Convergence of income distributions: Total and inequality-affecting changes in the EU

Kvedaras, Virmantas; Cseres-Gergely, Zsombor

Convergence of income distributions: Total and inequality-affecting changes in the EU☆

Virmantas Kvedaras *, Zsombor Cseres-Gergely

European Commission, DG Joint Research Centre, Via Fermi 2749, I-21027, Ispra (VA), Italy

ARTICLE INFO

ABSTRACT

By adapting the statistical framework suggested by Székely and Rizzo (2004) and considering the convergence of income distributions instead of aggregate (e.g., average) income, we exploit the scale-independence property of proper inequality metrics to evaluate not only the total but also the inequality-affecting (shape-influenced) convergence of income distributions. We illustrate the application using Monte Carlo experiments and characterizing the convergence of net equivalized income distributions among European Union member states.

1. Introduction

In this paper, we consider the convergence of income distributions by directly comparing their yearly changes derived from individual-level sample data, rather than inferring about convergence from some stochastic process of aggregate summary measures like average or median income.[1] The yearly measurement allows us to observe the dynamics of the convergence, whereas considering the whole distribution opens up the possibility of evaluating not only the convergence of income but also convergence of income inequality within the same framework.

One of the critical properties of proper metrics of income inequality is their scale-independence or, in other words, their invariance to a common rescaling of everyone's income. Therefore, changes in properly rescaled income might be informative about convergence in terms of inequality. One might be tempted to use the difference in (or ratio of) averages or medians (as, e.g., Handcock et al., 1996, as well as Handcock and Morris, 1998, and Handcock and Morris, 1999) for such purpose, which is however an arbitrary choice. We propose to use a more general rescaling based on the minimization of the distance between the (quantile functions of) analysed income distributions and the pooled income distribution.

To evaluate the multi-country convergence (divergence) of income distributions, we adapt Székely and Rizzo's (2004) approach of statistical testing for equal distributions. To be able to use the statistical test of convergence of distributions for inference about income convergence, we not only apply it year-wise but also change the perspective of how the test statistic is exploited. From the statistical point of view, convergence to the limiting distribution under the null hypothesis of equal distributions takes place as the number of observations (n) increases to infinity, and diverges under the alternative. On the other hand,

[1] For the discussion of the latter methodology, see, e.g., Phillips and Sul (2007), Young et al. (2008), Durlauf et al. (2009), and Johnson and Papageorgiou (2020); the recent examples of application to the EU are Alcidi et al. (2018), and Cabral and Castellanos-Sosa (2019); Ravallion (2003), Bleaney and Nishiyama (2003), and Chambers and Dhongde (2016) are examples of the application of such methodology to the analysis of convergence of income inequality instead of aggregate income.

if we keep n fixed at some positive integer,[2] then all changes observed in the expectation of the test statistic would stem from the variation in the underlying data generating process (DGP): This is what underlies the (economic) evaluation of convergence and divergence of income over years. Using the same number of observations[3] that we achieve through independent and random sampling of realizations from (consistent estimates of) income distributions also avoids the incomparability problem related to a varying population/sample size over years and among countries.

We illustrate the functionality of the proposed methodology with Monte Carlo (MC) simulations and use it to characterize the EU-wide income convergence during 2007–2014. Here, the underlying distributions of real annual equivalized net income are derived separately for each country and year from the harmonized European Union Statistics on Income and Living Conditions (EU-SILC) survey database.[4]

2. Multi-country convergence: Evaluation and testing

Consider Székely and Rizzo's (2004) approach to testing for equal distributions (possibly in a high dimension) using the generic notation that, for a while, ignores the potential time-varying nature of distributions. Namely, let $\{\mathcal{A}_i\}_{i=1}^k$ denote independent random samples of random vectors (in $\mathcal{R}^d$) from the corresponding distributions[5] $\{F_i\}_{i=1}^k$. Let the respective sample sizes be denoted by $\{n_i\}_{i=1}^k$ and $n := \sum_{i=1}^k n_i$. The k-sample test statistic is given by

$$\xi_n = \sum_{1 \le i < j \le k} e(\mathcal{A}_i, \mathcal{A}_j), \tag{1}$$

where for any pair $\mathcal{A} = \{a_1, a_2, \ldots, a_{n_1}\}$ and $\mathcal{B} = \{b_1, b_2, \ldots, b_{n_2}\}$,

$$e(\mathcal{A}, \mathcal{B}) = \frac{n_1 n_2}{n_1 + n_2} \left(\frac{2}{n_1 n_2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \|a_i - b_j\| \right.$$
$$\left. - \frac{1}{n_1^2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_1} \|a_i - a_j\| - \frac{1}{n_2^2} \sum_{i=1}^{n_2} \sum_{j=1}^{n_2} \|b_i - b_j\| \right). \tag{2}$$

Eq. (2) satisfies the triangle inequality, and therefore, $\xi_n = 0$ if all elements in the samples coincide, and $\xi_n > 0$ otherwise. Székely and Rizzo (2004) show that, under the null hypothesis,

$$H_0: \quad F_1 = F_2 = \cdots = F_k,$$

ξ_n has a well-defined limiting distribution, whereas under the composite alternative,

$$H_1: \quad \exists i, j, \ F_i \ne F_j,$$

the expectation $E[\xi_n]$ is asymptotically a positive constant times n, provided that $n_i/n \to c_i \in (0, 1)$. Since the limiting distribution under H_0 depends on $\{F_i\}_{i=1}^k$, Székely and Rizzo (2004) suggest bootstrapping from the pooled sample to derive the critical values.

Let $\kappa_{1-\alpha}^{(b)}$ stand for the α-size critical value from the bootstrap (of size $b \in \mathbb{N}$). $\xi_n > \kappa_{1-\alpha}^{(b)}$ or, equivalently,[6] $\tau_{1-\alpha} := \xi_n / \kappa_{1-\alpha}^{(b)} > 1$ would reject H_0. We use the latter, as it avoids the scale dependence on $\kappa_{1-\alpha}^{(b)}$. Separately for each year, the empirical analogue of $E(\tau_{1-\alpha})$, denoted by $\bar{\tau}_{1-\alpha}$, is obtained from the following simulations indexed by $s \in \{1, \ldots, S\}$. First, after the consistent non-parametric estimation of the distribution functions with their inverse (the quantile function), we draw random samples of size $n_0 \in \mathbb{N}$ for each country[7] using the procedure suggested by Hutson and Ernst (2000): first generating independent random samples from the uniform distribution on [0,1] and then mapping them through a quantile function to the respective independent and identically distributed samples of the variable of interest (income). Then, after implementing the earlier described bootstrap procedure that delivers the critical value $\kappa_{1-\alpha}^{(b)}$, we use the obtained repetitive samples to estimate $E[\tau_{1-\alpha}]$ by averaging simulation realizations $\tau_{1-\alpha}^{(s)} := \xi_n^{(s)} / \kappa_{1-\alpha}^{(b)}$ over the outcomes of repeated samples (thus yielding $\bar{\tau}_{1-\alpha}$). Under H_0, the probability[8]

$$P\{\xi_n < \kappa_{1-\alpha}^{(\infty)}\} = P\{\xi_n / \kappa_{1-\alpha}^{(\infty)} < 1\} \to 1 - \alpha, \ \text{as } n \to \infty. \tag{3}$$

Therefore, in cases where the equality of income distributions under evaluation cannot be rejected, the distribution of $\xi_n^{(s)} / \kappa_{1-\alpha}^{(b)}$, connected with the generated samples as described above, should approximately satisfy condition (3) for not too small values of n (as well as a sufficiently large number of bootstrap replications and MC sampling iterations), and the $1 - \alpha$ quantile of $\xi_n^{(s)}$ approximately coincides with $\kappa_{1-\alpha}^{(b)}$, while of $\xi_n^{(s)} / \kappa_{1-\alpha}^{(b)}$ with 1.

Whenever n is fixed over time, the changes in $E[\xi_n / \kappa_{1-\alpha}^{(\infty)}]$ are driven by the underlying DGP. Hence, decreasing (increasing) values of $E[\xi_n / \kappa_{1-\alpha}^{(\infty)}]$ would point to the presence of convergence (divergence) in economic terms. Since the discussed statistic is not directly observable, we will use the respective estimate $\bar{\tau}_{1-\alpha}$ with its confidence bands (i.e., of the average). We shall further use the notion of *full convergence* to characterize the state of insignificant difference between the distributions, and not the potential shift in their difference. Such full convergence will be checked by evaluating the coincidence of the $1 - \alpha$ quantile of $\xi_n^{(s)} / \kappa_{1-\alpha}^{(b)}$ with 1.

The income convergence evaluation above looks at the *total convergence* using the original $\{F_i\}_{i=1}^k$ without any potential adjustment for the scale differences of income. To obtain the *scale-independent* evaluation of convergence using the above procedure, one just needs to replace $\{F_i\}_{i=1}^k$ with $\{\widetilde{F}_i\}_{i=1}^k$, where $\widetilde{F}_i$ denotes a CDF of the rescaled income $\widetilde{y}_{i,r} = \widetilde{b}_i y_{i,r}$, where $y_{i,r}$, $r \in \{1 : N_i\}$ denotes the original income observations underlying F_i in country i with its sample size N_i, and

$$\widetilde{b}_i = \underset{b \in \mathbb{N}}{argmin} \|b \cdot F_i^{-1} - \bar{F}^{-1}\| \tag{4}$$

where $\bar{F}$ stands for some reference CDF, and in particular, we will use the pooled income CDF derived from observations of all countries (in a fixed year).[9]

---

[2] It cannot be very small in order to have empirical power against the null hypothesis.

[3] It can be also increasing, just uniformly across all years to retain the discussed interpretation.

[4] Further details on the employed data are provided in Cseres-Gergely and Kvedaras (2019).

[5] The usual cumulative distribution function (CDF) of a univariate real-valued continuous random variable Y_i will be used, as denoted by $F_i := F_{Y_i}(y) = P(Y_i \le y)$, $y \in \mathcal{Y} \subset \mathcal{R}$, where P is the probability measure. We will assume that the underlying CDFs are absolutely continuous, monotonically strictly increasing, and twice differentiable.

[6] It should be noted that, in order to avoid heavy notation, we use generic notation dropping for the time, country, bootstrap size, and n size indexes, i.e., $\tau_{1-\alpha}$ is used instead of $\tau_{1-\alpha}^{(b,n)}(i, t)$. Where it becomes important, we explicitly discuss this aspect of the potential change in time and/or variation over the cross-section.

[7] Thus implying $n = k \cdot n_0$.

[8] Eq. (3) holds for all $\alpha \in (0, 1)$ implying the uniform distribution of p-values of the test statistic under the null hypothesis, and this could be tested further. Due to computational intensity, in the empirical application, we restrict our attention to only one size level, $\alpha = 0.05$, by evaluating if the 95% quantile of $\tau_{1-\alpha}^{(s)} \left(= \xi_n^{(s)} / \kappa_{1-\alpha}^{(b)} \right)$ coincides with 1.

[9] In the empirical application, we will use the L^1 norm.

(a) Non-deviant countries vs DGP-A
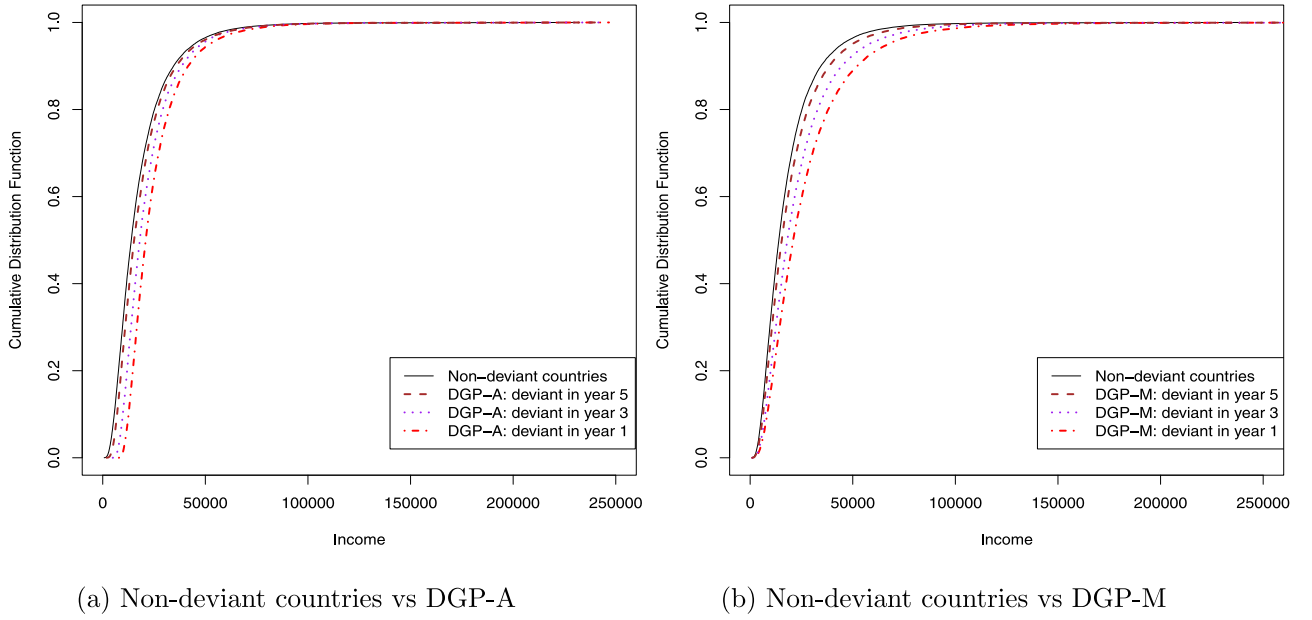

(b) Non-deviant countries vs DGP-M

Fig. 1. Empirical CDFs of DGP-based realizations of income.

Notice that a usual requirement for an income inequality metric is its scale-independence, so multiplication of all persons' income by a positive constant would not change the inequality level. Hence, if no (significant) difference remained between the distributions after the described rescaling based on the scaling constants from Eq. (4), inequality of income, as measured by any scale-independent metric, would be the same. Furthermore, the above minimization of the differences between quantile functions[10] implies that the difference remaining after such a transformation cannot be removed without affecting the value of the scale-independent inequality metric of income. Therefore, using $\{\widetilde{F}_i\}_{i=1}^k$ in the earlier convergence evaluation procedure is informative about the inequality-affecting income convergence that is independent from the scale differences of $\{F_i\}_{i=1}^k$.

To finalize, we point out that the notation above was generic, whereas such analysis is performed separately for each year with all quantities being time-variant.

3. Monte Carlo illustration of convergence evaluation

For simulations, we set up a DGP that resembles the empirical data: The estimated log-normal EU-wide income distribution (F_L) is the basis, with the mean and standard deviation being 9.5 and 0.7, respectively. In the DGP, for all countries but the deviant, each year's income is given by random realizations from F_L, i.e., $y_{i,t} \sim F_L(9.5, 0.7)$ for all periods indexed by t. All countries that have the same income distribution are indexed by $i \neq i^*$, where i^* is reserved for a country that has a potentially different distribution of income.

Let $q_{0.5}$ denote the median of F_L. For the deviant country indexed by i^*, we consider two types of deviations from F_L: additive (DGP-A) and multiplicative (DGP-M).

$$\text{DGP-A:} \quad y_{i^*,t}^{(a)} \sim \begin{cases} F_L(9.5, 0.7) + q_{0.5} \cdot \frac{6-t}{10}, & t \in \{1, 2, \dots, 5\}, \\ F_L(9.5, 0.7), & t \geq 6. \end{cases}$$

(5)

$$\text{DGP-M:} \quad y_{i^*,t}^{(m)} \sim \begin{cases} F_L(9.5, 0.7) \cdot \left(1 + \frac{6-t}{10}\right), & t \in \{1, 2, \dots, 5\}, \\ F_L(9.5, 0.7), & t \geq 6. \end{cases}$$

(6)

In DGP-A, during the first five years,[11] the absolute deviation, given by the non-zero second term on the top-right part of Eq. (5), is decreasing linearly and proportionally to the median of F_L (from 0.5 to 0.1 of $q_{0.5}$), while during the last three years, the distribution of $y_{i^*,t}^{(a)}$ is the same as for the rest of the countries.

In DGP-M, during the first five years, the multiplicative factor is decreasing proportionally to values of F_L (from 1.5 in the first year to 1.1 in the fifth). As in the case of DGP-A, during the last three years, the distribution of $y_{i^*,t}^{(m)}$ remains the same as for the rest of the countries.

Fig. 1 plots the (empirical) CDFs of the simulated data, restricting the plotting range for better visibility. Both the (decreasing) additive and the multiplicative deviations are observable.

Fig. 2 plots the results of the implemented convergence evaluation and testing procedure. Black dots represent the evaluation statistic $\bar{\tau}_{0.95}$ with its 95% confidence bounds around it in grey. The upper dashes in blue stand for the 95% quantile of the simulated realizations $\tau_{0.95}^{(s)}$ (see the discussion below Eq. (3)), which allows us to infer about the acceptability of the null hypothesis of equal income distributions. The top and bottom panels of the figure correspond to DGP-A and DGP-M, respectively, whereas the left and right panels represent the testing without and with the rescaling adjustment, correspondingly.

In both the additive and the multiplicative cases, there is a clearly identifiable pattern of convergence and stabilization after the fifth year observed with non-rescaled data (see Fig. 2(a) and (c)), just as was embedded into the underlying DGPs with a statistically significant reduction in $\bar{\tau}_{0.95}$ values. As far as one can judge from these few realizations, the limiting condition in Eq. (3) also seems to work well enough, with the 95% quantile of $\tau_{0.95}^{(s)}$ (blue dashes) being very close to one.

The results of the evaluation using the scale-adjusted data seem to be also quite reasonable (see the right panel of Fig. 2). As

---

[10] The usage of quantile functions is convenient here: Their support is always the same and bounded, i.e., [0, 1].

[11] In the simulations, we use eight periods, which coincides with the number of years to be considered in the empirical application.

(a) DGP-A: no scale adjustment

(b) DGP-A: with scale adjustment

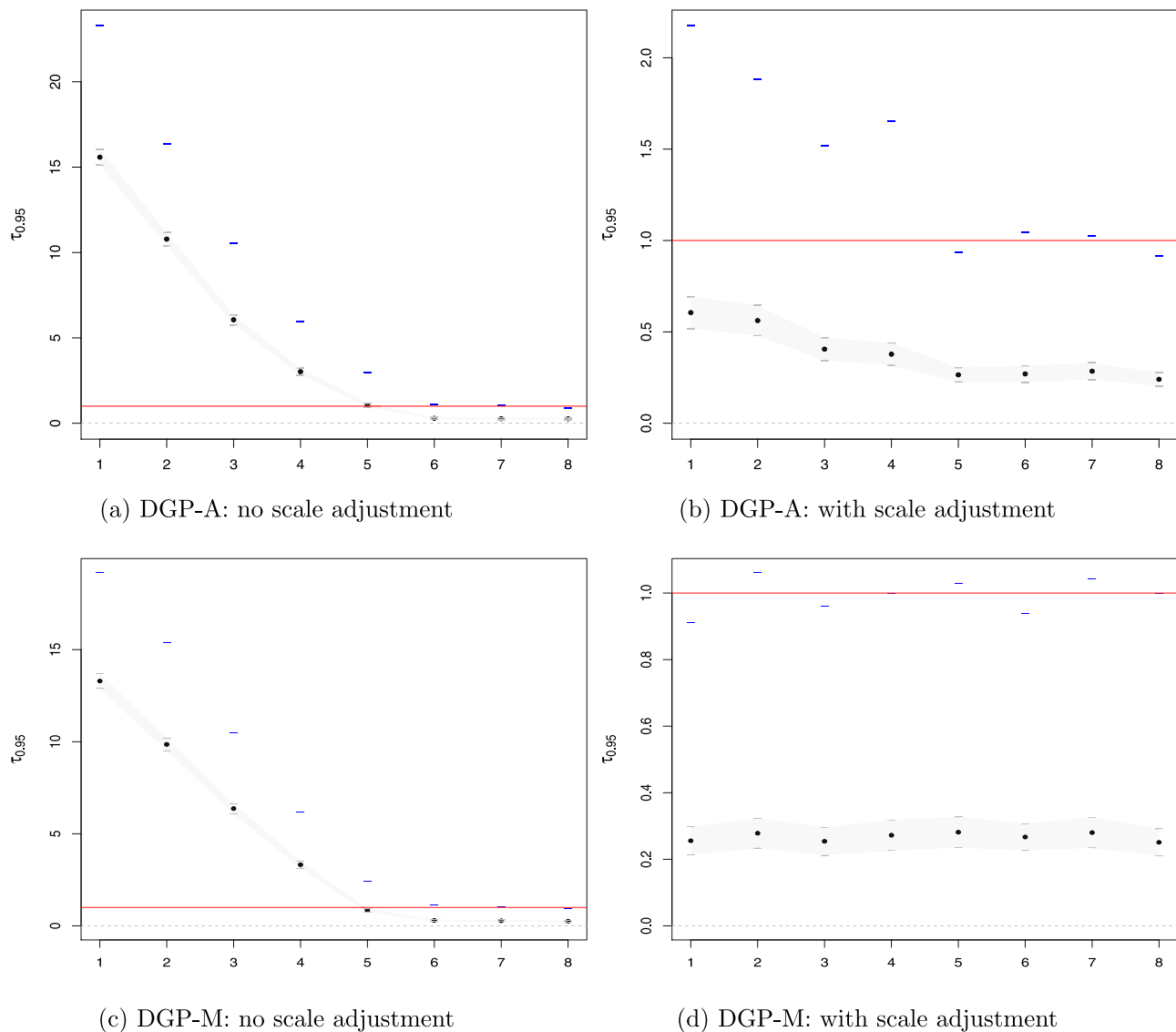(c) DGP-M: no scale adjustment

(d) DGP-M: with scale adjustment

Fig. 2. Monte Carlo simulations of convergence evaluation.

expected, in DGP-M with the multiplicative perturbation of realizations, which does not affect the inequality characteristics of an economy, the scale-adjusted $\bar{\tau}_{0.95}$ is steady (see Fig. 2(d)). In DGP-A with the additive perturbations, it is significantly shrinking over time (see Fig. 2(b)). However, the performance of the limiting condition given by Eq. (3) seems to be less precise in the DGP-A case for smaller values of deviation observed in the fourth and fifth years. This can be connected to quite moderate MC iterations $S = 500$, as well as $n_0 = 250$ and $b = 200$ (within each iteration) used in the simulations,[12] leading to a more substantial variance of potential realizations. It might also be implicitly caused by the optimization-based pre-estimation of rescaling constants $\{\tilde{b}\}$.

The MC results reveal that the suggested methodology can be informative about the total and scale-independent (inequality-affecting) convergence of income distributions.[13]

4. Convergence of income distributions in the EU

We first evaluate the convergence of income distributions among the considered 27 EU countries[14] (EU27). Despite the original diversity of EU countries, the harmonization of institutions, together with the cohesion policy and structural funds, is supposed to create adequate capacity, mechanisms, and means fostering their economic convergence. Fig. 3(a) plots the statistic $\bar{\tau}_{0.95}$ and the 95% quantile of the realized $\tau_{0.95}^{(s)}$ for unadjusted income.

The convergence of income distributions among the EU countries from 2007 to 2014 is clearly statistically significant, even in the aftermath of the financial crisis, which is consistent with the findings of Cabral and Castellanos-Sosa (2019). The largest convergence within a single year took place just after the financial crisis. The convergence seems to have become slower for several years afterwards, but it accelerated again after 2012, i.e., during the latest years of recovery in the EU.[15]

---

[12] The same parametrization will be applied in the empirical application; therefore, the chosen values are relatively low due to significant computational costs whenever 27 EU countries are under consideration.

[13] This is in spite of the relatively small size of the population used in the deviating country: The proportion is the same as that of the population of Spain relative to the whole EU.
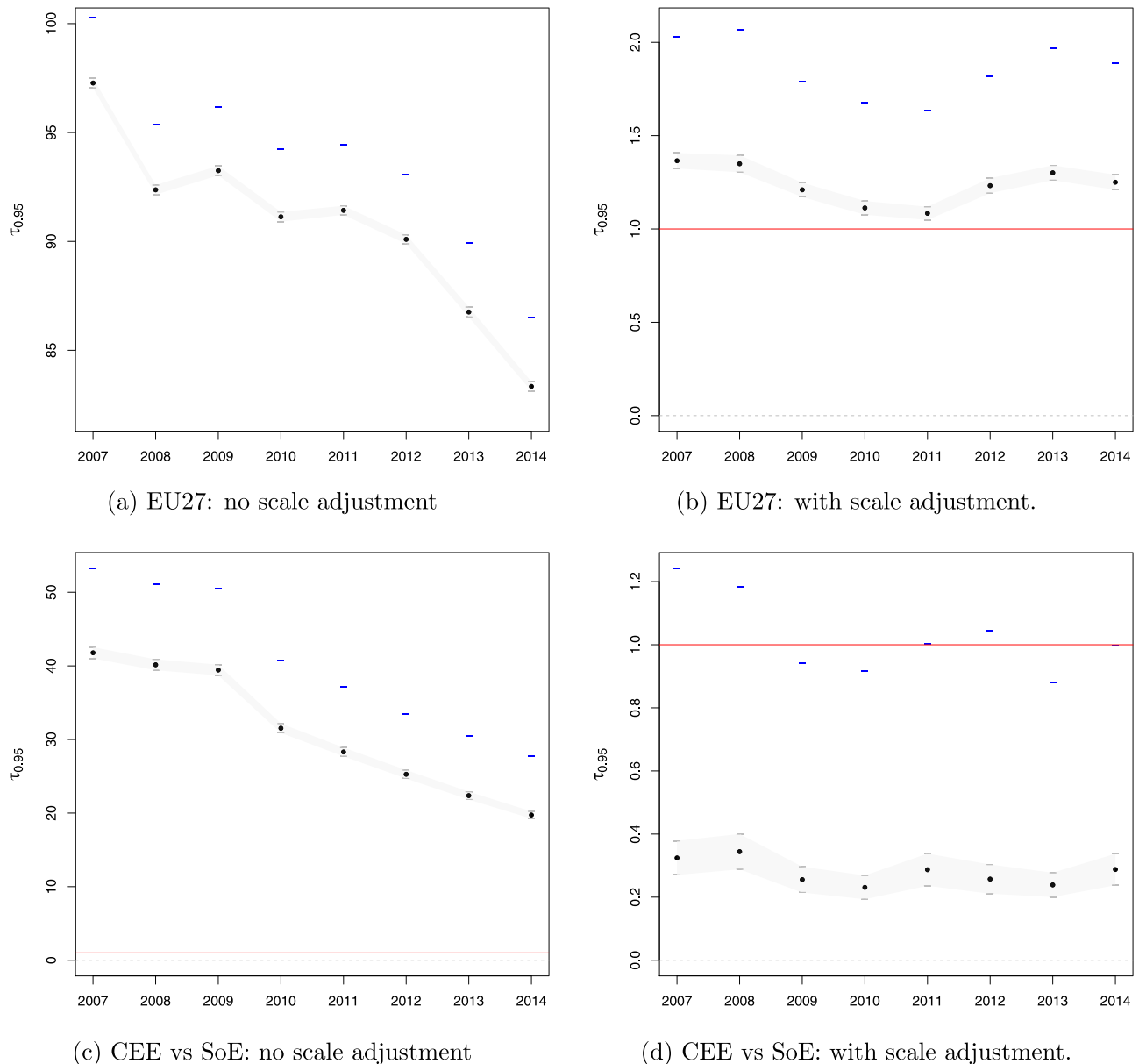
---

[14] Does not include Croatia.

[15] The largest convergence in 2008 is mostly associated with the fact that the high-income North-Western (NW) economies in the EU were among the

(a) EU27: no scale adjustment

(b) EU27: with scale adjustment.

(c) CEE vs SoE: no scale adjustment

(d) CEE vs SoE: with scale adjustment.

Fig. 3. Dynamics of divergence of empirical income distributions.

There is less convincing evidence on convergence of the inequality-affecting part of income distributions (see Fig. 3(b)): $\bar{\tau}_{0.95}$ had an initial downswing after the crisis, with an upswing in the later years that seems to be inversely connected with the economic situation during this period.

We finalize the illustration with an intriguing example of convergence of income distributions between Central and Eastern Europe (CEE) and Southern Europe (SoE) (see the lower panel of Fig. 3). Considering the income distributions within these large macro-regions of the EU,[16] not only the convergence did take

place during 2007–2014, but, using the scale-adjusted data, the hypothesis of full convergence in 2014 (and even earlier) cannot be rejected at the 5% significance level (see Fig. 3 (d)). Therefore, any scale-invariant inequality metric would become very similar for the two regions (for such evidence see, e.g., Benczur et al., 2017, and JRC, 2020). Thus, irrespective of the remaining differences in the scale of income, the CEE and SoE regions taken as a whole became quite similar in terms of income distribution.

References

Alcidi, C., Ferrer, J.N., Salvo, M.D., Musmeci, R., Pilati, M., 2018. Income Convergence in the EU: A tale of two speeds. CEPS Commentary, 9 January, 2018.

Benczur, P., Cseres-Gergely, Zs., Harasztosi, P., 2017. EU-wide income inequality in the era of the Great Recession. Working papers in economics and finance 2017–14. Joint Research Centre, European Commission.

Bleaney, M., Nishiyama, A., 2003. Convergence in income inequality: Differences between advanced and developing countries. Econ. Bull. 4 (22), 1–10.

Cabral, R., Castellanos-Sosa, F.A., 2019. Europe's income convergence and the latest global financial crisis. Res. Econ. 73 (1), 23–34.

first to experience the real consequences of the financial crisis in terms of slower economic growth. Most lower-income countries from other EU regions experienced a larger slump of economic growth relative to the NW region later, mostly in 2009, thus resulting even in some divergence (see Fig. 3(a)). The speeding up of convergence since 2012 is mostly associated with the recovery of growth (in absolute and relative terms) in the lowest-income EU countries from Central and Eastern Europe, especially, as compared with the Southern European countries.

[16] Cseres-Gergely and Kvedaras (2019) contains the analogous evaluations of convergence between (and also within) other sub-groups of EU countries.

Chambers, D., Dhongde, S., 2016. Convergence in income distributions: Evidence from a panel of countries. Econ. Model. 59, 262–270.

Cseres-Gergely, Zs., Kvedaras, V., 2019. Change and convergence of income distributions in the European Union during 2007–2014. Working papers in economics and finance 2019–13. Joint Research Centre, European Commission.

Durlauf, S.N., Johnson, P.A., Temple, J.R.W., 2009. The econometric of convergence. In: Mills, T.C., Patterson, K. (Eds.), Palgrave Handbook of Econometrics. Palgrave MacMillan, London.

Handcock, M.S., Morris, M., 1998. Relative distribution methods. Sociol. Methodol. 28, 53–57.

Handcock, M.S., Morris, M., 1999. Relative Distribution Methods in the Social Sciences. Springer-Verlag Inc., New York.

Handcock, M.S., Morris, M., Bernhardt, A.D., 1996. A Distributional Approach to Measuring Changes in Economic Inequality. Mimeo, Department of Statistics and Operational Research, New York University, USA.

Hutson, A.D., Ernst, M.D., 2000. The exact bootstrap mean and variance of *L*-estimator. J. R. Stat. Soc. Ser. B Stat. Methodol. 62, 89–94.

Johnson, P., Papageorgiou, C., 2020. What remains of cross-country convergence? J. Econ. Lit. (forthcoming).

JRC, European Commission, 2020. Beyond Averages — Evidence and Perspectives for an EU-Wide Fairness Discourse. Fairnes Report, Joint Research Center, European Commission (forthcoming). DOI: http://dx.doi.org/10.2760/261169 (online). DOI(2): http://dx.doi.org/10.2760/20683 (print).

Phillips, P.C., Sul, D., 2007. Transition modeling and econometric convergence tests. Econometrica 75 (6), 1771–1855.

Ravallion, M., 2003. Inequality convergence. Econom. Lett. 80 (3), 351–356.

Székely, G.J., Rizzo, M.L., 2004. Testing for equal distributions in high dimension. InterStat November 5.

Young, A.T., Higgins, M.J., Levy, D., 2008. Sigma convergence versus beta convergence: Evidence from U.S. county-level data. J. Money Credit Bank. 40 (5), 1083–1093.