On the comparison of classical and special relativistic space-time

Balzer, Wolfgang

W.Balzer[1)]

ON THE COMPARISON OF CLASSICAL AND SPECIAL RELATIVISTIC SPACE-TIME

One way of comparing two theories T,T´ is to reduce T to T´ in a formal sense. Much has been written about different intuitions on reduction and several meta-scientific concepts of reduction have been proposed; but few examples have been used by way of detailed examination in order to throw light on those meta-scientific concepts.[2)] Of course, there are numerous examples of reduction in the ordinary (i.e."non-meta") scientific literature. But usually meta-scientific concepts of reduction cannot be directly applied to such examples: the "ordinary" treatments may be too vague, too sloppy, too incomplete, or they may use special assumptions so that actually only very small fragments of the theories are involved.

This problem of application is well known from ordinary science, and usually part of its solution consists in an interplay between reality (as given by examples) and scientific concepts. At the beginning there are usually a few examples which an author uses as paradigms in order to introduce his concepts. But once the concepts are presented there are attempts to apply them to other "new" cases as well. If difficulties arise then either the concepts may be kept unchanged and the new examples have to be "twisted", or the new example can be taken as "experimentum crucis", and the concepts have to be adjusted.

This interplay also takes place at the meta-level of the philosophy of science, and I believe that with respect to reduction we are still at a rather early stage of it. Much attention will have to be given to examples, and the present volume is only a first attempt in that direction.

W. Balzer et al. (eds.), Reduction in Science, 331–357.

My aim in this paper is to present a formally elaborated example of reduction in which the theory of classical space and time (CT) is reduced to the theory of special relativistic space-time (RT). Since the reduction to be employed will be strict, i.e.not approximative,the question of adequacy arises very pressingly because "every physicist" will say that the appropriate reduction relation of CT to RT has to be an approximative one.In the presence of this considerable opposition I will try to defend my example as a genuine case of reduction by considering the various objections that might be raised.In this way I hope to shed some light on the general concept of reduction without subscribing to any one of the existing formal notions.Also,the discussion will contribute to a clarification of the concept of classical space-time and its relation to Galilei-invariance.As far as I know,this is the first comparison of space-times on the axiomatic level (as opposed to the "group theoretic" level).The surprisingly easy way of defining a reduction relation ρ in this setting should be regarded as an argument for paying more attention to axiomatic analysis which in investigations of space-time at the moment is completely suppresed in favour of group theoretical methods.

I GENERAL NOTIONS

Today in physics space-time structures are characterized with respect to their corresponding invariances.Roughly and generally, one starts with some structure $x=\langle D,R_1,\ldots,R_m\rangle$ consisting of a set D and relations R_i on D.Automorphisms of x are those bijective functions $\varphi:D\rightarrow D$ which preserve all R_i,i.e. $\forall a_1\ldots a_n\in D(\ R_i(a_1,\ldots,a_n)\leftrightarrow R_i(\varphi(a_1),\ldots,\varphi(a_n)))$,provided R_i is n-ary.The set of all automorphisms of x together with the concatenation operation of functions is a group, called the <u>transformation group</u> of x. If $D=\mathbb{R}^n$ and if the R_i are specified (e.g. for n=1, $R_1=<$,$R_2=+$ etc.) then the corresponding transformation groups are well known and can be characterized easily.

These characterizations are then "transferred" to non-mathematical structures by means of group isomorphisms. A structure x is identified by means of its transformation group being isomorphic to some well known transformation group of a given mathematical structure. For instance, some structure is a Galileian space-time iff its transformation group is isomorphic to the group formed by Galilei-transformations on $\mathbb{R}^3$ plus affine transformations of $\mathbb{R}$. In order to demonstrate that some "direct" characterization (as opposed to an indirect via transformation groups) is adequate it is sufficient to show that the transformation group of a model thus characterized is isomorphic to the corresponding mathematical group accepted by physicists.

It turns out that such direct proofs are complicated, and it is easier to show that any structure x under consideration is isomorphic to a given mathematical structure y which has the known transformation group. For if this is so then the two automorphism groups (of x and of y) are isomorphic, too.

I will use a slightly more general set-up which is a version of Bourbaki's "species of structures".[3] What has just been outlined then takes the following form.

A theory T consists of a class of <u>potential models</u> M_p and a class of (proper) <u>models</u> M:

$$T=\langle M_p, M\rangle \quad \text{where } M \subseteq M_p.$$

All potential models have the form

$$\langle D_1,\ldots,D_k;A_1,\ldots,A_l;R_1,\ldots,R_m\rangle$$

where $k,l,m \in \mathbb{N}$ are fixed, $D_1,\ldots,D_k$ are sets, called <u>base sets</u>, $A_1,\ldots,A_l$ are sets of mathematical objects (called <u>auxiliary base sets</u>) and $R_1,\ldots,R_m$ are relations of given set-theoretic types $\tau_1,\ldots,\tau_m$ "over" $D_1,\ldots,D_k,A_1,\ldots,A_l$.[4] For instance, A_1 may be $\mathbb{R}$ and $R_1: D_1 \times D_2 \to \mathbb{R}$ a function. The auxiliary base sets represent some mathematical "part" of the model which always has the same (standard) interpretation. Let T and T′ be given so that the types of the potential models of T and T′ and the mathematical parts involved are the same, and let $x=\langle D_1,\ldots,D_k;A_1,\ldots,A_l;R_1,\ldots,R_m\rangle \in M_p$ and $x'=\langle D'_1,\ldots,D'_k;A_1,\ldots,A_l;R'_1,\ldots,R'_m\rangle \in M'_p$. We say that x and x′ are <u>isomorphic</u> iff there are bijective functions $\varphi_i : D_i \to D'_i$ $(i \leq k)$ such that for all appropriate arguments $a_1,\ldots,a_n$ and all $j \leq m$:

$$R_j(a_1,\ldots,a_n) \leftrightarrow R'_j(\varphi_{i_1}(a_1),\ldots,\varphi_{i_n}(a_n)).$$

$\varphi=\langle\varphi_1,\ldots,\varphi_k\rangle$ is called an <u>automorphism</u> of x if φ is an isomorphism from x to x. By Aut(x) we denote the group of automorphisms of x (with group operation defined by $\varphi \circ \psi = \langle\varphi_1 \circ \psi_1,\ldots,\varphi_k \circ \psi_k\rangle$. The result indicated above holds in this more general setting, too: if x and x′ are isomorphic then so are Aut(x) and Aut(x′).

II CLASSICAL THEORY OF SPACE AND TIME (CT)

<u>D1</u> x is a <u>potential model of</u> CT $(x \in M_p(CT))$ iff $x=\langle S,T;\mathbb{R};\lessdot,\tau,\delta\rangle$ and
1) S and T are non-empty sets, and disjoint
2) $\lessdot \subseteq T \times T$
3) $\tau : T \times T \to \mathbb{R}$
4) $\delta : T \times S \times S \to \mathbb{R}$

S is the set of points of space, T the set of instants. The intended meaning of $\lessdot$, τ and δ is this. $t \lessdot t'$ means that t is earlier than t′, $\tau(t,t')=\alpha$ means that the period of time between t and t′ (as measured by some clock) has length α, and $\delta(t,a,b)=\alpha$ means that at time t the distance between a and b is α.

If N is a set and $d : N \times N \to \mathbb{R}$ then $\underline{bet}_d \subseteq N^3$ and $\equiv_d \subseteq N^4$ are defined by

$$\underline{bet}_d(a,b,c) \text{ iff } d(a,b)+d(b,c)=d(a,c)$$

$$ab \equiv_d a'b' \text{ iff } d(a,b)=d(a',b').$$

If $\delta : T \times S \times S \to \mathbb{R}$ and $t \in T$ then $\delta(t) : S \times S \to \mathbb{R}$ is defined by $\delta(t)(a,b)=\delta(t,a,b)$. The meaning of $\underline{bet}_d(a,b,c)$ is that b is between a and c, and $ab \equiv_d a'b'$ means that the pairs $<a,b>$ and $<a',b'>$ are congruent.

<u>D2</u> x is a <u>model of</u> CT $(x \in M(CT))$ iff
1) $x=<S,T; \mathbb{R}; \lessdot, \tau, \delta> \in M_p$
2) for all $t \in T$: $<S, \delta(t)>$ and $<T,\tau>$ are metric spaces, and $\lessdot$ is a linear order
3) $<T, \underline{bet}_\tau, \equiv_\tau>$ is a 1-dimensional Euclidean geometry
4) for all $t \in T$: $<S, \underline{bet}_{\delta(t)}, \equiv_{\delta(t)}>$ is a 3-dimensional Euclidean geometry[5)]
5) for all $t,t' \in T$: $\delta(t)=\delta(t')$

We can best imagine a model as a "series" of identical copies of 3-dimensional spaces where T provides the indices. T can be visualized by a straight line on which an ordering $\lessdot$ and a distance τ is given. At each instant t the corresponding space $<S, \underline{bet}_{\delta(t)}, \equiv_{\delta(t)}>$ satisfies all the axioms of Euclidean geometry. If we omit T from δ then we would just have two metric spaces put together. This couldn't be called a "space-time" because in such a structure we could not formulate expressions of the form "<u>at</u> t the distance of a and b is α". By making δ dependent on t we obtain the possibility of formulating such expressions. On the other hand the time-dependence of δ is immediately withdrawn by means of D2-5) which requires δ, in fact, <u>not</u> to depend on t properly. The effect is a "rigid" space-time consisting essentially of the cartesian product of "space" and "time".

Some further comments may be helpful. First, in a model the (relativistic) set E of events could be explicitly defined by $E=S \times T$. I have chosen not to use E as a primitive in order to do justice to the historical situation before Minkowski. Second, I have not included any notions and requirements concerning the orientation of space. So, reflections are not excluded from the corresponding transformation group. The system could be easily adjusted to obtain the proper transformation groups. I have chosen not to exclude reflections because this would make things more complicated without adding new aspects to the reduction relation. Third, if at $t \in T$ we choose coordinate systems K for $\langle S, \delta(t)\rangle$ and K′ for $\langle T, \tau\rangle$ then the content of D2) is represented equivalently by the structure $\langle S \times T, \psi\rangle$, where $\psi : S \times T \times \mathbb{R}^3 \to \mathbb{R}$, $\psi(\langle b,t\rangle) = \langle \psi_1(b), \psi_2(t)\rangle$ and $\psi_1(b)$, $\psi_2(t)$ are the coordinates of b and t relative to K and K′.[6)]

I will next describe the corresponding transformation group and only afterwards discuss the question of adequacy. Let $\mathscr{G}$ be the "elementary group"[7)], i.e. $\mathscr{G}$ is defined as the direct product $\mathscr{G} = \mathscr{G}_S \otimes \mathscr{G}_T$ where $\mathscr{G}_S$ is the group of dilatations, translations and rotations of $\mathbb{R}^3$ and $\mathscr{G}_T$ is the affine group of $\mathbb{R}$. Let $\mathscr{G}'$ be obtained from $\mathscr{G}$ by including reflections in $\mathscr{G}_S$ and by omitting dilatations from $\mathscr{G}_T$ and $\mathscr{G}_S$.

<u>T1</u> If $x \in M(CT)$ then Aut(x) is isomorphic to $\mathscr{G}'$.

Proof: Let < be the usual "smaller than" relation on $\mathbb{R}$, and let $|\cdot|$, $\|\cdot\|$ be the Euclidean distance functions on $\mathbb{R}$ and $\mathbb{R}^3$, respectively. Then the structure $y = \langle \mathbb{R}^3, \mathbb{R}; \mathbb{R}; <, |\cdot|, \delta_R\rangle$ has the same type as our models of CT if we define

$\delta_R : \mathbb{R} \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ by $\delta_R(\alpha, \mathfrak{x}, \mathfrak{y}) = \|\mathfrak{x} - \mathfrak{y}\|$.

(The second occurrence of "$\mathbb{R}$" in y indicates the use of $\mathbb{R}$ as range of $|\cdot|$ and δ_R in the status of an auxiliary base set). It is well known that Aut(y) is isomorphic to $\mathscr{G}'$, so by what was said in Sec. I it is sufficient to show that any model $x \in M(CT)$ is isomorphic to y. Let $x = \langle S, T; \mathbb{R}; \prec, \tau, \delta\rangle \in M(CT)$. Then an isomorphism $\varphi = \langle \mu, \eta\rangle$ with $\mu : S \to \mathbb{R}^3$ and $\eta : T \to \mathbb{R}$ is obtained by introducing coordinate systems for S and T respectively in the well known way ∎

The physical meaning of the differences between $\mathscr{G}$ and $\mathscr{G}'$ is clear. Spatial reflections cannot be actively performed in reality, and the passive possibility of looking at physical systems through some mirror has played no role up to now. Dilatations correspond to the freedom of choice of a unit. A treatment including dilatations in the transformation group of CT would have to start at the qualitative level of bet and $\equiv$. It is achievable, but more complicated than the present formulations. I think in spite of these small deviations one can say that CT is "essentially" represented by $\wp$, and thus is "essentially" Newtonian space-time.

The immediate objection now is that "classical space-time" has to be Galilei-invariant so that the full group of Galilei-transformations, and not its sub-group $\wp$, is the appropriate transformation group. The objection has three parts. First, as a sociological statement, one simply observes that most physicists today hold that classical space-time is Galilei-invariant. Second, from an historical point of view, one may argue that in the period leading to and including the introduction of classical mechanics space-time was regarded as Galilei-invariant. Third, from a systematic point of view, a comparison of CT with RT (or other theories) may suggest that we look for Galilei-transformations as a counterpart to Lorentz-transformations. I will consider the three items in turn.

As to the first point, I agree that physicists today require classical space-time to be Galilei-invariant. But philosophy of science is not the same as sociology of science and what constitutes an unshakable fact for the latter may be of less importance for the former. I believe that this first part of the objection is the least important one, and is outweighed by the other two. I will argue that with respect to the other parts classical space-time should not be Galilei-invariant but only be invariant under the elementary group.

From an historical point of view it seems to me that Galilei was the first to point out that mechanical (i.e. dynamical) events will be the same if taking place in or being perceived from two different frames of reference moving relative to each other with constant velocity. During the development of Newtonian mechanics, too, Galilei-invariance in this special sense always turned up with considerations of mechanical systems ("dynamics"). In the course of such considerations space-time was always presupposed, i.e. the properties of space and time were assumed to be already known.

Space was represented by Euclidean geometry and time by a straight line (if at all), and there was no idea of the relevance of the state of motion of an observer for the properties of space and time. The latter statement is compatible with the fact that the discussion of inertial frames of reference as a matter of dynamics preceded the advent of special relativity. I am not in a position to give a detailed historical account of this topic. But unless historical arguments to the contrary are put forward I conclude that, historically, classical space-time is adequately represented by the elementary group.

Third, my formulation of RT in the next section is especially suited to making clear why there is a systematic drive for Galilei-invariance on the classical side. Any model of RT "implicitly" contains some frame of reference W. But W is not uniquely determined by the other parts of the model, and a change of W in general will not leave unaffected the validity of the axioms. So in RT it is natural to consider transformations of coordinates relative to different frames of reference which are possible in one model. This leads to Lorentz-transformations. One is tempted to look for a similar feature at the side of CT. Things look differently from different frames of reference, and one would like to know how the coordinates transform under changes of the frame of reference. In order to perform such investigations it is necessary on the classical side to introduce different frames of reference. This can be done, but only at the price of introducing a new basic concept. In the models of CT only one frame of reference can be defined in analogy to W in RT, namely $\{\{\langle a,t\rangle / t \in T\} / a \in S\}$. If we want to talk about different classical frames we are forced to use further concepts not available in CT. Thus there is a formal distinction between RT and CT. For a potential model of RT there are many different possible frames of reference W which make it into a model. For a potential model of CT there is only one possible frame, namely the one defined above, and this frame is not necessary for stating the axioms. Intuitively, in RT the basic stuff the models are formed of (E and $\prec$) has to be enriched by further entities (W) if we want to express the full complexity of the models. In CT no such additional entities are needed. Again, what was said here is compatible with stating that the status of Galilei-transformations in <u>mechanics</u> is independent of relativistic theories.

One way of obtaining a Galilei-invariant theory from CT is to enrich CT by frames of reference. Models then would have the form $<S,T;\mathbb{R};\lessdot,\tau,\delta,F>$ where F is a partition of $S \times T$ satisfying further requirements to the effect that F is just a "bundle" of parallel straight lines so that the lines are not "orthogonal" to T. It is clear that different F's can make some given model of CT into a model of the new theory, and all these F's can be obtained from each other by Galilei-transformations. It is not difficult to show that this theory in fact is represented by the group of Galilei-transformations in the sense of Sec.I. But it is also clear that the new concept of a frame of reference is not linked in any interesting way to the "old" concepts; it is added ad hoc. There is no intrinsic connection between F and the other concepts, in contrast to the connection between W and E,$\prec$ in RT. The only systematic reason for Galilei-invariance of the classical theory comes from the search for an analogue to relativistic frames of reference. But any theory created by this analogy is an artificial construct which has no standing on its own -in contrast to CT.

As far as I can see all other arguments for Galilei-invariance of classical space-time can be traced back to the three just mentioned.For instance, it may be said that space-time theory and the full theory of classical mechanics form an inseparable unit, so that the invariances of mechanics are also relevant for the underlying space-time. This is the same kind of reasoning by analogy from RT as we just met before. Again, on closer inspection, this view imposes features on the classical theory which seem to be added ad hoc after the invention of RT.

To summarize,then,I would say that historical and systematical considerations favour classical space-time as being represented by the elementary group, that these two aspects are more important than the sociological one, and that, therefore, CT is an adequate reconstruction of classical space-time.

III THE SPECIAL RELATIVISTIC THEORY OF SPACE-TIME (RT)

D3 x is a potential model of RT ($x \in M_p(RT)$) iff $x = \langle E; \prec \rangle$ and
1) E is a non-empty set
2) $\prec \subseteq E \times E$

E is the set of events and $\prec$ is the so called "causal relation". $e \prec e'$ means that a signal can be sent from event e to event e'.[8)] Some notation needs to be fixed for the following. Let $x = \langle E; \prec \rangle \in M_p(RT)$. If $X \subseteq E$ and $e_1, e_2 \in E$ we write "$e_1 \preccurlyeq e_2$" for "$e_1 \prec e_2$ or $e_1 = e_2$". We say that e is an upper (lower) bound of X iff for all $e_1 \in X$: $e_1 \preccurlyeq e$ ($e \preccurlyeq e_1$) We write $e = \inf_{\prec} X$ ($e = \sup_{\prec} X$) iff e is a lower (upper) bound of X and for all lower (upper) bounds e_2 of X: $e_2 \preccurlyeq e$ ($e \preccurlyeq e_2$). X is called bounded iff X has an upper and a lower bound. We write "$e_1 \rightsquigarrow e_2$" for "$\exists e, e' \in E(\neg e \prec e' \wedge \neg e' \prec e \wedge e \neq e' \wedge e_1 \prec e \prec e_2 \wedge e \prec e' \prec e_2)$" which means that a signal slower than light can be sent from e_1 to e_2.

If $x \in M_p(RT)$ we say that W is a frame for x iff (1) W is a partition of E, (2) for each $w \in W$ there are functions $f_w : E \to w$ and $g_w : E \to w$ so that for all $e \in E$:

$$f_w(e) = \begin{cases} e & \text{if } e \in w \\ \inf_{\prec} \{e' \in w / e \prec e'\} & \text{if } e \notin w \end{cases}$$

$$g_w(e) = \begin{cases} e & \text{if } e \in w \\ \sup_{\prec} \{e' \in w / e' \prec e\} & \text{if } e \notin w \end{cases}$$

If W is a frame for x then $\underline{bet}_x \subseteq W^3$ and $\equiv_x \subseteq W^4$ are defined by

$\underline{bet}_x(u,v,w)$ iff $f_u \circ f_v \circ f_w = f_u \circ f_w$ and

$uv \equiv_x u'v'$ iff $f_{u'} \circ f_u \circ f_v \circ f_u = f_{u'} \circ f_{v'} \circ f_{u'} \circ f_u$.

Intuitively, W can be imagined as a bundle of parallel straight lines running "time-like" (wrt. $\prec$) through E. Each line $w \in W$ represents a possible path of some free particle and is called a world line. $f_w(e)$ is the event of arrival of a flash of light at world line w which is emitted at e. Similarly, $g_w(e)$ is the event on w determined by the condition that a flash of light omitted at $g_w(e)$ would hit e. $\underline{bet}_x$ and $\equiv_x$ have the usual meaning of relations of betweenness and congruence, respectively: only that the objects they are defined for are world lines.

<u>D4</u> a) x is a <u>model of</u> RT <u>relative to</u> W iff

1) $x = \langle E; \prec \rangle \in M_p(RT)$ and W is a frame for x
2) $\prec$ is transitive and $e \prec e'$ implies $\neg\, e' \prec e$
3) for all $w \in W$ and all $X \subseteq w$: if X is bounded then there are e_1, e_2 so that $e_1 = \inf_{\prec} X$, $e_2 = \sup_{\prec} X$, $e_1 \in w$ and $e_2 \in w$
4) for all $w \in W$ and all $e, e' \in E$: if $e \in w$ and $e' \in w$ then $(e = e' \vee e \rightarrow e' \vee e' \rightarrow e)$
5) for all $w \in W$ and $e \in E$ there are e_1, e_2 so that $e_1 \prec e \prec e_2$ and $e_1, e_2 \in w$
6) for all $v, u \in W$: if $u \neq v$ then $f_{u/v}$ and $g_{v/u}$ are inverse to each other
7) $\langle W; \underline{bet}_x, \equiv_x \rangle$ is a 3-dimensional Euclidean geometry
8) for all $u, v, u', v' \in W$: $(f_w \circ f_v \circ f_u) \circ (f_w \circ f_{v'} \circ f_{u'})_{/w} = (f_w \circ f_{v'} \circ f_{u'}) \circ (f_w \circ f_v \circ f_u)_{/w}$ and $(f_w \circ f_v \circ f_u)_{/w} = (f_w \circ f_u \circ f_v)_{/w}$
9) for all $w \in W$ and $e, e' \in w$: if $e \prec e'$ then there is $v \in W$ so that $v \neq w$ and $f_w(f_v(e)) \prec e'$
10) for all $w \in W$ there is a countable and dense (wrt. $\prec$) subset of w
11) for all $u, v, w \in W$: if there is $e \in w$ so that $f_u(f_v(e)) = f_u(e)$ then $\underline{bet}_x(u, v, w)$

b) x is a model of RT ($x \in M(RT)$) iff there is some W so that x is a model of RT relative to W

The present axiomatization is essentially due to A.Kamlah who made precise Reichenbach's original version in (Kamlah,1979),pp.436. Three deviations from Kamlah's system should be mentioned.First, I require W to exhaust all of E, second I have added D4-a-3) which I cannot prove in Kamlah's system. Third, I have added D4-a-11) which is essential for the proof of T2).

Axiom 3) is needed in order to prove that the defining conditions for f_w and g_w, in fact, guarantee uniqueness,that is, f_w and g_w, in fact, are functions. 4) requires each world line to run through the time-like sections of the light cones, and 5) rules out absolute boundaries wrt. $\prec$. $f_{u/v}$ in 6) denotes the restriction of f_u to v. Requirement 6) is of more technical character. The functions f_w are well defined (on the basis of the previous axioms) and uniquely determined in x. So the betweenness and congruence relations $\underline{bet}_x$ and $\equiv_x$ in 7) are well defined,too. 8) expresses a kind of invariance. It makes no difference whether a signal travels via world lines u',v',w,u,v to w or alternatively via u,v,w,u',v': the event of arrival at w will be the same in both cases. Similarly, it makes no difference to go to w by way of u and v or via v and u. These conditions also guarantee that the world lines of W are "straight". Condition 9) requires that the world lines are dense in E (wrt. $\prec$): in each neighbourhood of each f_w there is another f_v. 10) guarantees that world lines have the right cardinality (used for mapping them on $\mathbb{R}$ bijectively). Requirement 11), finally, enforces that all world lines of W are "parallel" to each other.

In models of RT we can introduce clocks, simultaneity and a "space-like" metric as follows.

<u>D5</u> Let $x=\langle E;\prec\rangle \in M_p(RT)$, let W be a frame for x and $w \in W$.

a) Φ_w is a <u>clock for</u> x (relative to w) iff

$\Phi_w : w \to \mathbb{R}$ is bijective and for all $u \in W$ and $e, e' \in w$: 1) $e \prec e'$ iff $\Phi_w(e) < \Phi_w(e')$
2) $\Phi_w(f_w(f_u(e))) - \Phi_w(e) = \Phi_w(f_w(f_u(e'))) - \Phi_w(e')$

b) If Φ_w is a clock for x relative to w then $sim_{\Phi,w} : w \to Pot(E)$ is defined by: $e' \in sim_{\Phi,w}(e)$ iff $e=e'$ or there exist $v \in W$ and $e_1, e_2 \in w$ such that (1) $e' \in v$, (2) $e_1 \prec e \prec e_2$, (3) $e' = f_v(e_1)$ and $e_2 = f_w(e')$, (4) $\Phi_w(e) = 1/2(\Phi_w(e_2) + \Phi_w(e_1))$

c) If $v \neq w, v \in W$ then $d_{x,v,w}$ is <u>a metric for</u> x relative to v,w iff $d_{x,v,w} : W \times W \to \mathbb{R}$ is a metric such that for all $u_1, \ldots, u_4 \in W$:
1) $\underline{bet}_x(u_1, u_2, u_3)$ iff $d_{x,v,w}(u_1, u_2) + d_{x,v,w}(u_2, u_3) = d_{x,v,w}(u_1, u_3)$
2) $u_1 u_2 \equiv_x u_3 u_4$ iff $d_{x,v,w}(u_1, u_2) = d_{x,v,w}(u_3, u_4)$
3) $d_{x,v,w}(v, w) = 1$

$\Phi_w(e)$ is intended to denote the time (as measured on w) at which event e takes place. $sim_{\Phi,w}(e)$ is the class of all events of E which are simul taneous to e (with respect to Φ_w), and is called the simultaneity class of e (wrt. Φ_w). $d_{x,v,w}(u, u')$ is the spatial distance between world lines u and u'.

With respect to RT the question of adequacy is easier to settle. There is common agreement that the causal Minkowski-structure
$\langle \mathbb{R}^4 ; <_c \rangle$ with $\langle \alpha_1, \ldots, \alpha_4 \rangle <_c \langle \beta_1, \ldots, \beta_4 \rangle$ iff

$$\alpha_4 < \beta_4 \wedge \left(\sum_{i \leq 3} (\alpha_i - \beta_i)^2 \right)^{1/2} \leq c \cdot (\beta_4 - \alpha_4)$$

is a model (indeed, <u>the</u> model) of RT. The automorphism group of this structure is the group of Lorentz-transformations, as was indicated already by Weyl.[9)] Our scheme of Sec.I, however, cannot be directly applied to these structures. For $\mathbb{R}^4$ gets its standard meaning only through additional relations (like $<,+,\cdot,0,1$) which are not mentioned in $\langle \mathbb{R}^4; <_c \rangle$, and these relations are kept fixed when Lorentz-transformations are considered. So $\mathbb{R}^4$ here has the status of an auxiliary base set, and therefore no proper base set at all is involved. But without a base set there are no automorphisms in the sense of Sec. I.

Fortunately, it is easy to modify the structure $\langle \mathbb{R}^4; <_c \rangle$ in an equivalent way so that the modified version will fit into the Bourbaki scheme. Consider the structure $x = \langle R; <_R \rangle$ where R is a set and the axiom for $<_R$ is

$$\exists c\, \exists \varphi : \mathbb{R} \to \mathbb{R}^4 (\, \varphi \text{ is bijective} \wedge \forall a,b \in R\ (a <_R b \leftrightarrow \varphi(a) <_c \varphi(b))).$$

Let me call x a <u>Bourbaki model</u> of RT. If L is the group of automorphisms of $\langle \mathbb{R}^4; <_c \rangle$, i.e. the group of bijective mappings $\varphi : \mathbb{R}^4 \to \mathbb{R}^4$ preserving $<_c$, then L is isomorphic to $\mathrm{Aut}(x)$. An isomorphism $\Delta_\varphi : L \to \mathrm{Aut}(x)$ is given by $L \ni \lambda \mapsto \Psi = \varphi^{-1} \circ \lambda \circ \varphi \in \mathrm{Aut}(x)$ where φ is as required in the definition of $<_R$. For, by the definition of L, $<_R$ and Ψ, we have $\varphi(a) < \varphi(b)$ iff $(\lambda \circ \varphi)(a) <_c (\lambda \circ \varphi)(b)$ iff $(\varphi^{-1} \circ \lambda \circ \varphi)(a) <_R (\varphi^{-1} \circ \lambda \circ \varphi)(b)$ iff $\Psi(a) <_R \Psi(b)$. So by what was said in Sec. I it is sufficient to show that each model of RT is isomorphic to some Bourbaki model $\langle R; <_R \rangle$.

<u>T2</u> If $x \in M(RT)$ then x is isomorphic to some Bourbaki model $\langle R; <_R \rangle$.

Proof: If $x = \langle E; \prec \rangle$ let $R := E$ and $<_R := \prec$. From the proof of TIV-9-b) of (Balzer, 1982) (p.251) it follows that there is some c and some bijective $\varphi : E \to \mathbb{R}^4$ so that for all $e, e' \in E$: $e \prec e' \leftrightarrow \varphi(e) <_c \varphi(e')$. So by the definition of $<_R$ and R, $y = \langle R; <_R \rangle$ is a Bourbaki model of RT and, trivially, x and y are isomorphic ∎

This shows that our axiomatization of RT is adequate.

IV REDUCTION OF CT TO RT

We define a reduction relation ρ as follows.

D6 a) If $x=\langle S,T;\mathbb{R};\lessdot,\tau,\delta\rangle \in M_p(CT)$, $y=\langle E;\prec\rangle \in M_p(RT)$ and W is a frame for y then we set $x\,\rho_W y$ iff there are v,w,Φ_w and $d_{y,v,w}$ so that

1) $v,w \in W$ and $v \neq w$
2) Φ_w is a clock for y relative to w
3) $d_{y,v,w}$ is a metric for y relative to v,w
4) $S=W$
5) $T=\{sim_{\Phi,w}(e)/e \in E\}$
6) for all $t,t' \in T$: $t \lessdot t'$ iff $\exists e,e' \in w(t=sim_{\Phi,w}(e) \wedge t'=sim_{\Phi,w}(e') \wedge e \prec e'$
7) for all $t,t' \in T$ and $\alpha \in \mathbb{R}$: $\tau(t,t')=\alpha$ iff $\exists e,e' \in w(t=sim_{\Phi,w}(e) \wedge t'=sim_{\Phi,w}(e') \wedge |\Phi_w(e)-\Phi_w(e')|'=\alpha$
8) for all $t \in T$ and $a,b \in S$: $\delta(t,a,b)=d_{y,v,w}(a,b)$

b) A relation $\rho \subseteq M_p(CT) \times M_p(RT)$ is defined by $x\,\rho\,y$ iff there is a frame W for y such that $x\,\rho_W y$

Note that, once v,w,Φ_w and $d_{y,v,w}$ in D6-a) are given, requirements 4)-8) have the form of explicit definitions of $S,T,\lessdot,\tau$ and δ. Intuitively, S is identified with the set W of world lines, and T with the set of simultaneity classes. That is, classical points of space are identified with world lines (paths of free particles) and classical instants with classes of simultaneous events. The ordering of classical instants in x is given by the ordering induced in simultaneity classes by $\prec$ (D6-a-6). Classical time-distance is defined by the time-distance read off from the relativistic clock on w (D6-a-7), and classical spatial distance is defined by the distance function induced on W in y. Roughly, for given v,w Φ_w and $d_{y,v,w}$, x is defined in terms of the components of y. We obtain the following theorems.

T3 If y is a model of RT relative to W and $x \rho_W y$ then x is a model of CT.
T4 For all $y \varepsilon M(RT)$ there is x so that $x \rho y$ and $x \varepsilon M(CT)$.
T5 For all $x \varepsilon M(CT)$ there is $y \varepsilon M(RT)$ so that $x \rho y$.
T6 Not: for all x,y, if $y \varepsilon M(RT)$ and $x \rho y$ then $x \varepsilon M(CT)$.
T7 Not $(\forall x,x',y(<x,y> \varepsilon \rho \wedge <x',y> \varepsilon \rho \rightarrow x=x'))$ and not $(\forall x,y,y'(<x,y> \varepsilon \rho \wedge <x,y'> \varepsilon \rho \rightarrow y=y'))$.

For the proofs let y,W,v,w, Φ_w, $d_{y,v,w}$ be given as in D6) and let y be a model of RT relative to W.

Lemma 1 For each $w \varepsilon W$ there is a clock Φ_w for y relative to w, and Φ_w is uniquely determined up to linear transformations.
Proof: See (Kamlah, 1979), pp.448 in a slightly different notation ■

Lemma 2 $\{sim_{\Phi,w}(e)/e \varepsilon E\}$ is a partition of E. For all $e \varepsilon E$ and $v \varepsilon W$: $sim_{\Phi,w}(e) \cap v$ is a singleton, and in particular $sim_{\Phi,w}(e) \cap w=\{e\}$.
Proof: (1) It is easy to show that the relation defined by $e \simeq e'$ iff $\exists e_1(e,e' \varepsilon sim_{\Phi,w}(e_1))$ is an equivalence relation on E. (2) $e' \varepsilon sim_{\Phi,w}(e_1)$ and $e' \varepsilon sim_{\Phi,w}(e_2)$ imply $e_1=e_2$, because the events required to exist in D5-b) are uniquely determined and because Φ_w is bijective. (3) We show that there is an e' such that $sim_{\Phi,w}(e) \cap v=\{e'\}$.
Case 1) v=w. From $e' \varepsilon sim_{\Phi,w}(e)$ and $e \neq e'$ it follows that there is an e_1 such that $e'=e_1 \prec e$, and in the same way, that there is an e_2 such that $e \prec e_2 = e'$. From these two statements we obtain $e'=e_1 \prec e \prec e_2=e'$ which is impossible. So $e=e'$ and $sim_{\Phi,w}(e) \cap w=\{e\}$

which proves the special case, too. Case 2): $v \neq w$. There are $e_1^+, e_2^+ \varepsilon w$ such that $e_1^+ \prec e \prec e_2^+$ and $\exists e^+ \varepsilon v (f_v(e_1^+) = e^+ \wedge f_w(e^+) = e_2^+)$. Choose $e_1, e_2 \varepsilon w$ so that $e_1 \prec e_2$, $\Phi_w(e_2) - \Phi_w(e_1) = \Phi_w(e_2^+) - \Phi_w(e_1^+)$ and $\Phi_w(e_2) - \Phi_w(e) = \Phi_w(e) - \Phi_w(e_1) = 1/2(\Phi_w(e_2) - \Phi_w(e_1))$. The definition of Φ_w implies $e_2 = f_w(f_v(e_1))$ and therefore (with $e' = f_v(e_1)$): $e' \varepsilon\, sim_{\Phi,w}(e)$, that is, $sim_{\Phi,w}(e) \cap v \neq \emptyset$. Suppose $e', e'' \varepsilon\, sim_{\Phi,w}(e) \cap v$. Then $e' = f_u(e_1)$ and $e_2 = f_w(e')$, so $e' \varepsilon\, u \cap v$, and, because W is a partition, $u = v$. Thus we obtain $e' = f_v(e_1) \wedge \Phi_w(e) = 1/2(\Phi_w(e_2) + \Phi_w(e_1))$, and in the same way: $e'' = f_v(e_1'') \wedge \Phi_w(e) = 1/2(\Phi_w(e_2'') + \Phi_w(e_1''))$. Suppose $e' \prec e''$. Then $f_v(e_1) \prec f_v(e_1'')$, so $e_2 \prec e_2'' \wedge e_1 \prec e_1''$, from which we obtain $\Phi_w(e_2) < \Phi_w(e_2'') \wedge \Phi_w(e_1) < \Phi_w(e_1'')$. But this together with $\Phi_w(e_2) < \Phi_w(e_2'')$ and $\Phi_w(e_1) < \Phi_w(e_1'')$ is impossible, so not $e' \prec e''$. In the same way we obtain not $e'' \prec e'$ from which it follows, finally, that $e' = e''$ ∎

<u>Lemma 3</u> There is precisely one metric $d_{y,v,w}$ for y relative to v,w.
Proof: This is the well known Representation Theorem for Euclidean geometry ∎

Proof of T3: By lemma 2) $v \cap sim_{\Phi,w}(e) = \{e'\}$ and by D4-5) $v \neq sim_{\Phi,w}(e)$. If $S \cap T$ were not empty then for some $b \varepsilon S \cap T$, we would obtain $b = v = sim_{\Phi,w}(e)$ which yields a contradiction. Again by lemma 2), if $sim_{\Phi,w}(e_1) = t = sim_{\Phi,w}(e_2)$ and $e_1, e_2 \varepsilon w$ then $e_1 = e_2$. So in D6-a-6), e and e´ are uniquely determined by t and t´. Hence τ, as defined in D6-a-6) is a function. Also, by lemma 3), $d_{y,v,w}$ is uniquely determined, which proves D1-3). D2-1) is trivial. That τ is a metric is checked with the help of lemma 2) and the triangle inequality in $\mathbb{R}$. That $\delta(t)$ is a metric follows from the fact that

$d_{y,v,w}$ is a metric, and that D6-a-7) does not depend on t. $\lessdot$ is a linear order because $\prec$ on w is a linear order and because of lemma 2). We prove D2-3): $<T; \underline{bet}_\tau, \equiv_\tau>$ is a 1-dimensional Euclidean geometry. From lemma 2) and by direct calculation we obtain (1) $\underline{bet}_\tau(t_1,t_2,t_3)$ iff $\exists e_1,e_2,e_3 \varepsilon w (t_i = sim_{\Phi,w}(e_i) \wedge e_1 \preccurlyeq e_2 \preccurlyeq e_3)$ and (2) $t_1t_2 \equiv_\tau t_3t_4$ iff $\exists e_1 \ldots e_4 \varepsilon w(t_i = sim_{\Phi,w}(e_i) \wedge |\Phi_w(e_1) - \Phi_w(e_2)| = |\Phi_w(e_3) - \Phi_w(e_4)|)$. Now let bet and $\equiv$ on w be defined by: $bet(e_1,e_2,e_3)$ iff $e_1 \preccurlyeq e_2 \preccurlyeq e_3$ and $e_1e_2 \equiv e_3e_4$ iff $|\Phi_w(e_1) - \Phi_w(e_2)| = |\Phi_w(e_3) - \Phi_w(e_4)|$. From (1) and (2) it follows that if $<w; bet, \equiv>$ is a 1-dimensional Euclidean geometry then so is $<T; \underline{bet}_\tau, \equiv_\tau>$. Thus it is sufficient to show that $<w; bet, \equiv>$ is a 1-dimensional Euclidean geometry, and this is proved directly by using lemma 1). D2-4) follows immediately from the definitions of $\underline{bet}_{\delta(t)}$, $\underline{bet}_{d_{y,v,w}}$ etc. and from D6-a-7).D2-5) also follows from D6-a-7) directly ∎

Proof of T4: Let $y = <E; \prec> \varepsilon M(RT)$, i.e. there is a frame W for y so that y is a model for RT relative to W. Let $v,w \varepsilon W$. By lemma 1) there is a clock Φ_w for y relative to w and by lemma 3) there is a metric $d_{y,v,w}$ for y relative to v,w. Define $x = <S,T; \mathbb{R}; \lessdot, \tau, \delta>$ by conditions 4)-8) of D6-a). By lemma 2) $\{e\} = w \cap sim_{\Phi,w}(e)$. So $sim_{\Phi,w}(e_1) = t = sim_{\Phi,w}(e_2) \wedge e_1,e_2 \varepsilon w$ implies $e_1 = e_2$, i.e. α in D6-a-6) is uniquely determined and therefore τ is a function. By lemma 3) $d_{y,v,w}$ is uniquely determined, so δ is a function, too, i.e. $x \varepsilon M_p(CT)$. By the definition of x: $x \rho_W y$, and so by T3): $x \varepsilon M(CT)$ ∎

Proof of T5: Let $x = <S,T; \mathbb{R}; \lessdot, \tau, \delta> \varepsilon M(CT)$. Define $<E; \prec>$ as follows: $E = S \times T$ and $<a,t> \prec <b,t'>$ iff $t \lessdot t' \wedge \tau(a,b) \leq \delta(t,t')$. Define $W \subseteq Pot(E)$ by $w \varepsilon W$ iff $\exists a \varepsilon S(w = \{<a,t>/t \varepsilon T\})$. It is then easily checked that $\langle E;\prec \rangle$ is a model of RT relative to W (compare (Balzer,1982),pp.222,TIV-5-b) in a slightly different set-up)∎

Proof of T6 and T7: By construction of mathematical counter examples∎

Our claim about ρ is that ρ constitutes a reduction of CT to RT, and therefore ρ is a "reduction relation". Such a claim can be attacked on various lines, and I will consider several objections in turn.

A first objection against ρ as a reduction relation is that it is *strict* -as opposed to "approximative". In one way this objection may be seen as another version of the objection of Sec. II against CT not being Galilei-invariant. The reasoning seems to be this. *If* classical space-time is Galilei-invariant *then* it can only be approximatively reduced to RT.So Galilei-invariance on the classical side seems to be sufficient for approximative features of reduction. This is, I think, the intuitive basis of the objection though I do not know how to substantiate it in the absense of generally accepted conditions on all possible forms of reduction. But it is clear why the reasoning has so much credit: because of the approximative relation between the corresponding groups of Galilei- and Lorentz-transformations.

This kind of objection is just a corollary to the one in Sec.II, and *if* it is conceded that CT is adequate (without being Galilei-invariant as I have argued in Sec.II) then the present objection becomes pointless. To put it differently: if there is a strict reduction relation ρ between CT and RT (as the one just presented) then its strictness need not count as an inadequacy of ρ but can be seen as an inadequacy of CT to represent classical space-time (being not Galilei-invariant). The same point is reinforced by observing that the Galilei-invariant extension of CT mentioned in Sec.II can be reduced to RT in an approximative way. (it is tempting to add "and only in an approximative way" but, again, such a statement seems difficult to substantiate.) Anyway, this possibility again shows that approximation of reduction goes together with Galilei-invariance on the classical side.

Now opponents might concede that CT is an adequate reconstruction and still insist that my ρ is not a reduction relation. This amounts to saying that the ρ employed does not have the properties which an adequate relation should have. Since there is a considerable variety of different concepts of reduction, it will not suffice to show that ρ can be subsumed under one of them: the objection might be sustained by using a different concept of reduction. Given this situation I will go through some of the different requirements proposed by different authors and comment on their bearing on the present example.

First, there is the traditional condition of derivability of the laws of the reduced theory from those of the reducing theory "after translation" which by Adams[10] was expressed as follows:

(1) $$\forall x,y(y \in M' \wedge x \rho y \rightarrow x \in M)$$

where ρ reduces T to T', $\rho \subseteq M_p \times M'_p$. T3) above attempts to establish this condition for the present example but it does not succeed completely. Strictly, (1) fails, for the frame W employed in establishing the relation $x \rho_W y$ may be different from the one which makes y a model of RT (see T6) above). If W is chosen perversely enough then "$x \in M(CT)$" does not follow any longer.[11] This is a puzzling result, and it would be helpful to see whether the difference between (1) and T3) has to do -and if so, in which way precisely- with the difference between "strict" and "approximative" reduction. For someone taking condition (1) as necessary for reduction, my ρ cannot be a reduction relation.

Second, there is Sneed's condition of uniqueness:[12]

(2) $$\forall x,x',y\ (x \rho y \wedge x' \rho y \rightarrow x=x').$$

T7) above says that (2) is not satisfied in the present example. But I doubt whether this condition can be imposed generally. Intuitively, Sneed justifies the requirement as expressing that the reducing theory gives a more detailed picture of reality. But this property is not equivalently expressed by (2). Condition (2) may be sufficient for T′ giving a more detailed picture than T but (2) certainly is not necessary for this property to hold. The "finer" picture of T′ may be "coarsened" in different ways so that the outcomes still are of the same structure (as shown by the present example). In general, I see no argument for condition (2) to be satisfied for all reduction relations, and I would hesitate to exclude non-unique relations on apriori grounds.

Third, there is the condition that to each model of the reduced theory there corresponds -via ρ - a model of the reducing theory:

(3) $\forall x(x \varepsilon M \rightarrow \exists y(y \varepsilon M' \wedge x \rho y))$.

Requirement (3) is essential for Mayr′s account and can be traced back to Suppes.[13] T5) above shows that this condition is satisfied.

Fourth, there is a kind of "converse" of (3), namely

(4) $\forall y(y \varepsilon M' \rightarrow \exists x(x \varepsilon M \wedge x \rho y))$,

i.e. each model of the reducing theory via ρ gives rise to a model of the reduced theory. The spirit of condition (4) is this. Given a model y of the reducing theory T′ we can construct or define in y a structure which is a model of the reduced theory T. This is the basic idea of Bourbaki′s "procedure of deduction of a structure of species Θ from a structure of species Σ ".[14] T4) above shows that condition (4) is satisfied. It should be noted that (4) also can be regarded as expressing a "derivability requirement" as mentioned in connection with (1).

A fifth formal condition is that each model of the reduced theory can be embedded into a model of the reducing theory:

$$(5) \qquad \forall x\ (x \in M \rightarrow \exists y (y \in M' \wedge x \sqsubset y))$$

where "$x \sqsubset y$" means "x is a substructure of y".(5) represents the kernel of Ludwig's notion of "Einbettung".[15] It is obvious that this condition does not apply to the case at hand, but I doubt whether it should. Intuitively, I would interpret (5) as saying that T is a <u>specialization</u> of T', and specialization and reduction are two different intertheoretic relations which can and should be kept apart.[16]

Sixth, there is the condition of ρ "preserving invariances".[17] A weak version of this condition is the following:

$$(6) \qquad \forall x,y (\ x \rho y \rightarrow [x]\ \bar{\rho}\ [y]'\ )$$

where [x] and [y]' denote the equivalence classes of x and y given by the corresponding invariances of T and T', and $X \bar{\rho} Y$ is a shorthand for $\forall x \in X\ \exists y \in Y (x \rho y) \wedge \forall y \in Y\ \exists x \in X (x \rho y)$. In (Pearce & Rantala, 1983a) a condition similar to (6) is considered as an aspect of continuity in scientific change. In my view (6) is a "special law" of the "theory of reduction" which will be satisfied only in special cases. It is possible to construct a pair of models $\langle x,y \rangle \in M(CT) \times M(RT)$ for which [ ] and [ ]' are given by elementary transformations and Lorentz-transformations, respectively, and for which (6) is false. My hypothesis is that typically (6) will be false for pairs of theories which are incommensurable but are nonetheless connected by some reduction relation.

Last but not least there is the condition of translatability of the language of the reduced theory into the language of the reducing theory, inherent in the received view and recently substantiated by Pearce.[18] Without going into technical details this requirement can be nicely illustrated with my example. Consider the atomic expressions "$t \lessdot t'$", "$\tau(t,t') = \alpha$" and "$\delta(t,a,b) = \alpha$" of CT. In D6-a) these are "defined" in terms of the primitives of RT "up to the choice of v,w and the units for the clock". But this "up to" prevents us from finding proper translations of these expressions in the language of RT. Also,the relations among pairs of the form <a,t> in CT "is simultaneous with" and "occupies the same point of space as" can be easily defined in CT but cannot be translated into expressions of RT (this is why RT is called "relativistic"). It is clear therefore,that translatability in the usual sense does not obtain in my example. This might be regarded as an argument against the adequacy of ρ. But it might as well be regarded as an argument against requiring translatability as a condition necessary for <u>all</u> reduction relations. What was said in the last paragraph applies here, too.Translatability characterizes only a special subclass of the class of all reduction-pairs -though a very interesting one. Typically,translatability will not obtain in cases of incommensurable theories, the two theories considered here constituting a commonly accepted example of incommensurability.

To summarize these considerations: It seems to me that condition (4) is the most central one for reduction. It combines the intuition of a derivation of the laws of the reduced theory from those of the reducing theory with the formal achievements of Bourbaki's work.[19)] All the other conditions will be satisfied only in special cases but not in general. If this view is not completely misled then it is difficult to see how and why my ρ-relation should be inadequate,i.e.no reduction relation proper.

A last line of attack against ρ is to say that it does not satisfy the informal requirement that all intended applications of the reduced theory correspond via ρ to intended applications of the reducing theory:[20)]

$$\forall x \in I \, \exists y \in I' (x \rho y) \tag{7}$$

where I and I′ are the sets of intended applications of T and T′, respectively. In the present case, A.Kamlah has pointed out that probably among the intended applications of CT there are systems which are considered from <u>accelerated</u>

frames of reference which are ruled out by RT and therefore do not belong to I(RT). Then (7) would fail, provided we accept that the y required there is given by "the same" accelerated system which gave rise to the classical x we start with. The crucial point in this argument is, I think, that reference is made to some systematic concept not available in both theories: "acceleration". And this, in turn, leads to the question of how the intended applications of a theory are determined in general. Is it the case that scientists in the course of achieving agreement on whether some system is an intended application for T use concepts from theories systematically dependent on T? A clear cut "no" would be dogmatic. Scientific practice as far as it is documented by historians will perhaps yield the answer "in most cases not". At least this is the answer one would expect from systematic reflections on the determination of I of I.[21] According to Sneed and Stegmueller[22] I is determined "paradigmatically", i.e. one gives a list of "paradigms" forming a set $I_o \subseteq I$, and systems not in I_o will belong to I if they are sufficiently similar to those of I_o. If "sufficient similarity" cannot be decided on easily then the theory itself will be used as a criterion. Some new system x will be regarded as an intended application iff it can be successfully subsumed under T.

The paradigmatic method, if applied to CT, yields as intended applications systems which are either in direct contact with the earth or consist of stars or planets as seen from the earth. It is hard to come across space-time systems (as distinct from mechanical systems) described from frames of reference which are accelerated relative to the earth. The question certainly deserves a more detailed analysis but for the moment the above remarks will have to suffice. At least they make plausible the claim that (7) need not constitute a definite refutation of CT being reducible to RT.

In total, then, it seems to me that the present example should be regarded as a proper case of reduction. Both theories involved are physically adequate and based on operationally accessible notions. The reduction relation, too, makes physical sense and has formal properties which fit some of the general definitions of reduction already available.

NOTES

1) I am indebted to A.Kamlah,D.Pearce and H.-J. Schmidt for many remarks and suugestions on an earlier draft.
2) Compare the examples enlisted in Moulines' contribution to this volume. Recent notions of reduction can be found in (Ludwig,1978), (Mayr, 1976),(Pearce,1979),(Pearce & Rantala,1983a), (Sneed,1971), and (Balzer & Sneed,1977/78).
3) See (Bourbaki,1968),pp.259.
4) For further explanations compare my set-theoretic (as opposed to Bourbaki's rather idiosyncratic "syntactic") treatment of species of structures in (Balzer,1984).
5) See (Tarski,1959). I assume here that Tarski's A13) is always replaced by the corresponding second-order version, namely the formula on p.18 loc.cit. By an appropriate change of the axioms of dimensionality we easily obtain the axioms for 1-dimensional Euclidean geometry used in D2-3). More precisely,we have to omit A11) and A12) and add (in Tarski's notation): $\forall xyz[\beta(xyz) \vee \beta(yxz) \vee \beta(xzy)]$.
6) Compare (Balzer,1982),Chap.III.
7) Terminology is taken from (Weyl,1923),p.142. According to (Ehlers,1973) this group is characteristic for Newtonian spacetime.
8) For intuitive explanations of the following formalism see (Balzer,1982),Chap.IV.
9) An exact proof is found in (Zeeman,1964).
10) See (Adams,1959).
11) It is possible to modify RT so that each model contains a frame W explicitly (compare (Balzer, 1982),Chap.IV). By using such a modified RT, condition (1) for reduction can be proved for some ρ' modified along the same lines. The resulting version of RT, however, is open to criticism concerning its adequacy for the automorphism groups of its models are not pre-

cisely isomorphic to the Lorentz-group.

12) See (Sneed,1971),p.221,D51-2).

13) See (Mayr,1976),p.289,Definition (2.12-ii) and (Suppes,1957),p.271.

14) See (Bourbaki,1968),p.267.

15) See (Ludwig,1978),p.88.

16) Compare (Balzer & Sneed,1977/78).

17) This condition was first suggested to me by H.-J.Schmidt at an informal meeting.

18) See (Pearce & Rantala,1983a) and (Pearce,1979).

19) The treatment of reduction given in (Balzer & Sneed,1977/78) as essentially covered by (1) -though expressing the first intuition- falls short of exhibiting all the formal advantages of (4).

20) See (Sneed,1971),p.229,D54-A-2).

21) Compare (Balzer,1982),pp.28.

22) (Stegmueller,1973),pp.198.

W.Balzer
Seminar fuer Philosophie,Logik und Wissenschafts-theorie,Universitaet Muenchen
Ludwigstr.31
D-8000 München 22

REFERENCES

Adams,E.W.: 1959, 'The Foundations of Rigid Body Mechanics and the Derivation of its Laws from those of Particle Mechanics',in: The Axiomatic Method, Henkin,Suppes,Tarski (eds.),Amsterdam

Balzer,W.& Sneed,J.D.: 1977/78, 'Generalized Net Structures of Empirical Theories', Studia Logica 36 and 37

Balzer,W.: 1982, Empirische Theorien: Modelle,Strukturen,Beispiele,Braunschweig-Wiesbaden

Balzer,W.: 1984, 'On a New Definition of Theoreticity',to appear in Dialectica

Bourbaki,N.: 1968, Theory of Sets, Paris

Ehlers,J.: 1973, 'The Nature and Structure of Spacetime',in: J.Mehra (ed.), The Physicist's Conception of Nature,D. Reidel

Kamlah,A.: 1979, 'Erläuterungen',in: A.Kamlah and M.Reichenbach (eds.): Hans Reichenbach: Gesammelte Werke,Band 3, Braunschweig-Wiesbaden
Ludwig,G.: 1978, Die Grundstrukturen einer physikalischen Theorie, Berlin-Heidelberg-New York
Mayr,D.: 1976 , 'Investigations of the Concept of Reduction I', Erkenntnis 10
Pearce,D.: 1979, Translation,Reduction and Equivalence: Some Topics in Intertheory Relations, Dissertation,University of Sussex
Pearce,D.& Rantala,V.: 1983a 'Logical Aspects of Scientific Reduction',in: Epistemology and Philosophy of Science,Proceedings of the 7th International Wittgenstein Symposium 1982,Wien
Pearce,D.& Rantala,V.: 1983b, 'The Logical Study of Symmetries in Scientific Change', in: Epistemology and Philosophy of Science, Proceedings of the 7th International Wittgenstein Symposium 1982,Wien
Sneed,J.D.: 1971, The Logical Structure of Mathematical Physics,D. Reidel,second edition 1979
Stegmueller,W.: 1973, Theorie und Erfahrung,Zweiter Halbband,Berlin-Heidelberg-New York
Suppes,P.: 1957, Introduction to Logic,Princeton
Tarski,A.: 1959, 'What is Elementary Geometry?',in: Henkin,Suppes,Tarski (eds.): The Axiomatic Method,Amsterdam
Weyl,H.: 1923, Raum,Zeit,Materie, 5th ed.,Berlin
Zeeman,E.C.: 1964, 'Causality Implies the Lorentz-Group', Journal of Mathematical Physics 5