)\left(\bar{X}_{k}-\bar{X}_{k^{\prime}}\right) \beta_{\Omega}^{*}(h)$ where $\bar{X}_{k}$ and $\bar{X}_{k^{\prime}}$ are vectors of group mean characteristics, $\beta_{\Omega}^{*}(h=1)=\beta_{\Omega}(h=1) / 2$ for the worst observed health state, $\beta_{\Omega}^{*}(h=C)=\beta_{\Omega}(h=C-1) / 2$ for the best health state, and $\beta_{\Omega}^{*}(h)=\left(\beta_{\Omega}(h)+\beta_{\Omega}(h-1)\right)$ $/ 2$ otherwise. This pairwise decomposition is exact but not symmetric, i.e. the contribution of the difference in group means for any particular characteristic $x_{v}(v=1, \ldots \mathrm{~V})$ to $\Delta_{k k^{\prime}}^{I}$ will not be the same as to $\Delta_{k^{\prime} k}^{I}$. The detailed decomposition is based on the average of these two estimates since $S^{I}$ is obtained by aggregation over all pairwise combinations of groups:

$$
\begin{equation*}
S^{I}=\sum_{k=1}^{K} \sum_{k^{\prime}=1}^{K} p_{k} p_{k^{\prime}}\left|\Delta_{k k^{\prime}}^{I}\right|=\sum_{k=1}^{K} \sum_{k^{\prime}=1}^{K} p_{k} p_{k^{\prime}} \operatorname{sgn}\left(\Delta_{k k^{\prime}}^{I}\right)\left(\sum_{h=1}^{C} P\left(H_{k}^{I}=h\right)\left(\bar{X}_{k}-\bar{X}_{k^{\prime}}\right) \beta_{\Omega}^{*}(h)\right) \tag{6}
\end{equation*}
$$

where the sign function takes a value of 1 if $\Delta_{k k^{\prime}}^{I} \geq 0$ and -1 otherwise. For the probit link, the essential non-linearity of $\Phi(z)$ prevents the use of this approach so the computationally intensive nested Shapley (i.e. Owen) decomposition procedure (Shorrocks 2013) is used instead. Stratification due to the variation in each individual sociodemographic characteristic about the corresponding population average is eliminated in turn, with the final estimates obtained as averages of the marginal contributions over all admissible elimination pathways.

## 4 Empirical analysis

The new analytical framework was used to investigate differences in adult health between the regions of England. Our empirical analysis made use of the most recently available data from both the Family Resources Survey (FRS: Department for Work and Pensions (DWP) et al. 2018) and Households Below Average Incomes (HBAI: DWP 2018a), which related to the 2016-17 financial year at the time of writing. The FRS is an annual cross-sectional survey that collects information about the incomes and living circumstances of a representative sample of approximately 20,000 private households in the United Kingdom. The FRS provides the primary data for both HBAI, which is considered to be "the foremost source of UK data and information about household net income and poverty" (ONS 2019), and EU-Statistics on Income and Living Conditions (EU-SILC).

The study is based on the nine Government Office Regions in England, which constitute NUTS 1 statistical regions. The analysis was limited to the HBAI sample of surveyed individuals aged 16
and over, unless defined as a dependent child, to ensure that all observations had complete data on age, sex and income. Sample weights were used throughout the analysis with these being given by Survey of Personal Incomes (SPI)-adjusted HBAI grossing factors (DWP 2018b), modified to allow for missing health and education attainment data using inverse probability weights (Wooldridge 2002). Bootstrap standard errors for all statistics were generated using the procedure recommended in DWP (2014), which is based on the simple random resampling of households. This procedure is acknowledged by DWP to yield imperfect estimates of the extent of uncertainty because it fails to take account of the complexity of the FRS sampling design and ex-post weighting methodology, but the direction of bias is unclear having been shown to depend on the statistic of interest (Brewer et al. 2017). The final sample consisted of 18,877 individual observations with no missing data in 13,658 household clusters.

### 4.1 Health measures

FRS respondents were asked to say in general whether their health was very good, good, fair, bad or very bad, from which we derived a self-reported health status (SRHS) variable by inverting the numerical coding so that higher scores correspond to better outcomes. Self-reported measures provide insight into how individuals experience their own health, which is important for their wellbeing, and have been widely used in the health economics literature to explore the relationship between income and health (see O'Donnell et al. 2015). Nevertheless, it should be borne in mind that such measures do not provide objective indicators of general health status, with a number of recent studies providing evidence that reporting biases may be correlated with income and other sociodemographic characteristics (see, e.g. Davillas et al. 2017).

Table 1 reports population proportions by region and by SRHS response category within each region, with the latter data plotted in the accompanying Fig. 1. Roughly $70 \%$ of the English population assessed their own health as either 'good' or 'very good', $20 \%$ as fair and $10 \%$ as either 'bad' or 'very bad'. Clear-cut comparisons of reported health between pairs of regions can be made on the basis of first-order stochastic dominance in only $72 \%$ of cases. Nevertheless, the criterion does serve to identify a distinct North-South divide, with the health of all five 'Northern' regions (North East, North West, Yorkshire \& Humberside, West Midlands and East Midlands) unambiguously worse than that of any of the remaining four regions of 'Southern' England.

### 4.2 Sociodemographic Variables

Table 2 provides summary statistics for the set of sociodemographic variables that were employed in the indirect standardisation procedure. INCOME is defined as weekly net (disposable) equivalised household income before housing costs (DWP 2018b), which is equal to the total weekly income of all household members after deductions of income tax and other contributions, adjusted to reflect average survey-year prices and equivalised using the OECD scale to take account of household composition. 'Very rich' individuals in the FRS are assigned income levels derived from the SPI, as the latter are deemed to give a more accurate indication of the level of high incomes than the FRS. Individuals with zero recorded income were assigned a value of $£ 1 /$ week. AGE is age in years at last birthday, top coded at 80 for confidentiality reasons. MALE is a dummy variable coded 1 for males. NONWHITE is an ethnicity dummy variable coded zero for whites and one otherwise. The three qualification variables HIQ12, HIQ345 and HIQ678 are derived from the FRS highest academic or vocational qualification variable DVHIQUAL, with responses banded together using Regulated Qualifications Framework levels (GOV.UK 2019) into four roughly equal sized

Table 1 Population proportions by region and self-reported health status

|  | SRHS |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Percentages | Population <br> share | Very bad <br> $[1]$ | Bad <br> $[2]$ | Fair <br> $[3]$ | Good <br> $[4]$ | Very good <br> $[5]$ |  |
| Region |  |  |  |  |  |  |  |
| London | 1.34 | 5.09 | 17.81 | 38.26 | 37.50 |  |  |
| South East | 16.0 | 1.61 | 5.42 | 21.79 | 39.17 | 32.01 |  |
| East of England | 11.1 | 1.78 | 5.53 | 21.64 | 40.18 | 30.88 |  |
| South West | 10.2 | 1.33 | 6.89 | 22.37 | 38.72 | 30.68 |  |
| West Midlands | 10.4 | 1.95 | 7.27 | 22.78 | 40.10 | 27.90 |  |
| East Midlands | 8.4 | 1.91 | 7.83 | 22.80 | 39.06 | 28.39 |  |
| Yorks \& Humber | 9.8 | 1.97 | 6.80 | 24.48 | 38.29 | 28.46 |  |
| North East | 4.9 | 2.41 | 8.37 | 23.55 | 37.09 | 28.58 |  |
| North West | 13.0 | 2.93 | 8.06 | 22.27 | 39.21 | 27.53 |  |
| England | 100.0 | 1.86 | 6.55 | 21.80 | 39.00 | 30.80 |  |

Regions listed in descending order of comparative health (see Table 3). Source: Own calculations from HBAI and FRS data
groups: none and entry level (e.g. literacy and numeracy certificates) - the omitted reference category; levels 1 and 2 (e.g. lower secondary school qualifications); levels 3,4 and 5 (e.g. upper secondary school and sub-degree qualifications); and levels 6,7 and 8 (e.g. first and higher university degrees).

### 4.3 Empirical results

This section presents our findings on the extent of adult health differences between English regions, before reporting the results of the multivariate decomposition analysis based on the indirect standardisation procedure.


Fig. 1 Cumulative population proportions of English regions by self-reported health status

Table 2 Regional means of sociodemographic variables

| Variable name Region | Equivalent household income | Age | Sex | Ethnicity | Highest RQF qualification level |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | 1 or 2 | 3,4 or 5 | 6,7 or 8 |
|  | INCOME <br> £/week | AGE <br> years | $\begin{aligned} & \text { MALE } \\ & \% \end{aligned}$ | NWHITE <br> \% | $\begin{aligned} & \text { HIQ12 } \\ & \% \end{aligned}$ | $\begin{aligned} & \text { HIQ345 } \\ & \% \end{aligned}$ | $\begin{aligned} & \text { HIQ678 } \\ & \% \end{aligned}$ |
| London | 740.85 | 43.61 | 50.30 | 34.65 | 15.60 | 20.14 | 44.34 |
| South East | 679.61 | 48.91 | 48.97 | 8.70 | 20.08 | 27.21 | 33.94 |
| East of England | 620.35 | 49.07 | 48.94 | 5.75 | 22.78 | 26.40 | 28.57 |
| South West | 629.50 | 49.89 | 49.47 | 3.77 | 21.35 | 28.23 | 30.54 |
| West Midlands | 552.49 | 48.33 | 49.26 | 13.83 | 23.55 | 25.70 | 23.69 |
| East Midlands | 570.91 | 48.86 | 49.37 | 8.11 | 22.11 | 27.05 | 25.49 |
| Yorks \& Humber | 533.46 | 48.15 | 49.27 | 6.80 | 23.15 | 28.08 | 22.70 |
| North East | 514.22 | 48.62 | 49.13 | 4.71 | 21.94 | 29.94 | 22.51 |
| North West | 532.64 | 48.29 | 49.02 | 9.20 | 22.79 | 29.15 | 25.73 |
| England | 613.84 | 47.95 | 49.34 | 12.19 | 21.07 | 26.39 | 30.16 |

Regions listed in descending order of comparative health (see Table 3). RQF stands for the Regulated Qualifications Framework with "none and entry level" the omitted category. Source: Own calculations from HBAI and FRS data

### 4.3.1 Adult health differences between English regions

Table 3 reports the skew-symmetric matrix of pairwise health difference indices $\Delta_{\text {row,col }}$, where $\Delta_{\text {col, row }}=-\Delta_{\text {row, col }}$ by definition and regions are listed in descending order of comparative health. Thus, for example, the $\{\mathrm{NW}, \mathrm{LO}\}$ entry of 0.138 in the bottom left hand corner implies that if one person had been randomly chosen from each region then there was a $13.8 \%$ difference in the chances that the healthier of the pair would have been from London rather than the North West, with the Londoner healthier in 42.0 \% of such comparisons, the North Westerner in $28.2 \%$ and the pair being equally healthy in the remaining $29.7 \%$ of matches. It follows that the string of positive values in the $\{\mathrm{LO}\}$ column imply that SRHS was significantly better in London than in every other English region. Conversely, the string of positive entries in the $\{\mathrm{NW}\}$ row imply that SRHS in the North West was worse than in all other regions, although not all pairwise indices are significantly different from zero.

All of the entries below the leading diagonal are positive, implying that the statistical preference relation results in a strict total ordering of the regions in terms of SRHS, with London having the best SRHS followed in turn by South East, East, South West, East Midlands, West Midlands, Yorkshire \& Humberside, North East and North West. Taking the population-weighted average of the pairwise indices in any column yields the summary measure of the health of that region compared to England as a whole. For example, there was a 8.9 \% difference in the chances that a randomly chosen Londoner would have been more rather than less healthy than a person chosen at random from anywhere in England (including London itself), while there was a $5.0 \%$ difference in the chances that a North Westerner would have been less rather than more healthy in a similar comparison.

The headcount stratification index $S$ is reported in the bottom right hand corner of Table 4: the value of 0.050 implies that there was a $5.0 \%$ difference on average in the chances that the healthier of any randomly chosen pair from the population of England was from the region with the better rather than worse self-reported health, with London making a disproportionate
Table 3 Pairwise measures of differences in self-reported health status by Government Office Region

Regions listed in descending order of comparative health. The matrix is skew-symmetric with $\Delta_{\text {col,row }}=\Delta_{\text {row,col }}$ by definition. $\Delta_{\text {EvG,col }}=\sum_{\text {row }} p_{\text {row }} \Delta_{\text {row,col }}$ and $S_{\text {col }}=\mathrm{p}_{\text {col }} \sum_{\text {row }} \mathrm{p}_{\text {row }}\left|\Delta_{\text {row,col }}\right|$ are the regional comparative health and headcount stratification indices respectively, with $\Delta_{\mathrm{ENG}, \mathrm{ENG}}=0$ and $\mathrm{S}_{\mathrm{ENG}}=\mathrm{S}$ by construction. Household-clustered bootstrapped standard errors based on 500 replications are in italics. ${ }^{*} p<0.05,{ }^{* *} p<0.01$. Source: Own calculations from HBAI and FRS data
contribution to this total given how good its SRHS was compared to all other regions. Multicategory health data are commonly dichotomised to yield a binary measure amenable to analysis in terms of the proportion of respondents in good (as opposed to not good) health (Ziebarth 2010), but the choice of cut-off is arbitrary and information is inevitably discarded in the process. Dichotomisation of SRHS reduces $S$ to between 0.5 and $3.5 \%$ depending on the cut-off point, but in all cases resulting in an underestimate of the variation in SRHS between regions.

### 4.3.2 Multivariate decomposition analysis

The multivariate decomposition analysis is based on fixed effects estimates of the LPDRM for England reported in Table 4. The dummy dependent variable $I(h \leq c)$, which takes a value of one if the SRHS of the individual is no better than category $c(c=1,2,3,4)$ and zero otherwise, is specified as a function of age, age squared, sex, ethnicity, the logarithm of income and highest qualification. The use of the fixed effects estimator additionally allows for any regionspecific factors that may influence SRHS. The probabilities of having no better than fair or good health are predicted to increase with age over the whole of the normal lifespan but peak at about 75 and 65 years old respectively for bad and very bad health, presumably due to the effects of selective mortality beyond these ages (see Mirowsky and Ross 2008). Gender and ethnic group effects were largely insignificant, with the exception that non-whites were significantly more likely than whites to report having very bad health. Higher incomes were associated with better outcomes across the whole of the health distribution, implying that the health profiles of higher income classes would have first-order stochastically dominated those

Table 4 Fixed effects estimates of the linear probability distribution regression model for England

|  | SRHS no better than: |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | very bad $I(h \leq 1)$ | bad $I(h \leq 2)$ | fair $I(h \leq 3)$ | good $I(h \leq 4)$ |
| AGE | $0.00115^{* *}$ | $0.00418^{* *}$ | $0.00661^{* *}$ | $0.01003^{* *}$ |
|  | 0.00033 | 0.00071 | 0.00128 | 0.00157 |
| AGE squared | $-0.00001^{*}$ | $-0.00003^{* *}$ | -0.00002 | $-0.00005^{* *}$ |
|  | 0.00000 | 0.00001 | 0.00001 | 0.00001 |
| MALE | 0.00277 | -0.00698 | -0.00293 | -0.00502 |
|  | 0.00192 | 0.00393 | 0.00707 | 0.00801 |
| NWHITE | $-0.00641^{*}$ | -0.00418 | -0.00335 | 0.02547 |
|  | 0.00269 | 0.00634 | 0.01277 | 0.01639 |
| log(INCOME) | $-0.00389^{* *}$ | $-0.01299^{* *}$ | $-0.03976^{* *}$ | $-0.02200^{* *}$ |
|  | 0.00103 | 0.00205 | 0.00453 | 0.00545 |
| HIQ678 | $-0.02809^{* *}$ | $-0.10842^{* *}$ | $-0.22006^{* *}$ | $-0.15049^{* *}$ |
|  | 0.00351 | 0.00731 | 0.01224 | 0.01263 |
| HIQ345 | $-0.02381^{* *}$ | $-0.08916^{* *}$ | $-0.15080^{* *}$ | $-0.08643^{* *}$ |
|  | 0.00359 | 0.00739 | 0.01212 | 0.01192 |
| HIQ12 | $-0.02392^{* *}$ | $-0.07655^{* *}$ | $-0.10758^{* *}$ | $-0.04760^{* *}$ |
| Observations | 0.00368 | 0.00778 | 0.01260 | 0.01185 |
| Household clusters | 18877 | 18877 | 18877 | 18877 |
| $R^{2}$ | 13658 | 13658 | 13658 | 13658 |
| Pseudo-R2 | 0.013 | 0.046 | 0.070 |  |

Region fixed effects not reported. Robust household-clustered standard errors are in italics. ${ }^{*} p<0.05$, ${ }^{* *} p<$ $0.01 . \mathrm{R}^{2}=1-(\mathrm{RSS} / \mathrm{TSS})$ is the conventional linear regression measure. Pseudo- $\mathrm{R}^{2}=1-\left(\mathrm{L}_{\mathrm{M}} / \mathrm{L}_{0}\right)$, calculated using only observations with predictions in the unit interval. Source: Own calculations from HBAI and FRS data
of lower income classes (cf. Zheng 2011). Higher levels of education and training were similarly associated with better health outcomes compared to the reference category of those with no or entry level qualifications, with the size of the effects greatest for those with the highest qualifications. Menard (2000) has shown empirically that the conventional $R^{2}$ statistic is highly correlated with the mean of the dependent variable in discrete choice models, concluding that the McFadden (1974) pseudo- $R^{2}$ based on the log likelihood ratio provides "the most generally applicable and consistently useful" analogous measure. All the regressions have significant explanatory power.

Table 5 reports unstandardized and indirectly standardised measures of comparative regional health, where the former are repeated from Table 3 and the latter are based on the health levels predicted by the LPDRM given the sociodemographic composition of each region and the 'grand mean' constant. The differences between these two measures are attributable to differences in health outcomes conditional upon sociodemographic characteristics, providing residual estimates of the relative health 'performance' of each region compared to England as a whole after controlling for differences in sociodemographic characteristics. Thus, for example, the good comparative health of London was because of both the favourable sociodemographic composition of the region and better than average conditional health outcomes. Indirectly standardised comparative health was only significantly positive in London, but significantly negative in all the regions of 'Northern' England. Only the relative health performance of London was significantly above average, and only that of the North West was significantly worse.

Table 6 reports the results from the decomposition of the headcount stratification index $S$ based on the LPDRM estimates. The value of the indirectly standardized index $S^{I}$ implies that the difference in the chances that the healthier of any randomly chosen pair was from the region with the better rather than worse SRHS would only have been $2.3 \%$ on average if the conditional distribution

Table 5 Comparative health of English regions

|  | Unstandardised <br> Region <br> $\Delta_{\text {ENG, row }}$ | Indirectly standardised | Difference |
| :--- | :--- | :--- | :--- |
|  |  | $\Delta_{\text {ENG }, \text { row }}^{I}$ |  |
| London | $0.089^{* *}$ | $0.048^{* *}$ | $0.041^{* *}$ |
| South East | 0.015 | 0.007 | 0.014 |
|  | 0.021 | 0.008 | 0.013 |
| East of England | 0.012 | 0.005 | 0.011 |
|  | 0.011 | -0.008 | 0.019 |
| South West | 0.013 | 0.005 | 0.013 |
|  | -0.002 | -0.005 | 0.003 |
| West Midlands | 0.015 | 0.007 | 0.016 |
| East Midlands | $-0.033^{*}$ | $-0.020^{* *}$ | -0.013 |
|  | 0.015 | 0.005 | 0.013 |
| Yorks \& Humber | $-0.035^{*}$ | $-0.017^{* *}$ | -0.018 |
|  | 0.015 | 0.006 | 0.014 |
| North East | $-0.036^{*}$ | $-0.019^{* *}$ | -0.017 |
| North West | 0.014 | 0.006 | 0.013 |
|  | $-0.048^{*}$ | $-0.020^{*}$ | -0.027 |

Regions listed in descending order of comparative health (see Table 3). Indirect standardisation based on LPDRM estimates. Household-clustered bootstrapped standard errors based on 500 replications are in italics. $* p<0.05,{ }^{* *} p<0.01$. Source: Own calculations from HBAI and FRS data
of health had been the same in all regions as in England as a whole. Hence $46.8 \%(=0.023 / 0.050)$ of the overall stratification in SRHS is 'explained' by multivariate differences in sociodemographic composition, with the 'unexplained' residual of $53.2 \%$ due to systematic differences in the conditional health distributions.

Table 6 also presents the detailed decomposition analysis in which the contribution of each factor reflects both the nature of the relationship between that factor and individual health as estimated by the distribution regression model, and the association at the regional level between the factor and indirectly standardised health as predicted by the model (cf. Equation (6)). Thus age made a net positive contribution to stratification because health was estimated to unambiguously deteriorate as a function of age over most, if not quite all, of the lifespan and older people were concentrated in regions with worse predicted SRHS given their overall sociodemographic composition. In total, $14.4 \%$ (= 27.7 - 13.4) of headcount stratification was attributable to regional differences in age profiles. Sex had a negligible impact on stratification while ethnicity had only a small negative impact despite the highly uneven regional distribution of the non-white population. Income made a positive contribution to stratification given the strictly positive relationship between income and health and the tendency for more prosperous regions to have better predicted SRHS. But the size of this contribution was only $5.7 \%$ of overall stratification, suggesting that income differences per se were not a major driver of regional health differences despite the very strong positive association between the ranking of regions by comparative health and average income evident from Table 2. Finally, education and training also had a net positive impact on individual health but whereas regions with better indirectly standardised health tended to have a disproportionate number of inhabitants with university-level qualifications, those regions with worse indirectly standardised health had above average proportions of inhabitants with only lower secondary school or advanced qualifications. Overall, educational differences emerge as the most important predictor of regional health differences, accounting for $28.4 \%$ (= 41.6-(7.0 + 6.2)) of overall stratification.

Table 6 Decomposition of the headcount stratification index $S$

|  |  |  |
| :--- | :--- | :--- |
| Unstandardised index: $S$ | $0.050^{* *}$ | Share of $S(\%)$ <br> Indirectly standardised index: $S^{I}$ |
|  | 0.006 |  |
| of which due to: $A G E$ | $0.023^{* *}$ | 46.8 |
| AGE squared | 0.003 |  |
| MALE | $0.014^{* *}$ | 27.7 |
| NWHITE | 0.004 | -13.4 |
|  | $-0.007^{*}$ | 0.0 |
| log(INCOME) | 0.003 | -1.8 |
|  | 0.000 | 5.7 |
| HIQ678 | 0.000 |  |
| HIQ345 | -0.001 | 41.6 |
| HIQ12 | 0.001 |  |
| Residual: $S-S^{I}$ | $0.003^{* *}$ | -7.0 |
|  | 0.001 | -6.2 |

Decomposition based on LPDRM estimates. Household-clustered bootstrapped standard errors based on 500 replications are in italics. ${ }^{*} p<0.05,{ }^{* *} p<0.01$. Source: Own calculations from HBAI and FRS data

Finally we examine the robustness of the indirect standardisation findings to the choice of link function in the distribution regression model, with supplementary results based on a probit specification reported in the Appendix. Table 7 reports maximum likelihood estimates of the marginal effects from a fixed effects probit distribution regression model, where these are largely similar to the LPDRM coefficients reported in Table 4. Moreover the probit-based indirectly standardised comparative health estimates reported in Table 8 are virtually identical to those in Table 5. As a result, the probit-based estimate of $S^{I}$ in Table 9 is very close to our preferred LPDRM-based estimate in Table 6, implying a similar aggregate decomposition of the overall level of stratification into compositional and conditional health effects. And the detailed decomposition results are broadly similar to those based on the LPDRM in terms of the relative contributions of the various sociodemographic characteristics to $S$, although the absolute size of these effects tends to be somewhat smaller.

## 5 Discussion

The paper offers a new analytical framework to both measure differences in ordinal health outcomes between population subgroups and account for such health disparities in terms of the individual-level socioeconomic and demographic characteristics of their members. Our approach builds on the concept of statistical preference to motivate the definition of readily intelligible summary measures of comparative subgroup health and between-group variation in health, calculation of which require neither that all groups share a common median health state nor the prior dichotomisation or cardinalisation of the health variable. In particular, the headcount stratification index satisfies an exchange condition akin to the Pigou-Dalton principle of transfers if the ordering of groups is transitive, providing a measure that is equal to twice the between-group absolute Gini index for binary health status indicators but is also well-defined for polytomous categorical variables.

The use of the framework is illustrated by an investigation of SRHS differences between the nine English Government Office Regions in 2016/17. Unlike first-order stochastic dominance, the statistical preference relation provided a strict total order of regional health profiles, with higher levels of comparative SRHS in the four regions of 'Southern' England than in the remaining five regions of 'Northern' England. On average, there was a statistically significant $5.0 \%$ difference in the chances that the (strictly) healthier of any two individuals was from the region with the better rather than the worse population health. But there was an even larger 7.5 \% difference in the chances that a randomly chosen 'Southerner' would have reported higher rather than lower SRHS than a randomly chosen 'Northerner', providing clear evidence of the existence and extent of the North-South SRHS divide within England.

The paper also develops indirect standardisation techniques based on the estimation of the conditional distribution of ordinal health outcomes in order to identify the aggregate compositional and conditional health components of our summary measure of health variation between regions. Our procedure is straightforward to implement in practice and has the advantage over direct standardisation methods, such as the DiNardo et al. (1996) reweighting procedure, that it also permits a detailed breakdown of the aggregate composition effect by sociodemographic characteristics. Our empirical results imply that roughly half of stratification in SRHS was attributable to differences in the sociodemographic composition of regions, with this factor accounting for a significant part of both the comparatively good SRHS of London and the relatively poor SRHS of all the regions of 'Northern' England. More specifically, regional differences in age, highest
qualification and income profiles were all significantly associated with higher levels of stratification, with younger, more qualified and higher income individuals more likely to have both reported better health and lived in regions with better predicted SRHS according to the distribution regression model. Finally, the remaining half of stratification was due to regional differences in conditional SRHS outcomes with this factor also helping to explain both the comparatively good health of London and poor health of the North West. Further work is required to understand the causes of the variation in conditional health outcomes across regions with a view to identifying potential health policy strategies to improve health of the population in the worst performing regions (cf. van Doorslaer and Koolman 2004).

The empirical study illustrates how the procedures developed in the paper may be used to investigate the population geography of SRHS. It would also be of interest to analyse patterns of regional stratification in other survey measures of health and subjective well-being - such as mental and physical disabilities, longstanding illnesses, life satisfaction, meaningfulness, happiness and anxiety - and the extent to which these may also be associated with differences in sociodemographic composition. Further work is also required to examine the extent and drivers of changes in regional health stratification over time, with Whitehead (2014) emphasising the persistence of the root causes of observed differences in general health between regions. More generally, the analytical framework may be employed to investigate health differences between population subgroups classified on the basis of class, gender or race not just region.

## Appendix. Supplementary indirect standardisation results based on fixed effects probit distribution regression model for England

Table 7 Marginal effect estimates from the probit distribution regression model

|  | SRHS no better than: |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
|  | very bad $I(h \leq 1)$ | bad $I(h \leq 2)$ | fair $I(h \leq 3)$ | good $I(h \leq 4)$ |
|  |  |  |  |  |
| AGE | $0.00139^{* *}$ | $0.00569^{* *}$ | $0.00910^{* *}$ | $0.00802^{* *}$ |
| AGE squared | 0.00031 | 0.00077 | 0.00148 | 0.00158 |
|  | $-0.00001 * *$ | $-0.00004^{* *}$ | $-0.00004^{* *}$ | $-0.00003^{*}$ |
| MALE | 0.00000 | 0.00001 | 0.00001 | 0.00002 |
|  | 0.00226 | -0.00643 | -0.00374 | -0.00536 |
| NWHITE | 0.00158 | 0.00366 | 0.00771 | 0.00824 |
| log(INCOME) | -0.00643 | -0.00250 | 0.00025 | 0.02420 |
| HIQ678 | 0.00330 | 0.00695 | 0.01479 | 0.01609 |
|  | $-0.00310^{* *}$ | $-0.01252^{* *}$ | $-0.04122^{* *}$ | $-0.02229^{* *}$ |
| HIQ345 | 0.00062 | 0.00163 | 0.00446 | 0.00576 |
|  | $-0.01935^{* *}$ | $-0.08601^{* *}$ | $-0.21969^{* *}$ | $-0.16366^{* *}$ |
| HIQ12 | 0.00249 | 0.00581 | 0.01223 | 0.01379 |
|  | $-0.01380^{* *}$ | $-0.06077^{* *}$ | $-0.13682^{* *}$ | $-0.10329^{* *}$ |
| Observations | 0.00223 | 0.00549 | 0.01166 | 0.01346 |
| Household clusters | $-0.01373^{* *}$ | $-0.04761^{* *}$ | $-0.09160^{* *}$ | $-0.06466^{* *}$ |
| $R^{2}$ | 0.00236 | 0.00545 | 0.01176 | 0.01374 |
| Pseudo-R $R^{2}$ | 18877 | 18877 | 18877 | 18877 |

Notes: Region fixed effects not reported. Any bias due to the incidental parameter problem (see Arellano and Hahn, 2007) is likely to be negligible as there are only nine regions and more than a thousand observations on each region. Robust household-clustered standard errors are in italics. ${ }^{*} p<0.05,{ }^{* *} p<0.01 . \mathrm{R}^{2}=1-(\mathrm{RSS} / \mathrm{TSS})$ is the conventional linear regression measure. Pseudo- $\mathrm{R}^{2}=1-\left(\mathrm{L}_{\mathrm{M}} / \mathrm{L}_{0}\right)$

Table 8 Comparative health of English regions

|  | Comparative health |  |  |
| :--- | :--- | :--- | :--- |
|  | Unstandardised | Indirectly standardised | Difference |
| Region | $\Delta_{\text {ENG, row }}$ | $\Delta_{\text {ENG,row }}^{I}$ |  |
|  |  |  |  |
| London | $0.089^{* *}$ | $0.048^{* *}$ | $0.042^{* *}$ |
| South East | 0.015 | 0.007 | 0.014 |
|  | 0.021 | 0.008 | 0.013 |
| East of England | 0.012 | 0.005 | 0.011 |
|  | 0.011 | -0.007 | 0.019 |
| South West | 0.013 | 0.005 | 0.013 |
| West Midlands | -0.002 | -0.005 | 0.003 |
| East Midlands | 0.015 | 0.007 | 0.016 |
| Yorks \& Humber | $-0.033^{*}$ | $-0.020^{* *}$ | -0.013 |
|  | 0.015 | 0.005 | -0.013 |
| North East | $-0.035^{*}$ | $-0.016^{* *}$ | 0.018 |
| North West | 0.015 | -0.006 | -0.017 |
|  | $-0.036^{*}$ | $0.019^{* *}$ | 0.013 |
|  | 0.014 | -0.027 |  |

Notes: Regions listed in ascending order of comparative health (see Table 3). Indirect standardisation based on probit distribution regression model estimates. Household-clustered bootstrapped standard errors based on 500 replications are in italics. ${ }^{*} p<0.05,{ }^{*} * p<0.01$

Table 9 Decomposition of the headcount stratification index $S$

|  |  | Share of $S(\%)$ |
| :--- | :--- | :--- |
| Unstandardised index: $S$ | $0.050^{* *}$ | 100.0 |
| Indirectly standardised index: $S^{I}$ | 0.006 | 46.1 |
| of which due to: $A G E$ | $0.023^{* *}$ |  |
| AGE squared | 0.003 | 21.1 |
| MALE | $0.010^{* *}$ | -4.0 |
|  | 0.002 | 0.0 |
| NWHITE | $-0.002^{* *}$ | -0.7 |
|  | 0.001 |  |
| log(INCOME) | 0.000 | 4.3 |
|  | 0.000 |  |
| HIQ678 | -0.000 | 30.5 |
| HIQ345 | 0.001 | -2.5 |
| HIQ12 | $0.002^{* *}$ | -2.5 |
| Residual: $S-S^{I}$ | 0.001 | -2.5 |
|  | $0.015 * *$ | 53.9 |

Notes: Decomposition based on probit distribution regression model estimates. Age, gender, ethnicity, income and educational qualifications were treated as separate groups in the first stage of the detailed decomposition procedure, with the contributions of the individual age and qualification variables calculated in the second stage. Household-clustered bootstrapped standard errors based on 500 replications are in italics. ${ }^{*} p<0.05,{ }^{* *} p<0.01$

## Springer

Acknowledgements The author bears sole responsibility for the further analysis and interpretation of the Family Resources Survey and Households Below Average Income datasets employed in this study.

## Declarations

Ethics The paper raises no ethical issues

Conflict of interest The author declares that he has no conflict of interest.

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