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Working Paper Series

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No.: FW15V2/05
First Draft: 2005-02-21
This Version: 2005-09-13

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Systematic Credit Cycle Risk

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by Marc Gürtler* and Dirk Heithecker**

Abstract. According to the new capital adequacy framework (Basel II) finally adopted by the Basel Committee in June 2004 the eligibility of collaterals, especially financial collaterals, is extended in comparison to the existing rules. However, financial assets are valued conservatively in the credit context which suggests a strong correlation between collaterals and credit default rates. This paper discusses the impact of the dependency of financial collaterals and default rates on credit risk. Therefore, a general calculation framework for the loss rate of collateralized loans is given and an analytical solution for the valuation of financial collaterals is presented. Finally, the model is applied on empirical data of German insolvencies and German capital markets.

Keywords: Basel II; Capital Adequacy Requirements; Value at Risk; Loss Given Default; Probability of Default; Collateral; Collateral Valuation

JEL-Classification: G21, G28

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1. Introduction

During the last 10 years credit portfolio models have improved significantly. This evolution has been enforced by the Basel Committee on Banking Supervision (BCBS) during the design process of the measure of credit risk in the International Convergence of Capital Measurement and Capital Standards (Basel II).[1] Whereas measuring the default probability of an individual debtor has been well investigated before,[2] most of research has been carried out in modelling and empirically investigating co-movements in the default frequency to determine the number of defaulted loans in a portfolio. Well known credit risk models that have been scientifically evaluated to a great extend are CreditPortfolioView, CreditRisk+, CreditPortfolioManager, and CreditMetrics, which use different approaches for modelling the correlation between defaults, but lead to homogeneous results when parameterized.[3]

However, not only the default rate of credits in a portfolio is of interest, but also the heaviness of loss in the event of default quoted as the loss rate of the amount outstanding (exposure). Most credit models take this so called Loss Given Default (LGD) or the corresponding Recovery (R, with LGD = 1 – R) of a credit exposure as an exogenously given constant parameter or stochastic variable whose value depends on easily identifiable characteristics, like the existence of a security, the seniority of the liability, or the rating of the borrower. However, the LGD does not dependent on the actual default rate of the credit portfolio. There are mainly two caveats of this treatment of the LGD: on the one hand the LGD need not to be independent from the actual default rate - indeed there are many empirical studies disproving this assumption. On the other hand the LGD cannot be analysed due to it components like recovery from a collateral and recovery from other firms assets.

Furthermore, if banks implement the Internal Ratings Based (IRB) Approach for regulatory credit risk measurement own estimates for the LGD have to be made, which ought to consider

[1] See Basel Committee on Banking Supervision (1999a, 2004).

[2] See Altmann and Saunders (1997) for a survey of research over the last 20 years.

[3] See Crosbie and Bohn (2003), Credit Suisse Financial Products (1997), Wilson (1997a,b) and Gupton, Finger and Bhatia (1997) for details about these models. Conceptual, empirical or numerical comparisons of these models are presented by Koyluoglu and Hickmann (1998a, b), Gordy (2000), Crouhy, Galai, and Mark (2000a) as well as Hamerle and Rösch (2004).

the relationship between the LGD, the collateral and the default rate. Concretely, it is specified that the bank[4]

(1) "[…] *must take into account the potential of the LGD […] to be higher […] during a period when credit losses are substantially higher than average*",

(2) "[…] *must consider the extent of any dependence between the risk of the borrower and that of the collateral or collateral provider*."

Additionally, the BCBS recently has concretized its standards for the quantification of the LGD parameter, that consists of the following components:[5]

(3) *"Identification of appropriate downturn conditions for each supervisory asset class within each jurisdiction."*

(4) *"Identification of adverse dependencies, if any, between default rates and recovery rates."*

(5) *"Incorporation of adverse dependencies, if identified, between default rates and recovery rates so as to produce LGD parameters for the bank's exposures consistent with identified downturn conditions."*

According to these requirements LGD-models should at least give answers to the potential existence of a link between default rates and recovery rates as well as appraise its impact on estimating an appropriate LGD. With respect to (2) this is essential for a collateral security in particular.[6]

This paper makes contributions to the ongoing research on understanding and modelling LGD focussing on the recovery from collaterals under downturn conditions. Firstly, we present a model that enables us to examine correlations between the collateral value and the observed default rates. Basically, we use the "one-factor" approach of CreditMetrics that provides the basic principle for measuring the regulatory capital requirements in Basel II.[7] Especially, we extend this model according to the approach of Frye (2000a, b), but we show, how recovery concerning the collateral can be separated from total recovery. For collaterals with a short workout-period, like financial collaterals, an analytical solution is given. Secondly, we demonstrate how the relationship between the value of collaterals and the "*downturn conditions*",

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[4] See Basel Committee on Banking Supervision (2004), paragraphs 468 and 469.

[5] See Basel Committee on Banking Supervision (2005), pp. 3-4.

[6] Precisely, significant negative correlation between the collateral value and the default rate serves as a sufficient constraint for the identification (4), see Committee on Banking Supervision (2005), p. 3.

[7] See e.g. Finger (2001) and Wilde (2001) for details of the models and the relation to Basel II.

here represented by the observed historical insolvency quotes, can be identified and quantified. Our analysis focuses on financial collaterals, like stocks, bonds, commodities, and cash in foreign currencies and examines the dependency on a systematic level by using merely indices. Finally, we show that our results lead to a similar reduction of regulatory credit risk like mentioned in the rules for financial collaterals specified in the IRB foundation approach of Basel II.[8] However, we also may give reasons for a slightly higher reduction as well.

The paper is organized as follows: The next section gives a brief overview on the LGD-related literature.[9] Section 3 describes the model that especially addresses the standards (4) and (5) when examine downturn effects with respect to (financial) collaterals. In Section 4 we present an empirical analysis to identify systematic risk between capital market and the cycle of insolvency quotes in Germany to meet standard (3). This Section ends with a discussion of the results about the consequences with respect to Basel II. Section 5 provides some ideas of potential future research topics.

2. Literature Review

During the recent five years there has been made main effort in ascertain the factors identifying the LGD of loans and bonds through different empirical investigations. Especially there exist many analyses on default databases made available by Moody's. The studies from Moody's itself[10] as well as from other authors like Frye (2003) or Hu and Perraudin (2002) suggest that the LGD depends on the seniority, the existence of a security, and on the yearly default rates. Studies by Acharya, Bharath, and Srinivasan (2004) as well as Düllmann and Trapp (2004) on datasets provided by Standard & Poor's and Portfolio Management Data retrieve the same results. However, other aggregated time series data examined by Altman, Bradi, Resti, and Sironi (2004), or Carey and Gordy (2003) show that the relationship between yearly default rates and LGD might be less strong and depend on the decade investi-

[8] See Basel Committee on Banking Supervision (2004), paragraphs 154-161. Furthermore, according to Franks, Servigny and Davydenko (2004) as well as Davydenko and Franks (2004) financial collaterals can be classified as an important collateral, since it is the second most often posted collateral besides real estate and can be liquidated by the bank itself shortly after default even in case of insolvency.

[9] A more detailed survey about empirical findings in LGD datasets and about modelling LGD is given by e.g. Schuermann (2004) and Altmann, Resti, and Sironi (2003). In this paper, we only highlight the recent results.

[10] See e.g. Hamilton, Cantor, and Ou (2002), Hamilton, Cantor, West, and Fowlie (2002), Keenan, Hamilton, and Berthault (2000), and Gupton, Gates, and Carty (2000).

gated. Although all studies approve the importance of collaterals, investigations on a periodic, e.g. yearly, dependency of collaterals are rather scare and controversial: The study of Araten, Jacobs, and Varshney (2003), which is based on default data of the JP Morgan Chase Wholesale Bank, finds no evidence that the LGD of a collateralized loan is linked to the economic cycle. In contrast Franks, Servigny, and Davydenko (2004) analyse default data of ten banks in the U.K., France, and Germany and point out that the realisation of collaterals is affected by the economic cycle. However, both studies use different measures for the "state of economy": Araten, Jacobs, and Varshney refers to the yearly default rates, Franks, Servigny, and Davydenko to the gross domestic product.

In comparison to the number of LGD studies there are only a few approaches concerning LGD modelling in credit risk models. They can be divided into two groups. The first category has its seeds in the framework presented by Frye (2000a, b). In such models the LGD is stochastic and driven by a single systematic factor which also serves as a systematic factor for the default process. The models only differ in the distribution of the factors: Frye uses the normal distribution, Pykthin and Dev (2002) as well as Pykthin (2003) apply the log-normal distribution, and Schönbucher (2003) makes use of the logit distribution.[11] The second category is recently presented by Tasche (2004). His framework assumes the LGD to be stochastic but co-monotonous with the probability of default. He also gives a numerical example in which the LGD is assumed to be beta distributed. In summary, both model categories agree in the assumption that the LGD is stochastically described by a single distribution function, i.e. the collateral is not evaluated separately.

One can conclude that so far there are no empirical studies concerning the dependency between the performance of collaterals and the default rate. One reason of this fact is the non-existence of an LGD-model, which distinguishes between recovery payments either taken from the collateral or received from other assets of the debtor.

3. The Model

In the following subsections we outline the framework of our model that builds the fundamental idea of the relationship between collateral value and default rate in a credit portfolio. The model is based on the two-state-one-factor return generating process from CreditMetrics that

[11] All these models are implemented on empirical data by Düllmann and Trapp (2004).

has its origin in the seminal firm-value framework developed by Merton (1974) and was extended to a portfolio approach by Vasicek (1987, 1991, see also 2002) and Finger (1999). It has mainly influenced the risk weighting function of the IRB-Approach of Basel II.[12] In the following subsection 3.1 we describe the framework to model default and loss rate of an individual loan.[13] Subsection 3.2 discusses the loss rate in a portfolio context. An analytical calculation formula of the VaR that especially fits with financial collaterals as a security is presented in subsection 3.3.

3.1 Default and Loss Rate of an Individual Loan

We assume a discrete-time model where the values of the collateral[14] $\tilde{C}_t$ and of the (other) firms asset $\tilde{A}_t$ during each period t are functions of the normally distributed (standardized) returns $\tilde{c}_t$ and $\tilde{a}_t$ with mean zero and unit variance.[15] We expect these "normalized" returns to be linked due to a correlation $\operatorname{corr}(\tilde{a}_t, \tilde{c}_t) = \rho_{AC}$. Without loss of generality we can rewrite the return due to principal component analysis as

$$\begin{aligned} \tilde{a}_t &= \sqrt{\rho_A} \cdot \tilde{x}_t + \sqrt{1-\rho_A} \cdot \tilde{\varepsilon}_{A,t}, \\ \tilde{c}_t &= \sqrt{\rho_C} \cdot \tilde{x}_t + \sqrt{1-\rho_C} \cdot \tilde{\varepsilon}_{C,t} \text{ if } \rho_{AC} \geq 0 \text{ or } \tilde{c}_t = -\sqrt{\rho_C} \cdot \tilde{x}_t + \sqrt{1-\rho_C} \cdot \tilde{\varepsilon}_{C,t} \text{ if } \rho_{AC} < 0, \end{aligned} \tag{1}$$

with $\tilde{x}_t \sim N(0,1)$, $\tilde{\varepsilon}_{A,t} \sim N(0,1)$, and $\tilde{\varepsilon}_{C,t} \sim N(0,1)$ all being independently, identically, and normally distributed with mean zero and unit variance. Additionally, we assume the independence of all $\tilde{\varepsilon}_\bullet$'s between different points in time t and $t + \Delta t$ in which $\Delta t \geq 0$ is a constant.[16] The variable $\tilde{x}_t$ is identified as the systematic factor, because it affects the value of

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[12] See e.g. Finger (1999, 2001) and Wilde (2001).

[13] Our approach is similar to Jokivuolle and Peura (2000), and Pykthin (2003), but in contrast to them we expect recovery in the case of default from the collateral as well as from the other firm assets.

[14] To keep track of the model, stochastic variables are marked with a tilde "~".

[15] In literature it is often assumed that assets follow a geometric Brownian Motion and therefore are considered as log-normally distributed with mean μ and standard deviation σ of the logarithmic returns. See e.g. Merton (1974), Jokivuolle and Peura (2000), and Pykthin (2003). Then, the asset value $\tilde{Y}_t$ depends on its normalized return $\tilde{y}_t$ according to $\tilde{Y}_t = Y_{t-1} \cdot \exp(\mu + \sigma \cdot \tilde{y}_t)$.

[16] Since default risks are often considered to be cyclical, see e.g. Basel Committee on Banking Supervision (2004), paragraphs 452, and Rösch (2003), the corresponding stochastic variables are auto correlated, and thus follow an auto regressive (AR) process. For the latter statement see Franke, Härdle, and Hafner (2004). Consequentially, the assumption of a geometric Brownian Motion (see footnote 15) would not hold. With the more

the collateral as well as the other assets simultaneously. For interpretation, $\tilde{x}_t$ builds up the common risk scenario for the collateral and the other firms assets. The $\tilde{\varepsilon}_\bullet$'s are the collateral and asset specific (idiosyncratic) risk factors. If the common factor $\tilde{x}_t$ is fixed the returns are independently distributed. Additionally, the two parameters ρ_A and ρ_C describe the fractions of risk expressed by the variance that can be explained by the systematic factor. For this reason, the identity $|\rho_{AC}| = \sqrt{\rho_A \cdot \rho_C}$ is valid.

Due to the one-factor approach of CreditMetrics the borrower defaults when his normalized return $\tilde{a}_T$ at time $t = T$ falls short of a threshold $b_A = N^{-1}(PD_A)$ in which PD_A is the expected default probability. Since b_A (in literature often denoted as the default point)[17] is exogenously given this model belongs to the category of so-called "ratings-based" approaches for quantifying credit risk.[18] Conditional on the systematic factor $\tilde{x}_T$ the expected default probability of the loan becomes

$$P(\tilde{a}_T < b_A \mid \tilde{x}_T) = N\left(\frac{N^{-1}(PD_A) - \sqrt{\rho_A} \cdot \tilde{x}_T}{\sqrt{1-\rho_A}}\right) =: p_A(\tilde{x}_T).^{19} \tag{2}$$

In order to evaluate expectation of the uncertain loss rate $\tilde{\ell}^{(d)}_{C,T+\delta T}$ of the secured loan in the event of default (signed with index "d") we allow for a workout period δT which in turn implies recovery to take place at $t = T + \delta T$. In addition, we make the following assumptions:[20]

(A) In the event of default the total liabilities B hold by numerous creditors are payable, including the exposure L of the loan of the bank. During the work-out period δT no additional claims will be added.

general approach of only normally distributed "normalized" returns in a discrete time context (with constant Δt) the model still is valid even if auto correlation is present.

[17] See Crosbie and Bohn (2003), p. 7, Crouhy, Galai, and Mark (2000b), p. 373, or Ong (1999), p. 85. Additionally, it has to be mentioned that due to Basel II default occurs before realising a collateral security, see Basel Committee on Banking Supervision (2004), paragraph 452. Therefore the collateral has to be neglected when modelling default.

[18] See e.g. Gordy (2003).

[19] See e.g. Vasicek (2002), p. 160.

[20] These assumptions are made with respect to the investigation of the collateral in the recovery process and are therefore kept strong for simplification. Less strong assumptions are possible.

(B) After liquidation of the collateral the bank receives a claim on a fraction of the firms assets that is equal to the exposure of the loan after liquidation of the collateral with respect to the residual total liabilities of the borrower.

(C) No other collaterals (of other lenders) have to be considered.

(D) In the event of default the value of the collateral is small in comparison with the total liabilities B of the borrower.

Conditional on the systematic factor $\tilde{x}_{T+\delta T}$ at $t = T + \delta T$ the value $\tilde{C}_{T+\delta T}$ of the collateral is independent from the other firms assets $\tilde{A}_{T+\delta T}$. Therefore, in the event of default the conditional expected loss rate $LGD_{sec}(\tilde{x}_{T+\delta T})$ of the secured loan becomes[21]

$$LGD_{sec}(\tilde{x}_{T+\delta T}) := E\left(\tilde{\ell}^{(d)}_{sec,T+\delta T} \mid \tilde{x}_{T+\delta T}\right) = \frac{1}{L} \cdot E\left(\tilde{L}_{eff}(\tilde{x}_{T+\delta T})\right) \cdot LGD_{unsec}(\tilde{x}_{T+\delta T}) \tag{3}$$

with $LGD_{unsec}(\tilde{x}_{T+\delta T}) := E\left(\tilde{\ell}^{(d)}_{unsec,T+\delta T} \mid \tilde{x}_{T+\delta T}\right)$ and $\tilde{L}_{eff}(\tilde{x}_{T+\delta T}) := \max\left(L - \tilde{C}_{T+\delta T} \mid \tilde{x}_{T+\delta T}; 0\right)$, (4)

where $LGD_{unsec}(\tilde{x}_{T+\delta T})$ stands for the expected loss rate of an unsecured loan and $\tilde{L}_{eff}(\tilde{x}_{T+\delta T})$ labels the "effective" exposure that the bank claims from the other firms assets $\tilde{A}^{(d)}_{T+\delta T}$ (e.g. in the insolvency procedure), both conditional on $\tilde{x}_{T+\delta T}$. Thus, the conditional expected loss rate $LGD_{sec}(\tilde{x}_{T+\delta T})$ can be calculated from the (conditional) expected loss rate $LGD_{unsec}(\tilde{x}_{T+\delta T})$ of an unsecured credit and the expected effective exposure $\tilde{L}_{eff}(\tilde{x}_{T+\delta T})$.[22] Consequentially, the effect of a collateral on the expected (conditional) loss rate is quite simple in the sense that it just reduces the exposure of the loan from L to $\tilde{L}_{eff}(\tilde{x}_{T+\delta T})$. Thus, the collateral can be evaluated separately in the credit context.

From equations (1) and (2) we get that a low value of the systematic factor $\tilde{x}_T$ at $t = T$ implies a low return of the firms asset which in turn leads to a high default probability of the borrower. Due to equations (1), (3) and (4) in the case of a positive correlation $\rho_{AC} \geq 0$ (between the return of the collateral and the return of the firm) the return of the collateral $\tilde{c}_{T+\delta T}$ for low values of the systematic factor $\tilde{x}_{T+\delta T}$. Thus, both – the default probability of the firm and the

[21] See appendix A.1 for details.

[22] This result is similar to the approach made within the framework of Basel II, see Basel Committee on Banking Supervision (2004), paragraph 291.

value of the collateral – are simultaneously affected negatively.[23] Since this property is caused by the common systematic factor it can be named "systematic collateral credit risk". However, it is worth mentioning that if there is a negative correlation $\rho_{AC} < 0$ the expected value of the collateral rises if the default probability increases. In this case the collateral "hedges" the losses.

3.2 Modelling Systematic Collateral Credit Risk

After discussing the framework of an individual loan we will analyse how the "systematic collateral credit risk" affects the credit risk within a portfolio. Therefore, we again refer to the analytical solution of the model of CreditMetrics,[24] based on the assumption that

(E) there is only one systematic risk factor for all borrowers i in the portfolio $i \in \{1,\ldots,n\}$,

(F) the loan portfolio is "infinitely homogeneous", i.e. it consists of an infinite number of credits, and the parameters (with respect to default) of each obligor in the portfolio are assumed to be the same.[25]

With this simplifications the impact of an individual loan on the portfolio default rate DR_T^∞ and the portfolio loss rate LR_T^∞ can be evaluated easily, because both depend on the sensitivity of the individual loan with respect to the systematic risk factor but not on the composition of the portfolio.[26]

Precisely, in order to evaluate the default rate DR_T^∞ of the portfolio due to assumption (F) all borrowers i with $i \in \{1,\ldots,n\}$ are of the same type as described in section 3.1 and thus the default process is identical. Therefore, their normalized returns $\tilde{a}_{i,t}$ are represented by the one-factor approach (see equation (1)) with equal parameters ρ_A, PD_A, a joint systematic fac-

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[23] Additionally, for this effect a positive autocorrelation between $\tilde{x}_T$ and $\tilde{x}_{T+\delta T}$ has to be assumed.

[24] See Vasicek (1991, 2002), and Finger (1999). For a model extension on collaterals see Frye (2000a, b), Pykthin and Dev (2002), Pykthin (2003), and also Düllmann and Trapp (2004). However, Frye does not distinguish between recovery payments from the collateral and from other assets. For an interpretation of this model see Altmann, Resti and Sironi (2004), p. 10.

[25] The more general case of an so-called "infinitely granular" portfolio is discussed by Gordy (2003) and could be implemented here as well. However, as in Gordy (2000) and Rösch (2003), for parameter estimation borrowers are grouped in homogeneous risk buckets, e.g. industry sectors, which are treated as homogenous portfolios.

[26] This characteristic can be called "portfolio invariant". For a detailed derivation of this issue see Gordy (2003), Bluhm, Overbeck, and Wagner (2003), pp. 83-85, or Schönbucher (2003), pp. 305-307.

tor $\tilde{x}_t$ and independent idiosyncratic factors $\tilde{\varepsilon}_{\bullet}$'s. Default at t = T occurs if $\tilde{a}_{i,T}$ falls short of a threshold b_A. Conditional on $\tilde{x}_T$ all normalized returns $\tilde{a}_{i,T} \mid \tilde{x}_T$ are independently distributed. Since the portfolio is assumed to be "infinitely homogeneous" the idiosyncratic risk is diversified completely and the default rate $DR_T^{\infty}(\tilde{x}_T)$ of the portfolio in period T becomes (almost surely)[27]

$$DR_T^{\infty}(\tilde{x}_T) = \lim_{n\to\infty}\left(\frac{1}{n}\cdot\sum_{j=1}^{n} I\left(\tilde{a}_{j,T} < b_A \mid \tilde{x}_T\right)\right) = E\left[I\left(\tilde{a}_{i,T} < b_A \mid \tilde{x}_T\right)\right] = p_A(\tilde{x}_T) =: N(DP_T^{\infty} \mid \tilde{x}_T) \tag{5}$$

in which $DP_T^{\infty} \mid \tilde{x}_T$ denotes the default point of the portfolio with

$$DP_T^{\infty} \mid \tilde{x}_T = \frac{N^{-1}(PD_A) - \sqrt{\rho_A}\cdot\tilde{x}_T}{\sqrt{1-\rho_A}} = b_A(\tilde{x}_T). \tag{6}$$

Because the periodic change of the systematic factor $\tilde{x}_T$ obviously controls the periodic change of the default rate $DR_T^{\infty}(\tilde{x}_T)$ of the portfolio (or an investigated risk bucket) it can be interpreted as a measure of the "state of the credit cycle".

Similarly, the idiosyncratic risk of each borrower i does not have any impact on the loss rate of the portfolio during the workout-period from T to T + δT which can be calculated as the exposure weighted sum of the expected conditional loss rates of each borrower i with exposure L_i conditional on the systematic factor $\tilde{x}_{T+\delta T}$. Precisely, we get (almost surely)[28]

$$LR_T^{\infty}(\tilde{x}_{T+\delta T}) := \lim_{n\to\infty}\left(\sum_{i=1}^{n}\omega_i\cdot LGD_i(\tilde{x}_{T+\delta T})\right) \tag{7}$$

$$\text{with } \omega_i = L_i \Big/ \sum_{k=1}^{n} L_k \text{ and } LGD_i(\tilde{x}_{T+\delta T}) := E(\tilde{\ell}_{i,T+\delta T}^{(d)} \mid \tilde{x}_{T+\delta T}). \tag{8}$$

Again, for the portfolio loss rate only the individual expected conditional loss rates $LGD_i(\tilde{x}_{T+\delta T})$ and the weight w_i of the individual loan are of interest.

From equation (2) and (5) as well as (3) and (8) we can conclude for the individual loan investigated in subsection 3.1 that the possible positive linkage between the normalized

[27] See e.g. Gordy (2003) and Vasicek (2002) for details. For the following equation I(S) indicates the indicator function with I(S) = 1, if statement S is true, and I(S) = 0 otherwise.

[28] The following notation is similar to Bluhm, Overbeck, and Wagner (2003), p. 88. Precisely, $\lim_{n\to\infty}\left(\sum_{i=1}^{n}\omega_i\cdot\tilde{\ell}_{i,T+\delta T}^{(d)} \mid \tilde{x}_{T+\delta T} - \sum_{i=1}^{n}\omega_i\cdot E\left(\tilde{\ell}_{i,T+\delta T}^{(d)} \mid \tilde{x}_{T+\delta T}\right)\right) = 0$ is valid.

(lagged) return of its collateral $\tilde{c}_{T+\delta T}$ and $\tilde{x}_T$ (because of $\rho_{AC} \geq 0$ and positive autocorrelation between $\tilde{x}_T$ and $\tilde{x}_{T+\delta T}$) negatively affects the loss rate $LR_T^{\infty}(\tilde{x}_{T+\delta T})$ and positively affects the default rate $DR_T^{\infty}(\tilde{x}_T)$ of the portfolio. Again, the "systematic collateral credit (cycle) risk" can be observed: simultaneously, due to the systematic risk factor, the number of defaults in the portfolio rises while the loss lowering effect of the collateral declines. Also, if there is a negative correlation between $\tilde{c}_{T+\delta T}$ and $\tilde{x}_T$ the loss lowering impact of the collateral will strengthen when defaults increase.

3.3 An Analytical Approach for Estimating the VaR

For quantification of the impact of the "systematic collateral credit risk" on the evaluation of collaterals it is possible to derive an analytical formula for the risk measure Value at Risk (VaR). Therefore, we make the following additional assumptions:[29]

(G) The collateral is re-evaluated permanently and therefore possible changes in the value only have to be considered over the work-out period.[30]

(H) The collateral asset is log-normally distributed with drift parameter $\mu_{C,\delta T} \approx 0$ and volatility $\sigma_{C,\delta T} \ll 1$.[31]

(I) The work-out period δT for the collateral is short and no changes in the systematic factor have to be expected.

With respect to assumptions (G) and (H) the value C_T of the collateral at default is known and it becomes

$$\tilde{C}_{T+\delta T} = C_T \cdot \exp\left(\mu_{C,\delta T} + \sigma_{C,\delta T} \cdot \tilde{c}_{T+\delta T}\right) \tag{9}$$

at the end of the work-out period. Due to assumption (I) we get $\tilde{x}_{T+\delta T} \approx \tilde{x}_T$, and $\tilde{x}_T$ serves for the default probability and for the loss rate in the event of default as the (only) systematic factor. We get for the "normalized" return of the collateral

$$\tilde{c}_{T+\delta T} \approx \pm\sqrt{\rho_C} \cdot \tilde{x}_T + \sqrt{1-\rho_C} \cdot \tilde{\varepsilon}_{C,T+\delta T} . \tag{10}$$

[29] Especially financial collaterals due to the comprehensive approach of Basel II fulfil these assumptions. See Basel Committee on Banking Supervision (2004), paragraphs 145-181.

[30] Due to Basel II for financial collaterals a work-out period of only 20 days has to be considered if the financial asset is re-evaluated daily. See Basel Committee on Banking Supervision (2004), paragraph 167.

[31] This assumption directly follows from assumption (G) and is often used in VaR-approaches using Delta-approximation. See Jorion (2001), p. 255.

Accordingly, we refer to a "true" one factor model like it is explained in Gordy (2003) or Vasicek (2002) as well as Finger (1999). The VaR contribution of each credit to the portfolio VaR can be calculated by using the expected loss rate conditional on the α-quantile $q_\alpha(\tilde{x}_T)$ of $\tilde{x}_T$, i.e. $P\left(\tilde{x}_T \le q_\alpha(\tilde{x}_T)\right) = \alpha$.[32] Resulting from this, the effective exposure L_{eff}^{VaR} of the collateralized loan under a VaR-measure can be written as[33]

$$L_{eff}^{VaR} \approx \max\left(L - C_T \cdot \left(1 + H_{sen}\right);0\right) \tag{11}$$

with $H_{sen} = \sqrt{\rho_C} \cdot \sigma_{C,\delta T} \cdot q_\alpha(\tilde{x}_T)$ if $\rho_{AC} \ge 0$ and $H_{sen} = -\sqrt{\rho_C} \cdot \sigma_{C,\delta T} \cdot q_\alpha(\tilde{x}_T)$ if $\rho_{AC} < 0$. (12)

Therefore, the effective exposure of the loan can be interfered from the exposure L minus the actual value of the collateral adjusted by a haircut H_{sen}. In the presented formula, the haircut especially relies on the correlation parameter ρ_C, meaning that only the standard deviation of the collateral value with respect to the systematic credit cycle is of interest. We therefore call this haircut the "(systematic) credit cycle sensitive haircut" because it only reflects the systematic risk of the collateral subject to the credit cycle. Consequentially, the haircut is negative if the value of the collateral is expected to decline with the default rate ($\rho_{AC} \ge 0$), i.e. if "systematic collateral credit risk" is present. However, if the collateral is likely to rise if the default rate declines ($\rho_{AC} < 0$) it may be positive.[34]

In contrast the effective exposure under a VaR-approach with assumption (G) and (H) using a haircut that accounts for systematic as well as idiosyncratic risk of the collateral would be

$$L_{eff}^{VaR} \approx \max\left(L - C_T \cdot \left(1 + H_{con}\right);0\right) \text{ with } H_{con} = \sigma_{C,\delta T} \cdot q_\alpha(\tilde{x}_T).^{35} \tag{13}$$

With respect to our model we call this a "conservative haircut" since it reflects the worst case in which the return of the collateral is perfectly correlated with the returns of the other assets

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[32] In credit risk portfolio models, the VaR with confidence level (1-α) is defined as the (1-α)-Quantile (or α-Fraktile) of the portfolio loss distribution. See Gordy (2003), p. 205.

[33] For details on the derivation see appendix A.2.

[34] For both cases we imply that $\alpha < 0{,}5$ and $q_\alpha(\tilde{x}_T) < 0$. Additionally, from a conservative risk measurement prospective, positive haircuts should not be used, at most a haircut of zero would be appropriate. With respect to Basel II, positive haircuts are forbidden, since the LGD under downturn conditions shall not be lower than the expected ("long-run default-weighted") LGD. See Basel Committee on Banking Supervision (2004), paragraph 468, and Basel Committee on Banking Supervision (2005), p. 4.

[35] This formula for the haircut also results from the instructions of Basel II for estimating own haircuts for financial collaterals, since "*a 99th percentile, one tailed confidence interval is to be used*". See Basel Committee on Banking Supervision (2004), paragraph 156.

and therefore with the default rate of the portfolio ($\rho_{AC} = 1$). From a conservative point of view this approach might be most appropriate.

4. Empirical Results

It has been shown in the previous section that the impact of the collateral value (or its normalized return $\tilde{c}_t$) on the "state of the credit cycle" denoted by $\tilde{x}_t$ plays an important role for the evaluation of the collateral in the credit portfolio.[36] The following subsections deal with the detection of "(systematic) collateral credit cycle risk" from empirical data. Concretely, we focus on the quantification of the positive correlation $\rho_{AC} \geq 0$. In our analysis we use insolvency rates from Federal Statistical Office of Germany (FSO) to determine credit cycles and German indexes of the capital market as systematic variables for financial collaterals.[37] In the next subsection 4.1 the estimation framework is presented. The data series are described in subsection 4.2 and 4.3. The correlation analysis is carried out in subsection 4.4 and its impact on haircut measurement is shown in subsection 4.5.

4.1 Parameter Estimation

In order to estimate the influence of the "systematic collateral credit risk" on financial collaterals we use a two-step procedure to determine the correlation between the systematic factor and the collateral return. The framework is similar to Düllmann and Trapp (2004), Pykthin (2003) and Frye (2000b), but we suggest slightly different formulas based on a point estimator.

[36] With respect to Basel II, a dependency between those two parameters is of special interest when identifying adverse relationships between recovery rates and default rates in order to estimate LGDs under economic downturn conditions. See Basel Committee on Banking Supervision (2005), p. 3, and also citation (4) in section 1.

[37] Therefore, our analysis only accounts for correlation between (financial) collaterals and the credit cycle using highly aggregated data. Thus, this analysis only detects a systematic dependency because we do not rely on specific loan portfolios with collaterals. However, banks often fail to present long data series to examine a systematic relationship between default rates and collateral values over a long horizon. Our approach uses a time horizon of up to 40 years and therefore may serve as a secondary study for verification of a adverse dependency between LGD and default rates, that the BCBS claims to be investigated. See Basel Committee on Banking Supervision (2005), p. 3.

We derive parameters using a Method of Moments[38] framework. In this framework the correlation ρ_A and the probability of default PD_A can be calculated by the formulas[39]

$$\hat{\rho}_A = \frac{\hat{\sigma}^2_{DP}}{1+\hat{\sigma}^2_{DP}} \text{ and } \widehat{PD}_A = N\left(\frac{\hat{\mu}_{DP}}{\sqrt{1+\hat{\sigma}^2_{DP}}}\right) \tag{14}$$

in which $\hat{\sigma}^2_{DP}$ is the estimated variance of the default point and $\hat{\mu}_{DP}$ the estimated expectation of the default point. In this context we have to mention that we use default points DP_T^∞ instead of default rates DR_T^∞ because due to equation (6) default points $\widetilde{DP}_T^\infty$ are normally distributed for all T:

$$\widetilde{DP}_T^\infty \sim N\left(\mu_{DP};\sigma^2_{DP}\right) \text{ with } \mu_{DP} = N^{-1}(PD_A)/\sqrt{1-\rho_A} \text{ and } \sigma^2_{DP} = \rho_A/\left(1-\rho_A\right). \tag{15}$$

To have reasonable estimators the default points $\widetilde{DP}_T^\infty$ are assumed to be independent. This assumption is of course critical since default rates (and therefore default points) are possibly cyclical.[40] However, our procedure is in line with the well established estimation approach for $\hat{\rho}_A$ and $\widehat{PD}_A$.[41] It can be justified by the fact that $\widehat{PD}_A$ represents the expected (long-run) default probability and deviations from this expectation value only stem from realisations of the systematic factor $\hat{x}_T$.[42] Since $\hat{x}_T$ is directly linked to the observed default rate DR_T^∞ (due

[38] In contrast to the general application of the Method of Moments estimators, see e.g. Greene (2003), pp. 525-557. We use the sample variance instead of the empirical variance for the second central moment. Our estimator is similar to the one of Bluhm and Overbeck (2003), but uses default points instead of default rates and thus can be derived analytically.

[39] See appendix A.3 for details.

[40] See e.g. Löffler (2003), who assumes an autoregressive process of the order of two (AR(2)-process) for the default rates. The BCBS stresses, that parameter estimations for LGD have to be done over at least one complete economic cycle. See Basel Committee on Banking Supervision (2004), paragraph 472.

[41] See the Maximum Likelihood Estimator by Gordy and Heitfield (2002), p. 7, Rösch (2003), p. 312, Demey, Jouanin, Roget, and Roncalli (2004), p. 105, Düllmann and Trapp (2004), p. 8, Hamerle, Liebig, and Rösch (2003), p. 78 as well as the Method of Moments estimator by Gordy (2000), p. 133, and Bluhm and Overbeck (2003), p. 40.

[42] In the literature the underlying approach for this interpretation is called “point in time”, see Düllmann and Trapp (2004), p. 8, Footnote 9. A “point-in-time” approach for credit risk modelling does not take into account the actual state of the “business cycle”, i.e. it is assumed for the risk measure, that no information of the current “point in the credit cycle” is available. The model of Basel II follows such an approach. For further information see Basel Committee on Banking Supervision (1999b), p. 28.

to equation (5)), $\hat{x}_T$ serves as a factor to identify “downturn” and “upturn” conditions of the portfolio like mentioned in citation (3) in section 1.[43]

According to (14) we only have to determine the estimators $\hat{\mu}_{DP}$ and $\hat{\sigma}^2_{DP}$. Here we use the empirical estimators[44]

$$\hat{\mu}_{DP} = \frac{1}{\tau} \cdot \sum_{T=1}^{\tau} DP_T^{\infty} \text{ and } \hat{\sigma}^2_{DP} = \frac{1}{\tau - 1} \cdot \sum_{T=1}^{\tau} \left(DP_T^{\infty} - \hat{\mu}_{DP} \right)^2 \tag{16}$$

with $T \in \{1,\ldots,\tau\}$ an arbitrary point in time of a time series of defaults (with length τ). Finally, from equation (6) the estimated realisation of the systematic factor $\hat{x}_T$ can be inferred from the estimates $\hat{\rho}_A$, $\widehat{PD}_A$ (for ρ_A, PD_A), and the observed default points DP_T^{∞} for all points in time $T \in \{1, \ldots, \tau\}$.

At the first glance the estimated values $\hat{x}_T$ for the systematic factor (as well as $\hat{\rho}_A$) seem to depend on the default probability PD_A (or its estimation $\widehat{PD}_A$). In contrast, the default probability PD_A in the model is not linked with the asset correlation ρ_A. Furthermore, ρ_A (> 0) is the only parameter that is responsible for a deviation between default rate and PD_A (see equation (2)). However, the estimators (14) and (16) possess the relevant characteristic that the expectation of $\hat{x}_T$ (and $\hat{\rho}_A$) does not depend on PD_A.[45] Therefore the parameters are unbiased.

In addition (see subsection 4.3), we estimate the parameters of the collateral for the lagged periods $T + \delta T \in \{1 + \delta T, \ldots, \tau + \delta T\}$ since during $T + \delta T - 1$ and $T + \delta T$ the collaterals of the defaulted loans are liquidated. We assume the value of the collateral to be log-normally distributed (see equation (9), here using a period of length T) with the parameter $\hat{\mu}_{C,T}$ for the

[43] Precisely, we identify “downturn” and “upturn” conditions directly with the (historical) default rates like it is suggested by the BCBS, see Basel Committee on Banking Supervision (2005), p. 3. However, other sources, e.g. economic variables like the GDP growth and the unemployment rate, should be used further. Since this approach is already well known, see e.g. Altmann, Resti, and Sironi (2005), we here present a more “direct” approach.

[44] See appendix A.3 for details and for the calculation of the asymptotic covariance matrix to compute the standard error of the estimation.

[45] See appendix A.4 for a proof of this statement.

drift and $\hat{\sigma}_{C,T}$ for the volatility. We use a similar estimation framework as in equations (14) and (16).

Finally (see subsection 4.4), we calculate the normalized returns $\tilde{c}_{T+\delta T}$ during the liquidation period, and analyse their correlations with the systematic (credit cycle) factor $\tilde{x}_T$.

4.2 Credit Risk Cycles for Germany

For the first step of the analysis as described in section 4.1 we use a broad database of yearly insolvency rates from 1962 until 2003 made available by the Federal Statistical Office of Germany (FSO). For a more detailed analysis it will be accounted for six major industry sectors due to NACE-code:

(1) Energy and Mining
(2) Manufacturing
(3) Construction
(4) Wholesale and retail trade
(5) Transport, storage and communication
(6) Financial intermediation.

This sectional data series are obtainable from 1965 to 2003. All rates are related to the territory of former West Germany.[46]

It has to be stated that insolvency is only one of the default attributes defined in Basel II.[47] For simplification we identify insolvency and default as it has also been done by Rösch (2003). Deviations between insolvency rates and Basel II default rates would not lead to different results as long as they are systematic since the long-term default probability PD_A is not important for estimating the “state of the credit cycle”. Only the deviation from this mean is essential for this state. Further, the data of the FSO might serve as an unbiased proxy for the credit cycle in Germany, especially for small and medium sized companies, because on average 2,000,000 enterprises are included and more than two third are personal undertakings. As a result of this statement, the idiosyncratic risk is negligible and the data base can be treated as “infinitely homogeneous”. Additionally, resulting from the long time series a reliable analysis can be outlined.

[46] A similar database for determining the credit cycle of Germany is used in Rösch (2003). See also Hamerle, Liebig, and Rösch (2003) and Bögelein, Hamerle, Rauhmeier, and Scheule (2002).

[47] See Basel Committee on Banking Supervision (2004), paragraph 452.

As a start, we check up on the insolvency rate, if it fits with the general assumptions of the default rate DR_T^∞ and default point DP_T^∞ in the Vasicek-model (see equation (5) and (6)). The data of Entire Germany is shown in Figure 1.

Figure 1 Default Rates and Default Points for Entire Germany, 1962 – 2003

From various statistical tests the default point DP_T^∞ of the insolvency rates can be characterized by

(I) incorporating a positive time trend,

(II) being strongly auto correlated but trend-stationary,

(III) being normal distributed (after detrending).

Assumption (I) can be verified by regressions of the DP_T^∞ and the time de-trended[48] default point $DP_{T,\text{detrended}}^\infty$ using an Augmented Dickley Fuller (ADF) test with and without time trend as well as by an autoregressive regression of DP_T^∞ and $DP_{T,\text{detrended}}^\infty$.[49] In each of both cases the two compared regressions only slightly differ (except for the time trend), that serves as a strong hint, that a time trend is present. We receive assumption (II) from tests on auto correlation by Ljung-Box-Pierce, Godfrey-Breusch (Lagrange Multiplier), and Durbin-Watson,[50] that are all significant on a 1 % level. The hypothesis of stationary could be maintained at least on a 10 % level using the test from Kwiatkowski, Phillips, Schmidt, and Shin (KPSS).[51] Assumption (III) might hold, since the hypothesis of $DP_{T,\text{detrended}}^\infty$ to be normal distributed can be maintained at least on a 1% level for the tests of Jarque-Bera, Kolmogorov-Smirnov-

[48] For detrending, the function $DP_{T,\text{detrended}}^\infty = \left(DP_T^\infty + \beta\right) - \alpha \cdot \tau$ with $\tau \in \{-21, \ldots, 20\}$ was used. Here α represents the time trend and $\left(DP_T^\infty + \beta\right)$ the constant. The parameters α and β are fitted to the data using least squares.

[49] See appendix A.5 for the results of the regressions. As can be seen from the regression analysis the trend variable is highly significant. This might be sufficient for discovering a time trend due to a t-test. See e.g. Hamilton (1994), p. 461.

[50] See Greene (2003), pp. 268-271. The results are reported in appendix A.6.

[51] See Franke, Härdle, and Hafner (2000), pp. 168-170. The results are reported in appendix A.7. However, the hypothesis of non-stationary in the ADF-test could not be rejected on a 90 % level, see the result in appendix A.5. Since the ADF-test may lead to loose results when testing highly auto regressive AR processes (see the discussion of unit root tests e.g. in Hamilton (1994), pp. 447-448, or Chatfield (2004), pp. 262-264, and since the stationary constraints of AR process are fulfilled in any case in the autoregressive regression (see appendix A.5), we prefer the results of the KPSS test.

Lilliefors, Geary, Sharpiro-Wilk and D'Agostino-Person.[52] Therefore, the assumption of CreditMetrics as well as within the Basel II framework holds that the normalized return $\tilde{a}_T$ (and therefore the default point $\widetilde{DP}_T^{\infty}$) for discrete time observations $T \in \{1, \ldots, \tau\}$ is normal distributed with cyclical patterns.

TABLE 1 Estimations of $\hat{\rho}_A$ and $\widehat{PD}_A$ for Entire Germany and the Six Sectors

However, when estimating the parameters $\hat{\rho}_A$ and $\widehat{PD}_A$ in order to calculate a time series for the systematic credit cycle factor $\tilde{x}_T$, the detrended default point $DP^{\infty}_{T,detrended}$ is to be used. This is necessary since the (normalized) systematic factor $\tilde{x}_T$ in the one-factor model of CreditMetrics (see equation (2)) does not reflect an increase of PD_A but a fluctuation around PD_A.[53] The results for $\hat{\rho}_A$ and $\widehat{PD}_A$ are shown in Table 1 and are all highly significant. The asset correlations vary through the sector between 0.72 % for sector 5 (Transport, Storage, and Communication) and 1.83 % for sector 2 (Energy and Mining).[54]

FIGURE 2 Detrended Default Rates and the Systematic Credit Cycle Factor for Entire Germany

Finally, we calculate the estimated realisation of the systematic factor $\hat{x}_T$ from the received parameters. As it is shown in Figure 2 for entire Germany the realisations of $\hat{x}_{T,GER}$ are nearly linear inversely linked to the default rate $DR^{\infty}_{T,detrended}$ since these default rates are rather low. The cyclicality of $\hat{x}_T$ is readily identifiable in Figure 2 as it is expected from theory, and

[52] See Zar (1999), Thode (2002), and Thadewald and Büning (2004) for details. The statistics is presented in appendix A.7.

[53] Although the time trend is not relevant for the calculation of $\hat{\rho}_A$ and $\hat{x}_T$ this trend is important when calculating PD_A since the time trend influences the expectation of the default rate.

[54] The result for $\hat{\rho}_A$ is rather low compared to those of Basel II, that allows for asset correlations of 0.12 up to 0.24. See Basel Committee on Banking Supervision (2004), paragraph 273. However, in the literature these distinctive low values are often reported when using insolvency rates. See Düllmann and Scheule (2003) or Rösch (2003) for Germany, Dietsch and Petey (2002) for France as well as Hamerle, Liebig, and Rösch (2003) for Germany and U.S. This might be caused by the large number of enterprises incorporated in insolvency studies leading to low one-factor dependencies. When determining credit cycles for correlation analysis, as in the present paper, this is not a problem since correlation is standardised by the variance anyway.

more distinctive than reported from other data (e.g. from Moody's, see Bluhm, Overbeck, and Wagner (2003), p. 120). This may be a result of the large number of enterprises included in the data that reduces the influence of idiosyncratic factors. The cyclicality can be verified by tests of autocorrelation due to Ljung-Box-Pierce, Godfrey-Breusch, and Durbin-Watson.[55]

4.3 Data Series for Financial collaterals

To evaluate the systematic credit cycle risk of different types of financial collaterals we investigate stocks, bonds, commodities and foreign exchange rates. We merely use German indexes since their performances seem to be good indicators for the market systematic risk of the considered collaterals. The following time series (performed by DataStream/DS from Thompson Financial Services) are included:

• **Stocks**		• **Bonds**
DAX, MDAX, SDAX (DS calculated)		Citigroup Bond Index Germany (CGBI)
Prime Sectors (DS calculated):		REX General Bond Price Index
(1) Basic Resources	(4) Retail	DB German Fully Taxed Bonds
(2) Industrial	(5) Telecommunication	– Yield of all Bank Bonds Outstanding
(3) Construction	(6) Financial Services	– Yield of all Corporate Bonds Outstanding
• **Exchange Rates**		• **Commodities**
EUR to USD, GBP, JPY		Goldman Sachs Commodity Index (GSCI)
		Gold Bullion in USD, EUR

The data series nearly fits with the model assumption of normal distributed yearly geometric returns $\tilde{c}_t$, that can be verified by tests of Jarque-Bera, Kolmogorov-Smirnov-Lilliefors, Geary, Shapiro-Wilk, and D'Agostino-Person with at least 1 % of significance. Additionally, the returns do not show autocorrelation due to tests of Ljung-Box-Pierce, Godfrey-Breusch (Lagrange Multiplier), and Durbin-Watson at 1 % level.[56]

TABLE 2 Correlation Analysis of the Geometric Returns of Financial Indexes and Prices

The correlations between the time series of the financial data is shown in Table 2. Especially the yearly geometric returns of stock indices (series (1) to (3)) as well as bond indices and

[55] See appendix A.8 for details.

[56] See appendix A.9 for the results of the tests. Only the geometric returns of Gold prices may assumed to be normal distributed only to certain extend since seven out of ten test are not significant at 1 % level.

interest rates (series (4) to (7)) are correlated close to unit,[57] whereas the time series of commodities (series (8) to (10)) and exchange rates (series (11) to (13)) show high significant correlation but far from being unit. While analysing the correlations between the asset classes we found that the stock market seems to be modestly positively correlated with the bond market[58] but both are inversely correlated with commodity prices and exchange rates at low levels of significance. Additionally, the bond market may not be linked with commodities and exchange rates since the levels of significance are rather low (and except for the GBP it does not exceed the 35 % level), but prices for commodities are correlated with exchange rates up to a 1 % level of significance.

4.4 Collateral Credit Cycle Risk

In order to evaluate the possibility of a dependency between the systematic credit cycle factor $\hat{x}_T$ and the standardized "collateral performance" $\tilde{c}_{T+\delta T}$ we investigated the correlation coefficient due to Spearman ρ_{Spear}, Kendall τ_B, both based on the rank order of the variables, and Pearson-Bravais ρ_{Pear} that examines the linear correlation between two parameters and directly leads to an estimate for the parameter $\pm\sqrt{\rho_C}$ in the case of $\delta T = 0$.[59] However, the results reported here refer to a work-out period δT of one month that is well in line with the 20 days suggested by Basel II for financial collaterals.[60] The findings do not change significantly if we use different work-out periods from zero up to two month. This may be caused by the fact that autocorrelation of $\hat{x}_T$ with a lag of one year is high, and presumably even higher for only a few months.[61]

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[57] Of course, the yield is inversely linked with the bonds indices since an increase of the interest rate would lead to a decrease of bond prices. See e.g. Jorion (2001), pp. 206-207, Crouhy, Galai and Mark (2000b), pp. 218-224, or Ilmanen, McGuire, and Warga (1994) for simple modelling of price changes of fixed income securities.

[58] Although our analysis on the relationship between asset classes is not very detailed since it is behind the scope of this paper, the stock-bond/interest-market relationship is in line with the literature. See e.g. Connoly, Striver, and Sun (2005), p. 161, or Andreou, Desiano, and Sensier (2001).

[59] It has do be added that with this analysis the correlation coefficients rather serve as indicators for a relationship between the market of the collateral and yearly default rates (like it is mentioned in Basel II) than they are able to explain dynamics between financial markets and insolvency rates. For the latter, the auto correlation of the default points has to be handled in the analysis e.g. by removing it via auto regression.

[60] See Basel Committee on Banking Supervision (2004), paragraph 167.

[61] See appendix A.5 for the autoregressive regression of the default points.

All three coefficients of correlation should not differ too much. Differences between ρ_{Spear} and τ_B could result from outliers in the rank order that lead to a lower value for ρ_{Spear}. Deviations of both parameters from ρ_{Pear} could stem from metric outliers and should be investigated in detail since risk management should especially account for those. Additionally, it could be a hint for an asymmetric relationship.

4.4.1 Stock Indices

Firstly, we investigate the correlation between $\hat{x}_{T,GER}$ for entire Germany and the three main German indices DAX, MDAX, and SDAX between 1974 and 2003 (SDAX: only since 1990). We analysed the whole period, and also the three decades separately. The results are shown in Table 3.

TABLE 3 Results of the Correlation-Analysis between the Systematic Factor and Returns of Stock Indices

Obviously, over the period of 30 years the correlations are close to zero and not significant. Nevertheless, the values vary tremendously over time. During the first decade from 1974 to 1983 the linear correlations between credit cycle and DAX as well as credit cycle and MDAX were negative (−0.67 for the DAX and −0.49 for the MDAX) with significance levels of 0.9 % (DAX) and 14.9 % (MDAX). For the last two decades the negative relationship is less strong and not significant, since the p-value does not exceed 59,4 % (p-value for ρ_{Pear} for the SDAX during 1994 and 2003). One may conclude, that there is a weak relationship between stock market and insolvency rates.

FIGURE 3 Correlations between Sector Systematic Factors and Prime Indexes during 1994 and 2003

In order to get more distinct results we examine the relationship between the six sectors and corresponding Prime Sector Indices of the Deutsche Börse AG. The results for the linear correlation ρ_{Pear} over last decade (1994-2003) and their p-values are shown in Figure 3.[62] All segments except sectors 3 (Constructions) and 4 (Wholesale and Retail Trade) show positive correlation with the systematic credit cycle factor. An explanation for sector 3 and 4 might be

[62] We report only the last decade since stocks of small and medium enterprises in Germany have been traded not much longer than ten years. This might have strengthened the relationship.

that these two sectors incorporate most enterprises (more than 240,000 and 600,000 on average, respectively) including more than 60 % personal undertakings. Especially the credit cycle of those small enterprises might not be conform with the performance of their Prime Sectors only including big companies. However, all relationships are not very significant with levels varying from 42.2 % to 67.8 %.

One can conclude that the relationship between the stock market and the German credit cycle in total is only slight and might be negative at a stretch. This might be due to the fact that most enterprises included in the study are rather small and are not directly linked to the capital market via equity. However, the dependence should be considered to be positive for some sectors, if only the last decade is accounted for. The highest correlation measured is 0.29 (between Prime Index Industrial and $\hat{x}_{T,\mathrm{Manufacturing}}$ from 1994 to 2003).

4.4.2 Bond Indices

In this section we investigate the relationship between the credit cycle and the fixed income market. Therefore we analysed the time series of two bond indices for the German Bond Market from the Deutsche Bundesbank (REX) and the Citigroup (CGBI) as well as changes in yields of Bank Bonds (y_{Bank}) and Corporate Bonds ($y_{\mathrm{Corp.}}$) as reported by the Deutsche Bundesbank (fully taxed, outstanding), because the yield is inversely linked with the value of bonds.[63] The results for the whole period as well as for the available decades are reported in Table 4.

TABLE 4 Results of the Correlation-Analysis between the Systematic Factor and Returns of Fixed Income Securities Indexes and Yields

Consequentially, the bond market is inversely correlated with the systematic factor $\hat{x}_{T,\mathrm{GER}}$ because the corresponding correlations of both indices are negative and of both yields are positive. Furthermore, the correlations are low significant (for indices up to 7.6 %, for yields up to 2.8 %). All absolute values of the (Pearson-)correlations for the REX and the yields over the whole period do not vary much from 0.27 to 0.34. Concerning the decades, during 1974 and 1983 the dependency seems to be stronger than during the other decades. On the other

[63] For a simple modelling of price changes of fixed income securities please refer to footnote 57 and the literature cited there.

hand, the low values during the last decade for indices, especially the CGBI, suggest that there might be no negative dependency.

FIGURE 4 Correlations between Sector Systematic Factors and Yield during the Entire Period

Additionally, we made an investigation concerning the six sectors and the yield on corporate bonds ($y_{Corp.}$, for sector 6 we used y_{Bank}). As can be seen from Figure 4 the correlations of sectors 2 to 5 are highly significant (up to 0.9 % for sector 5) with values varying from 0.31 (Construction) to 0.41 (Transport, Storage and Communication). Only for sector 1 (Energy and Mining) the correlation is negative, i.e. the bond market tends to turn down when insolvency quotes are high. However, the level of significance for this sector as well as for sector 6 (Financial Intermediation) is low compared to the other sectors.

FIGURE 5 Correlations between Sector Systematic Factors and Maturity

Finally, we analysed the relationship with respect to the bond maturity. Figure 5 displays the results for the considered sub-indexes of the REX (1 to 10 years) and for the yield y_{all} (all bonds outstanding, 1-2 to 9-10 years). This figure shows the consistent result that the correlation between systematic factor $\hat{x}_{T,GER}$ and REX and between systematic factor $\hat{x}_{T,GER}$ and y_{all}, respectively, drops when shifting to higher maturity, i.e. the relationship between bond market and credit cycle is less strong for securities with longer maturity.

To conclude, we have to point out that there seems to be a negative relationship between bond market and credit cycle and thus bonds appear to be capable to hedge losses in a credit portfolio. The value of the negative correlation depends on the sector of the borrower as well as on the maturity of the bonds. The highest absolute value specified in the examination is –0.35 for ρ_{Pear} between $\hat{x}_{T,GER}$ and the REX (maturity 1 year). This result might be in line with the general observation that interest rates fall when the economy, here quantified by the credit cycle factor, deteriorates.[64] Since the investigated bond indices only represent bonds from issues with highest degree of creditworthiness only the interest rates influence its value but not

[64] Interest rates are generally assumed to move with the business cycle. For a discussion see DeStefano (2004), pp. 533-535.

changes in the default rate. However, during the last decade there is no strong evidence for a negative relationship.

4.4.3 Commodities

In order to investigate the dependency between commodities and the credit cycle we tested the Goldmann Sachs Commodity Index (GSCI) and the price of Gold Bullion (per once) in USD and EUR for correlation with the systematic factor $\hat{x}_{T,GER}$ during the period from 1970 to 2003 (GSCI only from 1971 on).

TABLE 5 Results of the Correlation-Analysis between the Systematic Factor and Returns of Commodities

The results presented in Table 5 show that correlations between these types of collaterals are of low significance but positive. Over the whole period the GSCI has highest significance reaching a p-value of 14.6 % with a positive linear correlation of 0.25, and additionally it is positive for all decades separately. For the gold price the relationship is less strong and close to zero over the whole period. However, over the decades the values change both in value and significance. For Gold in USD the Pearson-correlations vary from 0.42 (p-value = 22.3 %) to –0.46 (p-value = 17.8 %). For Gold in EUR these correlations are positive during all decades but low significant since the lowest p-value is 20.9 % (from 1974 to 1983).

Concerning the six sectors we briefly want to review the results for "Gold in USD"[65] since gold is the only eligible commodity collateral within the Basel II framework and exchange rate risk is especially accounted for. Over the last thirty years the dependency between the systematic factor and the performance of gold was close to zero at low significance levels for all sectors except 1 (Energy and Mining) and 6 (Financial Intermediation). Especially for sector 1 the Pearson correlation coefficient was high (0.36) and significant (with a p-value of 4.2 %), for sector 6 it was still reliable positive (0.14 with a p-value 41.8 %). If we only consider the decades separately, the dependency varies tremendously for all sectors. For the last decade (1994 to 2003) it was negative for all sectors with highest values for sector 1 (Energy and Mining, $\rho_{Pear} = -0.71$ and p-value = 2.1 %) and sector 5 (Transport, Storage and Communication, $\rho_{Pear} = -0.62$ and p-value = 5.5%). During 1974 to 1983 it was positive for all sectors

[65] We do not present the results in detail here.

being most significant for sector 1 (Energy and Mining, ρ_{Pear} = 0.53 and p-value = 11.3 %) and 2 (Manufacturing, ρ_{Pear} = 0.46 and p-value = 14.8 %).

However, obviously the dependency between commodities in general and the credit cycle is rather low but positive. Gold prices do not show any strong relationship over the whole period, being reliable positive only for two sectors especially during 1974 to 1983. This result might fit with the common statement that gold is a stable asset especially when the economy deteriorates.

4.4.4 Foreign Exchange Rates

Finally, we examined the relationship between the credit cycle and the change of the United States Dollar (USD), British Pound (GBP) and the Japan Yen (JPY) to the Euro (EUR) from 1974 to 2003. As it is presented in Table 6 the results differ tremendously between the currencies.

TABLE 6 Results of the Correlation-Analysis between the Systematic Factor and Changes of Exchange rates

Considering the whole period USD and GBP show nearly zero correlations with the EUR, whereas JPY indicates a negative dependency. The relationships have low significance for the JPY (the p-value is 38.7 %) and nearly no significance for the USD and GBP (the p-values are 94.4 % and 54.3 %, respectively). The last two decades from 1984 to 2003 indicate a negative relationship for all currencies. In case of the USD it has been even significant (the p-value for ρ_{Pear} is 7.6 %) for the last decade (1994-2003). In contrast, at least for the USD and the JPY from 1974 to 1983 the dependency is positive.

Moreover, we examined the six sector for each currency separately:[66] For the USD, the correlation is close to zero over the whole period. The highest level of significance is 64.0 % (sector 2, Manufacturing). Again, we get different results from the decades. Especially from 1994 to 2003 it is negative for all sectors incorporating highest absolute Pearson correlations for sector 6 (Financial Intermediation, ρ_{Pear} –0.70 and p-value = 2.5 %) and sector 4 (Wholesale and Retail Trade, ρ_{Pear} = –0.65 and p-value = 4.2 %). However, from 1973 to 1983 it is posi-

[66] We do not display the results in detail here.

tive being most significant for sector 6 (Financial Intermediation, ρ_{Pear} = 0.67 and p-value = 3.5 %) and for sector 5 (Transport, Storage and Communication, ρ_{Pear} = –0.66 and p-value = 3.8 %). Similar, the return of the exchange rate of the GBP (to the EUR) does seemingly not depend on the credit cycle except for sector 1 (Energy and Mining) that has a positive linear correlation of 0.33 at a level of significance of 7.0 %. On the other hand, for the other sectors significance levels ranging from 50.0 % (sector 4, Wholesale and Retail Trade) to 98.0 % (sector 6, Financial Intermediation) are low. If we only investigate the decade 1994 to 2003 it is negative even for sector 1 (Energy and Mining, ρ_{Pear} = –0.20 and p-value = 58.0 %) incorporating highest (negative) Pearson correlation for sector 6 (Financial Intermediation, ρ_{Pear} = –0.69 and p-value = 2.5 %) and sector 4 (Wholesale and Retail Trade, ρ_{Pear} = –0.65 and p-value = 4.2 %). The JPY has a high positive linear correlation especially with sector 1 (Energy and Mining, ρ_{Pear} = 0.29 and p-value = 11.0 %) and sector 6 (Financial Intermediation, ρ_{Pear} = 0.36 and p-value = 4.2 %). Worth mentioning, that during 1994 to 2003 the relationship was merely negative (except for sector 1, Energy and Mining) being most significant for sector 5 (Transport, Storage and Communication) at a level of 3.9 % (ρ_{Pear} = –0.66) and sector 2 (Manufacturing) at a level of 10.7 % (ρ_{Pear} = –0.54).

Obviously, there is no strong relationship between the credit cycle and the change in exchange rates. Indeed, the dependency varies not only in values but also in the sign when examining different decades. A sector analysis shows for sector 1 (Energy and Mining) positive correlations for the GBP and JPY. In total, this result could be explained by the fact that the exchange rate might only be of high importance for enterprises with an international focus. Caused by the numerous databases (especially for sectors 2 to 6) this attribute is more likely to be idiosyncratic than systematic.

4.5 Implications for Credit Risk Measurement concerning Basel II

In the last subsection we made a comprehensive analysis of a possible linkage between the performance of financial collaterals as it can be observed at the capital market, and the systematic credit cycle factor $\tilde{x}_T$ denoted by the insolvency quotes in Germany. From the results one can conclude that on the one hand there is no intense positive link between those two parameters,[67] at best for the Bond market there is some evidence that the two parameters are inversely related. On the other hand these findings should be handled with care since they vary across the periods under consideration. This is in line with an analysis of Carey and Gordy (2003) on recovery rates who pointed out that investigations of short periods often add up to high negative correlations between default rates and recovery.

From the portfolio-perspective one could conclude that financial collaterals serve as stable assets for the bank to protect the portfolio against high losses, because

(1) their market value can be observed permanently and they can be liquidated easily,
(2) their systematic credit cycle risk is low, i.e. if the credit cycle deteriorates and the number of defaults in the portfolio rises a downturn of the collaterals needs not to be expected.

In order to evaluate the second characteristic specifically, we calculate the haircuts for each assets classes under consideration as announced in equation (12) and (13). We use the one-month standard deviation $\sigma_{C,1\text{ Month}}$ of the logarithmic return during the last five years of the analysis (1999-2003). For the credit cycle sensitive haircut (see equation (12)) we provide two estimates: Firstly, we determine a risk-adjusted value $H_{sen}^{(reas)}$ that might be most reasonable for risk management assessing that the coefficients vary tremendously over time. It is based on $\rho_{Pear}^{(reas)}$ that is the highest positive (Pearson) correlation within a 95^{th} percentile interval considering each decade separately. Secondly, a neutral haircut $H_{sen}^{(neutr)}$ is calculated by using $\rho_{Pear}^{(neutr)}$ (if its level of significance is lower than 15 %) or is identified with zero (if the level of significance is not lower than 15 %). The parameter $\rho_{Pear}^{(neutr)}$ stands for the linear correlation over the whole period. Additionally, we estimate conservative haircuts H_{con} on the basis of the 99th percentile of the standard normal distribution (see equation (13)). The results and standard supervisory haircuts H_{BII} of Basel II (if available) are shown in Table 7.

TABLE 7 Haircuts of Financial Collaterals

Obviously, our results for the haircut H_{con} do not differ very much from the supervisory haircuts H_{BII} although we used indices and not individual stocks and bonds. However, as expected from theory the haircuts decline if only the standard deviation with respect to the credit cycle

[67] This result is not very surprising since in Germany small and medium sized companies mostly are not traded on the capital markets.

is taken into consideration for calculating haircuts $H_{sen}^{(reas)}$ and $H_{sen}^{(neutr)}$. For the risk adjusted, reasonable haircut $H_{sen}^{(reas)}$ the discounts drop merely by half whereas for the neutral estimate $H_{sen}^{(neutr)}$ in most cases a haircut of zero seems to be appropriate. Especially this statement stresses that the link between the capital market and insolvency rates can not be identified confidently.

We can conclude that existing valuation procedures of financial collaterals do not account for the independency of capital markets and credit cycle of individual or small enterprises. We showed how the results from the empirical investigation in section 4.4 can be integrated in the valuation procedure of financial collaterals. Lately, without sticking to the presented study one could raise the striking argument for lower haircuts than proposed by Basel II because a systematic linkage between the performance of national and international capital markets and the performance of individual or small enterprises (acting more locally) is doubtful. The evaluation of financial collaterals should account for that.

5. Conclusion

Correlations are the main drivers for financial risk that arise from credit portfolios. In the present paper we investigate the systematic relationship between financial collaterals and default rates that we call "(systematic) credit cycle risk" of collaterals and that specially has to be considered when own LGD-estimates according to Basel II are determined. To our knowledge such an investigation is carried out for the first time, and therefore was partitioned into three parts: modelling, empirical analysis, and its implications for haircut calculations. Firstly, we present a simple model that distinguishes between recovery from the collateral and from other firms assets. Because we were mainly interested in the relationship between the collateral and the cyclicality of defaults within a credit portfolio we give an estimation procedure for evaluating the correlation between a systematic factor for the credit cycle and the performance of collaterals. We claim that our model is capable to build a theoretic framework in order to identify dependencies between recovery rates, especially those of collateralized loans, and default rates, like they are mentioned by the BCBS.

In order to show how an investigation can be carried out for loan portfolios we empirically implement our model to analyse a large database of German corporate insolvencies containing mostly individual or small enterprises, and financial collaterals. We find out that correlations are far from being strong or at a stretch being unit: especially the bond market seems to be inversely correlated to the credit cycle. Using an analytical approximation of the model, we suggest haircuts for financial collaterals that account for the less strong linkage between capital market and insolvency rates.

A. Appendix

A.1 Derivation of Equations (3) and (4)

Firstly, in the event of default the bank receives payments $\tilde{C}^{(d)}_{T+\delta T}$ from the liquidation of the collateral. The loss rate $\tilde{\ell}^{(d)}_{C,T+\delta T}$ of the exposure L after the liquidation at $t = T + \delta T$ is

$$\tilde{\ell}^{(d)}_{C,T+\delta T} = \max\left(1 - \frac{\tilde{C}^{(d)}_{T+\delta T}}{L}; 0\right) = \frac{\tilde{L}_{eff}}{L}, \text{ with } \tilde{L}_{eff} := \max\left(L - \tilde{C}^{(d)}_{T+\delta T}; 0\right). \tag{A1}$$

Secondly, the bank acquires a title on a fraction $\Delta = \tilde{L}_{eff} / \tilde{B}_{eff}$ of the firms assets $\tilde{A}^{(d)}_{T+\delta T}$ depending on the remaining exposure $\tilde{L}_{eff}$ and on the remaining total liabilities (after the liquidation of the collateral) $\tilde{B}_{eff} = \max\left(B - \tilde{C}^{(d)}_{T+\delta T}; 0\right) \overset{(D)}{\approx} B$. We can calculate the loss rate $\tilde{\ell}^{(d)}_{unsec,T+\delta T}$ from the unsecured exposure $\tilde{L}_{eff}$ of the credit after emergence of the collateral as

$$\tilde{\ell}^{(d)}_{unsec,T+\delta T} = \max\left(1 - \frac{\Delta \cdot \tilde{A}^{(d)}_{T+\delta T}}{\tilde{L}_{eff}}; 0\right) = \max\left(1 - \frac{\tilde{A}^{(d)}_{T+\delta T}}{\tilde{B}_{eff}}; 0\right) \approx \max\left(1 - \frac{\tilde{A}^{(d)}_{T+\delta T}}{B}; 0\right).^{68} \tag{A2}$$

Resulting from assumption (D), the proportional loss from the unsecured part of the credit with collateral and from an unsecured credit is identical.

Conditional on the systematic factor $\tilde{x}_{T+\delta T}$ at $t = T + \delta T$ $\tilde{C}^{(d)}_{T+\delta T}$ and $\tilde{A}^{(d)}_{t+\delta t}$ are independent (and therefore the collateral does not depend on the default state anymore). The expected loss rate of the secured loan in the event of default conditional on $\tilde{x}_{T+\delta T}$ becomes

$$E\left(\tilde{\ell}^{(d)}_{sec,T+\delta T} \mid \tilde{x}_{T+\delta T}\right) = E\left(\tilde{\ell}^{(d)}_{C,T+\delta T} \cdot \tilde{\ell}^{(d)}_{unsec,T+\delta T} \mid \tilde{x}_{T+\delta T}\right) = \frac{1}{L} \cdot E\left(\tilde{L}_{eff} \ \tilde{x}_{T+\delta T}\right) \cdot E\left(\tilde{\ell}^{(d)}_{unsec,T+\delta T} \mid \tilde{x}_{T+\delta T}\right) \tag{A3}$$

where $\tilde{\ell}^{(d)}_{sec,T+\delta T}$ ($\tilde{\ell}^{(d)}_{unsec,T+\delta T}$) stand for the loss rate of the secured (unsecured) loan in the event of default.

[68] This formulation of the loss rate is a stylized, simplified notation of common loss assumptions in credit derivatives pricing models, see e.g. Briys and Varenne (1997) or Klein (1996). A literature review can be found in Uhrig-Homburg (2002). A similar approach with log-normally distributed assets $\tilde{A}^{(d)}_{T+\delta T}$ is used by Pykthin and Dev (2002).

A.2 Derivation of Equation (11)

Let $q_\alpha(\tilde{x}_T)$ denote the α-quantile of $\tilde{x}_T$, i.e. $P\left(\tilde{x}_T \le q_\alpha(\tilde{x}_T)\right) = \alpha$. Provided that[69] the assumptions

(1) the portfolio is "infinitely homogeneous", and

(2) $E\left(\sum_{i=1}^{n} w_i^{(n)} \cdot \tilde{\ell}_{i,T+\delta T} \mid \tilde{x}_T\right)$ is decreasing in $\tilde{x}_T$

are fulfilled the VaR of the portfolio default rate DR_T^∞ and the portfolio loss rate LR_T^∞ can be determined by using equations (5) and (8) with $\tilde{x}_T = q_\alpha(\tilde{x}_T)$. Since assumption (1) is already stated (see assumption (F) in subsection 3.2), only assumption (2) has to be revised carefully.

In our context by using equations (2) and (3) the expected loss at $T + \delta T$ of an individual credit conditional on $\tilde{x}_T$ is

$$E\left(\tilde{\ell}_{\sec,T+\delta T} \mid \tilde{x}_T\right) = E\left(\tilde{\ell}^{(d)}_{\sec,T+\delta T} \mid \tilde{x}_T\right) \cdot p_A(\tilde{x}_T) = \frac{1}{L} \cdot E\left(\tilde{L}_{eff}\ \tilde{x}_T\right) \cdot LGD_{unsec} \mid \tilde{x}_T \cdot p_A(\tilde{x}_T) \tag{A4}$$

whereas $p_A(\tilde{x}_T)$ is specified by equation (2) and $LGD_{unsec} \mid \tilde{x}_T$ (the loss rate of an unsecured part of the loan) may be determined by the models of Frye (2000a), Pykthin and Dev (2002), Pykthin (2003), or Tasche (2004). Both terms $p_A(\tilde{x}_T)$ and $LGD_{unsec} \mid \tilde{x}_T$ are decreasing in $\tilde{x}_T$, and thus, assumption (2) is valid.

The effect of the collateral can be determined (see equation (4) in subsection 3.1) as

$$E\left(\tilde{L}_{eff}\ \tilde{x}_T\right) = E\left(\max\left(L - \tilde{C}_{T+\delta T}\ \tilde{x}_T; 0\right)\right). \tag{A5}$$

Since the value of the collateral at time T+δT can be calculated by equation (9) and (10) we get for equation (A5)[70]

$$E\left(\tilde{L}_{eff}\ \tilde{x}_T\right) = N(d \mid \tilde{x}_T) - \frac{E\left(\tilde{C}_{T+\delta T}\ \tilde{x}_T\right)}{L} \cdot N\left(d\ \tilde{x}_T - \sigma_{C,\delta T} \cdot \sqrt{1-\rho_C}\right) \text{ with} \tag{A6}$$

$$E\left(\tilde{C}_{T+\delta T}\ \tilde{x}_T\right) = C_T \cdot \exp\left(\mu_{C,\delta T} \pm \sigma_{C,\delta T} \cdot \sqrt{\rho_C} \cdot \tilde{x}_T + \frac{\sigma^2_{C,\delta T} \cdot \sqrt{1-\rho_C}}{2}\right), \tag{A7}$$

$$d\ \tilde{x}_T = \frac{\ln\left(C_T\right) - \ln(L) - \mu_{C,\delta T} \pm \sigma_{C,\delta T} \cdot \sqrt{\rho_C} \cdot \tilde{x}_T}{\sigma_{C,\delta T} \cdot \sqrt{1-\rho_C}}. \tag{A8}$$

[69] See e. g. Gordy (2003), especially p. 205, assumptions (A-2), and p. 207, assumptions (A-3) and (A-4).

[70] See e.g. Hull (2003), pp. 268-270.

With the well established approximation $\mu_{C,\delta T} \approx 0$ and $\sigma_{C,\delta T} \ll 1$ we may use the following simplifications[71]

$$E\left(\tilde{C}_{T+\delta T} \mid \tilde{x}_T\right) \approx C_T \cdot \left(1 \pm \sigma_{C,t_M} \cdot \sqrt{\rho_C} \cdot \tilde{x}_T\right), \tag{A9}$$

$$N(d \mid \tilde{x}_T) \approx 0 \text{ and } N\left(d \mid \tilde{x}_T - \sigma_{C,\delta T} \cdot \sqrt{1-\rho_C}\right) \approx 0 \text{ if } C_T \cdot \left(1 \pm \sigma_{C,t_M} \cdot \sqrt{\rho_C} \cdot \tilde{x}_T\right) \geq L, \tag{A10}$$

$$N(d \mid \tilde{x}_T) \approx 1 \text{ and } N\left(d \mid \tilde{x}_T - \sigma_{C,\delta T} \cdot \sqrt{1-\rho_C}\right) \approx 1 \text{ if } C_T \cdot \left(1 \pm \sigma_{C,t_M} \cdot \sqrt{\rho_C} \cdot \tilde{x}_T\right) < L. \tag{A11}$$

Concluding from this we are able to write

$$E\left(\tilde{L}_{eff} \mid \tilde{x}_T\right) \approx \max\left(L - C_T \cdot (1+H); 0\right) \text{ with } H = \pm\sqrt{\rho_C} \cdot \sigma_{C,t_M} \cdot \tilde{x}_T. \tag{A12}$$

Under a VaR approach, the effective exposure L_{eff}^{VaR} of the credit can be calculated as the expected effective exposure conditional on $\tilde{x}_T = q_\alpha(\tilde{x}_T)$:

$$L_{eff}^{VaR} := E\left(\tilde{L}_{eff} \mid q_\alpha(\tilde{x}_T)\right) \approx \max\left(L - C_T \cdot (1+H_{sen}); 0\right) \text{ with } H = \pm\sqrt{\rho_C} \cdot \sigma_{C,\delta T} \cdot \tilde{x}_T = q_\alpha(\tilde{x}_T). \tag{A13}$$

As long as H < 0 the assumption (2) is valid anyway since $E\left(\tilde{L}_{eff} \mid \tilde{x}_T\right)$ is decreasing in $\tilde{x}_T$. If H > 0, assumption (2) may still hold, since it has to be valid on a portfolio basis. In this case, further proofs on the portfolio expected loss has to be carried out.[72]

A.3 Equations for the Estimation Procedure

Since the default point is normal distributed, i.e.

$$\widetilde{DP}_T \sim N\left(\mu_{DP}; \sigma_{DP}^2\right) \tag{A14}$$

with $\mu_{DP} = N^{-1}(PD_A)/\sqrt{1-\rho_A} =: \mu(PD_A, \rho_A)$ and $\sigma_{DP}^2 = \rho_A / \left(1-\rho_A\right) =: \sigma^2(\rho_A)$ (A15)

that can also be written as

$$\widetilde{DP}_T = \frac{N^{-1}(PD_A)}{\sqrt{1-\rho_A}} + \sqrt{\frac{\rho_A}{1-\rho_A}} \cdot \tilde{x}_T \text{ with } \tilde{x}_T \sim N(0,1), \tag{A16}$$

we get from a simple point estimation the sample mean and sample variance[73]

$$\hat{\mu}_{DP} = \frac{1}{\tau} \cdot \sum_{T=1}^{\tau} DP_T \text{ and } \hat{\sigma}_{DP}^2 = \frac{1}{\tau-1} \cdot \sum_{T=1}^{\tau} \left(DP_T - \hat{\mu}_{DP}\right)^2 \tag{A17}$$

---

[71] For the following expressions we used the approximations $\exp(y) \xrightarrow{y \ll 1} 1+y$, $N(a/y) \xrightarrow{a>0, y \ll 1} 1$ and $N(a/y) \xrightarrow{a<0, y \ll 1} 0$.

[72] See also Gordy (2003), p. 207 assumption (A-4), especially number (ii) and number (iii) and the discussion there.

[73] See e.g. Jorion (2001), p. 123, or Greene (2003), pp. 886-888.

and the estimated asymptotic covariance matrix

$$\underline{\underline{\Sigma}}_N = \begin{pmatrix} \frac{\hat{\sigma}^2_{DP}}{\tau} & 0 \\ 0 & \frac{2 \cdot \hat{\sigma}^4_{DP}}{\tau - 1} \end{pmatrix} =: \begin{pmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix}, \tag{A18}$$

$$\text{i.e. } \operatorname{Var}\left(\hat{\mu}_{DP}\right) = \frac{\hat{\sigma}^2_{DP}}{\tau} \text{ and } \operatorname{Var}\left(\hat{\sigma}^2_{DP}\right) = \frac{2 \cdot \hat{\sigma}^4_{DP}}{\tau - 1}. \tag{A19}$$

For the calculation of the estimated parameters $\hat{\rho}_A$ and $\widehat{PD}_A$ and their estimated asymptotic covariance matrix it has to be accounted for the nonlinear functions $PD_A(\mu, \sigma^2)$ and $\rho_A(\mu, \sigma^2)$.[74] From (A15) we can conclude

$$\hat{\sigma}^2_{DP} = \frac{\hat{\rho}_A}{1 - \hat{\rho}_A} \Leftrightarrow \hat{\rho}_A = \frac{\hat{\sigma}^2_{DP}}{1 + \hat{\sigma}^2_{DP}} \tag{A20}$$

and

$$\hat{\mu}_{DP} = \frac{N^{-1}(\widehat{PD}_A)}{\sqrt{1 - \hat{\rho}_A}} \Leftrightarrow \widehat{PD}_A = N\left(\frac{\hat{\mu}_{DP}}{\sqrt{1 + \hat{\sigma}^2_{DP}}}\right). \tag{A21}$$

We use Delta-Method to calculate the asymptotic covariance matrix and therefore derive the Jacobian Matrix of ρ_A and PD_A

$$\begin{aligned} \underline{\underline{J}} &= \begin{pmatrix} \partial PD_A / \partial \mu_{DP} & \partial PD_A / \partial \sigma^2_{DP} \\ \partial \rho_A / \partial \mu_{DP} & \partial \rho_A / \partial \sigma^2_{DP} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{\sqrt{1 + \sigma^2_{DP}}} \cdot n\left(\frac{\mu_{DP}}{\sqrt{1 + \sigma^2_{DP}}}\right) & -\frac{1}{\left(1 + \sigma^2_{DP}\right)^{\frac{3}{2}}} \cdot n\left(\frac{\mu_{DP}}{\sqrt{1 + \sigma^2_{DP}}}\right) \\ 0 & \frac{1}{\left(1 + \sigma^2_{DP}\right)^2} \end{pmatrix} =: \begin{pmatrix} J_{11} & J_{12} \\ 0 & J_{22} \end{pmatrix}. \end{aligned} \tag{A22}$$

In order to calculate the asymptotic covariance matrix

$$\underline{\underline{\Sigma}}_{DP} = \underline{\underline{J}}\,\underline{\underline{\Sigma}}_N \underline{\underline{J}}^T = \begin{pmatrix} J_{11} & J_{12} \\ 0 & J_{22} \end{pmatrix} \begin{pmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix} \begin{pmatrix} J_{11} & 0 \\ J_{12} & J_{22} \end{pmatrix} = \begin{pmatrix} J_{11}^2 \Sigma_{11} + J_{12}^2 \Sigma_{22} & J_{12} J_{22} \Sigma_{22} \\ J_{12} J_{22} \Sigma_{22} & J_{22}^2 \Sigma_{22} \end{pmatrix}. \tag{A23}$$

From that we can conclude

$$\operatorname{Var}\left(\hat{\rho}_A\right) = J_{22}^2 \Sigma_{22} = \frac{2}{\tau - 1} \cdot \frac{\hat{\sigma}^4_{DP}}{\left(1 + \hat{\sigma}^2_{DP}\right)^4} \tag{A24}$$

and

[74] See e.g. Greene (2003), p. 70 and p. 870, for a brief overview about the following proceeding.

$$\mathrm{Var}\left(\widehat{PD}_A\right) = J_{11}^2\Sigma_{11} + J_{12}^2\Sigma_{22} = \left[n\left(\frac{\hat{\mu}_{DP}}{\sqrt{1+\hat{\sigma}_{DP}^2}} \right) \right]^2 \cdot \left(\hat{\sigma}_{DP}^2 \cdot \frac{(\tau-1)\cdot\left(1+\hat{\sigma}_{DP}^2\right)^2 + \tau\cdot\hat{\sigma}_{DP}^2}{\left(1+\hat{\sigma}_{DP}^2\right)^3\cdot\tau\cdot(\tau-1)} \right). \tag{A25}$$

A.4 Unbiasedness of the Estimators

It has to be shown, that the estimated expectation of the asset correlation $\hat{\rho}_A$ as well as the estimated expectation of the realization $\hat{x}_T$ of systematic factor do not depend on PD_A but on the fluctuation of the default rate around PD_A.

According to our model, the default point can be re-written as

$$\widetilde{DP}_T^\infty = \mu_{DP} + \sigma_{DP}\cdot\tilde{x}_T \text{ with } \mu_{DP} = \frac{N^{-1}(PD_A)}{\sqrt{1-\rho_A}} \text{ and } \sigma_{DP} = \sqrt{\frac{\rho_A}{1-\rho_A}}. \tag{A26}$$

The variance of the default point

$$\sigma_{DP}^2 = \mathrm{Var}\left(\widetilde{DP}_T^\infty\right) = \mathrm{Var}\left(\mu_{DP} + \sigma_{DP}\cdot\tilde{x}_T\right) = \rho_A \big/ \left(1-\rho_A\right) \tag{A27}$$

does obviously not depend on PD_A.

The estimator $\hat{\rho}_A = \hat{\sigma}_{DP}^2 \big/ \left(1+\hat{\sigma}_{DP}^2\right)$ for ρ_A relies only on the empirical variance $\hat{\sigma}_{DP}^2$. Since its expectation is equal to $E\left(\hat{\sigma}_{DP}^2\right) = \sigma_{DP}^2$, the expectation of $\hat{\rho}_A$ also do not depend on PD_A. Additionally, the estimation $\hat{x}_T$ of a realization x_T of the systematic factor at time T is related to the observed realization DP_T^∞ using $\hat{\rho}_A$ and $\widehat{PD}_A$ due to

$$\frac{DP_T^\infty - \hat{\mu}_{DP}}{\hat{\sigma}_{DP}} \underset{(A26)}{=} \hat{x}_T = \frac{N^{-1}(\widehat{PD}_A)}{\sqrt{\hat{\rho}_A}} - \frac{\sqrt{1-\hat{\rho}_A}}{\sqrt{\hat{\rho}_A}}\cdot DP_T^\infty. \tag{A28}$$

Thus, the observed default point is normalized by $\hat{\mu}_{DP}$. Since the expectation of estimated value $\hat{\mu}_{DP}$ is equal to the expected value of $\widetilde{DP}_T^\infty$, $E\left(\hat{\mu}_{DP}\right) = \mu_{DP} = E(\widetilde{DP}_T^\infty)$, $\hat{x}_T$ also does not depend on μ_{DP} and therefore not on PD_A.

A.5 ADF-Regression and Autoregressive Regression of the Default Point and Detrended Default Point

TABLE 8 Statistics of the ADF-Regression of the Default Point and Detrended Default Point

TABLE 9 Statistics of the Autoregressive Regression of the Default Point and Detrended Default Point

A.6 Tests of Autocorrelation and Stationary (KPSS) of the Default Point and Detrended Default Point

TABLE 10 Test of Autocorrelation of the Default Point and Detrended Default Point

TABLE 11 KPSS-Test of Stationary of the Default Point and Detrended Default Point

A.7 Tests of Normality of the Detrended Default Point

TABLE 12 Statistics of the Tests of Normality of the Detrended Default Point

A.8 Tests of Autocorrelation of the Systematic Credit Cycle Factor

TABLE 13 Tests of Autocorrelation of the Systematic Credit Cycle Factor

A.9 Tests of Normality and Autocorrelation of the Logarithmic returns of Financial Collaterals

TABLE 14 Tests of Normality of the Logarithmic returns of Financial Collaterals

TABLE 15 Tests of Autocorrelation of the Logarithmic returns of Financial Collaterals

References

Acharya, V. V., Bharath, S. T., and Srinivasan, A. (2004): Understanding the Recovery Rates on Defaulted Securities, Working Paper, *www.defaultrisk.com*.

Altman, E., Brady, B., Resti, A., and Sironi, A. (2004): The Link between Default, and Recovery Rates: Theory, Empirical Evidence, and Implications, *Journal of Business*, forthcoming.

Altman, E., Resti, A., and Sironi, A. (2004): Default Recovery Rates in Credit Risk Modeling: A Review of the Literature, and Empirical Evidence, Working Paper, *www.defaultrisk.com*.

Altman, E., and Saunders, A. (1997): Credit Risk Measurement: Developments Over the Last Twenty Years, *Journal of Banking and Finance* **21**, pp. 1721-1742.

Andreou, E., Desiano, R., and Sensier M. (2001): The Behaviour of Stock Returns and Interest Rates over the Business Cycle in the US and UK, *Applied Economics Letters* **8**, p. 233-238.

Araten, M., Jacobs, M. Jr., and Varshney, P. (2004): Measuring LGD on Commercial Loans: An 18-Year Internal Study, *The RMA Journal* **86** (4), pp. 28-35.

Basel Committee on Banking Supervision (1999a): *A New Capital Adequacy Framework*, Bank for International Settlements.

Basel Committee on Banking Supervision (1999b): *Credit Risk Modelling: Current Practises and Applications*, Bank for International Settlements.

Basel Committee on Banking Supervision (2004): *International Convergence of Capital Measurement, and Capital Standards*, Bank for International Settlements.

Basel Committee on Banking Supervision (2005): *Guidance on Paragraph 468 of the Framework Document*, Bank for International Settlements.

Bluhm, C. and Overbeck, L. (2003): Systematic Risk in Homogeneous Credit Portfolios, *Credit risk : Measurement, Evaluation and Management*, Heidelberg.

Bluhm, C., Overbeck, L., and Wagner, C. (2003): *An Introduction to Credit Risk Modelling*, Chapmann & Hall/CRC, London.

Bögelein, L., Hamerle, A., Rauhmeier, R., and Scheule, H. (2002): Modelling Default Rate Dynamics in the CreditRisk+ framework, *Risk* **15** (10), pp. 24-28.

Briys, E., and Varenne, F. de (1997): Valuing Risky Fixed rate Debt: An Extension, *Journal of Financial and Quantitative Analysis* **32** (2), S. 239-248.

Charfield, C. (2004): The Analysis of Time Series, Chapman & Hall, Boca Raton et. al.

Carey, M., and Gordy, M. (2003): *Systematic Risk in Recoveries on Defaulted Debt*, Working Paper, Federal Reserve Board, Washington.

Connoly, R., Striver, C., and Sun,L. (2005): Stock Market Uncertainty and the Stock-Bond Return Relation, *Journal of Financial and Quantitative Analysis* **40** (1), pp. 161-194.

Credit Suisse Financial Products (1997): *CreditRisk+ – A Credit Risk Management Framework*, London.

Crosbie, P., and Bohn, J. (2003): *Modeling Default Risk*, Moody's KMV Company.

Crouhy, M., Galai, D., and Mark, R. (2000a): A Comparative Analysis of Current Credit Risk Models, *Journal of Banking and Finance* **24**, pp. 59-117.

Crouhy, M., Galai, D., and Mark, R. (2000b): *Risk Management*, McGraw-Hill, New York.

Davydenko, S., and Franks, J. (2004): Do Bankruptcy Codes Matter? A Study of Defaults in France, Germany, and the U.K., Working Paper, *www.defaultrisk.com*.

Demey, P., Louanin, J.-F.Roget, C. and Roncalli, T. (2004): Maximum likelihood estimate of default correlations, *Risk* 17(11), pp. 104-108.
DeStefano,M. (2004): Stock Returns and the Business Cycle, *The Financial Review* **39**, pp. 527-547.

Dietsch, M., and Petey, J. (2004): Should SME exposures be treated as retail or corporate exposures? A comparative analysis of default probabilities and asset correlations in French and German SMEs, *Journal of Banking and Finance* **28**, pp. 773-788.

Düllmann, K., and Trapp, M. (2004): Systematic Risk in Recovery Rates – An Empirical Analysis of U.S. Corporate Credit Exposures, Working Paper, *www.defaultrisk.com*.

Dülmann, K., and Scheule, H. (2003): Asset Correlation of German Corporate Obligors: Its Estimation, its Drivers and Implications for Regulatory Capital, Working Paper, *www.defaultrisk.com*.

Finger, C. C. (1999): Conditional Approaches for CreditMetrics Portfolio Distributions, *CreditMetrics Monitor* (4), pp. 14–33.

Finger, C. C. (2001): The One-Factor CreditMetrics Model In The New Basel Capital Accord, *RiskMetrics Journal* **2** (1), pp. 8-18.

Franke, J., Härdle, W., and Hafner, C. M. (2004): *Statistics of Financial Markets*, Springer, Berlin.

Franks, J., Servigny, A. de, and Davydenko, S. (2004): *A Comparative Analysis of the Recovery, and Recovery Rates for Private Companies in the U.K., France, and Germany*, Standard, and Poor's Risk Solution.

Frye, J. (2000a): Collateral Damage, *Risk* **13** (4), pp. 91-94.

Frye, J. (2000b): Depressing Recoveries, *Risk* **13** (11), pp. 108-111.

Frye, J. (2003): A False Sense of Security, *Risk* **16** (11), pp. 63-68.

Gordy, M. (2000): A Comparative Anatomy of Credit Risk Models, in: *Journal of Banking and Finance* **24**, pp. 119-149.

Gordy, M. (2003): A Risk-Faktor Model Foundation for Ratings-Based Bank Capital Rules, in: *Journal of Financial Intermediation* **12**, pp. 199-232.

Gordy, M. and Heitfield, E. (2002): Estimating Default Correlations from Short Panels of Credit Rating Performance Data, Working Paper, Federal Reserve Board.

Greene, W. H. (2003): *Econometric Analysis*, Prentice Hall, New Jersey.

Gupton, G. M., Finger, Christopher C., and Bhatia, Mickey (1997): *CreditMetrics – Technical Document*, Morgan Guarantee Trust Co.

Gupton, G. M., Gates, D., and Carty, L. V. (2000): *Bank Loan Loss Given Default*, Moody's Investor Service.

Hamerle, A., Liebig, T., and Rösch, D. (2003): Benchmarking Asset Correlation, *Risk* **14** (11), pp. 77-81.

Hamerle, A., and Rösch, D. (2004): Parameterizing Credit Risk Models, Working Paper, *www.defaultrisk.com*.

Hamilton, J. D. (1994): Time Series Analysis, Princeton, Chilchester.

Hamilton, D. T., Cantor, R., and Ou, S. (2002): *Default, and Recovery Rates of Corporate Bond Issuers*, Moody's Investor Service.

Hamilton, D. T., Cantor, R., West, M., and Fowlie, K. (2002): *Default, and Recovery Rates of European Corporate Bond Issuers, 1985-2001*, Moody's Investor Service.

Hu, Y.-T., and Perraudin, W. (2002): The Dependence of Recovery Rates, and Defaults, Working Paper, *www.defaultrisk.com*.

Hull, J. (2003): *Options, Futures,, and Other Derivatives*. Prentice Hall, New Jersey.

Ilmanen, A., McGuire, D., and Warga, A. (1994): The Value of Duration as a Risk Measure for Corporate Debt, *Journal of Fixed Income* **4** (6), pp. 70-76.

Jokivuolle, E., and Peura, S. (2000): A Model for Estimating Recovery Rates, and Collateral haircuts for Bank Loans, Bank of Finland Discussion Papers, *www.defaultrisk.com*.

Jorion, P. (2001): *Value at Risk*, McGraw-Hill, New York.

Keenan, S. C., Hamilton, D. T., and Berthault, A. (2000): *Historical Default Rates of Corporate Bond Issuers, 1920-1999*, Moody's Investor Service.

Klein, P. (1996): Pricing Black-Scholes Options with correlated Credit Risk, *Journal of Banking and Finance* **20**, pp. 1211-1229.

Koyluoglu, H. U., and Hickman, A. (1998a): Reconcilable Differences, *Risk* **11** (10), pp. 56-62.

Koyluoglu, H. U., and Hickmann, A. (1998b): A Generalized Framework for Credit Portfolio Models, Working Paper, *www.defaultrisk.com*.

Löffler, G. (2003): The effects of Estimation Error on Measure of Portfolio Credit Risk, *Journal of Banking and Finance* **27**, pp. 1427-1453.

Merton, R. C. (1974): On the Pricing of Corporate Debt: the Risk Structure of Interest Rates, *Journal of Finance* **29**, pp. 449-470.

Ong, M. K. (1999): *Internal Credit Risk Models – Capital Allocation, and Performance Measurement*, Risk Waters, London.

Pykthin, M. (2003): Unexpected Recovery Risk, *Risk* **16** (8), pp. 74-78.

Pykthin, M., and Dev, A. (2002): Analytical Approach to Credit Risk Modelling, *Risk* **15** (3), pp. S26-S32.

Rösch, D. (2003): Correlations, and Business Cycles of Credit Risk: Evidence from Bankruptcies in Germany, in: *Financial Markets and Portfolio Management* **17**, pp. 309-331.

Schönbucher, P. J. (2003): *Credit Derivatives Pricing Models: Models, Pricing, and Implementation*, Jon Wiley & Sons, Chichester.

Schuermann, T. (2004): What Do We Know About Loss-Given-Default?, Working Paper, *www.defaultrisk.com*.

Tasche, D. (2004): The Single Risk Factor Approach to Capital Charges in Case of Correlated Loss Given Default Rates, Working Paper, *www.defaultrisk.com*.

Thadewald, T., and Büning, H. (2004): *Jaque-Bera Test, and its Competitors for Testing Normality – A power Comparison*, Working Paper, Freie Universität Berlin.

Thode, H. C. (2002): *Testing for Normality*, Dekker, New York.

Uhrig-Homburg, M. (2002): Valuation of Defaultable Claims – a Survey, *Schmalenbach Business Review* **54** (1), pp. 24-57.

Vasicek, O. A. (1987): *Probability of Loss on Loan Portfolio*, KMV Corporation.

Vasicek, O. A. (1991): *Limiting Loan Loss Probability Distribution*, KMV Corporation.

Vasicek, O. A. (2002): Loan Portfolio Value, *Risk* **15** (12), pp. 160-162.

Wilde, T. (2001): IRB Approach Explained, *Risk* **14** (5), pp. 87-90.

Wilson, T. (1997a): Portfolio Credit Risk I, *Risk* **10** (9), pp. 111-119.

Wilson, T. (1997b): Portfolio Credit Risk II, *Risk* **10** (10), pp. 56-62.

Zar, J. H. (1999): *Biostatistical Analysis*, Fourth Edition, Prentice Hall, Upper Saddle River, NJ.

Figures and Tables

Figure 1 Default Rates and Default Points for Entire Germany, 1962 – 2003

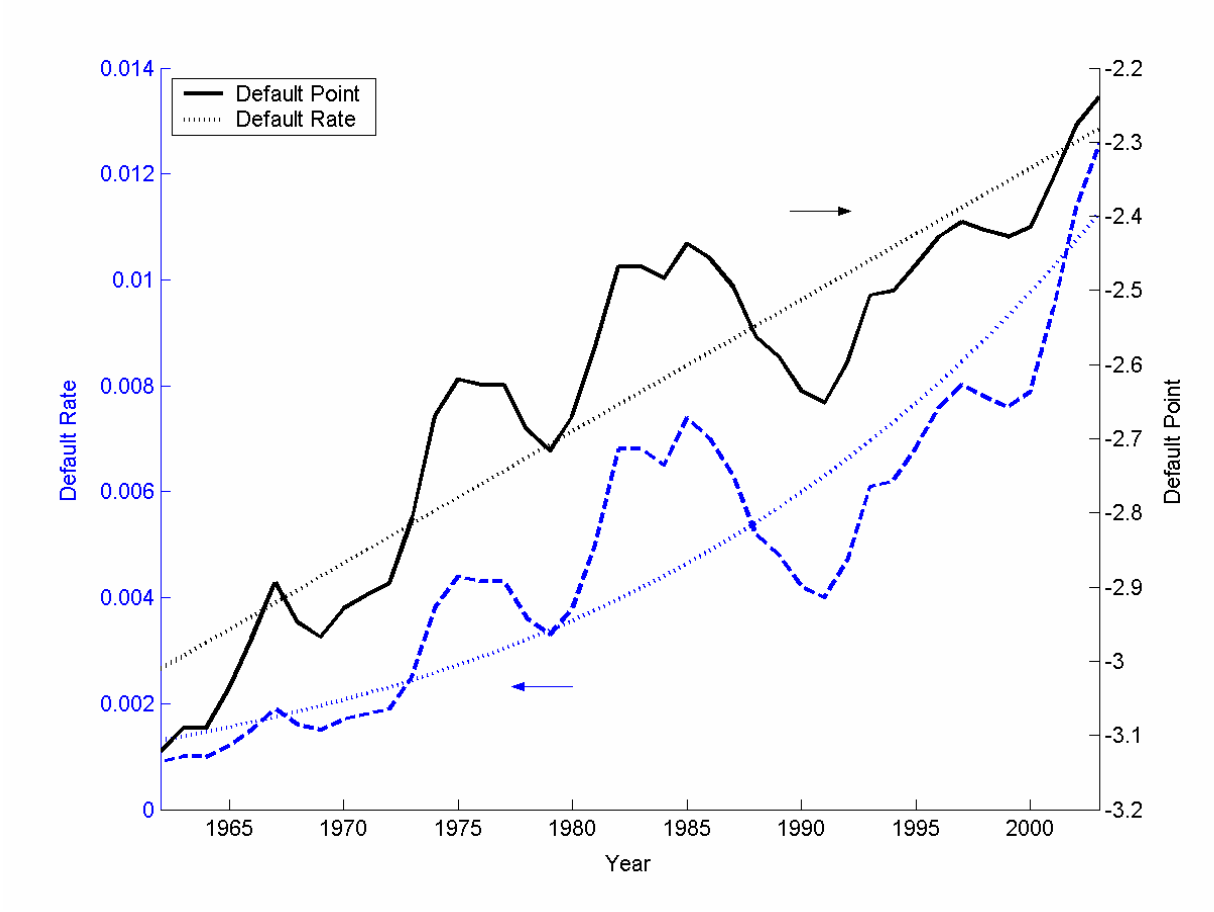


Figure 2 Detrended Default Rates and the Systematic Credit Cycle Factor for Entire Germany

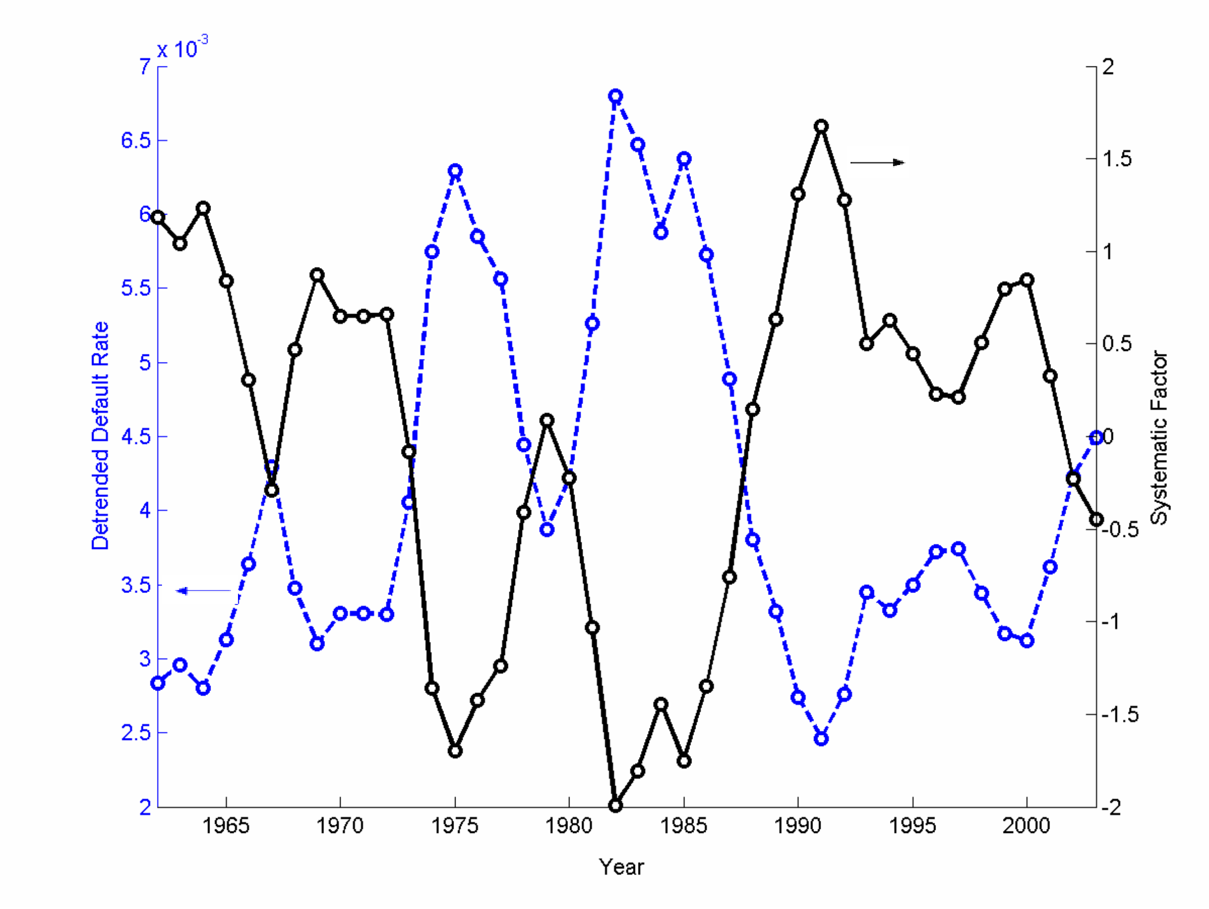
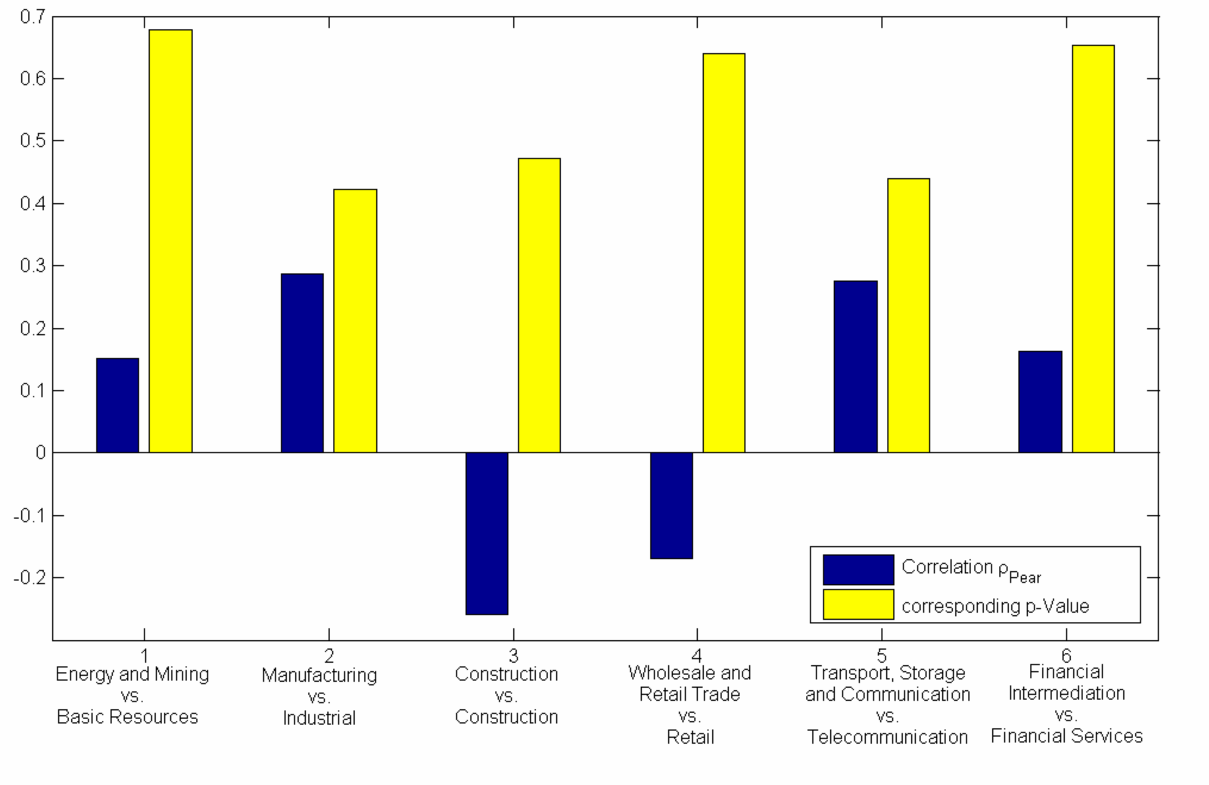
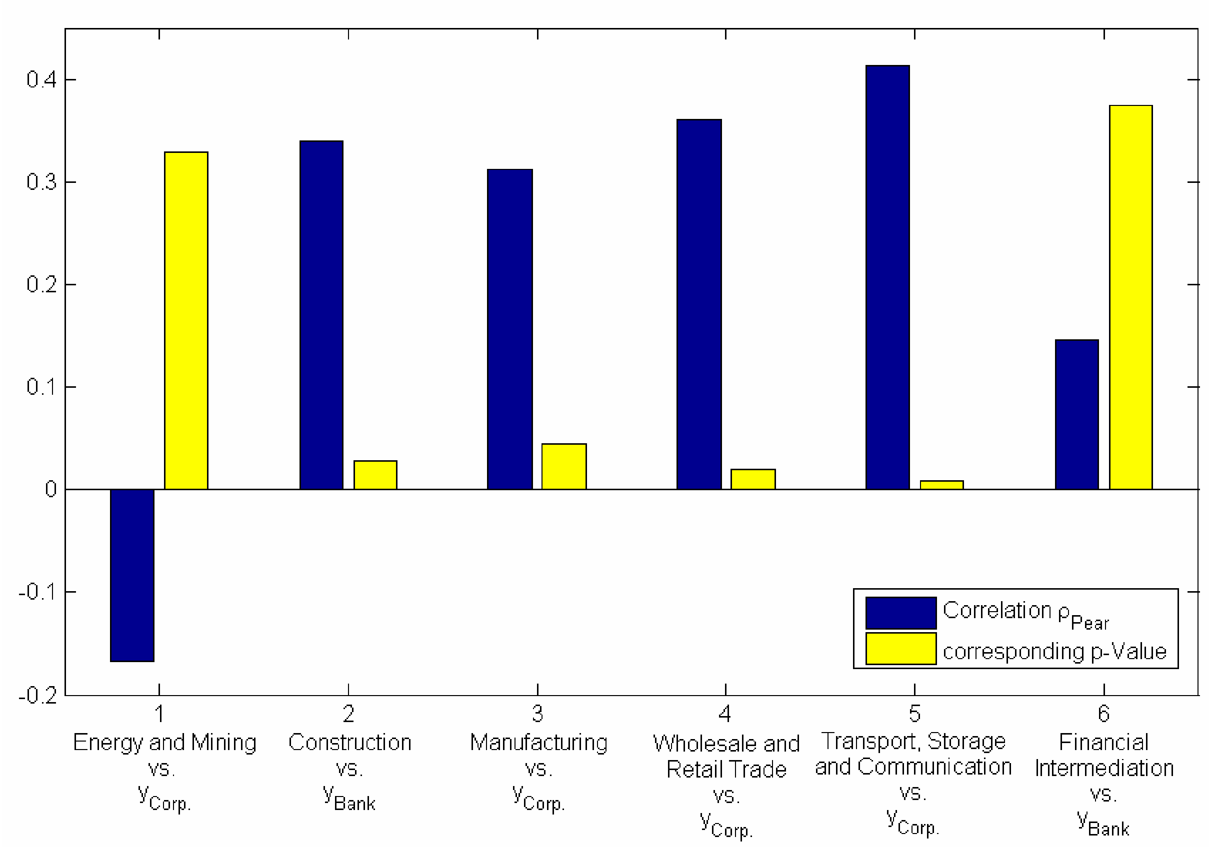


Figure 3 Correlations between Sector Systematic Factors and Prime Indexes



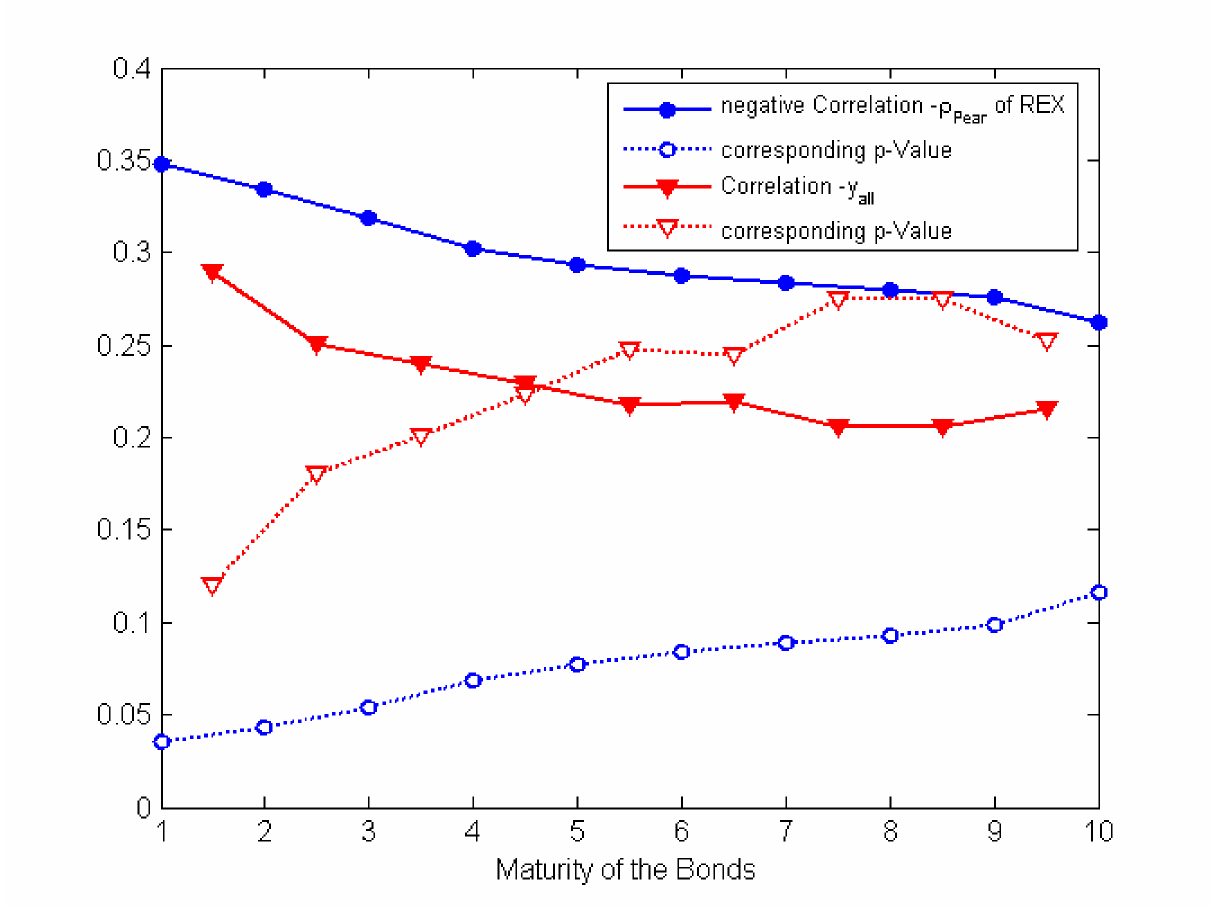
The linear correlation coefficients due to Pearson-Bravais (ρ_{Pear}) are shown. For the hypothesis of correlation coefficients being zero (no correlation) using t-statistics the p-values are reported. Only the period from 1994to 2003 is considered.

Figure 4 Correlations between Sector Systematic Factors and Yield



The linear correlation coefficients due to Pearson-Bravais (ρ_{Pear}) are shown. For the hypothesis of correlation coefficients being zero (no correlation) using t-statistics the p-values are reported. The period from 1965 to 2003 is considered.

Figure 5 Correlations between Sector Systematic Factors and Maturity



The linear correlation coefficients due to Pearson-Bravais (ρ_{Pear}) between the systematic factor (entire Germany) and the return of the REX and the yield on German Bonds outstanding are shown. For the hypothesis of correlation coefficients being zero (no correlation) using t-statistics the p-values are reported. The period from 1962 to 2003 is considered.

Table 1 Estimations of $\hat{\rho}_A$ and $\widehat{PD}_A$ for Entire Germany and the Six Sectors

Sector		$\hat{\rho}_A$	$\widehat{PD}_A$
	Entire Germany	0.0087††† (0.0019)	0.0013 … 0.0113††† (0.0058)
1	Energy and Mining	0.0183††† (0.0043)	0.0003 … 0.0011†††
2	Manufacturing	0.0104††† (0.0023)	0.0013 … 0.0113††† (0.0091)
3	Construction	0.0129††† (0.0028)	0.0026 … 0.0250††† (0.0070)
4	Wholesale and Retail Trade	0.0049††† (0.0011)	0.0011 … 0.0096††† (0.0043)
5	Transport, Storage and Communication	0.0072††† (0.0036)	0.0010 … 0.0207††† (0.0058)
6	Financial Intermediation	0.0162††† (0.0016)	0.0014 … 0.0160††† (0.0054)

Standard Errors for $\widehat{PD}_A$ and $\hat{\rho}_A$ are in parentheses, for $\widehat{PD}_A$ they refer to a standardized level of $PD_A = 0.5$. All parameters are significant at 1%(†††) level. Since the expected default probability $\widehat{PD}_A$ varies due to the time trend, we report the lowest and highest value here.

Table 2 Correlation Analysis of the Geometric Returns of Financial Indexes and Prices

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(1) DAX	1 1 1												
(2) MDAX	0.87††† 0.69††† 0.88†††	1 1 1											
(3) SDAX	0.8††† 0.71 0.87†††	0.89††† 0.73††† 0.88†††	1 1 1										
(4) REX	0.05*** 0.08*** 0.12***	0.03*** 0.03*** 0.07***	-0.24*** -0.24*** -0.33***	1 1 1									
(5) CGBI	-0.03 -0.02 -0.08	-0.08 -0.1 -0.13	-0.29* -0.22** -0.31**	0.96††† 0.85††† 0.96†††	1 1 1								
(6) y_{Bank}	0.03 -0.06 -0.07	0.04 0.00 -0.03	0.27** 0.3** 0.43**	-0.96††† -0.87††† -0.97†††	-0.92††† -0.77††† -0.91	1 1 1							
(7) $y_{Corp.}$	-0.2 -0.18 -0.27	-0.19 -0.15 -0.23	-0.11* 0.03*** 0.04***	-0.88††† -0.77††† -0.9†††	-0.75††† -0.66††† -0.77†††	0.89††† 0.78††† 0.91	1 1 1						
(8) GSCI	-0.24** -0.2** -0.28***	-0.14* -0.17** -0.21**	-0.14 -0.24 -0.31	-0.07*** -0.1*** -0.15***	-0.04*** -0.1*** -0.1***	0.02*** 0.08*** 0.11***	0.04*** 0.13*** 0.17***	1 1 1					
(9) Gold (USD)	-0.2** -0.14*** -0.2***	-0.18* -0.13** -0.18**	0.02 -0.03** -0.03**	0.02 -0.02* -0.01*	0.09 0.06 0.06	-0.05 0.01 0,00	-0.05 -0.00** -0.01*	0.55††† 0.35††† 0.55†††	1 1 1				
(10) Gold (EUR)	-0.14** -0.14** -0.25**	-0.12* -0.17* -0.26*	-0.16 -0.26 -0.32	0.25 0.14 0.22	0.25 0.16 0.25	-0.24 -0.18 -0.27	-0.29 -0.16 -0.25	0.47††† 0.27†† 0.37††	0.4†† 0.21† 0.28***	1 1 1			
(11) JPY	-0.3* -0.18** -0.28**	-0.25 -0.15** -0.22**	-0.07 -0.05** -0.05**	-0.11** 0.06** 0.05**	0.24** 0.16* 0.23*	0.05** -0.03*** -0.05***	0.11*** -0.03** -0.04**	0.41†† 0.15*** 0.23***	0.42†† 0.41 0.57†††	0.11*** 0.15*** 0.21***	1 1 1		
(12) USD	-0.23*** -0.15** -0.23***	-0.19** -0.07** -0.14**	-0.09 -0.09 -0.1	-0.13 0.07 0.08	0.16* 0.17* 0.22*	0.08 -0.05 -0.06	0.15 -0.02 -0.05	0.19*** -0.02** -0.01**	-0.01*** 0.02*** 0.03***	-0.06 0.03** 0.06**	0.91 0.62††† 0.79†††	1 1 1	
(13) GBP	-0.22** -0.08** -0.11**	-0.14** 0.01 0.00*	0.11 0.14 0.23	-0.43†† -0.27†† -0.4††	-0.3 -0.15* -0.22*	0.4†† 0.25†† 0.38††	0.32† 0.17 0.26	0.08** 0.07 0.11	0.14 0.10 0.15	-0.05 -0.01 -0.05	0.48††† 0.25†† 0.37††	0.47††† 0.27†† 0.37††	1 1 1

The indication of the columns is carried out only in terms of the specification of the numbers that are dedicated to the variables in the first column. The correlation coefficients are those due to Pearson-Bravais (linear correlation), Kendall (rank correlation) and Spearmann (linear rank correlation). For the hypothesis of correlation coefficients being zero (no correlation) significance is reported at level of 1%(†††) / 5% (††) / 10% (†) / 15% (***) / 30% (**) / 50% (*).

Table 3 Results of the Correlation-Analysis between the Systematic Factor and Returns of Stock Indices

$\hat{x}_{T,GER}$	whole period	1974-1983	1984-1993	1994-2003
DAX	$\rho_{Spear} = -0.1346^{*}$ $\tau_B = -0.1225^{*}$ $\rho_{Pear} = -0.1496^{*}$	$\rho_{Spear} = -0.7697^{\dagger\dagger}$ $\tau_B = -0.6444^{\dagger\dagger\dagger}$ $\rho_{Pear} = -0.6707^{\dagger\dagger}$	$\rho_{Spear} = -0.1757$ $\tau_B = -0.1555$ $\rho_{Pear} = -0.1497$	$\rho_{Spear} = -0.1757$ $\tau_B = -0.1111$ $\rho_{Pear} = 0.0977$
MDAX	$\rho_{Spear} = -0.1209$ $\tau_B = -0.0838$ $\rho_{Pear} = -0.0911$	$\rho_{Spear} = -0.4787^{**}$ $\tau_B = -0.3333^{**}$ $\rho_{Pear} = -0.4917^{***}$	$\rho_{Spear} = -0.1393$ $\tau_B = -0.1555$ $\rho_{Pear} = -0.1393$	$\rho_{Spear} = -0.2242$ $\tau_B = -0.1555$ $\rho_{Pear} = -0.0427$
SDAX	$\rho_{Spear} = -0.0642$ $\tau_B = -0.0476$ $\rho_{Pear} = -0.1987^{*}$			$\rho_{Spear} = -0.0666$ $\tau_B = -0.0222$ $\rho_{Pear} = -0.1925$

The correlation coefficients are those due to Spearmann (ρ_{Spear}), Kendall (τ_B), and Pearson-Bravais (ρ_{Pear}). For the hypothesis of correlation coefficients being zero (no correlation) significance is reported at level of 1%(†††) / 5% (††) / 10% (†) / 15% (***) / 30% (**) / 50% (*).

Table 4 Results of the Correlation-Analysis between the Systematic Factor and Returns of Fixed Income Securities Indexes and Yields

$\hat{x}_{T,GER}$	whole periode	1964-1973	1974-1983	1984-1993	1994-2003
REX	ρ_{Spear}= - 0.2743*** τ_B= - 0.1952† ρ_{Pear}= - 0.2945†		ρ_{Spear}= - 0.7818†† τ_B= - 0.600†† ρ_{Pear}= - 0.7818††	ρ_{Spear}= - 0.1515 τ_B= - 0.1555 ρ_{Pear}= - 0.1988	ρ_{Spear}= - 0.0909 τ_B= - 0.1111 ρ_{Pear}= - 0.2631*
CGBI	ρ_{Spear}= 0.0421 τ_B= - 0.0058 ρ_{Pear}= - 0.0557				ρ_{Spear}= - 0.0787 τ_B= - 0.0666 ρ_{Pear}= - 0.1834
y_{Bank}	ρ_{Spear}= 0.2887† τ_B= 0.1848† ρ_{Pear}= 0.2782†	ρ_{Spear}= 0.4303** τ_B= 0.3333** ρ_{Pear}= 0.2622*	ρ_{Spear}= 0.7818†† τ_B= 0.6000†† ρ_{Pear}= 0.7568†	ρ_{Spear}= 0.2606* τ_B= 0.2000* ρ_{Pear}= 0.2294	ρ_{Spear}= 0.2000 τ_B= 0.1111 ρ_{Pear}= 0.3450*
$y_{Corp.}$	ρ_{Spear}= 0.3373†† τ_B= 0.2218†† ρ_{Pear}= 0.3393††	ρ_{Spear}= 0.5636† τ_B= 0.422*** ρ_{Pear}= 0.4372**	ρ_{Spear}= 0.7818†† τ_B= 0.6000†† ρ_{Pear}= 0.7964†††	ρ_{Spear}= 0.2121 τ_B= 0.2000* ρ_{Pear}= 0.2047	ρ_{Spear}= 0.4545* τ_B= 0.3777** ρ_{Pear}= 0.5758†

The correlation coefficients are those due to Spearmann (ρ_{Spear}), Kendall (τ_B), and Pearson-Bravais (ρ_{Pear}). For the hypothesis of correlation coefficients being zero (no correlation) significance is reported at level of 1%(†††) / 5% (††) / 10% (†) / 15% (***) / 30% (**) / 50% (*).

Table 5 Results of the Correlation-Analysis between the Systematic Factor and Returns of Commodities

$\hat{x}_{T,GER}$	whole period		1974-1983		1984-1993		1994-2003	
GSCI	ρ_{Spear} =	$0.2965^{\dagger}$	ρ_{Spear} =	0.3939^{**}	ρ_{Spear} =	0.3212^{*}	ρ_{Spear} =	0.1636
	τ_B =	0.1871^{***}	τ_B =	0.2444^{*}	τ_B =	0.2000^{*}	τ_B =	0.1111
	ρ_{Pear} =	0.2542^{***}	ρ_{Pear} =	$0.6216^{\dagger}$	ρ_{Pear} =	0.4345^{***}	ρ_{Pear} =	-0.0245
Gold (USD)	ρ_{Spear} =	-0.0857	ρ_{Spear} =	0.2121	ρ_{Spear} =	-0.2363	ρ_{Spear} =	-0.2848^{*}
	τ_B =	-0.0571	τ_B =	0.1555	τ_B =	-0.2000^{*}	τ_B =	-0.2000^{*}
	ρ_{Pear} =	0.0019	ρ_{Pear} =	0.4231^{**}	ρ_{Pear} =	-0.2733^{*}	ρ_{Pear} =	-0.4620^{**}
Gold (EUR)	ρ_{Spear} =	-0.0054	ρ_{Spear} =	0.2727^{*}	ρ_{Spear} =	0.3818^{**}	ρ_{Spear} =	0.0424
	τ_B =	0.0222	τ_B =	0.2000^{*}	τ_B =	0.3777^{**}	τ_B =	-0.0222
	ρ_{Pear} =	-0.0128	ρ_{Pear} =	0.4078^{**}	ρ_{Pear} =	0.1731	ρ_{Pear} =	0.1335

The correlation coefficients are those due to Spearmann (ρ_{Spear}), Kendall (τ_B), and Pearson-Bravais (ρ_{Pear}). For the hypothesis of correlation coefficients being zero (no correlation) significance is reported at level of 1%(†††) / 5% (††) / 10% (†) / 15% (***) / 30% (**) / 50% (*).

Table 6 Results of the Correlation-Analysis between the Systematic Factor and Changes of Exchange rates

$\hat{x}_{T,GER}$	whole period	1974-1983	1984-1993	1994-2003
USD	$\rho_{Spear} = 0.0744$	$\rho_{Spear} = 0.2242$	$\rho_{Spear} = -0.2121$	$\rho_{Spear} = -0.3454^{*}$
	$\tau_B = 0.0443$	$\tau_B = 0.2000^{*}$	$\tau_B = -0.1555$	$\tau_B = -0.1555$
	$\rho_{Pear} = -0.0130$	$\rho_{Pear} = 0.3190^{*}$	$\rho_{Pear} = -0.4275^{**}$	$\rho_{Pear} = -0.5851$†
GBP	$\rho_{Spear} = -0.1114$	$\rho_{Spear} = -0.4181^{**}$	$\rho_{Spear} = -0.1393$	$\rho_{Spear} = 0.0667^{*}$
	$\tau_B = -0.0725$	$\tau_B = -0.3333^{**}$	$\tau_B = -0.1111$	$\tau_B = 0.0667$
	$\rho_{Pear} = -0.1115$	$\rho_{Pear} = -0.4500^{**}$	$\rho_{Pear} = -0.1569$	$\rho_{Pear} = -0.1449$
JPY	$\rho_{Spear} = 0.1928^{**}$	$\rho_{Spear} = 0.2363$	$\rho_{Spear} = -0.1515$	$\rho_{Spear} = -0.2484^{*}$
	$\tau_B = 0.1491^{**}$	$\tau_B = 0.1555$	$\tau_B = -0.1111$	$\tau_B = -0.1555$
	$\rho_{Pear} = 0.1581^{*}$	$\rho_{Pear} = 0.1848$	$\rho_{Pear} = -0.0480$	$\rho_{Pear} = -0.4104^{**}$

The correlation coefficients are those due to Spearmann (ρ_{Spear}), Kendall (τ_B), and Pearson-Bravais (ρ_{Pear}). For the hypothesis of correlation coefficients being zero (no correlation) significance is reported at level of 1%(†††) / 5% (††) / 10% (†) / 15% (***) / 30% (**) / 50% (*).

Table 7 Haircuts of Financial Collaterals

	$\sigma_{C,1\ Month}$	$\rho_{Pear}^{(reas)}$	$\rho_{Pear}^{(neutr)}$	H_{BII}	H_{con}	$H_{sen}^{(reas)}$	$H_{sen}^{(neutr)}$
Stocks							
DAX	0.0836	0.6852	0	15%	19.45%	13.33%	0%
MDAX	0.0519	0.6031	0	15%	12.07%	7.28%	0%
Bonds							
CGBI (1-3 years)	0.0041	0.7966	0	0,5-2%	0.95%	0.76%	0%
CGBI (3-5 years)	0.0086	0.6770	0	2%	2.00%	1.35%	0%
REX (1 year)	0.0021	0.4727	-0.3475	0.5%	0.49%	0.23%	-0.17%
REX(5 years)	0.0102	0.5025	-0.2938	2%	2.37%	1.19%	-0.70%
REX(10 years)	0.0153	0.5073	0	4%	3.56%	1.81%	0%
Commodities							
GSCI	0.0632	0.8993	0.2543		14.70%	13.22%	3.74%
Gold [EUR]	0.0426	0.8255	0	15%	9.91%	8.18%	0%
Exchange Rates							
USD	0.0299	0.7900	0	8%	6.96%	5.50%	0%
GBP	0.0203	0.5333	0	8%	4.72%	2.52%	0%
JPY	0.0321	0.7296	0	8%	7.47%	5.45%	0%

The standard deviations $\sigma_{C,1\ Month}$ are based on the monthly geometric returns during the period 1999 to 2003. For the reasonable linear correlation $\rho_{Pear}^{(reas)}$ the highest upper 95%-quantil of all decades is taken. The neutral linear correlation $\rho_{Pear}^{(neutr)}$ represents the value, that is measured over the whole period. It is zero, if the level of significance is less than 15 %. The four haircuts are those due to Basel II (H_{BII}), own estimates using a 99%-quantil without reduction of the standard deviation (H_{con}) , with reduction resulting from $\rho_{Pear}^{(reas)}$ ($H_{sen}^{(reas)}$) and from $\rho_{Pear}^{(neutr)}$ ($H_{sen}^{(neutr)}$).

Table 8 Statistics of the ADF-Regression of the Default Point and Detrended Default Point

	γ	β	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	DW
				Entire Germany				
ΔDP_T^{∞}	$-0.17^{\ddagger\ddagger\ddagger}$	$0.0035^{\dagger\dagger}$	$0.65^{\dagger\dagger}$	-0.26^{**}	0.13^{*}			$1.94^{\ddagger}$
$\Delta DP_T^{(\infty,\text{detrended})}$	$-0.20^{\ddagger\ddagger\ddagger}$		$0.69^{\dagger\dagger\dagger}$	-0.26^{***}	0.13^{*}			$1.98^{\ddagger\ddagger\ddagger}$
				Energy and Mining				
ΔDP_T^{∞}	$-0.90^{\ddagger\ddagger\ddagger}$	$0.0073^{\dagger}$	$0.18^{\ddagger}$	-0.29^{**}	$0.58^{\dagger}$	0.37^{**}	$0.67^{\dagger\dagger\dagger}$	1.85
$\Delta DP_T^{(\infty,\text{detrended})}$	$-0.69^{\ddagger\ddagger\ddagger}$		-0.08	0.25^{***}	$0.39^{\dagger\dagger}$			1.81
				Manufacturing				
ΔDP_T^{∞}	$-0.20^{\ddagger\ddagger\ddagger}$	$0.0042^{\dagger\dagger}$	$0.58^{\dagger\dagger\dagger}$	-0.17^{*}				$1.99^{\ddagger\ddagger\ddagger}$
$\Delta DP_T^{(\infty,\text{detrended})}$	$-0.27^{\dagger\dagger}$		$0.57^{\dagger\dagger\dagger}$	-0.14^{*}				1.86
				Construction				
ΔDP_T^{∞}	$-0.25^{\dagger}$	$0.0052^{\dagger\dagger\dagger}$	$0.52^{\dagger\dagger\dagger}$					$1.95^{\ddagger\ddagger}$
$\Delta DP_T^{(\infty,\text{detrended})}$	$-0.26^{\dagger}$		$0.50^{\dagger\dagger\dagger}$					$1.96^{\ddagger\ddagger\ddagger}$
				Wholesale and Retail Trade				
ΔDP_T^{∞}	$-0.26^{\dagger\dagger}$	$0.0044^{\dagger\dagger\dagger}$	$0.53^{\dagger\dagger\dagger}$					$1.93^{\ddagger\ddagger\ddagger}$
$\Delta DP_T^{(\infty,\text{detrended})}$	$-0.26^{\dagger\dagger}$		$0.53^{\dagger\dagger\dagger}$					$1.98^{\ddagger\ddagger\ddagger}$
				Transport, Storage and Communication				
ΔDP_T^{∞}	$-0.37^{\dagger\dagger}$	$0.0077^{\dagger\dagger\dagger}$	$0.36^{\dagger\dagger}$					$2.00^{\ddagger\ddagger\ddagger}$
$\Delta DP_T^{(\infty,\text{detrended})}$	$-0.36^{\dagger\dagger}$		$0.38^{\dagger\dagger}$					1.92
				Financial Intermediation				
ΔDP_T^{∞}	$-0.37^{\dagger}$	$0.0102^{\dagger\dagger}$						$2.00^{\dagger\dagger\dagger}$
$\Delta DP_T^{(\infty,\text{detrended})}$	$-0.38^{\dagger}$							$2.00^{\dagger\dagger\dagger}$

We used the following ADF-Regression: $\Delta DP_T^{\infty} = m + \gamma \cdot DP_{T-1}^{\infty} + \beta \cdot T + \sum_{j=1}^{n} \Phi_j \cdot \Delta DP_{T-j} + \varepsilon_T$ with $\Delta DP_T^{\infty} = DP_T^{\infty} - DP_{T-1}^{\infty}$ for ΔDP_T^{∞}. For $\Delta DP_T^{(\infty,\text{detrended})}$ β is set to zero. For the hypothesis of non-stationary (unit root, $\gamma = 0$) using Philips-Perron statistics, of regression coefficients being zero ($\beta, \Phi_1, \ldots, \Phi_5 = 0$) using t-statistics and the hypothesis of no autocorrelation of the residuals (DW = 2) using Durban h statistics are reported at 1%(†††) / 5% (††) / 10% (†) / 15% (***) / 30% (**) / 50% (*) / 80% (‡) / 85% (‡‡) / 90% (‡‡‡) level of significance.

Table 9 Statistics of the Autoregressive Regression of the Default Point and Detrended Default Point

	m	β	Φ_1	Φ_2	DW	F-value	$\overline{R}^2$
Entire Germany							
DP_T^{∞}	-0.54	$0.0037^{\dagger\dagger}$	$1.37^{\dagger\dagger\dagger}$	$-0.57^{\dagger\dagger\dagger}$	1.76	$382.47^{\dagger\dagger\dagger}$	0.97
$DP_T^{(\infty,\text{detrended})}$	-0.00		$1.37^{\dagger\dagger\dagger}$	$-0.57^{\dagger\dagger\dagger}$	1.76	$90.96^{\dagger\dagger\dagger}$	0.82
Energy and Mining							
DP_T^{∞}	-1.71	0.0027^{*}	0.16^{*}	0.32^{**}	2.09	$6.65^{\dagger\dagger\dagger}$	0.34
$DP_T^{(\infty,\text{detrended})}$	-0.00		0.15^{*}	0.26^{*}	1.95	1.43^{*}	0.03
Manufacturing							
DP_T^{∞}	-0.66	$0.0049^{\dagger\dagger\dagger}$	$1.29^{\dagger\dagger\dagger}$	$-0.54^{\dagger\dagger\dagger}$	1.77	$276.93^{\dagger\dagger\dagger}$	0.96
$DP_T^{(\infty,\text{detrended})}$	-0.00		$1.30^{\dagger\dagger\dagger}$	$-0.55^{\dagger\dagger\dagger}$	1.77	$65.03^{\dagger\dagger\dagger}$	0.77
Construction							
DP_T^{∞}	-0.60	$0.0052^{\dagger\dagger\dagger}$	$1.28^{\dagger\dagger\dagger}$	$0.53^{\dagger\dagger\dagger}$	1.96	$263.48^{\dagger\dagger\dagger}$	0.95
$DP_T^{(\infty,\text{detrended})}$	-0.00		$1.28^{\dagger\dagger\dagger}$	$-0.53^{\dagger\dagger\dagger}$	1.96	$64.22^{\dagger\dagger\dagger}$	0.76
Wholesale and Retail Trade							
DP_T^{∞}	-0.71	$0.0044^{\dagger\dagger\dagger}$	$1.26^{\dagger\dagger\dagger}$	$-0.53^{\dagger\dagger\dagger}$	1.93	$446.6^{\dagger\dagger\dagger}$	0.97
$DP_T^{(\infty,\text{detrended})}$	-0.00		$1.27^{\dagger\dagger\dagger}$	$-0.53^{\dagger\dagger\dagger}$	1.93	$63.52^{\dagger\dagger\dagger}$	0.76
Transport, Storage and Communication							
DP_T^{∞}	-0.97	$0.0077^{\dagger\dagger\dagger}$	$0.99^{\dagger\dagger\dagger}$	$-0.35^{\dagger\dagger}$	2.00	225.31	0.95
$DP_T^{(\infty,\text{detrended})}$	-0.00		$1.00^{\dagger\dagger\dagger}$	$-0.37^{\dagger\dagger}$	2.00	$24.86^{\dagger\dagger\dagger}$	0.57
Financial Intermediation							
DP_T^{∞}	-0.97	$0.0099^{\dagger\dagger}$	$0.61^{\dagger\dagger\dagger}$	0.02	1.91	$96.62^{\dagger\dagger\dagger}$	0.89
$DP_T^{(\infty,\text{detrended})}$	-0.01		$0.62^{\dagger\dagger\dagger}$	0.02	1.91	$10.685^{\dagger\dagger\dagger}$	0.35

We used the following autoregressive regression: $DP_T^{\infty} = m + \beta \cdot T + \Phi_1 \cdot DP_{T-1} + \Phi_2 \cdot DP_{T-2} + \varepsilon_T$ for DP_T^{∞}. For $DP_T^{(\infty,\text{detrended})}$ β is set to zero. For the hypothesis of regression coefficients being zero ($\beta,\Phi_1,\Phi_2=0$) using t-statistics and of all regression coefficients ($\beta=\Phi_1=\Phi_2=0$) jointly being zero using F statistics significance is reported at 1%(†††) / 5% (††) / 10% (†) / 15% (***) / 30% (**) / 50% (*) level.

Table 10 Test of Autocorrelation of the Default Point and Detrended Default Point					
Sector			Q	LM	DW
	Entire Germany	DP_T^{∞}	35.77 (0.00)	39.20 (0.00)	0.00 (< 0.01)
		$DP_T^{(\infty,\text{detrended})}$	32.53 (0.00)	30.85 (0.00)	0.26 (< 0.01)
1	Energy and Mining	DP_T^{∞}	10.75 (0.00)	10.49 (0.00)	0.00 (< 0.01)
		$DP_T^{(\infty,\text{detrended})}$	1.36 (0.24)	1.42 (0.23)	1.48 (> 0.05)
2	Manufacturing	DP_T^{∞}	35.16 (0.00)	38.62 (0.00)	0.00 (< 0.01)
		$DP_T^{(\infty,\text{detrended})}$	29.28 (0.00)	28.30 (0.00)	0.33 (< 0.01)
3	Construction	DP_T^{∞}	35.47 (0.00)	38.54 (0.00)	0.00 (< 0.01)
		$DP_T^{(\infty,\text{detrended})}$	29.97 (0.00)	28.49 (0.00)	0.33 (< 0.01)
4	Wholesale and Retail Trade	DP_T^{∞}	37.01 (0.00)	39.50 (0.00)	0.00 (< 0.01)
		$DP_T^{(\infty,\text{detrended})}$	30.54 (0.00)	28.14 (0.00)	0.34 (< 0.01)
5	Transport, Storage and Communication	DP_T^{∞}	33.80 (0.00)	35.95 (0.00)	0.00 (< 0.01)
		$DP_T^{(\infty,\text{detrended})}$	21.77 (0.00)	20.71 (0.00)	0.51 (< 0.01)
6	Financial Intermedia-tion	DP_T^{∞}	30.01 (0.00)	33.78 (0.00)	0.00 (< 0.01)
		$DP_T^{(\infty,\text{detrended})}$	15.50 (0.00)	14.86 (0.00)	0.73 (< 0.01)

The tests used are the Ljung-Box-Pierce test (Q), Lagrange Multiplier test by Godfrey-Breusch (LM), and Durbin-Watson test (DB) for 1 lag. Tested is the hypothesis of no autocorrelation, in parentheses are the corresponding p-values (“>” or “<” if the p-value is not specified exactly).

Table 11 KPSS-Test of Stationary of the Default Point and Detrended Default Point

Sector			ω = 4	ω= 8	ω= 12
	Entire Germany	DP_T^{∞}	0.1546†††	0.1260††	0.1191††
		$DP_T^{(\infty,\text{detrended})}$	0.1546†	0.1260†	0.1191†
1	Energy and Mining	DP_T^{∞}	0.0835†††	0.0956†††	0.1723†
		$DP_T^{(\infty,\text{detrended})}$	0.0838†††	0.0964†††	0.1749†††
2	Manufacturing	DP_T^{∞}	0.1676†	0.1421††	0.1272††
		$DP_T^{(\infty,\text{detrended})}$	0.1676†††	0.1421†††	0.1272†††
3	Construction	DP_T^{∞}	0.1362††	0.1231††	0.1186†††
		$DP_T^{(\infty,\text{detrended})}$	0.1362†††	0.1231†††	0.1186†††
4	Wholesale and Retail Trade	DP_T^{∞}	0.1145†††	0.1090†††	0.1153†††
		$DP_T^{(\infty,\text{detrended})}$	0.1145†††	0.1090†††	0.1153†††
5	Transport, Storage and Communication	DP_T^{∞}	0.0749†††	0.0823†††	0.1032†††
		$DP_T^{(\infty,\text{detrended})}$	0.0749†††	0.0823†††	0.1032†††
6	Financial Intermedia-tion	DP_T^{∞}	0.1746†	0.1287††	0.1251††
		$DP_T^{(\infty,\text{detrended})}$	0.1746†††	0.1287†††	0.1251†††

For the KPSS test the regression: $DP_T^{\infty} = m + \beta \cdot T + k \cdot \sum_{i=1}^{T} \varepsilon_i + \eta_T$ for DP_T^{∞} is used. For $DP_T^{(\infty,\text{detrended})}$ β is set to zero. For the hypothesis of stationary (k = 0) with the reference point ω using KPSS statistics significance are reported at 1%(†††) / 5% (††) / 10% (†) level.

Table 12 Statistics of the Tests of Normality of the Detrended Default Point

Sector		KSL	JB	G	DP	SWR
	Entire Germany	0.1315 (0.07)	3.3419 (0.18)	1.4398 (0.15)	3.6273 (0.16)	2.171 (0.02)
1	Energy and Mining	0.1204 (> 0.2)	2.1755 (0.34)	1.900 (0.06)	2.1809 (0.33)	1.3887 (0.08)
2	Manufacturing	0.0903 (> 0.2)	2.1253 (0.35)	0.6546 (0.52)	2.202 (0.33)	1.0808 (0.14)
3	Construction	0.1505 (0.03)	2.5343 (0.28)	0.2918 (0.77)	2.5493 (0.28)	1.7818 (0.04)
4	Wholesale and Retail Trade	0.1787 (< 0.01)	3.5778 (0.17)	0.3462 (0.7292)	4.4721 (0.11)	2.3510 (0.01)
5	Transport, Storage and Communication	0.0775 (> 0.2)	1.1480 (0.56)	0.3757 (0.71)	1.0884 (0.58)	-0.02 (0.49)
6	Financial Intermediation	0.1415 (0.05)	2.3700 (0.31)	2.7738 (0.01)	2.0296 (0.36)	1.7354 (0.04)

The tests used are the Kolmogorov-Smirnov-Lilliefors (KSL), Jarque-Bera (JB), Geary (G) and D'Agostino-Pearson (DP) and Sharpiro-Wilk-Royston (SWR). We tested the hypothesis, that the data is normal distributed.. In parentheses are the corresponding p-values (">" or "<" if the p-value is not specified exactly).

Table 13 Tests of Autocorrelation of the Systematic Credit Cycle Factor

Sector		Q	LM	DW
	Entire Germany	32.5339 (0.00)	30.8478 (0.00)	0.2617 (< 0.01)
1	Energy and Mining	1.3570 (0.24)	1.4119 (0.23)	1.4780 (> 0.05)
2	Manufacturing	29.2754 (0.00)	28.2957 (0.00)	0.3308 (< 0.01)
3	Construction	29.9652 (0.00)	28.4902 (0.00)	0.3271 (< 0.01)
4	Wholesale and Retail Trade	30.5381 (0.00)	28.1433 (0.00)	0.3413 (< 0.01)
5	Transport, Storage and Communication	21.7719 (0.00)	20.7121 (0.00)	0.50984 (< 0.01)
6	Financial Intermediation	15.5046 (0.00)	14.8602 (0.00)	0.73107 (< 0.01)

The tests used are the Ljung-Box-Pierce test (Q), Lagrange Multiplier test by Godfrey-Breusch (LM), and Durbin-Watson test (DB) for 1 lag. Tested is the hypothesis of no autocorrelation, in parentheses are the corresponding p-values (“>” or “<” if the p-value is not specified exactly).

Table 14 Tests of Normality of the Logarithmic returns of Financial Collaterals

Type	Data Series	KSL	JB	G	DP	SWR
Stocks	DAX	0.04315 (>0.2)	0.7224 (0.70)	0.0487 (0.96)	1.3853 (0.50)	-1.0191 (0.15)
	MDAX	0.0877 (>0.2)	0.5849 (0.75)	0.3915 (0.70)	1.3192 (0.52)	-0.2400 (0.41)
	SDAX	0.1501 (>0.2)	0.4137 (0.8131)	0.2540 (0.80)	0.3402 (0.84)	-0.7302 (0.23)
Bonds	REX	0.1062 (>0.2)	1.7168 (0.43)	1.5450 (0.1223)	1.6252 (0.44)	0.4365 (0.33)
	CGBI	0.0906 (>0.2)	0.8097 (0.66)	1.2816 (0.41)	0.9019 (0.64)	-1.5623 (0.06)
	y_{Bank}	0.0734 (>0.2)	0.0900 (0.96)	0.6854 (0.49)	0.5912 (0.74)	-1.7315 (0.04)
	$y_{Corp.}$	0.0589 (>0.2)	0.4895 (0.7829)	0.1446 (0.89)	0.4982 (0.78)	-1.1972 (0.37)
Commodities	GSCI	0.1417 (0.09)	2.1078 (0.35)	1.3685 (0.17)	3.5608 (0.17)	1.2326 (0.11)
	Gold (USD)	0.1047 (>0.2)	23.2864 (0.00)	2.2427 (0.025)	18.7839 (0.00)	2.7481 (0.00)
	Gold (EUR)	0.1623 (0.03)	35.7018 (0.00)	2.6382 (0.01)	24.262 (0.00)	3.575 (0.00)
Exchange Rates	JPY to EUR	0.0849 (>0.2)	0.0704 (0.97)	0.0849 (0.43)	0.9922 (0.61)	-1.4283 (0.08)
	USD to EUR	0.1827 (<0.01)	42.1253 (0.35)	1.6762 (0.09)	4.6052 (0.4127)	1.3416 (0.09)
	GBP to EUR	0.1978 (<0.01)	2.5827 (0.27)	2.1134 (0.03)	3.0967 (0.21)	2.1763 (0.01)

The tests used are the Kolmogorov-Smirnov-Lilliefors (KSL), Jarque-Bera (JB), Geary (G) and D'Agostino-Pearson (DP) and Sharpiro-Wilk-Royston (SWR). We tested the hypothesis, that the data is normal distributed.. In parentheses are the corresponding p-values (">" or "<" if the p-value is not specified exactly).

Table 15 Tests of Autocorrelation of the Logarithmic returns of Financial Collaterals

Type	Data Series	Q	LM	DW
Stocks	DAX	0.9516 (0.76)	0.0870 (0.77)	1.9263 (> 0.05)
	MDAX	1.8325 (0.18)	1.7665 (0.18)	2.2733 (> 0.05)
	SDAX	0.5906 (0.44)	0.8877 (0.35)	1.9219 (> 0.05)
Bonds	REX	0.1868 (0.67)	0.1700 (0.68)	2.1077 (> 0.05)
	CGBI	0.6117 (0.43)	0.5261 (0.47)	0.6928 (< 0.01)
	y_{Bank}	0.5806 (0.45)	0.5286 (0.46)	2.2143 (> 0.05)
	$y_{Corp.}$	0.0008 (0.97)	0.0010 (0.97)	1.8671 (> 0.05
Commodities	GSCI	0.02340 (0.88)	0.0213 (0.88)	1.8998 (> 0.05)
	Gold (USD)	0.3231 (0.57)	0.2908 (0.59)	1.7106 (> 0.05)
	Gold (EUR)	0.1623 (0.69)	0.1487 (0.70)	1.7257 (> 0.05)
Exchange Rates	JPY to EUR	0.1579 (0.69)	0.1405 (0.71)	1.9024 (> 0.05)
	USD to EUR	3.0667 (0.08)	2.8609 (0.09)	1.3504 (> 0.05)
	GBP to EUR	1.6278 (0.20)	1.4845 (0.22)	2.4013 (> 0.05)

The tests used are the Ljung-Box-Pierce test (Q), Lagrange Multiplier test by Godfrey-Breusch (LM), and Durbin-Watson test (DB) for 1 lag. Tested is the hypothesis of no autocorrelation, in parentheses are the corresponding p-values (">" or "<" if the p-value is not specified exactly).