

## Drifting together or falling apart? The empirics of regional economic growth in post-unification Germany

Funke, Michael; Curran, Declan; Colavecchio, Roberta

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**DRIFTING TOGETHER OR FALLING APART? THE EMPIRICS OF REGIONAL ECONOMIC GROWTH IN POST-UNIFICATION GERMANY**

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4 REGIONAL ECONOMIC GROWTH IN POST-UNIFICATION GERMANY  
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9  
10 Roberta Colavecchio

11 Department of Economics  
12 Hamburg University  
13

14  
15  
16 Declan Curran

17  
18 National Institute for Regional and Spatial Analysis  
19 National University of Ireland, Maynooth  
20

21  
22 Michael Funke\*

23  
24 Department of Economics  
25 Hamburg University  
26

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30 Hamburg, Revised September 2008  
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35 **Abstract**

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37 The objective of this paper is to address the question of convergence across German districts in the  
38 first decade after German unification by drawing out and emphasising some stylised facts of regional  
39 per capita income dynamics. We achieve this by employing non-parametric techniques which focus  
40 on the evolution of the entire cross-sectional income distribution. In particular, we follow a  
41 distributional approach to convergence based on kernel density estimation and implement a number  
42 of tests to establish the statistical significance of our findings. This paper finds that the relative  
43 income distribution appears to be stratifying into a trimodal/bimodal distribution.  
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46  
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50 Vienna (August 2006). We would like to thank the editor and an anonymous referee for helpful  
51 comments on an earlier draft. The usual disclaimer applies.  
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56 **Keywords:** Regional economic growth, Germany, convergence clubs, density  
57 estimation, modality tests

58 **JEL-Classification:** C14, R11, R12  
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\* Corresponding author; Department of Economics, Hamburg University, Von-Melle-Park 5, 20146 Hamburg, GERMANY, [funke@econ.uni-hamburg.de](mailto:funke@econ.uni-hamburg.de).

## 1. Introduction

When, on October 3<sup>rd</sup> 1990, the 60 million Germans in the West were formally re-united with the 16 million Germans in the East, the two parts could hardly have been more different. Despite a common culture and language, after forty years of development with radically different economic institutions and incentives, the Federal Republic of Germany and the German Democratic Republic (GDR) were characterized by substantial disparities in physical and human capital, labour productivity, incomes and wealth. According to Sinn and Sinn (1992), GDP per person in East Germany in 1989 was only 60 percent of the West German level. The West was one of the technologically most advanced and richest countries in the world; the East was economically shattered after four decades of communism and nearly bankrupt. In the years leading up to unification, real GDP growth was steady in the former West Germany and the unemployment rate was stable. After unification, the western states experienced sharper business cycle fluctuations: a modest upturn in 1990-91 was followed by a sharp recession in 1992-93, both of which were mainly due to the unification process. The initial economic boom was led by "exports" to the eastern states, where consumers were switching to cheaper and better quality goods produced in the West. The subsequent recession was also closely related to unification. Restrictive measures were implemented to reduce the fiscal deficit and the Bundesbank tightened monetary policy to cap the rising inflation. These policy responses, coupled with a contraction in foreign demand, had a dampening effect on the economy and the post-unification boom gradually turned into a deep recession, with GDP growth rates well below the historical average for western Germany.

The problem of uneven regional developments has been closely monitored in post-unification policy debates and in recent years there has been a surge in empirical work on growth and convergence.

When considering regional convergence, various empirical approaches have been implemented in the literature: from simple plots of measures of dispersion over time to intra-distributional dynamics using Markov chains applied to GDP per capita. Numerous studies have revealed persistent differences in per capita income among regions. Evidence shows that some regions managed to sustain high per capita income over a long time span while other regions seemed to be trapped in a low income growth path. These persistent differences are strikingly at odds with the standard neoclassical growth model, which predicts that poorer countries usually develop faster than richer ones and that there is a tendency toward convergence in levels of GDP per capita. On the other hand there exists an opposing growth paradigm [see, for example, Azariadis and Drazen (1990)] explaining multiple steady states in the growth rate of per capita income. According to Azariadis and Drazen (1990) and Aghion and Howitt (1998, chapter 10), multiple locally stable equilibria can be attributed to differences in initial conditions. Faini (1984) has initially considered multiple steady states in the context of regional development issues. In all these models, different initial conditions may cause regions to get stuck at different self-perpetuating levels of economic activity. As

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3 suggested by Quah (1996, 1997) and Paap and van Dijk (1998), this may lead to a polarisation into  
4 clubs of rich and poor countries or regions.<sup>1</sup>

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6 Research on convergence has accommodated cross-regional heterogeneity in a sequence of stages.  
7  
8 At first, conventional cross-section analysis [see, for example, Barro (1991) and Mankiw et al.  
9 (1992)] assumed complete homogeneity in steady state growth rates. Recently, Lee et al. (1997,  
10 1998) allowed complete heterogeneity in steady state growth rates. However, as pointed out by  
11 Islam (1998), extensions that allow varying growth rates run the risk of robbing the concept of  
12 convergence of any economic meaning. Instead of assuming complete heterogeneity, we set a  
13 structure of an intermediate form: we advocate techniques which focus on the evolution of the entire  
14 cross-sectional distribution in addressing the question of convergence across German districts in the  
15 first decade after German unification.<sup>2</sup> In this context, a convergence process occurs if, for instance,  
16 a bimodal density is detected at the beginning of the sample period and over time there is a tendency  
17 in the distribution to move towards unimodality. Alternatively, if there already is a unimodal  
18 distribution after German unification, convergence occurs when the dispersion of this density and  
19 therefore per capita income declines over time. To the best of our knowledge, no papers have  
20 attempted to formally test the convergence club hypothesis across East and West German regions  
21 after unification.<sup>3</sup> It is our purpose to detect whether clubs exist and which regions are associated  
22 with which clubs. A natural approach to assess the evolution over time of the dispersion of the  
23 regional per capita income is to estimate the cross-section distributions by using kernel density  
24 estimation.  
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27 The remainder of the paper is structured as follows. Section 2 describes the data set used for this  
28 study together with the non-parametric estimates of the per capita regional GDP over time. To  
29 support the visual impression given by kernel density estimates, and to provide further insight on the  
30 features of the underlying density, we have performed multimodality tests, whose results are  
31 presented in section 3. Section 4 concludes.  
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52 <sup>1</sup> The obvious difficulty here is to figure out in the data which countries are in the bad and which ones are in  
53 the good equilibrium. Barrier to getting out of such a trap can be the lack of a "big push" [see Murphy et al.  
54 (1989)]. Rodrik (1996) has argued that the East Asian miracle may have depended on a state-assisted process  
55 of overcoming coordination failure, and a consequent shift between two different equilibrium output levels (or  
56 a virtuous circle). It is also worth noting that the possibility of non-uniqueness is discussed informally even in  
57 Solow's (1956) original exposition of the neoclassical growth model.

58 <sup>2</sup> In this paper we add to the contributions of Bianchi (1997), Corrado et al. (2005), Laurini et al. (2005),  
59 Lopez-Bazo et al. (1999) and Pittau (2005) testing for "two-club" or "twin-peak" convergence of GDP per  
60 capita across countries and EU regions by analysing data which do not overlap with the data of existing  
papers.

<sup>3</sup> Funke and Niebuhr (2005) have demonstrated the existence of two clubs across West German regions prior  
to unification using threshold estimation techniques.

## 2. Data Issues and Empirical Evidence from Non-Parametric Density Estimation

The opportunity to assess spatial disparity trends in per capita income indicators is limited by the availability of consistent and comparable data. Long and dense time series for small geographic units are difficult to obtain, and in many cases not existent. In this section, we briefly present the spatial distribution of our data which are at the heart of our analysis. There are three levels of administration in Germany: (1) the Federal Republic at the national level; (2) 16 federal states (*Bundesländer*) on the regional level and (3) 439 districts (*Kreise*) or towns with autonomous administration (*kreisfreie Städte*), both on the local level. Smaller municipalities belong to the districts. In our empirical work below we focus on these 439 districts covering the entire economy.<sup>4</sup> Our data run from 1992 to 2001. Data for 1993 are missing. The source of our data is the “Arbeitskreis Volkswirtschaftliche Gesamtrechnung der Länder”.<sup>5</sup> The GDP per capita data are at constant 1995 prices and are obtained dividing the GDP of the German districts by their population. Ideally, we should deflate district-level per capita incomes using district-level deflators but, since district-level price indices are not available, we follow the usual practice and simply use the 16 state-level GDP deflators.

Nonparametric density estimations can reveal several features of the data and therefore help to capture the stylised facts that need explanation, exploring which specifications match with the data. The kernel estimator for the density function  $f(x)$  at point  $x$  is

$$(1) \quad \hat{f}(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x_i - x}{h}\right)$$

where  $x = x_1, x_2, \dots, x_n$ , is an independent and identically distributed sample of random variables from a probability density  $f(x)$  and  $K(\cdot)$  is the standard normal kernel with window width  $h$ . The window width essentially controls the degree to which the data are smoothed to produce the kernel estimate. The larger the value of  $h$ , the smoother the kernel distribution. A crucial issue is the selection of this smoothing parameter. In order to solve the trade-off between oversmoothing and undersmoothing, i.e. the trade-off between bias and variance, we have first used Silverman's (1986,

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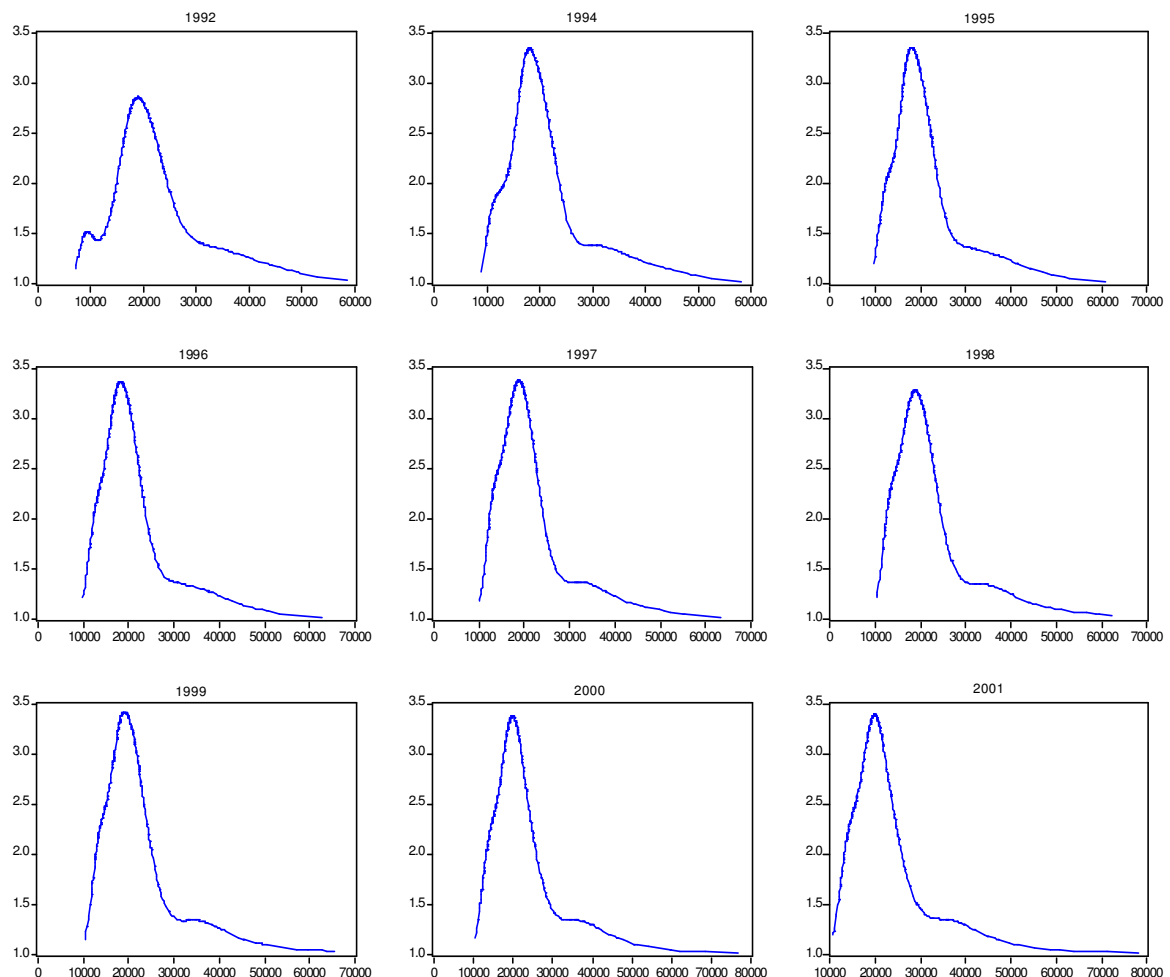
<sup>4</sup> We focus on district-level data because state-level data tend to „aggregate away” important differences between smaller geographic entities within the 16 states. For example, in the dataset that we analyse below, the ratio of GDP per capita between the richest (*Hamburg*) and the poorest state (*Sachsen-Anhalt*) was 2.63 in 2001, while the corresponding ratio for the richest (*Landkreis München*) and the poorest district (*Mittlerer Erzgebirgskreis*) was 7.30. On the other hand, one has to be aware that district-level GDP per capita figures may be affected by a commuting bias. Especially, commuters could overstate GDP per capita in agglomerations and city regions. Hamburg and Berlin are classified as a single region. This was forced on us because of lack of district-level data for both states. We also run the Kernel estimates excluding Berlin and Hamburg. Qualitatively, results are unchanged and the pattern is not much affected.

<sup>5</sup> See [http://www.statistik-bw.de/Arbeitskreis\\_VGR/publikationen.asp](http://www.statistik-bw.de/Arbeitskreis_VGR/publikationen.asp).

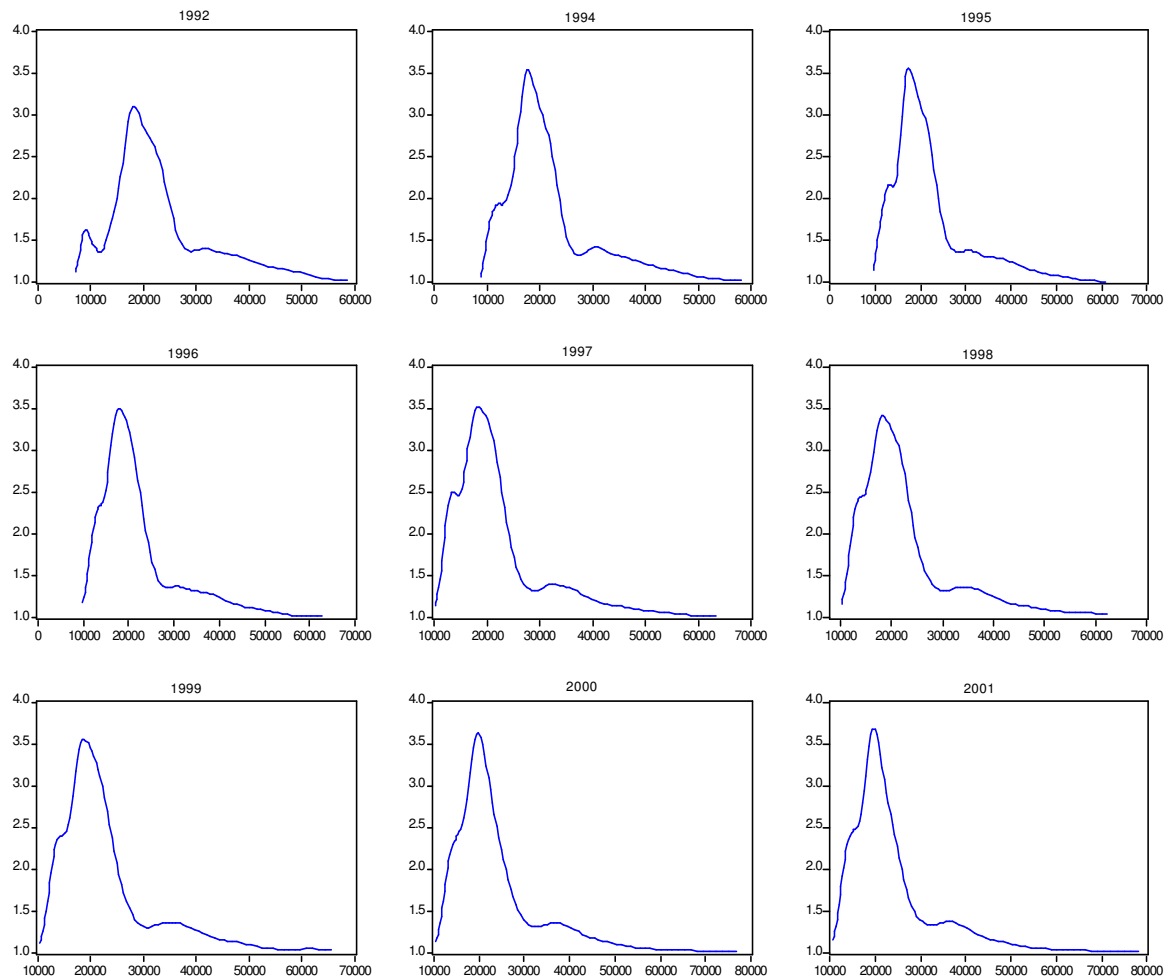
p. 48) “first generation” rule-of-thumb for a Gaussian kernel. Additionally, we consider the two-stage direct plug-in bandwidth selection method of Sheather and Jones (1991).

The distributions have been fitted to the logarithm of real per capita income. In Figures 1 and 2 are plotted the kernel density estimations for the (log) GDP from 1992 to 2001 obtained using the two abovementioned bandwidth selection methods and by transforming the income variable to the original scale. The figures show similar patterns, validating the fact that the estimates are robust with respect to the bandwidth specification. Nevertheless, as expected, the Silverman (1986) rule of thumb returns a slightly larger optimal smoothing parameter and therefore the relative density estimate (Figure 1) appears oversmoothed compared to the one obtained used the Sheather and Jones (1991) plug-in method (Figure 2).

**Figure 1: Non-parametric densities of per capita GDP (constant 1995 prices) across German districts using Silverman’s (1986) “first generation” rule-of-thumb**



**Figure 2: Non-parametric densities of per capita GDP (constant 1995 prices) across German districts using the plug-in bandwidth selection method of Sheather and Jones (1991)**



A preliminary inspection of the estimated densities reveals several noteworthy aspects. First, the snapshots show pronounced triple peakedness at the beginning of the considered time span. This evidence indicates that the German districts in 1992 can be separated into three groups, poor, rich and middle income. Second, as time passes this triple peakedness becomes less visible as the mode corresponding to low-income level recedes somewhat, without disappearing entirely as Figure 1 would have us believe.<sup>6</sup> As we will see, this bimodal/trimodal ambiguity recurs later when we utilize statistical tests for multimodality. Either way, this smoothing of the third mode is indicative of an improvement in economic conditions of the German poorest districts. In particular, this smoothing of the initial trimodality supports the notion of a catching-up process of eastern Germany at the beginning of the 1990s, i.e. the poorest districts did not stay as poor as they were immediately after unification. That said, despite the tendency of initially poor units to increase relative incomes, on

<sup>6</sup> A “mode” is meant here to be a point on the empirical density estimate around which the tangent to the curve changes its slope from positive to negative.



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3 average, over the considered decade, several districts experienced negative growth rates.<sup>7</sup> Third,  
4 there is a visible tendency for the remaining two peaks to move apart, with the third mode moving to  
5 the right towards a higher income level. Moreover, the variability of the "low-mean distribution" has  
6 been declining over the decade from 1992 to 2001 and in 2001 appears to be considerably smaller  
7 than the spread of the "high-mean distribution". This evidence reveals that cross regional income  
8 disparity has become larger rather than smaller as predicted by absolute convergence.  
9

10 Furthermore, we use the methodology of distributional dynamics to model the evolution of the  
11 relative distribution of per capita incomes for Germany districts. This approach models directly the  
12 evolution of relative income distributions by constructing transition probability matrices that track  
13 changes over time in the relative position of districts within the distribution. This is an exercise that  
14 a number of authors have undertaken (see Quah, 1996a, 1996b, 1997). The modelling of distribution  
15 dynamics assumes that the density distribution  $\phi_t$  has evolved in accordance with the following  
16 equation:  
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$$(2) \quad \phi_{t+1} = M \phi_t,$$

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26 where  $M$  is an operator that maps the transition between the income distributions for the periods  $t$   
27 and  $t+1$ . Since the density distribution  $\phi$  for the period  $t$  only depends on the density  $\phi$  for the  
28 immediately previous period, this is a first-order Markov process. In our estimates below we have  
29 assumed that the distribution  $\phi$  has a finite number of states. For the Markov transition matrices we  
30 assume that the probability of variable  $s_t$  taking on a particular value  $j$  depends only on its past value  
31  $s_{t-1}$  according to the first-order Markov chain  
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$$(3) \quad P\{s_t = j \mid s_{t-1} = i\} = P_{ij},$$

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43 where  $P_{ij}$  indicates the probability that state  $i$  will be followed by state  $j$ . As  
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$$(4) \quad P_{i1} + P_{i2} + \dots + P_{in} = 1$$

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50 we may construct the so-called transition matrix  
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58 <sup>7</sup> In particular, the growth rates of the real GDP per capita over the decade from 1992 to 2001 were negative in  
59 66 districts. Out of these 66, seven districts (Delmenhorst, Landkreis Holzminden, Landkreis Sigmaringen,  
60 Landkreis Soltau-Fallingborstel, Landkreis Unterallgäu, Neustadt an der Weinstrasse, Wilhelmshaven) have  
even experienced two-digit negative growth rates. Following Jones (1998, p. 4) these districts might be labeled  
"growth disasters".

$$(5) \quad P = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \dots & \dots & \dots & \dots \\ P_{n1} & P_{n1} & \dots & P_{nn} \end{bmatrix}$$

where line  $i$  and column  $j$  give the probability that state  $i$  will be followed by state  $j$ . In our modelling approach, the probability  $P_{ij}$  measures the proportion of districts in regime  $i$  during the previous period that migrate to regime  $j$  in the current period.

The transition probability matrix in Table 1 reports transitions between the 1992 and 2001 distributions of GDP per capita relative to the German average. The main diagonal of the matrix gives the proportion of districts that were in the same range of the distribution immediately after German unification as a decade later. Table 1 also provides information about  $n$ , the number of districts that begin their transitions in a given state. Furthermore, we provide the classes that divide up the state space.

The salient characteristics of the transition probability matrix in Table 1 reveal a number of noteworthy behavioural patterns in the distribution of real GDP over time. First, as indicated by the first element of the main diagonal (0.03), districts which originally reside in the lowest range of the distribution (i.e. with a GDP per capital of 50% or less of the German average) appear to be very unlikely to remain in this category at the end of the period in question. Such districts display a strong tendency to either move forward to the second category (0.68) or jump to the third category (0.27). Second, the third and fourth elements of the main diagonal (a real GDP of 65%-80% and 80%-100% of the German average, respectively) indicate a relatively high probability for the regions within this range to maintain their status quo over the period. That said, regions in the third category appear to be relatively open to backward or forward movements (with probabilities of 0.13 and 0.22 respectively) while those in the fourth seem decidedly more backward looking, as illustrated by the 0.26 probability of moving a step back but only a mere 0.04 probability of moving forward one step. Finally, the districts residing in the fifth category (with a real GDP of 100-125% of the average) appear to be more likely to either retain this position or fall back by one category. These districts marked inability to move forward (a probability of 0.02) suggests there comes a point where incremental increases in real *GDP* become harder and harder to sustain. Furthermore, those districts that reside in the highest income category at the beginning of the time period display a very high probability (0.83) of consolidating their position of affluence.

The forementioned characteristics support the findings of kernel density estimation, namely: the tendency of the poorest districts to catch-up; the middle income districts retaining their status quo (despite a small number of their ranks back-peddling); and the consolidation of the richest districts of their position.

**Table 1: Transition Probability Matrix Relative to the German Average**

		GDP PER CAPITA 2001						
GDP PER CAPITA 1992	$n$		4	61	79	128	74	73
	63	[0-0.5]	0.03	0.68	0.27	0.02	0.00	0.00
	30	[0.5-0.65]	0.07	0.40	0.33	0.17	0.03	0.00
	45	[0.65-0.8]	0.00	0.13	0.53	0.22	0.11	0.00
	106	[0.8-1.00]	0.00	0.00	0.26	0.70	0.04	0.00
	90	[1.00-1.25]	0.00	0.00	0.00	0.41	0.57	0.02
	85	[1.25-∞]	0.00	0.00	0.00	0.01	0.15	0.83
			[0-0.5]	[0.5-0.65]	[0.65-0.8]	[0.8-1.0]	[1.0-1.25]	[1.25-∞]

In Table 1, the operator  $M$  has been constructed by assuming that the distribution  $\phi_t$  has a finite number of states. This discrete modelling approach leads to the problem that the researcher has to determine the number of intervals and the limit values of each interval in an arbitrary and ad hoc way. Furthermore, the discretisation process may eliminate the property of Markovian dependence in the data, as Bulli (2001) has pointed out. The solution addressing these shortcomings consists of carrying out a continuous analysis of transition, which avoids discretisation through the use of conditional densities that are estimated non-parametrically and referred to as stochastic kernels. A stochastic kernel amounts to a transition matrix with an infinite number of infinitely small ranges. The results from this tool are displayed as three-dimensional graphs in Figure 3 and a two-dimensional contour map in Figure 4.

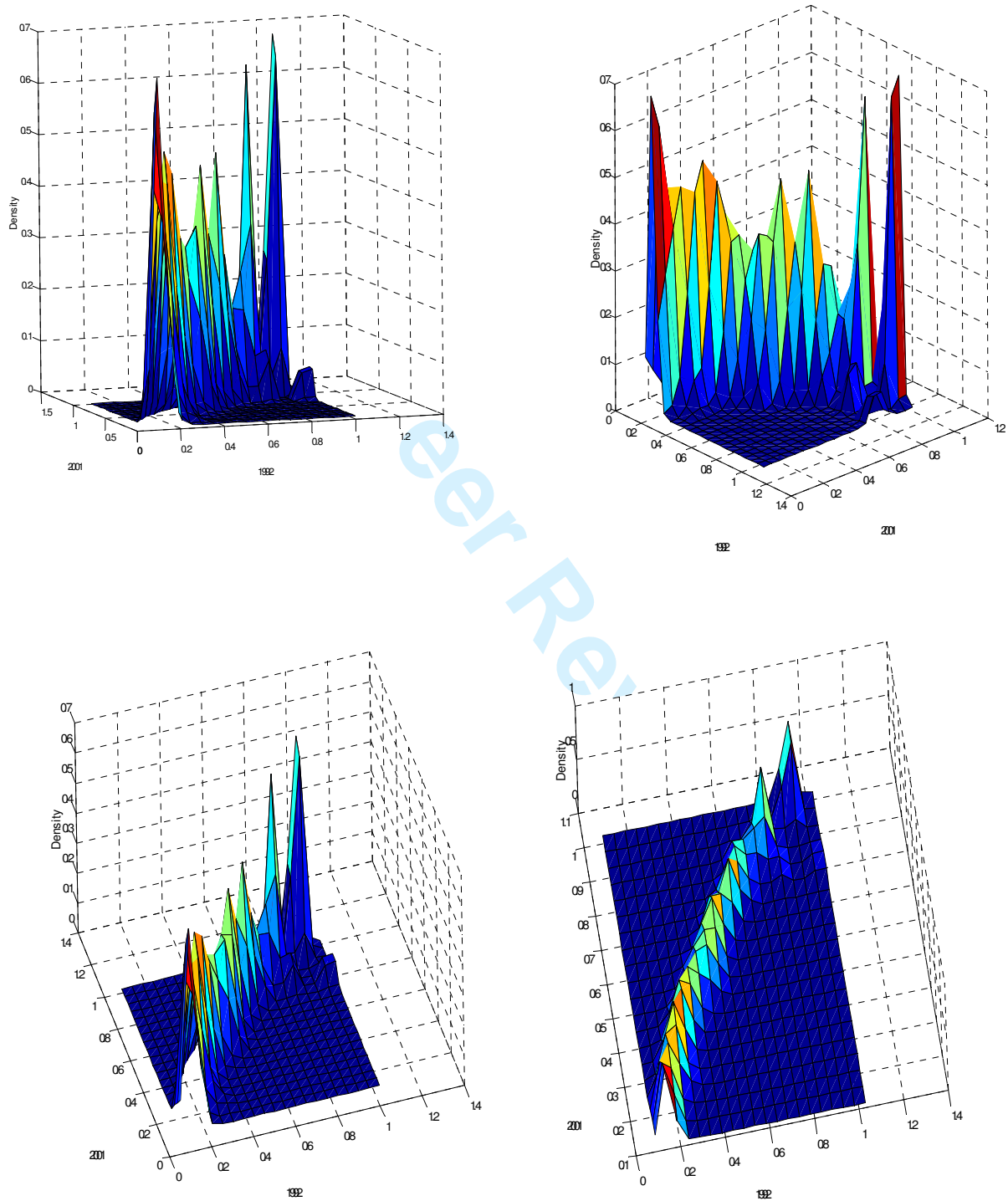
The three dimensional stochastic kernel graph yields a number of valuable insights, which are both additional and complementary to those of the static kernel illustrations of Figures 1 and 2. In order to fully exploit the information content of this construct we firstly adjust the viewer's perspective by rotating the illustration (Figure 3, top left and top right). We then provide further insights by tilting the graph downwards, as if looking down on the three dimensional distribution from above. This "aerial view" is further enhanced by means of contour images of the distribution (Figure 4). Rotating the graphs in Figure 3 (top right and top left) highlights two features: the pronounced peaks at the beginning and end of the distribution; and middle section of the distribution which, while relatively lower, still suggests the possibility of either slippage or enhancement of one's relative position.

This aerial view of the income distribution, highlighting as it does the diagonal pattern of the distribution over time, illustrates the tendency of regions residing in low income categories in 1992 to remain there in 2001, while high income regions retain their affluent status throughout the period in question. That said, a further more subtle nuance can be gleaned from Figures 3 and 4. The hint of concavity visible in both the three dimensional graph (bottom right) and contour representation are

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3 indicative of upward movement in the status of the lower income regions; a finding which is  
4 explored further below.  
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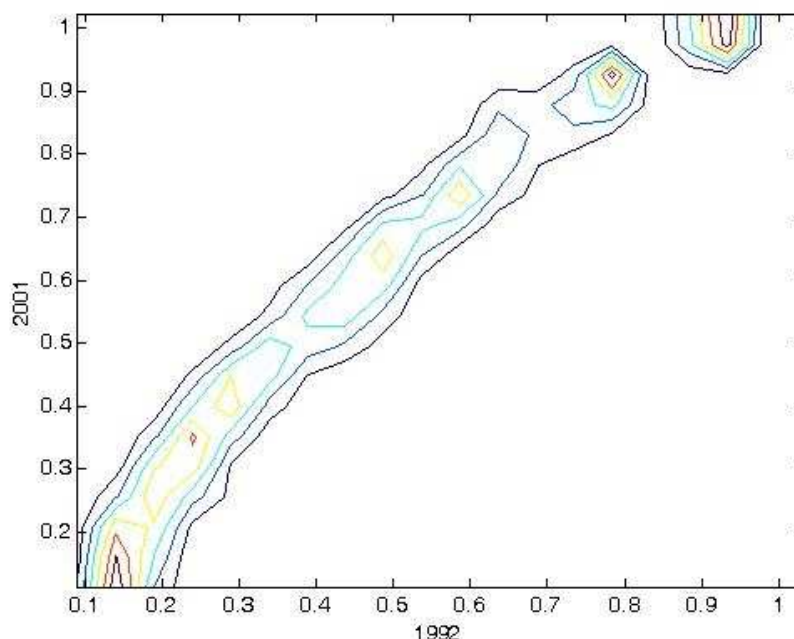
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**Figure 3: Stochastic Kernel Estimates, 1992 - 2001**



Note: In Figure 3 and 4 we have used the region with the highest per capita income as a numeraire. The choice is arbitrary but has no impact upon the qualitative results.

**Figure 4: Stochastic Kernel Contours**



### 3. Tests for Multimodality

The discussion above has relied heavily on the visual impression and shape of the non-parametric income densities. In practical terms, looking at Figure 1 - 4, the first question to ask is: are those districts randomly drawn from an unimodal distribution, a bimodal distribution or is there any kind of trimodality? In order to ascertain the significant number of modes present in our estimated density functions, we employ Silverman's (1981, 1986) nonparametric test for multimodality and peakedness.<sup>8</sup>

The non-parametric procedure tests the null hypothesis that a density  $f$  has  $k$  modes (or peaks, bumps) against the alternative that  $f$  has more than  $k$  modes, where  $k$  is a non-negative integer. The test statistic in this case is the critical window width, defined by

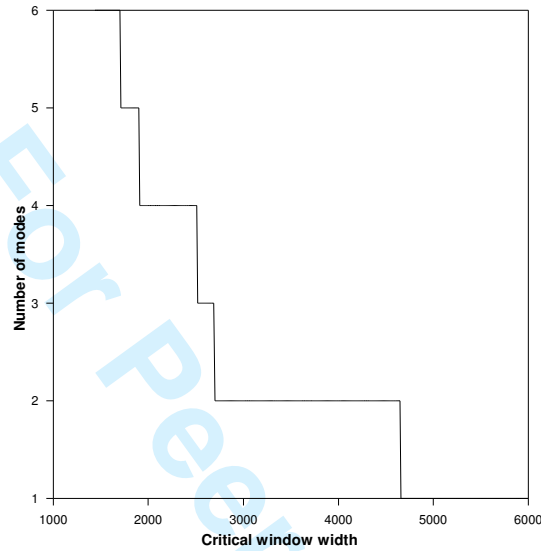
$$(6) \quad h_{crit}(k) = \inf \{h \mid \hat{f} \text{ has at most } k \text{ modes}\}.$$

For  $h < h_{crit}(k)$ , the estimated density has at least  $k+1$  modes. The basic idea of the test is intuitive and simple. Specifically, if the series has  $k$  modes, then  $h_{crit}(k-1)$  should be 'large' because substantial

<sup>8</sup> This test of multimodality has been used by Bianchi (1997) to test the hypothesis of income convergence for a group of 119 countries between the years of 1970 and 1989. Bianchi (1997) rejects the hypothesis of convergence in favour of the formation of convergence clubs.

smoothing is required to generate a  $(k-1)$ -mode density. For example, if the data possess two strong modes, a large value of  $h$  will be needed to obtain an unimodal estimate. An illustrative calculation of the critical window widths  $h$  and the corresponding number of modes (peaks) in the kernel density estimates for the year 1999 is plotted in Figure 5.

**Figure 5: Number of Modes in the Kernel Density Estimate as a Function of the Window Width Size  $h$ , 1999**



Thus, the technique forms a natural hypothesis-testing framework since large numbers of  $h_{crit}(k)$  indicate more than  $k$  modes. The crucial question, then, becomes how large is “large” when the chosen bandwidth is concerned. The value of  $h_{crit}(k)$  is computed through a binary search algorithm, and its significance level can be assessed by the bootstrap procedure attributable to Efron (1979). In particular, the bootstrap test requires a statistic test  $t(x)$  and an estimated null distribution for the data under  $H_0$ . Given these, the  $p$ -value of the test is

$$(7) \quad p_{\text{bootstrap}} = \text{prob}_{\hat{F}_0} \{t(x^*) > t(x)\}$$

where

$$(8) \quad x^* = (x_1^*, x_2^*, \dots, x_n^*)'$$

is the bootstrap drawn from the null distribution  $\hat{F}_0$ . To approximate  $p_{\text{bootstrap}}$ , bootstrap samples have to be drawn from a rescaled density estimate obtained by setting

$$(9) \quad x_i^* = \bar{y}^* + \sqrt{1 + \frac{h^2}{\hat{\sigma}^2}} (y_i^* - \bar{y}^* + h\varepsilon),$$

where  $\sqrt{1 + h^2/\hat{\sigma}^2}$  is the rescaling factor,  $y_i^*$  are sampled with replacement from the original sample,  $\bar{y}^*$  its mean,  $\hat{\sigma}^2$  its variance and  $\varepsilon$  is assumed to be distributed as a standard normal since the kernel is Gaussian.<sup>9</sup> In the spirit of the analysis of Hall (1992), the bootstrap method treats the available sample as the population, and through repeated re-sampling of this sample, obtains the distribution of statistics of the test. A sample is taken of the original series (with replacement) and transformed to have the same first and second moments. Critical values are then obtained by generating a large number of samples.<sup>10</sup> This is not a nested test and its results should therefore be interpreted as a hierarchical set of significance tests.

We execute the Silverman (1981, 1986) test for each year, with null hypotheses of one, two and three modes (hence alternative hypotheses of more than one, more than two and more than three modes).

**Table 2: Silverman's Multimodality Test**

YEAR	CRITICAL BANDWIDTHS AND <i>P</i> -VALUES			<i>k</i> *
	$h_{crit(1)}$	$h_{crit(2)}$	$h_{crit(3)}$	
1992	2490 [0.00]	2240 [0.00]	1780 [0.16]	3
1994	2530 [0.00]	2170 [0.12]	1600 [0.21]	2
1995	3120 [0.00]	2960 [0.08]	1590 [0.19]	3
1996	3760 [0.00]	2640 [0.13]	1810 [0.26]	2
1997	3060 [0.00]	3200 [0.06]	1930 [0.10]	3
1998	3910 [0.07]	2570 [0.19]	1860 [0.35]	2
1999	4660 [0.00]	2700 [0.10]	1860 [0.08]	2
2000	3875 [0.00]	2530 [0.10]	1710 [0.14]	2
2001	3620 [0.00]	3130 [0.05]	2710 [0.09]	3

Notes: Bootstrapped *p*-values in [-].

<sup>9</sup> Rescaling is necessary since the kernel estimation artificially increases the variance of the estimate [see Efron and Tibshirani (1993)]. Since the procedure samples from a smooth estimate of the population, it is called smooth bootstrap.

<sup>10</sup> In our simulations we set the number of bootstrap replications to 3000.

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5 The results are listed in Table 3, with the first row for any given year indicating the values of  $h_{crit}(k)$ ;  
6 the  $p$ -values associated with the corresponding critical value widths are given in parentheses; and  $k^*$   
7 representing the number of modes detected. Taken together, the Silverman test suggests a persistent  
8 ambiguity between trimodality and bimodality over the time period, consistent with the “eye-ball  
9 evidence” drawn from Figures 1 - 4.  
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13 In order to geographically illustrate the clusters detected in the Kernel density estimations, we have  
14 produced a set of maps, Figures 6 and 7, which create a visual impression of the spatial structure of  
15 the real GDP per capita across German districts. The categories are defined such that in each income  
16 range there resides an equal number of districts. To be consistent with the results of our empirical  
17 analysis we have chosen to identify three and six categories of the real GDP per capita in 1992 and  
18 in 2001, the first and the last year of the considered time span. The presented maps provide evidence  
19 that spatial clusters do exist for the variable under consideration.<sup>11</sup> In particular, as one would  
20 expect, the poorest district are concentrated in East Germany. In 1992, all districts, except Berlin,  
21 Kreisfreie Stadt Potsdam and Kreisfreie Stadt Erfurt, belong to the “poorest” group,<sup>12</sup> whereas only  
22 10 percent of the West districts are included in this low-income cluster. By 2001, the proportion of  
23 eastern districts that still reside in this same cluster has shrunk to 80 percent of the total eastern  
24 districts. However, a number of districts have switched to the richer groups, showing an  
25 improvement in their relative income level. In particular, the districts Landkreis Teltow-Fläming and  
26 Landkreis Dahme-Spreewald in the greater Berlin area have moved from the low- to the middle-  
27 income group. Furthermore, 8 percent of the eastern German districts (Kreisfreie Stadt Dresden,  
28 Kreisfreie Stadt Rostock, Kreisfreie Stadt Cottbus, Kreisfreie Stadt Neubrandenburg, Kreisfreie  
29 Stadt Jena, Kreisfreie Stadt Erfurt, Kreisfreie Stadt Schwerin, Kreisfreie Stadt Zwickau, Kreisfreie  
30 Stadt Potsdam) have gained a foothold amongst the richest elite by 2001.  
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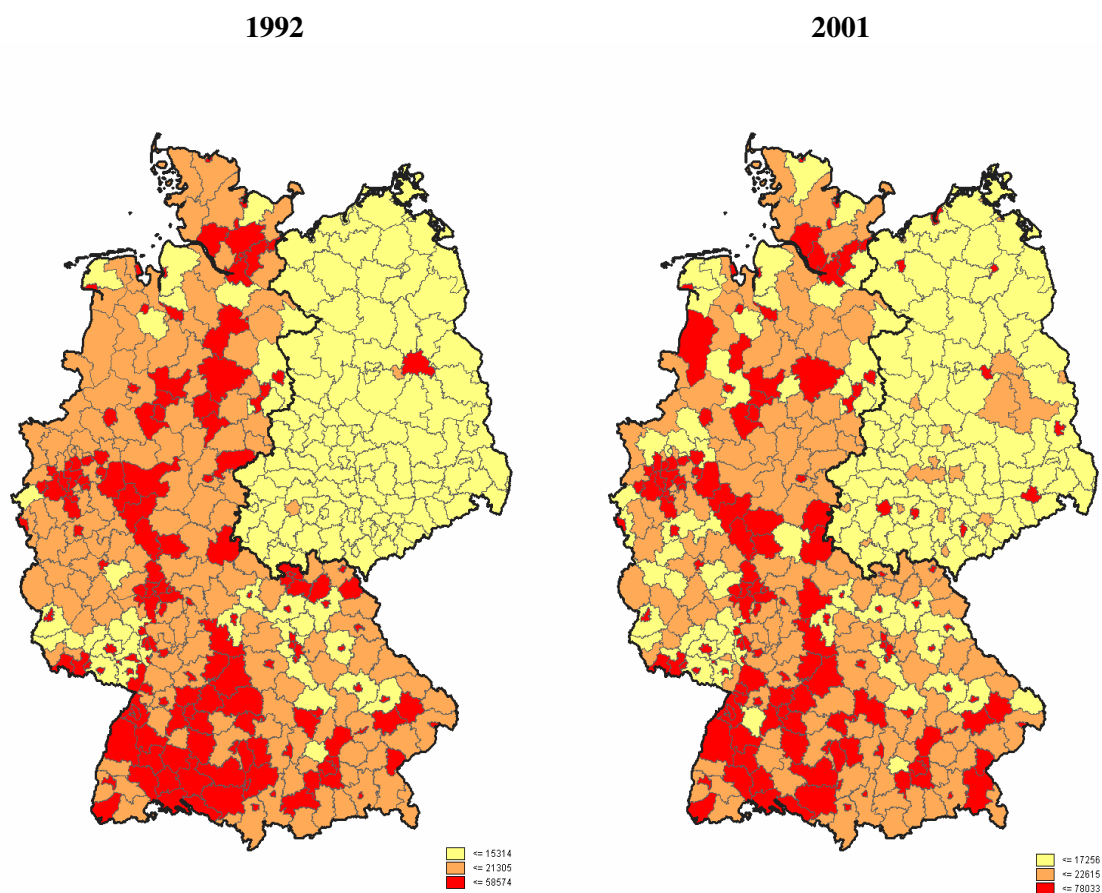
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56 <sup>11</sup> For the correct interpretation of the maps it is important to bear in mind that they are not suitable to assess  
57 the absolute growth performance of the 439 German districts: in particular, it is not possible to say whether  
58 over the last decade the poorest areas caught up with the richest ones or whether some areas got richer or  
59 poorer as they switched from a cluster to another. (The reason for that is that the thresholds defining the  
60 identified categories have changed over time). Looking at those thresholds – which all rose considerably – it is  
indeed possible to state that the average German GDP has risen between 1992 and 2001.

<sup>12</sup> Data for Mecklenburg-Vorpommern (18 Kreise) is not available for 1992. However, their per capita income level is found to reside within the lowest income category as soon as these figures become available in 1996.



Figure 6: Real GDP per capita 1992 and 2001



Notes: The thin lines indicate the regional boundaries of the 439 districts, the thick lines indicate the East German states and districts.

A further piece of information that can be gleaned from the visual inspection of the maps is that approximately 40 percent of West Germany belongs to the high-income cluster, both in 1992 and in 2001, with a high concentration of rich districts localized in the Hamburg area to the North, as well as in the western and southern parts of West Germany. One surprising feature that emerges is the marked downturn in the fortunes of 24 western districts, who experienced an erosion of their per capita GDP from above 21,300 euros in 1992 to 17,200 euros in 2001.<sup>13</sup>

All in all, the comparison between 1992 and 2001 shows that the spatial structure of the real GDP per capita of German districts over the last decade has indeed changed. Figure 6 shows that the relative income position of the East German districts has remained at the bottom of the ranking whereas districts located in the South-West and in the Hamburg area were still included in the richest group. In other words, the relatively “poorer” districts have remained clustered in the eastern

<sup>13</sup> In 2001 the share of western districts included in the low-income group over the total number of West German districts rose to 18 percent (58 out of a total of 326 West German districts) from 10 percent in 1992.

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3 part of Germany and the “wealthier” areas have remained localized in the South-West. That said, the  
4 emergence of a number of wealthier eastern districts concurrently with the fall-back experienced by  
5 a pocket of western regions suggests that the overall picture may be more complex than first  
6 thought.  
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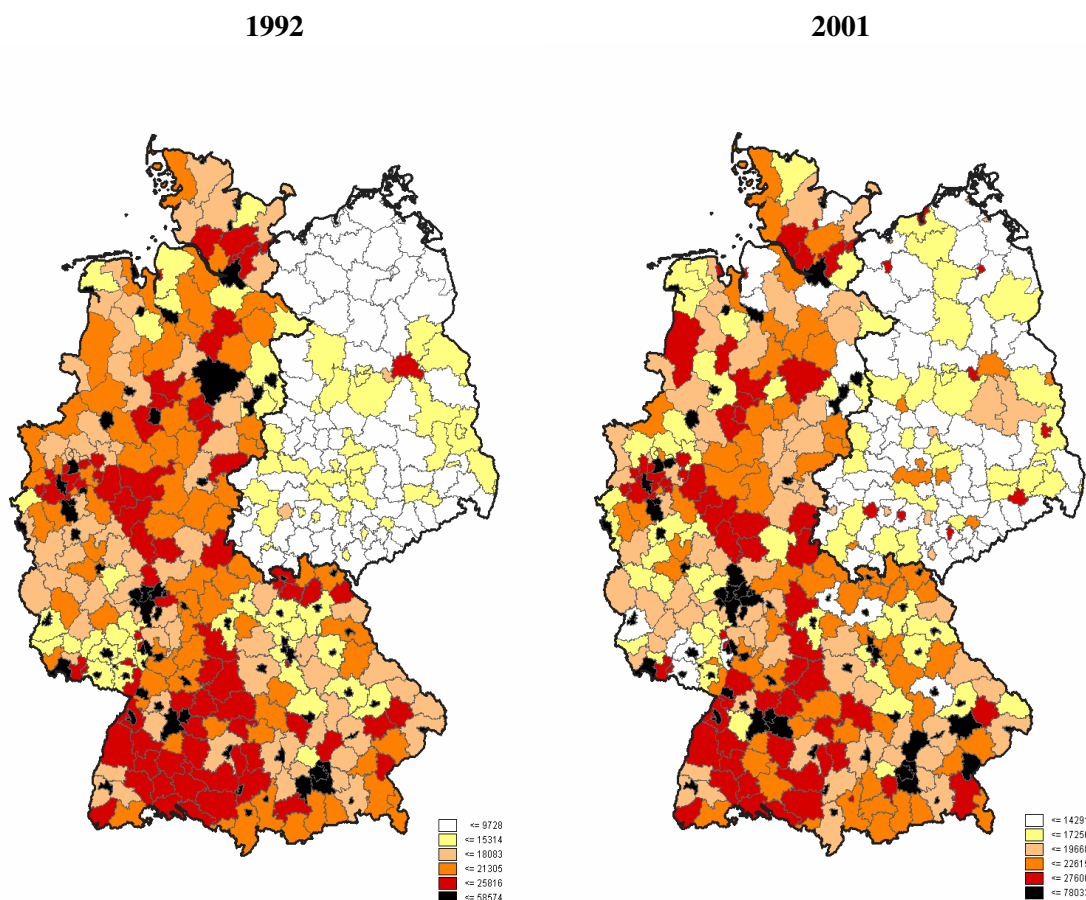
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10 Figure 7 paints the same picture in greater detail as six different income groups are identified. This  
11 6-category map allows a more precise view of the spatial structure of real GDP per capita across the  
12 German districts, while retaining a natural consistency with the 3-category map of Figure 6. Of the  
13 two lowest income ranges, the very poorest range is observed only in East Germany in 1992. By  
14 2001, however, it is apparent from the more detailed 6-category maps that a number of West  
15 German states now reside in this lowest category, particularly in the north and south-west. Within  
16 the East there is also a discernible movement from the lowest income range to the second lowest,  
17 over the period in question. In the middle income ranges there has been a perceptible emergence of  
18 middle-income category districts in the East German states over the 1992-2001 time period, whereas  
19 in the West those regions residing in the middle income ranges in 1992 have broadly retained their  
20 status throughout the period. Similar to the trends observed in Figure 6, the relatively wealthier  
21 regions tend to be concentrated in the west and southern areas of the country in both 1992 and 2001.  
22 One can also discern the emergence of a sprinkling of relatively wealthy regions in the East by 2001,  
23 due perhaps to real GDP growth associated with urban, commercial areas such as Berlin and  
24 Dresden.  
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28 Taken as a whole, the visual impression created in Figures 6 and 7 of the spatial structure of the real  
29 GDP per capita leads one to conclude the following: over the period 1992-2001 there has been a  
30 noticeable catching-up process in terms of the real GDP of East German regions; West German  
31 regions that have been residing in middle income ranges tend to have retained this status throughout  
32 the period in question, though as illustrated by the 6-category maps a small number of western  
33 regions which were in the lower income categories in 1992 have fallen back somewhat by 2001; the  
34 relatively richer clusters in the western and southern areas of the country have consolidated their  
35 position over the period in question, while a sprinkling of relatively wealthy regions has also  
36 emerged in the East.<sup>14</sup>  
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<sup>14</sup> These results are consistent with the empirical evidence in Bayer and Jüßen (2007) and Villaverde and Maza (2008) showing moderate speed of  $\beta$ -convergence across German and European regions.

Figure 7: Real GDP per capita 1992 and 2001



Notes: The thin lines indicate the regional boundaries of the 439 districts, the thick lines indicate the East German states and districts.

#### 4. Conclusions and Further Comments

The objective of this paper is to address the question of convergence across German districts in the first decade after German unification by drawing out and emphasising some stylised facts of regional per capita income dynamics, rather than estimating any particular economic model. We achieve this by employing techniques which focus on the evolution of the entire cross-sectional income distribution. In particular, we follow a distributional approach to convergence based on non-parametric kernel density estimation and implement a number of tests to establish the statistical significance of our findings. The visual inspection of the estimated densities indicates the following: the presence of trimodality in 1992; in subsequent years less pronounced trimodality, supporting the notion of a catching-up process of eastern Germany in the early 1990s; and a tendency for the

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3 remaining two peaks to move apart, resulting in a swelling of the middle income mode and a more  
4 pronounced high income mode.<sup>15</sup>  
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6 An alternative approach to investigating the presence of convergence clubs would be to track in  
7 more detail the performance of each geographical unit. This may provide another dimension of  
8 disparity that is relevant for economic policy making. From a policy perspective, besides having  
9 information about the entire cross-section of observations, it is also important to know how likely is  
10 each district to improve its conditions, how many did so and what are their characteristics. In other  
11 words, whether or not districts that were rich (poor) a decade ago are the same ones that are rich  
12 (poor) now has relevant policy implications. If the poor regions are persistently poor, one may want  
13 to consider public programs aimed at enhancing the performance of these districts. On the other  
14 hand, if the incomes per capita are rotating over time, one would be less concerned about overall  
15 geographical income distribution. Our approach has not conceptualised this alternative mixing or  
16 ranking change aspect of disparity. Further consideration should be given to such indicators in future  
17 research.  
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51 <sup>15</sup> The exact nature of multimodality, however, is still surrounded by some degree of uncertainty. At first  
52 glance, it might seem promising to consider growth model with multiple equilibria in the tradition of Aghion  
53 and Howitt (1998, Chapter 10), Azariadis (1996), Drazen and Azariades (1990) and Matsuyama (1991) when  
54 trying to explain "job-poor" versus "job-rich" growth experiences. In such models, a country may be trapped in  
55 a "job-poor" equilibrium when, in principle at least, an alternative and superior equilibrium is also feasible.  
56 However, the recent literature has cast doubts on the robustness of multiple equilibria. Frankel and Pautner  
57 (2000) analyse a two sector model with increasing returns, based upon Matsuyama (1991). They show that if  
58 the wage is stochastic and arrives as a Poisson process, the multiplicity property may be eliminated because  
59 some of the deterministic equilibria are more robust to perturbations than others. A similar conclusion has  
60 been established by Herrendorf et al. (1999) for heterogeneous agents. They show that sufficient heterogeneity  
of agents will lead to a refinement in the set of observable equilibria and uniqueness in models like that of  
Matsuyama (1991).

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