The orgin and role of invariance in classical kinematics

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THE ORIGIN AND ROLE OF INVARIANCE IN CLASSICAL KINEMATICS

By classical kinematics (CK) I understand the theory the models of which have the form $\langle P, \mathbb{R}, s \rangle$, where P is a non-empty set (of points or 'parti= cles'), $\mathbb{R}$ is the set of real numbers,and $s: P \times \mathbb{R} \rightarrow \mathbb{R}^3$ is smooth in its second argument. s is called position function and $\mathbb{R}$ denotes time, so '$s(p,t) = \langle \alpha_1, \ldots, \alpha_3 \rangle$' has to be read as 'point p at time t is in position $\langle \alpha_1, \alpha_2, \alpha_3 \rangle$'.[1)]

It is well known that models of classical kinematics are invariant under spatial- and time-displacements,spatial rotations,and combinations of these three kinds of transformations.More pre= cisely,if $\langle P, \mathbb{R}, s \rangle$ is a model and if $s': P \times \mathbb{R} \rightarrow \mathbb{R}^3$ is defined by

there are a real,orthogonal 3×3-matrix $\mathfrak{a}$, $\mathfrak{f} \in \mathbb{R}^3$ and $b \in \mathbb{R}$ such that for all $t \in \mathbb{R}$ and $p \in P$: $s'(p,t) = \mathfrak{a} s(p,t+b) + \mathfrak{f}$, (1)

then $\langle P, \mathbb{R}, s' \rangle$ again is a model of classical kine= matics.Let me call transformations of this kind 'space-time transformations'.What has been just said then can be expressed by saying that CK is invariant under space-time transformations.

A philosopher of science would be satisfied with stating that space-time transformations are just those transformations under which CK's axioms are invariant.But this is not the case.CK in fact is invariant under a much bigger class of transformations.How then does it come that physicists have concentrated just on space-time transformations? I try to answer this question by investigating the origin and role of space-time transformations.It turns out that philosophers of science can be satisfied by showing CK to be a

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theoretization of underlying theories of space and time. If those underlying theories are taken into account when formulating CK's axioms all but space-time transformations are excluded.[2)]

Intuitively, I can summarize my results on the origin and role of invariance under space-time transformations in CK in form of a thesis.

> The invariance of CK under space-time transformations originates from concrete observations that distances in space and time remain the same under spatial- and time-displacements as well as under spatial rotations of 'space-time'-coordinate systems. The actual role of this invariance consists of two components. First, invariance gives us a certain freedom of choice for coordinate systems. Second, very often one can execute 'active' counterparts of transformations of coordinate systems -i.e. active transformations of the physical system itself relative to a fixed coordinate system- with the effect of not changing the system. Thus invariance serves as a guide in the application of active space-time transformations.

I will first tell a science fiction story about how theories of space and time might have developed in order to optimally please philosophers of science thinking about space-time invariance. Only after this story I will try to see how it increases the probability of my thesis.

1. A SCIENCE FICTION STORY

In some culture on our planet the art of dealing with distances and motions might have developed as follows. By using a certain kind of measuring instruments, say 'rigid rods', in order to compare distances people recognize that what they are doing can be described by 'bringing ends of (or marks on) rigid rods to coincide with marks or particles or marks on bodies', 'testing whether rods are straight' and by 'testing whether rods are equally long'. People find out that certain statements about these operations regularly turn out as true. They introduce some abbreviations by referring to a set P of points (or particles or marks), a relation $\underline{b}$ ('$\underline{b}abc$' meaning that point b on a rod is situated between points a and b), and to a relation $\equiv$ ('$ab \equiv a_1 b_1$' meaning that the rod with end points a and b is just as long as the rod with end points a_1 and b_1). The statements which turn out to be true are just those statements which in Tarski (1959) are taken as axioms for a system of Euclidean geometry.[3] In short, people find out that their handling spatial distances yields a realization (modulo idealization) of some structure $\langle P, \underline{b}, \equiv \rangle$ which is a model of Euclidean geometry.

At the same time mathematics is developed and people come to know real numbers. It turns out as very effective to compare lengths of rigid rods (or distances) by assigning to each rod a real number such that the congruence relation under this assignment is 'represented' by equality of real numbers, and the betweenness relation is 'represented' by a kind of additivity of the numbers assigned to the rods.[4] Finally, philosophers of science come into being -wondering about and questioning this development. They prove that, up to the conventional choice of assigning the real number 1 to some arbitrary rigid rod, such an assignment of real numbers to rods is uniquely determined by the statements about $\underline{b}$ and $\equiv$ and the abovementioned representation properties. They introduce a theory, the 'theory for metrization of space' the models of which are entities of the form $\langle P, \underline{b}, \equiv, d \rangle$, where $\langle P, \underline{b}, \equiv \rangle$ is a model of Euclidean geometry, and $d: P^2 \rightarrow \mathbb{R}$ is such that (1) $\langle P, d \rangle$ is a metric space and (2) for all $a, b, a', b' \in P$: $\underline{b}aba' \leftrightarrow d(a,b)+d(b,a')=d(a,a')$ and $ab \equiv a'b' \leftrightarrow d(a,b)=d(a',b')$. Here, d is the 'assignment of numbers to rods' referred to above. d is called distance function and the set of models $\langle P, \underline{b}, \equiv, d \rangle$ of the theory for metrization of space is denoted by M1.

People further recognize that spatial descriptions become much simpler if, instead of stating all distances between all points involved, they state for each point its distances to three distinguished, eventually new, points which are the same for the whole system. They introduce coordinate systems (CS) for models in M1. A CS for $x=\langle P,\underline{b},\equiv,d\rangle$ is just a tuple

$$y=\langle a_o,\ldots,a_3\rangle \in P^4$$

such that (1) for $0\leq i\neq j\leq 3$: $a_i\neq a_j$, (2) for $1\leq i\leq 3$ $d(a_o,a_i)=1$, and (3) for $1\leq i\neq j\leq 3$:

$$d(a_i,a_o)^2+d(a_o,a_j)^2=d(a_i,a_j)^2.$$

a_o is called the origin and $a_1,\ldots,a_3$ are points forming right angles with a_o (requirement (3) above), and having distance 1 from a_o. Right angles are expressed by Pythagoras' formula. The abovementioned simplification of spatial description then is achieved by introducing in $x=\langle P,\underline{b},\equiv,d\rangle$ via a CS $y=\langle a_o,\ldots,a_3\rangle$ a unique function $s:P\to\mathbb{R}^3$ such that (1) $s(a_o)=0$, (2) $s(a_i)=\mathfrak{u}_i$ for $1\leq i\leq 3$, and (3) $\|s(a)-s(b)\|=d(a,b)$ for all $a,b\in P$. Here $\mathfrak{u}_i$ are the unit vectors and $\|\cdot\|$ is the Euclidean norm on $\mathbb{R}^3$. s is unique because in a model of Euclidean geometry some point $a\in P$ with respect to y uniquely determines the four distances $d(a,a_i)$ $(i=0,\ldots,3)$, and these, by conditions (1)-(3), can be used to solve the equations $\|s(a)-s(a_i)\|=d(a,a_i)$ for $s(a)$. In this way, instead of describing spatial relations by stating all distances among all points, people have to give only one vector $s(a)$ for each point a in order to obtain the same amount of information. And as the number of particles increases, so does the amount of simplification gained by this method.

Now people make the following experience. If they describe spatial relations of some concrete object via a distinct CS y and afterwards via another CS y' which does not move relative to y then the resulting functions s and s' are connected by a space-transformation, i.e. there exists a real, orthogonal 3×3-matrix α and a vector $b \in \mathbb{R}^3$ such that for all $a \in P$: $s'(a) = \alpha s(a) + b$. Note that this fact is observed, not deduced.

After these achievements there develops a need for comparing processes or changes of various kinds. By choosing a certain sort of measuring instruments, for instance 'clocks', people are able to compare different processes with each other by referring to the initial and final events of those processes. They recognize that what they are doing can be described by 'bringing into coincidence the initial and final events of processes', 'deciding whether an event in a process preceeds another one in that process', 'comparing different processes' and 'concatenating two processes in order to obtain a longer one'. They introduce some abbreviations: a set T of initial events, final events, and other clearly distinguishable events occurring in processes, a relation $\lessdot$ on such events ('$t \lessdot t_1$' meaning that event t preceeds event t_1 in some process), a relation $\preccurlyeq$ among processes ('$tt' \preccurlyeq t_1 t_2$' meaning that the process with initial and final events t and t' is not longer than a process with initial and final events t_1 and t_2), and an operation $\circ$ among processes ('$tt' \circ t_1 t_2 = t_3 t_4$' meaning that the process with initial and final events t_3 and t_4 is the result of concatenating the processes with initial and final events tt' and $t_1 t_2$, respectively). By investigating various kinds of processes people find out certain regularities. Using their abbreviations the statements expressing these regularities are roughly the axioms for positive, closed extensive systems (see Krantz et al. (1971), p. 73). As in the case of theories about space they find it convenient to assign real numbers to processes and to compare the latter by means of the former. Again, this assignment has to 'represent' the essential properties of the relations among processes in a natural way by real numbers: $\preccurlyeq$ is represented by $\leq$ and $\circ$ by +. And again, mathematicians can prove that such assignments are uniquely determined up to choice of a unit. In short, a 'theory for metrization of time' is intro=

duced the models of which have the form $\langle T, \lessdot, \precsim, \circ, \Upsilon\rangle$, where (1) T is a set containing at least two elements, (2) $\lessdot \subseteq T \times T$ is a linear ordering which is dense, 'continuous' and 'sepa= rable', (3) $\precsim \subseteq T^4$ is such that for all $t,t',t'' \in T$: $\langle t',t''\rangle \precsim \langle t,t\rangle \rightarrow t'=t''$ and $\langle t,t\rangle \precsim \langle t',t''\rangle$, (4) $\circ : T^4 \rightarrow T^2$ is such that for all $t,t',t'' \in T$: $\langle t',t''\rangle \circ \langle t,t\rangle = \langle t',t''\rangle$ and $\langle t,t\rangle \circ \langle t',t''\rangle = \langle t',t''\rangle$, (5) for $D := T^2 \setminus \Delta(T)$: $\langle D, \precsim_{/D}, \circ_{/D}\rangle$ is a positive, closed extensive structure in the sense of Krantz et al. (1971), p.73, (6) $\Upsilon : T \times T \rightarrow \mathbb{R}$ is such that $\langle T, \Upsilon\rangle$ is a metric space and for all $t,t',t_1,t_2,t_3,t_4 \in T$: (6.1) $t \lessdot t' \lessdot t_1 \rightarrow \langle t,t'\rangle \circ \langle t',t_1\rangle = \langle t,t_1\rangle$, (6.2) $\langle t,t'\rangle \precsim \langle t_1,t_2\rangle \leftrightarrow \Upsilon(t,t') \leq \Upsilon(t_1,t_2)$, and (6.3) $\langle t,t'\rangle \circ \langle t_1,t_2\rangle = \langle t_3,t_4\rangle \leftrightarrow \Upsilon(t,t') + \Upsilon(t_1,t_2) = \Upsilon(t_3,t_4)$.
Here $\Delta(T)$ is the diagonal in T and $f_{/D}$ denotes f, restricted to D. Let M2 be the set of all such models.

As a matter of economy people introduce coordinate systems which in this case are very simple: a CS for $\langle T, \lessdot, \precsim, \circ, \Upsilon\rangle$ consists of just two events $\langle t^o, t^1\rangle$ such that $t^o, t^1 \in T$ and $t^o \lessdot t^1$ (t^o is something like 'the birth of christ' and t^1 marks the end of the first unit of time 'after' t^o). With the help of a CS they can com= pare the lengths of processes not by referring to the 'distances' between initial and final events but by referring to distances of events from t^o. If $\langle t^o, t^1\rangle$ is a CS for $x = \langle T, \lessdot, \precsim, \circ, \Upsilon\rangle \in$ M2 then there exists a unique function $\Theta : T \rightarrow \mathbb{R}$ such that (1) $\Theta(t^o) = 0$, (2) $\Theta(t^1) = 1$, (3) for all $t,t' \in T$: $t \lessdot t' \leftrightarrow \Theta(t) < \Theta(t')$, and (4) for all $t \in T$: $|\Theta(t)| = \Upsilon(t,t^o)$. Θ here is the analogue to the s intro= duced before in connection with space. That is, $\Theta(t)$ gives the 'position' of event t with respect to the CS $y=\langle t^o,t^1\rangle$ for x.So by means of a CS statements about comparison and lengths of pro=cesses can be formulated in terms of events and 'coordinates' of events.

When the theory for metrization of time and coordinate systems are introduced the following is observed.If some concrete process and its de=velopment in time is described from a CS $\langle t^o,t^1\rangle$ and also from a different CS $\langle \bar{t}^o,\bar{t}^1\rangle$ then the resulting functions Θ and Θ' are connected by a time-transformation,i.e.there is some $\alpha\in\mathbb{R}$ such that for all $t\in T$: $\Theta'(t)=\Theta(t)+\alpha$.This is an experimental result.

As soon as these two theories exist peoples' interests turn to motions.Motions are certain kinds of processes involving particles which in themselves do not change.Such a process consists of a relative change of different particles constituting the process relative to each other. With the help of spatial- and time-measurements they can find out spatial distances between the particles at various 'times'.On the other hand, people also can measure these times and their 'distances' and start to talk about the rate of spatial change with respect to a certain process or the length of a process (or an 'interval of time').But when they try to formulate this more precisely in terms of the distance functions already available they run into difficulties.If they express spatial change by differences of distances and periods of time by distances of events how then they assure that the events are precisely those at which the distances were mea=sured? Since they have no term to express this in their vocabulary and since they want to do so they introduce a new term,a four-place function $d:P^2\times T^2\rightarrow\mathbb{R}^2$ with the following meaning. '$d(a,b,t,t')=\langle\alpha,\beta\rangle$' means that the spatial distance between a and b measured at event t is α and the 'time-distance' between t and t' measured at point a is β.Clearly,with this con=cept they can formulate all kinds of statements about motions and rates of spatial change in time. Immediately they recognize that this new d 'con=

tains' the old d and $\mathcal{T}$. For if the two time-arguments are fixed, say by t and t', one obtains a function $d_{tt'}: P^2 \to \mathbb{R}$, defined by $d_{tt'}(a,b) = \pi_1(d(a,b,t,t'))$ (π_i is the projection on the i-th component). And by fixing the two spatial arguments, say by a and b, one obtaines a function $d_{ab}: T^2 \to \mathbb{R}$, defined by $d_{ab}(t,t') = \pi_2(d(a,b,t,t'))$. Also it is noticed that the non-metrical concepts of space, namely $\underline{b}$ and $\equiv$, must be adjusted in order to describe actual operations when time matters. Thus a new theory is introduced, the 'theory for metrization of space-time'. Its models are of the form

$$\langle T, P, \lessdot, \preccurlyeq, \circ, \underline{b}, \equiv, d \rangle$$

where (1) T and P are sets, (2) $d: P^2 \times T^2 \to \mathbb{R}^2$, (3) for all $a,b \in P$: $\langle T, \lessdot, \preccurlyeq, \circ, d_{ab} \rangle$ is in M2, (4) $\underline{b} \subseteq T \times P^3$ and $\equiv \subseteq T \times P^4$, (5) for all $t,t',t'' \in T$: $d_{tt'} = d_{tt''}$ and $\langle P, \underline{b}_t, \equiv_t, d_{tt'} \rangle$ is in M1, (6) there are $t,t' \in T$ $(t \neq t')$ such that for all $a,b,c,e \in P$: $d_{ab}(t,t') = d_{ce}(t,t')$. Here, $\underline{b}_t$ and $\equiv_t$ are just $\{\langle a,b,c \rangle / \langle t,a,b,c \rangle \in \underline{b}\}$ and $\{\langle a,b,a',b' \rangle / \langle t,a,b,a',b' \rangle \in \equiv\}$. In this theory the concepts T, $\lessdot$, $\preccurlyeq$ and $\circ$ concerning time are just as before. On the other hand, the spatial relations $\underline{b}$ and $\equiv$ have got an additional argument for time instants. From these time dependent spatial relations one obtains the original ones by considering 'cuts' $\langle P, \underline{b}_t, \equiv_t \rangle$ at certain instants t. Requirement $d_{tt'} = d_{tt''}$ says that spatial relations as represented by d depend only -if at all- on the first time-argument. The set of models of this theory is called M3.

In models of M3 space and time are connected as far as necessary by the introduction of a time-argument for all spatial relations. The models thus can be imagined as continuous sequences of 'spaces', i.e. models of M1. All the spaces of such a sequence have the same set of points but the spatial relations can be different in each. The 'indices' by which the sequence is ordered are the time-instances and it is required (via $\lessdot$) that they in fact are ordered in a way isomorphic to the order of real numbers. On the other hand space and time are independent from each other to a physically desirable extent. The relations concern= ing time $\lessdot$, $\preccurlyeq$ and $\circ$ are independent from space by construction and the second component of d -which describes time-distances- is independent from space by requirement (6) above. It should be noted that models of M3 are logically much weaker than all kinds of Riemannian space-times because in M3 the spatial relations can undergo <u>all</u> kinds of changes -even non-continuous ones.

Now people in our fictitious culture of course do not want to give up the simplifications gained by coordinate systems. So they introduce 'space-time' coordinate systems. Such a CS for $\langle P,T,\lessdot,\preccurlyeq,\circ,\underline{b},\equiv,d\rangle \in$ M3 is just a tuple $\langle t^o,t^1,a_o,\ldots,a_3\rangle$ such that (1) $t^o,t^1 \in T, t^o \lessdot t^1$ and (2) for all $t,t' \in T$: $\langle a_o,\ldots,a_3\rangle$ is a CS for $\langle P,\underline{b}_t,\equiv_t,d_{tt'}\rangle$. Such a CS yields a unique 'coordinatization' ψ for it can be proved that if $x=\langle P,T,\lessdot,\preccurlyeq,\circ,\underline{b},\equiv,d\rangle \in$ M3 and $y=\langle t^o,t^1,a_o,\ldots,a_3\rangle$ is a CS for x then there exists a unique function $\psi: T \times P \rightarrow \mathbb{R}^4$ such that (1) $\psi_{a_o}(t^o)=0$ and $\psi_{a_o}(t^1)=1$, (2) for all $t,t' \in T$ and all $a,b \in P$: $t \lessdot t' \leftrightarrow \psi_{a_o}(t) < \psi_{a_o}(t')$ and $|\psi_{a_o}(t)|=d_{ab}(t,t^o)$, (3) for all $t \in T$ and $a,b \in P$: $\psi_t(a_o)=0$ and $\psi_t(a_i)=\nu_i$ (i=1,2,3) and $\|\psi_t(a)-\psi_t(b)\|=d_{tt^o}(a,b)$, (4) for all $t \in T$ and $a,b \in P$: $\psi_a(t)=\psi_b(t)$. Here, $\psi_a: T \rightarrow \mathbb{R}$ is defined by $\psi_a(t)=\pi_1(\psi(t,a))$, and $\psi_t: P \rightarrow \mathbb{R}^3$ is defined by $\psi_t(a)=\langle \pi_2(\psi(t,a)), \pi_3(\psi(t,a)),\pi_4(\psi(t,a))\rangle$. Roughly, ψ consists of just of the pair of the earlier Θ and s. Thus with the help of a CS people are able to describe motions by means of space-time coordinates. To each relevant event of the motion they assign four coordinates $\langle t, b_1, .., b_3 \rangle$ relative to CS $\langle t^o, t^1, a_o, .., a_3 \rangle$, where t gives the 'time-distance' to t^o and $b_1, .., b_3$ are the usual spatial coordinates relative to the spatial coordinate system $\langle a_o, ..., a_3 \rangle$.

Once again, independence from the choice of a special CS is empirically found out as follows. Whenever people investigate a concrete system yielding a model of M3 and whenever y and y' are two coordinate systems for x then the coordinatizations ψ and ψ' obtained by these coordinate systems are connected by a space-time transformation, i.e. there are $b \in \mathbb{R}$, $\not{b} \in \mathbb{R}^3$ and a real, orthogonal 3×3 matrix α such that $\psi' = \langle \pi_1(\psi)+b, \alpha(\langle \pi_2(\psi), \pi_3(\psi), \pi_4(\psi) \rangle) \rangle$. This experience is made as long as both coordinate systems y and y' in their spatial parts do not move relative to each other.

Now my story ends with the following observation. If $x = \langle P, T, \lessdot, \curlyeqprec, o, \underline{b}, \equiv, d \rangle \in M3$, if y is a CS for x and ψ is the unique coordinatization $\psi : T \times P \rightarrow \mathbb{R}^4$ given by y then s, defined by

$$s := \{ \langle p, \alpha, a, b, c \rangle / \exists t \in T (\langle t, p, \alpha, a, b, c \rangle \in \psi) \}$$

is a function $s : P \times \mathbb{R} \rightarrow \mathbb{R}^3$. If s is smooth then $\langle P, \mathbb{R}, s \rangle$ is a model of classical kinematics. And the class of particle systems obtained from $\langle P, \mathbb{R}, s \rangle$ by space-time transformations is identical with the class of all systems $\langle P, \mathbb{R}, s' \rangle$ obtained from one and the same $x \in M3$ by using different coordinate systems y' yielding different coordinatizations ψ', and consequently position functions s'. If we write $s_{x,y}$ to indicate that s comes from a model x of M3 via CS y in the way described above then the last statement can be formulated as follows. All and exactly all the space-time transforms of system $\langle P, \mathbb{R}, s \rangle$ can be obtained by fixing one space-time model $x \in M3$ and constructing all functions $s_{x,y}$ where y varies in the class of all possible coordinate systems for x.Expressed still differently we have: If $x=\langle P,T,\lessdot,\nless,\circ,\underline{b},\equiv,d\rangle\in M3$ and $M:=\{\langle P,\mathbb{R},s_{x,y}\rangle/$ y is a CS for x$\}$ then (1) any two members of M are connected by a space-time transformation and (2) if $z\in M$ and z' is obtained from z by a space-time transformation then $z'\in M$.

2. WHAT TO LEARN FROM THAT STORY?

1.) The story is not as fictituous as is suggested by the headline of Sec.1.In fact,the historical development on our planet led to Euclidean geometry.Although its axiomatic form and completeness of today was not available when CSs and CK entered the scene there is no doubt that the 'invention' of coordinate systems at the times of Descartes did presuppose the knowledge of Euclidean geometry in its not yet perfect form.Also, there is no historical doubt that clocks were constructed before CK became important.So the historical development from Euclidean geometry and its CSs via clocks and its CSs to kinematics roughly corresponds to that of our story.The only point I would admit to be totally fictituous is my treatment of time.I do not know of any historical sources (i.e.say,from before 185o) in which a foundational treatment of time is proposed which tends into the direction of extensive systems. (This is not so for space because if one examines Euclid's axioms one can find a number of axioms for extensive systems in his treatment of quantities,and the whole work is written in 'the spirit' of extensive systems.) Concerning time,however, foundational disputes have not ended up to now. For different reasons there _is_ no such commonly accepted theory of time as is Euclidean geometry for space.The development of CK as sketched above may also be fictituous in one certain aspect to which I will return in detail under 5.).But certainly it is correct in its rough logical structure which consists of bringing together independent concepts of space and time.Remember that

I have in mind only <u>classical</u> kinematics.

2.) The story shows how the problem mentioned in the introduction can be solved. The problem was that, since the models of CK are invariant under a class of transformations much bigger than that of space-time transformations, there should be some explanation of why physicists are interested only in space-time transformations. In the story the solution is very simple. We just have to think of CK as being constructed on a space-time theory of the form M3 via coordinate systems. If this underlying theory is considered as a part of CK we obtain a theory in which space-time transformations of the position function in fact are the only ones admitted. More precisely, if we take models of CK to be entities of the form $x = \langle P, T, \lessdot, \prec, \circ, \underline{b}, \equiv, d, s\rangle$ such that (1) $z := \langle P, T, \lessdot, \prec, \circ, \underline{b}, \equiv, d\rangle \in M3$ and (2) there exists a CS y for x such that s is connected with x via y in the way described above (after the introduction of 'space-time' CSs) then (3) if $\langle z, s\rangle$ is such a model and s' is a space-time transform of s then $\langle z, s'\rangle$ is a model, too, and (4) structures obtained from $\langle z, s\rangle$ by differ= ent transformations are no models, i.e. if $\langle z, s\rangle$ and $\langle z, s'\rangle$ are models then s and s' are connected by a space-time transformation.[2] Intuitively, we can say that the class of space-time transforma= tions is exactly the invariance class of position functions relative to underlying space-time structures and their being connected with these by CSs. Still more roughly, invariance under space-time transformations is the correct invariance for position functions allowed by underlying space-time structures. It is only for practical reasons (reasons of simplicity) that this connec= tion to underlying theories is (systematically?) suppressed in physical treatments.

3.) The story served to underline my thesis formu= lated in the introduction. The origin of CK's in= variance under space-time transformations lies in observations -as stressed in Sec.1- of changes of CSs. Whenever systems are observed from differ= ent CSs their descriptions by position functions are connected by space-time transformations. Thus it is observed that the metrical relations -expressed by the d of M3- are invariant under changing CSs which in turn yield space-time transforms of the position functions.

It might be objected that what I present here as an empirical finding also might have been deduced from other assumptions. It would have been possible as well, first, to observe that only certain changes of the CSs lead to identical results with respect to metrical relations. In this way people could have found a characterization of what might be called admissible CSs. And from the concept of admissible CSs space-time transformations might be deduced. In principle I would be satisfied with this story, too. For the only thing I want to stress is that invariance has empirical roots. And the alternative just mentioned certainly would have its origins in experience, too. For experience would be necessary to draw the distinction between admissible and non-admissible CSs. But I think the concept of an admissible CS -an admissible CS is not the same as an inertial system- did not play a central role in the actual development, and, since this alternative story does not yield more clarity, I stick to my story in Sec. 1.

Concerning the role of invariance the situation is a bit more difficult. The first aspect mentioned in the introduction, namely that invariance shows a certain freedom of choice for CSs is unproblematic. It even may seem trivial, although it is not. It is certainly of practical importance to know that some specified kinds of transformations of the CS do not affect the system under consideration. For instance, it is valuable to know that we can approach our object until we see it clear enough, or that we can go around, say, a picture in order to observe it from the front side and not from a very uninformative point of view situated on the wall were the picture is hanging.

But there is a second component in the role of invariance which, I must confess, has not been illuminated by my story. This role can be seen by passing from passive transformations to active transformations. Up to now we have always been thinking of passive transformations in the fo= llowing sense. It was assumed that the actual physical system under consideration remained un= affected while the CS -in its real representa= tion given by $\langle t^o, t^1, a_o, .., a_3 \rangle$- was thought to be changed. We obtain active transformations if the real CS $\langle t^o, t^1, a_o, .., a_3 \rangle$ is left unchanged and the physical system under consideration from the point of view of this CS is changed. Such a change can consist in bringing the physical system to another place or to 'start it' at a later time. As an ex= ample consider a pendulum swinging in a laboratory. If we pass from the CS given by the laboratory's walls to a CS installed immediately in front of the pendulum in form of some iron frame we have a passive transformation. If we take the pendulum, bring it to another corner of the laboratory, and there start it swinging again we have an active transformation.

Now although invariance primarily has come up with passive transformations its 'discovery' cer= tainly tempts to investigate the reverse situation of active transformations. And in fact there are quite a number of concrete situations in which space-time transformations can be executed acti= vely without much change of the system. This is true, for instance, for systems of solid bodies. It is also true for motions which in a certain sense can be controlled by humans, e.g. pendulae, cars and a wide class of technical applications. For this big class of phenomena with its overwhelming practical importance there is an empirical in= variance under active (not too extreme) space- time transformations. Since this kind of invariance in its theoretical description does not differ from the passive case it seems not unfair to take it as the second component of the role of invari= ance in CK.

4.) The story should please philosophers of science for two reasons. First, it contains a lo= gical reconstruction of the underlying theories of kinematics and therefore it makes explicit how classical mechanics is based or can be based on theories of space and time. By introducing suitable intertheoretic relations -as for instance <u>theoretization</u> (compare Balzer (1978))- we can build a small hierarchy of theories such that classical mechanics is the top element of this hierarchy and its basic elements are theories containing only qualitative basic terms. So we have one possible way of depicting how quantitative theories rest on qualitative, 'proto'-physical theories.

Second, the story throws some light on the connection between mechanics and measurement of space and time. For it contains conditions under which unique distance-values for space and time-distances can be obtained from qualitative operations -as expressed by the qualitative terms of M1 and M2. And it shows how these distance values -without essential additional assumptions- go into kinematical descriptions. I do not want to say, of course, that Euclidean geometry or extensive systems describe real measurements. But it seems plausible to expect descriptions of actual methods of measurement for space and time to yield substructures of models of geometry or extensive systems. Thus, the connection between actual measurement and mechanics can be worked out by studying sub-structures and their intertheoretic relations 'inside' the hierarchy mentioned above.

5.) A last point we can extract from the story is this. Space-time can be described in a way that makes no difference between points of space and particles. In order to understand precisely what I mean by this it is necessary to describe the alternative in which points and particles are treated differently. In this alternative approach spatial distances among <u>points</u> -in contrast to particles- are required to remain fixed in time. That is, the distances between any two points a,b are the same at all times t. In the language of M3 this amounts to requiring that for all $a,b \in P$ and all $t,t',t_1,t_2 \in T$: $d_{tt'}(a,b)=d_{t_1t_2}(a,b)$. Intuitively, this means that the spatial distances are independent of time. If the models in M3 are required to satisfy this additional condition then space-time becomes 'rigid'. By this I mean that the models of such a theory do not allow for motions. Remember that in M3 motion is described by change of spatial distances relative to time-distances. If the spatial distances are not allowed to change no motion is possible. These rigid models can be imagined as a sequence of models of geometry such that in any two spaces of the sequence all points are 'at the same place'. The visual picture of this situation is given by $\mathbb{R}^4$ (or rather by $\mathbb{R}^3$ with a two-dimensional space). For each $\alpha \in \mathbb{R}$ the cuts $\{\alpha\} \times \mathbb{R}^3$ are models of geometry, and all these cuts are neatly built one upon another such that 'vertically' there are no differences if we go through the cuts.

Kinematics in this approach is treated by introducing into such a rigid space-time finitely many particles moving around. This can be done, e.g. by introducing another set Q of objects -namely particles- and a function $i: Q \times T \rightarrow P$ assigning to each particle and each point of time a 'point of space', namely the point of space where the particle is at that time.

This way of reconstructing kinematics seems to be closer to physicists' present day thinking. It is just nice to have a neat, rigid space-time sharply to be distinguished from things moving around. Our considerations about invariance would remain essentially the same -perhaps with some slight complications- when we use this kind of reconstruction for CK. What we can learn from the story then is that this alternative way of reconstructing CK is not the only one. There are other possibilities, one of which consists in dropping the distinction between points of space and particles.[5)] In models of M3 points are allowed to change their distances during the flow of time. That is, there can be $t, t', t_1, t_2 \in T$ and $a, b \in P$ such that $d_{tt'}(a,b) \neq d_{t_1 t_2}(a,b)$. The disadvantage of this approach, in the physicist's view, consists of giving up a distinction, namely that between points of space and particles which seems intuitively clear. The question whether this intuition is sound I leave for Sec.3.

3. SOME POLEMIC REMARKS

I want to conclude with some more general remarks on classical space-time and present day physics which are likely to be felt as polemics because their origin is far away from present day para= digms -in Kuhn's sense- of physics.

Let me start by pointing out that today space-time physicists throughout use the language of manifolds and of representations by transforma= tion groups.My treatment in terms of 'old fashi= oned' logics comes really from an other world.I agree that modern representations of space-time are framed in a way which allows to depict vari= ous kinds of older and weaker theories in a way that is felt to be very elegant.As far as physics is concerned such a treatment is all right be= cause it shows how older theories are related to and how they can contribute to a better under= standing of the most recent theories.From the point of view of philosophy of science or history of science,however,this advantage may turn into a disadvantage.For what is gained by modernizing old theories -i.e.reformulate them in a modern frame- may be lost in understanding these older theories in an historically adequate way.It is of course difficult to say precisely what is an historically adequate representation.But twenty years of dis= cussion in the philosophy of science should have taught us that there is not only a possibility but even a certain probability of incommensurable world views.To stick to most modern reformulations of old theories as historically adequate therefore bears the risk to become rather one-sided -some even call it blind- in certain respects.

I do not want seriously to entertain the idea that a logical frame as used in Sec.1 could be used as an alternative to prevailing modes of description.But I do so only because obviously no= body has really thought about this possibility and because I have no precise elaboration.I do want,however,to say one more word about histori= cal adequacy.In the present example of space-time we can find a nice example of meaning-vari= ance -if not incommensurability- ,namely the term 'event'.Certainly this term has existed in Newton's times.But it was no technical term of mechanics.People were talking about events only in ordinary language as they still do today.Only after Minkowski's treatment of special relativity events became a technical term,in fact a central object-term of space-time theories.For now events are the basic objects out of which models consist.This role in classical theories was played by points (or particles) and points of time.It was essential to the notions of point and instant that both were independent from each other.Although the notion of a classical point of time is somehow unclear it can be hold for sure that it is different from our new concept of event.So the shift in the meaning of 'event' is that from an ordinary lang= uage term in which reference to space and time was not necessarily included to a technical term not only containing the concepts of space and time but essentially containing their depending on each other.[6] In the logical reformulation of Sec.1 events are not necessary as basic objects. So it has more chance to be historically adequate than modern physical formulations.This point be= comes even more drastic if we consider,say,Rie= mannian connections instead of events.Such things did not exist at the old times.Similarly,I would claim that to apply a four-dimensional frame is adequate only after Minkowski.

But in addition to this difference in lang= uage my treatment differs in content,as already indicated.For I have treated space-time not as rigid -allowing for motions 'in' space-time- whereas the tendency seems to be to distinguish space-time and kinematics and to introduce moving particles only after a rigid or at least in some sense complete space-time is at hand.[7]

Now besides having demonstrated that this must not necessarily be so I have three arguments in favour of non-rigid space-times.First,I have Ockham's razor.With this I cut off all terms which are not really necessary.And if the dis= tinction between points and particles is not really necessary -which is proved by showing how to do without it in Sec.1- one of these two is superfluous.Second,the 'rigid space-time view' has the problem of giving a physical meaning to the function i mentioned in 5.) of Sec.2.What does 'i(q,t)=p' mean? I do not want to say that it is impossible to give reasonable meaning to this statement.But it seems rather difficult to do so. The problem is to give meaning to the 'at point p' part independently from the fact that q at t is there.Third,and connected with the last remark,I can ask: What are points of space if they are not materialized or not materializable by particles? Do they exist? And how? I think,on a naive approach to the alternative "There is a clear distinction between points and particles" and "There is no sense to talk about points not materializable by particles" we need not nece= ssarily prefer the first one.But even if we pre= fer the first alternative the second one still provides a problem.This problem is not solved up to now.

Finally,in today's space-time theories as well as in other physical theories there is a tendency to seperate mathematics from physics which brings me to a point of theory-formation in general.What is the role of mathematics in empi= rical theories? A first move to answer this quest= ion is to look on the historical development and to say that mathematics -at least $\mathbb{R}^4$ and re= lated stuff- is the outcome of physical theories and therefore a kind of 'physical image' of some aspects of the world.But today those mathematical structures are used in quite different areas as well and the development of non-Euclidean geome= tries has shown that there is not one single true space-time structure.So one is led to admit a variety of mathematical 'images'.The result has been a total separation of mathematical formalism and 'physical reality',as expressed e.g.in Ludwig (1978).Such a separation,however,necessitates the introduction of additional correspondence prin= ciples relating the real and the mathematical parts of a theory to each other.Now what is the status of such correspondence principles? No clear examples can be found in the literature. No clear interpretation of such principles is available yet. In view of the fact that these difficulties come from the want for separation of mathematics from physics -and only from there- the alter=native of trying to diminish the need for such rules by diminishing the separation of mathematics from physics seems to become more and more attractive.

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NOTES

0) I am indebted to A.Kamlah for helpful discus=sions.
1) This notion of kinematics differs from that of McKinsey et al. (1953) in two respects. First, time is represented by the whole set of real numbers and not by an open interval. This is no real difference because both these things are isomorphic -at least topologically. Second, the set P of points or particles is not requir=ed to be finite. This difference is essential and the reason for it is discussed in 5.) of Sec.2.
2) This is not quite true because dilatations cannot be ruled out in this way. However, I will neglect this point since dilatations have a clear physical meaning -freedom of choice of the unit of measurement- which can be sharply distinguished from that of space-time trans=formations.
3) Of course 'true' here does not mean 'true in the sense of the usual inductive definition in all of the domain'. It just means that 'suffi=ciently many' instances of sufficiently 'de=quantified' forms of the axioms are true. In order to avoid complications axiom (13) of Tarski (1959) has to be replaced by the corres=ponding second order formula on page 18 l.c.
4) Since the betweenness relation is not a rela=tion among rods some interpretation is nece=

ssary here.$\underline{b}abc$ can be interpreted as saying that the rod with end points a,c is just the concatenation of the rods with end points a,b and b,c.

5) It is our neglect of this distinction that forces us to allow P to be infinite (compare footnote 1)).

6) This example is discussed in some more detail in Balzer (1978b).

7) In the case of general relativity this leads to a subtle situation.For there,space-time is not rigid at all.Rather it is 'formed' by the material particles.But it must be stressed that this is only a kind of 'surface' space-time because general relativity logically contains what I call a rigid and complete space-time.Such a space-time is implicit in the very mathematical formulation in the form of e.g. $\mathbb{R}^4$ which is needed to define 4-dimensional differantiable manifolds.

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