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Long Memory Time Series and Fractional Integration. A Cliometric Contribution to French and German Economic and Social History

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Abstract: This paper presents the fractional integrated processes which are the main models used to describe long memory phenomena (section 1). Section 2 defines briefly the concept of fractional integration, Shows its main properties and provides an overview of the estimation techniques. A survey of their extensions in order to model the cyclical long term movement is provided in section 3. Section 4 consists of an application to socio-economic time series for France and Germany in the nineteenth and twentieth centuries.

I. Introduction

There is a substantial gap between conceptual progress in economic theory and quantitative measurement of the concepts that it generates. It is true that measurement is difficult and the instruments available are far from matching our needs. However, this cannot be used as an argument for balking at a quantitative approach to reality. A concept is by nature an abstract image of reality, a

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mental composition that can only be linked to reality by measurement. Without potential or effective measurement it can only remain sterile, with no possibility of being converted into action in order to progress towards new fields of knowledge.

Thus, developments in statistics have resulted in progress in measurement. Without a doubt, this has made fresh progress in theoretical aspects possible. However, progress in observation has only been made for the present. Analysis of very long periods without corresponding statistical production has had to make do with existing indicators that are frequently very remote from the problems that researchers wish to address. The statistics produced obviously provide a significant perspective but are not sufficient. However, as we are dealing with the past, new statistical developments can only be produced with the material left to us by history. But this material was developed to meet the requirements of knowledge and queries that by definition are completely different from the questions we are asking today.

The aim of quantitative history is to use the material available as a basis for constructing the new observations that we need. The method in itself implies limits that are those of the traces left by past generations. However, a considerable volume of material is available and the question is rather that of the means to be used to exploit it. In fact, quantitative history uses the methods of retrospective national accounting (in particular the drawing up of satellite accounts). It aims at representing the economy of a country in a simplified form. The approach is intangible in appearance and focuses first and foremost on the observation and measurement of socio-economic facts. It then makes it possible to separate the complex ensemble of phenomena that make up economic and social activity. Finally it can render socio-economic facts comparable so that they can be classified in a limited number of categories in order to be studied as components of a homogeneous ensemble, that is to say as an aggregate. This being so, quantitative history stems from organized knowledge and engenders its own theoretical field – that of the double matching of measurement to the underlying concept and to the real data that it must report. It must be admitted that when historical reality is addressed in its long-term dimension our skills lag considerably behind the questions we are asking. For example, what is the meaning of the juxtaposition of instantaneous measurements in temporal series when the observed object itself changes. The comparison of levels at two fairly well-separated dates certainly has little meaning. In contrast, we should certainly award more importance to the movement that is described. We therefore measure trends.

This research approach applied to comparative analysis of education, economic growth and demographics series, for France and Germany, in the nineteenth and twentieth centuries, has enabled us to find the different cyclical components of the series studied. Our method is first driven by a search for maximum objectivity in the observation of time series and next the possibility of applying it to a large number of series. This double requirement is dictated to us by the concem to prevent the criticism generally formulated conceming statistical studies conceming cyclic movements of the economy.

Starting from there, this article presents the fractional integrated processes which are the main models used to describe long memory phenomena.[1]

Section 2 briefly defines the concept of fractional integration, Shows the fundamental properties and provides a short summary of the estimation methods. Section 3 consists of a survey of their extensions in order to model long term cycles. Section 4 presents an application to socio-economic series for France and Germany in the nineteenth and twentieth centuries.

II. Fractional integrated processes

1. The concept of fractional integration

The class of fractional integrated processes is an extension of the class of ARIMA processes stemming from Box and Jenkins methodology. One of their originalities is the explicit modelling of the long term correlation structure.[2]

According to the values of parameters, these processes will possess the long term dependence property or long memory introduced by Hurst (1957) and Mandelbrot (1968).

Let x_t. $t=1 \dots n$ a time series and be $\rho(k)$ its autocorrelation function: $\rho(k) = E[x_t\, x_{t-k}]$. The stationarity property is verified if:

$$\sum_{k=0}^{\infty} \rho(k)^2 \langle \infty$$

In this case, it is said that x_t has the long memory property if:

$$\sum_{k=0}^{\infty} |\rho(k)| \langle \infty$$

A way of representing such correlation structures is the use of fractional integrated processes. These models are defined from the fractional differentiation operator $[1\text{-}B^d]$where B is the usual backshift operator: $B^k \chi_k = \chi_{t-k}$. The fractional operator is broken down using a binomial series:

$$[1-B]^d = 1 - dB - \cdots - \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} B^k + o\left(B^{k+1}\right)$$

1 For the reader interested in a detailed presentation, cf. Diebolt & Guiraud (2000).

2 In the traditional methodology, the long term structures are often likened to non-stationarity and so filtered/eliminated from the first stage of the treatment processing. But, it is evident that the use of an unsuitable filter can introduce an artificial correlation structure into the series; that is called overdifferentiation (Granger & Joyeux (1980)).

This operator makes it possible to define fractional integrated processes. It is assumed that process χ_t, $t=1 \ldots n$ (assumed to be centred for the purpose of simplicity, $E[\chi]=0$) follows an Auto Regressive Fractional Integrated Moving Average (ARFIMA) process if:

$$[1-B]^d \chi_t = u_t$$

where u_t is a usual ARMA(p,q) process:

$$\phi_p(B)u_t = \theta_q(B)\varepsilon_t$$

where ε_t is a white noise with zero mean and variance σ_{ε}^{2} We assume that u_t verifies the stationarity and invertibility conditions. This assumption is necessary to establish the following properties. One can demonstrate that:

- the process χ_t is stationary if $d < 1/2$,
- the process χ_t is invertible if $d > -1/2$. Odaki (1993) spread this interval to $d > -1$ by using a weak invertibility concept.
- the stationary process χ_t will have long memory if $0 < d < -1/2$.

For -1/2<d<0, the process is always characterized by the slow decay of autocorrelation, but it does not possess the long memory property (the autocorrelations have alternate signs). In this case, it is said that the series is "antipersistent" (cf. Hurst). This behaviour is often associated with an overdifferentiation of the series by the first difference filter.

For 1/2 < d < 1 , the property of stationarity is not verified. However, the asymptotic expression of the infinite MA decomposition coefficients ever tends to zero. This case is called "non stationary mean-reverting". The effects of a random shock will tend to decrease with the time, unlike to the unit root case (also called infinite memory, where the influence of a shock lasts indefinitely). Figure 1 shows the different properties according to the values of d.

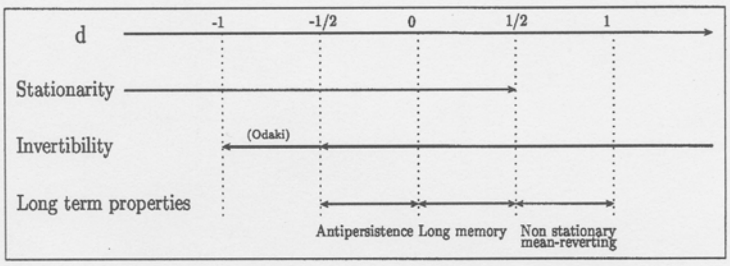

Figure 1: Properties of a fractional integrated process

Hosking (1981) demonstrated the fundamental properties of the ARFIMA (p,d,q) process. Asymptotically, these properties are given by these of the ARFIMA (O,d,0). Only the latter are presented here.

The stationary ARFIMA(0,d,0) process allows an infinite MA representation:

$$\chi_t = \sum_{k=0}^{\infty} \psi_k B^k \varepsilon_t$$

where $\psi_k = \Gamma(k + d)/\Gamma(d)\Gamma(k + 1) \sim c_1 k^{d-1}$ (c_1 positive constant). The coefficients of MA(∞) development decay at a hyperbolic rate. It is different from the exponential rate, characteristic of ARMA processes. The influence of a random shock will tend to vanish with time but at a relatively low Speed.

The spectral density of the ARFIMA(O,d,0) process is:

$$f_\chi(\omega) = \left[4\sin^2(\omega/2)\right]^{-2d} f_u(\omega)$$

where $\omega \in [0,\pi]$ and $f_u(\omega)$ is the spectral density of the ARMA(p,q) process:

$$f_x(\omega) = \frac{\sigma_\varepsilon^2 \left|\theta\left(e^{-i\omega}\right)\right|^2}{2\pi\left|\phi\left(e^{-i\omega}\right)\right|^2}$$

for $\omega \rightarrow 0$, $f_\chi(\omega) \sim c_2.\omega^{-2d}$ (c_2 positive constant). The spectral density has a peak at zero frequency. Such a spectral shape is often connected with the presence of a non-stationary component. But here the stationarity property is verified. There is a risk of interpretation error when such a spectral shape is automatically associated with the non-stationarity of the series.

The algebraic expression of the ARFIMA(p,d,q) autocorrelation function is complex and presents of limited interest. Hosking (1981) demonstrate that its asymptotic behaviour is $c_3 k^{k2d-1}$ (c_3 positive constant). One will find decay at hyperbolic rate, characteristic of fractional integrated processes.

It can be noted that if d > 1 / 2 , applying the first difference filter to the series will produce a series characterized by a fractional integration coefficient $\tilde{d}$ for which one can verify: $\tilde{d} = d - 1$. One can then study differentiated series and deduce the properties of the raw series. Fractional integrated processes can thus model not only the phenomena of long memory but can also be used to implement a tool of time series analysis. For this, Beran (1999) proposes a model called SEMIFAR to model jointly deterministic long term tendencies (treated by a non parametric approach) and stochastic long term structures (processed by the parametric fractional model presented here).

2. An overview of estimation techniques

The GEWEKE & PORTER-HUDAK (GPH) estimator.

This method is used to estimate the value of fractional integration coefficient by means of a simple procedure. However, the non consideration of a possible ARMA structure leads to a considerable risk of bias. After some transformations, the spectral density of the ARFIMA(p,d,q) process is:

$$\ln\left(I_n\left(\omega_j\right)\right)=\ln\left(f_u(0)\right)-d\ln\left(4\sin^2\left(\omega_j/2\right)\right)+\ln\frac{I_n\left(\omega_j\right)}{f_\chi\left(\omega_j\right)}+\ln\frac{f_u\left(\omega_j\right)}{f_u(0)}$$

where I„ (co) is the periodogram of the series x_t (estimator of the spectral density at the frequency $\omega_j=2\pi j/n: I_n\left(\omega_j\right)=\frac{1}{2\pi}\sum_{k=-n+1}^{n-1}\rho_k\exp^{-i\omega jk}$). If only the first *m* frequencies are used, with *m* such that as $m = g(n) = n^{\alpha}, 0 < \alpha < 1$, the influence of the short term component can be considered constant and *d* can be estimated with a linear regression of $y_j = \ln(I_n(\omega_j))$ from the deterministic regressor $X_j = 4\sin^2(\omega_j / 2)$. It can be demonstrated that the estimator obtained is asymptotically normal and its variance is known.[3] A significance test of the fractional integration coefficient can be set up.

Maximum likelihood methods.

These methods make it possible to estimate simultaneously all the parameters of the process, the fractional integration coefficient and the parameters of an ARMA structure. The estimator of the exact maximum likelihood proposed by Sowell (1992) is the vector $\hat{\beta}=\left(\hat{d},\hat{\phi}',\hat{\theta}'\right)'$ which maximizes the log-likelihood function *L(ß):*

$$L(\beta)=-(n/2)\ln(2\pi)-(1/2)\ln(R)-(1/2)x'R^{-1}x$$

where *R* is the variance-covariance matrix of the process.

This matrix *R* is a complicated algebraic expression and is difficult to calculate. We therefore use methods based on an approximation of the likelihood

[3] Cf. Agiakloglou, Nebolt & Wohar (1993), Chen, Abraham & Peiris (1994), Cheung (1993), Hassler (1993), Hurvich & Ray (1995) and Reisen (1994) for a discussion of the choice of the spectral window and its effects an the estimator as well as the bias generated by the presence of a short term component.

function. Two main techniques available are the spectral approximation of Fox & Taqqu (1986) and the minimization of the conditional sum of squared residuals proposed by Cheung & Baillie (1993). Asymptotically, these two methods converge on the exact maximum likelihood estimator.

The estimator suggested by Fox and Taqqu is the vector $\hat{\beta}^{SA}$ which maximizes the following expression:

$$L^{\alpha}(\beta)=\sum_{j=1}^{n-1}\left[\ln\left(2\pi f_x(\omega_j)\right)+\frac{I_T(\omega_j)}{f_x(\omega_j)}\right] \tag{8}$$

This expression is easier to use but can display a bias in small samples. On the other hand, it is effective when the value of the mean of the process is unknown.

The estimator proposed by Cheung and Baillie is the vector $\hat{\beta}^{CSS}$ which minimizes the quantity:

$$S=\frac{1}{2}\ln\sigma_{\varepsilon}^{2}+\frac{1}{2\sigma_{\varepsilon}^{2}}\sum_{t=1}^{n}\varepsilon_{t}^{2} \tag{9}$$

This estimator can easily be modified in order to introduce an estimator of the mean into the parameter vector. It can be developed to take into account on heteroskedasticity in the residuals.

III. Generalization of fractional integration

In his paper, Hosking (1981) notes that taking the fractional power of a second order polynomial makes it possible to describe long term structures of periodic shape. Thus, Gray, Zang & Woodward (1989) proposed the process called Generalized ARMA (GARMA). According to the values of the parameters, this process can possess a cyclical and persistant structure. Woodward, Chen & Gray (1998) extended this model to the case in which the series have k cyclical persistant component. This latter model is the most successful shape of fractional integration model. Formally, the series $x_t, t=1 \ldots n$ process if:

$$\prod_{j=1}^{k}\left[1-2\eta_j B+B^2\right]^{d_j}x_t=u_t \tag{10}$$

with the same notations as above, in particular u_t is a stationary and invertible ARMA(p,q) process. Obviously, the model specification must be such that $\eta_j \neq \eta_j' \forall \neq j' = 1\ldots k$.

The long term structure now depends on two parameters. The parameter ŋ indicates the long term periodicity while parameter d is linked to the "intensiveness" of long term structure. The quantity 2π / $\cos^{-1}$(ŋ) gives the periodicity in temporal terms. If ŋ → 1, one can verify that the cycle period tends toward infinity (ARFIMA case).

- The k-GARMA process is stationary if:

(i) $|\eta_j| < 1$ and $d_j < 1/2$, or

(ii) $|\eta_j| = 1$ and $d_j < 1/4$, $\forall_j = 1 \ldots k$;

- The k-GARMA process is invertible if:

(i) $|\eta_j| < 1$ and $d_j > -1/2$, or

(ii) $|\eta_j| = 1$ and $d_j > 1/4, \forall_j = 1 \ldots k$;

- The stationary k-GARMA process will have long memory property if:

(i) $|\eta_j| < 1$ and $0 < d_j < -1/2$, or

(ii) $|\eta_j| = 1$ and $0 < d_j < 1/4, \forall_j = 1 \ldots k$.

It can be seen that for $k=1$ and $\eta = 1$, the process corresponds to an ARFIMA (p,2d,q) process. Besides, if $\eta_j = 2\pi j / s, j = 0 \ldots [s/2]$, one finds the seasonal ARFIMA process proposed by HASSLER (1994).

The spectral density of k-GARMA process is:

$$f_x(\omega) = \prod_{j=1}^{k} \left[2|\cos(\omega) - \eta_j|\right]^{-2d_j} f_u(\omega) \tag{11}$$

Let $\varpi_j = \cos^{-1}(\eta_j)$. For $\omega \to \varpi_j, f_x(\omega) \sim |\omega^2 - \varpi_j^2|^{-2d_j}$. The spectral density shows k peaks at the frequencies ϖ_j.

The estimation method proposed by Chung (1996a,1996b)[4] is based on the minimization of the conditional sum of squared residuals (noted below CSS).

[4] The method proposed by Chung is relative to simple GARMA processes (k=l). The proofs can however be extended for k>l.

However, parameter estimation of this class of process is delicate. In case *k=1,* Chung (1996a, 1996b) shows that the estimator of η obtained by CSS minimization converges at a greater speed than the other parameters. This rules out the use of gradient-based methods on the whole Set of parameters.[5] It is therefore advisable to use an alternative method based on an incremental search (or gridsearch). This method is very slow if the grid-search corresponds to [-1,1]. It is then more effective to restrict the search interval to a neighbourhood of frequencies relative to the strongest values of the periodogram. For *k=1*, Chung demonstrates that the distribution of the estimators of d, as well as those of a possible ARMA structure obtained by minimization of the CSS function, is normal. Parameters significativity can be tested by a significance Student test. Moreover, this author Shows that the law of parameter η is known and proposes tabulated values ordinary to construct a confidence interval (cf. Chung (1996a)). We pursue our analysis with the presentation of results stemming from our empirical study.

IV. Empirical study of long term structures for French and German socio-economic series in the nineteenth and twentieth centuries

In this work, series studied are annual data observed over the period 1820-1996 for France (preceded by the letter F) and 1820-1989 for Germany (preceded by the Letter A). The series used are the GDP (GDP), the total population (POP), school population (SCOL), current (DECR) and constant (DECS) educational expenditures. Sources of series in levels, in differences as well as spectral densities and autocorrelation functions are given in appendices. A preliminary examination of the series Shows:

1) a strong increase of all the series at the end of the observation period (from 1950), which will require transformation in logarithmic data. However, series are strongly non stationary and the obtaining of coherent results is difficult.
2) the "evident non-stationarity" of series transformed. The differences of size between the first and last observations leads to suspecting that the use of a model with a long memory runs into the problem of non-stationarity for series in levels.

[5] The estimator of 17 converges at the rate T-1(1 t7 j< 1) or T-z (r7 = 1) while the other parameters converge at the usual rate $T^{-1/2}$. So the Cross terms of the hessian relatives to 17 parameter will quickly tend toward zero. From an analytical point of view, some information matrix terms will be infinites. From an empirical point of view, the Inversion of the Hessian matrix is then impossible.

The autocorrelation functions of level series display high and very weakly falling values. Periodograms are concentrated, with very low frequencies. Such observations first lead to developing a test of unit roots (Augmented Dickey and Fuller test). For reasons of conciseness, detailed results are not presented here. We indicate, however, that for all the series studied, the hypothesis of unit root is easily accepted against an alternative of a deterministic trend. In some cases, the use of a test strategy of the Henin-Jobert type leads to accepting simultaneously the presence of a unit root and a linear deterministic trend on first differences. This result may be at odds with intuition and, naturally, casts a doubt on the validity of the Dickey-Fuller test conclusions. The introduction of more elaborate alternatives, for example a trend with break, can lead to different results. Of course, the existence of a long term structure can lead to a certain number of diagnosis errors (cf. Diebold & Ruderbusch (1991)). However that may be, our series of first differences present significant short term structures of varying length. In the Same time, we note that the use of annual data results in diagnosis of a correlation structure shorter than ten year as a short term structure.

Statistical methods presented in the previous section are now used to discuss the shape of the long term trends in the series studied. Two methods are investigated. We first envisage the presence of a long memory component with the use of ARFIMA processes. We seek then the presence of a cyclic long term component with GARMA processes.

1. Search for long memory

The estimation method used here is a spectral approximation in the frequency domain (Fox & Taqqu (1986)). This algorithm requires starting values close enough to the optimal values. In order to produce them, we compute first the GPH estimator and the results are used as the initial values in the optimization algorithm. The results are shown in Table 1.

The stationarization of the series was obtained with first differences and so it is advisable to add the value 1 to the numbers below to obtain the values of fractional integration coefficient for series in levels.

For France, all the coefficients are significatively different from zero, positive for all the series except for FPOP. This result must be treated with caution because the GPH estimator is very sensitive to the presence of a short term component. Empirically, such a specification error can be suspected if the estimated values varies strongly when different number of ordinates are used in the periodogram regression (for different m or a). This seems to occur here.[6]

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[6] The results in Table 1 were obtained with m such as $m = T^a$ and $a = 0.5$. The same computations were performed for $a = 0.6$ and 0.7. For reasons of space, they are not presented here but are available from the authors.

We verify that the Orders of short term components are distinctly lower than whose obtained after application of a first differences filter only.

The maximum likelihood procedure confirms the previous results except for the FPOP series which now displays a positive fractional integration coefficient. The short term structure is now a MA(5) process for which the first and fifth terms are significant. The presence of a "troublemaker" short term component seems confirmed. The FDECR series is very close to non-stationarity and presents no more short term structures. All the series are persistent in first differences. The short term components are low for FGDP and FSCOL, non-existent for FDECR and of order 5 for FPOP and FDECS.

Data for Germany display similar results. The AGDP, APOP, ADELS and ASCOL series Show a significant and positive fractional integration coefficient. The short term structure of AGDP, ADELS and ASCOL are very close to the results obtained on French series. Only APOP does not display a significant short term structure while one Gould see a MA(5) on the French data. ADECR series display particular behaviour.

Table 1: ARFIMA model estimation

Series	d^{gph}	ARMA(p,q)	D^{MV}	ARMA(p,q)
AGDP	0.21 (2.60)	(0,3)	0.45 (6.22)	(3,0)
APOP	0.20 (2.65)	(1,0)	0.09 (2.04)	(0,0)
ADECS	0.54 (7.33)	(0,5)	0.29 (4.50)	(5,0)
ADECR	-0.37 (-14.65)	(0,7)	-0.43 (-6.73)	(0,7)
ASCOL	0.14 (2.65)	(0,4)	0.43 (7.37)	(0,3)
FGDP	0.29 (3.91)	(1,0)	0.27 (3.47)	(1,0)
FPOP	-0.28 (-4.19)	(0,5)	0.26 (5.36)	(0,5)
FDECS	0.50 (7.48)	(5,0)	0.38 (6.09)	(5,0)
FDECR	0.41 (6.16)	(0,0)	0.50 (10.58)	(0,0)
FSCOL	0.29 (4.38)	(0,3)	0.44 (5.94)	(0,1)

Note: values in brackets give Student statistics.

The optimization procedure developed on the first differentiated series does give any convergent results. The use of second differences gives an estimator close to -1/2. The raw series is therefore fractionally integrated of order approximately equal to 1.6 compared with the order 1.4 obtained in the French case. These two series are thus strongly non stationary. Their correlation structures are largely formed by very long term components that are difficult to exploit. This result underlines the difficulty of using current price data in a broad temporal framework. All the series Show long memory behaviour linked to their slowness of adaptation to a shock. At such a level of aggregation, this result is not a surprise. Relatively close forms inside every country lead to considering that a long term relation may exist among these variables. As a matter of fact, the similarities between the two countries lead us to thinking that the long term evolution was driven by comparable determinants.

2. Long term cycle study

2.1. Single component GARMA model

We now try to go further in the search for a possible long term component with finite periodicity. This study confirms the previous results (if the cycle frequency of the dominant long term component is infinite) or to complete them (if frequency is finite). Indeed, the presence of a cycle with frequency close to zero but significantly different will mislead the estimation procedure of an ARFIMA process (overestimation of the short term part, Chung (1996b)). Furthermore, if we do not find such a cycle, checking the relation $d^{ARFIMA} = 2d^{GARMA}$ obtained by two different optimization methods (spectral approximation of the log-likelihood function for ARFIMA and minimization of the sum of the squared conditional residuals for GARMA), makes it possible to confirm the results presented in the previous section. The results are shown in Table 2.

No finite cycle appears as a dominant long term structure. Coefficient η is not significantly different from 1 in all the series. Only the coefficient of series APOP seems not to be significant (its T statistic is lower than the 1.96 limit for a 5% test). All the series are persistent in differences.

We verify the relationship linking the two fractional integration coefficients at least roughly. Only the ADECR series Shows different results. We also retain a positive value for the fractional integration coefficient as well as a different short term structure.

Orders and values obtained for the short term parameters are very close to the optimal values presented in Table 1. The slight differences results from a settlement between the value of coefficient d and the variance of the residuals according to the two optimization methods.

Table 2: Single component GARMA estimation

Series	D^{GARMA}	η	ARMA(p,q)
AGDP	0.20 (3.43)	1	(3,0)
APOP	0.07 (1.69)	1	(0,0)
ADELS	0.139 (2.43)	1	(0,5)
ADECR	0.26 (2.10)	1	(0,3)
ASCOL	0.18 (2.79)	1	(0,3)
FGDP	0.12 (2.88)	1	(1,0)
FPOP	0.13 (2.37)	1	(0,5)
FDECS	0.17 (3.66)	1	(5,0)
FDECR	0.24 (4.50)	1	(0,0)
FSCOL	0.20 (4.57)	1	(0,1)

Notes: values in brackets give the Student statistics.

2.2. Two-component GARMA model

To conclude this work, we present the results of the 2-GARMA model estimation. The estimated values of parameters for every series are shown in Table 3. The selection of the optimal short term structure was made by repeating the estimation procedure for various values of the orders *p* and *q* and retaining as the optimal model that relative to the minimal sum of squared residuals.

First of all, we note that the method did not give stable results for the FPOP, ASCOL and ADELS series (value 0 indicates that no model 2-GARMA could satisfactorily be fitted on this series). For these series, and for the APOP series, the confidence interval of the parameter η *2* contains or is very close to the value 1. If there are effectively two fractional dynamics, they are too close for the method can not discriminate between them. On the contrary, everything seems to show that there is, for these series, only a single fractional component. For the other series, the value of the fractional integration coefficient for the parameter η *2* is fairly close to the value obtained by the estimation of the model 1-GARMA. This observation is evidence of the robustness of our result. The AGDP and FGDP series present a persistent structure and a value of parameter η *2* corresponding respectively to a cycle of 7(,4) years and 13(,2) years. ADECR, FDECR, FDECS and FSCOL series also seems to display a short term structure but the fractional integration coefficient is not significant for these series. However, the estimation results Show that a gain in term of the sum of squared residuals is brought by the introduction of the second constituent. A test with an of acceptance probability of 10 % makes it possible to retain these variables. So one can with reserve accept the presence of a second long term structure. They would correspond to cycles of period 7(,6) years for ADECR, 9(,8) years for FDECS, 9(,2) years for FDECR and 22 years for FSCOL.

Table 3: 2-GARMA model estimation

Series	η_2	d_1	η_2	d_2
AGDP	**1**	**0.15** **(1.96)**	**0.66** **[0.55,0.80]**	**0.13** **(1.96)**
APOP	**1**	*0.02* *(0.30)*	**0.934** **[0.97,1]**	*-0.02* *(-0.30)*
ADECR	**1**	**0.22** **(4.59)**	**0.68** **[0.51,0.84]**	*0.08* *(1.34)*
ADECS	**1**	0.17 (**)	**0.96** **[0.93,0.99]**	*-0.15* (**)
ASCOL	**1**	**0.29** **(2.17)**	0	0
FGDP	**1**	**0.26** **(4.27)**	**0.89** **[0.86,0.92]**	***0.26*** ***(2.16)***
FPOP	**1**	*0.01* (**)	**0.88** **[0.76,0.99]**	*0.06* (**)
FDECR	**1**	**0.221** **(6.594)**	**0.78** **[0.54,0.98]**	*0.04* *(1.45)*
FDECS	**1**	**0.18** **(2.52)**	**0.95** **[0.90,0.98]**	***-0.12*** ***(-1.26)***
FSCOL	**1**	*0.20* *(1.03)*	**0.96** **[0.92,1]**	***0.08*** ***(1.09)***

Notes

(i) The values printed in italics are these for which the nullity hypothesis is accepted, the values printed in ***bold italics*** are those for which a doubt remains. The values printed in **bold** are significant values.

(ii) The values in brackets give the Student statistic. The values in braces give the confidence interval obtained according to the values tabulated by Chung (1996a)

(iii) The sign ** indicates an algorithm failure to supply results.

V. Conclusion

A summary of the results obtained is given in the Table 4.

Table 4: Properties of the series

Series	Long term structure	Long term cycle	Short term structure
AGDP	Unit root and long memory	7 years	3 years (AR)
APOP	Unit root	none	none
ADECR	Unit root and long memory	7 years (doubt)	1 years (MA)
ADECS	Unit root and long memory	none	none
ASCOL	Unit root and long memory	none	2 years (MA)
FGDP	Unit root and long memory	13 years	1 years (AR)
FPOP	Unit root	none	5 years (MA)
FDECR	Unit root and long memory	9 years	none
FDECS	Unit root and long memory	20 years	5 years (AR)
FSCOL	Unit root and long memory	22 years	1 years (AR)

At the current state of knowledge, analysis by fractional integrated processes is the most appropriate method for describing the phenomena of long memory and, more widely, for identifying and/or clarifying certain theoretical hypotheses with view to modelling long term cyclic movements. At the point we have reached, it is therefore possible to shed light on the empirical results mentioned. The cliometric analysis of educational series, economic growth and demography, in France and in Germany, in the nineteenth and twentieth centuries leads us to a significant result: no long term cycle appears as the dominant constituent. Nevertheless, there are cycles which seem to affect some of the variables that we have studied. Others do not Show any particular movement. In Table 4, we distinguish first of all movements, close to the classic cycle of the Juglar type (average duration of which is between 7 and 11 years), for the AGDP, ADECR, FGDP and FDECR series. We then observe, for AGDP, ADECR, ASCOL, FGDP, FPOP, FDECS and FSCOL series, cycles of 5 years or less; these are similar to minor cycles of the Kitchin type (average duration 40 months). We also note Kuznets type cycles in the FDECS and FSCOL series. The latter found a periodicity of about 22 years for a period of a complete oscillation of production and 23 years for prices. Finally, we did not notice any cyclicity close to long movements such as Kondratieff cycles (whose average duration is between 48 and 60 years).

As in the myth of Sisyphus, the boulder is at the bottom of the hill again!

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Appendix

French series

In the Case of France, printed works are mentioned whose range is, a priori, national. However, it should be remembered that statistics concerning primary education were published regularly at the level of individual departments (until 1939 and sometimes later). In addition, part of the statistics collected but not published is housed in national or departmental archives. Consultation of these makes up for the lack of scholastic statistics from 1914 to 1958 (substantial collection of information was performed during this period but a comparatively small proportion was published, probably for reasons of budget). In a general manner, statistics concerning the education system in France in the nineteenth and twentieth centuries can be classified under two main headings. On the one hand, they consist of statistical data produced by one-off investigations performed at the initiative of a minister, a special commission or a group of members of parliament to examine certain aspects of the functioning of educational facilities. For example, they include statistics on primary, upper primary, secondary and higher education. They also comprise information collected during the everyday Management of educational institutions or of education as a whole and whose main (and sometimes sole) purpose was administrative. This concerns mainly data published in the statistical yearbooks of the SGF (Statistique Générale de la France), the Main basis for Most of the overall studies of the trends in school attendance in France. State accounts concerning education, as drawn up by L. Fontvieille (1990) and refined by A. Carry (1995) using the financial records of the Compte Général de 1'Administration des Finances can also be mentioned. Finally, reference should be Made the Sets of statistics published by the Institut National de Recherche Pédagogique, those produced by the Ministry of Education's Direction de 1'Evaluation et de la Prospective in the form of satellite accounts and those drawn up by C. Diebolt (1995, 1999abc) on the quantitative history of education in France. The series of the Gross Domestic Product (GDP) is calculated by J.-C. Toutain (1987, 1997) completed by the estimations of the INSEE. The annual total population series was established by C. Diebolt (1999c) from the legal population of France and structure by age as shown in censuses.

German series

in the case of Germany, mention should be Made first of all of two excellent collections of statistics. They form the best possible point of departure for the quantitative history of the German educational system from the first phases of the Industrial Revolution to the end of the Second World War. One is by D.K. Müller, B. Zymek and U. Herrmann (1987) and the other by H. Titze, H.-G. Herrlitz, V. Müller-Benedict and A. Nath (1987, 1995). The pioneer work of W.G. Hoffmann (1965), P. Lundgreen (1976) and D.K. Müller (1977) should also be noted on the subject of the long term changes in public expenditure on education, together with the work by C. Diebolt (1997) on hitherto neglected sectors (expenditure and its funding, the number of pupils according to education level, etc.). The GDP series was calculated by W.G. Hoffmann (1965), completed by the estimations of Statistisches Bundesamt. The annual total population series was established by C. Diebolt (1997) from the legal population of Germany and structure by age as shows in cencuses.