Can an Islamic model of housing finance cooperative elevate the economic status of the underprivileged?

Ebrahim, M. Shahid

Empfohlene Zitierung / Suggested Citation:

Terms of use:
This document is made available under the "PEER Licence Agreement". For more information regarding the PEER-project see: http://www.peerproject.eu This document is solely intended for your personal, non-commercial use. All of the copies of this documents must retain all copyright information and other information regarding legal protection. You are not allowed to alter this document in any way, to copy it for public or commercial purposes, to exhibit the document in public, to perform, distribute or otherwise use the document in public. By using this particular document, you accept the above-stated conditions of use.

Diese Version ist zitierbar unter / This version is citable under: https://nbn-resolving.org/urn:nbn:de:0168-ssoar-314407
Accepted Manuscript

Title: Can an Islamic Model of Housing Finance Cooperative Elevate the Economic Status of the Underprivileged?

Author: M. Shahid Ebrahim

PII: S0167-2681(09)00198-X
Reference: JEBO 2435

To appear in: Journal of Economic Behavior & Organization

Received date: 12-10-2008
Revised date: 31-7-2009
Accepted date: 4-8-2009

Please cite this article as: Ebrahim, M.S., Can an Islamic Model of Housing Finance Cooperative Elevate the Economic Status of the Underprivileged?, Journal of Economic Behavior and Organization (2008), doi:10.1016/j.jebo.2009.08.002

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.
Can an Islamic Model of Housing Finance Cooperative Elevate the Economic Status of the Underprivileged?

M. Shahid Ebrahim
University of Nottingham, U.K.

This Version: March 13, 2009.

Acknowledgments: This paper was refined during my sabbatical study at James Madison University (JMU). I appreciate the hospitality of JMU particularly that of Ehsan Ahmed. I have benefited from the critical comments of the participants of the seminars at James Madison University; University of Birmingham; University of Glasgow; the 2006 Conference on Computing in Economics and Finance (in Cyprus); the 2007 IIUM International Conference (in Malaysia); at the 2007 Workshop on Default Risk and Financial Distress (in Rennes, France), the 2007 Product Development and Management Association Conference (in Bangalore, India); the 2008 International Conference on Business and Finance (in Hyderabad, India); the 2008 International AREUEA Conference (in Istanbul, Turkey); the 2008 Workshop of European Network of the Economics of Religion (in Edinburgh, UK); and the 2008 Symposium on Religion, Markets and Society (in Nottingham, UK) on earlier drafts of the paper. I am also grateful to the following individuals for their helpful suggestions: Abdullah Al-Ghazali, Eyhab Al-Hajj, Aaref Al-Lawati, Bruce Brunton, Humayon Dar, Mohammad Omar Farooq, Diana Mitlin, Kelly Morris, Peter Oliver, Peer Smets, Ghulam Sorwar, Rafal Wojakowski and Robert Young. Finally, I appreciate the valuable comments provided by J. Barkley Rosser Jr. (the Editor) and two anonymous referees. All remaining errors are mine.

Correspondence Address: Professor M. Shahid Ebrahim
Chair in Financial Economics
Nottingham University Business School
Jubilee Campus, Wollaton Road
Nottingham NG8 1BB
United Kingdom
Tel: +44 (0) 115 846 7654
Fax: +44 (0) 115 846 6667
E-Mail: m.shahid.ebrahim@nottingham.ac.uk
Can an Islamic Model of Housing Finance Cooperative Elevate the Economic Status of the Underprivileged?

Abstract: A formal home loan is onerous to subprime borrowers in efficient markets. This can deter homeownership for financially strapped individuals, leading to a market failure. This paper proposes a special form of cooperative mortgage financing (practiced in Oman) to overcome this market failure. We integrate the literature of Mortgage Design with that of informal savings schemes (i.e., ROSCAs/ASCRAs) to illustrate that this mode of financing dissipates credit risk better than the formal mode of financing. It is also resilient to volatility of interest rates and allows prepayments without any additional charges. Finally, we verify the assertions of Besley et al. (1994), and Hart and Moore (1998) that cooperative mortgages are *pareto-superior* to formal mortgages in special cases.

Keywords: ASCRA, Asset Bubble, Mutual Bank, Inflation, Mortgage Design, and ROSCA.

JEL Classification Codes: C63 (Computational Techniques)  
G21 (Banks – Mortgages)  
G32 (Financing Policy – Capital and Ownership Structures)  
I31 (Welfare and Poverty – Basic Needs)  
O17 (Formal and Informal Sectors; Institutional Arrangements)  
P13 (Cooperative Enterprises)
1. Introduction

The profound argument made by Stiglitz (1994) is that market failure is a fundamental cause of poverty and financial market failures, which mainly arise from market imperfections, asymmetric information and the high fixed costs of small-scale lending, limit the access of the poor to formal finance, thus pushing the poor to the informal financial sector or to the extreme case of financial exclusion. In addition, it is argued that improving the access of the poor to financial services enables these agents to build up productive assets and enhance their productivity and potential for sustainable livelihoods (World Bank 2001). Hence the bottom line argument is that improving the supply of financial services to the poor can directly contribute to poverty reduction (Jalilian and Kirkpatrick, 2002).

(Green, Kirkpatrick and Murinde, 2005, p. 19)

Housing plays a vital role in the economy (see Sheng, 1997). This is due to its following attributes. First, a home is both consumption good as well as an investment (see Malpezzi, 1990). The investment aspect of homeownership helps to increase wealth (i.e., reduces poverty – see Buckley, 1994; Englehardt, 1994; Sheng, 1997; Haurin et al., 2002). Second, homeowners support their neighborhood more than renters, as they participate in crime prevention and support public schools. They are better citizens and vote at a higher rate (see Haurin et al., 2002). Homeownership fosters investment in local amenities and social capital, thus enhancing the status and quality of the community (see DiPasquale and Glaeser, 1999). Policy makers therefore have an obligation of ensuring access to this indispensable asset through an efficient financial intermediation system.¹

The ongoing subprime mortgage crisis (emanating from the U.S.) constitutes a financial market failure.² This is because financing a home for a subprime borrower poses a dilemma in an efficient capital market (see Fama, 1970; 1991). The ingenious mortgage bankers figured out an innovative way (which turned out to be disastrous as explained below) to get financially strapped

¹ The financial intermediation system has the capacity of rendering the economy vulnerable to risk, as it connects real estate prices with the macroeconomy (see Glaeser, 2000). This is because:
  (i) Regional home bubbles have negative impact on residential investment, and thus aggregate output (see Higgins and Osler, 1998).
  (ii) A sharp fall in house prices leads to a reduction in consumption (through the wealth effect – see Case et al., 2005).
  (iii) A significant decline in home prices leads to foreclosures and losses for lenders, thus straining the banking system (see Case, 2000).
  (iv) Endogenous developments in the credit markets are amplified and transmitted to the macroeconomy (through the financial accelerator effect – see Bernanke et al., 1999; and Aoki et al., 2004).

It is therefore imperative to design an efficient housing finance system to mitigate the vulnerability of the economy to risk (as discussed above). It also leads to a reduction in home prices relative to income, and bestows numerous economic benefits (see Malpezzi, 1990; Renaud, 2009).

² Note that subprime loans imply loans to borrowers who have sketchy credit history and are financially strapped or lack adequate income to qualify for a standard mortgage. They are thus lower in quality to prime loans.
individuals to qualify for a home loan using lax underwriting standards. This innovative way was through the use of “exploding” adjustable rate mortgages (ARMs) with unusually low introductory (i.e., “teaser”) rates, which reset to higher rates at the expiration of the introductory ones (see Gapper, 2007). This was supposed to facilitate access to a home for borrowers and to help them establish some credit history before qualifying for refinancing with a fixed rate mortgage. The implicit assumption was that the eventual appreciation of the home would bail out the borrower prior to the expiration of the “teaser” rate as he/she would be able to refinance and not be exposed to the shock of higher mortgage payments. Unfortunately, the opposite happened, and not only did payments increase drastically (with the termination of the teaser rate) but home prices fell too. This made it difficult for borrowers to keep up with their payments. Also, they could not refinance (or sell) their homes, as their values were significantly below their mortgage balance (leaving them with negative equity). They had no option but to default. It is estimated that more than 2.4 million American families have lost their homes through foreclosures (see Economist, 2007a; Ip and Paletta, 2007; and Mason and Rosner, 2007). This number is expected to go up to 9 million, as a second wave of defaults (stemming from the above exploding ARMs, along with negatively amortizing ARMs, which increase principal at the end of the low teaser, or optional payment periods) work their way through the system (see Wachter, 2008; Ward, 2009).

This crisis has drastically impacted the global economy as elaborated below.

---

3 The lax underwriting standards probably stem from a corner solution derived under the assumption of housing prices following a Log-Normal Random Walk (see the solution to Equations (5b) in Sections 2 and 3 in the form of Equations (5d) and (11) respectively. This issue is reiterated by Jaffee and Stiglitz (1990) in terms of the ad-hoc loan-to-value (LTV) ratio.

4 In general, ARMs are not appropriate for households with a large mortgage, volatile income, high default cost or low moving probability (see Campbell and Cocco, 2003).

5 Moral Hazard on the part of mortgage originators played a key role in the ongoing crisis. This is attributed to two major weaknesses of securitization. First, it encouraged careless lending (using ad-hoc standards based on loan-to-value ratio, payment-to-income ratio and credit guarantees). This allowed originators to conceal and convey the risk of the underlying properties to the lenders. The rating agencies, who were supposed to confirm the inherent risk of the mortgages, failed to do so due to the conflict of interest. This is because they were paid by the sellers of the securities as opposed to the buyers. Second, securitization allowed mortgage originators to get around their reserve capital requirements. This allowed them (and their off-balance-sheet vehicles) to lever up (see Crook, 2008).

Furthermore, in some cases, real estate professionals in the mortgage supply chain (such as real estate agents and appraisers to underwriters, lenders and lawyers) colluded to defraud the system. The FBI has launched an investigation dubbed as “Operation Malicious Mortgage” and has indicted 406 defendants in 144 cases involving $1bn in losses (see Kirchgaessner and Weitzman, 2008).

Finally, the credit guarantees also evaporated when the insurers themselves bought “tainted” assets like Collateral Debt Obligations (CDOs) backed by subprime loans (see Kroft, 2008).

---
The repercussions of the subprime “woes” are being felt (in both the real, as well as the financial sectors of the economy) globally. In the case of the real sector, the crisis has led to: (i) An increase in supply of homes for sale (due to repossessions), thereby depressing their prices and negatively impacting the construction sector, sales of durable goods, and thus the manufacturing sector (see Economist, 2007b; Spector, 2007; Economist 2009a); and (ii) An economic contraction (in the U.S.) impacting the economies of its trading partners through decreases in trade, investment and remittances, thus leading to a backlash against globalization (see Economist 2009b). In the case of the financial sector, the reduction in value of underlying collateral of mortgages (i.e., homes) has resulted in: (i) A loss in market value of more than $290 billion of bonds associated with subprime mortgages, devastating the capital base of major financial institutions on both sides of the Atlantic (see Economist 2008; and Barkley, 2008); (ii) The failure of more than 40 subprime lenders (see Authers, 2007); (iii) The inability of the U.S. government sponsored agencies (such as Fannie Mae and Freddie Mac) to provide some relief (in the crisis), as they themselves have been placed under conservatorship (see Crutsinger, 2008); (iv) The scrutiny of the remaining subprime lenders from state and federal regulators (see Ip and Paletta, 2007); (v) The tightening of credit to firms (in other industries, hedge funds, private equity groups, etc.) is anticipated to lead to a severe recession in the U.S. and a decline in value of American assets (see White et al. 2007); and (vi) The spreading of systemic problems from the U.S. to overseas, increasing capital market volatility, and “crimping” world growth (see Economist, 2007b; Gapper 2007). The International Monetary Fund estimates the total losses to reach $2.2 trillion (see Wolf, 2009).

---

6 The decline in the value of the U.S. dollar has led to a spike in the price of oil, gold and other commodities (denominated in the U.S. currency). This has impacted the cost of food production, as it increased fertilizer prices, fuel for tractors and farm machinery, and pesticides (which depend on oil prices). It has also diverted the use of arable land for the production of biofuels. Thus, in the face of increasing demand from a global population (emanating especially from the developing world) in conjunction with a drought in grain-producing areas (of the developed world), food prices have skyrocketed. The subprime crisis has therefore given way to the twin crisis of commodity and food inflation destabilizing the existing global social order (see Guardian Unlimited, 2008).

7 At the time of writing the current draft of this paper (in early March 2009), fears of a major global slowdown has resulted in an easing of commodity and food prices from its recent highs (see Blas, 2008). Nonetheless, the worsening of this crisis has prompted central banks on both sides of the Atlantic to support their respective banking systems with their bailout plans, along with a coordinated interest rate cut. This is designed to shore up confidence in the global financial system (Giles, 2008; Guha, 2008; Guha and Politi, 2008; Thal Larsen and Parker, 2008; and Willman, 2008). Meanwhile, in the epicenter of the crisis, the Obama administration has put forward a “Homeowner Stabilization Plan” to modify mortgage contracts to make them more affordable (see Paletta, 2009; and Simon, 2009). The U.S. Treasury is also planning to fund a quasi-private entity to acquire the most “toxic” subprime mortgages (whose deteriorating value brought down institutions such as Bear Stearns, Lehman Brothers, American International Group, etc.) under a “bad-bank” structure. This program is anticipated to cleanse the banking system to ease the flow of credit to the real sector of the economy (see Wolf, 2009).
The purpose of this paper is to present a novel way of home financing, using leverage \textit{endogenously} amongst underprivileged aspiring homeowners (via a housing finance cooperative) to avert subprime like financial market failure. This is radically different from the formal \textit{exogenous} form of financing via an intermediary (such as a bank, Savings and Loan Association or a mortgage company – see Jaffee and Renaud 1998). Our goal in calling for the formulation of a specialized institution (“circuits”) catering to help aspiring homeowners is in contrast to the trend towards integrating formal mortgage underwriters with capital markets (see Diamond and Lea, 1992; and Jaffee and Renaud, 1998).\textsuperscript{8} This is because the formal system is onerous to the underprivileged, especially in efficient financial markets where subprime borrowers are subject to a high cost of funding, as they are perceived to be more risky (as discussed above). Furthermore, inflationary shocks (or volatility of interest rates) on formal mortgages create a tilt in real payments, which makes aspiring homeowners ineligible by the income requirement of financial institutions (see Buckley, 1994). It is therefore not surprising that formal intermediaries are \textit{not} used by more than 70%-80% of homeowners in the developing world (see Okpala, 1994; and Ferguson, 1999).

The rationale behind the low underwriting rate of formal intermediaries (in the developing economies) is attributed by Jaffee and Renaud (1998) to the high costs of lending, especially when property rights, foreclosure procedures (needed for real estate to serve as collateral) and accurate methods of valuing property are not well established. Another strand of the literature (stemming from housing micro-finance) holds two factors primarily responsible for deterring (the underprivileged) from gaining access to formal mortgage finance (see Ferguson, 1999). One is the lack of affordability to legal buildings, as most of the dwellings in low-income settlements do not comply with building regulations, nor do they have formal land titles. This deters formal institutions from lending to these households. The second is the instability of income to secure repayment.

Since the goal of this paper is to establish a basic framework to increase the affordability of legal (formal) buildings, we assume that the government has already laid the necessary infrastructure of the following: law and regulation, information, risk pricing, payment and settlement, and financial stability (see Renaud, 2009). This assumption stems from the research that organizations and structure of the financial system plays a crucial role in the quality and rate of economic growth (see Goldsmith, 1969; and Renaud, 2009). We also assume that prospective

\textsuperscript{8} The rationale generally used against special “circuits” is as follows:
(i) They are not free-market oriented (i.e., they are dependent on government subsidies); or
(ii) They expose these specialized institutions to an inordinate amount of risk, as they are not well diversified, use a short funding strategy and tend to be fragile when subjected to economic shocks.
homeowners have stable incomes. This assumption is consistent with Levenson and Besley (1996), who find increasing ROSCA participation associated with income stability. We refrain from delving in the micro-financing of progressive housing (where households acquire land through purchase or invasion, thereby improving the structure and legal tenure incrementally and lobby for basic services), as the cost of micro-financing is higher than conventional banks, and thus still burdensome to the underprivileged (see Ferguson, 1999). Furthermore, this system of micro-financing is not sustainable in the long run due to high rates of default, as the mortgages are priced in an ad-hoc manner (see Lee, 1995). Thus, after establishing a basic framework to increase affordability of a formal home, we plan to extend it to the issue of income instability in future research.\footnote{The intricacies of the real world, especially pertaining to income stability, can be addressed by conducting a second optimization model on stochastic income (in addition to our primary one on stochastic home price) by ensuring that the probability of fixed payments exceeding it is negligible. The ultimate solution is derived as a combination of the two optimization models.}

The motivation behind our call for the establishment of cooperative financing institutions stems from the ad-hoc practice of clans in Oman to fund the purchase of homes of their poor brethrens with gratuitous (interest-free) loans (termed as qard hasan).\footnote{This custom of the Omani stems from the holy Qur'an, which exhorts Muslims to lend without interest classifying it as a loan to God Himself (see 2:245, 5:13, 57:11, 57:18, 64:17, and 73:20). This is reinforced in the tradition of Prophet Muhammad (PBUH) which ranks interest free loans higher than voluntary charity (sadaqah) (see Sunan Ibn Majah, Book of Ahkam, Chapter on Loan). For more information on this issue see Farooq (2008).} These are conducted along the lines of an Islamic endowment or trust (termed as waqf; plural: awqaf) which blends features of philanthropy with social service. The seed funding for this institution emanates from the cash contributions of well-to-do clan members from supplementary charitable sources of infaq (voluntary charity for a specific purpose) or sadaqah (voluntary charity) (see Cizakca, 2000).\footnote{Extended families (or clans – termed as Qabaail; singular: Qabeelah) have played a vital role in Oman’s social and political fabric for centuries. The largest clans boast of thousands of members stemming from hundreds of families. Nonetheless, they still function as close-knit units answering to their clan head (termed as Shaikh al-Qabeelah) and a small group of elders who advise him.} Bremer (2004) classifies this as a rebirth of the waqf model and explains it as follows:

---

9 The intricacies of the real world, especially pertaining to income stability, can be addressed by conducting a second optimization model on stochastic income (in addition to our primary one on stochastic home price) by ensuring that the probability of fixed payments exceeding it is negligible. The ultimate solution is derived as a combination of the two optimization models.

10 This custom of the Omani stems from the holy Qur'an, which exhorts Muslims to lend without interest classifying it as a loan to God Himself (see 2:245, 5:13, 57:11, 57:18, 64:17, and 73:20). This is reinforced in the tradition of Prophet Muhammad (PBUH) which ranks interest free loans higher than voluntary charity (sadaqah) (see Sunan Ibn Majah, Book of Ahkam, Chapter on Loan). For more information on this issue see Farooq (2008).

11 Extended families (or clans – termed as Qabaail; singular: Qabeelah) have played a vital role in Oman’s social and political fabric for centuries. The largest clans boast of thousands of members stemming from hundreds of families. Nonetheless, they still function as close-knit units answering to their clan head (termed as Shaikh al-Qabeelah) and a small group of elders who advise him.

---

Most of the clans refrain from meddling in politics and function predominately as a social net, helping members who fall on hard times. To help each other through the difficult economic environment, the clans operate a welfare fund in which every well-to-do family (or individual) is required to contribute a minimum amount each month. This gives the Shaikh sufficient annual budget to spend on funerals, weddings, study grants and interest-free home loans for destitute qabeelah members.

Since clans compete with each other economically, the prime goal of the Shaikh al-Qabeelah is to improve the lot of his clan members and bind them cohesively (under the clan/tribal banner). Granting interest-free loans serves both as an investment, as well as a means of uniting clan members cohesively. It is an investment, as it confers the poor clan members access to an indispensable asset (i.e., a home) which allows them an avenue to improve their economic lot and ultimately contribute for the upliftment of others. Members who receive home loans are also grateful to the clan and are loyal to it. This social net binds clan members.
Charities have played many critical functions in Islamic societies and have contributed to making these societies more just and fair through a number of mechanisms, in addition to the obvious one of providing service to the poor. Over and above their role in delivering services, Islamic charities served as a mechanism for narrowing social distances and reducing inequalities. Charities have particularly, served as a bridge between the haves and have-nots. They have provided a means by which the wealthier elements of society interact with poor individuals, come to know them as individuals, and recognize their obligations to assist them in combating poverty, its causes and effects. This linkage helps to keep low-income groups from becoming isolated from the social mainstream, strengthening the overall social order.

Charities, particularly awqaf, provided a source of support for institutions and interest groups independent of, and sometimes in opposition, to the state.

Islamic charities historically have played an additional role in society, that of promoter of decentralized economic development. Whether the charity is a waqf in the medieval Levant establishing commercial centers or building a khana for traveling business people, or an Indonesian zakat-funded charity teaching business management skills in today’s Indonesia, Islamic charities have been actively engaged in economic development for centuries. In this respect, they reflect the blending of the religious and the secular, the social and the economic, that is the key characteristic of the Islamic idea.

(Bremer, 2004, pp. 5-7)

She explicates the last sentence further in a footnote as follows:

This combination can be found present in the West in urban development-oriented civil society, as well, such as pro-poor non-profits that address inner-city economic development and civic business associations that promote the development of their respective cities. Generally, however, the mix of economic development and social service with charity is much more developed in the tradition of Islamic charities than in the more “purely” charitable tradition of Western society.

(Bremer, 2004, Footnote 4, p. 7)

The above stated practice of Omani clans is also in accordance with the prognosis of King and Levine (1993) and Levine (1997), as it facilitates in mitigating risk and reducing transaction costs for the underprivileged masses. It also has the potential of resolving the market failure which has provoked the ongoing subprime crisis. However, it has not been scrutinized until now by academics, as the data on this form of financing (conducted privately amongst clan members) is not in the public domain and hard to obtain. The reason for this is that tribal elders do not want to disclose their economic power. Furthermore, commercial banks also find it hard to compete against this practice and try to restrain it. They have not succeeded, as the average Omani is

Default risk is reduced by catering only to clan members, ensuring that there is adequate collateral in the form of assets or guarantees (co-signors) in addition to the immovable property being financed.
extremely loyal to his/her clan and is not willing to divulge any information. We have been quite fortunate to learn of this practice through our interaction with some Omanis, who have disclosed it for the purpose of this study. Thus, the specific issues of concern of this paper are as follows: (i) Can a specialized circuit (in the form of a financial cooperative – using *endogenous* leverage) alleviate credit risk better than the formal mode of prime sector financing (using *exogenous* leverage)? (ii) Can it alleviate inflation risks better than its formal counterpart? (iii) Can it be pareto-optimal over its formal counterpart, thereby uplifting the economic status of the underprivileged and stimulating economic development?

The three interrelated issues (stated above) encompass the optimal pricing of loans. This is a formidable task, as capital structure (i.e., choice of debt-equity) constitutes a major puzzle in finance (see Harris and Raviv, 1991). Typically, financial institutions price their loans in an ad-hoc manner using credit rationing (in the form of initial loan-to-value (LTV) ratio and income ratio – see Jaffee and Stiglitz, 1990). This is not efficient, as empirically demonstrated by the literature on banking crises and real estate cycles (see Lee, 1995; Herring and Wachter, 1999; and Malpezzi and Wachter, 2002). Furthermore, it exacts a huge toll on the macroeconomy, as Renaud (2003) and Hoshi and Kashyap (2004) estimate the costs of real estate crises in Indonesia, Thailand, Japan and United States to be roughly 65%, 45%, 20% and 3% of the GDP respectively.12

We investigate the above interrelated issues by blending two streams in the literature. First, we focus on Security (Mortgage) Design, which espouses that risk management through the optimal employment of secured debt and debt maturity reduces agency costs and enhances firm value (see Stanton, 1998; and Eisdorfer, 2008).13 This view is reinforced in Ebrahim and Mathur

---

12 The formal financial intermediaries encounter three risks in the underwriting of their mortgages: credit risk, interest rate risk and liquidity risk (see Jaffee and Renaud, 1998). They mitigate (i) interest rate risk by using a variable (adjustable) rate mortgage, by hedging through the use of derivative contracts or by securitizing mortgages (and selling them in the capital markets); and (ii) liquidity risk by securitizing the mortgages (as discussed in (i) before). However, these intermediaries have a major problem managing credit risk (through the ad-hoc use of credit rationing), as corroborated by Hester (1992) and Glauber (1992). This impacts their ability to securitize mortgages diligently as elaborated in Footnote 5.

13 The finance literature elaborates on two issues pertaining to the agency costs of debt. The first is the risk shifting (or asset substitution) issue, where owner-manager have an incentive to transfer the downside risk (of project) to the debt holders, while benefiting from the upside potential. A number of studies such as Smith and Warner (1979 a; b) and Barclay and Smith (1995b) have illustrated that risk management through the use of secured debt alleviates this issue. Other studies such as Barnea et al. (1980) and Barclay and Smith (1995a) demonstrate that debt maturity also mitigates this issue.

The second issue is the underinvestment one, where the owner-managers are motivated to reject positive net present value (NPV) projects if the wealth enhancement associated with undertaking it accrues mostly to debt holders (see Myers, 1977). Stulz and Johnson (1985) argue that secured debt can be employed to mitigate this issue.
(2007), who hypothesize that real estate mortgages have to be priced meticulously by adequately collateralizing them with the underlying tangible assets and income of the borrower. This condition is more stringent than the ad-hoc credit rationing ones, and ensures that the financier is not exposed to the risk of homeownership. That is, the mortgage is nearly risk-free.\textsuperscript{14} This condition follows from their (two period) model, that under rational expectations (symmetric information), a collateralized risk-free loan is pareto-superior over its risky counterpart. The rationale behind this result is as follows. In a world of symmetric information, a financier would rationally price risky mortgage to incorporate deadweight costs of default in the form of high interest rates to be transmitted to the borrower (prospective homeowner). This reduces the welfare of the homeowner. If the risky interest rate is high enough, prospective homeowners forgo a risky mortgage in lieu of a risk-free one (with a conservative debt ratio and low cost of financing). This has credence in the real world, as Singapore (which has the highest homeownership rate—of 84% in the world) does not allow excessive risk taking via mortgages on public housing units (see Edelstein and Lum, 2004).

Second, we focus on financial cooperatives, as they have played a crucial role in the economic development of Germany, United Kingdom, United States and many industrialized countries (see Shay, 1992). These originate from Accumulating Savings and Credit Associations (ASCRAs), which are interrelated to Rotating Savings and Credit Associations (ROSCAs) (see Grossman, 1992; and Bouman, 1995).\textsuperscript{15, 16, 17} Besley et al. (1993) find ROSCAs to be

The literature on the employment of secured debt (to mitigate risk shifting) depicts mixed results. On one hand, studies such as Smith and Warner (1979a; b) and Barclay and Smith (1995b) strongly support it. While on the other hand, Titman and Wessels (1988) find no evidence for it. The reasons for this discrepancy are attributed to three factors. One, secured debt is contingent on the quality of the asset being financed (see Shleifer and Vishny, 1992). Two, standards of underwriting debt (especially mortgages) are not scientific. They are based on ad-hoc credit rationing techniques (using initial LTV and income ratios—see Jaffee and Stiglitz, 1990). Finally, these ad-hoc underwriting criteria are not applied uniformly over the economic cycle (see Stanton, 1998; and Herring and Wachter, 1999). This is because credit is granted generously in the expansionary phase of the economic cycle and severely constrained in the contractual phase, thereby leading to a “credit crunch.”

\textsuperscript{14} We construe a nearly risk-free mortgage by resorting to an extremely high degree of confidence level at the underwriting stage. This pragmatic approach is empirically confirmed by Lacour-Little and Malpezzi (2003), who attest that homeowners do not “ruthlessly” default as soon as they go “underwater”. They normally wait until the negative equity on their home is extremely large. Our mortgage pricing approach is also supported by the assertion of the well-known economist Lawrence Summers:

\textit{Foreclosures are extremely costly. Between transaction costs that typically run at one-third or more of a home’s value and the adverse impact on neighbouring properties, foreclosures can easily dissipate more than the total value of the home being repossessed. They also inflict collateral economic damage, as reduced wealth and diminished borrowing capacity in homes reduces consumer spending, increases credit market fragility and depresses local taxes.}

(Summers, 2008, p. 11)

\textsuperscript{15} A ROSCA is basically an informal association of individuals, who pool in their resources to save. Members commit to contributing a fixed amount of money in a fund for each period in the tenure of the ROSCA. At each meeting, the entire fund is allocated to a particular member. The meetings continue with a different member of the group being granted the fund at each date. This process continues until every member of the ROSCA has
appealing, as they provide a pareto-superior solution to the problem of purchasing an indivisible (lumpy) good (such as a home). This is attributed by them to the improvement in social welfare stemming from intertemporal trade due to the mobilization of savings (under ROSCAs), which otherwise would have been idle under autarky. In contrast, Besley et al. (1994) establish that ROSCAs (in general) are less flexible and therefore less efficient than formal credit markets. However, there are special cases where a random ROSCA may increase welfare over a formal credit market due to the element of chance. Besley et al. (1994) therefore predict the decreasing role of ROSCAs with increasing economic development. It is thus a puzzle to see intermediaries with ROSCA roots (like Building Societies, Credit Unions and Mutual Savings Banks), which not only proliferate, but also compete head on with for-profit intermediaries (such as Commercial Banks, Stock Savings and Loans, etc.) in highly developed economies such as the Germany, U.K. and U.S. This is attributed in the literature to the (i) Inability of consumers to evaluate the quality of goods (or service) promised (or delivered) (see Hansmann, 1980); (ii) Inability of consumers to observe (or measure) the output (or benefit) (see Easley and O'Hara, 1983); (iii) Adaptability to a changing economic environment (see Emmons and Mueller, 1997); (iv) Narrowly defined activity where members have homogeneity of opinion (see Hart and Moore, 1998); (v) Mitigation of

received the fund once. The allocation of the fund is either through a lottery (Random ROSCA) or an auction (Bidding ROSCA).

ROSCAs are found in many parts of the world under different names such as chit funds in India, jamaiyah in Oman, hui in Vietnam, kye in Korea, pasanakus in Bolivia, paluwagon in Philippines, susu in West Africa, tanda in Mexico, and tontine in Senegal, among others.

16 ASCRAs pool savings just like ROSCAs. However, unlike ROSCAs, it accumulates them for a specific time after which it is distributed. The membership of ASCRAs is much wider than that of ROSCAs. The loan decision is not automatic, but subject to the consent of a managing board. Loan administration is generally quite elaborate, necessitating the need of up-to-date records and, in some cases, posting of collateral (see Bouman, 1995).

17 The German Cooperative Banks, U.K. Building Societies, U.S. Savings and Loans Associations (S&Ls) and Credit Unions originated from ASCRAs. The German Cooperative Banks (akin to the U.S. Credit unions specializing in consumer loans) and the U.K. Building Societies (akin to the U.S. S&Ls specializing in home loans) were established in 1778 and 1781 respectively. The first U.S. S&L was chartered in Philadelphia in 1831. The early S&Ls were cooperatives, but by the 1900s the bulk of them had evolved into stock (for profit) corporations. That is, they were associations in name only. The remaining S&Ls (in the Northeast and Wisconsin) retained their ASCRA roots. That is, they remained as mutual savings banks. These too succumbed to the pressures of demutualization by being bought out by commercial banks in the late 1980s and early 1990s (see Esty, 1997).

The U.S. Credit Unions were established in early 1900 (through the efforts of Alphonso Desjardins and Edward A Filene) to serve the average consumer with savings facilities and short-term loans (such as automobiles), while S&Ls focused on housing (until deregulation).

Mutual savings banks are organized differently from Credit Unions. Credit Unions are structured around “common bonds” or “fields of memberships” that are residential, professional, occupational or associational. A need for financial services (for Credit Union) does not constitute a “common bond.” In contrast, mutual savings banks operate under a cooperative ownership structure and use the “need for financial services” as a “common bond” (see Emmons and Schmid, 1999).
adverse selection (due to availability of adequate information on borrowers – see Buijs, 1998; and Smets, 2000); (vi) Alleviation of moral hazard (in the form of timeliness of payment and reduction of default – see Buijs, 1998; and Smets, 2000); and (vii) Relatively lower administrative and transaction costs (see Buijs, 1998).

A housing finance cooperative involves a group of people, who form a society to enable them to raise funds endogenously (among themselves). It serves as a specialized mutual savings bank for facilitating the purchase of a lumpy good (i.e., a house) for the members of the cooperative. The member (for whom the house is being purchased) repays the principal along with lending an additional amount (in lieu of an interest payment in a formal mortgage) to the cooperative. This simultaneous action allows members to offset the cost of borrowing with the benefit of lending, thus yielding a facility with a zero interest rate (assuming negligible administrative costs). The intuition behind the offsetting of interest rates is akin to the “barn raising” example of nineteenth century frontier farmers in the United States, as described in Besley et al. (1993). This was accomplished by forming a specialized ROSCA requiring in-kind contribution of member farmers in one region to help each other to build a barn. This specialized ROSCA was dissolved after every member in the group had a barn.

A housing finance cooperative can thus be construed as a special form of ASCRA, which is distinct from a ROSCA. This is because in this scheme of affairs one group of members do not benefit at the expense of the others. It is the cooperative which is the sole beneficiary or the benefactor (based on the net present values (NPV) of cash flows).

We integrate the above two streams of literature thereby assuming the existence of an information architecture, where property rights, foreclosure procedures (needed for real estate to serve as collateral) and accurate method of valuing property are well established (see Levine et al. 18

We assume that initial (seed) funding for a housing finance cooperative is available through either charitable sources (as stated earlier) or a mutual savings institution (such as a building society/ credit union/ mutual savings bank) or a governmental agency or a non-governmental organization (NGO) or a supranational agency like the World Bank. Once established and managed carefully, the cooperative can be self sustaining (see Buckley, 1999; Jones and Mitlin, 1999; Cizakca, 2000; and Bremer, 2004).

The current analysis ignores transaction costs, as the two mortgages are underwritten by entities, which are structured in contrasting organizational forms. A formal mortgage is typically offered by a profit-making intermediary (such as a bank or a stock-based S&L), while a cooperative mortgage is offered by a non-profit entity. Nonetheless, transaction costs in the “real world” are manifested in the above two mortgages in the form of interest rate spread and service fees respectively. The interest rate spread in a formal intermediary is priced to recompense labor/ management (in the form of salary, etc.) and capital (in the form of dividends). In contrast, the service fee in a cooperative is priced to recompense the various overhead incurred such as salary of employees, etc., while its management (comprised of committee members and board of directors) is on a voluntary basis (see Buijs, 1998).
2000; and Renaud, 2009). We initiate our study with a simple framework and extend it to study inflationary shocks, as well as prepayment options.

Our efforts yield four key results described as follows. First, we optimally price both the formal (fixed rate – prime sector) mortgage, as well as the cooperative home mortgage. The term “price” in our paper is used in a broad sense (consistent with Baltensperger, 1978) to include not only the interest rate (as in the formal mortgage) but also the loan-to-value ratio, as well as the tenure of the facility (see also Eisdorfer, 2008). Our pricing algorithm is more scientific than the ad-hoc credit rationing constraints used currently by banks. Risk control at the micro-level is important to arrest the volatility at the macro-level, in accordance with the prognosis of Sheng (1997) and Renaud (2009).

Second, we realize that the lien profile of a mortgage issued by a housing finance cooperative is linear, in contrast to the concave lien profile of a formal intermediary. This helps in reducing the tenure of the endogenous mortgage, building up an “equity cushion” faster, alleviating credit risk and the overall cost of financing.\(^{20}\)

Third, a housing finance cooperative is also able to control interest rate risk better than its formal counterpart. The endogenous use of leverage ensures that any increase in the cost of borrowing is offset by the benefit of lending. Inflationary shocks thus impact on the tenure of the cooperative facility instead of pricing out the prospective homeowner by increasing the front-end costs of owning a home (as in a formal prime sector mortgage).

Finally, we acknowledge that the gains from intertemporal trade make home financing through cooperatives pareto-superior [pareto-inferior] over its formal prime sector counterpart, depending on the characteristics of a home, that of a borrower and the underwriting standards adopted. This verifies the claim of (i) Besley et al. (1994) that efficiency of a cooperative is mixed when contrasted with formal credit markets; and (ii) Hart and Moore (1998) that a cooperative works well when it is focused on a limited scope of activities. This result does not incorporate the lower transaction costs of a cooperative stemming from its organization as a non-profit entity. Furthermore, we do not even incorporate (i) the relatively low default costs (stemming from the mitigation of adverse selection and moral hazard) in contrast to the high ones (stemming from subprime borrowers) (see Buijs, 1998; and Smets, 2000); or (ii) the prepayment advantage of a cooperative mortgage. If we were to do so, the results would overwhelmingly tilt in its favor. Nonetheless, it is a responsibility of a cooperative manager to structure its portfolio by catering to the disadvantaged (with low income), aspiring to purchase a home (with low initial value and

\(^{20}\) The reduction in the tenure partially alleviates a problem with cooperative financing, i.e., illiquidity.
medium to high risk) and using the following underwriting constraints: medium income multiplier and high confidence level. This would suffice in internally generating a capital surplus critical for sustaining its growth and ultimately improving the status of the underprivileged.

We thus conclude that an adaptation of an “informal” financial system of an ASCRA (blending philanthropy with social service), which is focused in a narrowly defined activity of funding homes (by mitigating adverse selection, moral hazard, administrative costs and transaction costs) can play a vital role in economic development by providing access to financial services to those shunned by the formal system. This result is consistent with Emmons and Mueller (1997), Buijs (1998), Hart and Moore (1998) and Smets (2000). It also supports the following assertion of Callier (1990):

*Informal finance persists and often flourishes because it resolves important problems that are handled poorly or not at all by most formal financial systems in developing countries.*

(Callier, 1990, p. 273)

The paper is organized as follows. Sections 2 and 3 illustrate the theoretical underpinnings behind the design of both formal and cooperative mortgages to contrast their efficiencies. Finally, Section 4 presents our concluding remarks.

2. Formal (Fixed Rate – Prime Sector) Mortgages

2.1. Model Development:

This section expounds on the mathematical design of an efficient risk-free formal mortgage (using exogenous leverage). Here, the financier has to ensure that the borrower does not transfer the asset (home) risk to him/her. This is a formidable task, as it entails controlling the conflict of interest (agency issue) between the borrower and the lender. This risk reduction is accomplished by collateralizing the loan, not only with the underlying asset but also with the income of the borrower.

The basic assumptions underlying an efficient nearly risk-free mortgage are as follows:

(i) The prospective homeowner (borrower) makes an initial deposit (ID) against purchase of a home (valued initially at \( P_0 \)) financed by a lender at the net amount (\( Q_0 \)). The terms of financing are as follows: cost of funds = \( r \), discount rate = \( \gamma \), and tenure of formal mortgage = \( T \). This implies:
\[ Q_0 = P_0 - ID \quad (1) \]
\[ r = \left( \frac{1 - \gamma}{\gamma} \right) \quad (2) \]

**FIGURE 1**
Payments over time

(ii) The borrower makes regular payments of an amount \( A \) from time \( t = 1 \) to tenure = \( T \) (see Figure 1) from an income stream, which is stable.\(^{22}\) This implies that the discounted value of it sums up to the amount financed \( (Q_0) \):

\[ Q_0 = \sum_{i=1}^{T} A \gamma^i = A \gamma \sum_{i=1}^{T} \gamma^{i-1} = A \gamma \left( \frac{1 - \gamma^T}{1 - \gamma} \right) \quad (3) \]

Furthermore, the amount owed to the lender \( (Q_t) \) at any time \( t \geq 0 \) is evaluated as the compounded value of initial loan \( (Q_0) \) reduced by the future value of the annuity (composed of the regular mortgage payments). That is,

\[ Q_t = Q_0 (1+r)^t - \sum_{i=1}^{t} A (1+r)^{t-i} \]

Substituting the value of \( r \) from Equation (2) and simplifying the above equation, we derive:

\[^{21}\text{We assume that the homeowners have ample funds to meet the initial downpayment (ID). If this assumption is violated, then ID has to be accumulated by making } (T_1+1) \text{ periodic payments of } A' \text{ in a personal account from time } (-T_1) \text{ to } 0. \text{ This is priced using the property of convergence of geometric series as follows:} \]

\[ ID = \sum_{i=0}^{T_1} A' (1+r)^i = \left( \frac{A'}{r} \right) [(1+r)^{(T_1+1)} - 1] \quad (1a) \]

\[ \Rightarrow A' = \frac{(ID \times r)}{[(1+r)^{(T_1+1)} - 1]} \quad (1b) \]

To contrast the two mortgage schemes, we select \( T_1 \) to be similar to that of the Cooperative Home Mortgage discussed in the next section (See Figure 3). This helps us contrast the tenure of “(T)” of the formal mortgage with “(T_2)” of the Cooperative alternative.

\[^{22}\text{In other words, we implicitly assume that the borrower's income is non-stochastic. We plan to extend our current framework to the case of stochastic income in the future.} \]

\[^{23}\text{The final version of the formula also uses the property of convergence of geometric series (see Hoy et al., 1996).} \]
Substituting the value of $A$ from Equation (3), we derive:

$$Q_t = Q_0 \left( \frac{1}{\gamma} \right)^t - \sum_{i=1}^t A \left( \frac{1}{\gamma} \right)^{t-i} = \left( \frac{Q_0}{\gamma} \right) - \left( \frac{A}{\gamma} \right) \left[ \frac{1 - \frac{1}{\gamma^t}}{1 - \frac{1}{\gamma}} \right]$$

Figure 2 portrays Equation (3a) to illustrate that the shape of the outstanding loan amount ($Q_t$) with respect to time is concave.

**FIGURE 2**
Asset/ Lien Value over time

**Formal Mortgage**

(iii) We assume that the home prices follow a Geometric Brownian Motion (also known as a Lognormal Random Walk). This assumption is consistent with the finance and the real estate finance literature (see Fama, 1970; Gau, 1987; Fama, 1991; and Szymanoski, 1994), and implies that the percentage changes in the prices of property are independent and identically distributed (iid). This implies that the future price changes are independent of past price movements and dependent only on the current price. This assumption requires all

Equation (3a) is akin to the standard annuity pricing formula derived in Hayre et al. (1995).
past information to be captured by the present price of a home, which is a consequence of the Efficient Market Hypothesis (EMH).\textsuperscript{25} This assumption helps us model the property price at time $t$ (given in months) as follows:

$$P_t = P_0 e^{\left(\mu - \frac{\sigma^2}{2}\right) t},$$  

where $\mu$ and $\sigma$ (described below) are the mean and standard deviation of the monthly appreciation of the property (see Figure 2).

We now move to the intricate design of the formal mortgage by evaluating the endogenous parameters ($Q_0$, ID, T) given the exogenous parameters stemming from the home characteristics (comprising of initial value $- P_0$, monthly appreciation $- \mu$, monthly risk $- \sigma$), the borrower characteristics (composed of his/her income $- y$) and the underwriting constraints (Confidence Level $- x\%$ -- explained below, Income multiplier $- b$ -- explained below).\textsuperscript{26}

(i) **Asset Value Constraint:** The financier ascertains that the outstanding loan ($Q_t$) is (at least) fully collateralized by the underlying home value at an $x\%$ confidence level. That is,

$$\text{Max. } \text{Probability} \left( \ln(P_t) - \ln(Q_t) \right) \geq \frac{x}{100} \quad \text{(in } t)$$

The above condition yields a safety margin, which is a multiple “$\alpha$” of the risk ($\sigma \sqrt{t}$); and helps us evaluate the minimum initial deposit ($ID_{\text{Min}}$) or maximum amount of loan ($Q_{0\text{Max}}$) made by the lender as follows.

$$\text{Max. } \frac{\ln(P_t) - \ln(Q_t)}{\sigma \sqrt{t}} \geq \alpha \quad \text{(in } t)$$

$$\Rightarrow \text{Max. } \left[ \ln(P_t) - \ln(Q_t) \right] \geq \alpha \sigma \sqrt{t} \quad \text{(5a)}$$

\textsuperscript{25} Note that the assumption of home prices following a Lognormal Random Walk helps us establish a simple framework to conduct our study. This framework can easily be extended to other house price distributions for further investigation. The rationale for employing a simple framework is to demonstrate a corner solution (to our loan pricing Equation (5b) in Sections 2 and 3), which probably leads to financial institutions using ad-hoc underwriting standards, as reiterated in Jaffee and Stiglitz (1990).

\textsuperscript{26} These underwriting constraints are interrelated, as demonstrated further in Equation (7c).

\textsuperscript{27} Equation (5) implicitly assumes that the time to repossess the property upon default is zero. However, if it takes “$L$” months to repossess it after default, then Equations (5), (5a)–(5g), (7) and (7a) have to be modified as follows: The time periods $t$ and $t^*$ in the suffix of property value ($P_t$ or $P_{t^*}$) have to be changed to $P_{t+L}$ or $P_{t^*+L}$. The same applies to Equations (5), (5a)–(5c), (11), (11a)–(11e) and (12c)–(12d) in Section 3.1.
This implies that at the optimum time $t^*$ ($t^* \in (0, T]$):

$$
\frac{1}{P_{t^*}} \left( \frac{\delta P_{t^*}}{\delta t^*} \right) - \frac{1}{Q_{t^*}} \left( \frac{\delta Q_{t^*}}{\delta t^*} \right) \geq \frac{\alpha \sigma}{2 \sqrt{t}} = \left( \frac{\alpha}{2} \right) \left( \frac{\sigma \sqrt{t}}{t} \right),
$$

and

$$
Q_{t^*} \leq P_{t^*} \left( e^{-\alpha \sigma \sqrt{t^*}} \right).
$$

The equality sign in Equation (5b) is observed in the case of interior solution, while the inequality sign is observed for a corner solution. The first and second terms on the left hand side (LHS) of this equation comprise of the rate of change in the value of the home minus that of the mortgage. The combination of the two terms estimates the rate of change of the equity in a home. Equation (5b) basically implies that for an optimum, the combined LHS terms should be greater than or equal to $\left( \frac{\alpha}{2} \right)$ times the rate of increase in risk of a home ($\frac{\sigma \sqrt{t}}{t}$).

For a formal home (with legal title), the first term on the LHS (of Equation (5b)) is positive, implying an appreciation over time. In contrast, for an amortizing mortgage, the second term is negative, implying depreciation in lien value over time. The combination of an appreciation term minus a depreciation term is nonetheless positive. If the mortgage is structured properly (encumbering an appreciating formal home), then one can ensure that the appreciation of equity over time is greater than or equal to $\left( \frac{\alpha}{2} \right)$ times the rate of change of risk of a home ($\frac{\sigma \sqrt{t}}{t}$) in the time interval $(0, T]$. Thus, Equation (5b) yields a non-binding inequality with a trivial solution $t^* > 0$ ($\forall \left( \alpha \sigma \right) > \mu > \sigma^2$). Simplifying Equation (5c) for $t^* = \varepsilon > 0$, we get:

$$
Q_{\varepsilon} \leq P_{\varepsilon} \left( e^{-\alpha \sigma \sqrt{\varepsilon}} \right).
$$

For an $\varepsilon$, which is slightly greater than zero, $Q_{\varepsilon} \approx Q_0$, $P_{\varepsilon} \approx P_0$; and $e^{-\alpha \sigma \sqrt{\varepsilon}} \approx 1$.

This yields $Q_0 \leq P_0$, thereby implying an LTV of 100%.

---

28 The second order condition for a maximum is automatically satisfied here, as Chiang (1984) illustrates that the maximization of a strictly concave and twice differentiable function (such as a natural logarithmic function) with a linear constraint (given by the income one described below) yields a negative number.

29 In case of a progressive home, the first term on the LHS of Equation 5(b) may not necessarily be positive. This is because in contrast to a formal home, a progressive home serves primarily as a consumption good with questionable investment value.

30 Note that Equations (5b) and (5c) hold true during normal times, but may not hold during periods of “bubbles” defined by Kindleberger (1978, p. 16) as “an upward price movement (of an asset) over an extended range that then implodes.” The bursting of bubbles has serious implications for the macroeconomy, as described in Footnote 1. Financial intermediaries have to proactively reconfigure their facilities during runaway asset prices when there is a significant risk in deflation of the home price bubble. This challenging issue needs to be investigated in the future.
The difficulty posed in the solution of Equations (5b) and (5c) leads to the financial intermediaries resorting to the use of ad-hoc LTV, as cited in Jaffee and Stiglitz (1990). In real estate booms, this can lead to a deterioration of underwriting standards where LTV hovers close to 100%, as cited in Wachter (2008). This solution poses a serious problem in the context of Shiller and Weiss (2000), as it exacerbates moral hazard on the part of a home-ownner. This is because it involves a zero initial deposit (ID), which serves as a “free” call option on the value of a home. A lack of initial capital at risk (also termed as “no skin in the game”) does not motivate a home-ownner to adequately maintain the property, as he/she has nothing to lose. It also makes it easier to walk away from the home in the event of declining home prices. Thus, to mitigate this issue of moral hazard, we curtail the LTV (i.e. \( Q_0 \)) to a maximum, which is a simple function of safety margin (such as \( e^{-\alpha \sigma} \)). This yields:

\[
Q_0 \leq P_0 \left( e^{-\alpha \sigma} \right) \tag{5d}
\]

\[
\Rightarrow (Q_0)_{\text{Max}} = P_0 \left( e^{-\alpha \sigma} \right) \tag{5e}
\]

Since ID = \( P_0 - Q_0 \), see Equation (1)

\[
\Rightarrow \text{ID} \geq P_0 [1 - e^{-\alpha \sigma}] \tag{5f}
\]

\[
\Rightarrow \text{ID}_{\text{Min}} = P_0 [1 - e^{-\alpha \sigma}] \tag{5g}
\]

(ii) Income Constraint: The lender ascertains that the borrower has adequate income (y) to meet his/her mortgage commitments comprising of the monthly payments (A). This implies curtailing the commitments such that they are a multiple (b) of income of the borrower. That is,

\[
\frac{\text{Income of Borrower (y)}}{\text{Mortgage Commitments (A)}} \geq \text{Income Multiplier (b)} \tag{6}
\]

\[
\Rightarrow \frac{y}{A} \geq b \tag{6a}
\]

\[
\Rightarrow A \leq \frac{y}{b} \tag{6b}
\]

\[
\Rightarrow (A)_{\text{Max}} = \frac{y}{b} \tag{6c}
\]

We can thus evaluate the optimal tenure (T) of the mortgage using Equation (3) as follows:

\[
(Q_0)_{\text{Max}} = \gamma (A)_{\text{Max}} \left( \frac{1 - \gamma T}{1 - \gamma} \right) \tag{3}
\]

31 This factor of \( e^{-\alpha \sigma} \) is derived by maximizing Equations (5) or (5a) but without the \( \sqrt{t} \) term.
Substituting for the values of \( (Q_0)_{\text{Max}} \) and \( (A)_{\text{Max}} \) from Equations (5e) and (6c), we derive:

\[
P_0 e^{-\alpha \sigma} = \left( \frac{\gamma y (1 - \gamma)^T}{b(1 - \gamma)} \right) \leq \left( \frac{\gamma y (1 - \gamma_{T_{\text{Max}}})}{b(1 - \gamma)} \right), \forall T \leq T_{\text{Max}}
\]

\[
\Rightarrow T \leq T_{\text{Max}} = \left\{ \frac{\ln[1 - P_0 (e^{-\alpha \sigma}) b(1 - \gamma)]}{\ln[\gamma]} \right\}
\]

(7a)

For the tenure “\( T \)” to be a positive real number, the term in the square brackets “[.]”(in the numerator) comprising of the natural logarithmic function has to be positive number between 0 and 1, yielding a negative number in the numerator. The reason behind our assertion stems from the fact that the denominator \( \ln[\gamma] < 0 \), as \( 0 < \gamma < 1 \) (see Equation (2)). This leads us to the following constraint for the numerator:

\[
\Rightarrow 0 < \left[ 1 - \frac{P_0 (e^{-\alpha \sigma}) b(1 - \gamma)}{\gamma y} \right] < 1.
\]

In other words,

\[
0 < \left\{ 1 - \left[ \frac{\gamma y}{P_0} \right] r \right\} < 1 \quad \text{using Equation 2}.
\]

(7b)

This yields the inter relationship between the two underwriting constraints after using Equations (2) and (5e):

\[
\Rightarrow 0 < b < \left[ \frac{\gamma y}{P_0} \right] (e^{\alpha \sigma}) = \frac{\gamma y}{r (Q_0)_{\text{Max}}}
\]

(7c)

Thus, the formal mortgage can be priced in terms of \( [Q_0, \text{ID, A, and T}] \) given the exogenous parameters \( [P_0, \mu, \sigma, \alpha, y, b, \text{and } \gamma] \) (and the satisfaction of Equation (7c)) as follows:

First, we evaluate \( (Q_0)_{\text{Max}} \) using Equation (5e).

Next, we evaluate \( A_{\text{Max}} \) using Equation (6c).

Next, we evaluate \( T \) using Equation (7a), rounding it off to the nearest integer.

Finally, we reevaluate \( Q_0 \) and ID using Equations (3) and (1) respectively.

2.2. Illustrative Examples.

We resort to numerical examples to elucidate the model further and to contrast the efficiency of the formal mortgage with that of its counterpart (i.e., the cooperative home mortgage), described in the following section.

---

32 Note also that \( r \) cannot be zero in this framework as \( r \to 0 \Rightarrow \gamma \to 1 \Rightarrow \ln[\gamma] \to 0 \Rightarrow T \to \infty \).
Example 1 [Formal Mortgage pareto-inferior to a Cooperative Home Mortgage]:

The exogenous values of a house are incorporated from Cannon et al. (2006) as follows: $\mu = 5.695\%/\text{year} = 5.695\% \times \frac{12}{12} = 0.4746\%/\text{month}$, $\sigma = 14.845\%/\text{year} = 14.845\% \times \sqrt{12} = 4.2854\%/\text{month}$. We assume that the lender wants to ensure risk-free loan status at 99.9999% confidence level, i.e. $x = 99.9999\%$. This implies $\alpha = 4.7537$. We further assume that $P_0 = $100,000, income of borrower $= y = $30,000/ year = $ 2500/ month, income multiplier $= b = 3.3333$, and $r = 5\%$ annual = 0.42% monthly.

We first solve for $(Q_0)_{\text{Max}}$, $A_{\text{Max}}$, and $T$ using Equations (5e), (6c) and (7a) as follows: $(Q_0)_{\text{Max}} = 81,569.64$, $A_{\text{Max}} = 750/\text{month}$, and $T = 145.1674$ months.

Rounding off $T$ to 145 months (i.e., 12 years, 1 month) and using $A_{\text{Max}} = 750/\text{month}$ yields $Q_0 = 81,501.10$ (from Equation (3)) and $ID = 18,498.90$ (from Equation 1).

Note the value of $T = 146$ months is not feasible, as it violates the upper bound of $Q_0$ given by Equation (5d).

Example 2 [Formal Mortgage pareto-superior to a Cooperative Home Mortgage]:

Here we retain the same exogenous parameters as in the above Example 1 with the exception of annual income of prospective borrower $= y$, which is selected as $20,000/\text{year} = 1666.67/\text{month}$. This yields $(Q_0)_{\text{Max}} = 81,569.64$, $ID = 18,430.36$, $A_{\text{Max}} = 500/\text{month}$, and $T = 273.843$ months (from Equations (5e), (6c) and (7a)). Rounding off $T$ to 273 months (22 years, 9 months) and using $A_{\text{Max}} = 500/\text{month}$ yields $Q_0 = 81,434.62$ (from Equation (3)).

2.3. Extension of the Model to the Case of Inflation.

Inflation is defined as a sustained increase in the price levels of goods and services in the economy. Its causes are highly disputed in the literature, stemming from the complex and dynamic

---

33 Our model mandates a high degree of precision to make the mortgage nearly risk-free. This also impacts on the comparative efficiency of a formal mortgage vis-à-vis a cooperative mortgage, as discussed in the following section.

34 If the homeowner has insufficient downpayment (of $18,498.90), then he/she can accumulate this in either 25 months (from $T_1 = -24$ months to $t = 0$) at $703.62/\text{month}$ (from Equation (1b)) or 31 months (from $T_1 = -30$ months to $t = 0$) at $560.27/\text{month}$ (from Equation (1b)). The two values of $T_1$ stem from the corresponding solutions of the Cooperative Mortgage in the following section.

35 We realize similar results when we: (i) increase the purchase price of a home ($P_0$) or income multiplier ($b$); or (ii) decrease the confidence level ($x\%$) or risk ($\sigma$). It should be noted (in the context of efficiency) that the underwriting constraints in the form of confidence level and income multiplier contradict each other.

36 Here too, the downpayment (of $18,430.36$) can be accumulated in either 38 months (from $T_1 = -37$ months to $t = 0$) at $448.63/\text{month}$ (from Equation (1b)) or 73 months (from $T_1 = -72$ months to $t = 0$) at $216.54/\text{month}$ (from Equation (1b)). The two values of $T_1$ stem from the corresponding solutions of the Cooperative Mortgage in the following section.
interactions of four groups of factors: the monetary (demand-side) shocks, the real (supply-side) shocks, the inertial (price adjustment) factors, and the institutional (political process) factors (see Kibritcioglu, 2002). Inflationary expectations affect the exogenous interest rate (and the discount factor – see Equation (2)) through the Fisher Effect. However, the performance of homes during inflationary periods depends on the simultaneous impact of changes in housing stock through depreciation, obsolescence and development, as demonstrated by Ebrahim and Mathur (2007) by employing numerical simulations. This is elaborated as follows. If housing stock increases [decreases] during inflationary periods, a house may underhedge [overhedge] inflationary impact. This changes the appreciation (µ) as well as the risk of it (σ).

Thus, we too evaluate the overall impact of an inflationary shock, using numerical simulation by extending the Examples 1 and 2 (described above in Section 2.2). We realize that it results in: (i) a marginal change in the down payment (see Equations (5e) and (5f)); and (ii) a substantial increase in the mortgage payment, which is restricted by the income constraint (see Equation (6c)). This binding income constraint leads to either (i) an inordinate tenure of mortgage (T) or (ii) a lack of solution (stemming from the violation of Equation (7c)), implying that the prospective homeowner is priced out of the property market.

2.4. Extension of the Model to the Case of Prepayment.

The prepayment provision in a formal fixed-rate mortgage constitutes of a call option, which allows the homeowner to revoke his/her debt at any time (prior to maturity) for an amount (known in advance, which is not affected by interest rates). The real estate finance literature illustrates various ways of pricing this option either as a higher interest rate or a higher mortgage initiation fee (see Hall, 1985).

Our numerical simulations (extending the above Examples 1 and 2 in Section 2.2) illustrate that when the prepayment option is priced in the form of a higher interest rate, it results in: (i) a negligible change in the down payment (ID); (ii) an increase in the mortgage payment (A); and (iii) an increase in the tenure (T).

In contrast, our simulation illustrates that when the prepayment option is priced in the form of a higher initiation fee, it results in: (i) a substantial increase in the down payment (ID); and (ii) negligible changes in the mortgage payments (A) and the tenure (T).

Nonetheless, both the above methods of pricing prepayments reduce the efficiency of formal mortgages vis-à-vis Cooperative Home Mortgages, as described in Section 3.4.
3. Cooperative Home Mortgages

3.1. Model Development:

This section expounds on the mathematical design of an efficient risk-free home mortgage financed by a cooperative (using endogenous leverage). That is, a mortgage where a borrower receives interest-free funding from the cooperative members at the time of purchase of property and repays the cooperative by funding other members at the same zero rate. Here too, the cooperative ensures that its members do not transfer the asset risk to it. This entails controlling for the conflict of interest (agency issue) between the borrower and the lender by collateralizing the loan, not only with the underlying asset but also with the income of the borrower.

The basic assumptions underlying an efficient (nearly) risk-free cooperative financing are as follows.

(i) On joining the association, the prospective homeowner (borrower) is required to accumulate an amount $S$ by making periodic payments from time $t = -T_1$ (prior to purchase of the house) until $t = T_2$. Here $T_2$ is the “real” tenure of the mortgage as explained below. These periodic payments (constituting Cooperative Dues) equal $(S/T_{1+T_2+1})$, and are used to finance properties of fellow cooperative members (see Figure 3).

![FIGURE 3 Payments over time](image)

**Note:** Cooperative Dues = $CD = (S/T_{1+T_2+1})$, Regular Principal Payment = $RPP = (P_0/T_3)$

37 The amount $S(T_{1+T_2+1})$ is akin to an initial deposit (ID) discussed in the case of formal mortgage (Section 2.1). See also Equation (9a).

38 Note that we use the term “Cooperative Dues” in an unconventional way to denote mandatory savings (accruing at zero interest) which are credited to the principal amount at the real tenure of the mortgage ($T_2$) as explained below. Any regular service fees can be added to these cooperative dues. See also Footnote 45.

39 Cooperative Dues (CDs) paid prior to assuming a mortgage (i.e., from $t = -T_1$ to 0) help in disseminating more information on the prospective borrower and to bind him/her cohesively to the cooperative (Buijs, 1998).
(ii) At $t = 0$, the cooperative buys a home valued at $P_0$ for the borrower and asks him/her to make additional principal payments of $\left( \frac{P_0}{T_3} \right)$ from $t = 1$ to $t = T_3$, where $T_3$ is the “notional” tenure of the mortgage. Thus, at time $t$, the gross amount owed to the cooperative equals $P_0(1 - \frac{t}{T_3})$.

We further assume that at time $t = T_2$, the capital accumulated by the borrower to help finance fellow cooperative members ($S$) offsets the liability the borrower owes to the association. Thus,

$$S = P_0(1 - \frac{T_2}{T_3})$$

(8)

In other words, the “real” tenure of the mortgage ($T_2$) is lower than the “notional” tenure $T_3$.

This helps to define the net amount owed ($Q_t$) to the association at time $t \geq 0$ as follows.

$$Q_t = P_0 - \sum_{i=1}^{t} \left( \frac{P_0}{T_3} \right) - \sum_{i=T_1}^{T_2} \left( \frac{S}{T_1+T_2+1} \right), \; \forall \; t \in [0, T_2]$$

$$\Rightarrow Q_t = P_0(1 - \frac{t}{T_3}) - S \left( \frac{T_1+t+1}{T_1+T_2+1} \right)$$

(9)

This implies:

$$Q_0 = P_0 - S \left( \frac{T_1+1}{T_1+T_2+1} \right)$$

(9a)

$$Q_{T_2} = P_0(1 - \frac{T_2}{T_3}) - S = 0 \text{ (using Equation (8)).}$$

(9b)

$$\Rightarrow S = P_0(1 - \frac{T_2}{T_3}) \Rightarrow \left( \frac{S}{P_0} \right) = (1 - \frac{T_2}{T_3})$$

(9c)

$$\Rightarrow \left( \frac{1}{T_3} \right) = \frac{1}{T_2} \left( 1 - \frac{S}{P_0} \right)$$

(9d)

Since $T_2 < T_3$ as assumed above $\Rightarrow S < P_0$.

(9e)

Substituting the value of $T_3$ from Equation (9d) in Equation (9) yields:

$$Q_t = P_0(1 - \frac{t}{T_2}) + S \left[ (\frac{t}{T_2}) - (\frac{T_1+t+1}{T_1+T_2+1}) \right]$$

(10)

Figure 4 portrays Equation (10) to illustrate that the shape of the outstanding loan amount ($Q_t$) is linear with respect to time $t$. This linear profile of $Q_t$ is very important, as it helps the
borrowers to build up equity in their homes at a faster pace than a formal mortgage with a concave profile (see also Figure 2). This feature of a cooperative mortgage is important to the lender (the cooperative) too, as a faster build up of an “equity cushion” avoids default better than that in the case of a formal mortgage.

**FIGURE 4**
Asset/ Lien Value over time
Cooperative Home Mortgage

(iii) Here too, we assume that the home prices follow a Geometric Brownian Motion to model the monthly property price at time $t$ as follows:

$$P_t = P_0 e^{(\mu - \frac{\sigma^2}{2})t}$$  \hspace{1cm} (4)

where $\mu$ and $\sigma$ are the mean and standard deviation of the monthly appreciation of the property (see Figure 4).

Finally, we move to the intricate design of the cooperative home mortgage by evaluating the endogenous parameters ($Q_0$, $S$, $T_1$, $T_2$, $T_3$) given the exogenous parameters stemming from the home characteristics (comprising of initial value $- P_0$, monthly appreciation $- \mu$, monthly risk $- \sigma$), the borrower characteristics (composed of his/her income $- y$) and the underwriting constraints (Confidence Level $- x\%$, Income multiplier $- b$).

(i) **Asset Value Constraint:** Here again, the financier ascertains that the outstanding loan ($Q_t$) is
(at least) fully collateralized by the underlying home value at an x% confidence level. That is,

\[
\text{Max. } \frac{\text{Probability} \left( \ln(P_t) - \ln(Q_t) \right)}{\sigma \sqrt{t}} \geq \frac{x}{100} \quad (in \ t)
\]

The above condition yields a safety margin, which is a multiple “a” of the risk \( \sigma \sqrt{t} \) and helps us evaluate the Cooperative Dues \( CD = \left( \frac{S}{T_1 + T_2 + T_3} \right) \) in conjunction with the time of joining the Cooperative \((-T_1)\).

\[
\text{Max. } \frac{\ln(P_t) - \ln(Q_t)}{\sigma \sqrt{t}} \geq a \quad (5)
\]

\[
\Rightarrow \text{Max. } [\ln(P_t) - \ln(Q_t)] \geq a \sigma \sqrt{t} \quad (5a)
\]

This implies that at the optimum time \( t^* \) \((t^* \in (0, T))\):

\[
\frac{1}{P_{t^*}} \frac{\delta P_{t^*}}{\delta t^*} - \frac{1}{Q_{t^*}} \frac{\delta Q_{t^*}}{\delta t^*} \geq \frac{a \sigma}{2 \sqrt{t}} = \left( \frac{a}{2} \right) \left( \frac{\sigma \sqrt{t}}{t} \right), \text{ and}
\]

\[
Q_{t^*} \leq P_{t^*} \left( e^{-a \sigma \sqrt{t^*}} \right) \quad (5c)
\]

Here too, the first and second terms of the LHS of Equation (5b) represent the rate of change of the value of a home minus that of a mortgage. The combination again estimates the rate of change of equity. Equation (5b) also implies that for an optimum, the combined LHS terms be greater than or equal to the product of \( (\frac{a}{2}) \) and the rate of increase in the risk of a home \( (\frac{\sigma \sqrt{t}}{t}) \).

For an appreciating formal home, the first term on the LHS is positive, while the second term (for an amortizing mortgage) is negative. The combination of both LHS terms is positive, as it represents a positive term minus a negative term. This again yields a non-binding inequality with a trivial solution \( t^* > 0 \) \((\forall (a \sigma) \gg \mu > > \sigma^2)\). This also results in an LTV of 100%, thereby aggravating the moral hazard on the part of homeowners, as they have no capital at risk. Thus, to alleviate this moral hazard, we restrict the LTV ratio to an equivalent function of safety margin \( (e^{-a \sigma}) \). This yields:

\[
Q_0 \leq P_0 (e^{-a \sigma}) \quad (11)
\]

Substituting the value of \( Q_0 \) from Equation (9a) in Equation (11), we derive:

---

40 These conditions hold true during normal times. However, when formal home prices constitute an asset bubble, then Equations (5b) and (5c) may not hold true.
\[ S \left( \frac{T_1+1}{T_1+T_2+1} \right) \geq P_0 (1 - e^{-\alpha \sigma}) \]  

\[ \Rightarrow \left( \frac{S}{P_0} \right) \geq \left( \frac{T_1+T_2+1}{T_1+1} \right) (1 - e^{-\alpha \sigma}) \]  

\[ \Rightarrow \left( \frac{S_{\text{Min}}}{P_0} \right) = (1 - e^{-\alpha \sigma}) \left( \frac{T_1+T_2+1}{T_1+1} \right) \]  

Equation (9e) states that \( S_{\text{Min}} < P_0 \). This implies that \( \left( \frac{S_{\text{Min}}}{P_0} \right) < 1 \). Substituting this in Equation (11c), we derive:

\[ (1 - e^{-\alpha \sigma}) \left( \frac{T_1+T_2+1}{T_1+1} \right) < 1 \]

\[ \Rightarrow T_1 > T_2 (e^{\alpha \sigma} - 1) - 1 \]  

Finally, substituting the value of \( \left( \frac{S_{\text{Min}}}{P_0} \right) \) from Equation (11c) in Equation (9d), we derive:

\[ \left( \frac{1}{T_3} \right) = \frac{1}{T_2} (1 - \left( \frac{S_{\text{Min}}}{P_0} \right)) = \left[ \frac{e^{-\alpha \sigma}}{T_2} - \frac{(1 - e^{-\alpha \sigma})}{(1+T_1)} \right] \]  

(ii) Income Constraint: The cooperative ascertains that the borrower has an adequate income (y) to meet his/her mortgage commitments comprising of both the Cooperative Dues as well as the Regular Principal Payments \( \frac{S}{T_1+T_2+1} \) and \( \frac{P_0}{T_3} \). This implies curtailing the commitments such that they are a multiple (b) of income of the borrower. That is,

\[ \frac{\text{Income of Borrower (y)}}{\text{Mortgage Commitments (CD+RPP)}} \geq \text{Income Multiplier (b)} \]  

\[ \Rightarrow \left\{ \frac{y}{S} \left( \frac{1}{T_1+T_2+1} + \frac{P_0}{T_3} \right) \right\} \geq b \]  

\[ \Rightarrow y \geq b \left( \frac{S}{T_1+T_2+1} + \frac{P_0}{T_3} \right) \]  

Substituting the values of \( \left( \frac{S_{\text{Min}}}{T_1+T_2+1} \right) \) and \( \left( \frac{1}{T_3} \right) \) from Equations (11c) and (11e), we derive:

\[ y \geq \frac{b P_0}{T_2} (e^{-\alpha \sigma}) \]
\[ T_2 \geq \frac{b \cdot P_0}{y} \left( e^{-\alpha \sigma} \right) \] (12d)

(iii) Efficiency Constraint: Finally, the cooperative ascertains that the discounted value of all payoffs should be greater than or equal to the net amount financed without interest \((Q_0)\). That is,

\[ \sum_{i=T_1}^{T_2} \left( \frac{S}{T_1+T_2+1} \right) (y^i) + \sum_{i=T_1}^{T_3} \left( \frac{P_0}{T_3} \right) (y^i) \geq Q_0 \] (13)

Since the summation signs (in the above equation) involve terms in geometric progression, they can be simplified further as follows:

\[ \left( \frac{S}{T_1+T_2+1} \right) (y^{-T_1}) \left[ \frac{1 - y^{T_1+T_2+1}}{1 - y} \right] + \left( \frac{P_0}{T_3} \right) \left[ \frac{1 - y^{T_3}}{1 - y} \right] \geq Q_0 \] (13a)

It should be noted that Equations (13) and (13a) (with equality signs) are akin to Equation (3) in the formal mortgage. That is, when Equations (13) and (13a) are satisfied with an equality sign, then a cooperative mortgage is pareto-neutral to its formal counterpart. However, when they are satisfied with an inequality sign, then a cooperative mortgage is pareto-superior to a formal mortgage. This implies that the cooperative is able to generate surplus capital to sustain its growth. In contrast, if they are violated, it implies that the mortgage is pareto-inferior and that the cooperative's capital base is being depleted. This efficiency of a cooperative is contingent on the characteristics of a home, the borrower and the underwriting standards adopted (as elaborated below).

The above Equations (13) and (13a) also distinguish the home financing cooperative from a ROSCA, as one group of members do not benefit at the expense of the others. It is the cooperative, which either benefits or confers benefits to others based on the NPV of cash flows.

Thus, the model can be solved for \([T_1, T_2, T_3, S_{\text{Min}}, \text{and } Q_0]\) given \([P_0, \mu, \sigma, \alpha, y, b, \text{and } \gamma]\) as follows:

First, we evaluate \(T_2\) using Equation (12d), rounding it off to the nearest integer.

Next, we evaluate \(T_1\) using Equation (11d), rounding it off to the nearest integer.

Next, we evaluate \(T_3\) using Equation (11e), rounding it off to the nearest integer.

Next, we evaluate \(S_{\text{Min}}\) using Equation (11c).

Next, we evaluate \(Q_0\) using Equation (9a).
Finally, we verify the above endogenous values by ascertaining that Equation (13a) holds true.

3.2. Illustrative Examples.

Example 1 [Cooperative Home Mortgage is *pareto-superior* to a Formal Mortgage]:

We retain the same exogenous parameters selected in Example 1 of the formal mortgage (Section 2.2). That is, \( P_0 = $100,000 \), \( \mu = 5.695\% / \text{year} = 0.4746\% / \text{month} \), \( \sigma = 14.845\% / \text{year} = 4.2854\% / \text{month} \), \( x = 99.9999\% \), \( \alpha = 4.75367 \), \( y = $ 2500/ \text{month} \), \( \text{Income Multiple (b)} = 3.3333 \), and \( r = 5\% / \text{year} = 0.42\% / \text{month} \).

Since the solutions for \( T_1 \) and \( T_2 \) involve inequalities, i.e., \( T_1 > 23.628 \) months (Equation 11d) and \( T_2 \geq 108.7595 \) months (Equation 12d), a unique mortgage solution is infeasible. We therefore depict two solutions to illustrate our point.

Solution (i):
Here we select, \( T_1 = 24 \) months (i.e., 2 years) and \( T_2 = 109 \) months (i.e., 9 years, 1 month). This implies \( T_3 = 8984 \) months (from Equation 11e), \( S_{\text{Min}} = $98,786.73 \) (from Equation 11c), \( Q_0 = $81,569.64 \) (from Equation 9a), Cooperative Dues (CD) = $737.21/ month (from Figure 3), Regular Principal Payment (RPP) = $11.13/ month (from Figure 3), Total monthly payments (CD+RPP) = $748.35, Discounted value of all payoffs = $84,833.54 > \( Q_0 = $81,569.64 \). This validates the efficiency constraint, i.e., Equation (13a).

Solution (ii):
Here we select, \( T_1 = 30 \) months (i.e., 2 years, 6 months) and \( T_2 = 109 \) months (i.e., 9 years, 1 month). This implies \( T_3 = 651 \) months (from Equation 11e), \( S_{\text{Min}} = $83,233.88 \) (from Equation 11c), \( Q_0 = $81,569.64 \) (from Equation 9a), Cooperative Dues (CD) = $594.53/ month (from Figure 3), Regular Principal Payment (RPP) = $153.61/ month (from Figure 3), Total monthly payments (CD+RPP) = $748.14, Discounted value of all payoffs = $85,063.41 > \( Q_0 = $81,569.64 \). This too validates the above efficiency constraint.

Example 2 [Cooperative Home Mortgage is *pareto-inferior* to a Formal Mortgage]:

This example illustrates the contrary case where the cooperative's capital base is depleted. That is, the cooperative mortgage is *pareto-inferior*. Here too, we retain the same exogenous parameters selected in the above Example 1 with the exception of an annual income of prospective homeowner (y), which is selected as $20,000/ year = $1666.67/ month. This yields \( T_1 > 36.06 \) months (from Equation 11d) and \( T_2 \geq 163.14 \) months (from Equation 12d). Here the efficiency
constraint is violated in the first solution given below. To restore efficiency, a high value of $T_1$ along with relaxation of the underwriting (income) constraint (of Equation 12d) is needed. This is illustrated in our second solution illustrated below.

**Solution (i):**
Here we select, $T_1 = 37$ months (i.e., 3 years, 1 month) and $T_2 = 164$ months (i.e., 13 years, 8 months). This implies $T_3 = 8087$ months (from Equation 11e), $S_{\text{Min}} = $97,971.91 (from Equation 11c), $Q_0 = $81,569.64 (from Equation 9a), Cooperative Dues (CD) = $485.01/month (from Figure 3), Regular Principal Payment (RPP) = $12.37/month (from Figure 3), Total monthly payments (CD+RPP) = $497.37, Discounted value of all payoffs = $78,935.48 < $81,569.64. This violates the efficiency constraint (i.e., Equation (13a)) indicating the depletion of the capital base of the cooperative.

**Solution (ii):**
To restore efficiency, a high value of $T_1 = 72$ months (i.e., 6 years) is needed. Here we opt for $T_2 = 159$ months (i.e., 13 years, 3 months), relaxing the income constraint imposed by Equation (12d). This yields $T_3 = 396$ months (from Equation 11e), $S_{\text{Min}} = $59,835.55 (from Equation 11c), $Q_0 = $81,172.43 (from Equation 9a), Cooperative Dues (CD) = $257.91/month (from Figure 3), Regular Principal Payment (RPP) = $252.53/month (from Figure 3), Total monthly payments (CD+RPP) = $510.44, Discounted value of all payoffs = $81,210.91 > $81,172.43. This restores efficiency to the system, i.e., validates Equation (13a).

In general, we observe the violation of the efficiency constraint for: (i) a high values of home ($P_0$) or income multiplier (b); or (ii) a low values of income (y) or risk ($\sigma$) or confidence level ($x\%$). Numerical illustrations of these are available from the authors upon request. It is imperative that the management of the cooperative structure their portfolio (around mortgages) which effectively generates surplus capital. Regulations such as the Community Reinvestment Act (CRA) of 1977 (in the U.S.) may necessitate that the cooperative underwrite some pareto-inferior mortgages in addition to the pareto-superior ones to sustain internal growth.41

Contrasting the above solutions with the formal mortgage derived in Section 2.2, we realize the following:

---

41 Currently, credit unions in the U.S. are not subject to CRA (1977), as their field of membership structure ensures that funds flow back to the community from where they take deposits.
(i) The cooperative home mortgage is marginally more efficient allocatively, as it yields an initial loan amount greater than that of the formal mortgage. This is in spite of the fact that both mortgages are subject to similar asset value constraints (Equations (5e), (9a) and (11c)). Furthermore, despite the higher initial outlay, the cooperative mortgage still involves marginally less total monthly payments with lower tenure. This seems contrary to intuition, as both mortgages face similar income constraints depicted by Equations (6b) and (12b) respectively. This discrepancy is resolved from the difference in profiles (linear versus concave) of both mortgages, where one is liable for principal payments only (for cooperative mortgage), while the other is liable for both principal and interest payments (for formal mortgage). This implies that a cooperative mortgage is less onerous than a formal mortgage.

(ii) The linear lien profile of a cooperative home mortgage (in contrast to the concave profile of a formal fixed rate mortgage) helps in building up an “equity cushion” at a faster pace and thus makes it less prone to default. This is attributed to the fact that a linear profile makes the cooperative lien less likely to intersect the convex profile of the asset value function (in a poor state of economy) in contrast to the concave profile of a formal mortgage (see Figures 4 and 2). This helps the cooperative mortgage to avoid the region of negative equity better than a formal mortgage, thus avoiding defaults.

(iii) The efficiency of a cooperative home mortgage is contingent on the characteristics of a home (comprising of initial price $- P_0$, underlying risk $- \sigma$), the borrower characteristics (comprising of income $- y$) and the underwriting constraints (comprising of confidence interval $- x\%$, and income multiplier $- b$). The cases with low initial home price, high income, low income multiplier, high confidence level and high risk constitute the instance cited in Besley et al. (1994) and Hart and Moore (1998), where a ROSCA (in our case the cooperative mortgage) is pareto-superior over a formal credit market (in our case the formal mortgage). However, this is reversed for high initial home price, low income, high income multiplier, low confidence level and low risk. It is imperative for the manager of a cooperative to structure its portfolio by: (i) catering to the disadvantaged (with low income) who aspire to purchase a home (with low initial value and medium to high risk), and (ii) using the following underwriting constraints: medium income multiplier and high confidence level. This would suffice in generating a surplus necessary for sustaining its internal growth and ultimately improving the economic status of the underprivileged.

---

42 This ignores the Solution (ii) of Example 2 obtained by infringing on the underwriting constraint of home multiplier.
3.3. Extension of the Model to the Case of Inflation.

Changes in the housing stock during inflationary periods lead to changes in the rate of appreciation ($\mu$) and the risk of it ($\sigma$), as demonstrated numerically in Ebrahim and Mathur (2007) (see Section 2.2). Our numerical simulations (extending the above Examples 1 and 2 in Section 3.2) illustrate that inflation leads to minor changes in the loan to value ratio. However, changes in the interest rates (and discount factor through the Fisher Effect) impact the left hand side of Equation (13a), and require meticulous selection of $T_1$, $T_2$ and $T_3$ to ensure its satisfaction. Nonetheless, for less drastic changes in the inflation rate, the solution is still feasible (unlike the case of formal prime sector mortgage) with minor changes in the LTV and periods ($T_1, T_2$). Thus, inflation does not drastically impact a cooperative, as the cost of borrowing offsets the benefit of lending.

3.4. Extension of the Model to the Case of Prepayment.

Prepayment of a cooperative home mortgage implies paying off the balance of the mortgage defined by Equation (9) prior to its real tenure $T_2$. That is at a tenure $T'_2 < T_2$. This impacts its relative efficiency with respect to formal mortgages (defined by Equation (13a)), as evaluated by employing numerical simulation (by extending the above Examples 1 and 2). This is described below.

Prepayment leads to an enhancement of the NPV of both the cooperative dues (CD) along with that of the regular principal payments (RPP). This is due to the acceleration of both payments which originally cancel each other in sum at $T_2$. This illustrates that prepayment is preferred here, as it allows the cooperative to recoup its capital lent at zero interest earlier. Thus, in the context of prepayment, a cooperative home mortgage is more efficient than a formal one.

The above result assumes that prior to the prepayment, the cooperative financing package is either equally or more efficient than the formal mortgage. If this is not true, then adjustment would have to be made with a high value of $T_1$ and relaxation of the income constraint (Equation 12d) to make it equally (or more) efficient with respect to formal mortgages, as illustrated in Example 2 (Solution (ii)).

4. Conclusion and Policy Implications

The formal housing finance system is onerous to the financially strapped individuals in an efficient capital market, as it prices the incremental risk of the subprime borrower. This has a capacity to cause a market failure, as the high cost of funding can deter homeownership. This paper proposes the establishment of a special circuit in the form of a cooperative to overcome this market failure, by integrating the two streams of the literature (comprising of mortgage design and
informal saving schemes). Intuitively, we propose a system akin to the “barn-raising” one in nineteenth century frontier U.S., where the in-kind contribution of prospective homeowner offsets the cost of borrowing such that the net contractual interest rate is zero. This form of financing is a special case of ASCRA and is practiced in a limited way by clans in Oman to help their destitute clan members. It is consistent with the prognosis of King and Levine (1993) and Levine (1997), as it facilitates in mitigating risk and reducing transaction costs for the underprivileged masses.

We implicitly assume that the cooperative has access to seed funding for its incorporation through either a charity or a mutual savings institution (such as a building society/credit union/mutual savings bank) or governmental agency or a non-governmental agency (NGO) or a supra national agency like the World Bank. Once it is established, it needs to be managed carefully for it to be self-sustaining.\footnote{The ability of a housing finance cooperative to grow is contingent on: (i) how efficient a cooperative (i.e., a credit union or mutual bank) is in contrast to its stockholder counterpart; and (ii) how it bootstraps itself by generating capital necessary for its growth internally by adopting a low risk strategy.}

We assume the existence of an information architecture and optimally price mortgages (in the spirit of Baltensperger (1978)) to contrast the one made by the housing finance cooperative with that of a formal intermediary in the prime sector. This is accomplished in a more scientific way instead of using the ad-hoc credit rationing constraints currently used by banks. Our efforts yield the following four key results.

First, a cooperative home mortgage is allocatively \textit{more} efficient than a formal prime sector mortgage, as the loan amount is marginally \textit{higher}. This is despite the fact that both are subject to similar asset value constraints. However, a cooperative mortgage involves marginally \textit{lower} total monthly payments with \textit{less} tenure. This is also in contrast to the fact that both are subject to similar income constraints. A cooperative mortgage is thus \textit{less} onerous than a formal mortgage, as its linear profile necessitates principal payments in contrast to a formal mortgage which necessitates principal plus interest.

Second, the linear lien profile of a cooperative home mortgage also makes it \textit{less} prone to defaults in contrast to the concave profile of a formal mortgage. This stems from the fact that a

\footnote{First, with respect to efficiency: The empirical evidence stemming from Altunbas et al. (2001) illustrate that mutual banks are not less efficient than their counterparts. Hansmann (1996) highlights the important role played by mutual banking institutions in the development of the U.S. financial system during the nineteenth century where they were equally competitive to stockholder owned banks. This result is not surprising, as the early credit unions and mutual banks in Europe and North America were affiliated with churches and religious institutions that strived to avoid usury by granting credit at rates lower than their profit-oriented counterparts (see El-Gamal, 2006).}

\footnote{Second, with respect to generating growth internally: Rasmusen (1988) illustrates that mutual banks chose less risky investment strategies, thereby providing good opportunities to uninformed depositor (cum shareholder), who have no resources for monitoring bank manager performance.}
cooperative mortgage builds equity faster for the homeowner, leaving a larger safety net for the financier. Thus, we conclude that a cooperative mortgage is a better alternative than a formal mortgage, as it has the potential of reducing macroeconomic volatility in accordance with the prognosis of Sheng (1997) and Renaud (2009).

Third, a housing finance cooperative performs better than its formal counterpart during periods of volatile interest rates (stemming from changes in inflationary expectations). This is attributed to the endogenous use of leverage, where the volatility in interest rates marginally impacts on the LTV and the tenure instead of pricing out the prospective homeowner due to the increase in affordability (in case of the formal mortgage).

Finally, the overall efficiency of a cooperative vis-à-vis a formal prime sector mortgage is contingent on the underlying characteristics of a home, that of the borrower and its underwriting standards. For some values of these parameters, a cooperative constitutes a special case reported in Besley et al. (1994) and Hart and Moore (1998), which is pareto-superior to the formal mortgage. However, for other values of the above parameters, a cooperative is pareto-inferior to a formal mortgage. It should be noted that our analysis does not incorporate the relatively low administrative, default and transaction costs embedded in the cooperative one in contrast to the subprime one (see Buijs, 1998; and Smets, 2000). Furthermore, there is no prepayment cost in a cooperative mortgage as opposed to a formal mortgage, where it results in a higher interest rate or a higher initiation fee (see Hall, 1985). If we were to incorporate these lower costs, a cooperative would still dominate in terms of its efficiency. This competitive advantage of cooperatives (in the form of credit unions) in the U.S. (despite their handicap described below) has subjected them to intense pressure from the banking industry, which threatens to contain its growth through legal and political means (see Wysocki, Jr., 2006). Nonetheless, a diligent manager of the cooperative should underwrite a portfolio which internally generates a capital surplus crucial for sustaining its growth. This implies: (i) catering to the disadvantaged (with low income) who aspire to purchase a home (with low initial value and medium to high risk), and (ii) using the following underwriting constraints: medium income multiplier and high confidence level.

The competitive advantages of the housing finance cooperative (as elaborated above) outweigh its disadvantages ensuing from its (i) illiquidity (as cooperatives are forced to hold the mortgages until maturity, in contrast to the formal intermediaries, who can securitize them in the

---

44 The reduction in default costs in a cooperative stems from alleviation of adverse selection and moral hazard as illustrated in Buijs (1998) and Smets (2000).

45 The pricing parameters (of a cooperative mortgage as demonstrated by Equations (12–12d) and (13–13a) in Section 3) are impacted by imposition of service fees. These are available from the authors upon request.
secondary markets), and (ii) inability to raise funds quickly due to inordinate demand (as charity funded cooperatives do not have the same ability as formal intermediaries in raising funds and are compelled to do so by appealing to their constituents, and meticulously managing its portfolio). Nonetheless, a cooperative serves as an exemplary special circuit that does not depend on government subsidies, and manages available resources more efficiently with reduced risk (in contrast to its formal counterpart). We therefore recommend it to be adopted globally to help the disadvantaged gain from its economic benefits (see Renaud, 2009). The ensuing benefit of homeownership will help in elevating the economic status of the underprivileged, fostering investment in local amenities and social capital, thus enhancing the quality of the community and stimulating economic development (see Malpezzi, 1990; Buckley, 1994; DiPasquale and Glaeser, 1999; and Haurin et al., 2002).
References


Chiang, A.C., 1984. Fundamental Methods of Mathematical Economics. New York:
Economist, 2007b. Finance and economics: Before the fall; Buttonwood, (February 17), 81.
Economist, 2009a. The collapse of manufacturing (February 19).
Economist, 2009b. Globalisation: Turning their backs on the world (February 19).


Guardian Unlimited, 2008. FAQ: The five factors that are driving up costs. (May 27), 4.


