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Irrationality in English Auctions*

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Abstract

This paper explores the effects of a particular form of irrational behaviour by participating bidders in a common value English auction. We allow bidders to update their expected valuation of the good as the current price increases, assuming that their opponents always play the symmetric Nash equilibrium. When only one bidder adopts this type of behaviour, it is ambiguous whether the final auction price is higher or lower than at the symmetric equilibrium. However, when both bidders behave irrationally, the final auction price is never lower than the symmetric equilibrium provided bidders "match" their strategies.

JEL Classification: D44

Keywords: English auctions, irrationality, asymmetric equilibrium.

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1 Introduction

Thus far, models of common value[1] English auctions have focused almost exclusively on rational bidders leading to Nash equilibria: each bidder plays the best response to his opponent's strategy and makes a positive *ex post* profit in equilibrium. Milgrom and Weber's (1982) important paper has focused on the symmetric equilibrium of this type of auction, whereas Bikhchandani and Riley (1991), following Milgrom's (1981) seminal results, have focused on the many asymmetric equilibria. Those theoretical results are questioned by a number of experimental studies[2]. For example, Avery and Kagel (1997) find that in a significant percentage of English auctions (around 30%) the winning bidder loses money because all bidders use aggressive and non-Nash equilibrium bidding strategies[3].

A common feature in experimental studies of English auctions is the nature of the alternative bidding rule (apparently) used by bidders. Levin et al. (1996) demonstrate that the bidding strategy that is best at explaining the data is a signal averaging rule, in essence a weighted average between a bidder's own signal and the last drop out price. Although this signal averaging rule yields (theoretically) the same average price as the Nash equilibrium strategy, it is not a Nash equilibrium. Furthermore, in their experiments, Levin et al. find that most auctions[4] were affected by overbidding: final auction prices were significantly higher than predicted by symmetric Nash equilibrium bidding and *ex post* profits were consequently lower.

Avery and Kagel conclude that the bidding strategy with more predictive power is also a "statistical" bidding rule[5]: a bidder appears to combine his own signal and the expected value of his opponent's signal, using the commonly known prior signal distribution. Again, such a bidding rule is not a Nash equilibrium. Bidders with signal realisations below the average of the prior distribution will overbid (compared to the Nash prediction), whereas bidders with signals above the average will underbid. Additionally, under that rule, the winning bidder will fall prey to the winner's curse and lose money with positive probability[6].

The experimental results in Gonçalves and Hey (2007) also suggest a "statistical" bidding rule as the most likely bidding function: bidders combine their own signal and the expected value of their opponent's signal using the prior signal distribution *given* the signal ranking, which was common knowledge before the start of each auction. This rule is equivalent to Avery and Kagel's bidding

[1]As well as private values.

[2]See Kagel (1995) for an overview of the experimental literature.

[3]This does not depend on bidders' experience; the percentage of auctions yielding negative profits for the winner is not significantly reduced as each bidder gains experience by participating in more auctions.

[4]With the exception of auctions with "superexperienced bidders".

[5]That they define as expected value bidding.

[6]The probability that *both* bidders have a signal realization below the distribution average and overbid compared to the Nash equilibrium prediction is 0.25 (Avery and Kagel assume the independent uniform distribution).

rule once account is taken of the additional information bidders had: the knowledge of the signal ranking. And this rule is also not a Nash equilibrium. As in Avery and Kagel, Gonçalves and Hey find systematic overbidding compared to the symmetric Nash equilibrium that results in lower *ex post* profits[7].

A common aspect of these alternative bidding rules is that they induce overbidding and the winner's curse: if all bidders adhered to those rules, final average prices would be at least as high as predicted by (symmetric) Nash bidding. Those rules also generate negative profits for the winner in a significant number of auctions. This paper proposes an English auction model (with two bidders) that attempts to explain these facts. In particular, the model incorporates the main feature of those alternative bidding rules: we assume that bidders may choose a bidding strategy based on their own signal and the expected value of their opponent's signal, which they will try to infer as the auction price increases. However, we also assume that bidders behaving in this way will incorrectly believe that their opponent will always play the symmetric Nash bidding strategy. These are the two main characteristics of what we consider to be an "irrational" bidder. We show that if a bidder is "irrational", his bidding strategy is more aggressive than the symmetric Nash bidding function.

We study two possibilities. First, we allow only one bidder to behave "irrationally". From the auctioneer's point of view, and with no further assumptions, the expected auction price may be higher or lower than the symmetric equilibrium depending on who holds the highest signal. However, and as a special case, we show that if the signal distribution is highly affiliated, the auctioneer's revenues are maximised at the symmetric equilibrium, a result that is similar in nature to those obtained by Bikhchandani and Riley (1991) and Avery (1998).

Second, we allow both bidders to behave "irrationally". In this case, there may exist multiple equilibria, including a mixed strategy equilibrium where both bidders bid aggressively with positive probability. This mixed strategy is an equilibrium because each bidder benefits from playing aggressively (an expected auction price lower than at the symmetric equilibrium) but faces a cost: the expected loss associated with the the probability that their opponent is playing aggressively as well (which implies an expected auction price higher than at the symmetric equilibrium). Depending on the probability distribution of the signals, other equilibria may exist, including pure strategy symmetric and asymmetric equilibria.

Our results show that the auctioneer receives a higher expected price than at the symmetric equilibrium provided both bidders play symmetrically (i.e. provided bidders "match" their strategies). Bikhchandani and Riley argue that it may be reasonable for symmetric bidders to behave symmetrically in one period games and therefore we would expect the auctioneer to benefit from

[7]In Gonçalves and Hey, 19% of all auctions resulted in negative profits for the winner, but the average profit earned by bidders was positive (although lower than under Nash bidding).

bidders' irrationality in single period auctions.

In a related paper, Eyster and Rabin (2005) propose the concept of "cursed equilibrium", where a player in a Bayesian game believes with probability χ that his opponents do not act in accordance with the private information they have (i.e. they do not act rationally). If $\chi = 0$, then all players are perfectly rational and the relevant equilibrium concept is that of Bayesian Nash equilibrum; however, if $\chi > 0$ then all players believe with positive probability that they are playing against not-fully rational players, and the relevant equilibrium concept is that of an χ-cursed equilibrium. In the context of common value English auctions, Eyster and Rabin show that seller revenue is increasing with χ and that if a sufficient number of bidders participate in the auction, the winner incurs the winner's curse and loses money. Our results partly corroborate these findings, but in our setup bidders may not only be "cursed" but may also choose "how cursed" they want to be (the probability of behaving irrationally). If both bidders behave irrationally with positive probability, they will overbid compared to the Nash equilibrium prediction and increase seller's revenue. They do not, however, expect to lose money.

The background of the model is presented in the next section, together with previous theoretical results. Section 3 analyses the case with only one irrational bidder and section 4 the case with two irrational bidders. Section 5 concludes. An appendix to the paper (available on the JEBO website) contains an illustrative example.

2 The English auction model

2.1 Definitions

We focus on a common value English auction where two symmetric risk neutral bidders compete for the purchase of one single indivisible good. We adopt the Japanese variant of the English auction used by Milgrom and Weber, where the price increases continuously and interested bidders must depress a button as long as they are interested in the good. When all but one bidder release the button, the auction finishes. The price, the number of bidders and the drop-out prices of all bidders are displayed for all to see. A strategy for a given bidder in this auction game, as explained by Milgrom and Weber, must specify, for any price level, whether he should remain active or quit given all the information available. In a model with only two bidders, such a strategy entails selecting a single number: the price at which that bidder will release the button and let his opponent win.

We assume the auctioneer has a reservation price of 0. The common value of this good, V, is *ex ante* unknown to both bidders. However, each bidder receives a signal $x_i \in [0, \overline{x}]$, $i = 1, 2$, of this value before the auction starts, which is known only to himself. In particular, we assume that each bidder's valuation takes the form $V_i = V = v(x_1, x_2) = x_1 + x_2$, $\forall i$. This particular functional form (called the Wallet Game) has been used by Klemperer (1998), and because it is so simple to work with, it provides valuable and intuitive insights into other valuation functions.

The signals are assumed to be affiliated and to have a joint density function $f(x_1, x_2)$ that is symmetric and continuous. Affiliation roughly means that the two signals are nonnegatively correlated. Milgrom and Weber show that in this particular case affiliation requires only that $g_{X_j|X_i}(x_j|\, x_i)$ satisfies the Monotone Likelihood Ratio Property (MLRP), where $g_{X_j|X_i}(x_j|\, x_i)$ is the conditional density of X_j given X_i. This implies that for all $x_j' > x_j$, and $x_i' > x_i$:

$$\frac{g_{X_j|X_i}\left(x_j\middle|\, x_i\right)}{g_{X_j|X_i}\left(x_j\middle|\, x_i'\right)} \geq \frac{g_{X_j|X_i}\left(x_j'\middle|\, x_i\right)}{g_{X_j|X_i}\left(x_j'\middle|\, x_i'\right)}. \tag{1}$$

2.2 Equilibria (symmetric and asymmetric)

Milgrom and Weber show that there exists a symmetric equilibrium of the English auction in which each bidder i's strategy, $S(x_i)$, is to remain active until the posted price reaches $S(x_i) = v(x_i, x_i) = 2x_i$ (see also Klemperer). This symmetric equilibrium is unique (Levin and Harstad (1986). However, Milgrom has shown the existence of a continuum of asymmetric equilibria. Let $h(.)$ be an increasing and surjective function[8]. Then, the following strategies are equilibrium bid functions of this model:

$$\begin{aligned} S_1(x) &= v(x, h(x)) = x + h(x) \\ S_2(x) &= v\left(h^{-1}(x), x\right) = h^{-1}(x) + x. \end{aligned} \tag{2}$$

Each function $h(.)$ will lead to a different asymmetric equilibrium, and hence there exists a continuum of asymmetric equilibria. In particular, one can see how each asymmetric equilibrium departs from the symmetric equilibrium. If $h(x) > x, \forall x$, then bidder 1 will be playing an aggressive asymmetric strategy, $S_1(x)$, because $S_1(x) > S(x)$. Given this, bidder 2 will then play a passive asymmetric strategy, $S_2(x)$ because $S_2(x) = v\left(h^{-1}(x), x\right) < S(x)$. [9,10]

2.3 Definition of "irrationality"

Suppose bidder 2 is playing his symmetric equilibrium strategy and remains active in the auction until the price reaches $S(x_2) = v(x_2, x_2) = 2x_2$. Let p denote the current price. Knowing that

[8] A function is surjective if its target coincides with its range.
[9] Note that if $h(x) > x$, $\forall x$, this implies that $x > h^{-1}(x)$, $\forall x$.
[10] For more details on asymmetric equilibria, see Milgrom (1981), Bikhchandani and Riley (1991) or Klemperer (1998).

bidder 2 is playing the symmetric equilibrium strategy gives bidder 1 the following information: at any price p, he knows that $S(X_2) \geq p$, or $X_2 \geq S^{-1}(p) = p/2$. In the symmetric equilibrium, this additional information released during the auction has no value and both bidders ignore it; given that bidder 2 is bidding up to $S(x_2)$, bidder 1's best reply is to bid up to $S(x_1)$. If $x_1 < x_2$, bidder 1 drops out at $p = S(x_1)$ and loses the auction. Even though he knows that the good is worth more than $p = S(x_1)$ but less than $S(X_2)$, this brings him no advantage in subsequent bidding. Given bidder 2's strategy, $S(x_2)$, winning the auction is only possible if he deviates from $S(x_1)$ and continues bidding. However, winning in such circumstances would yield a negative payoff: $v(x_1, x_2) - S(x_2) = x_1 + x_2 - 2x_2 < 0$.

The concept of "irrationality" used in this paper is based on this information released throughout the auction. First, we assume that an "irrational" bidder attempts to estimate his opponent's signal given all the information available to him in the auction: his own signal and the current price. This estimate is used to inform his strategy. Second, we assume that in doing so he presumes his opponent is playing the symmetric equilibrium strategy. Such an irrational bidder is effectively trying to outsmart his opponent by "estimating" his signal and thus obtaining a more accurate estimate of the good's true value. However, in doing so he does not anticipate that his opponent's best reply may no longer be the symmetric equilibrium strategy that formed the basis of his estimate.

The bidding strategy chosen by such an "irrational bidder" would be computed in the following way. Bidder 1 knows that at any price p, with both bidders still active, $S(X_2) \geq p$ or, equivalently, $X_2 \geq S^{-1}(p) = p/2$. At this price, bidder 1 updates his expectation of bidder 2's signal, assuming he is playing the symmetric equilibrium strategy, $S(X_2)$. Let $\phi_2(p, x_1)$ be the expectation by bidder 1 of bidder 2's signal, given that bidder 2 is active at a price of p and given bidder 1's own signal:

$$\begin{aligned}
\phi_2(p, x_1) &= E\left[X_2 | X_2 \geq S^{-1}(p), X_1 = x_1\right] \\
&= \frac{\int_{p/2}^{\overline{x}} x_2 . g_{X_2|X_1}(x_2 | x_1)\, dx_1}{\int_{p/2}^{\overline{x}} g_{X_2|X_1}(x_2 | x_1)\, dx_1},
\end{aligned} \quad (3)$$

where $\phi_2(p, x_1)$ is nondecreasing in both arguments[11]. Let $S_A(x_1, p) = x_1 + \phi_2(p, x_1)$ be bidder 1's asymmetric and irrational strategy. The irrational strategy is a function of the posted price, p.

[11] $\phi_2(p, x_1)$ is obviously nondecreasing in p. Milgrom and Weber show that for affiliated variables, $\phi_2(p, x_1)$ is also nondecreasing in x_1 (Theorem 5).

When bidders decide which strategies to play (before the auction starts), p is not known. Hence, given x_1, there exists a price level $p = p^*$ such that

$$p^* = x_1 + \phi_2 (p^*, x_1) . \tag{4}$$

Let $S_A (x_1) = p^*$ be bidder 1's asymmetric strategy, which is now only a function of x_1. The above equation is equivalent to

$$S_A (x_1) = x_1 + \phi_2 (S_A (x_1) , x_1) \tag{5}$$

where

$$\phi_2 (S_A (x_1) , x_1) = E \left[X_2 | \, X_2 \geq \frac{S_A (x_1)}{2}, X_1 = x_1 \right] . \tag{6}$$

Note how this strategy is constructed. Firstly, bidder 1 makes use of the additional information released during the auction, that is, he Bayesian updates his estimate of X_2 (his opponent's signal) by inverting the current price, p (he *irrationally* assumes that his opponent always plays the symmetric equilibrium strategy). Then, he (irrationally) uses this information to compute his bidding strategy. However, this strategy must be decided before the auction starts (at a time when p is not available). Hence, he calculates the price p^* at which his strategy would no longer be consistent with his irrational behaviour: the point at which his expectation of the value of the good conditional on all the information available is equal to the price he would pay if he happened to win at that particular point. At a price p lower than p^*, $x_1 + \phi_2 (p, x_1) > p$, which implies that at a price p, bidder 1 expects a positive payoff if he wins the auction. The highest price at which his expected payoff is not negative is p^*.

It is important to stress the departures from a model with fully rational bidders incorporated in our analysis. Rationally, this bidder should not use the Bayesian estimate of his opponent's signal in his bidding strategy, which is irrelevant at the symmetric equilibrium. We assume that this bidder *believes* this information to be relevant. This is our first departure from rationality. Second, in computing this estimate, bidder 1 assumes his opponent is playing the symmetric equilibrium strategy. In reality, if bidder 1 deviates from his symmetric equilibrium strategy, bidder 2's best response is also to deviate. By making this assumption, the Bayesian estimate of bidder 2's signal is always given by equation (3), even when bidder 2 is playing some other strategy. This is the second departure from the rational model.

An "irrational" bidder makes two mistakes. First, he is trying to outsmart his opponent, but in doing so he is asking himself the wrong question. Instead of asking himself "What is the value of the good conditional on my winning the auction?" (which would lead to the symmetric equilibrium outcome), he is asking "What is the value of the good conditional on my opponent being active at a given price p?". Second, asking the latter question, he is erroneously inferring the value of the good by assuming his opponent never deviates from the symmetric equilibrium.

We can show that this "irrational" strategy is always aggressive (i.e. $S_A(x_1) \geq S(x_1), \forall x_1$).

Proposition 1 *For any probability distribution over* $[0, \overline{x}]$, $S_A(.) \geq S(.)$ *(i.e. the "irrational" bidder's strategy is at least as aggressive as the symmetric equilibrium strategy).*

Proof. $S_A(.) = p^*$ is obtained from equation (4). Let $p = \lambda x_1$, where $\lambda \geq 0$. To prove this proposition, we need to show that $S_A(.) = p^* \geq S(.) = 2x_1$. Hence, we have to show that any $p = \lambda x_1$ with $\lambda \in [0, 2)$ cannot satisfy equation (4).

Under the assumptions outlined above, note that $\phi_2(p, x_1) \geq p/2$ for any p. Under the (conservative) assumption that $\phi_2(p, x_1) = p/2$ [12], when $\lambda \in [0, 2)$, equation (4) does not hold with equality:

$$\begin{aligned} p &< x_1 + p/2 \\ \lambda x_1 &< x_1 + \lambda x_1/2 \\ \lambda x_1 &< 2x_1. \end{aligned} \tag{7}$$

Hence, under our most conservative assumption, the *lowest* λ that satisfies equation (4) is $\lambda = 2$. This implies that $S_A(.) = p^* \geq S(.) = 2x_1$ (i.e. the asymmetric strategy used by an "irrational" bidder is at least as aggressive as the symmetric strategy). ■

If the irrational bidder's strategy, $S_A(.)$, is aggressive (Proposition 1), then following Milgrom (1981) and Bikhchandani and Riley (1991), we can show that

Proposition 2 *(Milgrom) The best response to* $S_A(.) \geq S(.)$ *is* $S_a(.) \leq S(.)$.

Proof. From Section 2.2 and the seminal result by Milgrom, we know that $S_A(x) = v(x, h(x)) \geq v(x, x) = S(x)$ if and only if $h(x) \geq x$, $\forall x$. The best response to $S_A(x)$ is $S_a(x) = v\left(x, h^{-1}(x)\right)$. Hence, if $h(x) \geq x$, $x \geq h^{-1}(x)$, and the best response is always less aggressive than the symmetric strategy: $S_a(x) \leq S(x)$.

From Proposition 1, we know that the particular asymmetric strategy to be played by an "irrational" bidder 1 is always aggressive (i.e. $S_A(.) \geq S(.)$, $\forall x$). This implies that $h(x_1) =$

[12] If $\phi_2(p, x_1) > p/2$, our result is strenghtened.

$\phi_2(x_1) \geq x_1$, $\forall x_1$. Consequently, the best response to $S_A(.) \geq S(.)$ by bidder 2 is a passive strategy, $S_a(.) \leq S(.)$. This implies that $x_2 \geq h^{-1}(x_2) = \phi_2^{-1}(x_2)$. ■

Given that the aggressive strategy takes the form $S_A(x_1) = x_1 + \phi_2(x_1)$, the passive strategy will be $S_a(x_2) = x_2 + \phi_2^{-1}(x_2)$, where $\phi_2^{-1}(.)$ is the inverse function of $\phi_2(.)$, given in equation (6). Thus, under the model assumptions, an "irrational" bidder will always bid at least as aggressively as in the symmetric equilibrium and his opponent more passively. Hence, "irrationality" is an implicit and credible threat of aggressive bidding, as shown in Propositions 1 and 2.

3 Asymmetric equilibrium with one "irrational" bidder

Let ε_i be the probability that bidder i chooses the aggressive bidding strategy associated with irrationality or, alternatively, the probability that bidder i is "irrational". Conversely, $(1-\varepsilon_i)$ is the probability that bidder i is rational. Assuming that only one bidder can be irrational, say bidder 1, would he choose $\varepsilon_1 > 0$? We can show that when $\varepsilon_2 = 0$ (bidder 2 is rational), bidder 1 plays the aggressive strategy with probability 1 (i.e. $\varepsilon_1^* = 1$) for probability distributions that lead to a sufficiently asymmetric pair of strategies. Define

$$C = E\left[2Y_2 - \phi_2^{-1}(Y_1) - \phi_2^{-1}(Y_2)\right] \tag{8}$$

where $Y_1 = \max[X_1, X_2]$ is the first order statistic and $Y_2 = \min[X_1, X_2]$ is the second order statistic.

Proposition 3 *When $\varepsilon_2 = 0$, bidder 1 plays $S_A(.)$ with probability $\varepsilon_1^* = 1$ for probability distributions such that $C > 0$. Otherwise, bidder 1 plays $S_A(.)$ with probability $\varepsilon_1^* = 0$.*

Proof. Suppose $x_1 > x_2$. If bidder 1 plays $\varepsilon_1 = 0$ (the symmetric equilibrium), he wins and makes a profit in equilibrium:

$$\begin{aligned} U_1 \mathbf{1}_{\{\varepsilon_1=0;\, x_1>x_2\}} &= x_1 + x_2 - S(x_2) \\ &= x_1 - x_2. \end{aligned} \tag{9}$$

If he plays $\varepsilon_1 = 1$, he wins and his payoff is

$$\begin{aligned} U_1 \mathbf{1}_{\{\varepsilon_1=1;\, x_1>x_2\}} &= x_1 + x_2 - S_a(x_2) \\ &= x_1 - \phi_2^{-1}(x_2) \\ &\geq x_1 - x_2. \end{aligned} \tag{10}$$

Note that because his signal is the highest, he wins in both cases and receives a higher payoff when he plays aggressively because his opponent drops out at a lower price, $S_a(x_2) \leq S(x_2)$.

If, on the other hand, $x_1 < x_2$, and bidder 1 plays $\varepsilon_1 = 0$, he loses and receives a 0 payoff, but if he plays $\varepsilon_1 = 1$, he may win the auction because of his aggressive strategy. Conditional on winning (i.e. conditional on $S_A(x_1) > S_a(x_2)$), his payoff is

$$\begin{aligned} U_1 \mathbf{1}_{\{\varepsilon_1=1;\ x_1<x_2;\ S_A(x_1)>S_a(x_2)\}} &= x_1 + x_2 - S_a(x_2) \\ &= x_1 - \phi_2^{-1}(x_2) \end{aligned} \tag{11}$$

which may be higher or lower than 0.[13]. *Ex ante*, before knowing the signal and using the symmetric distribution of the signals, his expected payoff from playing the symmetric equilibrium is

$$\begin{aligned} E\left[U_1 \mathbf{1}_{\{\varepsilon_1=0\}}\right] &= \frac{1}{2} E\left[X_1 - X_2 \mid X_1 > X_2\right] \\ &= \frac{1}{2} E\left[Y_1 - Y_2\right]. \end{aligned} \tag{12}$$

Ex ante, bidder 1 faces a 1/2 probability of holding the highest signal (the signal distribution is symmetric, thus $\Pr[X_1 = \max(X_1, X_2)] = 1/2$) and winning the auction at the symmetric equilibrium. With probability $\Pr[X_1 = \min(X_1, X_2)] = 1/2$, he holds the lowest signal and loses at the symmetric equilibrium, leaving him with a profit of 0.

His expected payoff from playing aggressively[14] is

$$\begin{aligned} E\left[U_1 \mathbf{1}_{\{\varepsilon_1=1\}}\right] &= \frac{1}{2} E\left[X_1 - \phi_2^{-1}(X_2) \mid X_1 > X_2\right] + \frac{1}{2} E\left[X_1 - \phi_2^{-1}(X_2) \mid X_1 < X_2\right] \\ &= \frac{1}{2} E\left[Y_1 - \phi_2^{-1}(Y_2)\right] + \frac{1}{2} E\left[Y_2 - \phi_2^{-1}(Y_1)\right]. \end{aligned} \tag{13}$$

Hence, bidder 1's best strategy is to play aggressively $(S_A(x_1))$ with probability $\varepsilon_1^* = 1$ if

$$E\left[U_1 \mathbf{1}_{\{\varepsilon_1=1\}}\right] > E\left[U_1 \mathbf{1}_{\{\varepsilon_1=0\}}\right]. \tag{14}$$

From equations (12) and (13), this inequality is satisfied for probability distributions such that

$$C = E\left[2Y_2 - \phi_2^{-1}(Y_1) - \phi_2^{-1}(Y_2)\right] > 0. \tag{15}$$

[13] If $S_a(x_2)$ is not very asymmetric, then it will be close to $S(x_2)$, and hence $\phi_2^{-1}(x_2)$ would be close to x_2; in this case, the payoff could be negative.

[14] This follows from Proposition 2.

Such distributions generate a sufficiently asymmetric pair of strategies. For probability distribution that do not satisfy this inequality, bidder 1 plays aggressively with probability $\varepsilon_1^* = 0$ and we obtain the symmetric equilibrium of Milgrom and Weber. ∎

In this setup, is the auctioneer better off than at the symmetric equilibrium? The expected auction price in the symmetric equilibrium is equal to the expectation of the second highest bidder's strategy:

$$\begin{aligned} E\left[P\mathbf{1}_{\{\varepsilon_1=\varepsilon_2=0\}}\right] &= E\left[S\left(X_i\right)\middle| X_i < X_j\right] \\ &= E\left[S\left(Y_2\right)\right] \\ &= 2E\left[Y_2\right], \end{aligned} \tag{16}$$

but for probability distributions such that bidder 1 plays $\varepsilon_1^* = 1$ and the equilibrium strategies are $\left(S_A\left(x_1\right), S_a\left(x_2\right)\right)$, the expected price depends on who holds the highest signal. With probability 1/2 bidder 1 holds the highest signal and wins the auction. In this case, the expected price is equal to the expectation of bidder 2's equilibrium strategy:

$$\begin{aligned} E\left[P\mathbf{1}_{\{\varepsilon_1=1;\ \varepsilon_2=0\}}\right] &= E\left[S_a\left(X_2\right)\middle| X_1 > X_2\right] \\ &= E\left[X_2 + \phi_2^{-1}\left(X_2\right)\middle| X_1 > X_2\right] \\ &= E\left[Y_2 + \phi_2^{-1}\left(Y_2\right)\right], \end{aligned} \tag{17}$$

but with probability 1/2 bidder 2 holds the highest signal and the expected price is equal to the expectation of the lowest bidding strategy (drop out price):

$$E\left[P\mathbf{1}_{\{\varepsilon_1=1;\ \varepsilon_2=0\}}\right] = \min\left[E\left[S_A\left(X_1\right)\middle| X_1 < X_2\right], E\left[S_a\left(X_2\right)\middle| X_1 < X_2\right]\right]. \tag{18}$$

As we will show later (section 4.1), *ex ante* the bidder playing $S_a(.)$ always expects to lose the auction[15]. Hence, the above equation becomes

$$\begin{aligned} E\left[P\mathbf{1}_{\{\varepsilon_1=1;\ \varepsilon_2=0\}}\right] &= \min\left[E\left[S_A\left(Y_2\right)\right], E\left[S_a\left(Y_1\right)\right]\right] \\ &= E\left[S_a\left(Y_1\right)\right] \\ &= E\left[Y_1 + \phi_2^{-1}\left(Y_1\right)\right]. \end{aligned} \tag{19}$$

[15] Because the auctioneer has the same information as the bidders before the auction starts (i.e. knowledge of the signal distribution), he also expects this bidder to lose.

Given these two expressions (equations (17) and (19)), it is ambiguous whether the auctioneer expects a higher price than in the symmetric equilibrium. In the first case, $2E[Y_2] \leq E\left[Y_2 + \phi_2^{-1}(Y_2)\right]$, which means that with probability 1/2 the auctioneer would (weakly) prefer the symmetric equilibrium. However, $2E[Y_2]$ could be higher or lower than $E\left[Y_1 + \phi_2^{-1}(Y_1)\right]$, which makes the auctioneer's preference generally dependent on the particular distribution we assume. Although this is generally true, the following result holds:

Proposition 4 *When the variables X_1 and X_2 are highly affiliated, the expected price in the symmetric equilibrium is unambiguously higher than the expected price in the asymmetric equilibrium with one "irrational" bidder, and the auctioneer is clearly worse off.*

Proof. When X_1 and X_2 are highly affiliated, the joint density $f(x_1, x_2)$ assigns very high probabilities to realisations of X_1 and X_2, which are close to one another. Hence, with highly affiliated variables, the difference between the first order statistic and the second approaches 0 :

$$(E[Y_1] - E[Y_2]) \simeq 0. \tag{20}$$

For such affiliated distributions, bidder 1 always plays $\varepsilon_1^* = 1$ because equation (15) is satisfied: $C = E\left[2Y_2 - \phi_2^{-1}(Y_2) - \phi_2^{-1}(Y_2)\right] > 0$. In the symmetric equilibrium, the expected price for the auctioneer becomes $E\left[P\mathbf{1}_{\{\varepsilon_1=\varepsilon_2=0\}}\right] = E[S(Y_2)] \simeq E[S(Y_1)] \simeq 2E[Y_1]$. In the asymmetric equilibrium, with probability 1/2 bidder 1 holds the highest signal. The expected price is

$$\begin{aligned} E\left[P\mathbf{1}_{\{\varepsilon_1=1;\ \varepsilon_2=0\}}\right] &= E\left[Y_2 + \phi_2^{-1}(Y_2)\right] \\ &\simeq E\left[Y_1 + \phi_2^{-1}(Y_1)\right]. \end{aligned} \tag{21}$$

With probability 1/2 bidder 2 holds the highest signal. The expected price is

$$E\left[P\mathbf{1}_{\{\varepsilon_1=1;\ \varepsilon_2=0\}}\right] = E\left[Y_1 + \phi_2^{-1}(Y_1)\right]. \tag{22}$$

Now note that the expected price in the symmetric equilibrium is at least as high as in the asymmetric equilibrium

$$E\left[P\mathbf{1}_{\{\varepsilon_1=\varepsilon_2=0\}}\right] \simeq 2E[Y_1] \geq E\left[P\mathbf{1}_{\{\varepsilon_1=1;\ \varepsilon_2=0\}}\right] \simeq E\left[Y_1 + \phi_2^{-1}(Y_1)\right] \tag{23}$$

because $E[Y_1] \geq E\left[\phi_2^{-1}(Y_1)\right]$, and the auctioneer weakly prefers the former. ■

This result is not a general one and is specific to distributions that result in high affiliation between the random variables. The intuition for this result is that in this case the bidder who sets the price becomes sufficiently less aggressive relative to the symmetric equilibrium, and this unambiguously reduces seller's revenue[16].

4 Equilibrium with two "irrational" bidders

It is perhaps more interesting to analyse a setup where both bidders are allowed to play aggressive strategies. Hence, we now allow both bidders to play $\varepsilon_i \geq 0$. Notice that if bidder 1 chooses to bid aggressively (by playing $\varepsilon_1 \geq 0$) and his opponent's reply is also to bid aggressively with positive probability ($\varepsilon_2 \geq 0$), whoever wins the auction may end up paying a higher price than at the symmetric Nash equilibrium.[17]

4.1 The auction game

The game is symmetric, so we can focus on bidder 1's strategy choice. In the auction game, bidder 1 can bid aggressively (A) or not aggressively (NA). His payoff depends on what bidder 2 does. If bidder 1 plays NA ($\varepsilon_1 = 0$) and 2 plays NA ($\varepsilon_2 = 0$) as well, the payoffs are those of the symmetric equilibrium. If bidder 1 plays NA but his opponent plays A, then we have the setup of section 3. Finally, both bidders may play A, that is, bidder 1 plays $S_A(x_1)$ and bidder 2 plays $S_A(x_2)$.

Before we present the payoff matrix, note that if $x_1 > x_2$, $S_A(x_1) > S_A(x_2)$, which implies that when both bidders play aggressively, the bidder holding the highest signal always wins, although the price may be different from that in the symmetric equilibrium. To prove this, remember that $S_A(x_1) = x_1 + \phi_2(x_1)$ and $S_A(x_2) = \phi_1(x_2) + x_2$. Affiliation implies that $\phi_i(.)$ is increasing in its argument[18], and hence if $x_1 > x_2$, this implies $\phi_2(x_1) > \phi_1(x_2)$, which in turn implies that $S_A(x_1) > S_A(x_2)$.

Bidder 1 will win at the symmetric equilibrium if $x_1 > x_2$, which happens with probability 1/2. In this case, and conditional on winning the auction, the payoff matrix for bidder 1 is given in Table 1. However, also with probability 1/2, bidder 1 loses at the symmetric equilibrium ($x_1 < x_2$), in which case the payoff matrix is given by Table 2.

If bidder 1 plays NA and bidder 2 plays A, bidder 1's payoff depends on which is biggest: $S_a(x_1)$ or $S_A(x_2)$. Before receiving the signals,

$$\begin{aligned} E\left[S_a\left(X_1\right)\right] &= E\left[X_1 + \phi_2^{-1}\left(X_1\right)\right] \\ &\leq E\left[S_A\left(X_2\right)\right] = E\left[S_A\left(X_1\right)\right] = E\left[X_1 + \phi_2\left(X_1\right)\right], \end{aligned} \tag{24}$$

[16] I thank an anonymous referee for this observation.

[17] When both bidders are "irrational", both bid aggressively and both assume their opponents are playing the symmetric equilibrium strategies. This leads to higher expected prices in the auction.

[18] See Theorem 5 in Milgrom and Weber.

		Bidder 2	
		NA	A
Bidder 1	NA	$x_1 - x_2$	$\begin{cases} x_1 - \phi_1(x_2), \text{ if } S_a(x_1) > S_A(x_2) \\ 0, \text{ if } S_a(x_1) < S_A(x_2) \end{cases}$
	A	$x_1 - \phi_2^{-1}(x_2)$	$x_1 - \phi_1(x_2)$

Table 1: Bidder 1's payoff matrix when $x_1 > x_2$

		Bidder 2	
		NA	A
Bidder 1	NA	0	0
	A	$x_1 - \phi_2^{-1}(x_2)$	0

Table 2: Bidder 1's payoff matrix when $x_1 < x_2$

which means that bidder 1 never expects to win when he plays NA and bidder 2 plays A. The *ex ante* payoff of playing NA when bidder 2 is also playing NA is given by equation (12) and the *ex ante* payoff of playing A when bidder 2 is playing NA is given by equation (13). Finally, the *ex ante* payoff of playing A when bidder 2 is also playing A is given by

$$\begin{aligned} E\left[U_1 \mathbf{1}_{\{\varepsilon_1=\varepsilon_2=1\}}\right] &= \frac{1}{2} E\left[X_1 - \phi_1(X_2) \middle| X_1 > X_2\right] \\ &= \frac{1}{2} E\left[Y_1 - \phi_1(Y_2)\right]. \end{aligned} \tag{25}$$

We summarize the *ex ante* payoff matrix for bidder 1 in Table 3.

4.2 Equilibria

Let C be defined as in equation (8) and define

$$D = E\left[Y_1 - \phi(Y_2)\right]. \tag{26}$$

Using the *ex ante* expected payoffs (see Table 3), we can show that:

Proposition 5 *For probability distribution such that $C > 0$, and $D > 0$, the dominant strategy equilibrium of the game is for both bidders to play A with probability $\varepsilon_i^* = 1$; if $C < 0$, and $D < 0$, the dominant strategy equilibrium is for both bidders to play NA with probability $(1 - \varepsilon_i^*) = 1$.*

		Bidder 2	
		NA	A
Bidder 1	NA	$\frac{1}{2}E\left[Y_1 - Y_2\right]$	0
	A	$\frac{1}{2}E\left[Y_1 + Y_2 - \phi_2^{-1}(Y_1) - \phi_2^{-1}(Y_2)\right]$	$\frac{1}{2}E\left[Y_1 - \phi_1(Y_2)\right]$

Table 3: Bidder 1's *ex ante* payoff matrix

For probability distributions such that only $C > 0$ is satisfied (thus $D < 0$), there are two pure-strategy Nash equilibria (bidder i plays A and bidder j plays NA) and one (symmetric) mixed strategy Nash equilibrium, where $\varepsilon^* = \frac{C}{C-D}$.

For probability distributions such that only $D > 0$ is satisfied (thus $C < 0$), there are two pure-strategy Nash equilibria (bidder 1 and bidder 2 both play NA or both play A), and one (symmetric) mixed strategy Nash equilibrium, where ε^* is given by the expression above.

Proof. The signal distribution is symmetric, which implies that $E\left[\phi_2^{-1}(.)\right] = E\left[\phi_1^{-1}(.)\right] = E\left[\phi^{-1}(.)\right]$. From Proposition 3, we know that for distributions such that $C > 0$, bidder 1 strictly prefers to play A. Looking at Table 3, it is easily checked that if $D > 0$, the dominant strategy equilibrium for both bidders is to play A. Conversely, if those inequalities are reversed, then the dominant strategy equilibrium is to play NA.

If $C > 0$ is satisfied, but $D < 0$, inspection of Table 3 will show that when bidder 1 plays A, bidder 2's best reply is to play NA, and bidder 1's best reply to bidder 2's strategy (NA) is also to play A. Therefore, this pair of strategies is a Nash equilibrium (symmetry tells us that bidder 1 playing NA and bidder 2 playing A is also a Nash equilibrium).

In order to obtain the (symmetric) mixed strategy equilibrium, we have to find the value of ε_1 that maximises his *ex ante* payoff in Table 3. Hence, bidder 1 must solve

$$\max_{\varepsilon_1} E[U_1] = (1-\varepsilon_1)(1-\varepsilon_2)\frac{1}{2}E[Y_1 - Y_2] + \varepsilon_1(1-\varepsilon_2)\frac{1}{2}E\left[Y_1 + Y_2 - \phi^{-1}(Y_1) - \phi^{-1}(Y_2)\right] + \\ + \varepsilon_1\varepsilon_2\frac{1}{2}E[Y_1 - \phi(Y_2)]. \quad (27)$$

The first order condition is

$$\frac{\partial E[U_1]}{\partial \varepsilon_1} = -(1-\varepsilon_2)E[Y_1 - Y_2] + (1-\varepsilon_2)\frac{1}{2}E\left[Y_1 + Y_2 - \phi^{-1}(Y_1) - \phi^{-1}(Y_2)\right] + \varepsilon_2\frac{1}{2}E[Y_1 - \phi(Y_2)], \quad (28)$$

which is set equal to zero and rearranged to yield

$$\begin{aligned} \varepsilon_2^* &= E\left[\frac{2Y_2 - \phi^{-1}(Y_1) - \phi^{-1}(Y_2)}{2Y_2 - \phi^{-1}(Y_1) - \phi^{-1}(Y_2) - Y_1 + \phi(Y_2)}\right] \\ &= \frac{C}{C-D}. \end{aligned} \quad (29)$$

Bidder 1 will be indifferent between playing A or NA when bidder 2 plays ε_2^*. The problem for bidder 2 is similar, and using the symmetry of the signal distribution, we conclude that a mixed strategy equilibrium exists such that $\varepsilon_1^* = \varepsilon_2^* = \varepsilon^*$. It is easily checked that $\varepsilon^* < 1$.

Finally, for probability distributions such that $D > 0$ but $C < 0$, inspection of Table 3 will show that having both bidders playing A or NA are Nash equilibria of the auction game. For such distributions, the (symmetric) mixed strategy of equation (29) is also a Nash equilibrium. In such an equilibrium, $0 < \varepsilon^* < 1$. ■

Depending on the signal distribution, the auction game may or may not have more than one Nash equilibrium. Unlike the simplified scenario of section 3, this raises the question of which equilibrium might realistically be played. Without further assumptions, it is not possible to predict which equilibrium would be more likely to be played. Allowing both bidders to play the irrational and aggressive strategy clearly enlarges the set of possible equilibria in the auction game.

It is worth pointing out that this equilibrium (as the equilibrium of Proposition 3, when only one bidder is "irrational") is based on *ex ante* expected profits (i.e. before the signal realisations become known to bidders). Whilst this may look like an inappropriate approach, we believe it is not[19]. If expected profits are calculated after the signals arrive, this would only affect the results in so far as the expected value of an opponent's signal (or any function of that signal) would have to be calculated using the density of X_j conditional on the observed value of x_i, and not the prior distribution (both bidders know their own signal as well as the signal distribution, but they do not know who holds the highest signal). Different results could well be obtained in such a setup, but if the auctions are repeated, bidders are likely to become increasingly reliant on their prior distribution and not the particular signal realisation of a particular auction: they know that signals will sometimes be high but sometimes be low. If a pattern of behaviour (rational or irrational) is to emerge from such repeated auctions, then it is likely that that pattern is based on the prior distribution, not the conditional distribution.

4.3 Expected price for the auctioneer

When two "irrational" bidders play aggressively with probability ε_i, we will show that the auctioneer is better off than at the symmetric equilibrium provided bidders "match" their strategies (i.e. both play A with the same probability ε^*), and this result holds for any probability distribution. From Proposition 5, the only equilibrium obtained that does not imply "matching" strategies is the equilibrium in which bidder i plays A and bidder j plays NA. In that case, we are back to the ambiguous result of section 3.

There are three possible cases of matching strategies. Firstly, when both bidders play the symmetric equilibrium (both play NA with probability $(1-\varepsilon)$), the expected price is $E[P] = 2E[Y_2]$ (see equation (16)). Secondly, when both bidders play aggressively (both play A with

[19] This objection was raised by an anonymous referee.

probability ε), the expected price is

$$\begin{aligned} E[P] &= E[X_i + \phi(X_i) | X_j > X_i] \\ &= E[Y_2 + \phi(Y_2)]. \end{aligned} \tag{30}$$

Thirdly, when one bidder plays A and his opponent plays NA (with probability $\varepsilon(1-\varepsilon)$), the auctioneer expects the bidder playing $S_a(.)$ to lose the auction, whether he holds the highest signal or not (see equation (24)). In this case, the expected price is

$$E[P] = \frac{1}{2}E\left[Y_1 + \phi^{-1}(Y_1)\right] + \frac{1}{2}E\left[Y_2 + \phi^{-1}(Y_2)\right]. \tag{31}$$

Hence, for the matching strategies equilibria, the expected price for the auctioneer is given by

$$\begin{aligned} E[P(\varepsilon)] &= (1-\varepsilon)^2 2E[Y_2] + 2\varepsilon(1-\varepsilon)\left[\frac{1}{2}E\left[Y_1 + \phi^{-1}(Y_1)\right] + \frac{1}{2}E\left[Y_2 + \phi^{-1}(Y_2)\right]\right] + \\ &+ \varepsilon^2 E[Y_2 + \phi(Y_2)], \end{aligned} \tag{32}$$

which can be simplified to

$$\begin{aligned} E[P(\varepsilon)] &= 2E[Y_2] + \left(E\left[Y_1 + \phi^{-1}(Y_1) + \phi^{-1}(Y_2) - 3Y_2\right]\right)\varepsilon + \\ &+ \left(E\left[2Y_2 + \phi(Y_2) - Y_1 - \phi^{-1}(Y_1) - \phi^{-1}(Y_2)\right]\right)\varepsilon^2. \end{aligned} \tag{33}$$

Proposition 6 *For any probability distribution, the expected price with two "irrational" bidders is always at least as high as the expected price at the symmetric equilibrium provided bidders "match" their strategies.*

Proof. We need to show that for any equilibrium value of ε^* (i.e. for any value of ε which results in a Nash equilibrium, with both bidders "matching" their strategies), the expression of equation (33) is always at least as high as the expression of equation (16).

Take the equilibrium value of ε^* in the mixed strategy equilibrium (equation (29)), and the definitions of C and D from equations (8) and (26) respectively. The expression in equation (33) becomes:

$$E[P(\varepsilon)] = 2E[Y_2] + (E[Y_1 - Y_2 - C])\varepsilon + (C - D)\varepsilon^2, \tag{34}$$

which after substituting $\varepsilon = \frac{C}{C-D}$ becomes

$$\begin{aligned} E\left[P\left(\varepsilon\right)\right] &= 2E\left[Y_2\right] + \left(E\left[Y_1 - Y_2\right]\right)\frac{C}{C-D} \\ &= 2E\left[Y_2\right] + \left(E\left[Y_1 - Y_2\right]\right)\varepsilon. \end{aligned} \tag{35}$$

This implies that in the mixed strategy equilibrium, the expected price for the auctioneer is always at least as high as at the symmetric equilibrium because $0 < \varepsilon^* < 1$ (Proposition 5) and $E\left[Y_1 - Y_2\right] \geq 0$ by the definition of the order statistics.

If both bidders play NA ($\varepsilon = 0$), the symmetric equilibrium strategies are played and the expected price is given by equation (16).

If both bidders play A ($\varepsilon = 1$), then the expected price is given by equation (30), which is at least as high as that obtained at the symmetric equilibrium (see Proposition 1). ∎

The expanded auction game, which allows both bidders to compute their bidding strategies "irrationally" and play them with positive probability may have multiple Nash equilibria, depending on the probability distribution of the signals. One of those Nash equilibria is the pure strategy asymmetric equilibrium where one bidder plays A and his opponent plays NA (see section 3). Without further assumptions, it is not possible to predict how likely it is for this equilibrium to be played. However, it appears implausible that symmetric bidders would end up in this equilibrium of the expanded auction game. As Bikhchandani and Riley argue, it is more "natural" to expect symmetric bidders to bid symmetrically.

5 Conclusion

Looking at experimental evidence of English auctions[20], we have noticed a behavioral pattern: not only is the Nash equilibrium bid function apparently not used by subjects, but the bid function with more predictive power is not a Nash equilibrium. Levin et al. point out that the Nash bidding function may not be as intuitive as economists believe, and therefore other (more intuitive) bidding rules could emerge naturally in experiments. Avery and Kagel expected more experienced bidders to learn from their mistakes (and from the winner's curse) and hence to converge towards Nash bidding as the experiment approached the end. They found very weak support for this claim. The "natural" rules that emerge in experiments seem to be fairly robust over time.

In this paper, we have proposed an extension to Milgrom and Weber's model that attempts to explain these findings. We have analysed the effects of introducing a particular type of irrationality in the English auction model. We assume an irrational bidder updates the estimate of his opponent's signal as the auction price increases. Moreover, we assume such a bidder believes his opponent is

[20] Levin et al. (1996), Avery and Kagel (1997) and Gonçalves and Hey (2006).

playing the symmetric Nash equilibrium strategy (i.e. he does not consider that his opponent's strategy may no longer be a best reply). This irrationality assumption (i) contributes towards explaining overbidding and the winner's curse in experimental auctions and (ii) provides testable predictions for further experiments.

We first looked at the possibility of having only one bidder behaving irrationally, and second, and more interestingly, of having both bidders behaving irrationally. In both cases, bidders choose to behave irrationally with positive probability. In this latter case, there may exist multiple Nash equilibria of the auction game. Nevertheless, we have shown that the expected price for the auctioneer is higher than at Milgrom and Weber's symmetric equilibrium provided bidders "match" their strategies, which appears to be a plausible assumption given the symmetry of bidders. In these equilibria, neither bidder expects to lose money, although *ex post* they may realise that they did.

We believe our conclusions have some empirical support. A large percentage (almost 30% in Avery and Kagel and 19% in Gonçalves and Hey) of experimental auctions result in negative profits for the winner. Additionally, final auction prices in Gonçalves and Hey are some 22% higher than predicted by the symmetric Nash equilibrium; in Avery and Kagel, final prices were 16% higher than predicted. Conversations with subjects in Gonçalves and Hey show that bidders understood perfectly that they would only receive money by winning auctions, and this may have triggered aggressive bidding and the winner's curse in a substantial proportion of cases.

Eyster and Rabin's application of the concept of "cursed equilibrium" to common value auctions, allowing bidders to believe with positive probability that they are playing against irrational bidders, is also an attempt to explain Avery and Kagel's results. They suggest that bidders in those auctions were indeed "cursed" with very high probability (probability 1 and 0.75 for inexperienced and experienced bidders respectively). This result is not totally satisfactory: under Eyster and Rabin's setup, any outcome (including the symmetric Nash equilibrium) could be justified[21]. By contrast, our model allows for experimental testing by predicting that bidders may, with some probability, play an aggressive strategy, depending on the signal distribution. Therefore, under our setup, it is possible to choose two or three signal distributions (say, uniform or normal) and calculate the respective aggressive (irrational) strategies that bidders would use if they behaved as we predict, as well as the probabilities that they would do so. In effect, our model predicts a particular value of χ (how cursed bidders are) for each signal distribution, and the validity of this prediction can

[21] In fact, Eyster and Rabin's conclusions are similar to Avery and Kagel's, suggesting that bidders used the prior distribution of signals to aid the estimation of the common value instead of playing the symmetric Nash equilibrium strategy. The latter define this to be expected value bidding (which is not a Nash equilibrium), whereas the former demonstrates that this could be an χ-cursed Bayesian-Nash equilibrium.

be experimentally tested against the symmetric Nash equilibrium and non-equilibrium statistical bidding rules. This is the next natural step of the approach suggested in this paper.

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