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Postprint / Postprint
Zeitschriftenartikel / journal article

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How often should you open the door?
Optimal monitoring to screen heterogeneous agents

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April 20, 2007

Abstract
This paper shows that monitoring a partner too much in the initial phase of a relationship may not be optimal if the goal is to determine his loyalty to the match and if the cost of ending the relationship increases over time. The intuition is simple: by monitoring too much we learn less about how the partner will behave when he is not monitored. Only by giving to the partner the possibility to mis-behave might he be tempted to do it, and only in this case is there a chance to learn his type at a time where separation would be possible at a relatively low cost.

JEL-Code: D2, D8, M5
Keywords: Monitoring, probation, screening, asymmetric information

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We are very grateful to Elena Argentesi, Heski Bar-Isaac, Esther Brügger, Pascal Courty, Winand Emons, Armin Falk, Eberhard Feess, Simon Lörtscher, Niko Matouschek, Margaret Meyer, Massimo Motta, Andreas Roider, Asa Rosen, Karl Schlag, Klaus Schmidt, Kathy Spier and Eyal Winter for helpful comments and discussions. The second author gratefully acknowledges financial support from the Swiss National Science Foundation (grant PA001–108965) as well as the hospitality of the Management & Strategy Department at Northwestern University’s Kellogg School of Management.
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1 Introduction

The need to test the reliability of potential partners at the beginning of a project is a feature typical of many human relationships, not only those of an economic nature. This need is particularly strong in the frequent cases in which, once the project starts, a separation from unreliable partners becomes more difficult. It is therefore not surprising that many partnerships feature, either explicitly or implicitly, an initial period of “probation” in which the persons involved monitor each other and decide, before it is “too late”, whether to go on with the relationship. What is perhaps more surprising is that monitoring partners permanently during these probation periods may not be optimal, and in this paper we want to show why.

The intuition is simple. Consider an engagement before marriage in which the woman wants to test the loyalty of the potential husband (or vice versa if you prefer). A simplification not too far from reality is to assume that there are two types of men: those who will never betray their partner and those who instead might fall to the temptation of a love affair if an attractive occasion materializes. In order to find out which type of men the potential husband is, the fiancée might try to spend as much time as possible with him, monitoring him closely in all his daily and night activities. In this way she would apparently learn a lot about him, but effectively she would not learn the most important thing to be learned, which is how the potential husband behaves when, as during marriage, he is not monitored continuously. Only by giving the partner the possibility to mis-behave might he be tempted to do it, and only in this case could his type possibly be revealed when separating would still be feasible at low cost.

A similar situation characterizes labor market contracts where probation-
ary periods are often specified explicitly. Note that the distinctive feature of these periods is not to make monitoring possible, but to be periods in which allow firing at a relatively low cost. Even when probation is not explicitly foreseen in a contract, various reasons (e.g. sunk costs or investments in job specific human capital) make it easier to fire a worker earlier in a career rather than later. In all these cases we argue that too much monitoring at the beginning of the relationship is not optimal because it prevents the firm from learning how the worker behaves when she is not monitored. As in the case of the engagement before marriage, the firm is typically interested in discriminating between two types of potential employees: those who are “unconditional cooperators” and who therefore exert a maximum level of effort in all instances and those who are instead “rational shirkers” and who would therefore indulge in laziness if the cost of effort is high and the probability of detection sufficiently low.\footnote{The distinction between those two types of agents is consistent with findings in a recent empirical literature: For example, Nagin et al (2002) analyze data from a field experiment in which the monitoring rate is varied to see how individuals react to it in terms of shirking. They find that although there are many “rational shirkers”, a significant proportion of agents does not take advantage when the monitoring rate goes down and can thus be classified as “unconditional cooperators”. In a similar vein, Ichino and Riphahn (2005) compare the absenteeism rate of newly hired workers during and after probation in a large Italian bank; 42% of them are never absent while among the others an increase in absenteeism is observed on average when incentives change at the end of probation.} Inasmuch as the “rational shirkers” can mimic the behavior of the “unconditional cooperators” during probation, continuous monitoring (“keeping the office door always open”) is suboptimal because the probability of detection would be too high and the “rational shirker” would never concede to the temptation of being “lazy”. As a result the two types of workers would be observationally identical during probation, and only when firing becomes costly would the true types be revealed. On the contrary, random monitoring (“opening the door not too often and with no specific pattern”) might be more revealing because the “rational shirker”
would be induced to take a chance to be lazy. Thus, only in this case would there be a chance that the two types of workers might be caught behaving differently.\(^2\)

Also in the case of trading between firms, and in general between “buyers” and “sellers”, the same type of result might apply. Most firms write long-term contracts with other firms to obtain inputs for their production process. The selection of partners for the provision of inputs may be subject to a trial period similar to the one that characterizes labor or marriage contracts. What this paper suggests is that these periods would be totally uninformative about the reliability of the trading partners if the buyer announced his willingness to monitor extensively the quality of the input acquired during trial. If the unreliable sellers knew that they would be fully monitored during trial, they would try to make their product indistinguishable from the product of reliable sellers. However, this would offer no guarantee that the product would be of high quality after the long-term contract is signed.

Note that our argument apply not only to long-term relationships but is also relevant for one-shot interactions in which the reliability of a partner is so important that it becomes crucial to test it before the main interaction starts, in order to screen out “bad types” from “good types”.

There is an interesting link between the mechanism highlighted in our paper and the recent growing literature on how economic incentives may “crowd out” intrinsic motivation.\(^3\) In this line of research, as well as in our paper, a

\(^2\)A framework in which it is unclear ex ante whether the right partner is chosen is also analyzed in Watson (1999), (2002). As in our paper, some of the potential partners are cooperators, while others are more opportunistic and would cheat whenever they find it profitable to do so. In equilibrium, interaction starts with small stakes only, while stakes are increasing over time as more information about the partner is revealed. Contrary to our paper, the role of monitoring is not considered, and therefore these models suggest an alternative channel (“gradualism”) through which large losses from doing business with the wrong partner can be avoided.

\(^3\)This literature is surveyed, among others, in Frey and Jegen (2001), Gneezy (2004),
principal’s use of monitoring and control to create effort incentives may be counterproductive. The literature has identified adverse effects of monitoring “good types” whose intrinsic motivation may be reduced by the feeling that the principal does not trust them, believing that explicit incentives are necessary to induce an effort that they would have been happy to exert in any case. Our paper, instead, focuses on the negative effect of monitoring on opportunistic “bad types” who are induced to mimic “good types”, which in turn hinders their detection.

In order to focus on the specific intuition that motivates our paper, we abstract from crowding-out aspects and assume that “good types” are indifferent to monitoring and control. Under this assumption, we analyze formally in Section 2 under what conditions principals can expect to be successful in filtering out unreliable partners during probation and what are the potential gains from setting the monitoring intensity optimally. Section 3 will relate and contrast our claim with other possible theories of the relationship between monitoring and productivity, including the above-mentioned literature on crowding-out. Section 4 concludes. All proofs are in the Appendix available on the JEBO website.

2 The model

2.1 Basic setup

We consider one principal facing $N \geq 1$ agents in a relationship lasting at most two periods $i = 1, 2$. Period 1 is a probation period while period 2 is the time after probation. In each period, agents can choose an action from $\{E, S\}$ where $E$ and $S$ denote “exerting effort” and “shirking”, respectively.
A shirking agent produces 0 output, while, when exerting effort, each agent produces $v_i$ in period $i$. We have in mind situations in which the output of an agent’s effort in period 1 is negligible compared to period 2, and thus we assume $v_1 = 0$ and $v_2 > 0$.\footnote{Assuming $v_1 > 0$ instead would not affect our results qualitatively; see section 3.1.} We interpret $v_2$ as a Net Present Value, thereby allowing period 2 to be of any length relative to the probation period. In particular, period 2 could last for some time so that $v_2$ would reflect the overall discounted output in a long-term relationship, or period 2 could be short, indicating, for example, that the principal benefits from the effort of the agent only at a single (but maybe particularly important) occasion.

Although all agents are equally valuable to the principal when exerting effort, they differ with respect to the (privately known) cost of doing so, which is represented by a parameter $\theta \in \{G, B\}$: “bad types”, denoted by B, have effort costs $c_i$ in period $i$, where $c_1 = c$, $c_2 = k \cdot c$ with $k > 0$, and where $c$ is drawn from an atomless distribution $H(c) \in C^2$ with support $[0,1]$ at the beginning of the game.\footnote{Our approach is isomorphic to a model with a continuum of types (cost realizations), where the type distribution has an atom at 0.} The parameter $k$ is simply a proportionality factor that allows for appropriately adjusting and discounting effort costs with respect to the length of period 2 relative to period 1. Furthermore, $k$ might also reflect potential differences in the intensity of effort required during and after probation. On the other hand “good types”, denoted by G, do not face any costs of exerting effort.\footnote{As explained in the Introduction, this view is for example broadly confirmed in a field experiment reported in Nagin et al (2002). Alternatively, we could assume that good types face effort costs that are lower on average than those of bad types. This would not change our arguments qualitatively.} In the population of agents, the shares of good and bad types are $\alpha$ and $(1 - \alpha)$, respectively, where $0 < \alpha < 1$. Concerning the informational environment, we assume that each agent privately learns his type at the beginning of the game and that $\alpha$ and

\begin{equation}
\begin{aligned}
\text{\footnotesize{4}} & \text{Assuming $v_1 > 0$ instead would not affect our results qualitatively; see section 3.1.} \\
\text{\footnotesize{5}} & \text{Our approach is isomorphic to a model with a continuum of types (cost realizations), where the type distribution has an atom at 0.} \\
\text{\footnotesize{6}} & \text{As explained in the Introduction, this view is for example broadly confirmed in a field experiment reported in Nagin et al (2002). Alternatively, we could assume that good types face effort costs that are lower on average than those of bad types. This would not change our arguments qualitatively.}
\end{aligned}
\end{equation}
$H(\cdot)$ are common knowledge.

Period 1 is a probation period in which the principal can monitor each agent at no cost.\footnote{Note that the mere objective of filtering out bad agents will be sufficient for our analysis, and assuming costless monitoring makes our point particularly striking. Therefore, our results are qualitatively different from the literature on law enforcement building on Becker (1968) where optimal detection probabilities are typically less than one due to convex enforcement cost functions (see also Besanko and Spulber (1989) and Polinsky and Shavell (2000)).} His choice variable is thus a monitoring frequency $q \in [0, 1]$. The outcome of the monitoring process is captured by a variable $M \in \{E, S, 0\}$ where $M = E$ and $M = S$ perfectly reveal whether or not shirking has occurred, while $M = 0$ denotes the case where an agent has not been monitored. In line with most of the literature, we assume that the principal is able to commit to her chosen monitoring rate.\footnote{Exceptions include Khalil (1997) and Khalil and Lawarree (2001) where the principal decides on her monitoring policy ex post, after agents have chosen their actions. As will become clear below, in our model, the principal would benefit from the ability to increase the monitoring rate ex post. However, from an ex ante point of view she would be better off with commitment, so credibility is an important issue for the principal to overcome this problem of time inconsistency. For example, one possibility to achieve this would be to commit only a limited amount of resources to the monitoring technology so that it would be technically unfeasible to monitor all agents at a higher rate than the announced one.}

After observing the outcome of the monitoring process and updating his beliefs appropriately by use of Bayes’ rule, the principal makes a firing decision $F \in \{0, 1\}$, where $F = 1$ means that an agent is fired. Firing costs in the trial period are zero, while in the second period they are prohibitively high.\footnote{All we need is that splitting becomes more costly for the principal as the length of the relationship increases, which is often the case in reality. For example, in many countries employment protection is increasing in tenure with the firm, such that the principal is interested in identifying bad agents as early as possible, in order not to remain stuck with them when firing becomes too expensive or virtually impossible. The same is true in situations of match-specific human-capital investments at the beginning of a relationship.} It is assumed that the population out of which the $N$ agents are drawn is sufficiently large such that, upon monitoring one agent, no inference can be made about the pool composition of the remaining $N - 1$ agents.

As for payments, we denote by $t_i$ the transfer from the principal to each
agent. During probation, the transfer $t_1$ could in principle be set equal to zero, although in most situations it would be natural to imagine that some kind of “show-up fee” $t_1 > 0$ is paid to the worker independently of the probation outcome. In the case of period 2, if the worker is hired at the end of probation, he cannot be fired later in the sense that he is in any case entitled to a transfer $t_2 > t_1$ independently of his performance. We also make the following assumption:

**Assumption 1.** $v_2 > t_2 \geq 1$,

which implies that the payoff for the principal from each agent who exerts effort in period 2 is positive ($v_2 > t_2$). Furthermore, as explained below, $t_2 \geq 1$ implies that in period 1, exerting effort is privately optimal for a bad agent for any value of $c$ when he is monitored with certainty.

Given Assumption 1, and coming back to the firing decision, we want the principal to wish to continue with an agent when his belief after the monitoring process is greater or equal to the prior $\alpha$.\textsuperscript{10} This will imply that the following assumption must hold:

**Assumption 2.** $\alpha \cdot (v_2 - t_2) + (1 - \alpha) \cdot (-t_2) > 0$.

Summarizing, the game has the following stages:

- At stage 0, each agent’s type is determined by a nature’s move and is only known to the agent.
- At stage 1, the principal sets and commits to a monitoring probability $q$ for the probation period.

\textsuperscript{10}We can alternatively assume that an agent cannot be fired without being monitored. This would for example be consistent with unjust-dismissal legislation in the US; see Krueger (1991).
At stage 2, period 1 begins and each agent independently decides on whether or not to exert effort. After the effort choice is made, each agent is monitored with probability $q$.

At stage 3, given the outcome of the monitoring procedure, the principal decides on which agents to fire. After the firing decision period 1 ends.

At stage 4, in period 2, all remaining agents again decide on whether or not to exert effort. Then the game ends.

2.2 Equilibrium analysis

2.2.1 Effort choice in period 2

Let us start the analysis of the game at stage 4, and denote by $a_i^\theta \in \{E, S\}$ the action chosen by type $\theta \in \{B, G\}$ in period $i = 1, 2$. Equilibrium values carry an asterisk *. We start with a good type. Since he has no effort costs, he is indifferent between exerting effort and shirking (both actions yield a payoff of $t_2$). Throughout we assume that both types exert effort when indifferent, so that good types will always choose $a_2^{G^*} = E$ in period 2. Contrary to that, in period 2, a bad type gets $(t_2 - k \cdot c)$ from choosing $E$ and $t_2$ from choosing $S$, so bad types will always shirk in period 2 (i.e. $a_2^{B^*}(c) \equiv S \forall c > 0$).

2.2.2 Firing decision

Let us now look at the principal’s optimal firing decision at stage 3 after monitoring has been carried out. Denote by $\beta \in [0, 1]$ the belief of facing a good type conditional on the outcome of the monitoring process:

$$\beta_M := Pr(\theta = G \mid M) \forall M = E, S, 0$$

It suffices to assume that the expected payoff for the principal in period 2 from having a good (bad) type is positive (negative).
Of course, $\beta_0 = \alpha$, and in (a Bayesian perfect) equilibrium, whenever possible, beliefs have to be consistently derived using Bayes’ rule from the equilibrium strategies of each type of agent at stage 2 (see e.g. Fudenberg and Tirole, 1991). Given that good types will exert effort while bad types will always shirk in period 2, the principal’s expected payoff from an agent for period 2 as a function of $\beta$ is given by

$$\beta(v_2 - t_2) + (1 - \beta)(-t_2),$$

which may be positive or negative. It follows that the principal will fire an agent, whenever monitoring “delivers” a belief for this agent to be a good type that is sufficiently low:

$$F^*(\beta) = \begin{cases} 
1 & \text{if } \beta < \frac{t_2}{v_2} \\
0 & \text{otherwise} \end{cases}.$$  

### 2.2.3 Effort choice in period 1

Now consider the optimal effort decision at stage 2 by each type for a given probability of monitoring $q$. In doing so, we directly look at the following equilibrium continuation and then see how it can be supported.

**Lemma 1.** At stage 2, for all $q < \bar{q} := \frac{1}{t_2}$, there exists an equilibrium continuation in which

- a) each good type chooses $a_{1}^{G*} = E$ independent of $q$,
- b) each bad type shirks whenever his realization of $c$ is sufficiently high. This happens with probability $(1 - e(q)) > 0,$\(^{12}\)
- c) the principal’s beliefs after monitoring has been carried out are given by

$$\beta_E^* = \frac{\alpha}{\alpha + (1 - \alpha)e(q)} > \alpha$$

$$\beta_S^* = 0.$$ 

\(^{12}\)The exact definition of $e(q)$ is given in Eqn. 9 below.
and she optimally fires all agents for whom \( M = S \) holds and keeps all other agents (including those who have not been monitored).

For an intuition for Lemma 1, let us start with a good type. On the equilibrium path, when choosing \( E \), he gets \( t_1 \) in period 1 and \( t_2 \) in period 2; if he is monitored, this will lead to \( M = E \), so the principal holds the belief \( \beta_E^* = \frac{\alpha}{\alpha + (1 - \alpha)q} > \alpha \) for which, by Assumption 2, \( F = 0 \) is optimal. If he is not monitored, the principal holds belief \( \beta^* = \alpha \), and he is not fired either. On the other hand, when choosing \( S \) in the first period, his payoff is still \( t_1 \) since he does not save in effort costs, but with probability \( q \), he is monitored, found to be shirking and, given belief \( \beta_S^* = 0 \), fired. It follows that his expected payoff for period 2 is only \((1 - q)t_2\) and, thus, a deviation is never profitable.

Now consider a bad type. When choosing \( E \), he gets \((t_1 - c)\) in period 1. When monitoring occurs, he is taken to be a good type and thus will also get \( t_2 \) in period 2, in which he will then shirk, so that he will not again incur any cost of effort in that period. On the other hand, when choosing \( S \), he gets \( t_1 \) in period 1 (thus saving on effort costs \( c \)), but with probability \( q \) he is found to be shirking and fired, so his expected payoff for period 2 is only \((1 - q)t_2\). It follows that \( S \) is preferred iff

\[
t_1 - c + t_2 < t_1 + (1 - q)t_2 \iff c > qt_2
\]

so that the optimal decision of a bad type as a function of \( q \) and \( c \) is given by

\[
a_{B^*}^*(q, c) = \begin{cases} 
S & \text{if } c > qt_2 \\
E & \text{otherwise}
\end{cases}
\]

(i.e. shirking occurs whenever effort costs are sufficiently high). This means that for \( q \geq \bar{q} := \frac{1}{t_2} \), all bad types choose \( E \) independent of their cost parameter \( c \). The threshold \( \bar{q} \) relates the benefit from employment in period
2 \( t_2 \) to the maximum cost of effort during probation, which is equal to 1. Clearly, when \( t_2 \) is relatively large, then \( \theta \) is low, which means that shirking in period 1 is undesirable for a bad agent given the prize at stake. Thus, even a relatively low level of \( q \) would induce all bad agents to exert effort as long as \( q \) is set greater or equal than \( \theta \). In this case, shirking would no longer occur on the equilibrium path, so there would be no information transmission in equilibrium and \( \beta_E^* = \alpha \) would hold. However, as will be shown below, it is indeed optimal for the principal to choose some \( q < \theta \) so that both possible actions, \( E \) and \( S \), occur with positive probability on the equilibrium path, so there is no leeway in forming off-equilibrium beliefs. Finally, note that the principal’s beliefs conditional on \( M \) are consistent with the equilibrium strategies of both types.

2.2.4 The principal’s optimal choice of \( q \)

The optimal choice of \( q \) at stage 1, under the assumption that the equilibrium continuation as derived in Lemma 1 is played subsequently remains to be determined. Clearly, the principal’s objective is to maximize her expected payoff. Let us analyze each part of it in turn.

**Good Types:** In period 1, there are \( \alpha \cdot N \) good types; none of them shirks in equilibrium, and therefore none of them is fired. Since for simplicity we assumed a negligible output during probation, they yield a negative payoff \((-t_1)\) to the principal, but the important point is that this payoff is independent of the monitoring probability \( q \). In period 2, there is again no shirking, and each of the \( \alpha \cdot N \) good types yields the principal \((v_2 - t_2) > 0\), which is again independent of \( q \). This means that the choice of \( q \) neither influences the number of good types in each period nor their choice of effort. Therefore, in what follows, we can neglect the payoff accruing from good types as it will
have no effect on the optimal level of $q$.

**Bad Types:** Recall that a bad type will shirk in period 1 whenever $c > qt_2$. Thus, from the principal’s point of view, the probabilities of shirking and exerting effort are given by, respectively,

$$
s(q) := Pr(c > qt_2) = \max(0, 1 - H(qt_2)), \tag{8}
$$

$$
e(q) := Pr(c \leq qt_2) = \min(H(qt_2), 1). \tag{9}
$$

Clearly we have $\frac{ds}{dq} \leq 0$, and $\frac{de}{dq} \geq 0$ and $s'(q) = -e'(q)$.

Since there are $(1 - \alpha) \cdot N$ bad types, the expected payoff generated by them in period 1 is:

$$
\pi_1(q) := (1 - \alpha) \cdot N \cdot (e(q) \cdot (-t_1) + s(q) \cdot (-t_1))
= (1 - \alpha) \cdot N \cdot (-t_1) \tag{10}
$$

because $e(q) + s(q) \equiv 1$. Note that this term is also independent of $q$. This highlights the fact that in period 1 the principal monitors workers not so much because she is interested in their output, which in this period is negligible, but mainly because she needs to detect bad workers.

In other words, from the principal’s point of view, monitoring in period 1 matters only because it influences the number of bad types in period 2. Precisely for this reason $q$ must be set in a way that induces some shirking in period 1; otherwise no bad type would be detectable. Formally, the number of bad types in period 2 is determined as follows. Each bad type shirks with probability $s(q)$ but is detected only with probability $q$ so that $(1 - \alpha) \cdot N \cdot (1 - q) \cdot s(q)$ bad types remain in period 2. Moreover, each bad type exerts effort with probability $e(q)$ and is thus not identified through monitoring, so another $(1 - \alpha) \cdot N \cdot e(q)$ of them survive the probation period. Taking this together, since each bad type generates a payoff of $(-t_2)$ to the principal,
her expected payoff from the bad types in period 2 is given by
\[ \pi_2(q) := (1 - \alpha) \cdot N \cdot ((1 - q) \cdot s(q) + e(q)) \cdot (-t_2) \quad \forall q \in (0, \bar{q}). \] (11)

We can therefore state the following result concerning the optimal monitoring frequency \( q^* \).

**Proposition 1.** Given the equilibrium continuation characterized in Lemma 1, the optimal monitoring frequency for the principal induces shirking on the equilibrium path (i.e. \( 0 < q^* < \bar{q} := \frac{1}{t_2} \)).

Note first that \( q^* < \bar{q} \) implies that the equilibrium continuation as determined in Lemma 1 also applies.

Intuitively when \( q \) is too low, only a few bad agents are identified while when \( q \) is too high, each bad agent is less likely to shirk in period 1 and thus cannot be identified through monitoring either. As bad types shirk with probability 1 in period 2 (yielding a negative payoff \(-t_2 < 0\), too much monitoring is not in the principal’s interest. Note that again this argument does not rely on monitoring costs, which are assumed to be zero, since with positive monitoring costs, the optimal monitoring rate would be even lower.

### 2.3 Comparative statics: Potential gains from monitoring optimally

We now analyze the potential magnitude of the gain induced by setting \( q \) optimally instead of monitoring “too often”. For this purpose, we perform a comparative statics analysis w.r.t. the distribution of effort costs and consider distribution functions of the class \( H(c) = c^n \) where \( n \geq 1 \). The case \( n = 1 \) is then just the uniform distribution, and by increasing \( n \), the function becomes more convex, and there is more probability mass on high realizations of effort costs.
With \( H(c) = c^n \) it then follows from Eqn. (9) that a bad agent exerts effort during probation with probability \( e(q) = q^n t_2^n \) for all \( q < \bar{q} \) so that from Eqn. (11) the objective function of the principal can be written as

\[
\max_q \pi_2(q) = (1 - \alpha) \cdot N \cdot (1 - q + q^{n+1} t_2^n) \cdot (-t_2).
\] (12)

Therefore the FOC is \( q^n t_2^n (n + 1) - 1 = 0 \) and the optimal monitoring rate is then given by

\[
q^*(n, t_2) = \frac{1}{t_2} \sqrt{\frac{1}{n + 1}}.
\] (13)

This leads to the following result.

**Proposition 2.** The optimal monitoring rate \( q^*(n, t_2) \) is strictly increasing in \( n \) satisfying \( \lim_{n \to \infty} q^*(n, t_2) = \frac{1}{t_2} (= \bar{q}) \).

Intuitively, when more and more probability mass is shifted towards high realizations of effort costs, more shirking takes place during probation, making monitoring more effective at the margin. Note, however, that for any finite \( n \), the optimal monitoring rate is bounded from above by \( \bar{q} \leq 1 \) such that it is never optimal for the principal to monitor with probability 1. In the limit, when virtually all mass is on the highest realization \( c = 1 \), if the monitoring rate were \( q \geq \bar{q} \), all bad agents would prefer to exert effort so that the principal would not learn anything at all. However, just slightly reducing the monitoring rate below \( \bar{q} \) would induce shirking with probability 1 so that the principal would learn a lot.

Using these results, we now show that, for \( n \) large, reducing \( q \) just slightly below \( \bar{q} \) will also make a big difference to the principal in terms of profit. As a benchmark, we take the case where monitoring is excessive \( (q \geq \bar{q}) \), which, in terms of profits for the principal, is also equivalent to the case \( q = 0 \). In both cases, she does not detect any of the bad agents, so her expected payoff.
from these bad agents in period 2 is a pure loss equal to

$$\pi_2(q = 0) = \pi_2(q \geq \overline{q}) = (1 - \alpha) \cdot N \cdot (-t_2) < 0. \quad (14)$$

On the other hand, when \( q^* \) is chosen instead, the probability that a bad type remains in period 2 is strictly less than 1 and given by \( 1 - q^* \cdot s(q^*) < 1 \).

Thus, the principal is able to filter out at least some of the bad types so that the loss caused by the remaining bad agents is

$$\pi_2(q = q^*) = (1 - \alpha) \cdot N \cdot (1 - q^* \cdot s(q^*)) \cdot (-t_2). \quad (15)$$

By taking the difference between (15) and (14), her absolute gain from choosing the optimal “interior” monitoring rate \( q^* \) compared to monitoring excessively at some \( q \geq \overline{q} \) is:

$$\Delta \pi := \pi_2(q = q^*) - \pi_2(q \geq \overline{q})$$

$$= (1 - \alpha) \cdot N \cdot (-q^* \cdot s(q^*)) \cdot (-t_2) > 0, \quad (16)$$

which can be usefully expressed in percentage terms as

$$\Delta \pi_% \ := \frac{\Delta \pi}{|\pi_2(q = 0)|} = q^* \cdot s(q^*). \quad (17)$$

With \( H(c) = c^* \) and \( q^*(n, t_2) \) as determined in Eqn. (13), this leads to the following result.

**Proposition 3.** The percentage gain for the principal from monitoring optimally is strictly increasing in \( n \) satisfying \( \lim_{n \to \infty} \Delta \pi_% = \frac{1}{t_2} (= \overline{q}) \).

When high realizations of effort costs become more likely, bad agents prefer to shirk more often for a given monitoring rate. We have seen in Proposition 2 that in this case, the principal prefers higher monitoring rates so that, while always remaining positive, the “distance” between \( \overline{q} \) and \( q^* \)
decreases. As Proposition 3 shows, while the difference \( \bar{q} - q^*(n) \) is decreasing in \( n \), the gain for the principal from choosing \( q^* \) instead of \( \bar{q} \) is increasing in \( n \) (i.e. there is a lot to be gained for the principal by not monitoring too much when there is a lot of probability mass on high realizations of effort costs). In the limit, when \( n \) becomes very large, the difference between \( \bar{q} \) and \( q^* \) becomes arbitrarily small, but the gain for the principal is maximum: at \( q = \bar{q} - \epsilon \), the principal induces shirking with probability 1, so that monitoring is highly effective, while at \( q = \bar{q} \), all bad agents exert effort so that monitoring is useless. Thus, as high realizations of effort costs become more likely, reducing the monitoring rate just by “an \( \epsilon \)” might have a strong effect on profits, and this effect is maximum when \( t_2 \) is small. It is in such situations where we would expect probation periods to be particularly useful for the principal. For the extreme case where, in addition to \( n \to \infty \), also \( t_2 \to 1 \), it can be shown that the principal would detect all bad agents with probability one.

3 Discussion

In the setting described above, monitoring during probation serves as a screening device that increases profits inasmuch as it allows the principal to identify and fire shirkers before this becomes too costly or even impossible. This is, however, not the only mechanism through which monitoring may affect profits. In this section we briefly discuss alternative mechanisms, contrasting them with the one identified by our paper.

3.1 Effect of monitoring on productivity during the monitoring period

One immediate modification of the basic model allows for a non-zero output in the probation period. In the simplified setting of section 2, monitoring has
no effect on productivity during probation simply because, by assumption, work done by agents in this period has no value for the principal.

It is straightforward to show that if \( v_1 > 0 \) instead, our argument survives as long as \( v_1 \) is not too large. From Eqn. (10), the expected payoff of the principal for period 1 becomes

\[
\pi_1(q) = (1 - \alpha) \cdot N \cdot (e(q) \cdot (v_1 - t_1) + s(q) \cdot (-t_1)),
\]

which is increasing in \( q \), since \( e'(q) > 0 \). Clearly, while increasing the optimal monitoring intensity, as long as output during the probation period is not too important to the principal, the effect we emphasize remains.

More subtle is instead the contrast with the results by Cowen and Glazer (1996) and Dubey and Wu (2001) who show that less monitoring might induce agents to exert more effort during the monitoring period such that a principal benefits from having a “less accurate picture” about an agent’s behavior.

In the first paper, an agent trying to exceed a given threshold, such as passing an exam, might exert more effort when he is monitored with low frequency only. For example, the principal announces that there will not be an exam question on each topic covered in class. The reason is that with fewer questions, the risk of failing is higher when the student has not studied the whole material. In the second paper, the principal wants to induce maximum effort by all agents. Agents have different skills and are monitored at random. The agent with the highest observed output receives a prize. To give the low ability agents incentives to exert effort also, the principal has an interest in keeping the sample of observations small (i.e. to monitor less often) since large samples would tend to favor high ability agents as they are more likely to produce a high output.
Both papers are static in the sense that a low monitoring rate in a given period might induce more effort and is therefore beneficial for a principal in this period. In contrast, our argument is dynamic in emphasizing the potential benefits from not monitoring extensively in early periods, inducing more shirking in these periods but serving as a screening device from which the principal benefits in later periods of a relationship.

3.2 Effect of monitoring on productivity in future periods

We now consider further channels through which monitoring in early periods may affect the principal’s future payoff and that are therefore potentially countervailing the selection mechanism we are emphasizing in this paper.

In particular, we consider the case where monitoring has a positive effect on an agent’s future productivity. For example, monitoring during probation might trigger a learning/training process or the formation of “good habits”, both of which may increase permanently the baseline productivity of all agents including the potential shirkers. In this case, the trade-off between the advantages and disadvantages of monitoring in the first period shifts in favor of increasing monitoring and against the claim of our paper.

Consider the simplest case where the productivity of a shirking agent in period 2 is given by $v^S_2(q) = a \cdot q$ where $a \in (0, t_2)$ measures the effect of monitoring in period 1 on an agent’s productivity in period 2, so that our basic model with $a = 0$ emerges as the limiting case.$^{13}$ The payoff of the principal from a shirking agent in period 2 is then simply $v^S_2(q) - t_2 = a \cdot q - t_2 < 0$, and thus the upper bound $a = t_2$ ensures that this payoff is still negative for all $q \in [0, 1]$; otherwise the selection motive vanishes and

$^{13}$The productivity of a non-shirking agent is still $v_2$. 

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the principal never wants to fire an agent after probation.

To make our points as clearly as possible, and to get closed-form solutions, we consider the case where \( H(\cdot) \) is uniform and where \( t_2 = 1 \) so that \( q^* = \frac{1}{2} \) and \( \overline{q} = 1 \) in the basic model (see Eqn. (13)). Then, the objective function for the principal becomes:

\[
\max_q \pi_2(q) = (1 - \alpha) \cdot N \cdot (1 - q + q^2) \cdot (aq - 1).
\]  

(19)

An interior solution will satisfy the first order condition and is given by

\[
q^{**}(a) = 1 + a - \sqrt{1 - a - 2a^2} \cdot \frac{3a}{a},
\]  

(20)

which is strictly increasing in \( a \) in the relevant range. This leads to the following result.

**Proposition 4.** As long as the beneficial effect of monitoring during probation on the productivity of shirking agents in period 2 is not too strong, the principal optimally still induces shirking on the equilibrium path. Formally, the optimal monitoring intensity is given by \( q^{**}(a) \) for all \( a \in (0, \overline{a}) \) and \( q = 1 \) otherwise, where \( q^{**}(\overline{a}) = \overline{q} \).

Note first that \( \lim_{a \to 0} q^{**}(a) = \frac{1}{2} = q^* \), and thus our result from the basic model emerges as a limit case. Moreover, because of the additional beneficial effect of monitoring, \( q^{**}(a) \) is strictly increasing in \( a \) and thus higher than in the basic model. However, \( q^{**}(a) \) is only consistent with the equilibrium continuation of Lemma 1 when below \( \overline{q} \) (i.e. as long as \( a \leq \overline{a} \)); otherwise, there is no more shirking on the equilibrium path.

When \( a \geq \overline{a} \), full monitoring (\( q = 1 \)) is optimal. In contrast to the basic model, the principal is not indifferent between \( q = 0 \) and all \( q \in [\overline{q}, 1] \); although none of the bad agents are filtered out for each of these monitoring
levels, the principal strictly prefers \( q = 1 \) because of the higher productivity of (shirking) agents.

Thus, whenever the optimal monitoring intensity is given by \( q^{**}(a) \), the optimal monitoring intensity tends to be higher compared to the basic model, but it is again not excessive in the sense that it does not prevent shirking overall since the selection motive still matters to the principal.

At a more general level, which purpose of monitoring prevails in a given context clearly depends on whether the heterogeneity of types tends to be exogenous and persistent, in which case screening becomes the main objective for the principal (as in our framework), or whether the principal can somehow influence the degree of heterogeneity over time such that learning or habit formation become more relevant. In our view, nothing seems to suggest that the screening purpose of monitoring should in principle be less relevant to the principal than these alternative purposes.

### 3.3 Endogenous pool of agents ex ante

A further potentially beneficial effect of a high monitoring frequency is suggested by Wang and Weiss (1998) who show that the prospect of being monitored intensively can deter potential shirkers workers from applying for the job.\(^{14}\)

Note first that in our model the parameter \( \alpha \) (i.e. the fraction of good types in the agent pool) can be interpreted as a proxy for the usefulness of the screening process of applicants. Obviously, from Eqn. (11), the principal’s equilibrium payoff is increasing in \( \alpha \).

Moreover it is also straightforward to extend our basic model to allow

\(^{14}\text{In their model, the productivity of agents is exogenous. The issue of mimicking, which is crucial in our framework, does therefore not arise.}\)
for an endogenous pool of agents: Assume that, prior to the effort choice, an agent can also choose whether or not to accept the job at all or quit, in which case she gets her reservation payoff \( W \).

From the incentive condition (6), an agent prefers to shirk in period 1 if \( c > \tilde{c} := q t_2 \). In this case his payoff along the equilibrium path is \( t_1 + (1 - q) t_2 \), and thus he will not quit in the first place as long as his outside option \( W \) is sufficiently small. Define the critical level \( \tilde{W} \) such that an agent with effort cost \( c > \tilde{c} \) is indifferent between quitting and shirking by

\[
\tilde{W} = t_1 + (1 - q)t_2.
\]  

(21)

He will thus prefer to quit for all \( W > \tilde{W} \), and since \( \frac{d}{dq} \tilde{W} = -t_2 < 0 \), a higher monitoring intensity tends to induce more agents with high effort costs (who would otherwise shirk) to refrain from working for the principal.

Therefore, as in section 3.2, the consideration of the mechanism considered by Wang and Weiss shifts the tradeoff in favor of more monitoring with respect to what implied by the basic version of our model. However, note that even in this case, our intuition would still apply inasmuch as some residual heterogeneity in the propensity to shirk remains in the pool of hired agents.\(^{15}\)

Moreover, in a world in which both mechanisms coexist, it is also interesting to consider their interaction. In this case, the principal’s ex ante commitment to excessive monitoring during probation to deter bad applicants may be time-inconsistent when the screening process is not perfect and the firm hires bad workers in equilibrium. The principal would still have an obvious incentive to reduce monitoring during probation, as suggested by our paper, in order to identify the remaining bad agents before the relevant productive activity starts. This reasoning leads to the preliminary conclu-

\(^{15}\)Wang and Weiss confine attention to the two-type case such that, in equilibrium, only agents with high productivity apply for the job.
sion that workers should be sceptical about a principal’s announcement of excessive monitoring when it is clear that if the principal maintained the commitment, she would not learn anything about the potential shirkers possibly still present among workers under probation.

3.4 Crowding-out of intrinsic motivation

Thus far we have maintained the assumption that only bad types react to the monitoring intensity, while the behavior of the good and intrinsically motivated types remains unaffected. This is in contrast to a large recent literature on the interaction of psychological and economic incentives arguing that the latter might crowd-out intrinsic motivation.\textsuperscript{16}

The underlying rationale for such dysfunctional responses is that explicit incentives may convert a human relationship into a purely economic one, and that this may reduce intrinsic motivation and behavior based on fairness. More concretely, it has been argued that economic incentives, in particular small ones, might simply insult individuals, and this may give rise to a non-monotonic relationship between economic incentives and performance (Gneezy and Rustichini 2000b and Gneezy 2004). Moreover, economic incentives might provide new information concerning the importance or cost of an individual’s activity (Gneezy and Rustichini 2000a and Benabou and Tirole 2003). Finally, economic incentives might not be in accordance with social norms of cooperation, fairness or trust (Fehr and Rockenbach 2003, Fehr and List 2004 and Sliwka 2007).

With respect to the intensity of supervisors’ control in human relationships, Frey (1993) and Falk and Kosfeld (2006) suggest that individuals might

---

\textsuperscript{16}See for example the seminal contributions by Deci (1971) and Titmuss (1970) in psychology and economics, respectively; for surveys see also Frey and Jegen (2001) and Gneezy (2004).
perceive control as a signal of distrust to which they respond with low effort. In such situations, a principal might benefit from foregoing the possibility to control agents too intensely.

In terms of our model, when such motives are present, then the behavior of the good types, which are intrinsically motivated, could be negatively affected by the monitoring intensity, while we have assumed that they are indifferent to monitoring. Clearly, in such an augmented framework, there would then be two reasons for principals not to monitor excessively: Not only would it induce bad types to make themselves non-identifiable during probation (as pointed out in our paper), but it might also reduce the intrinsic motivation to exert effort of the good types. In the light of these arguments, our result could then be interpreted such that excessive monitoring is not optimal for a principal even if crowding-out of intrinsic motivation is not a concern. When it is, then the optimal monitoring intensity would tend to be even lower.

4 Conclusion

We have shown in this paper that monitoring a partner too much in the initial phase of a relationship may not be optimal if the goal is to determine her loyalty to the match and if the cost of terminating the relationship increases over time. If too much monitoring induces the partner to behave well even if her inclination in the absence of monitoring would be to mis-behave, the principal does not learn what needs to be learned at the beginning of a relationship. Note that this mechanism is completely independent of the costs of monitoring, and thus the result holds even if monitoring is costless. This general intuition applies to many social relationships characterized by asymmetric information with respect to the types of agents such as for example,
labor and marriage contracts.

Furthermore, we show that the use of probation periods, together with some monitoring during such periods, is beneficial to the principal when agents’ the cost of exerting effort during probation tends to be high compared to the benefit of future interaction. In this case, bad agents give in to the temptation of shirking more frequently during probation, so that monitoring is an effective tool in filtering out these agents provided that it is not too frequent. On the contrary, when the costs of effort tends to be low, our model suggests that probation would be a waste of time as a method to detect bad agents through monitoring.

Our streamlined framework allows us to derive our basic result in its purest form at the cost of some simplifications. Apart from the further potential impacts of monitoring already discussed in section 3, we have also maintained the assumption that the principal cannot use incentive schemes to screen workers or elicit effort from bad types. Clearly such schemes would be costly for the principal because of informational rents, which will tend to be the higher, the larger the heterogeneity of types. Moreover, the screening process can often not be expected to work perfectly, possibly due to institutional constraints such as in the case frequently encountered in reality where workers in same job must also get the same (fixed) wage. This is the case we consider, but our results do not qualitatively depend on it; as long as some heterogeneity of types remains after some incentive structure has been implemented, the principal would still have an incentive to use also her monitoring policy as a screening device as stressed in our paper. After all, the fact that probation periods exist in many long-term relationships of different natures hints at the difficulty of relying solely on incentive contracts as a screening device. Monitoring during probation (but not too much) is therefore needed
for this purpose.
Appendix

A Proof of Proposition 1

We proceed along the following lines: since the objective function of the principal is continuous in the interval \([0, \overline{q})\),
i) we show that the expected payoff of the principal is strictly increasing at \(q = 0\) and strictly decreasing as \(q \to \overline{q}\), and
ii) we show that the absolute expected profit level is also higher at \(q = q^*\) than at \(q = 0\) and when \(q\) approaches \(\overline{q}\) (the payoff function of the principal is flat for all \(q \geq \overline{q}\)).

To do this, define

\[
Z(q) := [(1 - q)s(q) + e(q)](-t_2) = [(1 - q)(1 - e(q)) + e(q)](-t_2) \\
= [(1 - q + qe(q))(-t_2)]
\]

which

\[
Z'(q) = [-1 + e(q) + qe'(q)](-t_2).
\]

Recall that the expected payoff from the good agents and from the bad agents in period 1, respectively, is independent of \(q\). Furthermore, from (11), 
\[
\pi_2(q) = (1 - \alpha) \cdot N \cdot Z(q)
\]
so that \(q^*\) is uniquely determined by \(Z(q)\).

ad i): We need to show that \(Z'(q = 0) > 0\) and \(Z'(q \to \frac{1}{t_2}) < 0\):

\[
Z'(q = 0) = t_2 > 0,
Z'(q \to \frac{1}{t_2}) = (-t_2) \cdot [-1 + \frac{1}{t_2} \cdot e'(\frac{1}{t_2}) + 1] = -e'(\frac{1}{t_2}) < 0.
\]

ad ii): Note that we have

\[
Z(0) = Z(\frac{1}{t_2}) = -t_2,
Z(q^*) = (1 - e(q^*) + q^* \cdot e(q^*))(-t_2),
\]
and thus

\[ Z(q^*) - Z(0) > 0 \iff (1 - e(q^*) + q^* \cdot e(q^*))(-t_2) > (-t_2) \iff e(q^*) < 1, \]

which is true for all \( q^* < \bar{q} \). Thus, the principal’s payoff is strictly higher when an interior level of \( q \) is chosen.

**B Proof of Proposition 2**

Taking the derivative of \( q^*(n, t_2) \) with respect to \( n \) yields

\[
\frac{d}{dn} q^* = -\frac{1}{(n + 1)n^2} \left\{ n + (n + 1)(n \ln t_2 + \ln \frac{1}{t_2(n + 1)} \right\} 
\]

(24)

so that it has to be shown that the term in curly brackets is negative. We proceed in two steps:

Step 1: We first show \( X(n, t_2) := (n \ln t + \ln \frac{1}{1+n}) < 0 \) for all \( n \geq 1 \) and \( t_2 > 1 \). Note first that \( X(n, 1) = \ln \frac{1}{1+n} < 0 \) for all \( n > 0 \). Furthermore, we have \( \frac{d}{dt_2} (n \ln t) = \frac{n}{t_2} \). Finally,

\[
\frac{d}{dt_2} \ln \frac{1}{t_2(n + 1)} = -\frac{nt_2^{n-1}}{t_2^n} = -\frac{n}{t_2},
\]

(25)

so that \( X(n, t_2) \) is in fact constant in \( t_2 \) and equal to \( \ln \frac{1}{1+n} < 0 \).

Step 2: It has to be shown that \( Y(n) := n + (n + 1)X(n) < 0 \). Taking the derivative w.r.t. \( n \) yields

\[
\frac{dY}{dn} = \ln \frac{1}{n + 1}
\]

(26)

which is negative for all \( n > 0 \). It follows that \( Y(n) \) is maximum at \( n = 0 \) and that \( Y(0) = 0 \) so that \( Y(n) < 0 \) for all \( n \geq 0 \).
C Proof of Proposition 3

Recall that with $H(c) = c^n$, we have $e^*(q, n, t^2) = q^n t^2$. Inserting $q^*(n, t^2)$ from Eqn. (13) yields $e^*(n) = \frac{1}{n+1}$ and thus $s^*(n) = 1 - e^*(n) = \frac{n}{n+1}$. It then follows from Eqn. (17) that

$$\Delta_{\pi^m}(n, t^2) = \frac{1}{t^2} n \sqrt{\frac{1}{n+1}}.$$

(27)

Taking the derivative of (27) w.r.t $n$ yields

$$-\frac{1}{(n+1)}(n \ln t^2 + \ln \frac{1}{t^2(n+1)}) \sqrt{\frac{1}{n+1}} \frac{1}{t^2(n+1)}$$

(28)

where the first and the third term are clearly positive so that we have to show that the second term is negative. Notice that this amounts to verifying that $n \ln t^2 + \ln \frac{1}{t^2(n+1)} = X(n) < 0$, which was shown to hold in the proof of Proposition 2.

Furthermore as for the limit, we have $\lim_{n \to \infty} \frac{1}{t^2} n \sqrt{\frac{1}{n+1}} = \frac{1}{t^2}$ as claimed in the Proposition.

D Proof of Proposition 4

We first show that $q^{**}(a)$ as given in Eqn. (20) is increasing in $a$:

$$\frac{d}{da}q^{**}(a) = \frac{2\sqrt{1-a-2a^2} - 2 + a}{-6a^2 \sqrt{1-a-2a^2}},$$

(29)

Clearly, the denominator is negative, but the numerator is also negative in the relevant range: First note that it is equal to zero at $a = 0$. Moreover, it is strictly decreasing in $a$ as

$$\frac{d}{da}2\sqrt{1-a-2a^2} - 2 + a = -1 - 4a + \sqrt{1-a-2a^2} < 0,$$

(30)

which holds since the inequality holds for $a = 0$ and since the last two terms are also strictly decreasing in $a$. 
Finally note that, because $q^{**}(\frac{1}{2}) = 1 = \bar{q}$, the relevant range is $0 < a < \frac{1}{2}$ so that $\bar{a} = \frac{1}{2}$ in our case and that also $\sqrt{1 - a - 2a^2} > 0$ holds for all $a \in (0, \frac{1}{2})$.

It remains to show that, for $a < \bar{a}$, the principal’s equilibrium profit is also higher when choosing monitoring intensity $q^{**}(a)$ rather than $q = 1$. From Eqn. (19), when monitoring at rate $q^{**}(a)$, the principal’s payoff is

$$
\pi_2(q = q^{**}(a)) = \frac{(5a^2 - 2a + 2 + (a - 2)\sqrt{1 - a - 2a^2}) \cdot (-2 + a - \sqrt{1 - a - 2a^2})}{27a^2}.
$$

When monitoring with $q = 1$, the principal gets

$$
\pi_2(q = 1) = (1 - \alpha) \cdot N \cdot (a - 1) < 0.
$$

Taking the difference $\pi_2(q = q^{**}) - \pi_2(q = 1)$ yields

$$
\frac{12a^2 - 20a^3 + 3a - 2 - (4a^2 + 2a - a)\sqrt{1 - a - 2a^2}}{27a^2}, \quad (31)
$$

which is positive for all $a < \bar{a} = \frac{1}{2}$. Hence, the optimal monitoring policy is given by $q^{**}(a)$ for $a < \bar{a} = \frac{1}{2}$ and by $q = 1$ for $a \geq \bar{a}$.
References


