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Abstract

The Iterative Deferred Acceptance Mechanism

by Inácio Bó and Rustamdjan Hakimov[*]

We introduce a new mechanism for matching students to schools or universities, denoted Iterative Deferred Acceptance Mechanism (IDAM), inspired by procedures currently being used to match millions of students to public universities in Brazil and China. Unlike most options available in the literature, IDAM is not a direct mechanism. Instead of requesting from each student a full preference over all colleges, the student is instead repeatedly asked to choose one college among those which would accept her given the current set of students choosing that college. Although the induced sequential game has no dominant strategy, when students simply choose the most preferred college in each period (denoted the straightforward strategy), the matching that is produced is the Student Optimal Stable Matching. Moreover, under imperfect information, students following the straightforward strategy is an Ordinal Perfect Bayesian Equilibrium. Based on data from 2016, we also provide evidence that, due to shortcomings which are absent in the modified version that we propose, the currently used mechanism in Brazil fails to assist the students with reliable information about the universities that they are able to attend, and are subject to manipulation via cutoffs, a new type of strategic behavior that is introduced by this family of iterative mechanisms and observed in the field.

Keywords: Market Design, Matching, Iterative Mechanisms, College Admissions.

JEL classification: C78, C92, D63, D78, D82

1   Introduction

When considering centralized procedures for matching prospective students to universities or colleges, a common objective that policymakers have is for the matching generated to be fair: that is, matchings in which the reason a student may not be matched to a more preferred college is that all students who are matched to that college have higher priority than her. Balinski and Sönmez [1999] showed that the Gale-Shapley student proposing deferred acceptance procedure (DA) is characterized as the "best" fair mechanism, in that it is strategy-proof and Pareto dominates any other fair mechanism (that is, it is constrained efficient). In fact, variations of the DA mechanism are used in many real-life student matching programs around the world. College and secondary school admissions in Hungary [Biró, 2012], high school admissions in Chicago [Pathak and Sönmez, 2013] and New York City [Abdulkadiroğlu et al., 2009] as well as elementary schools in Boston [Abdulkadiroglu et al., 2006] are examples of the use of the DA mechanism. Other mechanisms, such as the college proposing DA, top trading cycles, the so-called "Boston mechanism" and the "Shanghai mechanism" are used to match millions of students to schools and colleges around the world [Chen and Kesten, 2015, Abdulkadiroğlu and Sönmez, 2003, Balinski and Sönmez, 1999].

In this paper, we analyze a mechanism currently being used to match students to public universities in Brazil, denoted SISU. The SISU mechanism differs from most of the others analyzed in the literature in that it does not require students to submit rank-ordered lists over colleges, but instead provides information on the tentative requirements for acceptance at each university and asks students to choose one college among them, producing an allocation after a fixed number of periods. We show that the SISU mechanism has some undesirable theoretical properties: it fails to give reliable information about where students could be accepted, and is subject to a new type of manipulation, denoted manipulation via cutoffs. We show, based on data obtained from the selection process that took place in 2016, that the first problem is empirically relevant, that the second is feasible, and provide anecdotal evidence that manipulation via cutoffs takes place in real life.

We propose a new mechanism for matching students to colleges, denoted Iterative Deferred Acceptance Mechanism (IDAM), based on a few simple modifications of the SISU mechanism. In each step of the IDAM mechanism, the period-specific acceptance requirement, in the form of a cut-off value for each university, is made public. Students who are not tentatively assigned to a college are given the option to choose from a menu of universities where the acceptance requirement in that period is such that the student would be accepted. At the end of each period, students' choices and tentative allocations are combined for each university and as a result some students may be rejected, therefore having to make another choice in the next period. An allocation is produced after a period in which no student is rejected. If students follow the simple strategy of choosing the most preferred college among those available at each step of the IDAM mechanism (denoted the *straightforward strategy*), the matching produced as an outcome is the Student Optimal Stable Matching, that is, the matching that is the most preferred by all students among all stable matchings.

While, unlike the standard Gale-Shapley Student-Proposing Deferred Acceptance (DA) mechanism, the IDAM does not have a dominant strategy, we show that stable outcomes are

equilibrium outcomes under both imperfect and perfect information under a robust equilibrium concept. More specifically, under imperfect information about other players' preferences and exam grades, following a straightforward strategy first-order stochastically dominates any other strategy at every subgame, when other players follow the straightforward strategy. Although under some extreme scenarios the number of steps that the mechanism takes until producing the allocation may be relatively high, we also show that if the number of steps is limited and students still follow the same strategy, the number of students involved in blocking pairs falls very quickly at each step. Finally, we show that, unlike the SISU and the mechanism currently used in the province of Inner Mongolia in China, the IDAM mechanism is not manipulable via cutoffs.

Proofs absent from the main text can be found in the appendix.

1.1 Related literature

Some recent papers have evaluated non-direct iterative mechanisms for matching students to colleges or schools. Dur et al. [2015] use the fact that the school choice mechanism used in the Wake County Public School System allows for students to interact multiple times with the procedure as a method for empirically identifying strategic players. Interestingly, the dynamic nature of the procedure, and the information that is made available during the process to the participants, makes it somewhat comparable to the IDAM mechanism.

Gong and Liang [2016] consider, both theoretically and experimentally, the mechanism currently in use to match students to universities in the province of Inner Mongolia in China. When running experiments, the authors find that, when compared to DA, the Inner Mongolia mechanism exhibits higher truth-telling rates in the environment with low preference correlation, but that this does not translate into a higher rate of stable outcomes. In the high preference correlation environment, on the other hand, there is a higher proportion of stable outcomes under DA. Although the dynamic mechanism used in Gong and Liang [2016] has some similarities to the IDAM, such as the availability of tentative cut-off grades, it is in fact a different mechanism, with different timing and incentives.

Three other papers evaluate experimentally the effects of iterative matching mechanisms. Echenique et al. [2015] consider a two-sided market, with DA being implemented dynamically. The authors found that 48% of outcomes are stable and, surprisingly, that the receiving side optimal stable matching is more likely to be reached than the proposing side. Klijn et al. [2016] compare dynamic versions of both the school-proposing and student-proposing versions of DA in one-sided settings of the school choice problem. The dynamic version of the student-proposing DA that they implement is equivalent to our IDAM-NC treatment. Finally, Bo and Hakimov [2016] evaluate experimentally how DA compares to the use of the IDAM mechanism, as well as a modified version of it in which less information is given about tentative allocations. They found that although the IDAM mechanism does not have a dominant strategy, the equilibrium strategy that we present in this paper is a better predictor of behavior than DA's dominant strategy.

2 Model

A **college matching market** is a tuple $\langle S, C, q, P_S, P_C \rangle$:

1. A finite set of **students** $S = \{s_1, \ldots, s_n\}$,

2. A finite set of **colleges** $C = \{c_1, \ldots, c_m\}$,

3. A **capacity vector** $q = (q_{c_1}, \ldots, q_{c_m})$,

4. A list of strict **student preferences** $P_S = (P_{s_1}, \ldots, P_{s_n})$ over $C \cup \{s\}$[1],

5. A list of strict **college preferences** over sets of students $P_C = (P_{c_1}, \ldots, P_{c_m})$[2]

An **exam-based college matching market** consists of a college matching market where:

1. Students have **vectors of exam scores** $z = (z(s_1), \ldots, z(s_n))$, where for each $s \in S$, $z(s) = (z_{c_1}(s), \ldots, z_{c_m}(s))$, are the exam scores that student s obtained, respectivelly, at college $c_1 \ldots, c_m$. We assume that for every $s, s' \in S$ and $c \in C$, $z_c(s) = z_c(s') \implies s = s'$,

2. Colleges have **minimum necessary scores** $\underline{Z} = (\underline{z}_{c_1}, \ldots, \underline{z}_{c_m})$.

3. Colleges' preferences over sets of students are **responsive to exam scores**, that is, for all $c \in C$ and $I \subseteq S$ such that $|I| < q_c$:

   (a) For all $s, s' \in S \backslash I$, $I \cup \{s\} P_c I \cup \{s'\} \iff z_c(s) > z_c(s')$,

   (b) For all $s \in S$, $I \cup \{s\} P_c I \iff z_c(s) \geqslant \underline{z}_c$.

We can also represent an exam-based college matching market by the tuple $\langle S, C, q, P_S, P_C, \underline{Z}, z \rangle$. The preference relation P_s for student s is over the set of colleges and the option of remaining unassigned, that is, $C \cup \{s\}$. Given a strict preference relation P_s, we can also derive the corresponding weak preference relation R_s, where $c R_s c' \iff c P_s c'$ or $c = c'$. We say that student s is **acceptable** for college c if $\{s\} P_c \varnothing$. We say that college c is acceptable for student s if $c P_s s$.

A **matching** μ is a function from $C \cup S$ to subsets of $C \cup S$ such that:

- $\mu(s) \in C \cup \{s\}$ and $|\mu(s)| = 1$ for every student s[3],

- $\mu(c) \subseteq S$ and $|\mu(c)| \leqslant q_c$ for every college c,

- $\mu(s) = c$ if and only if $s \in \mu(c)$.

---

[1]Here s represents a student remaining unmatched to any college.

[2]Whenever adequate, we abuse notation in the notation of preferences over singleton sets as $s P_c s'$ instead of $\{s\} P_c \{s'\}$.

[3]We abuse notation and consider $\mu(s)$ as an element of $C \cup \{s\}$, instead of a set with an element of $C \cup \{s\}$.

Denote by $\mathcal{M}$ the set of all matchings. A **random matching** is a probability distribution over $\mathcal{M}$. A matching is **individually rational** if for every student s, $\mu(s) P_s s$ and for every college $\mu(c) P_c \varnothing$. A matching μ is **blocked** by a student s and college c if student s is acceptable to college c, $c P_s \mu(s)$ and either $|\mu(c)| < q_c$ or there is a student $s' \in \mu(c)$ where $(\mu(c) \cup \{s\}) \setminus \{s'\} P_c \mu(c)$. A matching μ is **stable** if it is individually rational and is not blocked. In this model, as opposed to some of the literature in college admissions, colleges are not considered strategic. Since the actual real-life examples in which this family of mechanisms was observed were for college admissions where the rules determining admission criteria were decided by governments, that assumption fits the applications in mind, although it makes this problem closer to the assumptions in the school choice literature.

3 The SISU mechanism

Until 2010 college admissions in Brazil were essentially decentralized, with no central mechanism matching students to the programs in universities. In 2010 the ministry of education launched a new method for matching students to university programs,[4] denoted SISU. The SISU system represented a significant change in the way in which universities admitted students. First, it unified the acceptance criteria at the universities for the seats made available through the system: instead of a different exam for each university, a unified national exam was used.[5] Second, students were free to apply to any program in any university in the country (among those available in the SISU) without any extra cost, whereas before in some cases the student would have to travel to the university premises just to be able to apply. Third, and perhaps most importantly, the centralized system could allow a student to obtain information about which university programs would accept him.

During the period 2010 to 2016, the precise rules which define the SISU mechanism were changed multiple times. The version that we will consider for analysis is the one used for the year 2010, due to its simplicity, so whenever we refer to the *SISU mechanism* we are referring to this version of it. Although later versions have different modifications, as far as we know all the problems identified in this section are also present in the later versions.

The mechanism runs for four days. During the entire day t, for $t = 1, \ldots 4$ students may each submit a choice of a single college in C. If a student makes no choice, her last choice is repeated. At the end of each day $t < 4$, for each college c:

- If the number of students who chose college c and have an exam grade at c higher than $\underline{z_c}$ at day t is smaller than q_c, let the cut-off value at c for period t, ζ_c^t, be $\zeta_c^t = \underline{z_c}$.

- Otherwise, ζ_c^t is set to be the q_c^{th} highest grade at c among those who chose c at t.

- The cut-off values $\zeta_{c_1}^t, \ldots, \zeta_{c_m}^t$ are made public.

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[4]Different from countries like the US, in Brazil a student is accepted to a specific program in a university (for example, economics at the University of Brasilia).

[5]Different universities and programs could use different weights for the various parts of the exam. For example, economics programs could give a higher weight to the math section of the exam, while biology programs could give a higher weight to the biology section.

At the end of day $t = 4$, for each college c:

- The top q_c students who have an exam grade at c higher than $\underline{z_c}$ and chose c during day c are matched to college c.

- If the number of students who have an exam grade at c higher than $\underline{z_c}$ and chose c on day 4 is lower than q_c, all of them are matched to c.

- All students who chose c and were not matched to it will remain unmatched.

- All students who did not apply to a college during all days will also remain unmatched.

Although the potential ability to know which colleges a student might not be matched to before submitting their final choice seems like an interesting property, in fact that is not the case in general, as is noted in the following remarks.

Remark 1. Choices made during days $t = 1, \ldots, 3$ may have no direct effect on the final outcome. As a result, students have no clear incentive to make choices before day 5.

Of course, if some student s makes a choice in a day $t^* < 4$ and does not make a choice on day 4, her choice on day t^* will be the one considered when generating the outcome at the end of day 4. However, the outcome would be the same if we kept other players' choices and s made her choice only on day 4. The fact that this results in no clear incentive for students to make choices before day 4 makes the information available by the end of day 3, regarding which colleges student s could be matched, to even less reliable.

Remark 2. The cut-off values at some colleges may go down from one day to the next.

Since students may choose any college on any day, nothing prevents the cut-off values at some colleges from going down from one period to the next. For example, consider a scenario in which college c has only one seat. Let student s, where $z_c(s) = 200$, be the only student to choose college c during day 3. The cut-off value for c made public at the end of day 3 is therefore $\zeta_c^4 = 200$. If s chooses a different college during day 4 and no other student chooses c, then $\zeta_c^4 = \underline{z_c}$. That is, some student s' whose grade at c is greater than $\underline{z_c}$ but lower than 200, cannot take the cut-off value at college c, even at the end of day 3, as an indication that she had no chance at being accepted there by the end of day 4.

If the cut-off values go down from one day to another, then the use of those values as information that guides students' applications away from schools at which they will not be accepted becomes jeopardized. Moreover, if the cut-off values go down at some program from day 3 to 4, a student who may have preferred to go to that program and get accepted by the end of day 4 will not do so.

Another shortcoming of the SISU mechanism is that it is subject to a new type of manipulation, denoted manipulation via cutoffs, in which groups of students may induce other students to change their behavior in a way that may benefit some of the students in that group. This is denoted manipulation via cutoffs, and is explored in more details in section 7.

4 Empirical evidence

In order to evaluate the empirical relevance of the shortcomings of the SISU mechanism identified in the previous section, we analyze data for the selection process that took place in January 2016. In that year, more than 228,000 seats in public universities were offered, and a total of more than 2,500,000 students participated. The average competition level, therefore, was of more than 10 candidates per seat.

The data consists of the cut-off values for each of the 25,686 options available to the students, for each of the four days in which students were able to make choices. In Brazilian universities, students apply and may be accepted to specific programs in those universities, as opposed to joining the university as a whole. For example, a student must choose to apply to the daytime economics program at the Federal University of Rio de Janeiro, or to the nighttime computer science program at the same university. Although all programs use a national university entrance exam, different programs may give different weights for different parts of the exam (essay, math, literature, etc) when ranking students.

In the present analysis, we are interested in whether the cut-off values decrease from one day to another and, if so, by how much. As pointed out in section 3, a decrease in the cut-off values points to a failure of the SISU mechanism in providing information on the programs to which a student has no chance of being accepted and, moreover, lead students not to choose programs that they prefer and to which they would actually end up being accepted.

Figure 4 shows the proportion of programs available for the students at which the cutoffs increased, decreased or did not change from one day to the next. Some important facts to note are:

- The proportion of programs in which the cutoffs decreased is surprisingly high, on average 8.78% of them,

- The proportion of programs in which the cutoffs decreased increased over time,

- More than 10% of the final cutoffs were lower than those informed to the students on the last day in which they made choices.

In all but five of the 25,686 programs available the cut-off value by the end of day 4 were above zero. Figure 4.2 shows the histogram of the values of the cutoffs after they increased or decreased for each day. Although we cannot say that the distributions of cutoffs which decreased and those which increased are not distinguishable, it seems clear that the decreases or increases are not clustered around different values of cutoffs.

The next question is whether the changes in cut-off grades, when they decrease, are large enough to in fact affect students' beliefs and outcomes. If a cutoff decreases by a very small amount, for example, it may well be that no student could have been negatively affected by that change, since the number of students who become able to choose that program due to that decrease is small or even zero.

The measure that we use to evaluate the degree to which a cut-off value decreases is the change in the value of the empirical cumulative distribution function (CDF), for each program, from one day to the next. For example, say that the cut-off value at program p
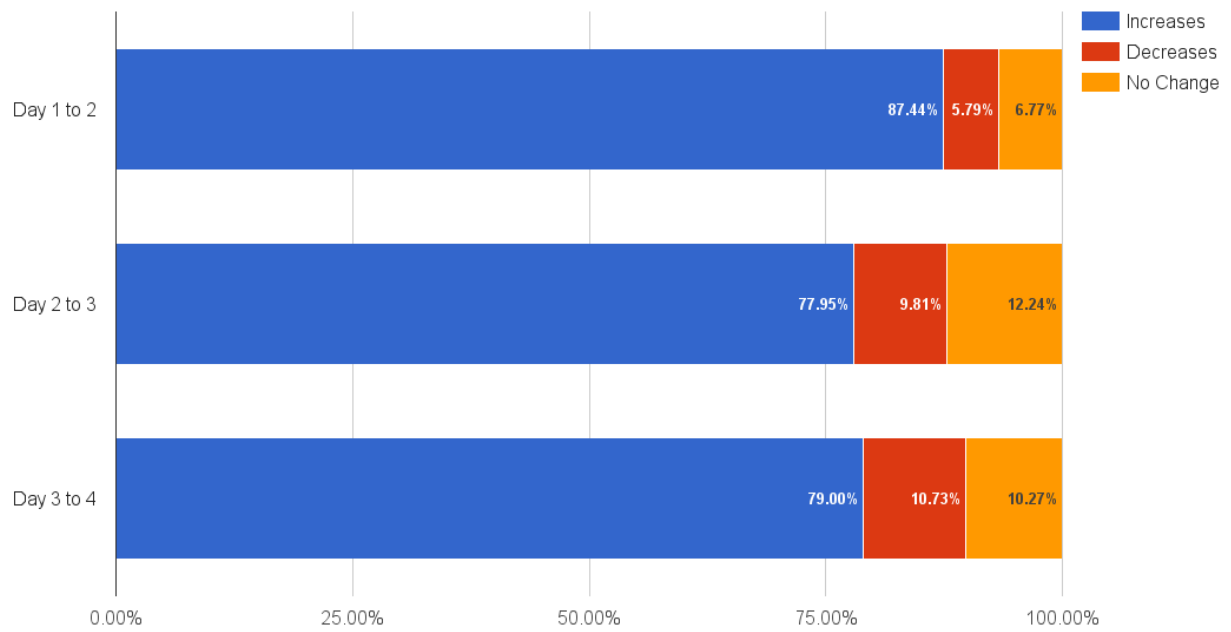
Figure 4.1: Proportion of programs at which the cut-off values increased, decreased or did not change from one day to the next
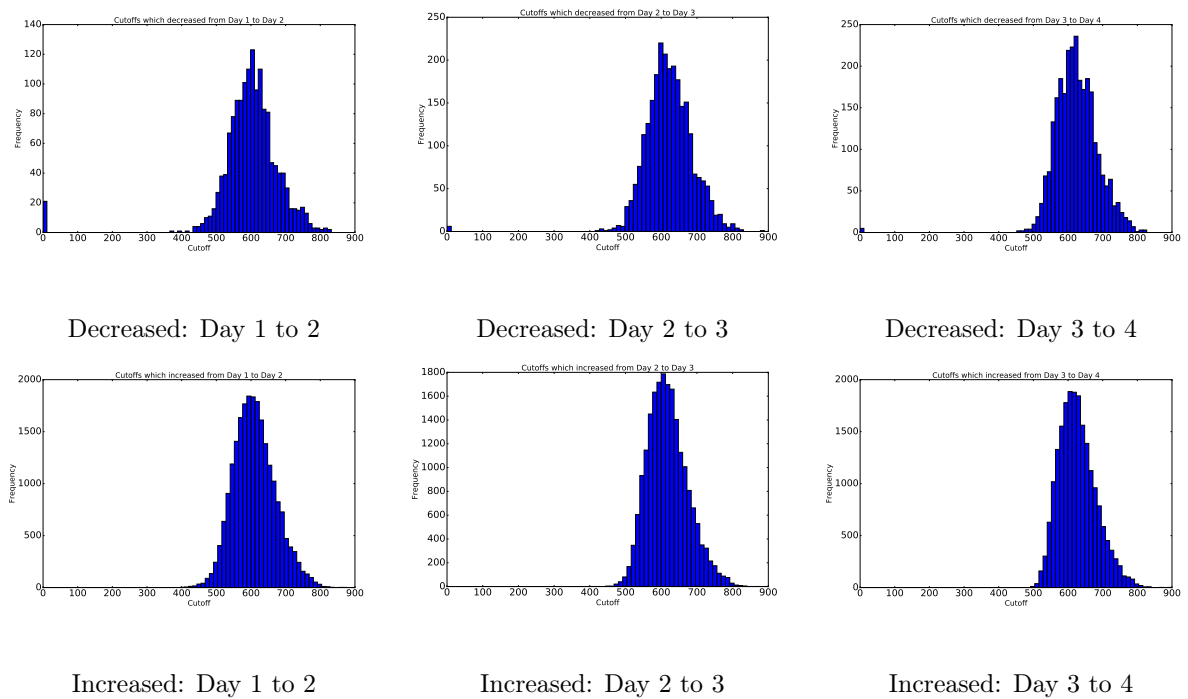
Figure 4.2: Cut-off values after they were decreased/increased from the previous day

| Day 1 to 2 | Day 2 to 3 | Day 3 to 4 |
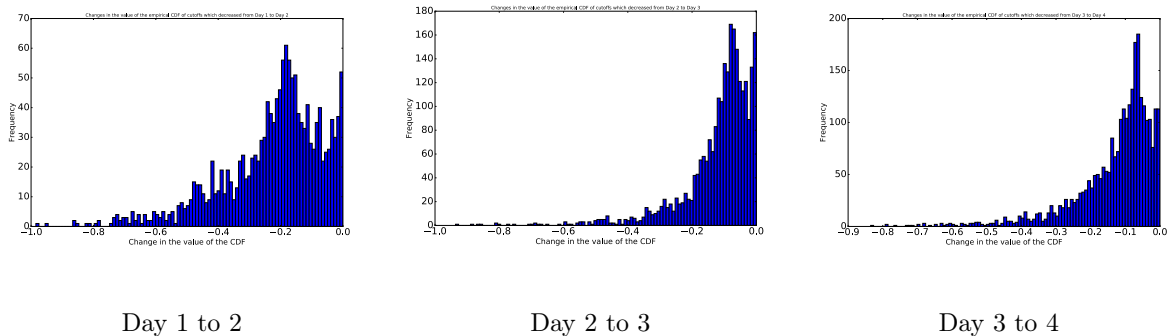
Figure 4.3: Change in the value of the empirical CDF for cut-off values that decreased

decreased from day 1 to day 2 from 550 to 500. If the value of the empirical CDF of all cut-offs on day 1, for program p. is of 0.3 and 0.2 on day 2, then that means that 30% of the cut-off values were below the one for program p on day 1, but only 20% of them were below the cut-off value of program p on day 2.

Figure 4.3 shows the frequency of the changes in the value of the empirical CDF for each pair of consecutive days.[6] For the programs that had their cut-off value reduced between these days, the graphs show that although the largest changes take place from day 1 to day 2, in all cases the proportion of large changes in the ranking is quite significant. In fact, the percentage of programs where the change in the value of the CDF was lower than -0.2 was 46.87%, 14.61%, and 19.39% for Day1/Day2, Day2/Day3 and Day3/Day4 respectively.

We can therefore conclude that the daily cut-off values which result from candidates interacting with the SISU mechanism fail to provide reliable information about the programs for which a student would not be accepted, since many of them are significantly reduced from one day to the next.

5 The Iterative Deferred Acceptance Mechanism

In this section we introduce the Iterative Deferred Acceptance Mechanism (IDAM). It essentially consists of the SISU mechanism with some important modifications, listed below. We will denote a student s as *tentatively accepted at college c by period t* if she chose college c at some period t^*, where $0 < t^* \leqslant t$ and for all t' such that $t^* \leqslant t' \leqslant t$, $\zeta_c^{t'} \leqslant z_c(s)$.

- **Commitment of choices**: Only students who are not tentatively accepted at some college in period $t-1$ are allowed to make a choice during period t. Moreover, when able to make a choice in period t, a student may only choose from colleges where the cut-off grade in period t is lower than her exam grade in that college.

- **Activity rule**: If a student s is allowed to make a choice in period t but does not, we consider that as choosing s (remaining unmatched) in that period.

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[6]All changes in the value of the CDFs were negative except for one, which had a change below 0.001 and was removed from the graphs for convenience.

- **Closing rule**: The mechanism ends after a period T in which every student is either tentatively accepted at some college or chose to remain unmatched at some previous period or when the number of periods reaches a predetermined number T^*.

Consider an exam-based college matching market $\langle S, C, q, P_S, P_C, \underline{Z}, z \rangle$ and a maximum number of steps $T^* \in \mathbb{N}$. The mechanism proceeds as follows:

- **Step** $t = 0$: Let $\mathcal{L}^0 = S$, $S^0 = \varnothing$, and for every $c \in C$, $\zeta_c^0 = \underline{z}_c$ and $\mu^0(c) = \varnothing$. Make public the values of $\zeta_{c_1}^0, \ldots, \zeta_{c_m}^0$.

- **Step** $0 < t \leqslant T^*$:

    - (a) Let $S^t \equiv \{s \in \mathcal{L}^{t-1} | \nexists c \in C : s \in \mu^{t-1}(c)\}$ and, for every $s \in S$, $\psi^t(s) \equiv \{c \in C : z_c(s) > \zeta_c^{t-1}\} \cup \{s\}$ if $s \in S^t$ and $\psi^t(s) = \varnothing$ otherwise.

    - (b) Request each student $s \in S^t$ choose an element of $\psi^t(s)$. Let $\mathcal{L}^t$ be all students in $\mathcal{L}^{t-1}$ minus those who chose s (that is, to remain unmatched) and define, for each $c \in C$, $\mathcal{L}^t(c)$ be the set of students who chose c at this step.

    - (c) For each college c, let $\mu^{t*}(c) \equiv \mu^{t-1}(c) \cup \mathcal{L}^t(c)$.

        * If $|\mu^{t*}(c)| < q_c$, let $\zeta_c^t = \zeta_c^{t-1}$ and $\mu^t(c) = \mu^{t*}(c)$.
        * If $|\mu^{t*}(c)| = q_c$, let $\zeta_c^t = \min_{s \in \mu^{t*}(c)} \{z_c(s)\}$ and $\mu^t(c) = \mu^{t*}(c)$.
        * If $|\mu^{t*}(c)| > q_c$, let $\mu^t(c)$ contain the top q_c students with respect to z_c in $\mu^{t*}(c)$, and $\zeta_c^t = \min_{s \in \mu^t(c)} \{z_c(s)\}$.

    - (d) Make the values of $\zeta_{c_1}^t, \ldots, \zeta_{c_m}^t$ public.

    - (e) If for every $c \in C$ it is the case that $\mu^{t*}(c) = \mu^t(c)$, stop the procedure.

- The function μ^t, for the highest value reached of t, is the outcome of the mechanism. Denote by T the last step executed in the procedure.

The following lemma shows that regardless of the choices made by the students when interacting with the IDAM mechanism, the cut-off values at each college never go down.

Lemma 1. *(Cut-off grades never go down) For every $0 \leqslant t \leqslant T$ and $c \in C$, $\zeta_c^t \geqslant \zeta_c^{t-1}$. Moreover, if for every $c \in C$ it is the case that $\zeta_c^{t^*+1} = \zeta_c^{t^*}$, then $T = t^* + 1$.*

One of the consequences of Lemma 1 is that the IDAM mechanism always ends in finite time. We define, formally "straightforward behavior" [Roth and Sotomayor, 1992] when interacting with the IDAM mechanism:

Definition 1. A student $s \in S$ presents **straightforward behavior with respect to** P^* when interacting with the IDAM if, whenever there is a period in which she is requested to make a choice over a set $I \subseteq C \cup \{s\}$, she chooses $c^* \in I$, such that $\forall c' \in I : c^* R^* c'$, where R^* is the weak preference derived from P^*.

Proposition 1. *If all students present straightforward behavior with respect to the preference profile P, there is a finite number of steps T for which the outcome of the IDAM mechanism is the student-optimal stable matching with respect to P.*

Proof. When all students present straightforward behavior, the steps of the IDAM mechanism are the same as the steps of the algorithm presented in Dubins and Freedman [1981] if students, each time they make a proposal, follow their preference ranking until being accepted by some college. Therefore, the outcome will be the student-optimal stable matching. □

If the maximum number of steps, T^*, is not high enough, the outcome of the IDAM mechanism may not be stable when students present straightforward behavior. As shown in the lemma below, however, in that case all blocking pairs will involve a college and an unmatched student.

Lemma 2. *Let all students present straightforward behavior with respect to the preference profile P and μ be the matching produced by the IDAM mechanism. If a student s blocks μ with some college c, then $\mu(s) = s$.*

Proof. If the IDAM mechanism is run for enough periods, Proposition 1 implies that μ is stable and therefore no student blocks μ with any college. Consider now the case in which the number of periods T^* is smaller than that, and suppose that there is a student s and a college c where $c P_s \mu(s)$, $\mu(s) = c'$ and student s and college c block μ. Since $\mu(s) = c'$, then at some period $t^* \leqslant T^*$, s chose college c'. Since s and c block μ, it must be that $\zeta_c^{T^*} < z_c(s)$. By Lemma 1, $\zeta_c^{t^*} \leqslant \zeta_c^{T^*}$. Therefore, in period t^* both colleges c and c' were available to s but she chose c'. A contradiction with straightforward behavior with respect to P. □

6 Incentives and equilibria under the IDAM mechanism

Although the outcome of the IDAM mechanism is the student-optimal stable matching when students present straightforward behavior, and differently from the GS-DA mechanism, [Dubins and Freedman, 1981, Roth, 1985], it is not the case here that students have a weakly dominant strategy in the game induced by the IDAM mechanism. In order to see this, we first need to formally define that game.

Fix a set of colleges C, with their capacities q and minimum scores $\underline{Z}$. The extensive game form G induced by the IDAM mechanism is a tuple (S, H, Φ, P, f) which consists of:

- A finite set of players $S = \{s_1, \ldots\}$.

- A finite set of actions $A = \{a_1, \ldots\}$.

- A set of finite histories H, which are sequence of actions, with the property that if $(a_i)_{i=1}^k \in H$, then for all $\ell < k$, $(a_i)_{i=1}^\ell \in H$. The null history, $h_\varnothing$ is also in H.

- At history h_0, nature draws the values of z and P from a joint distribution f, and each student s observes the realization of $z(s)$ and of P_s. The distribution f is common knowledge.

- Let Z be the set of terminal histories, that is, if $h \in Z$ where $h = (a_i)_{i=1}^k$, then there is no $h' \in H$, with $h' = (a_i')_{i=1}^\ell$ where $\ell > k$ and for all $i \leqslant k$, $a_i = a_i'$. Then $(a_i)_{i=1}^k \in Z \implies k \mod n = 0$.

- Φ is a player function. $\Phi : H \backslash Z \to S$. There exists an ordering of the players $(s_1, \ldots, s_n)$ such that, for all $h \in H$ such that $|h| \leqslant n$, $\Phi(h) = s_{|h|}$.[7]

  - Let $(a_i)_{i=1}^k \in H$, where $k \geqslant 1$. If $(a_i)_{i=1}^{k+n} \in H$, then $\Phi\left((a_i)_{i=1}^k\right) = \Phi\left((a_i)_{i=1}^{k+n}\right)$.[8]

- For each student s, $\mathcal{I}_s$ is a partition of $h : \Phi(h) = s$. Define $\zeta\left((a_i)_{i=1}^k\right)$ as the collection of lists of cutoff grades $((\zeta_c^0)_{c \in C}, (\zeta_c^1)_{c \in C}, \ldots)$ that result from the sequence of actions in $(a_i)_{i=1}^{k-(k \mod n)}$. Define $H_\ell^t \equiv \left\{(a_i)_{i=1}^k \in H : k \mod n = \ell \text{ and } k \div n = t-1\right\}$[9], and let $h, h' \in H_\ell^t$. The histories $h = (a_i)_{i=1}^k$ and $h' = (a_i')_{i=1}^k$ belong to the same member of the partition $\mathcal{I}_{s_\ell}$ if and only if:[10]

  - $|h| \mod n = |h'| \mod n$,
  - $\zeta(h) = \zeta(h')$,
  - $z(s_\ell|h) = z(s_\ell|h')$, that is, the realization of student s_ℓ's grades at the colleges are the same.
  - $a_i = a_i'$ for all i such that $i \mod n = \ell$ [11].

- $A(h)$ are the actions available at $h \in H$. For every $h_i \in H_\ell^t$, the set of actions depend on whether, given the history of actions until step t of the IDAM mechanism, student $s = \Phi(h_i)$ is offered a set of colleges to choose from, in which case $A(h_i) = \psi^t(s)$, or not, in which case we denote $A(h_i) = \{\Diamond\}$, where $\Diamond$ is simply a placeholder for an action when no action is requested from the student. We abuse notation and denote, for any $I_i \in \mathcal{I}_s$, $A(I_i)$ to be $A(h_i)$ for any $h_i \in I_i$ (remember that by definition all histories in I_i have the same set of actions associated with them).

- A (pure) strategy for player s is a function $\sigma_s(\cdot)$ that assigns an action in $A(I_i)$ to each information set $I_i \in \mathcal{I}_s$.

- The outcome function O assigns, to each strategy profile $\sigma = (\sigma_{s_1}, \ldots, \sigma_{s_n})$, a random matching that results from following the histories that result from following those strategies in the IDAM mechanism, given each realization of z and P.

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[7]That is, the first n actions consist of player s_1 playing first, s_2 second, and etc.

[8]This, combined with the previous item and the condition on terminal histories, implies that every player plays every n actions once.

[9]That is, H_ℓ^t are all histories that student s_ℓ could reach after t steps of the mechanism.

[10]Notice that this game form has perfect recall.

[11]That is, two histories belong to the same member of the partition if the student's grades at the colleges are the same, the history of cutoffs was the same, and the actions taken by that player were the same. That is, if player s_ℓ has the same grades and the same experience in both histories.

Since our solution concept will demand students' strategies to be rational at all possible information sets, we will need to consider how students' strategies act at each subgame. We first define a subgame:

Definition 2. A **subgame** of the game $G = (S, H, \Phi, P)$ at non-terminal history $h = (a_i)_{i=1}^k$, for $h \in H \backslash Z$ is a game $G|_h = (S|_h, H|_h, \Phi|_h, P|_h)$ where:

- $H|_h = \left\{ h' = (a_i')_{i=k}^l \text{ where } l \geqslant k \text{ and } (a_1, \ldots, a_{k-1}, a_k', \ldots, a_l') \in H \right\}$

- $S|_h = \{ s \in S : \Phi(h') = s \text{ for some } h' \in H|_h \backslash Z \}$

- $\Phi|_h : H|_h \to S|_h$ such that for all $h' \in H|_h$, where $h' = (a_i')_{i=k}^l$, $\Phi|_h(h') = \Phi(a_1, \ldots, a_{k-1}, a_k', \ldots, a_l')$

- For each $s \in S|_h$, $P_s|_h$ satisfies, for all $h', h'' \in H|_h$, $h' P_s|_h h'' \iff (a_1, \ldots, a_{k-1}, a_k', \ldots, a_l') P_s (a_1, \ldots$ The weak preference $R_s|_h$ is defined accordingly.

Finally, let $\sigma|_h = (\sigma_{s_1}|_h, \ldots, \sigma_{s_n}|_h)$ be the strategy profile σ restricted to the subgame $G|_h$. We can define analogously a subgame in terms of an information set instead of a single history. We will consider situations in which students present straightforward behavior. Therefore, we can define a straightforward strategy accordingly:

Definition 3. A strategy σ_s of student $s \in S$ is **straightforward with respect to** P^* if for every t and $h_s^t \in H_s^t$:

$$\begin{cases} \sigma_s(h_s^t | z(s), P^*) = \max_{P*}(A(h_s^t)) & \text{if } A(h_s^t) \neq \{\Diamond\} \\ \sigma_s(h_s^t | z(s), P^*) = \Diamond & \text{otherwise} \end{cases}$$

The first question that we make is whether a student has a dominant strategy at the game induced by the IDAM mechanism. This is a natural question, since the mechanism itself resembles the deferred acceptance procedure and truth-telling is a weakly dominant strategy under that direct mechanism. As we show below, that is not the case under the IDAM mechanism.

Proposition 2. *A student may not have a weakly dominant strategy under the IDAM mechanism*

The reason why not following a straightforward strategy may be profitable is that, in contrast to the case with the deferred acceptance direct mechanism, an agent may influence others' actions by modifying the signals received by the other agents, in the form of different cut-off grades or rejections. So if, for example, a student has a strategy that depends in some way on the cut-off grades then that fact could be exploited.

One interesting property of the IDAM mechanism, which is the driver of many of the theoretical results that will follow, is that although the combination of strategies that students may use is much richer than that of straightforward strategies, the sequence of interactions that the students have with the mechanism cannot be distinguished from interactions that result from all students following straightforward strategies.

Lemma 3. *Fix a realization of P and z and let σ be a strategy profile and h a history that results from that strategy profile. There is at least one strategy profile σ^*, where every student follows a straightforward strategy with respect to some preference profile P^*, which also results in history h.*

The result in Lemma 3 does not hold for the SISU mechanism, however.

Remark 3. There are sequences of actions that students may take under the SISU mechanism that cannot be produced by any profile of straightforward strategies.

To see why Remark 3 is true, consider a student who is the most preferred student by colleges c_1 and c_2, and in period 1 chooses college c_1, in period 2 chooses c_2, and in period 3 chooses c_1 again. This sequence of actions is not possible under the IDAM mechanism, cannot be the result of a straightforward strategy (since in all periods both colleges are available to her) and can take place under the SISU mechanism.

Although not having a dominant strategy can be seen as an undesirable characteristic of the IDAM mechanism, when compared to the property of strategy-proofness, we will now show that students following straightforward strategies is a robust equilibrium. First, we define our equilibrium concept.

Let A and B be two random matchings. We denote by $\succ_s$ the first-order stochastic dominance relation under P_s. That is, $A \succ_s B$ if for all $v \in C \cup \{s\}$, $Pr\{A(s) = v'|v'R_sv\} \geq Pr\{B(s) = v'|v'R_sv\}$.

Definition 4. A strategy profile σ is an **ordinal perfect bayesian equilibrium (OPBE)** of a game $G = (S, H, \Phi, P, f)$ if for all $I_i \in \mathcal{I}_s$, every $s \in S|_{I_i}$, every assessment μ over $\mathcal{I}_s$ and strategy $\sigma'_s|_{I_i}$ for player s in the subgame $G|_{I_i}$:

$$O_\mu\left(\sigma_s|_h, \sigma_{-s}|_h\right) \succ_s O_\mu\left(\sigma'_s|_h, \sigma_{-s}|_h\right)$$

The theorem below shows that students following straightforward strategies is an equilibrium.

Theorem 1. *Let σ^* be the strategy profile in which all strategies are straightforward. Then σ^* is an OPBE of the game induced by the IDAM mechanism.*

It is not the case, however, that every OPBE consists of every student following a straightforward strategy, as shown in the example below.

Example 1. Consider the following exam-based college matching market:[12]

$$
\begin{array}{ll}
S = \{s_1, s_2, s_3, s_4\} & C = \{c_1, c_2, c_3, c_4\}, \ q_i = 1 \\
\quad P_{s_1} : c_1 \ c_4 & \quad P_{c_1} : s_4 \ s_1 \ s_2 \ s_3 \\
\quad P_{s_2} : c_1 \ c_2 & \quad P_{c_2} : s_2 \ s_3 \ s_4 \\
\quad P_{s_3} : c_2 \ c_3 & \quad P_{c_3} : s_3 \ s_4 \ s_2 \\
\quad P_{s_4} : c_3 \ c_1 & \quad P_{c_4} : s_1 \ s_2 \ s_3 \ s_4
\end{array}
$$

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[12]The example is based on a market due to Kesten [2010].

Notice first that if all students follow the straightforward strategy, the outcome will be the matching μ as follows:

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_4 & s_2 & s_3 & s_1 \end{pmatrix}$$

Let students have perfect beliefs (that is, beliefs are degenerate in the true values). In this case, SPNE and OPBE are equivalent concepts. Let students s_2, s_3, s_4 follow the straightforward strategy and s_1 follows the strategy below:

1. In the first period, choose c_4.

2. In the following periods, follow the straightforward strategy.

The outcome of that strategy profile is μ', as follows:

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_2 & s_3 & s_4 & s_1 \end{pmatrix}$$

By Theorem 1, the strategy profile of all players following the straightforward strategy is an OPBE of the subgames that follow the first period. Now consider the first period. Since student s_1 is not acceptable at colleges c_2 and c_3, it is easy to see that any such deviation would lead to the outcome μ, which yields the same outcome for s_1 as following the proposed strategy. Moreover, any deviating strategy that consists of choosing c_1 in the first step will, at best, also lead to student s_1 being matched to c_4. This strategy profile is, therefore, an OPBE.

We proceed below with some further results, in which we consider the Nash equilibria of the game induced by the IDAM mechanism.

Proposition 3. *Every stable matching is a Nash equilibrium outcome of the game induced by the IDAM mechanism.*

Proof. Let μ be a stable matching. Make every student's strategy apply in the first period to their match under μ, and not apply anywhere else if they are rejected afterwards. That is an equilibrium. $\square$

Proposition 4. *Some Nash equilibrium outcomes are not stable.*

Proof. The example is based on a non-credible threat outside of the equilibrium path:

$$S = \{s_0, s_1, s_2, s_3\} \qquad C = \{c_1, c_2, c_3, c_4\}, \ q_i = 1$$

$$
\begin{array}{ll}
P_{s_0} : \boxed{c_1}\ c_2\ c_3\ c_4 & P_{c_1} : s_0\ s_1\ s_2\ s_3 \\
P_{s_1} : c_1\ \boxed{c_4}\ c_2\ c_3 & P_{c_2} : s_0\ s_1\ s_3\ s_2 \\
P_{s_2} : c_3\ \boxed{c_2}\ c_1\ c_4 & P_{c_3} : s_0\ s_1\ s_2\ s_3 \\
P_{s_3} : c_2\ \boxed{c_3}\ c_1\ c_4 & P_{c_4} : s_0\ s_1\ s_2\ s_3
\end{array}
$$

Consider the following matching:

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$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_0 & s_2 & s_3 & s_1 \end{pmatrix}$$

The matching μ is not stable, since (s_2, c_3) and (s_3, c_2) are blocking pairs. This outcome, however, is supported by the following strategy profile:

- σ_{s_0}: Apply to c_1 in step 1. If rejected, quit.

- σ_{s_1}: Apply to c_1 in step 1. If rejected:

    - and $z(c_3) = z(s_2)$ or $z(c_2) = 0$, apply to c_3. If then rejected, quit.
    - and $z(c_2) = z(s_3)$ or $z(c_3) = 0$, apply to c_2. If then rejected, quit.
    - otherwise, apply to c_4. If then rejected, quit.

- σ_{s_2}: Apply to c_2 in step 1. If then rejected, quit.

- σ_{s_3}: Apply to c_3 in step 1. If then rejected, quit.

Student s_0 gets her top choice, so she would not deviate. Given student s_0's strategy and the fact that she has top priority in c_1, student s_1 would not be able to be matched to c_1 and therefore has no profitable deviation. Consider now student s_2. Any profitable deviation strategy must apply to some school at step 1, otherwise she will remain unmatched. Moreover, any strategy that starts applying to c_2 will not change her outcome. We must then check all other possibilities:

- Apply to c_1 in the first step. Then s_2 is rejected from c_1 at step 1. Since $z(c_2) = 0$, student s_1 will then apply to c_3 and will remain matched there. Therefore, the only remaining options for s_2 would be to quit or to apply to c_2 or c_4. In both cases there is no improvement over μ.

- Apply to c_3 in the first step. Then s_2 is tentatively accepted at c_3. Since $z(c_3) = z(s_2)$, however, in step 2 student s_1 will apply to c_3, leading to the rejection of s_2. Again, the only remaining options for s_2 would be to quit or to apply to c_2 or c_4. In both cases there is no improvement over μ.

- Apply to c_4 in the first step. Since $z(c_2) = 0$, student s_1 will then apply to c_3 and will remain matched there. Student s_2 will not be rejected from c_4 and will therefore remain matched there. Since $c_2 P_{s_2} c_4$, that is not a profitable deviation.

The same analysis for s_3 would show that she also has no profitable deviation, and therefore μ is an equilibrium outcome for the game induced by the iterative mechanism. $\square$

One important fact to notice is that the schools' priorities in the example used above have an *Ergin-acyclic priority structure*. Haeringer and Klijn [2009] show that when the priority structure is Ergin-acyclic, the set of outcomes of the game induced by the SPDA mechanism equals the set of stable matchings. We can therefore conclude the corollary below.

Corollary 1. *The set of Nash equilibrium outcomes for the IDAM mechanism is not equal to the set of equilibrium outcomes for the SPDA.*

7 Manipulations via cutoffs

Other than the fact that under the SISU mechanism the cut-off values do not represent reliable information regarding the chances a student has of being accepted into a college, that mechanism is also subject to what we denote by *manipulation via cutoffs*. A manipulation via cutoffs occurs when a group of students artificially increase the cut-off values of some college, as a way of preventing applications from other students, and then in the last period vacate those seats so that students with a lower exam grade then take their place. The example below shows how manipulations via cutoffs can happen.

Example 2 (Manipulation via cutoffs). Consider the set of students $S = \{s_0, s_1, s_2, s_3\}$ and of colleges $C = \{c_1, c_2, c_3\}$, each with capacity $q_i = 1$ and minimum score zero. Students' preferences are as follows:

$$P_{s_0} : c_1 \ c_2 \ c_3$$
$$P_{s_1} : c_1 \ c_2 \ c_3$$
$$P_{s_2} : c_1 \ c_2 \ c_3$$
$$P_{s_3} : c_2 \ c_1 \ c_3$$

Students' exam grades at the colleges are as follows:

	c_1	c_2	c_3
s_0	100	100	100
s_1	200	200	200
s_2	300	300	300
s_3	400	400	400

Suppose that the SISU mechanism is going to be used, and students present straightforward behavior. The cut-off values, at the end of each period would then be as follows (remember that the cutoffs at $t = 4$ represent the final allocation cutoffs):

	c_1	c_2	c_3
$t = 1$	300	400	0
$t = 2, 3, 4$	300	400	200

The matching produced, therefore, will be μ:

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & \varnothing \\ s_2 & s_3 & s_1 & s_0 \end{pmatrix}$$

Suppose, however, that students s_0 and s_3 modify their behavior, and act instead as follows:

- During $t = 1, 2, 3$, student s_0 chooses college c_3 and student s_3 chooses college c_1.

- In period $t = 4$, student s_0 chooses college c_1 and student s_3 chooses college c_2.

Assuming that the other students present straightforward behavior, the cut-off values at the end of each period would be as follows:

	c_1	c_2	c_3
$t = 1$	400	$---$	100
$t = 2$	400	300	100
$t = 3$	400	300	200
$t = 4$	100	400	200

The matching produced will be μ':

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & \varnothing \\ s_0 & s_3 & s_1 & s_2 \end{pmatrix}$$

Student s_0 is significantly better off under μ' than under μ, while s_3 is matched to the same college in both cases.

Manipulations via cutoffs consists, in other words, of a set of students S^H "holding" seats in colleges and "releasing" them so that a set of students S^T can take them in the last period. In order for these types of manipulations to be successful, some conditions need to be satisfied.

First of all, the set of students S^H needs to be large enough when compared to the capacity of the college, and their exam grades in that college must be high enough. If the number of students in S^H is low when compared to the capacity of the college, the effect of them choosing that college in the value of the cutoff will be much less noticeable. To see that, consider the case in which, at a certain period, there are 100 students choosing college c, which has a capacity of 10 students, and for simplicity assume that those students' scores fill the range $\{1, 2, \ldots, 100\}$ (that is, one student has a score 1, one has a score 2, etc). Then, given those choices, the students who will be tentatively accepted will be those with scores 91 to 100, and therefore the cutoff value will be 91. Suppose that S^H has five students, with exam grades $\{300, 301, 302, 303, 304\}$. These are, of course, significantly higher than the other students'. If all of them choose college c in addition to the 100 students, all of them will be tentatively accepted in that period, but the change in the cut-off value will not be as significant: it will change from 91 to 96. If the capacity of the college was five, the change in the cutoff would be, instead, from 96 to 300. It is not necessarily the case that the number of students in S^H has to be equal to the college's capacity for the change in cutoff to be significant. Consider the case in which the exam scores of the 100 students choosing c are, instead, $\{252, 251, 250, 100, 99, 98, \ldots 4\}$, and the capacity is still five. The cut-off value for college c would be 99 in that period. If S^H has only two students, with exam grades $\{300, 301\}$, them choosing c would lead the cut-off grade at c to change from 99 to 250, instead.

Second, the other students have to respond in a straightforward way to the cut-off values in the last period. This can be considered a reasonably mild requirement. It does not require that the other students follow a straightforward strategy in all periods, but only that they do not choose, in the last period, a college where the cut-off value is above their grade in that college.

One may wonder how realistic the first condition is. After all, colleges typically accept hundreds or thousands of students every year, and a coalition of hundreds of high-achieving students performing these potentially risky manipulations does not seem realistic. In many

countries (including Brazil and China), however, students apply directly to specific programs in the universities, so even though the universities as a whole accept hundreds or thousands of students, the number of seats at each program is often below 100, and many times lower than 30 or 20. Moreover, even those seats are often subdivided. In China, the seats in each program are partitioned between seats reserved for candidates from specific provinces. In Brazil, federal universities partition the seats in the programs into five sets of seats, reserved for different combinations of ethnic and income characteristics. Finally, universities sometimes offer only a subset of the total number of seats in a program through the centralized matching process. In fact, *the median number of seats offered in each option available during the January 2016 selection process in Brazil, where more than 228,000 seats in public universities were offered, was five.*

There is evidence that this type of manipulation takes place in real life. In the Chinese province of Inner Mongolia, a mechanism which has some similarities to the SISU mechanism is used to match students to programs in universities. While the mechanism itself has significant differences, it is also vulnerable to manipulation via cutoffs. This fact seems to be exploited by students, as documented by China News:[13]

> (...) in fact, since 2008, the clearinghouse found that some high scored students applied to a college with lower cutoff score. For example, their score allows them to go to PKU or Tshinghua, but they chose Beijing Polytech first. On the other hand, some other students, from the same high school often, applied to college that their score would not allow them to go initially (...) [the] system shows that their rank is below the capacity — so they can't be admitted under usual terms — however they do not revise their choices.

Even more remarkably, there seems to be evidence that high schools are coordinating students' actions:

> (...) the clearing house noticed that, 2 or 3 min before the deadline, the ranking of students in the system is changing – this is the evidence that high schools are organizing their own high scored students to occupy seats for low scored students

Contrary to the SISU mechanism, the IDAM does not have this characteristic:

Remark 4. The IDAM mechanism is not manipulable via cutoffs.

It is easy to see why that is the case. In order for manipulations via cutoff to work, it is necessary for cut-off values to increase before the final allocation is determined, and for the final cut-off values (that is, the allocation cutoffs) to be lower. By Lemma 1, this cannot happen under the IDAM mechanism.

8 Convergence speed and stability

In this section we consider two related questions. As we saw in the description of the IDAM mechanism, the number of steps until it reaches the Student Optimal Stable Matching when

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[13]Source (in Chinese): http://edu.people.cn/n/2014/0904/c1053-25604075.html

students follow the straightforward strategy depends on preferences and exam grades. One question then is how many steps does it take for that result to be produced?

A second question is how "far" from a stable matching will the outcome be if the IDAM mechanism is run for a number of periods smaller than that necessary to produce the Student Optimal Stable Matching but students still follow the straightforward strategy? The measure of distance from a stable matching that we use is the number of individuals involved in blocking pairs.

For the results below, we consider exam-based college matching markets where the set of students and colleges can be partitioned as $S = \{S^1 \cup S^2 \cup \cdots \cup S^k\}$ and $C = \{C^1 \cup C^2 \cup \cdots \cup C^k\}$, where $\sum_{c \in C^i} q_c \leqslant |S^i|$ and colleges at C^i prefer students at S^i to those not in S^i. This is consistent with situations in which college exams have math and literature sections and students are good at either math or literature. Notice, however, that when $k = 1$ this definition accommodates any market in which the number of seats in colleges does not exceed the number of students. We also assume in this section that students follow the straightforward strategy.

Proposition 5. *If for every $i \in \{1, \ldots, k\}$, $c, c' \in C^i$ and $s, s' \in S^i$ it is the case that $sP_c s' \iff sP_{c'} s'$ and moreover for all $s \in S^i$, $c \in C^i$ and $c' \notin C^i$ it is the case that $cP_s c'$, then:*

1. *The maximum number of steps until stability is $\max_i \{|C^i|\}$.*

2. *If the IDAM mechanism runs for $T < \max_i \{|C^i|\}$ steps, the maximum number of individuals involved in blocking pairs is $n - \sum_{j=1}^{k} \sum_{i=1}^{T} q_i^j$, where for each j, $q_1^j \leqslant q_2^j \leqslant \cdots \leqslant q_{|C^j|}^j$ is the ordering of the capacities of the schools in C^j.*

The configuration of preferences used in Proposition 5 is consistent with scenarios in which the top preferences are mutually partitioned between students and colleges, and colleges share the selection criteria among their top students. One example would be a college admissions program that is based on national exams consisting of questions on different subjects and college programs that rank the students based on their grades in those different subjects. The stronger assumption in this case is that the partition is such that students are among the best at only one of the subjects. For example, if the partitioning of college programs is between medical sciences, STEM and humanities, a student who is among the top at humanities is not at STEM or medical subjects.

For the case of common preferences between all colleges, the result does not have to rely on some assumption on students' strategies.

Corollary 2. *When priorities are common across colleges and the IDAM mechanism runs for $T < m$ steps, the maximum number of individuals involved in blocking pairs is $n - \sum_{i=1}^{T} q_i$, where the capacities of the colleges in C are reordered such that $q_1 \leqslant q_2 \leqslant \cdots \leqslant q_m$.*

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A Appendix

A.1 Proofs

Lemma 1

Proof. We prove by induction. Let $t = 1$. From step $t = 0$, for every $c \in C$, $\zeta_c^0 = \underline{z}_c$. For each college $c \in C$, we have three cases:

- $|\mu^{1*}(c)| < q_c$, in which case $\zeta_c^1 = \zeta_c^0$

- $|\mu^{1*}(c)| = q_c$. Since $\mu^0(c) = \varnothing$, $\mathcal{L}^1(c)$ consists of the set of students who chose college c at step $t = 1$. Given the definition of $\psi^1(s)$, for every student $s \in \mathcal{L}^1(c)$, $z_c(s) > \zeta_c^0$. Therefore, $\zeta_c^1 = \min_{s \in \mu^{1*}(c)} \{z_c(s)\} > \zeta_c^0$.

- $|\mu^{1*}(c)| > q_c$. Since every student in $\mu^1(c)$ is also in $\mathcal{L}^1(c)$, $\zeta_c^1 = \min_{s \in \mu^1(c)} \{z_c(s)\} > \zeta_c^0$.

Now assume that for every $t = 0, \ldots, k$, where $k < T$ and for every $c \in C$, it is the case that $\zeta_c^t \geqslant \zeta_c^{t-1}$ and consider the step $k + 1$. For each college $c \in C$, we have three cases:

- $\left|\mu^{k+1*}(c)\right| < q_c$, in which case $\zeta_c^{k+1} = \zeta_c^k = \cdots = \zeta_c^{0}$ [14].

- $\left|\mu^{k+1*}(c)\right| = q_c$. We have two cases. If $\left|\mu^k(c)\right| < q_c$, then $\zeta_c^k = \zeta_c^0$. Note that since $\left|\mu^{k+1*}(c)\right| = q_c$, $\left|\mathcal{L}^{k+1}(c)\right| > 0$. By definition, for every student $s \in \mathcal{L}^{k+1}(c)$, $z_c(s) > \zeta_c^k = \zeta_c^0$. Since $\mu^{k+1}(c) = \mu^k(c) \cup \mathcal{L}^{k+1}(c)$, $\min_{s \in \mu^{k+1}(c)} \{z_c(s)\} \geqslant \min_{s \in \mathcal{L}^{k+1}(c)} \{z_c(s)\} > \zeta_c^0$. Therefore $\zeta_c^{k+1} > \zeta_c^0 = \zeta_c^k$. If, on the other hand, $\left|\mu^k(c)\right| = q_c$, since $\left|\mu^{k+1*}(c)\right| = q_c$ it must be the case that $\left|\mathcal{L}^{k+1}(c)\right| = 0$, and therefore $\min_{s \in \mu^{k+1}(c)} \{z_c(s)\} = \min_{s \in \mu^k(c)} \{z_c(s)\} = \zeta_c^k$, and therefore $\zeta_c^{k+1} = \zeta_c^k$.

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[14] Notice that $\left|\mu^{k+1*}(c)\right| < q_c$ together with the fact that students who are tentatively matched cannot change their submission implies that the cut-off grade will only be increased from ζ_c^0 once the number of tentatively accepted students reaches q_c.

- $\left|\mu^{k+1*}(c)\right| > q_c$. Since $\left|\mu^{k+1*}(c)\right| > q_c$ and $\mu^{k+1*}(c) \equiv \mu^k(c) \cup \mathcal{L}^{k+1}(c)$, $\left|\mathcal{L}^{k+1}(c)\right| > 0$. If $\left|\mu^k(c)\right| < q_c$, $\zeta_c^k = \zeta_c^0$ and since for every student $s \in \mathcal{L}^{k+1}(c)$, $z_c(s) > \zeta_c^k = \zeta_c^0$, $\zeta_c^{k+1} = \min_{s \in \mu^{k+1}(c)}\{z_c(s)\} > \zeta_c^0$. Otherwise if $\left|\mu^k(c)\right| \geqslant q_c$, $\zeta_c^k = \min_{s \in \mu^k(c)}\{z_c(s)\}$. That is, there are q_c students in $\mu^k(c)$ with exam grade at c greater than or equal to ζ_c^k. Moreover, by definition, for every $s \in \mathcal{L}^{k+1}(c)$, $z_c(s) > \zeta_c^k$. That is, there is at least one student in $\mathcal{L}^{k+1}(c)$ and all those students have an exam grade at c higher than the student in $\mu^k(c)$ who has the lowest exam grade at that college. Therefore, in $\mu^k(c) \cup \mathcal{L}^{k+1}(c)$ there are at least q_c students with exam grade at c strictly greater than ζ_c^k, and as a consequence the q_c^{th} highest exam grade in $\mu^{k+1*}(c)$ is strictly greater than ζ_c^k. Therefore, $\zeta_c^{k+1} = \min_{s \in \mu^{k+1}(c)}\{z_c(s)\} > \zeta_c^k$.

Now, for the second statement in the lemma, fix a $t \geqslant 0$ and suppose that for every $c \in C$ it is the case that $\zeta_c^{t+1} = \zeta_c^t$. We can use the two parts of the proof by induction above to conclude that there are two scenarios which are compatible with that assumption:

- $t = 0$ and for all $c \in C$, $\left|\mu^{1*}(c)\right| < q_c$. In this case, the definition of step 1(c) establishes that, for each c, $\mu^1(c) = \mu^{1*}(c)$. But then step 1(d) implies that the procedure will stop at step $t + 1$.

- $t > 0$ and for every $c \in C$, either $\left|\mu^{t+1*}(c)\right| < q_c$ or $\left|\mu^{t+1*}(c)\right| = q_c$ and $\mathcal{L}^{t+1}(c) = \varnothing$. In both cases, step $t+1$(c) implies that $\mu^{t+1}(c) = \mu^{t+1*}(c)$. Step $t+1$(d) then implies that the procedure will stop at step $t + 1$.

$\square$

Lemma 2

Proof. Consider some history $h \in H$. Given other players' strategies σ_{-s}, the history that results from the strategy profile (σ_s, σ_{-s}) consists, as described in the definition of the IDAM mechanism, of a series of periods in which each student has either only the action $\Diamond$ or some menu of options $\psi^t(s)$ to choose from. Therefore, given our strategy profile and student s, we can write down a list of pairs of menus given to student s and her choice.

For example, suppose that the set of colleges is $C = \{c_1, c_2, c_3, c_4\}$. A possible list could be the following:

$$\left(\left(\{c_1, c_2, c_3, c_4, s\}, c_2\right)_{t=1}, \left(\varnothing, \Diamond\right)_{t=2}, \left(\{c_1, c_3, s\}, c_3\right)_{t=3}, \left(\varnothing, \Diamond\right)_{t=4=T}\right)$$

That is, in the first step the student was offered the entire list of colleges and chose c_2. In the second step, she was not offered a menu and therefore performed the continuation action $\Diamond$. In the third step, the student was offered colleges c_1 and c_3. Finally, during the fourth and final step, no menu was offered. Notice that, even if we do not know the strategy that was followed by the student, it would be precisely the sequence of actions taken by a student following a straightforward strategy for the preferences $c_2 P_s c_4 P_s c_3 P_s c_1 P_s s$. In fact, there is a class of preferences that are consistent with that list of pairs.

In general, say that the sequence of menus offered and actions chosen for a student s up to history h are as follows:

$$\left(\left(\psi^1, a^1\right), \left(\psi^2, a^2\right), \ldots, \left(\psi^t, a^t\right)\right)$$

For simplicity, and without any loss of generality, assume that the sequence above has removed from the list the pairs $(\varnothing, \diamondsuit)$. Because of Lemma 1 and the definition of $\psi^t(s)$ in the description of the IDAM mechanism, if $a^t = c$, for all $t' > t$, $c \notin \psi^{t'}$, and therefore $a^i = a^j \implies i = j$, that is, there is no repetition of choices in a^i, $i = 1, \ldots, t$. Denote $\psi^i_- \equiv \psi^i \backslash \bigcup_{j=i}^t a^j$. We will show that this sequence could have been generated by a straightforward strategy of a student with a preference relation in the following class of preferences:[15]

$$S \backslash \psi^1 \ R_s^* \ a^1 \ P_s^* \ \psi^1_- \backslash \psi^2_- \ R_s^* \ a^2 \ P_s^* \ \psi^2_- \backslash \psi^3_- \ R_s^* \cdots R_s^* \ a^t \ P_s^* \ \psi^t_-$$

The notation above includes a class of strict preferences because some of its elements consists of sets of colleges. Any strict preference derived from some ordering over the elements of each of those sets belongs to the class of preferences that we are referring to. We will refer by P_s^* to some arbitrary element of those preferences. It is easy to see that each preference in that class is complete over the set of colleges and that no college appears more than once, since $\psi^t_- \subsetneq \psi^{t-1}_- \subsetneq \cdots \subsetneq \psi^1_- \subsetneq S$ and $a^i \notin \psi^j_-$ for all i, j.

Now, take some of the menus that were offered, ψ^i. We will now show that for all $a \in \psi^i$ where $a \neq a^i$, $a^i P_s^* a$. For that, it suffices to show that:

$$a \in \bigcup_{j=i+1}^t a_j \cup \bigcup_{j=i}^{t-1} \psi^j_- \backslash \psi^{j+1}_- \cup \psi^t_-$$

That is, we will show that a must be at some element to the right of a^i in the definition of P_s^*. Since $a \neq a^i$, this is equivalent to:

$$a \in \bigcup_{j=i}^t a_j \cup \bigcup_{j=i}^{t-1} \psi^j_- \backslash \psi^{j+1}_- \cup \psi^t_-$$

Since we defined $\psi^i_- \equiv \psi^i \backslash \bigcup_{j=i}^t a^j$, we can rewrite the condition as:

$$a \in \underbrace{\psi^i \backslash \psi^i_-}_{(i)} \cup \underbrace{\bigcup_{j=i}^{t-1} \psi^j_- \backslash \psi^{j+1}_-}_{(ii)} \cup \underbrace{\psi^t_-}_{(iii)}$$

Suppose not. Then a cannot be in (i), (ii) and (iii). By (i), it must be that $a \notin \psi^i \backslash \psi^i_-$. Since $a \in \psi^i$, that implies $a \in \psi^i_-$. By (ii), since $a \notin \psi^i_- \backslash \psi^{i+1}_-$, it must then be that $a \in \psi^{i+1}_-$. This reasoning can be repeated until finding that it must be that $a \in \psi^t_-$. But that is (iii), which leads to a contradiction.

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[15]Note that this class of preferences does not necessarily include all the preferences that are compatible with the choices made.

We therefore have that given σ_{-s}, the sequence $((\psi^1, a^1), (\psi^2, a^2), \dots, (\psi^t, a^t))$ is consistent with student s having a preference over colleges P_s^* and following a straightforward strategy up to step t, since for all $a \in \psi^i$ where $a \neq a^i$, $a^i P_s^* a$. If we follow the same exercise for every student, we may construct a preference profile $P^* = (P_{s_1}^*, \dots, P_{s_n}^*)$ where the students following straightforward strategies with respect to P^* will lead to history h. $\qquad \square$

Theorem 1

Proof. By Proposition 1, for any realization of z and P, the outcome of the strategy profile σ^* is μ^S, the student-optimal stable matching with respect to the preference profile P and college priorities z. By Lemma 3, if any student s uses some deviation strategy σ_s', each realization of z and P will lead to the student-optimal stable matching for a profile $(P_s^*(P, z), P_{-s}, z)$, where $P_s^*(P, z)$ is any preference profile that could generate the history that results from the strategy profile $(\sigma_s'(\cdot | z_s, P_s), \sigma_{-s}^*(\cdot | z_{-s}, P_{-s}))$. But Roth [1984] shows that the outcome of the student-optimal stable matching for the profile (P, z) is weakly preferred by s to that for $(P_s^*(P, z), P_{-s}, z)$. Therefore, for any realizations of z and P, student s obtains an outcome that is weakly better by following the straightforward strategy, given that others are following it. As a consequence, the lottery induced by the strategy profile σ^* stochastically dominates the one induced by $(\sigma_s'(\cdot | z_s, P_s), \sigma_{-s}^*(\cdot | z_{-s}, P_{-s}))$ for player s.

Given the definition of OPBE, we still need to show that following the straightforward strategy stochastically dominates any deviation strategy at subgames that follows a player's deviation from the straightforward strategy. In other words, supposing that a player did not follow the straighforward strategy up to period t, we need to show that following the straightforward strategy stochastically dominates any other continuation strategy, assuming that the other students follow that strategy. To see that this is true, it suffices to make two observations:

- Starting from period t, a student s's strategy is only relevant at that subgame from the moment that she is requested to make some choice at some period $t' \geqslant t$.

- At period t', from the perspective of that student, the induced subgame is indistinguishable from the IDAM mechanism that starts with students being unacceptable to schools that are not reachable anymore for them at period t'.

Since the stochastic dominance result above does not depend on whether a student is acceptable or not to different schools, it follows that the result also holds at subgames resulting from deviation strategies.

$\qquad \square$

Proposition 2

Proof. Consider the set of students $S = \{s_1, s_2, s_3\}$ and of colleges $C = \{c_1, c_2, c_3\}$, each with capacity $q_i = 1$. Student s_1, who will be the player to whom we will show no dominant strategy exists, has preferences $c_1 P_{s_1} c_2 P_{s_1} c_3$, and students' exam grades at those colleges are as follows:

	c_1	c_2	c_3
s_1	100	100	100
s_2	200	200	200
s_3	300	300	300

Suppose now that, conditional on the realized preferences and grades of student s_1, student s_3 follows a straightforward strategy with respect to the preference $c_3 P^3 c_2 P^3 c_1$. Notice that we are not stating those are student s_3's preferences. We are simply assuming that she will follow the *straightforward strategy with respect to P^3*. Next, we consider two strategies for student s_2 and show that no strategy is a common best response for these two possibilities.

Scenario 1

Suppose that student s_2's strategy is the following: in $t = 1$, choose c_3. If at some later point s_2 is asked again to make a choice, she will choose the college with the highest cut-off value at that period among the options available. In case of ties, she will choose the college with the lowest index number (for example, the index number of c_2 is 2). We will show that, given s_2 and s_3's strategies, the unique best response involves first choosing c_2. The sequence of steps will be as follows:

Step 1: Student s_1 applies to c_2. Students s_2 and s_3 apply to c_3. Student s_2 is rejected. Cutoffs $\left(\zeta_{c_1}^1, \zeta_{c_2}^1, \zeta_{c_3}^1\right)$ are $(0, 100, 300)$.

Step 2: Since $\zeta_{c_2}^1$ is the highest cutoff among the colleges offered to s_2, student s_2 applies to c_2. Student s_1 is rejected. Cutoffs $\left(\zeta_{c_1}^2, \zeta_{c_2}^2, \zeta_{c_3}^2\right)$ are $(0, 200, 300)$.

Step 3: Student s_1 is left with two options: choose c_1 or s. If she chooses s she will remain unmatched. If she applies to c_1, she will be accepted. Final cutoffs $\left(\zeta_{c_1}^3, \zeta_{c_2}^3, \zeta_{c_3}^3\right)$ would then be $(100, 200, 300)$ and the outcome would be the matching μ' as follows:

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

Student s_1 can therefore be matched to her most preferred college by first choosing c_2. We now show that by choosing first c_1 or c_3, s_1 will always be matched to a strictly inferior college. First, let her choose c_1 first:

Step 1: Student s_1 applies to c_1. Students s_2 and s_3 apply to c_3. Student s_2 is rejected. Cutoffs $\left(\zeta_{c_1}^1, \zeta_{c_2}^1, \zeta_{c_3}^1\right)$ are $(100, 0, 300)$.

Step 2: Since $\zeta_{c_1}^1$ is the highest cutoff among the colleges offered to s_2, student s_2 applies to c_1. Student s_1 is rejected. Cutoffs $\left(\zeta_{c_1}^2, \zeta_{c_2}^2, \zeta_{c_3}^2\right)$ are $(200, 0, 300)$.

Step 3: Student s_1 is left with two options: choose c_2 or s. If she chooses s she will remain unmatched. If she applies to c_1, she will be accepted. Final cutoffs $\left(\zeta_{c_1}^3, \zeta_{c_2}^3, \zeta_{c_3}^3\right)$ would then be $(200, 100, 300)$ and the outcome would be the matching μ' as follows:

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 \\ s_2 & s_1 & s_3 \end{pmatrix}$$

If s_1 chooses c_3 first instead, the following will happen:

Step 1: Students s_1, s_2 and s_3 apply to c_3. Students s_1 and s_2 are rejected. Cutoffs $\left(\zeta_{c_1}^1, \zeta_{c_2}^1, \zeta_{c_3}^1\right)$ are $(0, 0, 300)$.

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Step 2: Following her strategy and the fact that college c_1's index is lower than c_2, student s_2 applies to c_1. Student s_1 has three options: choose also c_1 and therefore be rejected and left to choose between c_2 and s in period $t = 2$, choose c_2 or choose s. In all cases she will either end up remaining unmatched or matched to c_2.

Scenario 2

Now suppose that student s_2 follows a similar strategy to scenario 1, but where instead of applying to c_3 and then to the college with the highest cut-off value, she will apply to the college with the *lowest* cut-off value, once again breaking ties based on the index of the college.[16] Following an exercise similar to the one above, it is easy to see that student s_1's strategies that involve choosing first c_2 or c_3 will lead her to either be unmatched or be matched to c_2, while choosing c_1 will match her to c_1, her most preferred college.

Since every best response strategy under scenario 1 is dominated by different strategies in scenario 2, we have shown that a student may not have a weakly dominant strategy at the game induced by the IDAM mechanism. $\qquad \square$

Proposition 5

Proof. Since students follow straightforward strategies, a student $s \in S^i$ will only apply to colleges that are not in C^i if the cutoffs at all colleges in C^i are above her exam grade. Moreover, since for every i the number of students who prefer any college in C^i to any college not in C^i is at least as big as the overall number of seats in these colleges, by the end of the execution of the IDAM mechanism all seats in those colleges will be occupied by students in S^i, and students in S^i who are not matched to colleges in C^i will be left unmatched (even though some of them may be tentatively accepted at some period during the execution of the mechanism). From the perspective of a student in S^i, therefore, a seat in a college in C^i which is being occupied by a student not in S^i is equivalent to an empty seat.

Consider any $i \in \{1, \ldots, k\}$ and let $q_1^i \leqslant q_2^i \leqslant \cdots \leqslant q_{|C^i|}^i$ be the ordered capacities of the colleges in C^i. We will denote by $\left\{ S_1^i, S_2^i, \ldots, S_{|C^i|}^i, S_-^i \right\}$ the partitioning of the students in S^i where S_1^i are the top q_1^i students in S^i in colleges C^i's preferences, S_2^i are the top q_2^i students after those in S_1^i in colleges C^i's preferences, etc. and S_-^i are the students in S^i below the top $\sum_{j=1}^{|C^i|} q_j^i$ students. By Proposition 1, when students follow straightforward strategies the final outcome of the IDAM mechanism is the student-optimal stable matching, and by Lemma 2, at any period in which there are blockings, those involve students who are not tentatively matched to any college. The number of students who are involved in a block is therefore maximized when the number of students tentatively accepted to a college in any period is minimal.

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[16] Although the strategies used in this proof for student s_2 may seem very arbitrary, they can be rationalized by two simple stories. Student s_2's strategy in scenario 1 is consistent with a student who knows that her top choice is c_3 but that has some uncertainty about which one among c_1 and c_2 is her second choice, and sees the cut-off grade as an indication of how competitive acceptance is at those colleges and therefore the perceived quality of those. The strategy in scenario 2 could be rationalized by a student who once again knows that her top choice is c_3 but that would otherwise prefer to go with a college with low-achieving peers, and uses the low cutoff as an indication of that fact.

Consider now period $t = 1$. Since every college in C^i has at least q_1^i seats, every student in S_1^i will be accepted at any college in that period. There is one case in which all other students will be rejected, though: if all students in S^i choose the same college with capacity q_1^i in period $t = 1$. In that case, $|S^i| - q_1^i$ students in S^i will be tentatively unmatched by the end of period 1, and therefore if the IDAM mechanism runs for just one period, that is, the maximum number of students in S^i who will be involved in blocking pairs. The same argument will follow at $t = 2$: given that the students in S_1^i are all matched to a school with capacity q_1^i, the number of students who are tentatively unmatched by period $t = 2$ is maximal when all the remaining students in S^i choose a college with capacity q_2^i.

If we consider all the colleges and students, this process will take place in parallel at each element of $S = \{S^1 \cup S^2 \cup \cdots \cup S^k\}$ and $C = \{C^1 \cup C^2 \cup \cdots \cup C^k\}$. That is, by the end of period 1, the maximum number of students involved in blocks in S^1 is $|S^1| - q_1^1$, in S^2 is $|S^2| - q_1^2$, etc. The result therefore extends to a maximum of $n - \sum_{j=1}^{k} \sum_{i=1}^{T} q_i^j$ students involved in blocks.

Finally, if we consider the maximum number of steps that it takes until the student-optimal stable matching is produced, we can ask about which preferences from the students minimize the number of students who are matched to their final allocation at each step. That is, by minimizing the number of students matched to their final allocation we allow for the maximum number of students who can still make choices. Here it is easy to see that the preferences considered above, in which all students apply to the colleges in order of increasing capacity, is also the one that at each step matches the minimal number of students to their final allocation. The overall process will in that case end when the last set in $\left\{S_{|C^1|}^1, S_{|C^2|}^2, \ldots, S_{|C^k|}^k\right\}$ is matched to their final allocation. That will be therefore be the one with the largest number of colleges. Thus, the maximum number of steps is $\max_i \{|C^i|\}$. □

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