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ABSTRACT

Repeated Contests with Asymmetric Information

by Johannes Münster *

The same contestants often meet repeatedly in contests. Behavior in a contest potentially provides information with regard to one's type and can therefore influence the behavior of the opponents in later contests. This paper shows that if effort is observable, this can induce a ratchet effect in contests: high ability contestants sometimes put in little effort in an early round in order to make the opponents believe that they are of little ability. The effect reduces overall effort and increases equilibrium utility of the contestants when compared with two unrelated one-shot contests. It does, however, also introduce an allocative inefficiency since sometimes a contestant with a low valuation wins. The model assumes an imperfectly discriminating contest. In extension I show that, qualitatively, results are similar in a perfectly discriminating contest (all pay auction).

JEL Classification: C72, D72, D74, D82, M52

ZUSAMMENFASSUNG

Wiederholte Wettkämpfe mit asymmetrischer Information

Dieselben Wettkämpfer treffen oft wiederholt in Wettkämpfen aufeinander. Aus dem Verhalten in einem Wettkampf können die Gegner Informationen über den Typ eines Wettkämpfers erhalten: seine Fähigkeit und Motivation zu gewinnen. Auf diesem Weg kann das Verhalten einen Einfluss auf das Verhalten der Gegenspieler in späteren Wettkämpfen haben. Dieser Aufsatz zeigt, dass dies zu einem Sperrklinken Effekt in wiederholten Wettkämpfen führen kann. Die Beteiligten strengen sich in einer frühen Runde manchmal nicht sehr an, um ihre Gegenspieler glauben zu lassen, dass sie nicht sehr an einem Gewinn interessiert sind, und so eine spätere Runde einfach gewinnen zu können. Dieser Effekt verringert die gesamte Leistung in dem Wettkampf. Er führt darüber hinaus zu einer allokativen Ineffizienz, da manchmal ein Spieler gewinnt, dem dies nicht viel Wert ist.

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1 Introduction

Contests or tournaments are ubiquitous. Examples include tournaments and, more generally, competition for advancement and promotion within firms, rent seeking contests, R&D races, election races, and appropriative conflicts. In the last decades an extensive literature concerning contests has been developed.[1]

However, little is known about repeated contests with asymmetric information. In many real world applications, the same contestants meet repeatedly in contests. For example, in many firms there are contests for becoming the "employee of the month", and in many universities there are "teacher of the year" awards. Moreover, important representative jobs, like chairmanship of a political organization, are typically held for only a limited time. Thus, the competition for getting such a job has the character of a repeated contest. Rent-seeking contests, too, are sometimes repeated.

This paper begins the study of repeated contests with asymmetric information using a stylized model of an imperfectly discriminating contest in the tradition of Tullock (1980). Two contestants compete in a contest that is repeated once. They observe the effort chosen by the rival in the first round before deciding on their own effort in the second round. The contestants have private information about how much they value winning, or about their abilities. Formally, I assume that each contestant has either a high or a low valuation, where the low valuation is zero. The model describes situations where the contestants do not know whether or not they face an active competitor, as in Myerson and Wärneryd (2006), Münster (2006), and Lim and Matros (2007). The valuations of the contestants are independent, but are constant over time. Of course, this highly stylized framework pre-

[1] This literature has been developed in several different fields. Excellent surveys are available. Rosen (1988) and Konrad (2007) are on contests generally. Lazear (1995) includes a chapter on labor market tournaments. See Nitzan (1994) on rent-seeking, and Baye and Hoppe (2003) on the strategic equivalence between rent-seeking contests and R&D races. Recent work on research tournaments includes Che and Gale (2003) and Fullerton and McAfee (1999). Szymanski (2003) surveys the design of sport tournaments. Skaperdas (2003) and Garfinkel and Skaperdas (2006) review the economic literature on appropriative conflict. In auction theory, there is a closely related literature on all pay auctions (in the language of contest theory, an all pay auction is a perfectly discriminating contest), see Baye, Kovenock and deVries (1996).

cludes consideration of many interesting topics. On the other hand, it yields a tractable model that leads to important insights into the effects that are present in repeated contests with asymmetric information.

The main insight is that there can be a ratchet effect in repeated contests: contestants with a high ability or a high valuation sometimes put in little effort in an early round in order to make the opponents believe that their ability is low - they are *sandbagging*. This reduces the resources spent in the contest. With regard to labor markets, this points to a drawback of relative performance compensation schemes. Moreover, sandbagging introduces an allocative inefficiency, since a contestant with a low valuation now sometimes wins in round one. However, I show that the net effect is beneficial for the contestants. Applied to rent-seeking, this means that, when rent-seeking activities are viewed as pure waste from a social point of view, rent dissipation is smaller. Thus, a repeated contest can actually be used to reduce the welfare loss due to rent-seeking. On the other hand, when rent-seeking activities are viewed as pure transfers, allocative efficiency is the only welfare criterion and the repeated contest therefore leads to higher welfare losses.

I show that there will be sandbagging in equilibrium if, and only if, the proportion of low valuation contestants is low. Otherwise, expected equilibrium effort and rent dissipation are like those in two unrelated one shot contests with asymmetric information.

As a robustness check, I also consider the case of a perfectly discriminating contest (an all pay auction). Results are qualitatively similar.

Contests with asymmetric information have been studied by Hurley and Shogren (1998a), who model one-sided asymmetric information, and by Hurley and Shogren (1998b) and Malueg and Yates (2004) who look at two-sided asymmetric information. Wärneryd (2003) is an interesting paper on the common value case. Myerson and Wärneryd (2006), Münster (2006), and Lim and Matros (2007) study contests where the contestants do not not how many competitors there are. None of these papers deals with *repeated* contests. My paper is also related to several papers on multi-stage contests. Rosen (1986) studies a sequential elimination tournament, and Gradstein and Konrad (1999) compare simultaneous contests with sequential contests. In contrast to the present paper, these papers look at sequential elimination contests where it is never the case that the *same* two contestants meet again in a later round. Most closely related to the present paper are Meyer (1991) and Meyer (1992) on the optimal design of a repeated contest between a pair of contestants, and Krähmer (2007), Mehlum and Moene (2006), and Amegashie (2006) on infinitely repeated contests between two contestants. In these papers, information is symmetric. Amegashie (2007) also discusses implications for signaling in contests. Moldovanu and Sela (2006) study an all-pay auction model of an elimination contest. They assume that the contestants who compete in a later round cannot directly observe the effort that their current rivals have chosen in an earlier round, thus abstracting away from the signaling issues at the heart of the present paper. In auction theory, the setup used by Jeitschko and Wolfstetter (2002) is close to my paper. My paper is also related to Hörner and Sahuget (2007) who study signaling in a dynamic auction. These papers study auctions with a deterministic allocation rule, whereas I look at an imperfectly discriminating contest, where there is some "noise" in the determination of the winner. Finally, my paper is related to Slantchev (2007), who studies incentives for feigning weakness in crisis bargaining with a potential escalation to war, which is modeled as a contest.

The paper is organized as follows. Section 2 sets out a simple model of repeated contests with asymmetric information. Section 3 looks at the second round contest. Section 4 studies the repeated contest. As a robustness check, section 5 discusses the case of a perfectly discriminating contest (all pay auction).

2 The model

There are two contestants $i = a, b$ and two rounds $t = 1, 2$. In each round, there is a prize to be won. There are two types of contestants. High valuation types value winning the prize with $v > 0$, while low valuation types have a valuation of zero. The low valuation types can also be thought of as having a budget cap of zero in each round, or infinite bid cost (zero ability). Hence the model is also applicable to a situation where an ability or endowment is needed in order to be able to compete at all, and only some of the contestants have this ability or endowment. Denote the type of i by v_i. Each contestant knows his own type but not the type of the other contestant. The valuations are independent across contestants but constant over time. In this model, a contestant does not know whether or not he faces any active rival, which is arguably a feature of many real world contests.

Let x_i^t denote the effort that contestant i chooses in round t. Contestant i wins in round t with probability

$$p_i^t = \begin{cases} \frac{x_i^t}{x_i^t + x_j^t}, & \text{if } x_i^t + x_j^t > 0, \\ \frac{1}{2}, & \text{if } x_i^t + x_j^t = 0. \end{cases} \tag{1}$$

This contest success function is commonly used in the literature. Microeconomic underpinnings have been developed by Mortensen (1982), Hirshleifer and Riley (1992), and Fullerton and McAfee (1999). For an axiomatization see Skaperdas (1996).[2]

In equation (1), it is assumed that a fair coin is flipped if no contestant puts in any effort. In some applications, it may be more natural to assume that a contestant cannot win if he chooses zero effort, and hence $p_i^t = 0$ for both $i = a, b$ if $x_a^t = x_b^t = 0$. Qualitatively, all the results below hold in this case, too.

Contestants are risk neutral and there is no discounting.[3] The objective function of contestant i is given by

$$u_i = \sum_{t=1}^{2} \left(p_i^t v_i - x_i^t \right). \tag{2}$$

For notational convenience, denote the utility gained in round t by $u_i^t := p_i^t v_i - x_i^t$.

The timing of the game is as follows. First, nature draws the types v_a and v_b independently from an infinite population. The fraction of low valuation types in the population is $\lambda \in (0, 1)$. Next each contestant learns

[2] In section 5 I consider an alternative specification: the perfectly discriminating contest where contestant i wins round t with probability one if $x_i^t > x_j^t$.

[3] Discounting would diminish the incentives to mislead opponents and thus increase the range of parameters where there is separation in round one. However, results would not change qualitatively.

his own type, but not the type of his rival. Then the first contest is played: the contestants simultaneously choose their first round efforts x_a^1 and x_b^1, respectively. Nature draws a winner according to the contest success function. The identity of the winner, as well as the efforts chosen, are revealed before the second round starts. Contestants update their beliefs according to Bayes' rule. In the second round, the contestants simultaneously choose x_a^2 and x_b^2, respectively. Again, nature draws a winner, and, finally, payoffs are received.

Note that the contest success function is discontinuous at $x_a^t = x_b^t = 0$. This leads to a technical problem concerning the existence of equilibria. In particular, the best response of a high valuation contestant to zero effort in round two is not well defined, since any strictly positive effort ensures victory. However, this problem is an artifact of the continuous strategy space. For example, if efforts are amounts of money, and there is a smallest unit of the currency, this problem does not show up. To avoid this rather uninteresting problem, I assume that, if a contestant puts in any effort at all, then he has to put in at least some strictly positive (but 'small') amount ε. That is, $x_i^t \in X := \{0\} \cup [\varepsilon, \infty)$ for some $\varepsilon > 0$. I will study the game for an arbitrarily small ε.[4]

In this game, a pure strategy consists of two functions x_i^1, x_i^2, where $x_i^1 : \{0, v\} \to X$ specifies i's effort in $t = 1$ as a function of i's type and $x_i^2 : \{0, v\} \times X^2 \times \{a, b\} \to X$ specifies i's effort in $t = 2$ as a function of i's type, efforts of both contestants in $t = 1$, and the winner in the first round. In addition to strategies, we also have to consider the beliefs of the con-

[4]There are several other ways of dealing with the problem; all lead to similar conclusions. (i) Following Blume and Heidhues (2006), one could allow additional efforts such as 0^+, which is identical to 0 except that 0^+ wins, with probability one, against 0. This approach will be used for the analysis of the perfectly discriminating contest in section 5 below. (ii) Another alternative is to use an endogenous tie-breaking rule, following Jackson et al. (2002). The only change of (1) concerns the case where $x_i^2 = x_j^2 = 0$. In the case of a tie at zero in $t = 2$, the contestant with the higher valuation wins with probability one. One might object to this tie-breaking rule because it depends on the valuations of the contestants, which are private information and thus not observable to the contest designer. But this difficulty can be solved by asking the contestants to report their types and breaking the tie according to the answers. Put more formally, in case of a tie at zero in $t = 2$, each contestant sends a message $s_i \in \{0, v\}$. If $s_i = v > s_j = 0$, then $p_i^2 = 1$. It will become clear that reporting one's true type is incentive compatible. There is one further subtlety concerning existence of optimal actions off the equilibrium path which will be dealt with below (in footnote 8).

testants. In the first round, each contestant thinks that his opponent has a low valuation with probability λ. Concerning the second round, let μ_a denote the probability that contestant a, after having observed x_b^1 and the identity of the winner of round one, ascribes to the event that his opponent has a low valuation ($v_b = 0$). Similarly, μ_b is the probability b ascribes to $v_a = 0$. I use perfect Bayesian equilibrium as the solution concept. A perfect Bayesian equilibrium consists of strategies and beliefs for each contestant such that (i) strategies are sequentially optimal, given the beliefs and the strategies of the opponent, and (ii) beliefs are updated according to Bayes' rule wherever possible.

Finally, I assume that social welfare is given by

$$w = \sum_{i=a,b} \sum_{t=1}^{2} \left(p_i^t v_i - \alpha x_i^t \right) . \tag{3}$$

The parameter α captures a value judgement concerning how the efforts should be evaluated. In applications to rent-seeking, α is the proportion of rent-seeking activities that are wasted from a social welfare point of view.[5] If $\alpha = 1$, rent-seeking is pure waste; welfare coincides with the contestants' utility. On the other hand, in some situations, it may be more reasonable to view rent-seeking activities as transfers to some third party. The case $\alpha = 0$ captures the case of pure transfers. Then, allocative efficiency is all that matters.

3 The second round contest

This section begins the analysis by studying the second round contest. Given beliefs μ_a and μ_b, the game in round two is identical to a one-shot contest with two-sided asymmetric information. This also provides a convenient benchmark for the behavior in the entire two stage game.

For a low valuation contestant, choosing $x_i^2 = 0$ is a strictly dominant strategy. Thus, if a contestant with a high valuation believes that his opponent has a low valuation with probability one, he will not put in any effort

[5] See, for example, Baye, Kovenock and deVries (1996).

except ε.

Consider now the case where $\mu_a < 1$ and $\mu_b < 1$. If a contestant has a high valuation, he will choose a strictly positive effort in equilibrium. To see this, suppose to the contrary that there is a contestant $i \in \{a, b\}$ who chooses $x_i^2 = 0$. Then the best reply of the high valuation type of the other contestant j is $x_j^2 = \varepsilon$; but, given that, the high valuation type of contestant i should choose some strictly positive x_i^2, a contradiction.

Hence the high valuation type of contestant a solves

$$\max_{x_a^2 \geq \varepsilon} \left(\mu_a + (1 - \mu_a) \frac{x_a^2}{x_a^2 + x_b^2} \right) v - x_a^2,$$

where x_b^2 denotes the effort of the high valuation type of contestant b. Similarly, the high valuation type of contestant b solves

$$\max_{x_b^2 \geq \varepsilon} \left(\mu_b + (1 - \mu_b) \frac{x_b^2}{x_a^2 + x_b^2} \right) v - x_b^2.$$

The first order conditions are

$$\begin{aligned} (1 - \mu_a) \frac{x_b^2}{(x_a^2 + x_b^2)^2} v &= 1, \\ (1 - \mu_b) \frac{x_a^2}{(x_a^2 + x_b^2)^2} v &= 1. \end{aligned}$$

The objective functions are concave, and hence the first order conditions are also sufficient for a maximum. Solving, we find that the efforts of the high valuation types are ($i = a, b$; $j \neq i$)

$$x_i^2 = \frac{(1 - \mu_j)(1 - \mu_i)^2}{(2 - \mu_i - \mu_j)^2} v. \tag{4}$$

If $\mu_a < 1$ and $\mu_b < 1$, these expressions are strictly positive and hence the constraints that $x_i^2 \geq \varepsilon$ do not bind.

Lemma 1 *Given beliefs $\mu_a < 1$ and $\mu_b < 1$, there is a unique equilibrium in the second round of the game, where the efforts of the high valuation types* *are given by equation (4). Expected utility of a high valuation type is*

$$u_i^2 = \mu_i v + \frac{(1-\mu_i)^3}{(2-\mu_i-\mu_j)^2} v \quad i = a, b; \ i \neq j.$$

Some special cases are particularly interesting. For example, if $\mu_a = \mu_b = 0$, we are basically in a full information contest between two high valuation contestants, and get the well known equilibrium where the effort of a contestant equals $v/4$. More importantly, the following corollaries are immediate and will be used frequently below.

Corollary 1 *Consider the symmetric case where $\mu_a = \mu_b = \lambda \in (0,1)$. Here, the effort of a high valuation contestant is*

$$x_i^2 = (1-\lambda)\frac{v}{4},$$

and expected utility of a high valuation contestant is

$$u_i^2 = \lambda v + (1-\lambda)\frac{v}{4}.$$

Corollary 2 *Consider the case where $0 < \mu_i < 1$ but $\mu_j = 0$ (contestant j believes with probability one that his opponent has a high valuation). The expected utility of the high valuation types equals*

$$\begin{aligned} u_i^2 &= \mu_i v + \frac{(1-\mu_i)^3}{(2-\mu_i)^2} v, \\ u_j^2 &= \frac{v}{(2-\mu_i)^2}. \end{aligned}$$

4 The repeated contest

4.1 Updating of beliefs

Before the contestants enter round two, they observe all actions taken in round one and the winner of the contest in round one, and update their beliefs about their rivals' type. Recall that types are drawn independently.

Moreover, the identity of the winner does not carry any additional information about the types once the first round efforts are known. Hence, the updated belief μ_i can be regarded as a function of the rivals' first round effort x_j^1 alone.

To simplify the exposition, I will impose the following simple reasonable belief refinement:

$$\mu_i = 0 \text{ if } x_j^1 > 0 \quad (i \neq j). \tag{5}$$

This seems reasonable because any $x_j^1 > 0$ is strictly dominated for the low valuation contestant.[6] If we do not impose (5), the set of equilibria is larger, but the additional equilibria differ from those that satisfy (5) only off the equilibrium path.[7]

4.2 Separation in round one

Here I look for perfect Bayesian equilibria (henceforth, equilibria) where both contestants "separate" in round one. A contestant i *separates in round one* iff $x_i^1(v) \neq x_i^1(0)$ with probability one. I will call an equilibrium an *equilibrium with separation in round one* if both contestants separate in round one.

Suppose that both contestants separate in round one. Then we have $x_i^1(v) > 0$ for $i = a, b$. From equation (5), for any $x_i^1 > 0$ we have $\mu_j(x_i^1) = 0$, and therefore i can not influence j's belief by choosing between different positive first round efforts. Hence the first round effort of a high valuation contestant must maximize his payoff for this round.

Therefore, in an equilibrium with separation in round one, $x_i^1(v)$ must

[6] Of course, any $x_i^t > 2v$ is strictly dominated by $x_i^t = 0$ for the high valuation type, too. However, this is not crucial - beliefs $\mu_i(x)$ for $x > 2v$ do not matter.

[7] In particular, if some behavior in round one cannot be supported as part of an equilibrium by beliefs that satisfy (5), it cannot be supported by *any* beliefs that are consistent with the strategies. This is due to the fact that, in the model, it is always good to be underestimated: if contestant i has a high valuation, his second round payoff increases in μ_j (see lemma 1). Suppose that μ_j does not satisfy (5). Then, after observing some $x_i^1 > 0$ off the equilibrium path, j may assign a positive probability to $v_i = 0$, and hence play less aggressively in round two. On the other hand, if $x_i^1 > 0$ is on the equilibrium path, consistency with strategies implies $\mu_j(x_i^1) = 0$, since any $x_i^1 > 0$ is strictly dominated for the low valuation type. This implies that the incentives to deviate in round one from some candidate equilibrium can only be stronger when beliefs do not satisfy (5).

solve

$$\max_{x_i^1 \geq \varepsilon} \left(\left(\lambda + (1-\lambda) \frac{x_i^1}{x_i^1 + x_j^1(v)} \right) v - x_i^1 \right)$$

for $i = a, b$, $j \neq i$. The solution to these problems is

$$x_i^1(v) = (1-\lambda) \frac{v}{4},$$

and gives a high valuation contestant $u_i^1 = \lambda v + (1-\lambda) v/4$.

Now consider the second round. Given beliefs, the optimal behavior of a high valuation type in the second round is easily described:

$$x_i^2 = \begin{cases} \varepsilon, & \text{if } x_j^1 = 0, \\ \frac{v}{4}, & \text{if } x_j^1 > 0. \end{cases}$$

Expected utility of a high valuation contestant from the second round equals $u_i^2 = \lambda v + (1-\lambda) v/4$ (for $\varepsilon \to 0$). Summing up, we have

Lemma 2 *If there is an equilibrium with separation in round one, then strategies in this equilibrium satisfy*

$$x_i^1 = \begin{cases} 0, & \textit{if} \quad v_i = 0, \\ (1-\lambda) \frac{v}{4}, & \textit{if} \quad v_i = v, \end{cases}$$

$$x_i^2 = \begin{cases} 0, & \textit{if} \quad v_i = 0, \\ \varepsilon, & \textit{if} \quad v_i = v \textit{ and } x_j^1 = 0, \\ \frac{v}{4}, & \textit{if} \quad v_i = v,\ x_i^1 > 0 \textit{ and } x_j^1 > 0. \end{cases}$$

Beliefs in round two are

$$\mu_i = \begin{cases} 0 & \textit{if} \quad x_j^1 > 0, \\ 1 & \textit{if} \quad x_j^1 = 0. \end{cases}$$

Expected utility equals

$$u_i = \begin{cases} 0, & \textit{if} \quad v_i = 0, \\ 2\left(\lambda v + (1-\lambda) \frac{v}{4}\right), & \textit{if} \quad v_i = v > 0. \end{cases}$$

By comparing this lemma with the results of the previous section, we immediately come to the following proposition.

Proposition 1 *In an equilibrium with separation in round one, all the actions taken in the first round are the same as those in a symmetric one-shot contest with two-sided asymmetric information. All the actions taken in the second round are the same as those in the corresponding one-shot contests with full information. Expected effort in each round equals the expected effort in a symmetric one-shot contest with two-sided asymmetric information.*

Proof. The equivalence of the actions in round one follows by comparing lemma 2 with corollary 1.

In an equilibrium with separation in round one, all private information is revealed in round one. Hence, in round two, the equilibrium actions must be the same as in the corresponding one-shot contests with complete information.

By lemma 2, total expected first round effort in an equilibrium with separation in round one equals $(1-\lambda)^2 v/2$, as in a symmetric one-shot contest with two-sided asymmetric information. Ex ante expected second round effort of one contestant equals

$$\lambda 0 + (1-\lambda)\left(\lambda\varepsilon + (1-\lambda)\frac{v}{4}\right) = (1-\lambda)^2 \frac{v}{4} + (1-\lambda)\lambda\varepsilon.$$

(With probability λ, the contestant has low valuation and chooses zero effort. With the remaining probability $1-\lambda$ he is a high valuation contestant. He chooses ε in the second round if his opponent is a low valuation contestant, which happens with probability λ. On the other hand, with $1-\lambda$ the opponent is also a high valuation contestant, and both choose $v/4$ in the second round.) Therefore, with $\varepsilon \to 0$, total expected effort of the second round is $(1-\lambda)^2 v/2$, too. ■

The following proposition 2 gives a necessary and sufficient condition for existence of an equilibrium with separation in round one.

Proposition 2 *An equilibrium with separation in round one exists if, and only if,* $\lambda \geq 1/2$.

Proof. A low valuation contestant never wants to deviate. A high valuation contestant can neither gain by deviating in round two only, nor by deviating to an $x_i^1 > 0$. Therefore, we only have to check whether a high valuation contestant wants to deviate to $x_i^1 = 0$. Denote the first round payoff from the deviation under consideration by $u_i^1 (dev)$. Here, $u_i^1 (dev) = \lambda v/2$. Because $x_i^1 = 0$, contestant j thinks i is a low valuation contestant: $\mu_j (0) = 1$.

Suppose that contestant j has a high valuation. Then he will play $x_j^2 = \varepsilon$ according to the equilibrium strategy. Contestant $i's$ best response to $x_j^2 = \varepsilon$ is $x_i^2 = -\varepsilon + \sqrt{\varepsilon v}$. With $\varepsilon \to 0$, we have $x_i^2 = \sqrt{\varepsilon v} - \varepsilon \to 0$, and

$$p_i^2 = \frac{-\varepsilon + \sqrt{\varepsilon v}}{(-\varepsilon + \sqrt{\varepsilon v}) + \varepsilon} = -\frac{\sqrt{\varepsilon}}{\sqrt{v}} + 1 \to 1.$$

Therefore, if j is a high valuation contestant, deviating gives contestant i a second round utility $u_i^2 (dev) = v$.[8]

On the other hand, if j is a low valuation contestant, $x_j^2 = 0$ and $x_i^2 = \varepsilon$, so with $\varepsilon \to 0$ we again have $u_i^2 (dev) = v$. Putting things together,

$$u_i (dev) = u_i^1 (dev) + u_i^2 (dev) = \lambda \frac{v}{2} + v.$$

Contestant i has no incentive to deviate if, and only if, $u_i \geq u_i (dev)$, that is,

$$2 \left(\lambda v + (1 - \lambda) \frac{v}{4} \right) \geq \lambda \frac{v}{2} + v.$$

This inequality holds if, and only if, $\lambda \geq 1/2$. ■

As proposition 2 shows, whether an equilibrium with separation in round one exists depends on the fraction of low valuation contestants, but not on the high valuation v.

To gain some intuition, consider the extreme case where $\lambda = 0$. Then both contestants think that their opponent is a high valuation contestant.

[8] As shown above, the assumption that $x_i^t \in X = \{0\} \cup [\varepsilon, \infty)$ ensures that there is a best reply to ε, i.e. a best reply to the best reply to zero. To ensure the existence of a best reply to the best reply to zero in the alternative approaches discussed in footnote 4 above, one can (i) introduce another effort level 0^{++}, which is identical to 0^+, except that 0^{++} wins against 0^+. (ii) With an endogenous tie-breaking rule, the following stipulation for a tie at $x_i^2 = x_j^2 = 0$ works: if the reports are $s_i = s_j = v$ and $x_i^1 = 0 < x_j^1$, then $p_i^2 = 1$. Note that reporting one's true type is incentive compatible.

By behaving according to the strategy described in lemma 2, a high valuation contestant gets twice the payoff of a one-shot full information contest, that is, $u_i = v/2$. If he deviates to $x_i^1 = 0$, he loses the first round with probability 1. On the other hand, he tricks his opponent into believing that he has a low valuation, and therefore wins the second round without any effort. Therefore, $u_i\left(dev\right) = v$ which is greater than $u_i = v/2$.

Now consider the case where $\lambda = 1$. Here, both contestants think their opponent has low valuation. By behaving according to the proposed strategy, a high cost contestant gets $2v$. If he deviates to $x_i^1 = 0$, he loses in round one with probability $1/2$. However, he does not gain anything in terms of second round payoff, because his opponent has a low valuation and will choose zero effort anyway. Therefore, the contestant does not gain by deviating.

Proposition 2 shows that these considerations generalize: an equilibrium with separation in round one exists if, and only if, λ is not too small.[9] Moreover, it will become clear below that this is the only symmetric equilibrium in this case.

4.3 Pooling in round one

A contestant i *pools in round one* iff $x_i^1\left(v\right) = x_i^1\left(0\right)$ with probability one. The following proposition is a negative result, which will turn out to be very useful later on.

Proposition 3 *There is (i) no equilibrium where both contestants pool in round one, and (ii) no equilibrium where one contestant pools in round one and the other contestant separates in round one.*

Proof. See appendix. ■

The intuition behind proposition 3 is straightforward. Suppose that, contrary to proposition 3, there is an equilibrium where both contestants pool in

[9] If a contestant cannot win without putting any effort, the incentives to deviate are lower, since he gets a payoff of zero from the first round if he chooses $x_i^1 = 0$. Therefore, the range of the parameter λ where an equilibrium with separation in round one exists is bigger. To be more precise, in the same way as in the proof of proposition 2, it can be proved that an equilibrium with separation in round one exists in this case if, and only if, $\lambda \geq 1/3$.

round one. Then a contestant with a high valuation can win the first round by spending only a tiny amount ε. The drawback is that, by doing this, the contestant shows that he has a high valuation; however, this is outweighed by the benefit of winning easily in round one. Thus, each high valuation contestant would like to deviate in round one. The same logic applies also to (ii): if only one contestant pools in round one and the other contestant separates in round one, then the high valuation type of the separating contestant will spend only ε in the first round, and hence the pooling contestant can still win the first round easily. The pooling contestant would like to deviate here, too.

4.4 Partial pooling in round one

A contestant i *pools partially in round one* iff i neither pools nor separates in round one. That is, we have $x_i^1(v) = x_i^1(0)$ with some probability $q \in (0,1)$, and $x_i^1(v) \neq x_i^1(0)$ with the remaining probability $1-q$. An *equilibrium with (symmetric) partial pooling in round one* is an equilibrium in which both contestants pool partially in round one (with the same probability q).

Clearly, in any equilibrium, the low valuation types choose zero effort with probability one. Thus, in any equilibrium with partial pooling in round one, the high valuation types play a non-degenerate mixed strategy which puts some mass on zero effort in the first round. With the remaining probability mass, they might, in principle, mix over several positive first-round efforts. However, given the updating of beliefs according to (5), a contestant cannot influence the belief of his rival by choosing between different strictly positive efforts in round one. Thus, if a high valuation type chooses a strictly positive effort in round one with a strictly positive probability, this effort must maximize his first round payoff. The first round payoff is a strictly concave function, whatever the strategy of the opponent may be. Thus it has a unique maximizer. Therefore, in equilibrium, high valuation types never mix between different strictly positive effort levels.

This implies that, in any equilibrium with symmetric partial pooling in round one, the high valuation types mix between zero and a unique strictly positive effort level in round one:

$$x_j^1(v) = \begin{cases} 0, & \text{with probability } q > 0, \\ x_j^1 > 0, & \text{with probability } 1-q, \end{cases} \qquad (j = a, b).$$

Thus, the optimal strictly positive first round effort of contestant $i = a, b$ $(i \neq j)$ solves

$$u_i^1 = \lambda v + (1-\lambda)\left(q + (1-q)\frac{x_i^1}{x_i^1 + x_j^1}\right) v \to \max_{x_i^1 \geq \varepsilon}$$

The first order conditions of these problems are

$$(1-\lambda)(1-q)\frac{x_j^1}{\left(x_i^1 + x_j^1\right)^2} v = 1 \quad i = a, b;\ i \neq j.$$

Solving, we get

$$x_i^1 = \frac{(1-\lambda)(1-q)}{4} v =: \hat{x} \tag{6}$$

Since $q > 0$, this is strictly positive, and hence the constraint $x_i^1 \geq \varepsilon$ is not binding.

Lemma 3 *If there is an equilibrium with symmetric partial pooling in round one, then strategies in this equilibrium are as follows:*
In t=1, a high valuation contestant i chooses $x_i^1 = 0$ with some uniquely defined probability $q \in (0, 1)$, and with the remaining probability $(1-q)$ he chooses $x_i^1 = \hat{x}$ defined in equation (6).
Beliefs in round two are

$$\mu_i\left(x_j^1\right) = \begin{cases} \mu, & \textit{if } x_j^1 = 0, \\ 0, & \textit{if } x_j^1 > 0, \end{cases} \tag{7}$$

where

$$\mu := \frac{\lambda}{\lambda + (1-\lambda) q}. \tag{8}$$

Effort of a high valuation contestant i in t=2 is

$$x_i^2 = \begin{cases} \frac{(1-\mu)}{4}v, & \textit{if } x_i^1 = x_j^1 = 0, \\ \frac{(1-\mu)^2}{(2-\mu)^2}v, & \textit{if } x_i^1 > x_j^1 = 0, \\ \frac{(1-\mu)}{(2-\mu)^2}v, & \textit{if } x_j^1 > x_i^1 = 0, \\ \frac{v}{4}, & \textit{if } x_j^1 > 0 \textit{ and } x_i^1 > 0. \end{cases}$$

Low valuation contestants always choose zero effort.

Proof. Strategies for round one follow from the discussion above. Strategies for round two follow from section 3. Beliefs must be consistent with the strategies; together with (5), this implies equation (7). ■

The following proposition 4 gives a necessary and sufficient condition for existence of an equilibrium with symmetric partial pooling in round one.

Proposition 4 *An equilibrium with symmetric partial pooling in round one exists if, and only if,* $\lambda < 1/2$.

Proof. See appendix. ■

To understand the logic behind proposition 4, let me briefly sketch the idea of the proof here. Suppose $l < 1/2$. By construction of the strategies, we only have to show that there is a $q \in (0,1)$ such that a high valuation contestant is indifferent between choosing $x_i^1 = \hat{x}$ and $x_i^1 = 0$ if his opponent behaves according to the strategies in lemma 3.

If q is (close to) zero, both contestants (almost) separate in round one. If $\lambda < 1/2$, then we know from proposition 2 that there is no equilibrium with separation in round one. That is, contestant i would strictly prefer to play $x_i^1 = 0$.

On the other hand, if q is (close to) one, then both contestants (almost) pool in round one. As we have seen above, there is no equilibrium with pooling in round one, since contestant i would prefer playing some small positive effort to playing $x_i^1 = 0$. Note that with $q \to 1$, $\hat{x}$ gets small. Thus, i strictly prefers $x_i^1 = \hat{x}$ to playing $x_i^1 = 0$.

I show in the appendix that the incentives to choose $x_i^1 = \hat{x}$ over $x_i^1 = 0$ are continuous and strictly increasing in q. Thus, we can use the intermediate value theorem to conclude that there exists a unique $q \in (0,1)$ such that i is indifferent between choosing $x_i^1 = \hat{x}$ and $x_i^1 = 0$. This shows that, if $\lambda < 1/2$, an equilibrium with symmetric partial pooling in round one exists and is unique in this class of equilibria. Moreover, there are no other symmetric equilibria in this case, since neither an equilibrium with pooling nor an equilibrium with separation in round one exists. The thick line in figure 1 below plots the equilibrium q as a function of λ.

On the other hand, if $\lambda \geq 1/2$, an equilibrium with separation in round one exists. Contestant i would strictly prefer to play $\hat{x}$ even in the case where j separates in round one ("mixes" with $q = 0$). A fortiori, if contestant j pools partially in period one, i is strictly better off with $x_i^1 = \hat{x}$ than with $x_i^1 = 0$. Thus no equilibrium with symmetric partial pooling in round one exists if $\lambda \geq 1/2$.[10]

These results indicate that, if the fraction of low valuation contestants is low, there is a ratchet effect in repeated contests. High valuation contestants are sometimes *sandbagging*: they sometimes choose low effort in order to make the opponent believe that they have a low valuation, which makes the opponent less aggressive in the second round. The overall effect is that total expected effort is decreased.

Proposition 5 *In an equilibrium with symmetric partial pooling in round one, expected overall effort is smaller than in two unconnected one-shot contests with two-sided asymmetric information.*

Proof. See appendix. ■

Expected effort in the first round is lower than in a one-shot contest for two reasons. First, the high valuation contestants sometimes choose zero effort. Second, even if they choose a positive effort, it is nevertheless smaller than the effort chosen in a one-shot contest:

$$\hat{x} = (1-q)\,\frac{(1-\lambda)\,v}{4} < \frac{(1-\lambda)\,v}{4}.$$

[10] In the same way, one can also show that, if one cannot win without putting in any effort, an equilibrium with symmetric partial pooling in round one exists if, and only if, $\lambda < 1/3$.

The reason is that the other contestant chooses zero effort with probability q even if he has a high valuation; hence the marginal benefit of first round effort is lower.

For the second round, the comparison is less straightforward. Depending on the efforts chosen in round one, effort in round two may be higher or lower than in a one shot contest. For example, if both contestants have a high valuation and both choose $\hat{x}$ in the first round, then they will both choose $v/4$ in the second round, more than in a one-shot contest with asymmetric information. However, if they choose $x_a^1 = x_b^1 = 0$, second round efforts will be lower:

$$x_a^2 = x_b^2 = \frac{(1-\mu)\,v}{4} < \frac{(1-\lambda)\,v}{4}.$$

The reason is that, after observing $x_i^1 = 0$, contestant $j = a, b$ thinks that $i \neq j$ has a low valuation with higher probability $\mu > \lambda$.

As proposition 5 shows, the overall effect is that expected effort in the repeated contest is unambiguously lower than in two unrelated one-shot contests. Turning to rent dissipation and welfare, it is clear that lower expected effort is beneficial for the contestants. But there is a countervailing effect: sometimes a low valuation contestant gets the prize in round one, and thus the allocation can be worse than in two unrelated one-shot contests. As the following proposition shows, for the contestants the beneficial effect dominates.[11]

Proposition 6 *Expected utility is higher in the equilibrium with symmetric partial pooling in round one than in two unrelated one shot contests.*

Proof. See appendix. ■

Another interesting benchmark for comparison is a single contest between two contestants, where each high valuation type values winning the prize by $2v$. In such a contest, expected effort and rent dissipation is exactly as in two unrelated one-shot contests where each high valuation type values winning one of the contests by v. Thus, in the equilibrium with symmetric partial pooling in round one, expected efforts are lower and equilibrium utilities are

[11] In the case where a contestant cannot win without putting in any effort, sometimes no one gets the prize in round one, although one, or even both, contestants have a positive valuation. However, it can be shown that the beneficial effect dominates in this case, too.

higher than in a single contest where each high valuation type values winning the prize by $2v$.

The effect on welfare depends on how the efforts are treated in the social welfare function. If $\alpha = 1$ in (3), welfare is the sum of the contestants' utilities and the repeated contest leads to higher expected welfare. On the other hand, if $\alpha = 0$, allocative efficiency is all that matters, and the repeated contest leads to lower expected welfare. Corollary 3 generalizes these observations.

Corollary 3 *There exists an $\alpha_0 \in (0, 1)$ such that expected welfare is higher in the equilibrium with symmetric partial pooling in round one than in two unrelated one-shot contests (or in a single contest where each high valuation type values winning the prize by $2v$) if, and only if, $\alpha > \alpha_0$.*

Proof. Let w_r denote expected welfare in the equilibrium with symmetric partical pooling in round one of the repeated contest. Moreover, let w_0 denote expected welfare in two unrelated contests. As argued above, $\alpha = 0$ implies that $w_r < w_0$, while $\alpha = 1$ implies $w_r > w_0$. By proposition 5, expected effort is lower in the repeated contest, thus by (3), $w_r - w_0$ is strictly increasing in α, and the result follows from the intermediate value theorem. ■

Thus the repeated contest is better if rent-seeking activities are considered sufficiently wasteful. This can be used to lessen the deadweight loss of rent-seeking, e.g. for a monopoly position. Suppose the monopoly is given to one of the rent seekers only for a limited amount of time, and then the question who is going to be the monopolist is opened up again. Although this adds a second rent-seeking contest, the total deadweight loss can be smaller due to sandbagging in the first round.

5 A perfectly discriminating contest

In this section I consider a perfectly discriminating contest (all pay auction) and show that, qualitatively, the results are the same as above. In a perfectly discriminating contest, the contestant who chooses the higher effort wins with probability one:

$$p_i^t = \begin{cases} 1, & \text{if } x_i^t > x_j^t, \\ \frac{1}{2}, & \text{if } x_i^t = x_j^t, \\ 0, & \text{if } x_i^t < x_j^t. \end{cases} \tag{9}$$

Like the lottery model (1), the contest success function (9) is frequently used in the literature (see, for example, Baye, Kovenock, deVries 1996 and Konrad 2007).

The assumption that there is a small minimum expenditure requirement $\varepsilon > 0$, which proved convenient for the analysis of the imperfectly discriminating contest, is not helpful for the analysis of the all pay auction. Instead, I will follow an approach which is used in auction theory (e.g. Blume and Heidhues 2006): I assume that x_i^t can be any non-negative real number and allow two additional effort levels 0^+ and 0^{++}. These efforts are identical to zero effort except that 0^+ wins against 0, and 0^{++} wins against 0^+ (and against 0). The role of 0^+ is similar with the role of the minimum expenditure requirement ε in the analysis above: to ensure existence of a best reply to zero effort in round two. The role of 0^{++} is to ensure existence of optimal actions off the equilibrium path: 0^{++} is the best reply to an effort of 0^+ in round two, it corresponds to $-\varepsilon + \sqrt{\varepsilon v}$, which, under (1), is the best reply to ε in $t = 2$ (see proof of proposition 2).[12] Both 0^+ and 0^{++} have a cost of zero, just as we studied the limit $\varepsilon \to 0$ above. The rival can discern efforts 0, 0^+, and 0^{++}. If we used these assumptions together with the contest success function (1), the analysis would proceed as presented above. These additional effort levels are just an innocuous way of dealing with difficulties caused by the continuous strategy space.[13]

The second round contest As above, let us start with round two. Consider first the symmetric case where $\mu_a = \mu_b = \gamma$. Here, high valua-

[12]Note that, under (9), there is no best reply to ε, since any higher effort wins with probability one. This is one reason why a minimum expenditure requirement is not helpful for the analysis of the all pay auction.

[13]Blume and Heidhues (2006) introduce 0^+ to circumvent the technical problem that there is no smallest real number above 0 and argue that this assumption is innocuous. One can also use the tie-breaking rule specified in footnotes 4 and 8 together with (9). This leads to the same results as the analysis presented in the main text.

tion types randomize uniformly on $(0, (1-\gamma) v]$, and the payoff of a high valuation types is γv. In particular, if $\gamma = 0$, we are basically in a complete information contest, where the all-pay auction results in complete rent dissipation and equilibrium payoffs of zero.

Now consider the case where $\mu_i \in (0,1)$ and $\mu_j = 0$ (j believes with probability one that his opponent i has a high valuation). In equilibrium, the high valuation type of contestant j randomizes uniformly on $(0, (1-\mu_i) v]$. The high valuation type of player i puts a mass point of μ_i on 0^+, and, with the remaining probability, randomizes uniformly on $(0, (1-\mu_i) v]$. Note that i wins if his opponent has a low valuation and thus plays $x_j^2 = 0$, which has probability μ_i. The expected utility of the high valuation types is $u_i = u_j = \mu_i v$. To gain some intuition, note that i can guarantee himself an expected utility of $\mu_i v$ by choosing 0^+. Therefore, i will never exert more effort than $(1-\mu_i) v$. But then the high valuation type of j can guarantee himself an expected utility arbitrarily close to $\mu_i v$ by choosing an effort just above $(1-\mu_i) v$. As usual in perfectly discriminating contests, any rents over and above these lower bounds are dissipated.

Separation in round one Suppose both contestants separate in round one. Then, in round one the high valuation types mix uniformly over $(0, (1-\lambda) v]$, and their payoffs from round one are λv. In stage 2, contestant i knows the type of his opponent. If $v_j = 0$, i gets v by choosing 0^+. If $v_j = v$, i gets zero. Thus i gets λv from the second round, and $u_i = 2\lambda v$ in total. Note that the actions and payoffs are exactly as in the corresponding one-shot contests - with two-sided asymmetric information for $t = 1$, and with complete information for $t = 2$. That is, proposition 1 holds.

When does an equilibrium with separation in round one exist? By construction, the only relevant consideration is whether it pays for a high valuation type of i to deviate to $x_i^1 = 0$. A deviation to $x_i^1 = 0$ gives a payoff of $\lambda v/2$ in $t = 1$. In $t = 2$, the opponent believes that i has a low valuation, i.e. $\mu_j = 1$; thus j chooses 0^+; the best reply of i is 0^{++} and i gets v.

Therefore, an equilibrium with separation in round 1 exists if, and only if, $2\lambda v \geq (\lambda v/2) + v$ or $\lambda \geq 2/3$. Hence, qualitatively, proposition 2 is robust: an equilibrium with separation in round 1 exists if, and only if, the fraction of low valuation types is sufficiently high. Note that the range of the parameter λ where an equilibrium with separation in round one exists is smaller than in the imperfectly discriminating contest. To understand why, note that the utility of a deviating contestant is the same under the contest success function (1) as under (9). But the utility in a (candidate) separating equilibrium is higher in the imperfectly discriminating contest (1), since a high valuation type has a strictly positive utility even if his opponent has a high valuation, too. In contrast, under (9), rent dissipation is complete if both contestants happen to have a high valuation.

Partial pooling in round one In any equilibrium with symmetric partial pooling in round one, the high valuation types play as follows. In $t = 1$, they choose 0 with some probability $q \in (0, 1)$ and randomize uniformly over $(0, (1 - \lambda)(1 - q) v]$ with the remaining probability $1 - q$. Thus the distribution of the first round effort of a high valuation type is

$$F(x) = q + \frac{x}{(1-\lambda)v} \text{ for } x \in [0, (1-\lambda)(1-q)v]. \tag{10}$$

Suppose contestant j follows this strategy. Then the expected first round utility of the high valuation type of i from $x_i^1 \in (0, (1 - \lambda)(1 - q) v]$ is

$$\begin{aligned} & \left(\lambda + (1-\lambda) F\left(x_i^1\right)\right) v - x_i^1 \\ = & \left(\lambda + (1-\lambda)\left(q + \frac{x_i^1}{(1-\lambda)v}\right)\right) v - x_i^1 \\ = & (\lambda + (1-\lambda) q) v. \end{aligned}$$

Now consider the second period utility from any $x_i^1 > 0$. If $x_j^1 = 0$, beliefs are updated to $\mu_j = 0$ and $\mu_i = \mu$, where μ is defined in (8). In this case, which has probability $\lambda + (1 - \lambda) q$, i gets μv. On the other hand, if $x_j^1 > 0$, beliefs are updated to $\mu_i = \mu_j = 0$, and i gets zero. Thus the expected second round utility from a strictly positive first round effort is $(\lambda + (1 - \lambda) q) \mu v = \lambda v$. Putting things together, the expected utility from $x_i^1 \in (0, (1 - \lambda)(1 - q) v]$ is $(2\lambda + (1 - \lambda) q) v$. Note that i has no incentive to choose 0^+ or 0^{++} in $t = 1$ since this gives the same expected utility.

Still assuming that j behaves according to (10), now suppose that i

chooses $x_i^1 = 0$. In $t = 1$, if j also chooses $x_j^t = 0$, i wins with probability $1/2$; i loses in all other cases. Thus, in the first round, i gets $(\lambda + (1-\lambda)\, q)\, v/2$. In the second round, if $x_j^1 = 0$, beliefs are updated to $\mu_a = \mu_b = \mu$ and i gets μv. On the other hand, if $x_j^1 > 0$, beliefs are $\mu_i = 0$ and $\mu_j = \mu$; again i gets μv. Thus the expected payoff of i from $x_i^1 = 0$ is $(\lambda + (1-\lambda)\, q)\, v/2 + \mu v$.

It follows that contestant i is indifferent between $x_i^1 = 0$ on the one hand, and any $x_i^1 \in (0, (1-\lambda)(1-q)\, v]$ on the other hand, if, and only if,

$$(\lambda + (1-\lambda)\, q)\, \frac{v}{2} + \mu v = (2\lambda + (1-\lambda)\, q)\, v \tag{11}$$

Using (8), it is straightforward to show that equation (11) cannot be satisfied by any $q \in (0,1)$ if $\lambda \geq 2/3$. On the other hand, if $\lambda < 2/3$, there is a unique $q \in (0,1)$ that solves equation (11), namely

$$q = \frac{1}{1-\lambda}\left(\sqrt{\lambda\,(\lambda+2)} - 2\lambda\right).$$

As above, an equilibrium with partial pooling exist if λ is sufficiently small; qualitatively proposition 4 is robust. Figure 1 compares the equilibrium q as a function of λ in the two different types of contests and shows that sandbagging occurs more often in the perfectly discriminating contest.

The expected payoff in an equilibrium with symmetric partial pooling is $v\sqrt{\lambda\,(\lambda+2)}$. Given $\lambda < 2/3$, this payoff is bigger than $2\lambda v$, which is the payoff in two unrelated one shot contests (or one single contest with a prize of value $2v$ for the high valuation types). Hence proposition 6 above applies to the perfectly discriminating contest as well.

As in the case of an imperfectly discriminating contest, the allocation of the object is inefficient in an equilibrium with partial pooling, while it is efficient in a one-shot contest. This effect decreases expected utility in the repeated contest. As we have seen, expected equilibrium utility is nevertheless higher. Therefore expected efforts must be lower. In other words, proposition 5 and corollary 3 hold as well.

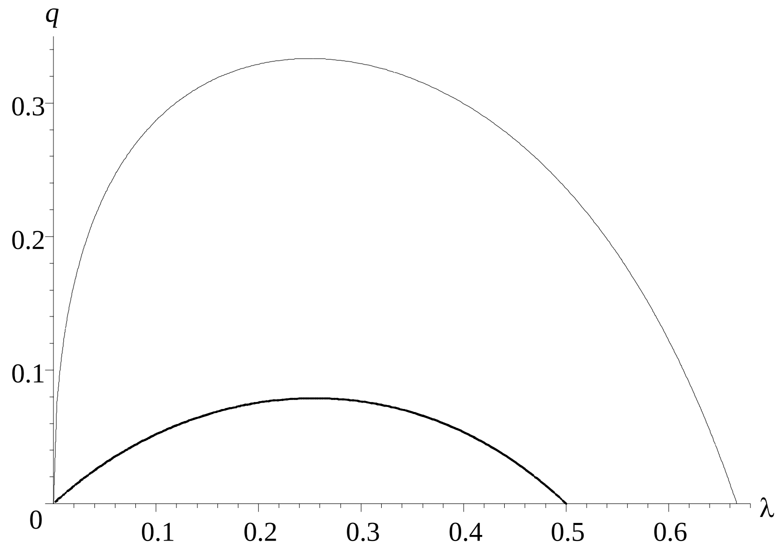

Figure 1: Equilibrium q as a function of λ in the imperfectly discriminating contest (thick line) and in the all-pay auction (thin line).

6 Conclusion

This paper has made a first step towards analyzing repeated contests with asymmetric information. It analyzed a highly stylized model, with two contestants of two types, who play a once repeated contest. The results show that the fraction of low valuation contestants is important. If the a priori probability of meeting a low valuation contestant is high, there will be separation in round one. Equilibrium play will be the same as in one-shot contests: in the first round, the same as in a one-shot contest with asymmetric information, in the second round, the same as in the corresponding one-shot contest with complete information. Expected effort and rent dissipation will be the same as in two unrelated one-shot contests with asymmetric information. On the other hand, if the a priori probability of meeting a low valuation contestant is low, there is no separation in round one. In this case, contestants with high valuations sometimes mimic the behavior of low valuation types in order to induce their opponents to believe that they don't care all that much about winning. This sandbagging reduces expected effort and rent dissipation.

Future research should go beyond the highly stylized two types, two contestants, two rounds framework analyzed here. When there are more than two contestants, separation in round one is more likely. When an opponent believes that a contestant has a low valuation this is beneficial for the contestant if, and only if, the opponent also thinks that there is no other high valuation contestant around - but this is less likely if there are many contestants. Thus, a higher number of contestants reduces incentives for sandbagging and thus makes separation in round one more likely.

There are many open questions on repeated contests with asymmetric information. One important simplifying assumption of the present paper is that the low valuation types have a valuation of zero. This assumption begs several interesting questions. For example, could it be that a low valuation type bluffs and imitates a high valuation type? Clearly, this is an important area for future work. In addition, it would be interesting to study the case where only the identity of the winner can be observed, but not the efforts chosen.

7 Appendix

7.1 Proof of proposition 3

(i) There is no equilibrium where both contestants pool in round one. Towards a contradiction, suppose there is an equilibrium where both contestants pool in round one. This implies $x_i^1(v) = x_i^1(0) = 0$, $p_i^1 = 1/2$, and $\mu_i(0) = \lambda$ since no information is revealed. The payoff of a contestant with high valuation is (see corollary 1)

$$u_i = \frac{v}{2} + \left(\lambda + \frac{(1-\lambda)}{4}\right) v.$$

Now consider what happens if a high valuation contestant deviates to $x_i^1 = \varepsilon$. Then he gets $u_i^1(dev) = v$ in the first round. In the second round, contestant j thinks that i has a high valuation, that is, $\mu_j = 0$. On the other hand, contestant $j's$ valuation is not known to i, and $\mu_i = \lambda$. The equilibrium payoff to contestant i is (by corollary 2 above)

$$u_i^2(dev) = \lambda v + \frac{(1-\lambda)^3}{(2-\lambda)^2} v.$$

Therefore,

$$u_i(dev) = v + \lambda v + \frac{(1-\lambda)^3}{(2-\lambda)^2} v.$$

It follows that

$$u_i(dev) - u_i = \frac{1}{4} v \frac{-3\lambda^3 + 9\lambda^2 - 12\lambda + 8}{(2-\lambda)^2} > 0 \text{ for all } \lambda \in [0,1].$$

Therefore, there is no equilibrium where both contestants pool in round one.

(ii) There is no equilibrium where one contestant pools in round one and the other contestant separates in round one. Suppose to the contrary that there is such an equilibrium. Assume without loss of generality that a pools[14], whereas b separates. Then we must have $x_a^1(v) = 0$ and $x_b^1(v) = \varepsilon$. Therefore, $u_a^1 = \frac{1}{2}\lambda v$ if $v_a = v$.

In the second round, a knows the type of b, and we have

$$u_a^2 = \begin{cases} v, & \text{if } v_a = v \text{ and } v_b = 0, \\ \frac{v}{(2-\lambda)^2}, & \text{if } v_a = v_b = v, \\ 0, & \text{if } v_a = 0. \end{cases}$$

(See corollary 2 for the second line.) Expected utility of contestant a therefore equals (if $v_a = v$)

$$u_a = \frac{1}{2}\lambda v + \lambda v + \frac{(1-\lambda)}{(2-\lambda)^2} v.$$

Now consider what happens if the pooling contestant a deviates to a positive $\tilde{x}_a^1$. The optimal positive effort is the best reply to $b's$ first round strategy. Hence

$$\tilde{x}_a^1 = \arg\max_{x_a^1 \geq \varepsilon} \left\{ \lambda v + (1-\lambda) \frac{x_a^1}{x_a^1 + \varepsilon} v - x_a^1 \right\} = \sqrt{(1-\lambda)\varepsilon v} - \varepsilon$$

[14] Here, and in the sequel, I omit "in round one" if this is clear from the context.

With $\varepsilon \to 0$, we get $\tilde{x}_a^1 \to 0$ and $p_a^1 \to 1$, and thus $u_a^1(dev) = v$. In the second round, we now have a game of complete information, and the ex ante expected second round payoff of contestant a is $u_a^2(dev) = \lambda v + (1-\lambda) v/4$. Therefore,

$$u_a(dev) = v + \lambda v + (1-\lambda)\frac{v}{4}.$$

It follows that

$$u_a(dev) - u_a = \frac{1}{4} v \frac{-3\lambda^3 + 17\lambda^2 - 28\lambda + 16}{(2-\lambda)^2} > 0 \text{ for all } \lambda \in [0,1].$$

Thus the pooling contestant a wants to deviate. Therefore, there is no equilibrium where one contestant pools in round one and the other contestant separates in round one.

7.2 Proof of proposition 4

(i) If $\lambda < 1/2$, the strategies and beliefs described in lemma 3 are an equilibrium for some unique $q \in (0,1)$. Consider the high valuation contestants. Deviating only in $t = 2$ does not pay. This follows from section 3. By construction, deviating to another strictly positive effort in round one does not pay, either.

It remains to show that there is a unique $q \in (0,1)$ such that a high valuation contestant is indifferent between 0 and $\hat{x}$ in the first round. Suppose contestant j behaves according to the proposed partial pooling strategy, and $v_i = v$. Let us first determine the payoff of i from playing $x_i^1 = 0$. If contestant i plays $x_i^1 = 0$, he gets

$$u_i^1 = \frac{1}{2}(\lambda + (1-\lambda) q) v,$$

since he wins in $t = 1$ with probability $1/2$ if contestant j also plays $x_j^1 = 0$, and loses for sure if contestant j plays $x_j^1 = \hat{x}$. Now consider the second round. If contestant j plays $x_j^1 = 0$, which happens with probability $\lambda + (1-\lambda) q$, we have a symmetric situation in $t = 2$: neither contestant knows the opponent's type, and they both have beliefs $\mu_i = \mu_j = \mu$ as defined in equation (8). In that case, the expected payoff of i is

$$E\left(u_i^2 \left| x_i^1 = x_j^1 = 0\right.\right) = \mu v + (1-\mu)\frac{v}{4}.$$

On the other hand, if contestant j plays $x_j^1 = \hat{x}$ (this happens with probability $(1-\lambda)(1-q)$), then we have an asymmetric situation where $\mu_i = 0$ while $\mu_j = \mu$. Then

$$E\left(u_i^2 \left| x_i^1 = 0, x_j^1 > 0\right.\right) = \frac{v}{(2-\mu)^2}.$$

Putting things together, the expected payoff of i if he plays $x_i^1 = 0$ is

$$\begin{aligned} u_i(0) \quad : \quad &= \frac{1}{2}(\lambda + (1-\lambda)q)v + \\ &(\lambda + (1-\lambda)q)\left(\mu v + (1-\mu)\frac{v}{4}\right) + (1-\lambda)(1-q)\frac{v}{(2-\mu)^2}. \end{aligned}$$

Now let us determine the payoff of i from playing $x_i^1 = \hat{x}$. In the first round, i gets

$$u_i^1 = (\lambda + (1-\lambda)q)v + (1-\lambda)(1-q)\frac{v}{4}.$$

Turning to the second round, if contestant j plays $x_j^1 = 0$, which happens with probability $\lambda + (1-\lambda)q$, we have an asymmetric situation in $t = 2$ where $\mu_j = 0$ and $\mu_i = \mu$. In this case,

$$E\left(u_i^2 \left| x_i^1 = \hat{x}, x_j^1 = 0\right.\right) = \mu v + \frac{(1-\mu)^3}{(2-\mu)^2}v.$$

On the other hand, if contestant j plays $x_j^1 = \hat{x}$ (this happens with probability $(1-\lambda)(1-q)$), then we have $\mu_a = \mu_b = 0$, and therefore $u_i^2 = v/4$. Putting things together, the expected payoff of i if he plays $x_i^1 = \hat{x}$ is

$$\begin{aligned} u_i(\hat{x}) \quad : \quad &= (\lambda + (1-\lambda)q)v + (1-\lambda)(1-q)\frac{v}{4} + \\ &+ (\lambda + (1-\lambda)q)\left(\mu v + \frac{(1-\mu)^3}{(2-\mu)^2}v\right) + \\ &+ ((1-\lambda)(1-q))\frac{v}{4}. \end{aligned} \tag{12}$$

Define $d(q) := u_i(\hat{x}) - u_i(0)$. We can establish the following lemma.

Lemma 4 *1. $d(q)$ is continuous for all $q \in [0,1]$.*

2. $d(0) < 0$ if, and only if, $\lambda < \frac{1}{2}$.

3. $d(1) > 0$.

4. $d(q)$ is strictly increasing in q for all $\lambda \in (0,1)$ and all $q \in (0,1)$.

Proof.

1. Obvious.

2. If $q = 0$, then $\mu = 1$. Therefore, $u_i(0) = \frac{1}{2}\lambda v + v$, and $u_i(\hat{x}) = 2\left(\lambda v + (1-\lambda)\frac{v}{4}\right)$. Hence $d(0) = \left(\lambda - \frac{1}{2}\right)v$ which is negative if, and only if, $\lambda < \frac{1}{2}$.

3. If $q = 1$, then $\mu = \lambda$. Therefore, $u_i(0) = \frac{3}{4}v + \frac{3}{4}\lambda v$, and $u_i(\hat{x}) = v\frac{5-3\lambda}{(2-\lambda)^2}$. Hence
$$d(1) = \frac{1}{4}v\frac{-3\lambda^3 + 9\lambda^2 - 12\lambda + 8}{(2-\lambda)^2}$$
which is positive for all $\lambda \in (0,1)$.

4. We can write $d(q) = d^1(q) + d^2(q)$ where
$$\begin{aligned} d^1(q) &= (\lambda + (1-\lambda)q)v + (1-\lambda)(1-q)\frac{v}{4} - \frac{1}{2}(\lambda + (1-\lambda)q)v, \\ d^2(q) &= (\lambda + (1-\lambda)q)\left(\mu v + \frac{(1-\mu)^3}{(2-\mu)^2}v\right) + ((1-\lambda)(1-q))\frac{v}{4} \\ &\quad - \left((\lambda + (1-\lambda)q)\left(\mu v + (1-\mu)\frac{v}{4}\right) + (1-\lambda)(1-q)\frac{v}{(2-\mu)^2}\right). \end{aligned}$$

To give an interpretation of these functions, $d^1(q) > 0$ is the first round gain from choosing positive effort, while $d^2(q) < 0$ is the second round loss which occurs since j knows that $v_i = v$ if i chooses positive effort in the first round.

In what follows, I will show that d^1 and d^2 are strictly increasing in q. Differentiating d^1, we have

$$d^{1\prime}(q) = \frac{1}{4}(1-\lambda)\,v > 0.$$

Substituting equation (8) into d^2, we get

$$d^2(q) = -\frac{1}{4}\lambda v \frac{\left(3\lambda - 3\lambda^2 + 6q\lambda^2 - 10q\lambda + 4q\right)}{\left(\lambda + 2q(1-\lambda)\right)^2}.$$

Differentiating,

$$d^{2\prime}(q) = \frac{1}{2}\lambda v(1-\lambda)\,\frac{\left(6\lambda^2 - 10\lambda + 4\right)q + (4-3\lambda)\lambda}{\left(\lambda + 2q(1-\lambda)\right)^3}.$$

Note that $(4-3\lambda)\lambda > 0$ for all $\lambda \in (0,1)$. Therefore, if $6\lambda^2 - 10\lambda + 4 \geq 0$, we have $d^{2\prime}(q) > 0$. On the other hand, if $6\lambda^2 - 10\lambda + 4 < 0$, then

$$\left(6\lambda^2 - 10\lambda + 4\right)q + (4-3\lambda)\lambda \geq \left(6\lambda^2 - 10\lambda + 4\right) + (4-3\lambda)\lambda = 3\lambda^2 - 6\lambda + 4 > 0$$

for all $q \in [0,1]$ and all $\lambda \in (0,1)$. Thus we have $d^{2\prime}(q) > 0$ in this case, too.

■

Now we can apply the intermediate value theorem to the preceding lemma: if $\lambda < 1/2$, there exists a unique $q \in (0,1)$ such that $d(q) = 0$. Therefore, if $\lambda < 1/2$, the strategies and beliefs described in lemma 3 are an equilibrium for the unique $q \in (0,1)$ that solves $d(q) = 0$.

(ii) If $\lambda \geq 1/2$, there is no equilibrium with symmetric partial pooling. If $\lambda \geq 1/2$, we have $d(0) \geq 0$, with strict inequality unless $\lambda = 1/2$. Since $d'(q) > 0$, we can conclude that $d(q) > 0$ for all $q \in (0,1)$. Hence, if contestant j partially pools, contestant i always has a strictly higher utility from playing $\hat{x}$ in t=1. Therefore no equilibrium with symmetric partial pooling in round one exists if $\lambda \geq 1/2$.

(iii) Uniqueness. Suppose there are other equilibria with symmetric partial pooling in round one. By lemma 3, these differ from the one described above only in the probability q with which a high valuation contestant chooses zero effort in round one, and in the related positive first round effort $\hat{x}$. However, as we have seen above, there is a unique q that solves $d(q) = 0$. Thus, the equilibrium is unique in the class of equilibria with symmetric partial pooling in round one. Moreover, when $\lambda < 1/2$, no equilibrium with separation in round one exists, and of course no pooling equilibrium. Hence the equilibrium is unique in the class of symmetric equilibria.

7.3 Proof of proposition 5

Expected efforts in the first round of the equilibrium with symmetric partial pooling in round one are

$$\begin{aligned} E\left(x_a^1 + x_b^1\right) &= 2\left(\lambda 0 + (1-\lambda)\left(q0 + (1-q)\,\hat{x}\right)\right) \\ &= (1-q)^2 (1-\lambda)^2 \frac{v}{2}. \end{aligned}$$

Now consider the second round effort of a contestant i. This is zero for a low valuation type. For a high valuation type, we have the following possibilities.

Case 1: contestant j has a low valuation. The second round effort of contestant i depends on his own first round effort:

- If $x_i^1 = 0$, then $x_i^2 = (1-\mu)\,v/4$. This subcase has probability $(1-\lambda)\,\lambda q$: contestant i has a high valuation, hence the factor $(1-\lambda)$; contestant j has a low valuation, hence the λ; and i has played $x_i^1 = 0$ in round one, hence the q.
- If $x_i^1 = \hat{x}$, then $x_i^2 = (1-\mu)^2 v/(2-\mu)^2$. This subcase has probability $(1-\lambda)\,\lambda\,(1-q)$.

Case 2: contestant j has a high valuation. The second round effort of contestant i depends on his own first round effort, and on the first round effort of contestant j:

- If $x_i^1 = x_j^1 = 0$, then $x_i^2 = (1-\mu)\,v/4$. This subcase has probability $(1-\lambda)^2 q^2$.

- If $x_i^1 = 0$ and $x_j^1 = \hat{x}$, then $x_i^2 = (1-\mu) v / (2-\mu)^2$. This subcase has probability $(1-\lambda)^2 q (1-q)$.
- If $x_i^1 = \hat{x}$ and $x_j^1 = 0$, then $x_i^2 = (1-\mu)^2 v / (2-\mu)^2$. This subcase has probability $(1-\lambda)^2 q (1-q)$, too.
- If $x_i^1 = x_j^1 = \hat{x}$, then $x_i^2 = v/4$. This subcase has probability $(1-\lambda)^2 (1-q)^2$.

Putting things together, it follows that the expected effort of a contestant is (each line corresponds to one of the bullet list items above)

$$\begin{aligned} E\left(x_i^2\right) &= (1-\lambda)\lambda q \frac{(1-\mu) v}{4} + \\ & (1-\lambda)\lambda(1-q)\frac{(1-\mu)^2 v}{(2-\mu)^2} + \\ & (1-\lambda)^2 q^2 \frac{(1-\mu) v}{4} + \\ & (1-\lambda)^2 q (1-q) \frac{(1-\mu) v}{(2-\mu)^2} + \\ & (1-\lambda)^2 q (1-q) \frac{(1-\mu)^2 v}{(2-\mu)^2} + \\ & (1-\lambda)^2 (1-q)^2 \frac{v}{4}. \end{aligned}$$

Substituting μ from equation (8) into the last equation we find that

$$E\left(x_i^2\right) = \frac{\left(\left(6\lambda^2 - 8\lambda + 4\right) q^2 + \left(4\lambda - 6\lambda^2\right) q + \lambda^2\right)(1-\lambda)^2}{4\left(\lambda + 2q(1-\lambda)\right)^2} v.$$

Total expected effort therefore equals

$$\begin{aligned} E\left(x_a^1 + x_b^1 + x_a^2 + x_b^2\right) &= (1-q)^2 (1-\lambda)^2 \frac{v}{2} + \\ & + \frac{\left(\left(6\lambda^2 - 8\lambda + 4\right) q^2 + \left(4\lambda - 6\lambda^2\right) q + \lambda^2\right)(1-\lambda)^2}{2\left(\lambda + 2q(1-\lambda)\right)^2} v. \end{aligned}$$

In two unconnected one-shot contests with two sided asymmetric information, expected effort equals $(1-\lambda)^2 v$ (see corollary 1). For notational convenience, define

$$f(q) := \left(4\lambda^2 - 8\lambda + 4\right) q^2 + \left(-12\lambda^2 + 20\lambda - 8\right) q + (11\lambda - 8)\lambda.$$

The difference in expected effort is

$$E\left(x_a^1 + x_b^1 + x_a^2 + x_b^2\right) - (1-\lambda)^2 v = (1-\lambda)^2 q \frac{f(q)\,q - 4\lambda^2}{2(\lambda + 2q(1-\lambda))^2} v.$$

We have to prove that this is negative for the unique q that solves $d(q) = 0$. It suffices to show that $f(q) < 0$ for all $q \in [0,1]$. Note that, for all $\lambda \in [0, 1/2]$, we have $f(0) = (11\lambda - 8)\lambda < 0$ and $f(1) = (\lambda + 2)(3\lambda - 2) < 0$. Moreover, $f(q)$ is convex in q for all $\lambda \in [0, 1/2]$. Therefore, $f(q) \leq \max\{f(0), f(1)\} < 0$ for all $q \in [0,1]$. This completes the proof.

7.4 Proof of proposition 6

Clearly, low valuation contestants always get zero utility. Denote the expected utility of a high valuation contestant in two unrelated one-shot contests with asymmetric information by u_0. From corollary one, we have

$$u_0 = 2\left(\lambda v + (1-\lambda)\frac{v}{4}\right).$$

In the equilibrium with partial pooling in round one, expected utility of a high valuation contestant is given by $u_i(\hat{x})$, see equation (12). The difference is

$$\begin{aligned} u_i(\hat{x}) - u_0 &= (\lambda + (1-\lambda) q) v + (1-\lambda)(1-q)\frac{v}{4} \\ &\quad + (\lambda + (1-\lambda) q)\left(\mu v + \frac{(1-\mu)^3}{(2-\mu)^2} v\right) \\ &\quad + ((1-\lambda)(1-q))\frac{v}{4} \\ &\quad -2\left(\lambda v + (1-\lambda)\frac{v}{4}\right). \end{aligned}$$

After substituting μ from equation (8) into the last equation, some tedious algebra leads to

$$u_i(\hat{x}) - u_0 = \frac{1}{2} qv \frac{\left(-6\lambda^3 + 18\lambda^2 - 18\lambda + 6\right) q^2 + 4\lambda (1-\lambda)^2 q + \lambda^2 (1-\lambda)}{(\lambda + 2q(1-\lambda))^2}$$

This is positive if, and only if, the numerator on the right hand side is positive. Note that $\left(-6\lambda^3 + 18\lambda^2 - 18\lambda + 6\right) > 0$ for all $\lambda < 1$. Hence $u_i(\hat{x}) > u_0$ for all $q \in (0,1)$, and thus for the equilibrium q. This completes the proof.

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