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## Economic Effects of VAT Reforms in Germany

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Economic Effects of VAT Reforms in Germany

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Abstract

In the tax policy debate, differentiation of value-added taxes is often justified by distributional concerns. Our quantitative analysis for Germany indicates that such concerns are misplaced. We find that the abolition of VAT differentiation has only negligible redistributive effects. Instead, reduced VAT rates are found to act as industry-specific subsidies. Whereas the overall welfare effects of pure VAT reforms are very small, a revenue-neutral introduction of a harmonised VAT combined with reductions in the marginal income tax rates or social security contributions turns out to yield substantial welfare gains for all households.

Keywords: VAT, tax reforms, distribution, efficiency, applied general equilibrium

JEL Code: D58, H22, H24

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1 Introduction

Consumption taxation through value-added taxes (VAT) is usually considered as a relatively efficient way of raising public funds. Theoretical analysis points to the neutrality of VAT with respect to intertemporal consumption decisions, whereas income taxes tend to distort the trade-off between consumption and savings.¹ On the other hand, a uniform VAT is often criticised on the basis of its allegedly regressive distributional effects. Reflecting distributional concerns, many countries apply VAT reductions to specific goods, which make up a larger share in the consumption of low-income households. In the EU, all countries but one use reduced VAT rates for specific consumption commodities. Especially in the old EU member states VAT reductions on food, water, medication, and public transport are quite common (European Commission, 2005).

This paper investigates the rationale for VAT differentiation on distributional grounds. Since VAT differentiation is not associated with the individual ability to pay of different consumers, it is an indirect device of distribution. Obviously, there are more direct instruments of distributive policy such as income taxation or monetary transfers. From an applied policy perspective, we must therefore be concerned with how large the redistributive effects of VAT differentiation are in practice, and whether or not alternative policy instruments are more effective as a means of redistribution. Answers to these questions cannot be provided by abstract theoretical considerations but depend on the precise type of products favoured by VAT reductions and the demand and supply conditions on the respective markets, which are determined by household preferences, production technologies, factor endowments, and market structures.

We adopt an applied general equilibrium (AGE) approach to investigate efficiency and distributional impacts of structural VAT reforms based on empirically observed data for Germany. The AGE approach provides a comprehensive framework for studying the effects of policy interference on all markets of an economy, rigorously based on microeconomic theory. The simultaneous consideration of the origin and

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¹In the public finance literature a number of reasons are mentioned why VAT differentiation might be justified under efficiency considerations: (i) administrative and compliance costs (Keen and Mintz, 2004), (ii) the role of the shadow economy, (iii) differences in price elasticities of goods, or (iv) complementarity of consumption goods with untaxed leisure activities. More recently, VAT reductions have also been proposed as a measure to stimulate employment in labour intensive service industries (Holmlund, 2002).
use of the agents’ income makes it possible to address both economy-wide efficiency as well as distributional impacts of policy regulation. This has made AGE models a standard tool for the quantitative analysis in many policy areas including fiscal, trade and environmental policy.

Building on the pioneering work by Shoven and Whalley (1972), AGE models have been widely used for the economy-wide impact assessment of tax policy reforms (for surveys see e.g. Shoven and Whalley 1984, 1992 or Kehoe and Kehoe 1996). A detailed evaluation of distributive effects of tax policy reforms with a disaggregation of the household sector has been undertaken in various country-specific AGE studies: Early examples include the seminal contribution by Ballard et al. (1985) for the USA, the analysis by Piggott and Whalley (1985) for England, or the work by Keller (1980) for the Netherlands. AGE analyses with a strong focus on distributive aspects have also evolved in the intersection of international trade and development economics such as Bourguignon et al. (2005) for Indonesia or Corra-ton and Cockburn (2007) for the Philippines (for a survey see Davies, 2003). Studies of the distributive consequences of income taxation and public transfers, however, have largely been performed in a partial equilibrium framework, typically based on microsimulation tools (Gupta and Kapur, 2000). Among the few exceptions that bridge the micro-macro gap for public transfer analysis within an AGE approach are Coady and Harris (2004) and Arntz et al. (2006).

The strand of AGE literature that is directed to the analysis of VAT reforms is relatively small. Ballard et al. (1987) analyse VAT in the USA as a possibility to increase the efficiency of the tax system. Hamilton and Whalley (1989) use a static AGE model to explore special intricacies of the interaction between federal and provincial taxes in Canada. Gottfried and Wiegard (1991) compare two different institutional settings for VAT implementation, i.e. tax exemption vs. zero rating, for the German economy. Dixon and Rimmer (1999) investigate VAT reforms in Australia with a special focus on the induced international trade effects. Åvitsland and Aasness (2004) combine an AGE model and a microsimulation model to assess VAT reform scenarios for Norway. Kaerney and van Heerden (2004) analyse the economic implications of a zero VAT rate on food in South Africa.

The German case is characterised by a standard and a reduced VAT rate. The latter applies predominantly to food, public transport, and print-media products. The analysis of VAT reforms thus requires a differentiation of the final demand system by consumption categories (e.g., Andrikopoulos et al., 2003). Furthermore, we must
distinguish different household types to quantify distributive impacts. The results of our AGE simulations confirm doubts about the effectiveness of reduced VAT rates as a redistributive instrument and point to welfare gains from uniform taxation. These welfare gains are boosted if taxes other than VAT are included in the tax reform even if we impose the requirement of distributional neutrality: The possibility of Pareto improvements from VAT harmonisation may be an important message to policymakers. Another key finding is that – from a sectoral (industrial) perspective – reduced VAT rates work mainly as distortionary subsidies to the respective final-goods producers and their intermediate-input suppliers.

The remainder of this paper is organised as follows. Section 2 gives an overview of the model structure and its parametrisation. Section 3 provides the results of the scenario simulations. Section 4 concludes.

2 Model and Parametrisation

For our simulation analysis we draw on a standard AGE model which has been refined to address central issues of VAT reforms.2 The most important extension in the VAT context refers to the disaggregation of the household sector into income terciles, where each tercile features a special income composition and consumption structure. For the empirical parametrisation of the model, various data sources including the German Input-Output Table for 1997, the production-consumption transition matrix – the so-called “Z-matrix”, and the German Income and Expenditure Survey (EVS) have been combined to form a consistent benchmark dataset.

In the following, we first summarise the basic features of our AGE model (Section 2.1). A detailed description of the household representation follows in Section 2.2. Finally, we discuss data and calibration issues (Section 2.3). A comprehensive algebraic summary of the model is provided in the appendix.

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2 Other issue-driven modifications of the standard model include recent analyses of climate policy (Böhringer and Lange, 2005) and labour market regulations (Böhringer, Boeters, and Feil, 2005).
2.1 Basic Model Structure

Firms and factors of production

The AGE model underlying our VAT reform analysis for Germany features 69 industries (production sectors). In each industry, output is produced from intermediate inputs, capital and labour. Production possibilities are characterised through nested constant-elasticity-of-substitution (CES) production functions, which describe the trade-off between various inputs. Perfect competition implies that there are no pure profits. The primary factors labour and capital are remunerated according to their respective marginal productivities. Cost minimisation by firms yields demand functions for production inputs at the industry level.

![Diagram of production structure for a representative industry]

Figure 1: Production structure for a representative industry

The domestic labour market exhibits frictions with equilibrium unemployment. More specifically, we adopt a wage-curve relationship where the rate of unemployment is explained by a wage-bargaining mechanism following Layard et al. (1991, Chapter 2). Labour is mobile across domestic industries but internationally immobile. Capital is mobile both across domestic industries and the national border. Foreign capital
exports and imports are fully elastic, i.e. the international interest rate is exogenous in our small open economy setting. In domestic production, we distinguish capital by domestic and foreign origin (see Figure 1). Outflows of domestic capital to the international capital market as well as inflows of foreign capital to the domestic capital market are not perfectly elastic – the calibration of the associated domestic capital export and import functions is laid out in Section 2.3.3. Technically speaking, the model includes only capital services, where capital stocks are implicit through a constant stock-to-yearly-services ratio.

In Figure 1 we use the following notation:

\[
\sigma^Y := \text{elasticity of substitution between the aggregate of intermediate production inputs and the input composite of labour, capital, and energy,}
\]

\[
\sigma^{KLE} := \text{elasticity of substitution between the capital-energy aggregate and labour,}
\]

\[
\sigma^M := \text{elasticity of substitution between intermediate inputs entering the industry composite of intermediate inputs,}
\]

\[
\sigma^{KE} := \text{elasticity of substitution between capital and aggregate energy,}
\]

\[
\sigma^K := \text{elasticity of substitution between domestic and foreign capital,}
\]

\[
\sigma^E := \text{elasticity of substitution between different energy carriers entering the aggregate energy input.}
\]

**Foreign trade**

Domestically produced goods are converted through a constant-elasticity-of-transformation function into goods destined for the domestic market and the export market, respectively. Export and import prices in foreign currency are considered as exogenous (small-open-economy assumption). Analogously to the export side, we adopt the Armington assumption of product heterogeneity for imports (Armington, 1969). A CES function characterises the trade-off between imported and domestically produced varieties of the same good. The Armington good enters intermediate and final demand. Foreign closure of the model is warranted through the balance-of-payment (BOP) constraint: The total value of exports equals the total value of imports accounting for an initial BOP deficit or surplus given by the base year statistics. The BOP constraint thereby determines the real exchange rate which indicates the (endogenous) value of the domestic currency vis-à-vis the foreign currency (the latter being exogenous in a small-open-economy setting).

---

3 Exports and imports include both goods and trans-border capital services.
Government budget

Given our focus on VAT reform, the model emphasises the role of consumption taxation. The VAT captures differences across consumption categories via three levels of the VAT rate (full rate, reduced rate, and tax exempt goods). Furthermore, we account for the indirect impact of value-added taxation in the production of goods which are tax exempt. Besides the VAT, direct taxes and social security contributions of households are incorporated. Social security contributions are assumed to be proportional to labour income while income taxation takes the form of a linear progressive schedule, i.e. a tax allowance combined with a constant marginal tax rate. Finally, the model features industry-specific output taxes and subsidies as well as import and export levies.

Private households

We distinguish three representative households capturing the lower, middle, and upper tercile of the income distribution. Each household takes a labour-leisure decision and chooses between different consumption goods. Details about the characteristics of the disaggregated households are provided in the following section.

2.2 Representation of the Household Sector

2.2.1 Household Disaggregation

The private household sector is disaggregated into three households representing, respectively, the lower, middle and upper income tercile of the households according to the German Income and Expenditure Survey (EVS). The EVS is a representative household survey by the German Federal Statistical Office. The 1998 sample comprises 62,000 households. The first part of the survey reports data on household structure, housing situation, financial and tangible assets as well as debt. The second part contains income and expenditure items adapted to the classification of the input-output accounts.

Households are grouped into the three income terciles according to “equivalent household income”. Household income is divided by the respective number of household members in order to compare households of different sizes. We use the square root of the household size as equivalence scale in order to compute the respective number of household members, thereby reflecting economies of scale due to fixed
costs in household consumption.\textsuperscript{4} The income and expenditure values of the three fictitious representative households are then set to the arithmetic mean of the respective income class.

Table 1 summarises basic characteristics of the household types. Disposable income varies substantially across the three terciles. Taking the first tercile as the basis of comparison, disposable income of the second tercile is higher by roughly one half, whereas the disposable income of the third tercile is three times as high. Less than two thirds of total gross income (the case of the first tercile even not more than one third) are made up of factor income. The residual income consists mainly of transfer payments, pensions and private credit (with “savings” meaning gross savings). The income tax schedule is progressive as stated by the differences between average and marginal tax rates\textsuperscript{5}; in addition, we report the implicit tax allowances associated with a linear progressive income tax scheme. Average social security contributions (SSC) are decreasing in income due to an assessment threshold for the SSC basis.

<table>
<thead>
<tr>
<th></th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disposable收入</td>
<td>1994 (89%)</td>
<td>3105 (83%)</td>
<td>6101 (84%)</td>
</tr>
<tr>
<td>Consumption</td>
<td>1738 (78%)</td>
<td>2317 (62%)</td>
<td>3674 (51%)</td>
</tr>
<tr>
<td>Savings</td>
<td>256 (11%)</td>
<td>788 (21%)</td>
<td>2427 (33%)</td>
</tr>
<tr>
<td>Taxes and SSC</td>
<td>237 (11%)</td>
<td>618 (17%)</td>
<td>1179 (16%)</td>
</tr>
<tr>
<td>Factor income</td>
<td>753 (34%)</td>
<td>2045 (55%)</td>
<td>4385 (60%)</td>
</tr>
<tr>
<td>Other income</td>
<td>1477 (66%)</td>
<td>1678 (45%)</td>
<td>2895 (40%)</td>
</tr>
<tr>
<td>Average tax rate</td>
<td>10.0%</td>
<td>12.8%</td>
<td>16.5%</td>
</tr>
<tr>
<td>Average SSC</td>
<td>23.5%</td>
<td>20.4%</td>
<td>12.7%</td>
</tr>
<tr>
<td>Marginal tax rate</td>
<td>14.9%</td>
<td>16.1%</td>
<td>22.1%</td>
</tr>
<tr>
<td>Implied tax allowance</td>
<td>247</td>
<td>417</td>
<td>1127</td>
</tr>
</tbody>
</table>

Rows (1) to (5): absolute values in € per month and percentage values as shares in gross income; rows (6) to (8): percentage of factor or labour income, respectively

Table 1: Household characteristics according to EVS

Table 2 reports the consumption shares of the household terciles by VAT categories. It can be seen that the share of the three VAT categories in consumption is rather stable (upper part of Table 2). This is especially the case for the tax-exempt goods,

\textsuperscript{4}Cf. e.g. Biewen (2000) or Atkinson et al. (1995, 18ff.) for alternative scales.

\textsuperscript{5}Tax rates are reported relative to gross factor income.
while the shares of the reduced-rate goods are slightly decreasing and those of the full-rate goods are slightly increasing in income. The figures in Table 2 already suggest that a differentiated VAT rate may not be well suited as a redistributive device. In relation to the disposable income (lower part of Table 2), the share of all VAT categories is decreasing, simply because of the increasing propensity to save.

<table>
<thead>
<tr>
<th></th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAT 0%</td>
<td>38.8%</td>
<td>37.2%</td>
<td>37.2%</td>
</tr>
<tr>
<td>VAT 7%</td>
<td>27.3%</td>
<td>25.6%</td>
<td>23.7%</td>
</tr>
<tr>
<td>VAT 16%</td>
<td>33.9%</td>
<td>37.2%</td>
<td>39.1%</td>
</tr>
</tbody>
</table>

Table 2: Household consumption structure

The expenditure shares of various consumption good categories (see Table 3) are calculated as fractions of the so-called “income available for expenditures”. The latter is defined as the sum of disposable income, sales of goods and property, pensions from private insurances, liquidation of financial and tangible assets and from bank and consumption credit loans. The expenditure categories of the EVS are adjusted in order to warrant consistency with the 12-goods-classification of the Z-matrix. In order to assess the impacts of a structural VAT reform, detailed EVS expenditure shares for each of the 12 categories have been grouped into categories with VAT of 16 percent, 7 percent or tax exemption, respectively – see Table 3 for an explicit overview.  

2.2.2 Consumption Structure

The consumption structure of the three representative households is reflected in the nesting of multi-level utility functions adopted within the numerical AGE model (see Figure 2). At the top level, we specify a consumption-savings decision under static expectations following Ballard et al. (1985). Current utility is then composed

6The data underlying Table 3 has been provided by the German federal statistical office in the form of a special Z-matrix differentiated by VAT rates.
<table>
<thead>
<tr>
<th>Consumption category</th>
<th>share in total consumption</th>
<th>share of VAT 0%</th>
<th>share of VAT 7%</th>
<th>share of VAT 16%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and beverages</td>
<td>12.4%</td>
<td>–</td>
<td>95.2%</td>
<td>4.8%</td>
</tr>
<tr>
<td>Alcohol and tobacco</td>
<td>3.5%</td>
<td>–</td>
<td>8.8%</td>
<td>91.2%</td>
</tr>
<tr>
<td>Clothing and footwear</td>
<td>5.8%</td>
<td>–</td>
<td>–</td>
<td>100.0%</td>
</tr>
<tr>
<td>Housing, water, energy</td>
<td>33.6%</td>
<td>82.3%</td>
<td>2.4%</td>
<td>15.3%</td>
</tr>
<tr>
<td>Household equipment</td>
<td>6.4%</td>
<td>5.9%</td>
<td>–</td>
<td>94.1%</td>
</tr>
<tr>
<td>Health</td>
<td>3.5%</td>
<td>85.4%</td>
<td>–</td>
<td>14.6%</td>
</tr>
<tr>
<td>Transport</td>
<td>10.5%</td>
<td>0.4%</td>
<td>14.5%</td>
<td>85.1%</td>
</tr>
<tr>
<td>Communication</td>
<td>2.7%</td>
<td>14.3%</td>
<td>–</td>
<td>85.7%</td>
</tr>
<tr>
<td>Recreation and culture</td>
<td>12.1%</td>
<td>28.1%</td>
<td>30.3%</td>
<td>41.6%</td>
</tr>
<tr>
<td>Education</td>
<td>0.5%</td>
<td>93.4%</td>
<td>–</td>
<td>6.6%</td>
</tr>
<tr>
<td>Restaurants and hotels</td>
<td>5.0%</td>
<td>–</td>
<td>49.4%</td>
<td>50.6%</td>
</tr>
<tr>
<td>Other goods and services</td>
<td>4.3%</td>
<td>65.8%</td>
<td>0.3%</td>
<td>33.8%</td>
</tr>
</tbody>
</table>

Table 3: VAT shares by consumption categories

of leisure and commodity consumption. Commodity consumption in turn is an aggregate of food consumption and other consumption goods (which are then further decomposed at the lower level). We explicitly represent food consumption because it is the most important consumption goods category to which the reduced VAT rate is applied in Germany. All consumption good categories are finally broken down into the three VAT categories (tax exempt goods, reduced and full VAT rate) according to their empirical shares (see Table 2).

In Figure 2 we use the following notation:

\[
\sigma^U \quad := \quad \text{elasticity of substitution between current and future consumption}, \\
\sigma^{CU} \quad := \quad \text{elasticity of substitution between current consumption and leisure}, \\
\sigma^C \quad := \quad \text{elasticity of substitution between food and the non-food consumption aggregate}, \\
\sigma^{LE} \quad := \quad \text{elasticity of substitution between leisure of high skilled and low skilled labour}, \\
\sigma^{VAT} \quad := \quad \text{elasticity of substitution between commodities subject to (three) different VAT categories}, \\
\sigma^{NF} \quad := \quad \text{elasticity of substitution between consumption commodities entering the non-food consumption composite}. 
\]
2.3 Parametrisation

2.3.1 Input-Output Data

In our comparative-static analysis, policy effects are assessed with respect to a reference situation – the benchmark – where no policy changes apply. The benchmark is typically determined by economic transactions in a particular benchmark year. As is customary in applied general equilibrium analysis, benchmark quantities and prices – together with exogenous elasticities – are used to calibrate the model. They determine the free parameters of the functional forms that capture production technologies and consumer preferences.

We use the input-output table of the German federal statistical office for the year 1997 as the central data source for model calibration. The first quadrant of the input-output table reports intermediate inputs for each sector. The second quadrant
provides information on final demand components: private and public consumption, investment, inventory changes, and exports. Factor payments to labour and capital (combined with profits in the row “operating surplus”) are included in the third quadrant which also reports the inflows of foreign goods and services to each production sector. With regard to taxes, the standard input-output table records product-specific taxes and subsidies as well as the VAT.

Output by production sector is linked to consumption by private households in terms of expenditure categories through the Z-matrix, see above.

2.3.2 Calibration of the Utility Function

The calibration of the parameters of the utility function (see Figure 2) requires the integration of empirical estimates of the elasticities of savings, labour supply, and consumption demand.

Savings elasticity

At the top level of the utility function (see appendix for the algebraic specification), the following elasticity of savings with respect to the interest rate can be derived for household $h$:

$$\varepsilon_{h,S} = \sigma^U_h (1 - \theta_{h,S}) + \theta_{h,K}$$

where

- $\theta_{h,S}$ := share of savings in extended income,
- $\theta_{h,K}$ := share of capital income in extended income.

Given household specific share parameters, we calibrate $\sigma^U_h$ in consistency with empirical estimates around 0.4 for the savings elasticity (see Boskin 1978, Elmendorf, 1996, Bernheim, 2001).

Labour supply elasticity

The implied uncompensated labour supply elasticity, $\varepsilon_{h,L}$ for household $h$ is:

$$\varepsilon_{h,L} = (\zeta_h - 1) \left[ \sigma^C_{h,CU} + (\sigma^U_h - \sigma^C_{h,CU}) \theta_{h,LE} \right] + (\sigma^U_h - 1) \theta_{h,CU} \theta_{h,LE} - \theta_{1,h}$$ (1)

where
\[ \zeta_h := \text{labour endowment as a multiple of actual labour supply}, \]
\[ \theta_{h,LE} := \text{share of leisure in value of current utility}, \]
\[ \theta_{h,CU} := \text{share of current utility in extended income}, \]
\[ \theta_{l,h} := \text{share of labour endowment in extended income}. \]

Given exogenous shares and calibrated elasticities \( \varepsilon_{h,L} \) and \( \sigma_h^U \), we can solve equation (1) for the unknown elasticity \( \sigma_{h,CU} \). As we set labour supply \( \langle \zeta_h \rangle \) uniformly to 1.75 (70 hours of weekly labour endowment relative to an average weekly working time of 40 hours). Our reading of the empirical literature on labour supply elasticities (for a survey see, e.g., Borjas, 2000) is that there are no strong results on income-bracket-specific labour supply elasticities and that uncompensated labour supply elasticities are centred around 0.15. In this vein, we calibrate the model to \( \varepsilon_{h,L} = 0.15 \) for all households.

**Consumption demand elasticities**

At the lower nests of the utility tree of Figure 2, we can solve recursively for the elasticities of interest, \( \sigma_C \) and \( \sigma_{NF} \), taking the upper-level elasticities as given:

\[
\sigma_{h,C} = -\frac{1}{1 - \theta_{h,F}} \left[ \varepsilon_F + \sigma_h^{CU} \theta_{h,F}(1 - \theta_{h,C}) + \sigma_h^U \theta_{h,C}(1 - \theta_{h,CU}) \right. \\
\left. + \theta_{h,F} \theta_{h,C} \theta_{h,CU} \right]
\]

\[
\sigma_{h,NF} = -\frac{1}{1 - \theta_{h,NF}} \left[ \bar{\varepsilon}_{NF} + \sigma_h^C \theta_{h,NF}(1 - \theta_{h,NF}) + \sigma_h^{CU} \theta_{h,NF} \theta_{h,NF}(1 - \theta_{h,C}) \right. \\
\left. + \sigma_h^U \theta_{h,NF} \theta_{h,NF} \theta_{h,C}(1 - \theta_{h,CU}) - \bar{\theta}_{h,NF} \theta_{h,NF} \theta_{h,NF} \theta_{h,C} \theta_{h,CU} \right]
\]

where

\[ \varepsilon_F := \text{own-price elasticity of food demand}, \]
\[ \bar{\varepsilon}_{NF} := \text{average own-price elasticity of demand for non-food goods}, \]
\[ \theta_{h,C} := \text{share of consumption in current utility}, \]
\[ \theta_{h,F} := \text{share of food in the consumption goods aggregate}, \]
\[ \theta_{h,NF} := \text{share of all non-food goods in the consumption goods aggregate}, \]
\[ \bar{\theta}_{h,NF} := \text{average share of individual non-food goods in the non-food aggregate}. \]

With respect to the price elasticities of consumption demand we draw on Chen (1999), who estimates consumption demand parameters for 42 OECD countries.
Chen uses Theil’s (1980) differential approach. His estimates for Germany are provided in Table 4 together with the mean of the estimates for the 42 OECD countries. Because of diverging definitions in product categories, we can only distinguish between the price elasticity of food (−.222) and the average price elasticity for the remaining 7 categories (−.563) in the model.7

<table>
<thead>
<tr>
<th>Product category</th>
<th>Germany</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>−.222</td>
<td>−.220</td>
</tr>
<tr>
<td>Clothing</td>
<td>−.423</td>
<td>−.422</td>
</tr>
<tr>
<td>Housing</td>
<td>−.426</td>
<td>−.432</td>
</tr>
<tr>
<td>Durable consumption goods</td>
<td>−.501</td>
<td>−.585</td>
</tr>
<tr>
<td>Health</td>
<td>−.844</td>
<td>−.734</td>
</tr>
<tr>
<td>Traffic</td>
<td>−.591</td>
<td>−.665</td>
</tr>
<tr>
<td>Recreation</td>
<td>−.608</td>
<td>−.628</td>
</tr>
<tr>
<td>Other</td>
<td>−.547</td>
<td>−.605</td>
</tr>
</tbody>
</table>

Table 4: Price elasticities of consumption demand

2.3.3 Calibration of Capital Supply and Demand

For the elasticity of domestic capital imports with respect to the domestic interest rate, \( \varepsilon_{KM} \), we can calculate from the production function (Figure 1 with the respective elasticities of substitution \( \sigma \)):

\[
\varepsilon_{KM} = \sigma^K (1 - \theta_{KM}) - \sigma^{KE} (1 - \theta_{KM})(1 - \theta_K) - \sigma^{KEL} (1 - \theta_{KM})\theta_K (1 - \theta_{KE})
\]

where

\[\sigma^K := \text{elasticity of substitution between domestic and foreign capital},\]

\[\theta_{KM} := \text{share of capital imports in domestic capital use},\]

\[\theta_K := \text{share of capital in the capital-energy sub-aggregate},\]

\[\theta_{KE} := \text{share of KE in the capital-energy-labour sub-aggregate},\]

We calibrate \( \sigma^K \) to match values of the capital import ratio, \( \theta_{KM} = 0.18 \) (French and Poterba, 1991), and the elasticity of capital imports with respect to the domestic interest rate \( \varepsilon_{KM} = 2.4 \) (de Mooij and Ederveen, 2001).

---

7 Price elasticities of food consumption in the same range have been identified for a number of countries more recently by Selvanathan and Selvanathan (2006).
The elasticity of domestic capital exports with respect to the domestic interest rate can be computed from the constant-elasticity-of-transformation function, splitting up domestic savings into capital exports and domestically used capital. Here we have:

\[ \varepsilon_{KX} = -\eta^{KS}(1 - \theta_{KX}) \]

where

\[ \eta^{KS} := \text{elasticity of transformation between capital exports and domestically used capital}, \]

\[ \theta_{KX} := \text{share of capital exports in domestic savings}. \]

\( \eta^{KS} \) is calibrated to values of the capital export ratio, \( \theta_{KX} = 0.21 \) (French and Poterba, 1991), and the elasticity of capital exports with respect to the domestic interest rate \( \varepsilon_{KX} = -2.4 \) (de Mooij and Ederveen, 2001).

## 3 Simulations

In our simulations of revenue-neutral VAT reforms for the German economy we replace the differentiated VAT rate by a uniform rate (while the treatment of tax-exempt goods remains unchanged). We perform three types of tax reforms. The first type is a pure VAT reform, where we introduce a uniform VAT rate at an endogenously determined level which keeps public revenues constant. The second type includes other taxes than VAT for revenue recycling. Here we set the rate for commodities with lower VAT (i.e. 7%) at the normal level (i.e. 16%) and balance the public budget through uniform reductions of either marginal income tax rates (MITR) or social security contributions (SSC), or through an increase in the income tax allowance (ITA).\(^8\)

The alternative instruments of revenue recycling will affect households differently. In order to separate efficiency considerations from distributional concerns, we therefore carry out a third type of simulations with non-uniform adjustments in recycling instrument, which – beyond revenue-neutrality – also warrant distributive neutrality across households. Table 5 provides a full list of the simulations.

---

\(^8\)It should be kept in mind that social security contributions are levied only on labour, whereas income taxes are levied on both labour and capital. See the discussion at the end of Section 3.1.
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pure value-added tax (VAT) reform</td>
</tr>
<tr>
<td>2</td>
<td>Uniform cut in marginal income tax rates (MITR)</td>
</tr>
<tr>
<td>3</td>
<td>Distributively-neutral cut in marginal income tax rates (MITR)</td>
</tr>
<tr>
<td>4</td>
<td>Uniform increase in income tax allowance (ITA)</td>
</tr>
<tr>
<td>5</td>
<td>Distributively-neutral increase in income tax allowance (ITA)</td>
</tr>
<tr>
<td>6</td>
<td>Uniform cut in social security contributions (SSC)</td>
</tr>
<tr>
<td>7</td>
<td>Distributively-neutral cut in social security contributions (SSC)</td>
</tr>
</tbody>
</table>

Table 5: Overview of simulations

3.1 Distributive Effects and Efficiency

Our discussion of simulation results starts with a scenario in which the differentiated VAT (16% and 7%, respectively) is replaced in a revenue-neutral way by a uniform VAT rate at an intermediate level (Scenario 1). Taking general equilibrium repercussions into account, the level of the post-reform VAT amounts to 14.1% (as compared to a normal VAT rate of 16% before).

The distributive effects of this pure VAT reform are reported in Table 6 both in terms of equivalent variation in percent of the benchmark income\(^9\) and in terms of absolute changes. Reflecting the higher share of goods with a reduced VAT rate in the expenditures of the lowest tercile (see Table 2), Scenario 1 has some regressive distributional effects. However, these effects from switching to a uniform VAT are rather small. Moreover, the gain for the upper tercile is higher only in absolute but not in relative terms.

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>EV in per cent</td>
<td>-0.19</td>
<td>-0.00</td>
<td>+0.14</td>
<td>+0.05</td>
</tr>
<tr>
<td>EV in € per month</td>
<td>-4.5</td>
<td>-0.1</td>
<td>11.7</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table 6: Pure VAT reform

Next, we analyse different varieties of tax reforms that use taxes other than the VAT itself in order to achieve the balanced budget. In Scenario 2, we uniformly (in percentage points) cut the marginal income tax rate (MITR) to warrant revenue neutrality.

\(^9\) Equivalent variation captures changes in all utility-generating items (see Figure 2), including leisure.

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<table>
<thead>
<tr>
<th>Scenario 2</th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-reform MINT</td>
<td>14.9%</td>
<td>16.1%</td>
<td>22.1%</td>
<td></td>
</tr>
<tr>
<td>post-reform MINT</td>
<td>14.1%</td>
<td>15.3%</td>
<td>21.3%</td>
<td></td>
</tr>
<tr>
<td>EV in per cent</td>
<td>-0.53</td>
<td>-0.04</td>
<td>+0.29</td>
<td>+0.07</td>
</tr>
<tr>
<td>EV in € per month</td>
<td>-12.2</td>
<td>-1.5</td>
<td>+24.3</td>
<td>+3.5</td>
</tr>
</tbody>
</table>

Table 7: Uniform cut in MITR

Table 7 indicates that the distributional effects are larger than for the case of a pure VAT reform. While the middle tercile is still virtually unaffected by the reform, the losses for the lower tercile and the gains for the upper tercile are more than double the respective figures of Scenario 1. Uniform cuts in the marginal income tax rate are favourable for the upper tercile because taxable income makes up the largest fraction of total income in this tercile.

In Scenario 3, we maintain the marginal income tax as the recycling instrument for balancing the public budget, but impose the restriction of distributive neutrality. The marginal income tax rate (MITR) is now endogenously adjusted so that the percentage change in EV is the same across all households. Table 8 summarises the implications of this (non-uniform) adjustment rule.

<table>
<thead>
<tr>
<th>Scenario 3</th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-reform MINT</td>
<td>14.9%</td>
<td>16.1%</td>
<td>22.1%</td>
<td></td>
</tr>
<tr>
<td>post-reform MINT</td>
<td>12.5%</td>
<td>15.2%</td>
<td>21.6%</td>
<td></td>
</tr>
<tr>
<td>EV in per cent</td>
<td>+0.07</td>
<td>+0.07</td>
<td>+0.07</td>
<td>+0.07</td>
</tr>
<tr>
<td>EV in € per month</td>
<td>+1.6</td>
<td>+2.7</td>
<td>+5.7</td>
<td>+3.3</td>
</tr>
</tbody>
</table>

Table 8: Distributively-neutral cut in MITR

The tax reduction of the MITR (in terms of the tax rate) is highest for the lower tercile (−2.4 p.p.) and lowest for the upper tercile (−0.5 p.p.). This results in a uniform increase in the equivalent variation of 0.07%.

Tax revenue recycling through the income tax may alternatively be based on the income tax allowance (ITA). Table 9 reports scenario results for the case of a uniform increase (in €) of the tax allowance for all households (Scenario 4).

In this scenario, all households face welfare losses, reflecting the implied increase in the marginal tax burden. Moreover, the losses are almost identical across terciles. This latter result can be explained by two countervailing effects that more or less
cancel each other out. On the one hand, a given increase of the ITA in € means a lower relative increase for the upper terciles. On the other hand, the upper terciles have higher marginal income tax rates, so that they benefit more from a given relative increase in the tax allowance.

The policy settings for Scenario 5 differ from those of Scenario 4 only in that tax allowances are differentiated endogenously in order to yield proportional welfare changes across all households. Distributional results are provided in Table 10. As Scenario 4 was almost neutral, distributionally, the results for Scenario 5 are very much alike.

### Table 9: Uniform increase in ITA

<table>
<thead>
<tr>
<th>Scenario 4</th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-reform ITA in €</td>
<td>247</td>
<td>417</td>
<td>1126</td>
<td></td>
</tr>
<tr>
<td>post-reform ITA in €</td>
<td>261</td>
<td>430</td>
<td>1140</td>
<td></td>
</tr>
<tr>
<td>EV in per cent</td>
<td>-0.13</td>
<td>-0.07</td>
<td>-0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>EV in € per month</td>
<td>-2.9</td>
<td>-2.6</td>
<td>-7.2</td>
<td>-4.2</td>
</tr>
</tbody>
</table>

### Table 10: Distributively-neutral increase in ITA

<table>
<thead>
<tr>
<th>Scenario 5</th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-reform ITA in €</td>
<td>247</td>
<td>417</td>
<td>1126</td>
<td></td>
</tr>
<tr>
<td>post-reform ITA in €</td>
<td>263</td>
<td>430</td>
<td>1139</td>
<td></td>
</tr>
<tr>
<td>EV in per cent</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>EV in € per month</td>
<td>-2.0</td>
<td>-3.5</td>
<td>-7.2</td>
<td>-4.2</td>
</tr>
</tbody>
</table>

Given distributional neutrality for Scenarios 3 and 5, both scenarios can be compared in efficiency terms. We can then see that Scenario 5 induces (small) welfare losses, while Scenario 3 leads to (small) efficiency gains. The reasoning behind this is that in Scenario 5 we essentially replace a lump-sum tax by a distortive tax, whereas in Scenario 3 we trade off two distortive taxes against each other.

The third instrument of tax revenue recycling considered in our analysis are the social security contributions (SSC). Again, we first show the case where the SSC are changed uniformly (Scenario 6), and then differentiate the SSC of the households endogenously to achieve proportional welfare gains across households (Scenario 7).

A uniform proportional decrease of the SSC – as in Table 11 – leaves the middle tercile substantially better off. The welfare gain for the upper tercile is also significant,
<table>
<thead>
<tr>
<th>Scenario 6</th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-reform SSC</td>
<td>23.5%</td>
<td>20.4%</td>
<td>12.7%</td>
<td></td>
</tr>
<tr>
<td>post-reform SSC</td>
<td>21.4%</td>
<td>18.5%</td>
<td>11.5%</td>
<td></td>
</tr>
<tr>
<td>EV in per cent</td>
<td>-0.29</td>
<td>+0.23</td>
<td>+0.21</td>
<td>+0.14</td>
</tr>
<tr>
<td>EV in € per month</td>
<td>-6.7</td>
<td>+9.1</td>
<td>+17.9</td>
<td>+6.8</td>
</tr>
</tbody>
</table>

Table 11: Uniform cut in SSC

whereas the lower tercile clearly loses. The fact that the middle tercile benefits most is explained by the highest share of labour income (the tax base for the SSC) in this group. In contrast, for the lower tercile, transfer income makes up a large part of total income, and for the upper tercile capital income gains in weight.

<table>
<thead>
<tr>
<th>Scenario 7</th>
<th>lower tercile</th>
<th>middle tercile</th>
<th>upper tercile</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-reform SSC</td>
<td>23.5%</td>
<td>20.4%</td>
<td>12.7%</td>
<td></td>
</tr>
<tr>
<td>post-reform SSC</td>
<td>19.9%</td>
<td>18.8%</td>
<td>11.7%</td>
<td></td>
</tr>
<tr>
<td>EV in per cent</td>
<td>+0.12</td>
<td>+0.12</td>
<td>+0.12</td>
<td>+0.12</td>
</tr>
<tr>
<td>EV in € per month</td>
<td>+2.8</td>
<td>+4.9</td>
<td>+10.2</td>
<td>+6.0</td>
</tr>
</tbody>
</table>

Table 12: Distributively-neutral cut in SSC

When we adjust the SSC in a way that assures distributive neutrality (see Table 12), the cut in SSC is highest for the lower tercile (−3.6 p.p.) and lowest for the upper tercile (−1.0 p.p.). In relative terms, however, the cut is now lowest for the middle tercile (to compensate for its high share of labour income). The uniform welfare increase amounts to 0.12%, which stands out as the highest value across all (distributionally neutral) scenarios and makes the SSC the most attractive candidate for actual tax reforms.

Social Security Contributions (SSC) are levied on labour only, whereas income taxes are levied on both labour and capital income. Revenue-neutral reductions of social security contributions (SSC) thus work as a direct cut of labour costs, whereas a reduction of the marginal income tax rate (MITR) will reduce the tax burden on labour as well as capital. The favourable results for SSC reflect that, in our model, capital taxation is associated with smaller distortions than labour income (connected to the tax elasticities of the respective income categories). However, our static AGE model does not explicitly consider the intertemporal dimensions of savings and investment. As a consequence, it is not well suited to compare the efficiency effects of
capital taxation with those of labour taxation. Caution is therefore warranted regarding the more favourable results for the SSC scenarios as compared to the MITR scenarios.\(^\text{10}\)

### 3.2 Macroeconomic Effects

Table 13 summarises the macroeconomic consequences across four of our scenarios: the pure VAT reform (Scenario 1) and the three scenarios based on alternative tax recycling instruments (Scenarios 3, 5 and 7) where distributional effects are compensated.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Tax recycling instrument</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td></td>
<td>0.12</td>
<td>0.36</td>
<td>-0.07</td>
<td>0.41</td>
</tr>
<tr>
<td>Employment</td>
<td></td>
<td>0.03</td>
<td>0.16</td>
<td>-0.21</td>
<td>0.33</td>
</tr>
<tr>
<td>Domestic capital use</td>
<td></td>
<td>0.25</td>
<td>0.64</td>
<td>0.12</td>
<td>0.51</td>
</tr>
<tr>
<td>Total consumption</td>
<td></td>
<td>0.03</td>
<td>0.03</td>
<td>-0.33</td>
<td>0.22</td>
</tr>
<tr>
<td>Total investment</td>
<td></td>
<td>0.14</td>
<td>0.57</td>
<td>0.22</td>
<td>0.54</td>
</tr>
<tr>
<td>Imports</td>
<td></td>
<td>-0.31</td>
<td>-0.21</td>
<td>-0.59</td>
<td>-0.05</td>
</tr>
<tr>
<td>Exports</td>
<td></td>
<td>-0.02</td>
<td>0.48</td>
<td>-0.40</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table entries are given as percentage changes.

Table 13: Macroeconomic effects of VAT reform

The results are consistent with the consideration of the EV values in Section 3.1 where the SSC and the MITR were the most favourable tax recycling instrument followed by the VAT and the ITA. As Table 13 shows this ranking is generally reflected by the figures for GDP and employment, domestic capital use, aggregate consumption and exports.

### 3.3 Industry Effects

Table 14 reports the impacts of VAT reforms on the output of individual industries. We condense the information on the 69 sectors of the model into a few aggregate

\(^{10}\)The theoretical literature on optimal taxation suggests that the excess burden of capital taxation is higher than that of labour taxation (e.g. Chamley, 1986, Atkeson et al., 1999).
indicators: the (unweighted) average rate of growth of the output across industries, the standard deviation of growth rates, the number of growing and shrinking industries, maximum and minimum of the industries’ growth rates as well as the 10th and 90th percentile.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tax recycling instrument</strong></td>
<td>VAT</td>
<td>MITR</td>
<td>ITA</td>
<td>SSC</td>
</tr>
<tr>
<td>Av. increase in production</td>
<td>0.10</td>
<td>0.42</td>
<td>-0.18</td>
<td>0.41</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.09</td>
<td>1.18</td>
<td>1.17</td>
<td>1.07</td>
</tr>
<tr>
<td>Number of growing industries</td>
<td>35</td>
<td>46</td>
<td>23</td>
<td>53</td>
</tr>
<tr>
<td>Number of shrinking industries</td>
<td>34</td>
<td>23</td>
<td>46</td>
<td>16</td>
</tr>
<tr>
<td>Maximum growth</td>
<td>5.47</td>
<td>5.66</td>
<td>5.42</td>
<td>5.59</td>
</tr>
<tr>
<td>Minimum growth</td>
<td>-1.89</td>
<td>-1.49</td>
<td>-2.54</td>
<td>-1.30</td>
</tr>
<tr>
<td>90th percentile</td>
<td>0.56</td>
<td>1.31</td>
<td>0.41</td>
<td>0.75</td>
</tr>
<tr>
<td>10th percentile</td>
<td>-0.63</td>
<td>-0.38</td>
<td>-1.06</td>
<td>-0.44</td>
</tr>
</tbody>
</table>

Entries are changes in percent (except number of growing/shrinking industries).

Table 14: Industry effects

The figures in Table 14 confirm our previous ranking of the tax recycling instruments, although MITR and SSC are almost indistinguishable, in terms of industry effects. Around the average values, there is a considerable spread in the industry-level outcomes. This spread is rather robust across the scenarios. At the disaggregated industry-level, the performance across sectors is also rather stable: Financial and Insurance Services as well as Research and Development are always among the industries that gain the most, whereas Communication and Media, Other Vehicles and Hotel and Catering Industry show the largest losses.\textsuperscript{11}

4 Conclusions

In this paper, we have investigated the economic effects of a structural VAT reform for the German economy. Based on an AGE framework tailored to the requirements of VAT reform analysis, we have simulated several revenue-neutral variants of abolishing the reduced VAT rate in Germany. We have analysed a pure VAT reform,\textsuperscript{11} A detailed table of industry level results is available upon request.
where the differentiated VAT is replaced with a uniform rate, that is about two percentage points lower than the standard rate of 16%, and further scenarios in which tax revenue is recycled through other taxes: the marginal income tax rate (MITR), the income tax allowance (ITA) or the social security contributions (SSC).

Our main findings can be summarised as follows: The abolition of the reduced VAT rate in itself has only a small distributional effect towards larger inequality. Therefore, VAT differentiation can hardly be considered as an important means of redistribution. When we combine the abolition of reduced VAT rates with revenue recycling through reduction of the marginal income tax rate or cuts in social security contributions, there is scope for significant gains in overall welfare. The income tax allowance, in contrast, produces welfare losses if used as a tax recycling instrument. These results also hold if we consider distributionally-neutral reforms where tax rates or social security contributions are adjusted differently across the income distribution, ensuring that all households equally benefit from the reforms.

Policy-induced changes in macroeconomic indicators like GDP, employment, domestic capital use, or aggregate consumption echo the welfare ranking of tax instruments. While the distributional effects of VAT reforms are within a relatively narrow range, the industry effects (in terms of variation in industry output) are much more pronounced. This indicates that the VAT rate differentiation should be viewed primarily as a subsidy to specific industries rather than an instrument of redistribution. From a political economy point of view, the different effects at industry level highlight lobbying interests of adversely affected sectors to work against changes of the actual VAT structure.
References


I  Appendix: Model Description

In this appendix we give a full algebraic description of the model. A list of all sets, indices, variables and parameters can be found in Sections I.1 to I.3. Section I.4 then presents the equations, classified into price and demand equations, market clearance conditions and household budget constraints.

I.1  Indices and Index Sets

I.1.1  Sets

\[ c \quad := \quad \text{consumption good index} \]
\[ h \quad := \quad \text{household index} \]
\[ s, ss \quad := \quad \text{industry indices} \]

I.1.2  Index Sets

\[ C \quad := \quad \text{all 12 consumption good categories} \]
\[ ELE \quad := \quad \text{one-element set: electricity} \]
\[ F \quad := \quad \text{one-element set: food consumption} \]
\[ FEN \quad := \quad \text{fossil energy sectors} \]
\[ NEN \quad := \quad \text{non-energy sectors} \]
\[ NF \quad := \quad \text{non-food consumption goods} \]
\[ S \quad := \quad \text{all 69 sectors of the German IOT 1997} \]
\[ VAT \quad := \quad \text{different VAT rates} \]

I.2  Variables

I.2.1  Quantities

\[ A_s \quad := \quad \text{Armitage good} \]
\[ A_{s,c} \quad := \quad \text{intermediate inputs for consumption (Z-matrix)} \]
\[ A_{s,G} \quad := \quad \text{intermediate input for government consumption} \]
\[ A_{s,I} \quad := \quad \text{intermediate input for investment goods} \]
\[ A_{s,ss} \quad := \quad \text{intermediate inputs for production} \]
\[ A_{s,STK} := \text{stock changes} \]
\[ C_c := \text{consumption goods} \]
\[ C_{c,h} := \text{consumption goods by household} \]
\[ C_{c,VAT} := \text{consumption goods by VAT category} \]
\[ C_h := \text{consumption good aggregate} \]
\[ CU_h := \text{current utility (goods consumption and leisure)} \]
\[ D_s := \text{deliveries to the domestic market} \]
\[ E_c := \text{energy aggregate in consumption} \]
\[ E_s := \text{energy aggregate in production} \]
\[ FE_c := \text{fossil energy aggregate in consumption} \]
\[ FE_s := \text{fossil energy aggregate in production} \]
\[ G := \text{government consumption} \]
\[ I := \text{aggregate investment} \]
\[ I_G := \text{government investment} \]
\[ K := \text{total capital employed domestically} \]
\[ K_s := \text{capital input} \]
\[ KD := \text{domestically invested capital} \]
\[ KD_{h} := \text{domestically invested capital by household} \]
\[ KE_s := \text{capital-energy aggregate in production} \]
\[ KEL_s := \text{quantity of capital-energy-labour aggregate} \]
\[ KM := \text{capital imports} \]
\[ KX_h := \text{capital exports} \]
\[ L_s := \text{labour input} \]
\[ LE_{h} := \text{leisure} \]
\[ M_s := \text{imports} \]
\[ NF_{h} := \text{non-food consumption} \]
\[ S_h := \text{savings} \]
\[ U_h := \text{utility index} \]
\[ X_s := \text{Exports} \]
\[ Y_s := \text{production in sector s} \]

### 1.2.2 Prices

\[ p_{A,s} := \text{price of Armington commodity} \]
\[ p_c := \text{price of consumption goods (gross of VAT)} \]
\begin{align*}
p_{C,h} & := \text{price of consumption goods aggregate} \\
p_{CU,h} & := \text{price of current utility aggregate} \\
p_{p,c} & := \text{production price of consumption good} \\
p_{D,s} & := \text{price of output delivered to the domestic market} \\
p_{E,c} & := \text{price of energy aggregate in consumption} \\
p_{E,s} & := \text{price of energy aggregate in production} \\
p_{F} & := \text{price of food consumption} \\
p_{G} & := \text{price index of government consumption} \\
p_{FE,c} & := \text{price of fossil energy aggregate in consumption} \\
p_{FE,s} & := \text{price of fossil energy aggregate in production} \\
p_{I} & := \text{price index for investment goods} \\
p_{K} & := \text{rental rate of capital} \\
p_{KE,s} & := \text{price of the capital-energy aggregate} \\
p_{KE,k,s} & := \text{price of the capital-energy-labour aggregate} \\
p_{K,d} & := \text{price of capital in the domestic market} \\
p_{KM} & := \text{price of capital imports} \\
p_{S,h} & := \text{price for capital supply of households} \\
p_{X,h} & := \text{price of capital exports} \\
p_{L} & := \text{wage (net of payroll tax)} \\
p_{LE,h} & := \text{price index for leisure} \\
p_{LS,h} & := \text{expected revenue from labour supply} \\
p_{M,s} & := \text{import prices (net of import tax)} \\
p_{NF,h} & := \text{price of non-food aggregate consumption} \\
p_{U,h} & := \text{price of utility aggregate (expenditure function)} \\
p_{S,h} & := \text{price of future consumption (savings)} \\
p_{X,s} & := \text{export prices} \\
p_{Y,s} & := \text{producer prices}
\end{align*}

I.2.3 Others

\begin{align*}
u & := \text{unemployment rate} \\
y_h & := \text{extended income of households}
\end{align*}
I.3 Parameters

I.3.1 Value Shares

\[ \theta^i := \text{value share of item } i \text{ in its respective sub-aggregate in the benchmark} \]

I.3.2 Taxes

\[ t_{I,h} := \text{marginal income tax rate} \]
\[ t_{KM} := \text{capital import tax} \]
\[ t_{PR} := \text{payroll tax (employer’s social security contributions)} \]
\[ t_{S,h} := \text{social security contributions of households} \]
\[ t_{VAT} := \text{value-added tax on consumption goods} \]
\[ t_{Y,s} := \text{output tax (sum of taxes and subsidies)} \]
\[ TA_h := \text{income tax allowance} \]

I.3.3 Elasticities

\[ \sigma^A_s := \text{EOS between domestic production and imports} \]
\[ \sigma^C_h := \text{EOS between food and non-food consumption} \]
\[ \sigma^{CU}_{hv} := \text{EOS between leisure and current consumption} \]
\[ \sigma^E_{c} := \text{EOS between electricity and fossil fuels} \]
\[ \sigma^S_{c} := \text{EOS between electricity and fossil fuels} \]
\[ \sigma^{FE}_{c} := \text{EOS between varieties of fossil fuels} \]
\[ \sigma^{FE}_{s} := \text{EOS between varieties of fossil fuels} \]
\[ \sigma^K := \text{EOS between domestic capital and capital imports} \]
\[ \sigma^{KE}_{s} := \text{EOS between K and E} \]
\[ \sigma^{KEL}_{s} := \text{EOS between KE and L} \]
\[ \eta^{KS} := \text{EOT between domestic and foreign investment} \]
\[ \sigma^{NE}_{c} := \text{EOS between NEN goods in consumption} \]
\[ \sigma^{NF}_{c} := \text{EOS between non-food goods} \]
\[ \eta^T_{s} := \text{EOT between domestic use and exports} \]
\[ \sigma^U_{h} := \text{EOS between future and current consumption} \]
\[ \sigma^{VAT}_{c} := \text{EOS between good varieties with different VAT rate} \]
\[ \sigma^Y_{s} := \text{EOS between intermediate inputs and KEL aggregate} \]

\( \sigma^U_h \) is calibrated to reproduce empirical savings elasticities.
\( \sigma^{CU}_h \) is calibrated to reproduce empirical labour supply elasticities.
\( \sigma^K \) and \( \eta^{KS} \) are calibrated to reproduce capital import and export elasticities. \( \sigma^C \) and \( \sigma^{NE} \) are calibrated to reproduce consumption good demand elasticities. The calibration procedures are explained in Sections 2.3.2 and 2.3.3 of the main text.

I.3.4 Others

\[
\begin{align*}
    b & := \text{unemployment benefits} \\
    \text{\textit{BOP}} & := \text{balance of payments surplus} \\
    \tilde{K}_h & := \text{capital endowment by household} \\
    \tilde{L}_h & := \text{time endowment by household} \\
    \text{\textit{\textit{TR}}}_h & := \text{benchmark transfers}
\end{align*}
\]

Any variable (or parameter in the case of taxes) with an upper bar denotes its benchmark value.

I.4 Model Equations

The model equations are split up into price and demand equations, market clearance conditions, budget constraints and auxiliary equations. There are no explicit production functions in the model, because all necessary information is contained in the dual price functions.

To maintain structural symmetry, the equations are written down in their most general form. In the actual numerical implementation of the model, considerable simplifications are achieved by normalising benchmark prices and quantities to unity where possible. Some of the CES functions collapse to Cobb-Douglas or Leontief functions by setting the elasticity of substitution to one or zero, respectively (see Section I.3.3).

I.4.1 Price Equations

Production is organised according to a nested CES production function. Subsets of the various industries’ inputs that are used to form sub-nests of the productions function can be found in Section I.1.1.

\[
\frac{p_{Y,s}(1-t_{Y,s})}{\bar{p}_{Y,s}(1-t_{Y,s})} = \left[ \theta^K_{\text{EL}} \left( \frac{p_{\text{EL},s}}{\bar{p}_{\text{EL},s}} \right)^{1-\sigma_Y^s} + \sum_{s \in NEN} \theta^{ss}_s \left( \frac{p_{A,ss}}{\bar{p}_{A,ss}} \right)^{1-\sigma_Y^s} \right]^{\frac{1}{1-\sigma_Y^s}}
\]  

(2)
\[
\frac{p_{KEL,s}}{\bar{p}_{KEL,s}} = \left[ \theta_s^{KE} \left( \frac{p_{KE,s}}{\bar{p}_{KE,s}} \right)^{1-\sigma_s^{KE}} + \theta_s^{L} \left( \frac{p_{L}(1+t_{PR})}{\bar{p}_{L}(1+t_{PR})} \right)^{1-\sigma_s^{KE}} \right]^{1\over 1-\sigma_s^{KE}} \tag{3}
\]

\[
\frac{p_{KE,s}}{\bar{p}_{KE,s}} = \left[ \theta_s^{K} \left( \frac{p_{K}}{\bar{p}_{K}} \right)^{1-\sigma_s^{KE}} + \theta_s^{E} \left( \frac{p_{E,s}}{\bar{p}_{E,s}} \right)^{1-\sigma_s^{KE}} \right]^{1\over 1-\sigma_s^{KE}} \tag{4}
\]

\[
\frac{p_{E,s}}{\bar{p}_{E,s}} = \left[ \theta_s^{ELE} \left( \frac{p_{A,ELE}}{\bar{p}_{A,ELE}} \right)^{1-\sigma_s^{E}} + \theta_s^{EC} \left( \frac{p_{FE,s}}{\bar{p}_{FE,s}} \right)^{1-\sigma_s^{E}} \right]^{1\over 1-\sigma_s^{E}} \tag{5}
\]

\[
\frac{p_{FE,s}}{\bar{p}_{FE,s}} = \left[ \sum_{i\in EN} \theta_s^{i} \left( \frac{p_{A,i}}{\bar{p}_{A,i}} \right)^{1-\sigma_s^{FE}} \right]^{1\over 1-\sigma_s^{FE}} \tag{6}
\]

Output is split into domestic use and exports through a CET function:

\[
\frac{p_{Y,s}}{\bar{p}_{Y,s}} = \left[ \theta_s^{DY} \left( \frac{p_{D,s}}{\bar{p}_{D,s}} \right)^{1+\eta_s^{U}} + \theta_s^{X} \left( \frac{p_{X,s}}{\bar{p}_{X,s}} \right)^{1+\eta_s^{U}} \right]^{1\over 1+\eta_s^{U}} \tag{7}
\]

Domestically produced goods and imports are combined to an “Armington good”:

\[
\frac{p_{A,s}}{\bar{p}_{A,s}} = \left[ \theta_s^{DA} \left( \frac{p_{D,s}}{\bar{p}_{D,s}} \right)^{1-\sigma_s^{A}} + \theta_s^{M} \left( \frac{p_{M,s}(1+t_{M,s})}{\bar{p}_{M,s}(1+t_{M,s})} \right)^{1-\sigma_s^{A}} \right]^{1\over 1-\sigma_s^{A}} \tag{8}
\]

Household utility is derived from future and current consumption:

\[
\frac{p_{U,h}}{\bar{p}_{U,h}} = \left[ \theta_h^{CU} \left( \frac{p_{CU,h}}{\bar{p}_{CU,h}} \right)^{1-\sigma_h^{U}} + \theta_h^{S} \left( \frac{p_{S,h}}{\bar{p}_{S,h}} \right)^{1-\sigma_h^{U}} \right]^{1\over 1-\sigma_h^{U}} , \tag{9}
\]

where the price of savings (future consumption) consists of the following components:

\[
\frac{p_{S,h}}{\bar{p}_{S,h}} = \frac{p_{I}}{\bar{p}_{I}} \frac{p_{C,h}}{\bar{p}_{C,h}} \frac{p_{K,h}}{\bar{p}_{K,h}} \tag{10}
\]

Current utility consists of consumption of goods and leisure:

\[
\frac{p_{CU,h}}{\bar{p}_{CU,h}} = \left[ \theta_h^{G} \left( \frac{p_{C,h}}{\bar{p}_{C,h}} \right)^{1-\sigma_h^{G}} + \theta_h^{LE} \left( \frac{p_{LE,h}}{\bar{p}_{LE,h}} \right)^{1-\sigma_h^{G}} \right]^{1\over 1-\sigma_h^{G}} , \tag{11}
\]

\[
\frac{p_{C,h}}{\bar{p}_{C,h}} = \left[ \theta_h^{E} \left( \frac{p_{E}}{\bar{p}_{E}} \right)^{1-\sigma_h^{E}} + \theta_h^{NF} \left( \frac{p_{NF,h}}{\bar{p}_{NF,h}} \right)^{1-\sigma_h^{E}} \right]^{1\over 1-\sigma_h^{E}} \tag{12}
\]

\[
\frac{p_{NF,h}}{\bar{p}_{NF,h}} = \left[ \sum_{i\in NF} \theta_h^{i} \left( \frac{p_{i}}{\bar{p}_{i}} \right)^{1-\sigma_h^{NF}} \right]^{1\over 1-\sigma_h^{NF}} \tag{13}
\]
\[
\frac{p_c}{\bar{p}_c} = \left[ \sum_{v \in NEN} \theta_c^v \left( \frac{p_{p,c}(1 + t_{vAT})}{\bar{p}_{p,c}(1 + t_{vAT})} \right)^{1 - \sigma_{p,c}^{vAT}} + \theta_c^E \left( \frac{p_{E,c}}{\bar{p}_{E,c}} \right)^{1 - \sigma_{E}^{p,c}} \right]^{-\frac{1}{1 - \sigma_{p,c}^{vAT}}} \quad \text{for } c \in F, NF
\] (14)

Consumption goods are produced from the output of the production sectors with a CES production function:

\[
\frac{p_{p,c}}{\bar{p}_{p,c}} = \left[ \sum_{s \in NEN} \theta_c^s \left( \frac{p_{A,s}}{\bar{p}_{A,s}} \right)^{1 - \sigma_{p,c}^{A,s}} + \theta_c^E \left( \frac{p_{E,c}}{\bar{p}_{E,c}} \right)^{1 - \sigma_{E}^{p,c}} \right]^{-\frac{1}{1 - \sigma_{p,c}^{A,s}}} \quad \text{(15)}
\]

\[
\frac{p_{E,c}}{\bar{p}_{E,c}} = \theta_c^{E E} \left( \frac{p_{A, E E}}{\bar{p}_{A, E E}} \right)^{1 - \sigma_{E}^{p,c}} + \theta_c^E \left( \frac{p_{E,E,c}}{\bar{p}_{E,E,c}} \right)^{1 - \sigma_{E}^{p,c}} \right]^{-\frac{1}{1 - \sigma_{E}^{p,c}}} \quad \text{(16)}
\]

\[
\frac{p_{F,E,c}}{\bar{p}_{F,E,c}} = \left[ \sum_{s \in FEN} \theta_c^s \left( \frac{p_{A,s}}{\bar{p}_{A,s}} \right)^{1 - \sigma_{F,E,c}^{A,s}} \right]^{-\frac{1}{1 - \sigma_{F,E,c}^{A,s}}} \quad \text{(17)}
\]

Government demand is composed of government investment and inputs from the production sectors in fixed proportions:

\[
\frac{p_{G}}{\bar{p}_G} = \frac{\theta_G}{\bar{p}_I} + \sum_s \theta_G \frac{p_{A,s}}{\bar{p}_{A,s}}
\] (18)

Investment goods are also produced with fixed production coefficients:

\[
\frac{p_{I}}{\bar{p}_I} = \sum_s \theta_I \frac{p_{A,s}}{\bar{p}_{A,s}}
\] (19)

Capital supply is transformed into domestic use and capital exports through a CET function:

\[
\frac{p_{K,S,h}}{\bar{p}_{K,S,h}} = \theta^K \left( \frac{p_{K,D}(1 - t_{I,h})}{\bar{p}_{K,D}(1 - t_{I,h})} \right)^{1 + \eta^{K,S}} + \theta^K \left( \frac{p_{K,X}}{\bar{p}_{K,X}} \right)^{1 + \eta^{K,S}} \right]^{-\frac{1}{1 + \eta^{K,S}}} \quad \text{(20)}
\]

Domestic and imported capital are imperfect substitutes in production:

\[
\frac{p_{K}}{\bar{p}_K} = \theta^K \left( \frac{p_{K,D}}{\bar{p}_{K,D}} \right)^{1 - \sigma^K} + \theta^K \left( \frac{p_{K,M}(1 + t_{K,M})}{\bar{p}_{K,M}(1 + t_{K,M})} \right)^{1 - \sigma^K} \right]^{-\frac{1}{1 - \sigma^K}} \quad \text{(21)}
\]

### I.4.2 Demand and Supply Equations

Demand for factors of production and intermediate inputs:

\[
\frac{A_{ss,s}}{\bar{A}_{ss,s}} = \frac{Y_s}{\bar{Y}_s} \left( \frac{p_{Y,s}(1 - t_{Y,s})}{\bar{p}_{Y,s}(1 - t_{Y,s})} \right)^{\sigma^Y_{ss}} \quad \text{for } ss \in NEN
\] (22)
\[
\begin{align*}
\frac{K_{EL_s}}{K_{EL_s}} &= \frac{Y_s}{Y_s} \left( \frac{p_{Y,s}(1-t_{Y,s})}{p_{Y,s}} \frac{\bar{p}_{KEL_s}}{\bar{p}_{KEL_s}} \right)^{\sigma_s} \\
\frac{L_s}{L_s} &= \frac{K_{EL_s}}{K_{EL_s}} \left( \frac{p_{KEL_s} \bar{p}_{L}(1+t_{PR})}{\bar{p}_{KEL_s} \bar{p}_{L}(1+t_{PR})} \right)^{\sigma_{KEL}} \\
\frac{K_{E_s}}{K_{E_s}} &= \frac{K_{EL_s}}{K_{EL_s}} \left( \frac{p_{KEL_s} \bar{p}_{K_E,s}}{\bar{p}_{KEL_s} \bar{p}_{K_E,s}} \right)^{\sigma_{KEL}} \\
\frac{K_s}{K_s} &= \frac{K_{E_s}}{K_{E_s}} \left( \frac{p_{K_E,s}}{\bar{p}_{K_E,s}} \frac{\bar{p}_K}{\bar{p}_K} \right)^{\sigma_{KE}} \\
\frac{E_s}{E_s} &= \frac{K_{E_s}}{K_{E_s}} \left( \frac{p_{K_E,s} \bar{p}_{E,s}}{\bar{p}_{K_E,s} \bar{p}_{E,s}} \right)^{\sigma_{KE}} \\
\frac{A_{ELE,s}}{A_{ELE,s}} &= \frac{E_s}{E_s} \left( \frac{\bar{p}_{E,s} \bar{p}_{A,ELE}}{\bar{p}_{E,s} \bar{p}_{A,ELE}} \right)^{\sigma_E} \\
\frac{FE_s}{FE_s} &= \frac{E_s}{E_s} \left( \frac{\bar{p}_{E,s} \bar{p}_{FE,s}}{\bar{p}_{E,s} \bar{p}_{FE,s}} \right)^{\sigma_E} \\
\frac{A_{ss,s}}{A_{ss,s}} &= \frac{FE_s}{FE_s} \left( \frac{\bar{p}_{FE,s} \bar{p}_{A,ss}}{\bar{p}_{FE,s} \bar{p}_{A,ss}} \right)^{\sigma_{FE}} \text{ for } ss \in FEN
\end{align*}
\]
\[
\frac{LE_h}{LE_h} = \frac{CU_h}{CU_h} \left( \frac{p_{CU,h} \bar{p}_{LE,h}}{p_{CU,h} \bar{p}_{LE,h}} \right)^{\sigma^C_h}
\]

(38)

\[
\frac{C_{F,h}}{C_{F,h}} = \frac{C_h}{C_h} \left( \frac{p_{C,h} \bar{p}_F}{p_{C,h} \bar{p}_F} \right)^{\sigma^C_h}
\]

(39)

\[
\frac{NF_h}{NF_h} = \frac{C_h}{C_h} \left( \frac{p_{C,h} \bar{p}_{NF,h}}{p_{C,h} \bar{p}_{NF,h}} \right)^{\sigma^C_h}
\]

(40)

\[
\frac{C_{c,h}}{C_{c,h}} = \frac{NF_h}{NF_h} \left( \frac{p_{NF,h} \bar{p}_c}{p_{NF,h} \bar{p}_c} \right)^{\sigma^N_h} \text{for } c \in NF
\]

(41)

\[
\frac{C_{c,h,VA}}{C_{c,h,VA}} = \frac{C_{c,h}}{C_{c,h}} \left( \frac{p_c \bar{p}_{c,1 + \bar{t}_{VA}}}{{p_c} - p_{c,1 + \bar{t}_{VA}}} \right)^{\sigma^VA_h} \text{for } c \in F, NF
\]

(42)

Demand of production output for consumption goods:

\[
\frac{A_{s,c}}{A_{s,c}} = \frac{C_c}{C_c} \left( \frac{p_{p,c} \bar{p}_{A,s}}{p_{p,c} \bar{p}_{A,s}} \right)^{\sigma^{NE}_c} \text{for } s \in NEN
\]

(43)

\[
\frac{E_c}{E_c} = \frac{C_c}{C_c} \left( \frac{p_{p,c} \bar{p}_{E_c}}{p_{p,c} \bar{p}_{E_c}} \right)^{\sigma^{NE}_c}
\]

(44)

\[
\frac{A_{ELE,c}}{A_{ELE,c}} = \frac{E_c}{E_c} \left( \frac{p_{p,c} \bar{p}_{A, ELE}}{p_{p,c} \bar{p}_{A, ELE}} \right)^{\sigma^{E}_c}
\]

(45)

\[
\frac{F_{E_c}}{F_{E_c}} = \frac{E_c}{E_c} \left( \frac{p_{p,c} \bar{p}_{F_{E,c}}}{p_{p,c} \bar{p}_{F_{E,c}}} \right)^{\sigma^{E}_c}
\]

(46)

\[
\frac{A_{s,c}}{A_{s,c}} = \frac{F_{E_c}}{F_{E_c}} \left( \frac{p_{F_{E,c}} \bar{p}_{A,s}}{p_{F_{E,c}} \bar{p}_{A,s}} \right)^{\sigma^{E^c}_c} \text{for } s \in FEN
\]

(47)

Government demand:

\[
\frac{I_G}{I_G} = \frac{A_{s,G}}{A_{s,G}} = G
\]

(48)

Demand for inputs for investment good production:

\[
\frac{A_{s,I}}{A_{s,I}} = \frac{I}{I}
\]

(49)

Demand for domestic and imported capital:

\[
\frac{K D}{K D} = \frac{K}{K} \left( \frac{p_K \bar{p}_{K D}}{p_K \bar{p}_{K D}} \right)^{\sigma^K}
\]

(50)

\[
\frac{K M}{K M} = \frac{K}{K} \left( \frac{p_K \bar{p}_{K M}(1 + \bar{t}_{K M})}{p_K \bar{p}_{K M}(1 + t_{K M})} \right)^{\sigma^K}
\]

(51)
Supply of capital to the domestic and foreign market:

\[
\frac{KD_h}{KD} = \left( \frac{\bar{p}_{K,S,h} p_{K,D}(1 - t_{I,h})}{\bar{p}_{K,S,h} \bar{p}_{K,D}(1 - t_{I,h})} \right)^{\nu^{KS}}
\]  

(52)

\[
\frac{KX_h}{KX} = \left( \frac{\bar{p}_{K,S,h} p_{K,X}}{\bar{p}_{K,S,h} \bar{p}_{K,X}} \right)^{\nu^{KS}}
\]  

(53)

I.4.3 Market Clearing Conditions

Armington good:

\[
A_s = \sum_{ss} A_{s,ss} + \sum_c A_{s,c} + A_{s,STK} + A_{s,G} + A_{s,I}
\]  

(54)

Capital:

\[
\sum_h K_h = \bar{K}
\]  

(55)

\[
K + KM = KD + KX + K = KX + K
\]  

(56)

\[
KD = \sum_h KD_h
\]  

(57)

\[
KX = \sum_h KX_h
\]  

(58)

\[
K = \sum_s K_s
\]  

(59)

Labour and leisure:

\[
(1 - u) \sum_h (\bar{L}_h - LE_h) = \sum_s L_s
\]  

(60)

Consumption goods:

\[
C_c = \sum_h C_{c,h}
\]  

(61)

Domestic investment:

\[
I = \sum_h S_h
\]  

(62)

Balance of payments:

\[
\text{BOP} = \sum_s (p_{X,s} X_s - p_{M,s} M_s) + p_{K,X} \sum_h KX_h - p_{K,M} KM
\]  

(63)

All other market clearing conditions are trivial, because they consist only of a single demand and a single supply component.
I.4.4 Household Budget Constraints

Budget constraints of private households (extended income, including correction term for savings):

\[ Y_h = p_{U,h}U_h = (p_{K,S,h}K + p_{L,S,h}L_h - T A_h) (1 - t_{I,h}) + TR_h + (p_{S,h} - p_I) S_h \]  

(64)

Government budget constraint:

\[ p_C G = \sum_s (t_{H,s} p_{V,s} Y_s + t_{M,s} p_{A,s} M_s) + \sum_s t_{RP} L_s \]

\[ + t_{KM} p_{KM} K M + \sum_{c,h,VAT} t_{VAT} p_c C_{c,h,VAT} \]

\[ + \sum_h t_{I,h} (p_{K,D,h} K D_h + (1 - u)(L_h - L E_h) p_L - T A_h) \]

\[ + \sum_h t_{S,h} (1 - u)(L_h - L E_h) p_L \]

\[ - \sum_h TR_h - \sum_s p_{A,s} A_{s,STK} - B O P - \sum_h u(L_h - L E_h) \frac{p_{C,h}}{p_{C,h}} \]  

(57)

I.4.5 Additional Equations for Unemployment

The supply price of labour is a weighted average of the after-tax wage and the unemployment benefit, which is indexed to the consumer price index:

\[ \frac{p_{L,S,h}}{p_{L,S,h}} = (1 - u) \frac{p_L}{p_L} (1 - t_{S,h} - t_{I,h}) + u \frac{p_{C,h}}{p_{C,h}} \]  

(66)

The unemployment rate is determined through a wage curve, which depends on the coefficient of residual income progression. We assume that the tax rates of the median household \((h = M)\) are the relevant ones:

\[ \frac{u}{\bar{u}} = \frac{1 - \bar{t}_{I,M} \left(1 - \frac{T_{M}}{Y_M}\right)}{1 - \bar{t}_{I,M}} \frac{1 - t_{I,M}}{1 - t_{I,M} \left(1 - \frac{T_{M}}{Y_M}\right)} \]  

(67)
Sensitivity analysis for
“Economic Effects of VAT Reforms in Germany”
by Stefan Boeters, Christoph Böhringer, Thiess Büttner, and Margit Kraus

Sensitivity analysis (1): Elasticity of labour supply doubled (0.3 instead of 0.15). We compare the macroeconomic effects (Table 13 in the paper):

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
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<tbody>
<tr>
<td><strong>Tax recycling instrument</strong></td>
<td>VAT</td>
<td>MITR</td>
<td>ITA</td>
<td>SSC</td>
</tr>
<tr>
<td>GDP</td>
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<td>-0.14</td>
<td>0.56</td>
</tr>
<tr>
<td>Employment</td>
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<td>0.13</td>
<td>-0.29</td>
<td>0.50</td>
</tr>
<tr>
<td>Domestic capital use</td>
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<td>0.70</td>
<td>0.06</td>
<td>0.64</td>
</tr>
<tr>
<td>Total consumption</td>
<td>0.05</td>
<td>0.10</td>
<td>-0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>Total investment</td>
<td>0.14</td>
<td>0.61</td>
<td>0.17</td>
<td>0.62</td>
</tr>
<tr>
<td>Imports</td>
<td>-0.29</td>
<td>-0.14</td>
<td>-0.67</td>
<td>0.13</td>
</tr>
<tr>
<td>Exports</td>
<td>0.01</td>
<td>0.60</td>
<td>-0.54</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Table entries are given as percentage changes.

Table 1: Higher elasticity of labour supply: macro effects
Sensitivity analysis (2): Elasticity of labour supply halved (0.075 instead of 0.15):

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax recycling instrument</td>
<td>VAT</td>
<td>MITR</td>
<td>ITA</td>
<td>SSC</td>
</tr>
<tr>
<td>GDP</td>
<td>0.12</td>
<td>0.42</td>
<td>-0.03</td>
<td>0.34</td>
</tr>
<tr>
<td>Employment</td>
<td>0.02</td>
<td>0.22</td>
<td>-0.16</td>
<td>0.25</td>
</tr>
<tr>
<td>Domestic capital use</td>
<td>0.25</td>
<td>0.62</td>
<td>0.16</td>
<td>0.46</td>
</tr>
<tr>
<td>Total consumption</td>
<td>0.02</td>
<td>-0.01</td>
<td>-0.29</td>
<td>0.14</td>
</tr>
<tr>
<td>Total investment</td>
<td>0.13</td>
<td>0.56</td>
<td>0.24</td>
<td>0.51</td>
</tr>
<tr>
<td>Imports</td>
<td>-0.32</td>
<td>-0.24</td>
<td>-0.55</td>
<td>-0.12</td>
</tr>
<tr>
<td>Exports</td>
<td>-0.03</td>
<td>0.42</td>
<td>-0.33</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table entries are given as percentage changes.

Table 2: Lower elasticity of labour supply: macro effects
Sensitivity analysis (3): Elasticity of substitution in production \((KE), L\) doubled (1 instead of 0.5):

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tax recycling instrument</strong></td>
<td>VAT</td>
<td>MITR</td>
<td>ITA</td>
<td>SSC</td>
</tr>
<tr>
<td>GDP</td>
<td>0.14</td>
<td>0.39</td>
<td>-0.05</td>
<td>0.41</td>
</tr>
<tr>
<td>Employment</td>
<td>0.02</td>
<td>0.14</td>
<td>-0.22</td>
<td>0.33</td>
</tr>
<tr>
<td>Domestic capital use</td>
<td>0.29</td>
<td>0.75</td>
<td>0.19</td>
<td>0.52</td>
</tr>
<tr>
<td>Total consumption</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.34</td>
<td>0.22</td>
</tr>
<tr>
<td>Total investment</td>
<td>0.14</td>
<td>0.59</td>
<td>0.22</td>
<td>0.54</td>
</tr>
<tr>
<td>Imports</td>
<td>-0.29</td>
<td>-0.17</td>
<td>-0.57</td>
<td>-0.05</td>
</tr>
<tr>
<td>Exports</td>
<td>0.04</td>
<td>0.64</td>
<td>-0.32</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Table entries are given as percentage changes.

Table 3: Higher elasticity of substitution in production: macro effects
Sensitivity analysis (4): Elasticity of subsitution in production \((KE, L)\) halved (0.25 instead of 0.5):

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax recycling instrument</td>
<td>VAT</td>
<td>MITR</td>
<td>ITA</td>
<td>SSC</td>
</tr>
<tr>
<td>GDP</td>
<td>0.12</td>
<td>0.34</td>
<td>-0.08</td>
<td>0.41</td>
</tr>
<tr>
<td>Employment</td>
<td>0.03</td>
<td>0.17</td>
<td>-0.20</td>
<td>0.32</td>
</tr>
<tr>
<td>Domestic capital use</td>
<td>0.23</td>
<td>0.57</td>
<td>0.09</td>
<td>0.51</td>
</tr>
<tr>
<td>Total consumption</td>
<td>0.03</td>
<td>0.03</td>
<td>-0.33</td>
<td>0.22</td>
</tr>
<tr>
<td>Total investment</td>
<td>0.13</td>
<td>0.56</td>
<td>0.21</td>
<td>0.54</td>
</tr>
<tr>
<td>Imports</td>
<td>-0.32</td>
<td>-0.24</td>
<td>-0.61</td>
<td>-0.05</td>
</tr>
<tr>
<td>Exports</td>
<td>-0.05</td>
<td>0.38</td>
<td>-0.45</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table entries are given as percentage changes.

Table 4: Lower elasticity of substitution in production: macro effects