

## The within-distribution business cycle dynamics of German firms

Doepke, Joerg; Weber, Sebastian

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### The within-distribution business cycle dynamics of German firms

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# The within-distribution business cycle dynamics of German firms\*

by

Jörg Döpke (Deutsche Bundesbank) and

Sebastian Weber (University of Hamburg)

## Abstract:

In this paper we analyse stylised facts for Germany's business cycle at the firm level. Based on longitudinal firm-level data from the Bundesbank's balance sheet statistics covering, on average, 55,000 firms per year from 1971 to 1998, we estimate transition probabilities of a firm in a certain real sales growth regime switching to another regime in the next period, e.g. whether a firm that has witnessed a high growth rate is likely to stay in a regime of high growth or is bound to switch in a regime of low growth in the subsequent period. We find that these probabilities depend on the business cycle position.

Keywords: business cycles, firm growth, Markov chains

JEL-Classification: E32, D21, D92

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# The within-distribution business cycle dynamics of German firms

## 1. Introduction

Schumpeter's (1942: pp83 ff.) interpretation of capitalism as a process of "creative destruction", formulated almost half a century ago, has recently been drawn to the attention of economists once again (see, e.g., the work of Aghion and Howitt, 1992, 1998). By stating that firms are the main driving factor in his theory of cycles and growth, Schumpeter (1951) emphasised that empirical research should be directed towards the business cycle behaviour of individual firms. As is well known, macroeconomics took a different approach. The representative firm became the workhorse in macroeconomic theory, and empirical research concentrated on the behaviour of aggregates. The assumption of a representative firm has been viewed with increasing criticism (see e.g. Kirman, 1992). Models with heterogeneous agents are gaining in popularity (see, e.g., Delli Gatti et al., 2003 or Ghironi and Melitz 2005).

On the empirical side, Higson et al. (2002, 2004) and Döpke et al. (2005) try to follow Schumpeter's suggestion and established stylised facts at the firm level. In particular, these papers document stylised facts for the cross-section distribution of real sales growth rates. According to these facts, the distribution of real sales growth depends on the business cycle position: anti-cyclical skewness is a pervasive finding in all three papers. Another key result of those analyses has been that the extreme percentiles (i.e. the rim percentiles) have reacted less

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6 sharply to business cycle conditions than the middle percentiles. Conclusions  
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8 from this fact with regard to the behaviour of single firms may be misleading to  
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10 some extent since only results for the percentiles themselves were obtained. This  
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12 is the motivation for the present paper.  
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16 – Figure 1 about here –  
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19 In the aforementioned literature, the analysis was centred on the overall  
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21 distribution of real sales growth rates. In the present paper, we take a closer look  
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23 at the within-distribution dynamics of real sale growth rates, i.e. at the behaviour  
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25 of individual firms, taking the movement of the distribution as given (as in Figure  
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27 1). The aim is to augment the already-established stylised facts with new ones in  
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29 the vein of Schumpeter. The analysis will be conducted by using non-  
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31 homogenous Markov chains and estimating the respective transition matrices.  
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34  
35 Our main results may be summarised as follows. We analyse stylised facts for  
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37 Germany's business cycle at the firm level. Based on longitudinal firm-level data  
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39 from the Bundesbank's balance sheet statistics covering, on average, 55,000 firms  
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41 per year from 1971 to 1998, we estimate transition probabilities of a firm in a  
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43 regime of a certain real sales growth switching to another regime in the next  
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45 period. We find that these probabilities depend on the position in the business  
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47 cycle.  
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50  
51 The remainder of the paper is as follows. In the next section we briefly explain  
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53 Markov chains and the estimation techniques employed. Section 3 discusses the  
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55 data set. Some descriptive results with regard to the cross section of transition  
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matrices are presented in section 4. Section 5 then deals with the impact of business cycle fluctuations on transition probabilities. Section 6 concludes.

## 2. Empirical methods

### 2.1 Transition probabilities and Markov-chains

A Markov chain is a stochastic process  $\{x_t\}$  with the property that for all  $t$  and all  $k \geq 1$

$$Pr(x_{t+1} | x_t, x_{t-1}, \dots, x_{t-k}) = Pr(x_{t+1} | x_t) \quad (1)$$

The variable  $x_t$  is a state, to be defined later, in which an object is at time  $t$ . All  $m$  possible states are elements of the vector  $\bar{x} \in R^m$ . The Markov property then states that the probability of being in a state at time  $t+1$ , i.e.  $x_{t+1}$ , depends only on the state which the object belonged to in the last period, i.e.  $x_t$ . The probabilities are summarised in a transition matrix  $P$  of dimension  $m \times m$  where each element has the interpretation<sup>1</sup>

$$P_{ij} = Pr(x_{t+1} = \bar{x}_j | x_t = \bar{x}_i) \quad (2)$$

Markov chains can be either homogenous or non-homogenous<sup>2</sup>. A Markov chain is said to be homogenous if, for every  $t$ , the transition matrix  $P(t) = P$ . In this

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1 For a more in depth discussion of Markov chains see Ljungqvist and Sargent (2000) chapter 1.

2 In earlier discussion this was termed stationary or non-stationary. Since nowadays these labels are associated with unit-root processes in time series

paper we necessarily assume that the Markov chain is non-homogenous, otherwise the change from one state to another would be purely random and, thus, a business cycle interpretation would be pointless. Therefore we will only consider the non-homogenous case. In this case  $p_{ij}(t)$  is the unobservable probability of moving from state  $\bar{x}_i$  to  $\bar{x}_j$  at time  $t$ . What is observable is the number of objects that move from  $\bar{x}_i$  to  $\bar{x}_j$  at time  $t$  denoted by  $n_{ij}(t)$ . The conditional distribution of  $n_{ij}(t), j = 1, \dots, m$  given  $n_{i\bullet}(t)$  is multinomially distributed:

$$\frac{n_{i\bullet}(t)!}{\prod_{j=1}^m n_{ij}(t)!} \prod_{j=1}^m p_{ij}(t)^{n_{ij}(t)} \quad (3)$$

Maximising equation (3) with respect to  $p_{ij}(t)$ , subject to the constraints

$$p_{ij}(t) \geq 0 \text{ and } \sum_{j=1}^m p_{ij}(t) = 1, \text{ gives us the maximum likelihood estimates for}$$

$$p_{ij}(t):$$

$$\hat{p}_{ij}(t) = \frac{n_{ij}(t)}{n_{i\bullet}(t)} \quad (4)$$

which is the frequency of movements out of a given state  $\bar{x}_i$  to  $\bar{x}_j$  (Anderson and Goodman, 1957).

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analysis, for clarity the terms homogenous or non-homogenous are preferable, despite sometimes being used in a different context.

## 2.2 Multinomial logit model

To gain further insights into the mechanisms that drive the transitions, one can subdivide the population into groups according to characteristics which supposedly influence the process. For each group the transition matrix can be estimated. The different matrices can then be compared. This is only possible with a limited number of discrete characteristics and without inference. A more promising approach therefore is to use regression analysis. The appropriate model for the present context is the multinomial logit model (McFadden 1974).

In this model, the data are divided into subsamples according to the state the observations were in at time  $t$ . Let us define a variable  $Y_{ki} = \{j \text{ if } x_{t+1} = \bar{x}_j \wedge x_t = \bar{x}_i\}$  for the  $k$ -th observation. The state  $j$  the  $k$ -th observation is in at  $t+1$ , conditional on the state  $i$  at  $t$ , is then a function of some independent variables  $z$ :  $Y_{ki} = z'_{kj} \beta_j + \varepsilon_{kj}$ . Assuming that the  $j$  error terms are independent and identically Gumbel distributed, the probability of being in state  $j$  is

$$Prob(Y_{ki} = j) = \frac{e^{z'_{kj} \beta_j}}{\sum_{j=1}^J e^{z'_{kj} \beta_j}} \quad (5)$$

This is the multinomial logit model. Unfortunately, this model is indeterminate, since adding a constant to the  $\beta$  vector results in the same probabilities. Therefore, the model is normalised by setting  $\beta_1 = 0$ , leading to the probabilities



$$Prob(Y_k = j) = \frac{e^{z'_k \beta_j}}{1 + \sum_{i=2}^I e^{z'_k \beta_i}} \quad (6)$$

This implies that we can compute j-1 log-odds ratios of the form

$$\ln \left[ \frac{p_{kj}}{p_{kh}} \right] = z'_k (\beta_j - \beta_h) \quad (7)$$

The parameters are calculated by maximising the log-likelihood function for (6).

The estimates then show the change in the probability of being in a particular state in t+1 relative to some base state in t.

Another method to model changes across regimes was suggested by Spilerman (1972). The sample is again divided into subsamples according to the state the observation is in at time t. A binary dependent variable is created with the properties

$$y_{ij} = \begin{cases} 1 & \text{if } x_{t+1} = \bar{x}_j \wedge x_t = \bar{x}_i \\ 0 & \text{if } x_{t+1} = \bar{x}_k, k \neq j \wedge x_t = \bar{x}_i \end{cases} \quad (8)$$

The definition means that a subset of the population is created consisting of all observations that are in a specific state at the start of the period. In this subset, every observation is denoted as 1 if it moves from state  $i$  to  $j$  and zero for all other movements. Spilerman suggested using OLS regressions; however, as we know, standard OLS regression leads to heteroscedastic standard errors and to values greater than one or less than zero for binary dependent variables. These problems can be avoided by using a logit regression. The elements of the transition matrix then consist of logistic functions  $A(\beta'x)$ :

$$P = \begin{pmatrix} y_{11} = \Lambda(\hat{\beta}'_{11}x) & \dots & y_{1m} = \Lambda(\hat{\beta}'_{1m}x) \\ \vdots & \ddots & \vdots \\ y_{m1} = \Lambda(\hat{\beta}'_{m1}x) & \dots & y_{mm} = \Lambda(\hat{\beta}'_{mm}x) \end{pmatrix} \quad (9)$$

Since the necessary condition for the maximum of the likelihood function is:

$$\frac{\partial \ln L}{\partial \beta} = \sum_{k=1}^{n_{i\bullet}} (y_{ijk} - \Lambda_k) x_k = 0 \quad (10)$$

and the vector  $x_i$  contains a constant term, it follows that

$$\sum_{k=1}^{n_{i\bullet}} y_{ijk} = \sum_{k=1}^{n_{i\bullet}} \Lambda_k \quad (11)$$

From the definition of  $y$  it follows that  $\sum_{i=1}^{n_{i\bullet}} y_{ijk} = n_{ij}$ , which implies:

$$\frac{\sum_{k=1}^{n_{i\bullet}} y_{ijk}}{n_{i\bullet}} = \hat{p}_{ij} = \frac{\sum_{k=1}^{n_{i\bullet}} \Lambda_k}{n_{i\bullet}} \quad (12)$$

This means that the average of the predicted probabilities from the regression is equal to the predicted transition probability for the whole population. As is clear, all probabilities of moving from one state to another have to add up to one for each starting state. Therefore, if we use a regression technique for each possible movement on its own, we are not taking this dependency into account explicitly. Thus, we estimate both the logit regressions as well as the multinomial logit regressions which, in turn, only give us the *relative* change in probabilities.

### 2.3 Stochastic kernel densities

In the previous discussion of the empirical approach, we assumed that the possible outcomes are discrete. For a continuous variable, any division into discrete states is necessarily arbitrary (Bulli 2001). In this case stochastic kernels can be used for evaluating the transition probabilities (Quah 1997). The stochastic kernel is a conditional kernel density estimate resulting in the conditional density function  $f(x_{t+1} = \bar{x}_j | x_t = \bar{x}_i) = p_{ij}$ . This function can be calculated, as usual, by dividing the bivariate kernel density estimate for  $x_{t+1}$  and  $x_t$  by the kernel

density estimate for  $x_t$ :  $f(x_{t+1} = \bar{x}_j | x_t = \bar{x}_i) = \frac{f(x_{t+1} = \bar{x}_j, x_t = \bar{x}_i)}{f(x_t = \bar{x}_i)}$  (Quah

2006, p 35). The result is a three-dimensional plot showing the conditional probabilities of being in a state in t+1 conditional on being in a certain state in t.

Having outlined our methodological set-up, we now turn to the empirical part of the paper. It proceeds as follows: after describing the data at hand (Section 3), we estimate the transition probabilities for discrete states (Section 4) and then use logit regression methodology to examine the business cycle impact (Section 5.1). Since the logit regression is statistically inaccurate, we check these results with the multinomial logit model in section 5.2. The results we will have attained by then are verified in section 5.3 by inspecting some of the stochastic kernel density estimates.

### 3. The data

For the following analysis we use the Bundesbank's corporate balance sheets statistics database (*Unternehmensbilanzstatistik*).<sup>3</sup> This is the largest database of non-financial firms in Germany. Its data were collected by the Bundesbank in the course of its rediscounting and lending operations. Credit institutions presented bills of exchange issued by non-financial firms to the Bundesbank. To verify the creditworthiness of a firm, the Bundesbank bills of exchange issued by non-financial firms were frequently presented to the Bundesbank by credit institutions. When a bill was presented for discounting, the creditworthiness of the issuing firm and all other firms that previously held this bill needed to be determined. In the case of default, liability for payment of the bill fell on any firm that had held the bill. By law, the Bundesbank could only accept bills backed by three parties known to be creditworthy. This procedure allowed the Bundesbank to collect a unique dataset of information stemming from the balance sheets and the profit and loss accounts of firms. Up to 60,000 annual accounts have been collected by the Bundesbank. Because of the creditworthiness requirements, the sample is not a random sample of German firms. This is illustrated by the fact that only 4% of the total number of enterprises in Germany is covered by the data set but about 60% of the total turnover of the corporate sector, resulting in underrepresentation of small firms (Stoess 2001). The latter fact also means that although the sample is non-random, it yet comprises of firms that are very important for the evolution of

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3 The data set has been used frequently and fruitfully for various scientific analysis. For more details regarding the data set, see Stoess (1998) and Deutsche Bundesbank (1998).

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6 German GDP.<sup>4</sup> It is noteworthy that all mandatory data collected for this data base  
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8 have been subject to double-checking by the Bundesbank's staff. Hence, for a  
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10 micro-data set, the data at hand should contain unusually few errors.  
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12  
13 Unlike previous studies, we were able to use data from 1971 to 1998 for most of  
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15 the analysis. In 1999, the introduction of the euro and the new refinancing  
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17 framework made the deals underlying the dataset less relevant. Therefore, we  
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19 have substantially fewer observations after 1998, and, thus, we omit this time  
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21 period in our analysis. Due to changes in the sector definitions, the dataset had to  
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23 be confined to the years 1971 to 1995 whenever industry dummies were used.  
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25 Since we are interested mainly in the pattern of real sales, we have relatively few  
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27 losses of data due to incomplete and inconsistent reporting. Real sales growth is  
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29 calculated for each firm by deflating the firms' sales with the deflator of real  
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31 GDP.<sup>5</sup> To take outliers into account we have employed a cut-off rate, i.e. a  
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33 fraction of +/- 50% growth rate is truncated from the data to take into account  
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35 mergers, for instance.<sup>6</sup>  
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45 4 This view is supported by the fact that the correlation coefficient between the  
46 GDP growth rate and the mean growth rate of the firms covered in the sample  
47 is about 0.89. Therefore, the following analysis should be interesting despite  
48 the underrepresentation of small firms. Caution is warranted with respect to  
49 extending the results beyond the enterprises covered in the sample.  
50

51 5 One might argue that each sector should be deflated with its respective deflator.  
52 With the exception of only some sectors, e.g. computer manufacturing, the  
53 sectoral deflators all move closely together; the GDP deflator hence appears to  
54 be a good approximation.  
55

56 6 The results also hold without any cut-off; we present the results with cut-off to  
57 show that they are not due to outliers. For a discussion of the cut-off with the  
58 present dataset see Döpke et al. (2005).  
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6 The next thing to consider is how to define the states for the firms according to  
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8 their real sales growth rate. One might choose an absolute criterion for the states  
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10 since we have restricted the range of possible values to the interval  $-50$  to  $50\%$ .  
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12 States such as  $-50$  to  $-40\%$ ,  $-40$  to  $-30\%$  and so on might be defined. The  
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14 problem with this definition is that distributional and within-distribution effects  
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16 are mixed. During a recession, the whole distribution moves to the “left”. This  
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18 means that many firms move from their original state to a lower state when the  
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20 states are defined as absolute values. The transition probabilities then would show  
21  
22 a lot of movement that is not within-distribution movement but a shift of the  
23  
24 distribution itself. Therefore states that move together with the distribution during  
25  
26 recessions have to be defined. Quantiles are natural candidates. By using  
27  
28 quantiles, we can disentangle the distributional shift (changing quantiles) from the  
29  
30 within-distribution movement (transition probabilities). Since the growth rate of  
31  
32 real sales is a continuous variable, the choice of the quantiles is somewhat  
33  
34 arbitrary. As a baseline scenario, we choose deciles as states. Choosing smaller  
35  
36 quantiles would lead to a large number of results in the subsequent analysis,  
37  
38 making interpretations difficult. To check for robustness we have performed the  
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40 same analysis for quintiles as well. The results for the deciles are confirmed by  
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42 the quintile definition of states.<sup>7</sup>  
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58 7 The definition of states also makes the analysis more robust. This would  
59 explain why using no cut-off does not change the results, as mentioned earlier.  
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#### 4. Descriptive cross-sectional results

Using a 50% cut-off, i.e. dropping all observations with absolute real sales growth rates above 50%, the deciles were calculated for each year. Figure 2 shows the evolution of the real growth rate of sales deciles over time.

Not surprisingly, the deciles move during business cycles, having lower values during recessions, examples being 1975, 1982 and 1993. As was described in the last section, each decile is regarded as a possible state for each firm. For every year each firm is assigned a state and from these assignments the transition probabilities are calculated for all year pairs. Conditional on the present state, we obtain probabilities of being in one of the ten possible states in the next year.

— Figure 2 about here —

In Figure 3 these conditional probabilities are plotted. Each single graph shows the transition probabilities conditional on the present state. In other words, if the graph is entitled “1.decile” the present state is the first decile, the x-axis shows all ten possible states next period. Furthermore, the y-axis measures the probability of moving from the first decile to another next year or staying in the same decile, i.e. each curve represents one row of the transition matrix for a given year.

A clear pattern emerges: for the lowest and highest deciles a u-shaped curve emerges irrespective of the year under review. The less extreme middle deciles show a clear hump-shaped pattern. Those patterns mean that firms with extreme growth rates are more volatile than firms with “normal” growth rates.

— Figure 3 about here —

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6 A look at the first decile graph in figure 3 shows us that the probabilities of  
7 staying in the first decile and moving to the tenth decile are the largest. This  
8 means that either the firm stays in the first decile, i.e. the firm will shrink also in  
9 the next period, or it will make a big jump and grow at an exceptionally fast rate.  
10  
11 The latter fact alone is not so surprising. When a firm is hit by a large negative  
12 idiosyncratic shock, it will experience a large negative real sales growth. Once it  
13 manages to return to old real sales levels, in the next period it will necessarily  
14 grow at a faster absolute rate than the rate by which it shrank the previous period -  
15 just by reaching the pre-shock level of real sales.  
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27 — Figure 4 about here —  
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30 The pattern for the first decile could therefore just be a statistical artefact.  
31 Interestingly, the pattern of either staying in the same state or making a big  
32 adjustment is also present in the tenth decile graph. Normally, one would expect  
33 firms entering a new market with exceptional growth potential to display high  
34 growth rates. After some time, the market becomes mature and the growth rates  
35 drop back to “normal” levels. In other words, one would expect a regression to the  
36 mean process. The transition probabilities for the first and tenth decile suggest a  
37 different story. Firms with extreme growth rates are extremely volatile, having  
38 high probabilities of staying in their extreme state or making a turnaround to the  
39 other extreme. Together with the hump-shaped pattern for the middle deciles, this  
40 suggests a two-class firm society. Firms with medium growth rates have high  
41 probabilities of staying in their respective state or making medium shifts to  
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neighbouring states. The other class of firm has extreme growth rates and highly volatile shifts of growth rates from one extreme to the other.

Figure 4 additionally considers whether there is a link between the sizes of the firm and the states, i.e. the average growth rates of the firms. The figure shows the average size of firms in each state measured by the level of real sales. We see that the average size is hump-shaped, i.e. highest for the middle states, peaking at the sixth and seventh states. In Figure 1, those are the deciles with “normal” growth rates between 0 and 10%. The extreme and volatile deciles have lower average sizes than the middle decile firms. This finding is in line with several analyses in the industrial organisation literature where an inverse relationship is found between the growth rate and the size of the firm as well as between the standard deviation of the growth rates of firms and the firm size (Sutton, 1997).

What is also apparent from Figure 5 is that the transition probabilities vary widely over the years. A  $10 \times 10$  transition matrix contains 100 elements and is therefore not easy to interpret, especially when comparing matrices from different years. One method is to use mobility indices to condense the information obtained from a transition matrix. One index, proposed by Shorrocks (1978, p. 1017), is

$$\hat{M}(P) = \frac{n - \text{trace}(P)}{n - 1}. \quad (13)$$

The index has a value of one for perfect mobility and zero for no mobility at all. The mobility index, together with the growth rate of real GDP, is shown in Figure 5.

— Figure 5 about here —

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6 The mobility index (indicated by the line with the cross as a symbol) is very high  
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8 and fluctuates around 0.95-0.96, indicating high mobility. During the first half of  
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10 the respective time period, the mobility index is pro-cyclical, while for the second  
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12 half a counter-cyclical pattern emerges. The simple mobility index therefore  
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14 shows no clear pattern over the business cycle. In the next section we take a closer  
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16 look at the single transition probabilities.  
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## 19 20 21 22 **5. Business cycle impact on transition probabilities** 23 24 25

### 26 27 **5.1 Results from transition probabilities and Markov chains** 28

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30 To gain insights into the behaviour of firms during business cycles, we use the  
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32 logit regression method introduced in part 2. We are interested in how business  
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34 cycle conditions influence the behaviour of transition probabilities and therefore  
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36 include the first difference of the growth rate of GDP as a regressor.<sup>8</sup> From the  
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38 industrial organisation literature, it is well known that the size and age of firms  
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40 affect their growth rate. We therefore include the absolute value of real sales as a  
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42 measure of firm size as a regressor. Unfortunately, the data set does not include  
43  
44 the age of firms. The discussion in the preceding section illustrated that the  
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46 behaviour of firms with extreme growth rates differs markedly from that of firms  
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48 with medium growth rates. This might be due to some sectors being more volatile  
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57 <sup>8</sup> First estimations with the present and lagged GDP growth rate showed that the  
58 first difference is the appropriate variable. The results are available upon  
59 request.  
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than other sectors. For this reason, we included a set of sectoral dummies as independent variables. The regression equation therefore looks like this:

$$Y_{it} = \Lambda[\alpha + \beta_1(\Delta GDP_t - \Delta GDP_{t-1}) + \beta_2 z_{it-1} + \sum_j \beta_j D_j + \varepsilon_{it}] \quad (14)$$

$Y_{it}$  is the binary dependent variable stating that a firm  $i$  at time  $t$  is in a certain state or not,  $\Delta GDP_t$  is the growth rate of real GDP at time  $t$ ,  $z_{it-1}$  is the value of real sales of firm  $i$  at time  $t$ .  $D_t$  is the sectoral dummy taking the value one if firm  $i$  belongs to sector  $j$  and zero otherwise.

The estimation is conducted by a logit regression, as explained in section 2. Since both heteroscedasticity and autocorrelation were present in the data, we have calculated consistent standard errors.<sup>9</sup> The regressions have been run for every possible dependent variable, i.e. one regression has been run for the variable staying in state one, another for the variable moving from state one to state two, and so on. The result for the coefficients of the differenced GDP growth rate is shown in Figure 6. Each single graph shows the value of the coefficient moving to the state indicated on the x-axis depending on the present state, which is indicated by the title of each graph. The lines around the dots represent a two-standard-error band around the coefficients.

— Figure 6 about here —

The outcome reveals an interesting pattern. For the firms in the first three states, i.e. firms with low growth rates, a boom, that is a positive change in the GDP

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<sup>9</sup> As a check for robustness we also used other model specifications such as OLS, Fixed Effects, Population-Averaged Logit with robust standard errors, etc. The

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6 growth rate, increases the probability of moving to a higher state, i.e. to a state  
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8 with higher growth rate, and reduces the probability of staying in the original  
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10 state.

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12 Interestingly, for the middle states the probability of moving to higher states as  
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14 well as to lower states is increased - implying that a boom phase is not necessarily  
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16 a phase of improvement for firms with medium growth rates but might, in fact,  
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18 lead to worse performance. This is particularly the case for firms in higher states  
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20 where the probability of moving to lower states, especially for moving to state  
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22 one, is positively affected by business cycle conditions.  
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27 This suggests that booms increase the mobility of firms offering both  
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29 opportunities for improvement as well as risks for performance. The reverse is  
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31 true for recessions, of course, meaning that recessions lower mobility.<sup>10</sup> One must  
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33 bear in mind, as was shown in Figure 1, that the deciles of growth rates  
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35 themselves move in accordance with business cycle conditions. This means that,  
36  
37 during a recession, the whole distribution of the growth rates of real sales shifts to  
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39 the left. The movement within the distribution is reduced. During the upswing the  
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41 distribution shifts to the right and the within-distributional movement is higher  
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43 than during the recession.  
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50 — Figure 7 about here —  
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54 results were all robust with respect to the different model specifications and are  
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56 available from the authors on request.

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58 <sup>10</sup> Since the coefficients are significant for most movements, it is clear that the  
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60 transition probabilities are indeed time-varying. This justifies our assumption  
of non-homogeneity.

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In Figure 7 the coefficients of the impact of the firms' size (measured in terms of the level of real sales) on transition probabilities are shown. A general pattern of convergence emerges: the larger the firm, the more likely it is to reside and remain in a medium decile. This finding is a standard result in industrial organisation literature showing that the discretisation of the continuous real sales growth at least can replicate other findings.<sup>11</sup>

## 5.2 Results from multinomial logit regressions

As mentioned in section 2, the results presented above do not ensure that the probabilities sum up to one and are, therefore, just approximations. We therefore present in Figure 8 a regression analysis with the multinomial logit model. The same set of regressors, with differenced GDP growth, has been used, and again consistent standard errors have been calculated. The graph is similar to the previous ones except for one feature: the number of states in  $t+1$  excludes the base state (the state in  $t$ ) since we only have results for the relative but not absolute change in the probabilities.

— Figure 8 about here —

Comparing Figure 8 with Figure 6, we see that the general pattern for the coefficient is the same for all graphs except for the 10<sup>th</sup> decile. Here, we have the problem that the multinomial logit model only shows relative changes. Since we know that the probabilities of a relative decline in all states in Figure 8 all have to

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<sup>11</sup> To take into account a possible endogeneity of real sales we have checked, whether taking into account the lagged value alters the results qualitatively,

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6 add up to one, the 10<sup>th</sup> decile graph means that the absolute probability of staying  
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8 in state 10 must have increased (contradicting the result in Figure 6). This  
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10 absolute rise in the probability of staying in state 10 means that we cannot say  
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12 whether the probability of moving to state 1 increases or decreases absolutely  
13  
14 while declining relative to state 10.  
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18 The result for the 10<sup>th</sup> decile in Figure 6 therefore is not robust, while the “right”  
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20 result is not interpretable in terms of absolute change. The rest of the graphs are  
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22 consistent, which leads us to the conclusion that the results in the previous section  
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24 show us the correct development with respect to their absolute change.  
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### 29 **5.3 Results from stochastic Kernel densities**

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31 As a final check of robustness, the results for the stochastic kernel density  
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33 estimates<sup>12</sup> are presented in Figure 9. The upper part of the graph shows the 3D  
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35 plot of transition probabilities and the corresponding contour plot for the boom  
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37 year of 1991. In the lower part, the respective graphs for the recession year of  
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39 1993 can be seen. Both graphs indicate that the extreme growth rates are indeed  
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41 more volatile than the middle growth rates. Comparing both graphs, we see that  
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43 extreme positive growth rates have a higher probability during recessions of  
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45 moving to negative growth rates while the opposite holds for extreme negative  
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47 growth rates.  
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53 — Figure 9 about here —  
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56 which is not the case. Details are available upon request fro the authors.

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58 12 For the estimation a Gaussian kernel was used. The bandwidth was selected  
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60 according to the Silverman bandwidth selection criterion (Silverman 1986).

## 6. Conclusions

We analyse stylised facts for Germany's business cycle at the firm level. Based on longitudinal firm-level data from the Bundesbank's balance sheet statistics covering, on average, 55,000 firms per year from 1971 to 1998, we estimate transition probabilities of a firm in a regime of a given real sales growth switching to another regime in the next period. We find that these probabilities depend on the business cycle position.

Two findings emerge from our analysis. Firstly, extreme states are prone to extreme movements across the states, i.e. firms with high absolute growth rates are more volatile than firms with medium growth rates; this result is confirmed by standard industrial organisation literature. Secondly, the change of business cycle and not the business cycle condition itself has a marked influence on the firms' within-distribution dynamics. Firms with low growth rates have a better chance of improving their position during changed business cycle conditions, while firms with high growth rates face an increased risk of decline. Firms with medium growth rates face both risks as well as chances.

These results are important improvements over the previous analysis (Döpke et al. 2005), which concentrated on the movement of percentiles rather than on the movement of firms themselves. Two important questions for further research emerge. The first question is that of causality.<sup>13</sup> The pattern of movements across

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<sup>13</sup> We estimated Granger causality tests for the transition probabilities and the differenced GDP growth rates but did not obtain a significant result. Progress towards answering this question is possible using quarterly data, which were not available for the present analysis.

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6 states could be the result of a macroeconomic shock affecting firms in a different  
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8 way. According to this interpretation, the movement of firms is the result of the  
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10 movement of GDP. The other possible explanation would reverse the causality. In  
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12 this case, idiosyncratic shocks, through some sort of spillover effect (e. g. credit  
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14 rationing due to bad debts for the banking sector, as proposed in Delli Gatti et al.  
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16 2003), would cause a movement of GDP. In this case, it is the differing  
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18 movements of firms which drive the GDP. A third explanation might be a non-  
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20 linear combination of both approaches. The distributional position of the firms is  
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22 more persistent during downturns and more volatile during upturns. One might  
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24 reason that the downturn is then due to a traditional macroeconomic shock while  
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26 the upswing is driven by idiosyncratic shocks since firms are affected in different  
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28 ways.  
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34 Another important question is the question of regression to the mean which is  
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36 usually found in industrial organisation literature. As has been shown in this  
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38 paper, the business cycle conditions affect the position of firms within the  
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40 distribution. Therefore, the business cycle effects should not be neglected when  
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42 dealing with questions of convergence between firms. It might well turn out that  
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44 the different reactions of firms during upswings might explain more about the  
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46 convergence process than the variables of size and age traditionally used.  
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Figure 1: Focus on individual firms rather than on distribution

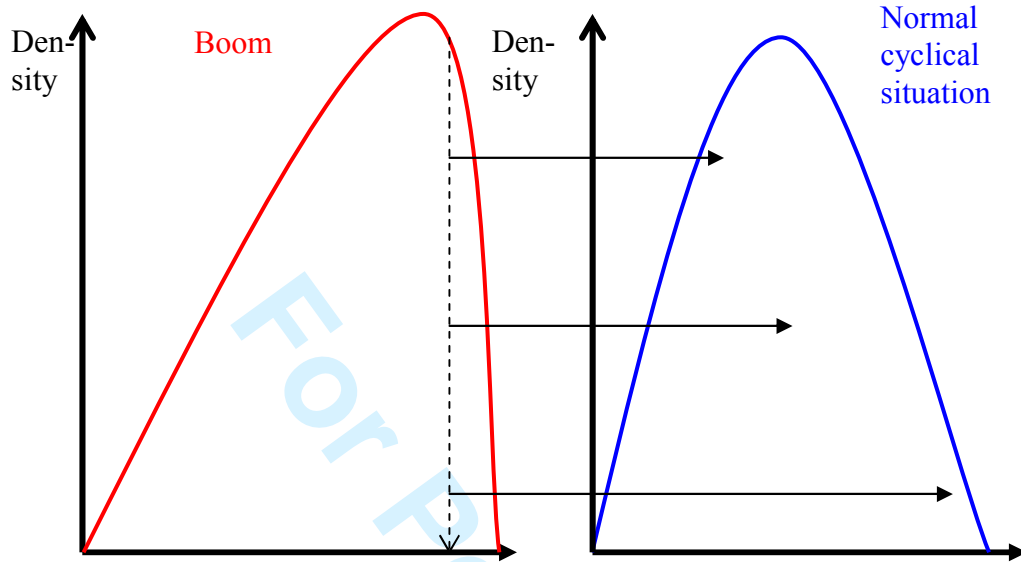
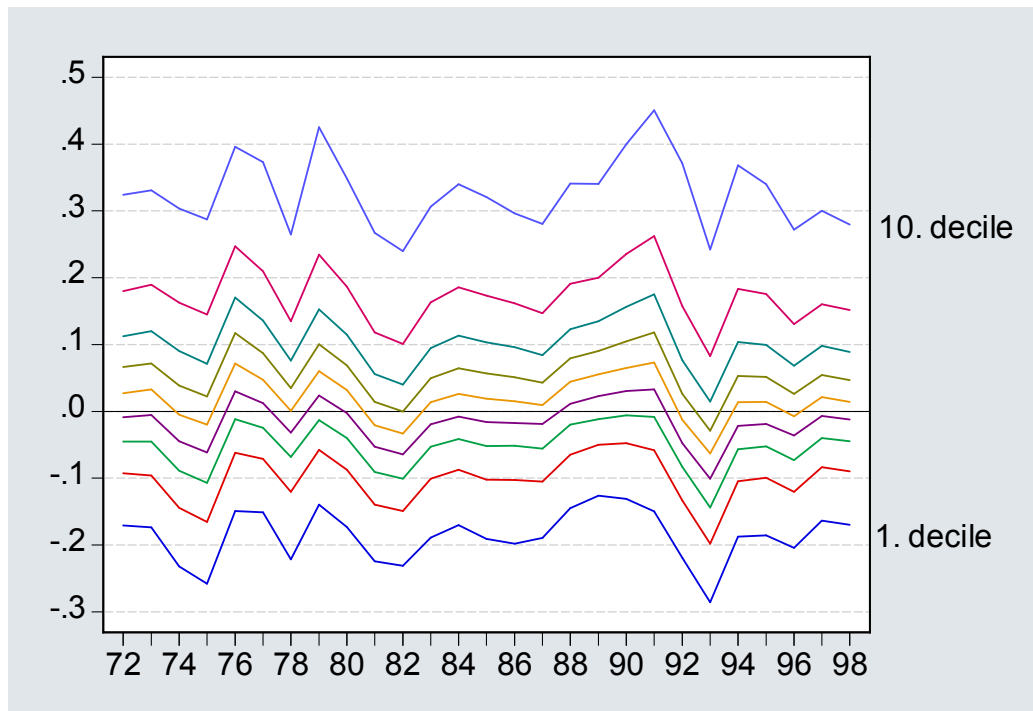
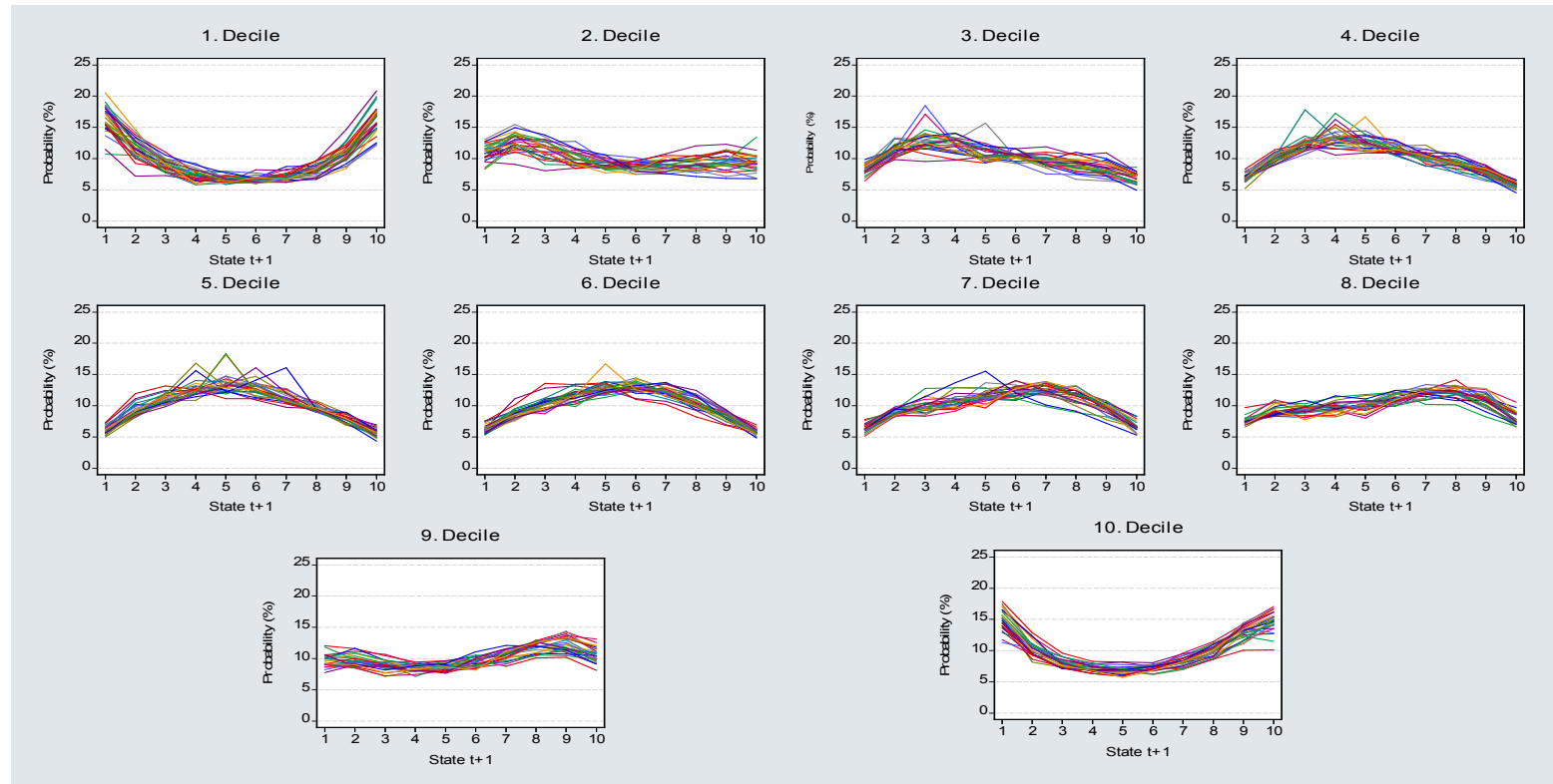


Figure 2: Year-on-year change in real sales, deciles, 1972 to 1998



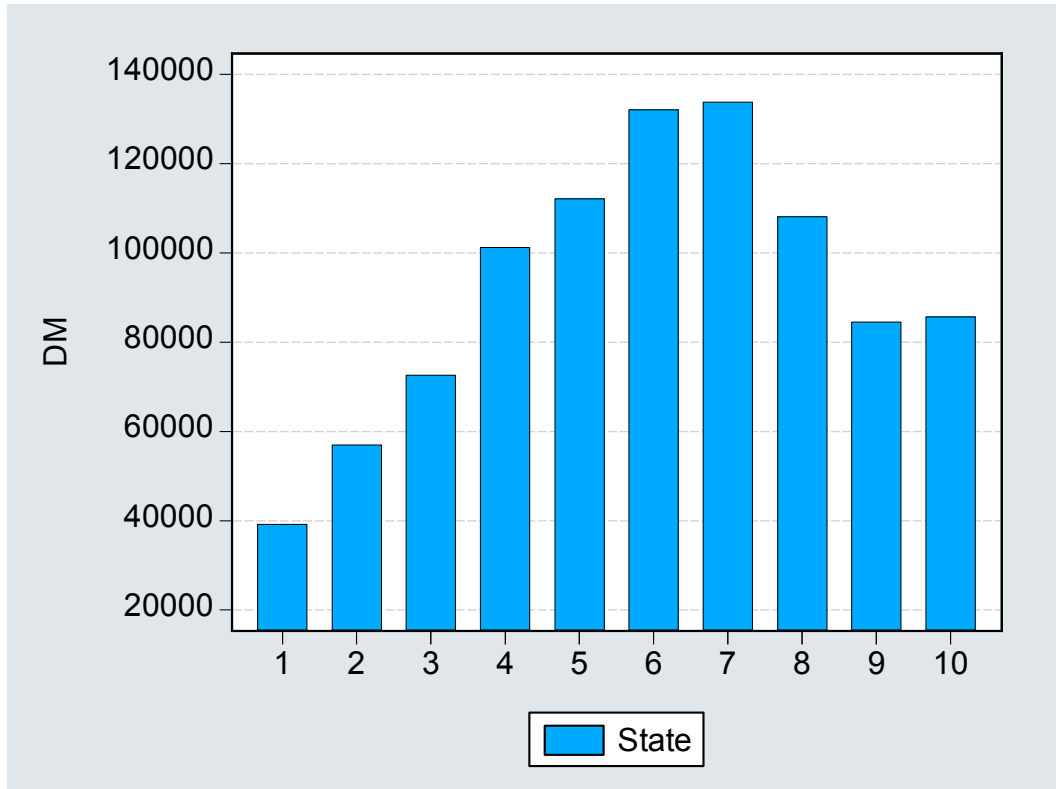
Note: The 10<sup>th</sup> decile denotes the firms with the largest increase in real sales.

Figure 3: Cross-sectional transition probabilities



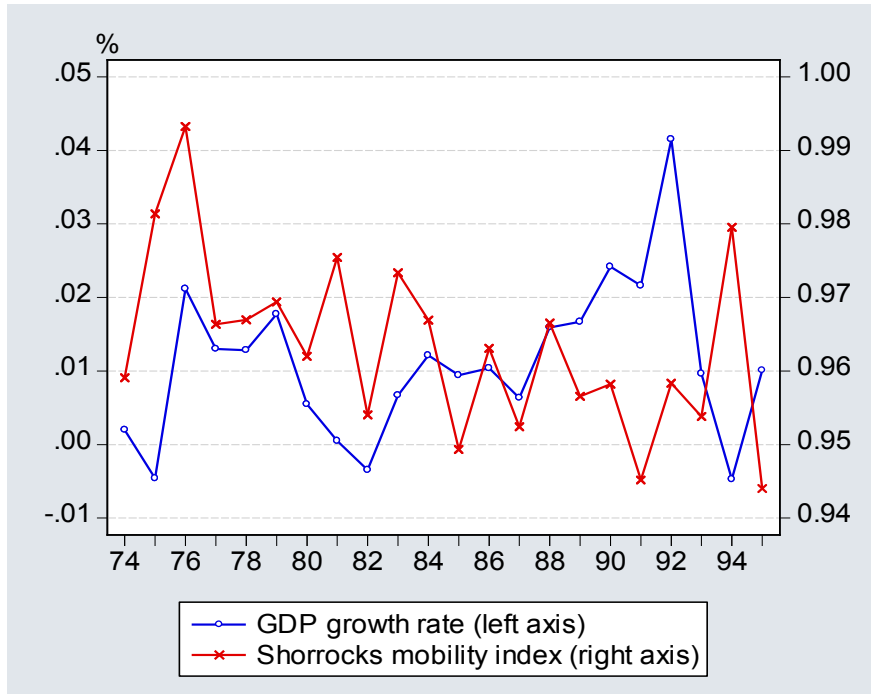
Note: the lines refer to the years under investigation.

Figure 4: Mean size of firms by states



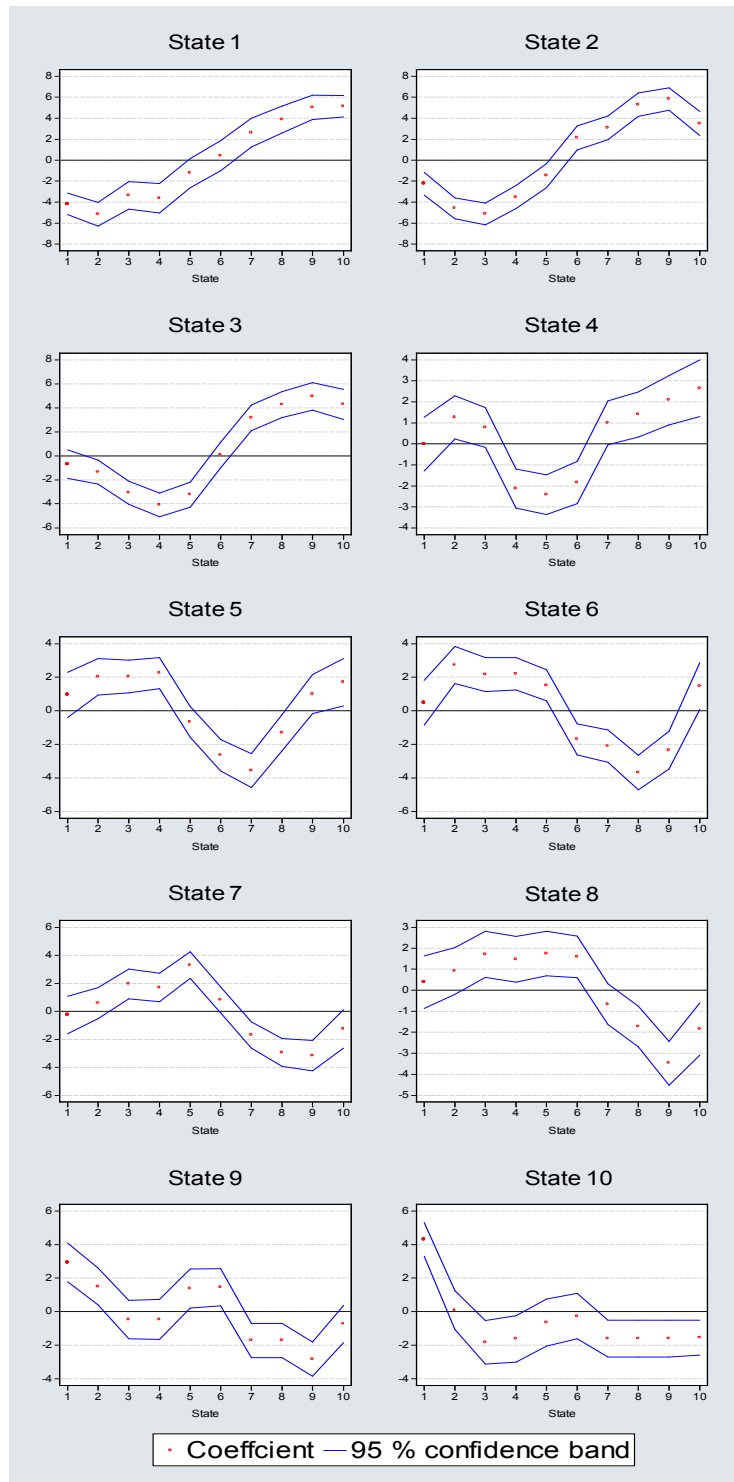
Note: "1" denotes the state of the firms with the lowest growth rate of real sales while "10" denotes the state of the firms with the highest growth rate of real sales.

Figure 5: Shorrocks' mobility index and GDP growth, 1973 to 1995



Notes: see main text for details.

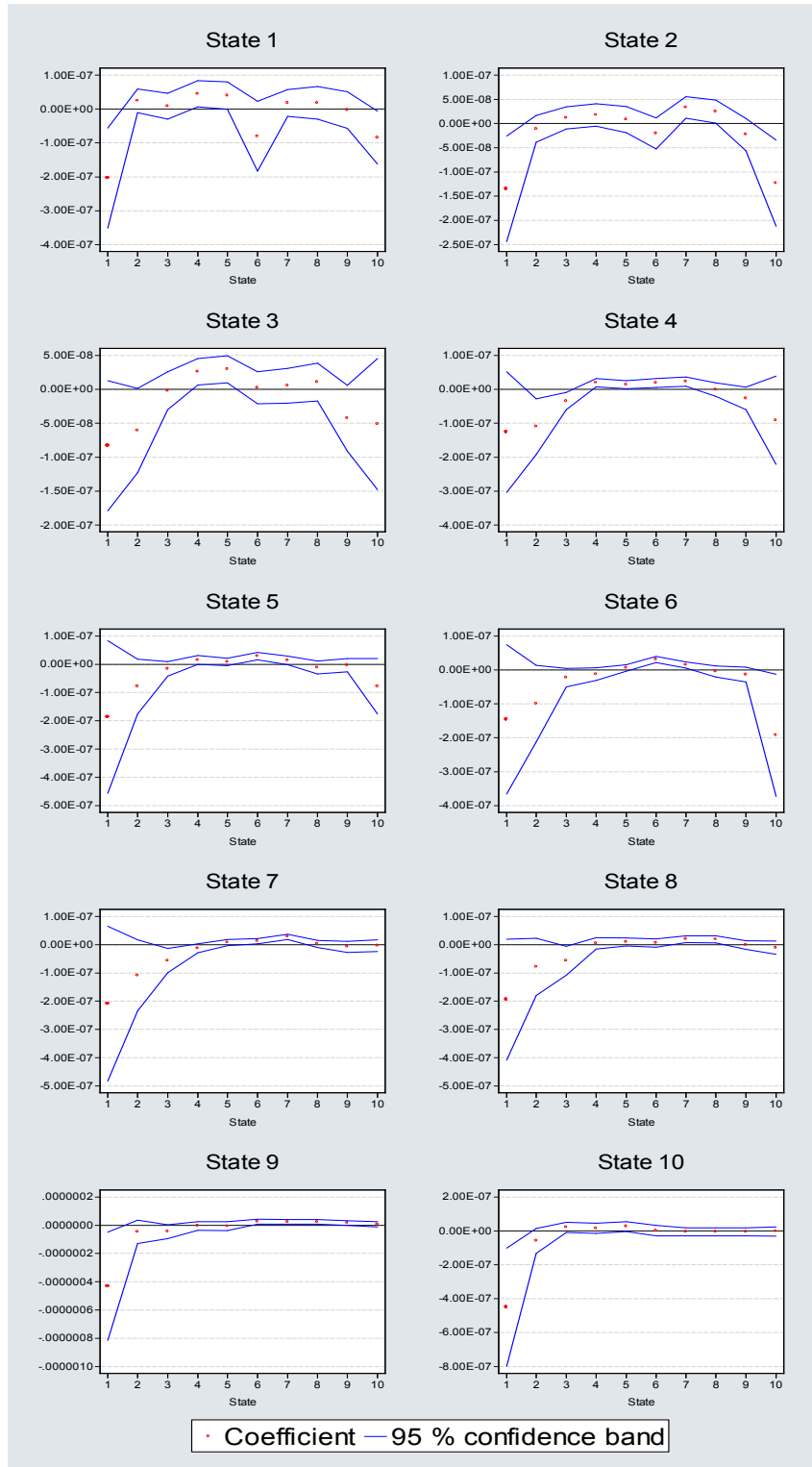
Figure 6: Influence of business cycle conditions on transition probabilities



Note: “1” denotes the state of the firms with the lowest growth rate of real sales while “10” denotes the state of the firms with the highest growth rate of real sales.

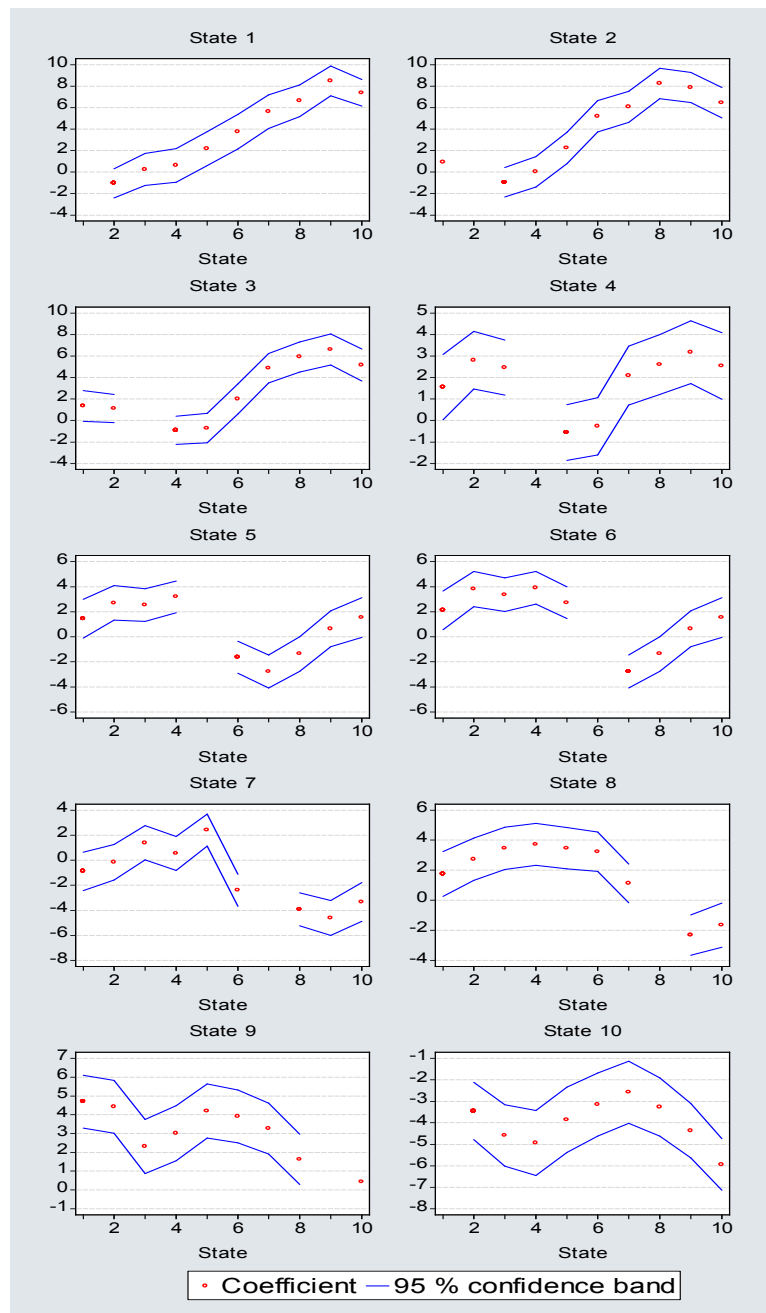


Figure 7: Influence of the firms' size on transition probabilities



Note: "1" denotes the state of the firms with the lowest growth rate of real sales while "10" denotes the state of the firms with the highest growth rate of real sales.

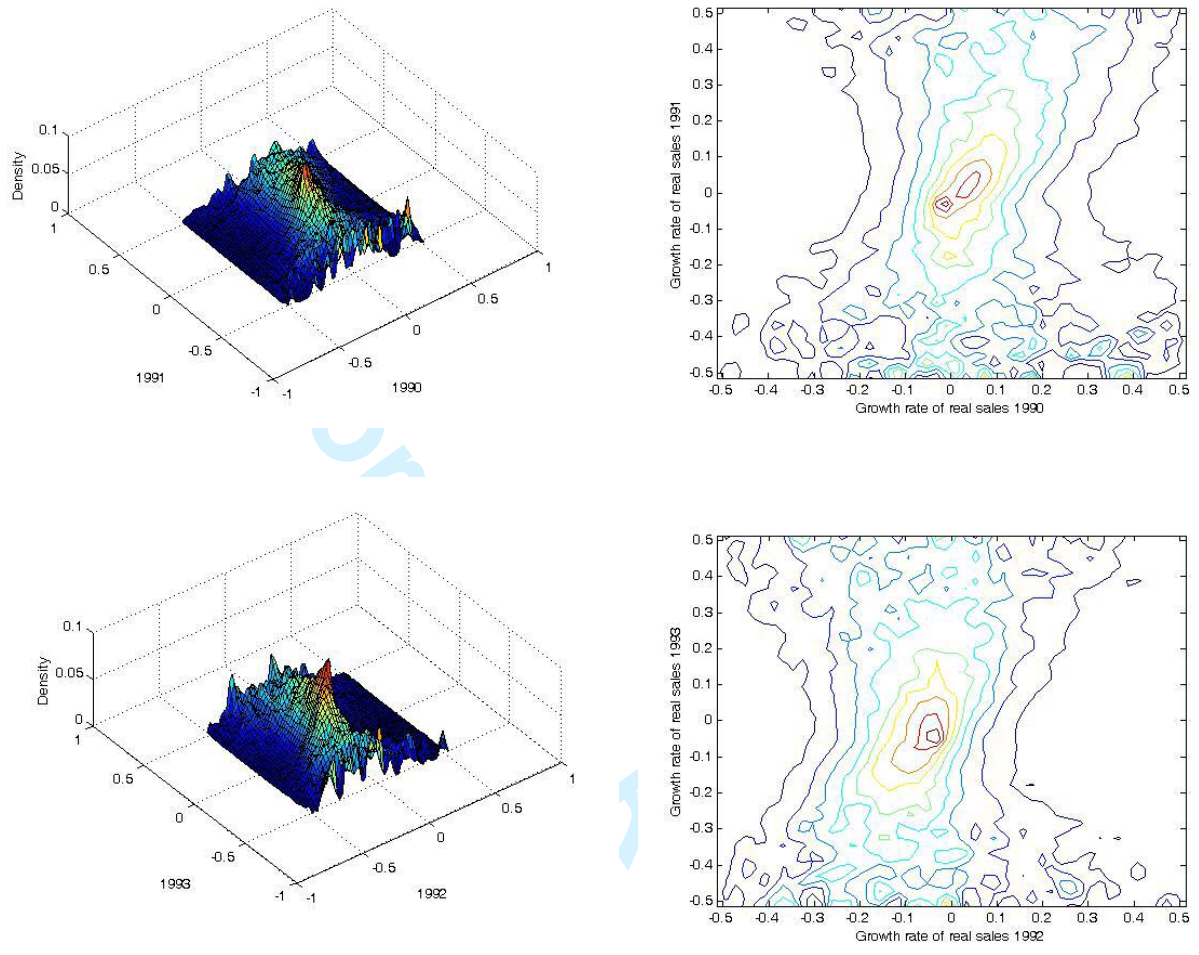
Figure 8: Results from multinomial logit regressions



Note: “1” denotes the state of the firms with the lowest growth rate of real sales while “10” denotes the state of the firms with the highest growth rate of real sales.

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Figure 9: Results from stochastic Kernel densities



**Appendix table: Descriptive statistics of the firms by state**

State	Sales (nominal)	Sales (real)	Freq.
1	30799.0 [368522.9]	39854.961 [463760.1]	92363
2	43839.4 [556983.4]	56941.7 [702584.5]	102719
3	55540.5 [588662.9]	72288.5 [758719.1]	105066
4	78173.7 [ 923295.1]	100785.4 [1127013.7]	105476
5	85765.5 [896151.8]	110838.9 [1108203.5]	105476
6	101097.2 [1286330.5]	129602.65 [1513562]	105662
7	100527.2 [1187542.1]	130986.3 [1456198.7]	104941
8	80657.6 [964734.0]	106069.3 [1207859.3]	102604
9	61365.4 [661377.7]	82217.5 [909628.3]	98650
10	46229.9 [530349.1]	62462.4 [692892.]	87775

Note: standard deviations in brackets.