A survey on cycles and chaos (part I)

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Claude Diebolt & Catherine Kyrtsou[*]

Abstract: There are two contracting viewpoints concerning the explanation of observed fluctuations in economic and financial markets. According to the first view (Newclassical) the main source of fluctuations is to be found in exogenous, random shocks to fundamentals. According to the second view (Keynesian) a significant part of observed fluctuations is caused by non-linear economic laws. Even in the absence of any external shocks, non-linear market laws can generate endogenous business fluctuations. The discovery of chaotic, seemingly random looking dynamical behaviour in simple deterministic models sheds important new light on this debate. In order to detect non-linear structures in economic and financial data a certain number of tests, some based on chaos theory, have been developed. In this paper, we will briefly discuss several statistical techniques devised to detect independence and non-linearity in time-series data (Part I). In a next issue of the journal, we shall also try to make a simple presentation of the basic notions of chaos, and then describe the related econometric tools (Part II).

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1. Introduction

A generally accepted definition of business cycles is one presented by Arthur F. Burns and Wesley C. Mitchell in their work *Measuring Business Cycles*. According to Burns and Mitchell:

> *«Business cycles are a type of fluctuation found in the aggregate economic activity of nations to organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles with amplitudes approximating their own.»* (Burns & Mitchell, 1946, p. 3).

The definition of business cycles advanced by Burns and Mitchell emphasises that cycles are *recurrent but not periodic*. For some, the term *business cycles* implies a certain rhythm of business activity. To describe the cycle as recurrent means that it possesses a repetitive pattern of development — a pattern of expansion, recession, contraction, and revival, followed by renewed expansion. The cycle, however, is uniform neither in time periods nor in amplitude. We cannot say that the expansion phase always lasts X months and measures of aggregate activity rise Y per cent above the preceding low point. There is a high degree of uniformity from one cycle to the next in the forces of cumulation. Upswings and downswings are self-reinforcing; they feed on themselves, possess similar characteristics, and show approximately concurrent movements in many different series. However, there is no evidence that they recur again and again in virtually the same form and amplitude. The completion of a cycle from trough to trough or peak to peak may take from approximately two to more then ten years, and the proportions of the upswing and downswing may vary all the way from mild to catastrophic.

By the late seventies and early eighties, the debate concerning the main source of business cycle fluctuations seemed to have been settled in favour of the exogenous shock hypothesis. An important critique on this hypothesis has been that it does not provide an economic explanation of observed fluctuations, but rather attributes these fluctuations to external, non-economic forces.

Due to the discovery of deterministic chaos however, a renewed interest in endogenous economic dynamics emerged. This notion captures exactly what at first sight seems to be a paradox. A deterministic, and thus perfectly predictable world becomes unpredictable when the initial states can only be measured with finite precision or when the system is subject to small disturbances. So, because of sensitive dependence on initial conditions, long run prediction of a chaotic system is impossible. Therefore, this does not mean that we cannot forecast such systems over short time periods.

A number of the early chaotic business cycle models are due to Day. Day (1982) showed that a version of Solow's (1956) familiar growth model, can generate chaotic output fluctuations around an unstable steady state growth path, that seemed much closer to real business cycles than the earlier endogenous business cycle models in the fifties[1] (Kaldor (1940), Hicks (1950), and Goodwin (1951)). These early examples were criticised, because the models had not been derived from underlying microeconomic foundations with profit or utility maximising agents. However, Benhabib and Day (1982) and Grandmont (1985) showed that in the overlapping generations model, the conflict between substitution and income effects, may lead to chaotic output fluctuations.

All examples discussed above are or can be reduced to an one-dimensional difference equation, and many of these examples apply the well known "Period three implies chaos" theorem of Li and Yorke (1975), for one-dimensional systems. From a qualitative point of view, the time series generated by these chaotic business cycle models are closer to actual, observed business cycles than the series generated by the early non-linear business cycle models. Therefore, quantitatively the chaotic time series are still quite different from actual observed business cycles. In particular, chaotic time series form an one-dimensional model frequently exhibit extremely large jumps, where a price or a quantity presents large decreases during one single period. Obviously, this is an unrealistic feature, which has never been observed in a yearly change of a capital stock or in aggregate output, or even in the fall of stock prices after a market crash. In addition, in one-dimensional model unrealistically strong non-linearities are needed to generate chaotic dynamics.

In order to generate time series that mimic more closely actual data, one has to look at higher dimensional models. Only very recently economists have started to build and explore the rich dynamical behaviour in two and higher dimensional non-linear endogenous business cycle models. For example, Medio and Negroni (1996) and De Vilder (1996) have shown that a two-dimensional version of the overlapping generations model can generate chaotic equilibrium paths, even when the two goods, current leisure and future consumption are gross substitutes.

The recent burst in non-linear dynamics has also led to a number of innovative approaches concerning the empirical and theoretical analysis of financial markets. In fact, the idea that what is going on in those markets is generated by high-dimensional chaotic processes, seems to be intuitively so attractive that many introductory pages of textbooks on non-linear dynamics cite financial

[1] See for example Lorenz (1993) or Day (1996) for a survey of other endogenous business cycle models generating chaos.

markets as a standard example governed by chaotic motion. Early works of Brock and Hommes (1998), Lux (1995, 1998), Malliaris and Stein (1999), Gaunersdorfer (2000), and Chiarella et al. (2000) show that structural non-linear financial markets models may lead to market instability and chaos. In these non-linear models, complex asset-price fluctuations are triggered by an interaction between a stabilising force driving prices back towards their fundamental value when the market is dominated by fundamentalists and a destabilising force driving prices away from their fundamental value when the market is dominated by speculative noise traders. The distribution of series derived from chaotic trajectories of the models (e.g., Lux (1998), Lux and Marchesi (1998), Iori (1999)) share important characteristics of true data: volatility clustering, high peaks around the mean, and reduced leptokurtosis under time aggregation. The introduction of endogenously transitions probabilities (Lux, 1998) or modelling using noisy chaotic systems (Malliaris and Stein (1999), Kyrtsou and Terraza (2001), Gaunersdorfer and Hommes (2000)), give a new dimension to high-dimensional chaos applications in finance.

As we have previously underlined, according to the chaotic approach, fluctuations are the results of endogenous changes. This statement, does not only mean that economic cycles or even stock prices are generated by a non-linear deterministic (or high-dimensional) chaotic system. The nature of economic agents' beliefs play also a very important role. According to the rational expectations hypothesis all agents hold the same belief and this is common knowledge. However, in reality agents have different interpretations of the available information. This endogenous heterogeneity may lead to market instability and complicated dynamics, such as cycles or even chaotic fluctuations, in financial markets (e.g Chiarella (1992), Day and Huang (1990), DeGrauwe et al. (1993), Lux (1995) and Sethi (1996)). Heterogeneity and social interactions are very natural non-linear effects which can play a key role in explaining fluctuations in real markets.

Recent non-linear stochastic modelling was applied by Skalin and Teräsvirta (1996) to study Swedish business cycles. They found that a certain number of Swedish macroeconomic series are non-linear and this non-linearity is characterised by STAR models. Cyclical variation at business cycle frequencies does not seem to be constant over time for all series, and it is difficult to find a Swedish business cycle. Only two series may be regarded as having genuinely asymmetric cyclical variation.

In order to detect non-linear structures in economic and financial data a certain number of tests, some based on chaos theory, have been developed. In the following sections, we will briefly discuss several statistical techniques devised to detect independence and non-linearity in time-series data. We shall also try to make a simple presentation of the basic notions of chaos, and then describe the related econometric tools.

2. Methodology

2.1 The White test (1989)

In White's test (1989), the time series is fitted by a single hidden-layer feed-forward neural network, which is used to determine whether any non-linear structure remains in the residuals of an autoregressive (AR) process fitted to the same time series. The null hypothesis for the test is linearity in the mean[2] relative to an information set.

The rationale for White's test can be summarised as follows: under the null hypothesis of linearity in the mean, the residuals obtained by applying a linear filter to the process should not be correlated with any measurable function of the history of the process. White's test uses a fitted neural net to produce the measurable function of the process's history and an AR process as the linear filter. Then, we test the hypothesis that the fitted function does not correlate with the residuals of the AR process. The resulting test statistic has an asymptotic χ^2 distribution under the null of linearity in the mean.

2.2 The Kaplan test (1994)

We begin the presentation of Kaplan's test (1994) by reviewing its origins in the chaos literature, although the test is currently being used as a test of linear stochastic process against general non-linearity, whether or not noisy or chaotic. In the case of chaos, a time series plot of the output of a chaotic system may be very difficult to distinguish visually from a stochastic process. However, plots of the solution paths in phase space (x_{t+1} plotted against x_t and lagged values of x_t) often reveal deterministic structure that was not evident in a plot of x_t versus t. A test based upon continuity in phase space has been proposed by Kaplan (1994).

The idea of Kaplan's test (Kaplan, 1994) is, that iterations from a deterministic processes should be closer to the preceding values than those from a purely stochastic dynamics. More formally, this amounts to testing whether for pairs of data points which are within some small distance $d_{ij} = \|y_i - y_j\| < r$, the average of the differences of their iterations $\varepsilon_{ij} = \|y_{i+1} - y_{j+1}\|$ is found to be smaller than some threshold value. The procedure involves producing linear stochastic process surrogates for the data and determining whether the surrogates or a noisy continuous non-linear dynamical solution path better describe the data. In general, we perform 20 replications with surrogate data and adopt two variants of this test: in the first, we compute the test statistic K as the average ε_{ij} from

[2] For a formal definition of linearity in the mean, see Lee, White and Granger (1993). Note that a process that is not linear in the mean is said to exhibit "neglected non-linearity". Also, a process that is linear is also linear in the mean, but the converse need not be true.

the 500 smallest distances d_{ij}, while in the second variant, we perform a linear regression on these smallest pairs (d_{ij}, ε_{ij}) and consider the intercept at $d_{ij}=0$. In both cases, the resulting test statistic K is compared to the minimum K from 20 time series of surrogate data. Linearity is rejected, if the value of the test statistic K from the surrogates is never small enough relative to the value of the statistic computed from the data.

2.3 The BDS test (1987)

The BDS test provides a preliminary step to determine whether a time series process does or does not have observations which are independently and identically distributed (i.i.d). In 1987 Brock, Dechert and Scheinkman (henceforth BDS) proposed a test of the i.i.d hypothesis based on the Grassberger and Procaccia correlation integral. We test the null hypothesis that a series is i.i.d against the alternative hypothesis that a series is linearly or non linearly correlated.

For a time series $\{X_t\}$ t=1,…T, we first consider the m-histories $X^m_t = (X_t, X_{t-1}, ..., X_{t+m-1})$, and then calculate the correlation integral :

$$C(\varepsilon,m) = \frac{1}{T_m(T_m-1)} \sum_{\substack{i,j=1 \\ i\neq j}}^{T_m} H(\|X_i^m - X_j^m\|) . \tag{2.3.1}$$

Here T_m = T-m+1 is the number of m-histories $X^m_t = (X_t, X_{t-1},…,X_{t+m-1})$ constructed from the sample of length

$$T;\ H(\|X-Y\|) = \prod_{s=1}^{m} H(|X_s - Y_s|) ,$$

and m the embedding dimension.

H is the Heaviside function given by:

$$H(\|X_i - X_j\|) = \{1 \quad \text{if} \quad \|X_i - X_j\| < \varepsilon \text{ and } \ 0 \quad \text{if} \quad \|X_i - X_j\| \geq \varepsilon\}.$$

Brock, Dechert and Scheinkman (1987) show the following theorem: Let X_t an i.i.d series and suppose that $\sigma^2_m > 0$; in that case

$$T_m^{1/2} [\ C(\varepsilon,m,T_m) - (C(\varepsilon,m,T_m))^m\ ] \xrightarrow{d} N(0,\sigma^2_m) \text{ with } T_m \to \infty$$

where the expression «$\xrightarrow{d} N(0,\sigma^2_m)$» means: «convergence in distribution to $N(0,\sigma^2_m)$» and $N(0,\sigma^2_m)$ denotes the normal distribution with mean 0 and variance , σ^2_m.

Considering that $C(\varepsilon,l)\xrightarrow[Tm\to\infty]{}[C(\varepsilon,l)]^m$ [i.e. Denker et Keller (1986, theorem 1 and (3,9))], equation (2.3.1) can be written as:

$$W(\varepsilon,m) = T_m^{1/2}\,[C(\varepsilon,m) - (C(\varepsilon,1\,))^m]\,/\sigma_m(\varepsilon). \tag{2.3.2}$$

Under the null hypothesis, X_t is i.i.d. and N(0,1). Note that W(ε, m) is a function of two unknowns: the embedding dimension m, and the radius ε. There is an important relation between the choice of m and ε concerning the properties of a small sample for the BDS statistic. For a given m, ε cannot be too small, because in the opposite case there are not enough pairs of points X_i, X_j which would make the maximum of distance between them to be inferior or equal to ε (necessary condition for the calculation of the correlation integral). These small values of ε yield a slope systematically equal to m, because of the problem of noise (noisy chaos)[Brock et Dechert (1987)]. Inversely, ε must not be too large because the correlation integral contains too many observations.

Barnett and Choi (1989) suggest selecting a small value for ε, without allowing it to reach zero. This implementation of a lower limit guards against noise in the data. Hsieh (1989) defines ε in terms of multiples of the series standard deviation. These multiples are 0.50, 0.75, 1.00, 1.25, 1.50. The same multiples given by Girerd-Potin and Taramasco (1994) are 0.25, 0.50, 0.75, 1.00, 2.00, and 4.00. Brock, Hsieh and LeBaron (1992) use instead the 0.25, 0.50, 1.00, 1.50, 2.00[3]. As Liu et al. (1993) indicate, the choice of ε is crucial insofar as different selected ranges of values of ε can lead to different conclusions. The authors suggest selecting m to belong to the interval [2,5].

Whatever the choice of ε and given the value of m, we calculate the W statistic. The obtained values of $|W|$ are to be compared with the theoretical value 1.96 of the normal distribution at the 5% level. If the estimated value is higher than 1.96, then the null hypothesis of data independence is rejected. This rejection can result from:

- either a non stationarity of the considered series, or
- a structure of dependence issued from a stochastic linear process (e.g. ARMA), or
- a structure of dependence issued from a nonlinear stochastic process (e.g. TAR, STAR, NMA, ARCH, GARCH, EGARCH), or
- a structure of dependence issued from a nonlinear deterministic process (e.g. tent map, Mackey-Glass equation).

In order to use the BDS test as a test of non-linearity (BDSL), it is necessary in the first place for the data to be stationary and to lack any linear structure. It is possible to eliminate any linear dependence by filtering the data and applying

3 They have reached the best results when the ratio ε/σ varies between 0.50 and 2.

the BDS test to the residuals of an autoregressive model estimated from the initial stationary series.

2.4 The Mizrach test (1995)

This test proposed by Mizrach (1995) is a simple nonparametric method for independence, which we can use in small samples. Mizrach defines a stochastic process to be locally independent of order p if the realisation X_t provides no information about the process p periods ahead. Therefore, we obtain the equality of the conditional and unconditional distributions. Let $(p_1,\ldots,p_{m-1})$ be a set of increasing integers on [1, L], L < T-m+1.

Local independence then implies:

$$\text{Prob}[\, X_{t+pm-1} < \varepsilon, \ldots, X_{t+p1} < \varepsilon, X_t < \varepsilon \,] = (\text{Prob}[X_t < \varepsilon\,])^m . \tag{2.4.1}$$

To estimate the joint, $F(X_t^m)$, and marginal, $F(X_t)$, distributions in (2.4.1), introduce the kernel function h: R $\longrightarrow$ R:

$$h\left(X_t\right) = H[X_t < \varepsilon] = \begin{Bmatrix} 1 & if\ \ X_t > \varepsilon \\ 0 & otherwise \end{Bmatrix} \equiv H\left(X_t, \varepsilon\right) .$$

The joint unconditional probability that m leads of the X's are less than ε is given by

$$\theta(m,\varepsilon) = \int_X \prod_{i=0}^{m-1} H\left(X_{t+p_i}, \varepsilon\right) dF\left(X_t\right) .^4 \tag{2.4.2}$$

A consistent estimator of (2.4.2) is the following U statistics:

$$\theta(m,T^*,\varepsilon) = \sum_{t=1}^{T^*} \prod_{i=0}^{m-1} H(X_{t+p_i}, \varepsilon)/T^*$$

where $T^* = T - \max[p_i]$ and T the number of observations.
The simple test for local independence is constructed using consistent estimators of the first two moments of this U-statistics.
Proposition: Let $\{X_t\}$ be locally independent for any $p_i \in [1,L]$, i = 1,…, m-1, $L<T_m$, then if $\theta(m,\varepsilon)>0$,

[4] Mizrach uses $p_o = 0$ for notational convenience.

$$\sqrt{T^*}\frac{\left[\theta\left(m,T^*,\varepsilon\right)-\theta\left(m-1,T^*,\varepsilon\right)\theta\left(1,T^*,\varepsilon\right)\right]}{\left[\theta\left(m-1,T^*,\varepsilon\right)\theta\left(1,T^*,\varepsilon\right)\left(1-\theta\left(m-1,T^*,\varepsilon\right)\right)\left(1-\theta\left(1,T^*,\varepsilon\right)\right)\right]^{0.5}}\xrightarrow{d}N(0,1)$$ [5]

where « $\xrightarrow{d}$ N(0,1)» means that the SNT statistic converges in distribution to N(0,1) as $T^*\to\infty$.

The SNT tests the null hypothesis of independence versus the alternative of linear or non-linear dependence in the process.

An application of the SNT in stock exchange returns series by Kyrtsou and Terraza (1998) has shown that even if we use short time series BDS remains robust. Thus, given the power of the BDS test, the SNT can be applied as a joint test.

2.5 The Granger, Maasoumi and Racine test (2000)

Granger *et al.* (2000) consider using the S_p statistic for testing serial independence of a time series against alternatives of dependence which can be of a general and non-linear nature.

The estimated value for this statistic is given as follows:

$$\hat{S}_\rho=\frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\sqrt{\hat{f}(\alpha,b)}-\sqrt{\hat{f}(\alpha)}\sqrt{\hat{f}(b)}]^2 d\alpha\ \mathrm{db}=$$

$$\frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\sqrt{\frac{1}{nh_\alpha h_b}\sum_{j=1}^{n}K\left(\frac{\alpha-\alpha_j}{h_\alpha},\frac{b-b_j}{h_b}\right)}-\sqrt{\frac{1}{nh_\alpha}\sum_{j=1}^{n}K\left(\frac{\alpha-\alpha_j}{h_\alpha}\right)}\sqrt{\frac{1}{nh_b}\sum_{j=1}^{n}K\left(\frac{b-b_j}{h_b}\right)}\right)^2 d\alpha\ \mathrm{db}$$

where $\hat{f}(\alpha,b)$ is the kernel estimator of the bivariate density of the random variables A and B evaluated at the point (α,b) based upon a sample of observations of size n. $\hat{f}(\alpha)$ and $\hat{f}(b)$ are the marginal densities evaluated at the points α and b. K(.) is a pth order univariate kernel function, where h_α and h_b are bandwidths. The critical values for the test are given by Granger *et al.* (2000). If $\hat{S}_\rho <$ critical value, then we accept the zero hypothesis for serial independence. If $\hat{S}_\rho >$ critical value, then we reject the zero hypothesis.

[5] Proof in Mizrach's (1995) working paper.

Granger et al. (2000) apply this test to a certain number of stochastic and chaotic models. For all series S_p statistic detects well linear or non-linear dependencies Nevertheless, it is important to note its power against chaotic dynamics. For example the logistic equation has autocorrelation function (ACF) and partial ACF which behave like white noise. The ACF incorrectly leads us to conclude that there is no dependence in the series, while the S_p metric correctly identify dependence in the series.

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