

## Real option value and random jumps: application of a simulation model

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## Real option value and random jumps: Application of a simulation model

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ABSTRACT. This paper studies how sensitive real option valuations are to incorrect assumptions about the stochastic process followed by the state variables. We design a valuation model which combines Monte Carlo simulation and dynamic programming and provides an appropriate framework to evaluate the effect of estimation errors on both the value of real options and their critical frontier. Although the model is flexible enough to value American-type options contingent on a wide range of stochastic processes, we focus on the analysis of the effect of stochastic jumps. We apply our model to the valuation of an investment in the car parts industry documented in previous literature. Our results clearly show that underestimating this type of jumps might lead to substantial misjudgements in a firm's decision-making processes. For instance, it may lead to profitable projects being rejected when jump diffusion is low, or negative expanded net present value projects being accepted.

## I. Introduction

In the spring of 2000, Tom E. Copeland predicted that the real option valuation, henceforth ROV, would eventually replace the discounted cash flow (DCF) model in under ten years (Copeland, 2000). This forecast may seem an inconceivably short space of a time, yet it entails a process of change which has lasted over 30 years, beginning in 1977 with Stewart C. Myers' proposal. This thirty year period should be more than enough time for the adoption of a valuation technique which is theoretically superior to any other known model, even more so when other more costly innovations have successfully been implemented in a significantly shorter period of time.<sup>1</sup>

Among the reasons put forward for the limited application of ROV, Newton and Pearson (1994) pointed to the operational complexity of its valuation techniques, Myers (1996) hinted at a lack of understanding of its underlying philosophy on the part of managers, and Lander and Pinches (1998) referred to the failure of mathematical tools to fulfil certain requirements. An even greater problem needs to be considered, namely the paradoxical lack of flexibility arising from the lack of any general model –no matter how complex its understanding or application may be– to be employed in the valuation of, if not all, then at least the more common real options.

Whereas the DCF formula may be applied directly to virtually all investment opportunities, ROV is bereft of any similar formula. What is more, ROV comprises a combination of analytical formulae and numerical techniques, each of which is appropriate to the valuation of a specific option on a particular underlying asset. Not

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<sup>1</sup> In addition to its theoretical superiority, a number of papers have found empirical insights regarding the relevance of real option in market values. Such is the case of Paddock *et al.* (1988); Berger *et al.* (1996); Danbolt *et al.* (2002) or Andrés-Alonso *et al.* (2005); among others.

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3 even the binomial model, perhaps the most flexible of all traditional models for  
4 valuing options, allows for direct treatment of stochastic processes other than  
5 continuous Brownian-type motions, or multiple sources of uncertainty.  
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10 This paper analyses the consequences arising from the use of ROV techniques  
11 inappropriate to the nature of the stochastic process for the state variable. We  
12 specifically investigate how sensitive values of American-type real options are to the  
13 effect of unusual discontinuities of the state variable. Valuation is approached by a  
14 Monte Carlo simulation model which is inspired by the proposal of Grant, Vora and  
15 Weeks (1996 and 1997) and Ibáñez and Zapatero (2004) for financial options.  
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24 Our model considers the specific nature of real investments which requires  
25 simultaneously determining the values of both real options and their underlying  
26 assets, from the state variable on which its cash flows depend. The main contribution  
27 of our model is that it directly estimates the values of the state variable that define  
28 the optimal exercise frontier, and thereby provides a clear decision rule to the holder  
29 of the option. Namely, comparing observed values of the state variable and the  
30 critical value at each exercise date it is possible to know whether exercise is  
31 recommended. For our analysis purposes, this feature represents an advantage  
32 compared to other simulation-dynamic programming methods, such as the powerful  
33 regression based procedure proposed by Longstaff and Schwartz (2001).<sup>2</sup>  
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48 We apply our model to the valuation of a foreign direct investment (FDI)  
49 undertaken by a Tier-One multinational supplier of automobile components, whose  
50 real options are well outlined in Azofra *et al.* (2004). The results of our valuation  
51 highlight the important consequences arising from mistakes in the estimation of the  
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59 <sup>2</sup> Our model's advantage decreases with the number of early exercise opportunities, since determining  
60 each critical value requires simulating new paths for the state variable, and hence the model costs -in  
terms of time and computing resources- grow exponentially.

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state variable stochastic process. Specifically, these findings underline the existence of significant differences in the value of call and put options when the foreseen evolution of the state variable advocates consideration of random discontinuities. The nature and scale of the errors depend both on the influence of the continuous motion as well as on jump diffusion, the greatest differences being observed when the continuous influence is lower and the jump diffusion is higher. Excluding discontinuities in these cases entails the appearance of bias in the evaluation which may lead to erroneous investment decisions being taken.

The rest of the paper is structured as follows. The next section addresses the problem of evaluating American options on discontinuous stochastic processes. Our valuation proposal is outlined in section three. Section four deals with the analysis of the valuation results for options to expand and contract embedded in an investment case in the car parts industry. The final section concludes and discusses relevant implications.

## **II. Valuing American real options contingent on discontinuous processes**

One challenge currently facing the real option approach is the development of a general model to enable valuation of a wide range of both call and put options, involving multiple exercise dates depending on the evolution of stochastic processes of an extensive nature.

The chance to exercise options at more than one future date is probably one of the most common features of corporate investment. For this reason, the same arguments put forward by the advocates of ROV when criticising DCF (McDonald and Siegel, 1986; Lee, 1988; Pindyck, 1991), are now applicable to “European” ROV. It is just as difficult to imagine a corporate investment opportunity whose

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3 exercise may not be postponed in the least, as it is to imagine a growth or  
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5 abandonment option with only one future date.<sup>3</sup>  
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8 As for the stochastic evolution of the state variable, most ROV models  
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10 assume geometric Brownian processes. These are processes which have been widely  
11  
12 used to describe the evolution of financial asset and commodities prices, but are hard  
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14 to upscale to other types of state variables on which real options may depend.  
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16 Variables such as demand, output prices or input costs, may adapt better to mixed  
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18 processes, which combine continuous Brownian motion with random discontinuous  
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20 jumps. Depending on each variable, these discontinuities could be brought about by a  
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22 change in customers' preferences, technological progress, or competitors' actions.  
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27 The occurrence of random jumps complicates –and indeed may render  
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29 impossible– valuation of American options by means of traditional solving  
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31 techniques: closed-form solutions, analytical approximations and lattice approaches.  
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33 Merton (1976) derives the analytical valuation formula for the European option when  
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35 the price of the underlying asset follows a mixed process, comprising a continuous  
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37 geometric Brownian motion contingent on discrete Poisson jumps. Merton's  
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39 proposal enables us to value options whose exercise is restricted to the expiration  
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41 date, but may not be used in the case of American options.  
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46 Analytical approximations together with the most common numerical  
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48 procedures –the binomial model (Cox, Ross and Rubinstein, 1979) and finite  
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50 differences (Brennan and Schwartz, 1978)– allow us to consider the possibility of  
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52 early exercise, although their computational application proves more complex when  
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<sup>3</sup> This does not reduce the advance of the ROV approach over traditional DCF models, but highlights interest in analysing the value to postpone the exercise of real options when possible. See Vandenbroucke (1999) for a comparison of the NPV approach and ROV approximations.

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3 involving stochastic processes other than the geometric Brownian family and  
4 multiple sources of uncertainty.  
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8 By contrast, models based on Monte Carlo simulation (Boyle, 1977) may be  
9 applied to the case of multiple state variables regardless of the type of stochastic  
10 evolution to which they are linked. The greater flexibility inherent in this method  
11 revolves around the fact that valuation is undertaken by directly determining the  
12 process of the underlying asset, meaning that the partial differential equation (PDE)  
13 need not be solved. The drawback is that the direct application of Monte Carlo  
14 simulation *is not* suitable for valuing American style options. At least this is what  
15 was believed to be true until fairly recently.<sup>4</sup>  
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27 Restrictions in simulation valuation are due to the very nature of this  
28 technique. Since exercising an option at a specific date prevents its subsequent  
29 exercise at a later date, the strategy which dictates optimal exercise of an American  
30 option relies not only on the prior evolution of its underlying variables, but also on  
31 their future values. Considering future events can only be performed through  
32 procedures involving *backward induction*, such as dynamic programming which  
33 resolves binomial and trinomial trees or finite difference procedures which do  
34 likewise with partial differential equations. By contrast, the Monte Carlo method is a  
35 *forward induction* process, generating future values for the variables based on  
36 previous values and therefore offering a suitable technique for assets whose cash  
37 flows at a given moment do not depend on subsequent events, as is the case of  
38 European options (Cortazar, 2001).  
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58 <sup>4</sup> A good example of this view is the second edition of Hull's handbook on financial options (Hull,  
59 1993), which on page 334 explains that "one of the drawbacks of the Monte Carlo approach is that it  
60 may only be used for European style derivatives". In this same sense, Hull and White (1993) postulate  
that "Monte Carlo simulation cannot deal with early exercise since there is no way of knowing whether  
this is optimal when a specific price is reached at a given moment".



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In order to overcome this restriction, recent research has proposed combining simulation with some backward induction procedure that may lead to a valuation model applicable to both European and American type options, whatever the number of state variables and the nature of the stochastic processes. The earliest attempt to apply simulation in American style options valuation may be found in Tilley (1993)<sup>5</sup>, who proposes a model for valuing financial options dependent on the stochastic evolution of a single state variable coinciding with its underlying asset. Tilley proposes sorting the simulated values of the underlying asset for each exercise date into groups or “bundles” for which a single value of keeping the option alive until the next period is assigned, as the mean of the continuation value of the whole of these paths.<sup>6</sup>

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Tilley’s approach has been followed by a growing number of papers that propose different combinations of simulation and backward induction procedures for valuing American-type financial derivatives. The resulting models approximate the early exercise frontier or conditional expectation function for the derivative. Prominent within this approach are the works of Barranquand and Martineau (1995) and Raymar and Zwecher (1997), who propose the use of a partitioning algorithm on the unidimensional space of the cash flows yielded by the option, rather than considering the multidimensional space of the underlying assets defined in Tilley (1993). Grant, Vora and Weeks (1996) as well as Ibáñez and Zapatero (2004) directly estimate the values of the state variables for which the value of keeping the

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<sup>5</sup> Some authors cite Bossaerts’ (1989) working paper as the earliest reference to the analysis of early exercise of American options through simulation.

<sup>6</sup> This procedure entails certain significant drawbacks, such as the need to store all the simulated paths –a time-consuming exercise– as well as the complexity linked to the sorting process when dealing with multiple sources of uncertainty.

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3 option alive until the following period matches the value of its immediate exercise at  
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5 each exercise date.  
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8 As an alternative proposal, Broadie and Glasserman (1997, 2004) and  
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10 Broadie, Glasserman and Jain (1997) advocate use of non-recombinatory simulated  
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12 trees<sup>7</sup> and stochastic meshes to determine two estimates of the option value, one  
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14 biased “upward” and another biased “downward”, both asymptotically unbiased and  
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16 convergent towards the certain value. Finally, Longstaff and Schwartz (2001) opt for  
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18 least square regressions as a method for approaching the expected value of keeping  
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20 the option alive at each decision point.  
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25 In the light of this type of proposal, recent corporate finance literature has  
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27 welcomed Monte Carlo simulation procedures for valuing real options. The  
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29 Barranquand and Martineau (1995) model and its subsequent development by  
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31 Raymar and Zwecher (1997) have been used to value American-type options where  
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33 the state variables evolve following conventional mean reverting and geometric  
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35 Brownian processes. Such is the case of Cortazar and Schwartz (1998) who resolve  
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37 optimal *timing* of oil reserves, and Cortazar (2001) who evaluates optimal operations  
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39 in a copper mine. Other papers apply the Longstaff and Schwartz (2001) algorithm in  
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41 valuing real options linked to patents and R+D projects (Schwartz, 2004; Miltersen  
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43 and Schwartz, 2004), dot-com companies (Schwartz and Moon, 2000; Schwartz and  
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45 Moon, 2001) and pharmaceutical companies (León and Piñeiro, 2003).  
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### 51 **III. The model**

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53 After reviewing previous literature, we address the problem of valuing real options –  
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55 both growth (call) as well as abandonment (put)– of limited duration,  $T^O$ , which may  
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<sup>7</sup> Unlike binomial and trinomial trees, the values that appear at each node are placed in the order in which they are generated and not following a hierarchic order.

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3 be exercised at one or more future points,  $\tau < 2\tau < \dots < N\tau = T^0$ , during the also  
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5 limited life of the underlying investment,  $T$ .<sup>8</sup> This investments yields cash flows  
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7 dependent on an uncertain variable,  $S_t$ , whose evolution over time we suppose to  
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9 follow a mixed process, comprising a geometric-Brownian type motion subject to  
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11 random jumps spread over a Poisson variable. This implies that the infinitesimal  
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13 variation of the state variable  $dS_t$  is given by:  
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$$16 \quad dS_t = (\alpha - \lambda k)S_t dt + \sigma S_t dz + (\pi - 1)S_t dq$$

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18 where  $\alpha$  and  $\sigma$  represent, respectively, the expected drift and volatility of the  
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20 continuous motion;  $\lambda$  is the mean frequency of discontinuous jumps per unit of time;  
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22  $(\pi - 1)$  and  $k$  are, respectively, the random variable measuring the size of the  
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24 proportional jump and its mean value;<sup>9</sup> and  $dz_t$  and  $dq_t$  represent stochastic Wiener  
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26 and jump processes, which we assume to be independent and characterised by their  
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28 usual expressions:  
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$$34 \quad dz = \xi \cdot \sqrt{dt}, \quad \xi \rightarrow N(0,1)$$

$$35 \quad dq = \begin{cases} 0 & \text{prob} = 1 - \lambda \cdot dt \\ 1 & \text{prob} = \lambda \cdot dt \end{cases}, \quad q \rightarrow \text{Poisson}[\lambda]$$

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37 Regarding the discontinuous variation, we assume each jump size to be  
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39 independent and  $\log(\pi)$  to follow a normal distribution with mean  $\mu_\pi$  and deviation  
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41  $\sigma_\pi$ , such that:  
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$$49 \quad k = E[\pi - 1] = \exp\left(\mu_\pi + \frac{\sigma_\pi^2}{2}\right) - 1$$

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51 Discontinuities are linked to “unusual” and significant events, which give rise  
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53 to upward and downward variations in the uncertain variable, while the continuous  
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60 <sup>8</sup> The equidistance ( $\tau$ ) of the exercise dates, derived from this formula, is assumed for clarification and explanation purposes only and does not condition the model in any way. Logically,  $T^0 < T$ .

process –the geometric Brownian motion– is linked to the idea of “normal” events. Following common practice, we suppose the direction of the jump to be unknown a priori and hence the effect of the jump in the trend to be zero, and therefore

$$\mu_{\pi} = -\sigma_{\pi}^2/2 \text{ y } k = 0$$

The method we propose to value both European and American style real options requires calculating, at each early exercise point,  $\tau < 2\tau < \dots < N\tau = T^O$ , the successive “critical” values  $S_{\tau}^*$ ,  $S_{2\tau}^*$ , ...,  $S_{N\tau}^*$  of the state variable. These critical values represent the exercise frontier, since for these values the payoff resulting from exercising the option is equal to the payoff of maintaining it until the next period. The exercise frontier is used to estimate both the option and underlying project values for a series of simulated paths. Solving the valuation problem is thus performed in two stages: i) an initial phase which consists of estimating the successive “critical” values by combining Monte Carlo simulation with dynamic programming; and ii) a second phase which involves determining the value of the option from the previous critical values and which merely requires the use of simulation techniques.

### *III.1. Stage one: Estimating the critical value frontier*

Assuming complete markets, the expression for the future equilibrium value for the state variable is the following:<sup>10</sup>

$$S_t = S_0 \exp \left[ (r - \delta - 0.5\sigma^2)\Delta t + \sigma z_0 \sqrt{\Delta t} + \sum_{i=1}^q \left( \sigma_{\pi} z_i - \frac{\sigma_{\pi}^2}{2} \right) \right]$$

where  $r$  and  $\delta$  represent, respectively, the continuous risk-free rate of return and the convenience yield,<sup>11</sup>  $z_0$  represents a standard normal random variable linked to the

<sup>9</sup> Hence, the mean growth rate caused by the discrete jumps is  $\lambda k$ .

<sup>10</sup> This expression derives from applying the risk-neutral valuation approach to the “twin” financial asset, which is perfectly correlated to the state variable.

continuous diffusion process;  $z_i$  are standard normal independent variables determining the size of each jump and, as mentioned,  $q$  reflects the number of discrete jumps generated by a Poisson distribution with frequency  $\lambda$ .<sup>12</sup>

Each of the paths of the state variable determines a pair of values for the underlying project contingent on the decision to exercise,  $V_{t,exercise}^i(S_t^i)$ , and to continue without exercise,  $V_{t,no}^i(S_t^i)$ , where the superscript  $i$  indicates the number of simulations performed. The value of the project when the option is exercised at  $t$  (with  $t \leq T^O$ ) is obtained adding the cash flow generated at that point,  $F_{t,exercise}^i$ , to the expected value of the project at the following period,  $E[V_{t+\tau,exercise}^i]$ , also taking into account the exercise price,  $X$ , obtained or paid for the sale or purchase of the project, depending on whether it is a put or a call option:

$$V_{t,exercise}^i(S_t^i) = \pm X + F_{t,exercise}^i + E[V_{t+\tau,exercise}^i] \exp(-r\tau)$$

where  $\tau$  is the time interval into which the lifespan of the option has been divided.

To calculate the value of the underlying project should the option not be exercised, we need to distinguish between the option expiration date and the remaining points at which early exercise is allowed. Hence, when  $t$  is the expiration date ( $t = T^O$ ), the value of the underlying is the present value of cash flows derived from expiration of the option without exercise,

$$V_{T^O,no}^i(S_{T^O}^i) = F_{T^O,no}^i + E[V_{T^O+\tau,no}^i] \exp(-r\tau)$$

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<sup>11</sup> Following Merton (1976) we assume the risk associated to the discontinuous jump of the state variable to be diversifiable. The risk-neutral simulation would then show a continuous modified drift,  $r-\delta$ , rather than the initial  $\alpha$ . This is the equivalent of subtracting from the continuous drift the risk premium of the corresponding asset (Trigeorgis, 1996: 102).

<sup>12</sup> The simulation of the number of discrete jumps at a time interval,  $\Delta t$ , is obtained from applying the Monte Carlo method to the accumulated probability function  $P[q \leq X]$ .

When  $t$  is a point prior to expiration,  $t < T^O$ , the project value is determined by considering the possibility of adopting a new contingent decision at a later point. The value of the underlying would be calculated from the expected value of the project at the following period –including optimal decisions taken up to expiration of the option– and the cash flow generated at that point.

$$V_{t,no}^i(S_t^i) = F_{t,no}^i + E[V_{t+\tau}^i] \exp(-r\tau)$$

We initiate the estimation of the critical values series by simulating  $M$  values of the state variable at the option expiration date,  $S_{T^O}^{i=1,2,\dots,M}$ , taken from the initial value,  $S_0$ , and according to the stochastic process assumed.<sup>13</sup> By definition, the critical value at the option expiration date  $S_{T^O}^*$ , is the one for which the value of the underlying, assuming immediate exercise of the option,  $V_{T^O,exercise}(S_{T^O}^*)$ , coincides with the value of non-exercise,  $V_{T^O,no}(S_{T^O}^*)$ .

Estimating these critical values requires simulating  $K$  paths of the state variable up to expiration of the underlying project,  $T$ ,  $S_{T^O+\tau}^{i,j}$ ,  $S_{T^O+2\tau}^{i,j}$ , ...,  $S_T^{i,j}$  for  $i=1, 2, \dots, M$  and  $j=1, 2, \dots, K$ , as Fig. 1 details:

**[Insert Fig. 1]**

Each of these paths enables us to estimate both cash flows derived from the exercise and non-exercise generated from the option expiration date up to the end of the project ( $T^O$ ,  $T$ ). Discounting each of these cash flows at the option expiration date,  $T^O$ , yields the corresponding contingent value of the investment,

<sup>13</sup> The simulation may begin at any moment and for any value of the state variable (Grant, Vora and Weeks, 1996). However, when dealing with American options, whose optimal exercise at each moment depends on future expectations, the first critical value to be calculated must correspond to expiration.

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$V_{T^O, decision}(S_{T^O}^i)$ , and the comparison of these  $M$  pairs allows us to pinpoint the required critical value,  $S_{T^O}^*$ .

Having calculated the critical value of the state variable at the option expiration date, we go back to the immediately prior exercise point,  $T^O - \tau$ , for which we repeat the process of estimating the critical value of the state variable,  $S_{T^O - \tau}^*$ . The procedure for determining  $S_{T^O - \tau}^*$  again requires generating a new set of  $M$  state variable values,  $S_{T^O - \tau}^{i=1, \dots, M}$ , from which other  $K$  paths are simulated up to the end of the project – values of  $S_{T^O}^{i,j}$ ,  $S_{T^O + \tau}^{i,j}$ ,  $S_{T^O + 2\tau}^{i,j}$ ,  $\dots$ ,  $S_T^{i,j}$  for  $i=1, 2, \dots, M$  and  $j=1, 2, \dots, K$ . These paths in turn serve to determine the cash flows generated from that date and for both exercise and non-exercise cases.

The optimization process requires not only considering whether the option is exercised or not at  $T^O - \tau$ , but also possible exercise at a later date, which will mainly affect the future expected value of the project. If the option is exercised at  $T^O - \tau$ , the expected value of the project at the following point,  $E[V_{T^O, exercise}^i]$ , must be calculated considering the exercise which has already occurred, in turn preventing any new exercise decisions at subsequent dates. However, if the option is not exercised at  $T^O - \tau$ , the expected value of the project at the following period requires comparing the simulated value of the variable  $S_{T^O}^{i,j}$  with the critical value obtained during the previous step,  $S_{T^O}^*$  in order to incorporate the possibility of adopting a new decision.

Hence, determining the value of the project at  $T^O - \tau$ , assuming non-exercise at this date, merely involves adding the current cash flow to the discounted expected value of the project at  $T^O$ ,  $E[V_{T^O}^i]$ , which is calculated by averaging the  $K$  simulated values for both exercise or non-exercise at  $T^O$  cases. Finally, it just remains to

compare the value of the exercise of the option,  $V_{T^O-\tau, exercise}^i$ , with the value of keeping it alive until the following period,  $V_{T^O-\tau, no}^i$ , so as to identify the critical value at  $T^O-\tau$ ,  $S_{T^O-\tau}^*$ .

The procedure to determine  $S_{T^O-\tau}^*$ , is repeated for each of the prior dates where exercise is possible until the remaining values that make up the optimal exercise frontier are found. Logically, as we go back to the initial moment and although the logic for estimation always remains the same, the complexity and number of operations involved in determining critical values multiplies.

### *III.II. Stage two: Estimating the current value of the option*

Having determined the critical values of the state variable at the different moments when early exercise is possible,  $S_{\tau}^*$ ,  $S_{2\tau}^*$ , ...,  $S_{T^O-\tau}^*$ ,  $S_{T^O}^*$ , the value of the American option may be estimated by conventional simulation as if it were a European option.

In this case, simulation involves estimating a sufficient number of state variable paths from the current moment to the project expiration date and we estimate the moment of optimal exercise along each path in accordance with the optimal early exercise frontier. Finally, we obtain the present value of the option discounting the resulting payoff from each path, and then taking the average of all the paths.

## **IV. Valuation of an investment in the car parts industry**

In order to evaluate both interest in flexibilising valuation of real options and the proposed simulation model, we analyse the results of its application to the case of an investment undertaken by a Tier-One multinational supplier of automobile components. A previous valuation of this investment and its real options, using the



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2  
3 binomial model, is documented in Azofra *et al.* (2004). The project involves a  
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5 foreign direct investment in the acquisition of production capacity in Brazil. As  
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7 evidenced in Azofra *et al.* (*op. cit.*), the project is one of those opportunities which  
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9 despite yielding a negative NPV is undertaken due to their strategic value, which is  
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11 shown to emerge from its real options.  
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16 Through this commitment, the investing firm aimed to cater for the demand  
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18 for car parts manufacturers in Brazil and, at the same time, achieve a strategic  
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20 foothold in the South American car parts market so as to access future growth  
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22 opportunities. The initial outlay of 38 million dollars was enough to meet sales  
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24 forecasts for the initial five years, leading to the supply of components for some  
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26 500,000 vehicles during the first year of operations and nearly 850,000 units in each  
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28 of the next four.  
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31  
32 Table 1 shows the expected cash flows for these five years. The  
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34 corresponding NPV varied between a loss of seven million dollars, in the case of a  
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36 five year period and capital cost of 13.66%, and a loss of 21 million dollars, in the  
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38 case of perpetual life and a capital cost of 28.4%. In either case, DCF valuation  
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40 advised against the investment.  
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44 **[Insert Table 1]**  
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47 Considering growth and flexibility options offers quite a different valuation  
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49 result. Azofra *et al.* (*op. cit.*) value European growth options, capabilities to  
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51 redistribute resources towards more profitable uses, and early abandonment of the  
52  
53 project –the latter as an American-type option–. Estimation is performed by applying  
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55 a binomial model, in which the uncertain variable is defined as the number of cars  
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57 produced in Brazil, which is assumed to follow a geometric Brownian process. The  
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59 study confirmed the significance of these options in different scenarios for future  
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3 evolution of the state variable, providing empirical evidence to support the relevance  
4 of real options when explaining the investment decision.  
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8 Taking the information from this case as a reference, we explore the  
9 consequences of neglecting possible discontinuities in the future evolution of the  
10 state variable when valuing American-type options. Our analysis focuses on valuing  
11 options to expand and contract the initial size of the project. The expiration date of  
12 these options coincides with the end of the fourth year of operations and may be  
13 exercised at three different dates, at the end of the second, third and fourth annual  
14 periods.  
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24 The option to expand the project resembles a call option with a strike price  
25 equal to the cost of the required assets. Exercising the option involves an increase of  
26 50% of both the market share and the maximum production capacity and entails an  
27 outlay equal to 40% of the initial investment.<sup>14</sup> The option to contract resembles a  
28 put option with a strike price equal to the book value of the assets affected.  
29 Exercising the option entails the reducing the maximum production capacity by 50%.  
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39 The values of these options are estimated using two different stochastic  
40 processes for the state variable: a pure geometric Brownian motion, on the one hand,  
41 and a mixed process comprising the previous Brownian process and discontinuous  
42 jumps linked to a Poisson type motion, on the other. The values of the parameters  
43 used in the evaluation are: i) for the pure geometric Brownian process, we assayed  
44 with alternative volatilities of 7%, 13% and 20% and annual average growth rates of  
45 0%, 7% and 15%; and ii) for the discontinuous part of the mixed process, the  
46 volatility values of the jump considered are 25%, 50%, 200%, 400% and 500%  
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<sup>14</sup> In order to simplify the analysis we have not considered the car-makers' control of the value chain and its implications for the option valuation. For a detailed study, see Azofra *et al.* (*op. cit.*).

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3 which are combined with a single value of the expected number of jumps equal to  
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5 0.2 ( $\lambda = 0.2$ ).<sup>15</sup>  
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8 We also assume that the jumps belong to the category of diversifiable or non-  
9 systematic risk. Following Azofra *et al.* (2004), the initial value of the state variable  
10 is 1 629,000 vehicles, for which there is a maximum value acting as an absorbing  
11 barrier at 4 000,000 units. Besides, initial configuration of the project implies a  
12 maximum production capacity of 1 200,000 vehicles that can be modified by its  
13 options to expand and contract. Based on S&P-500, the beta coefficient of the state  
14 variable is 1.035, and the risk-free return and the market premium are 6.59% and  
15 6.85%, respectively.  
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18 Simulation is undertaken in annual subintervals, valuing the possibility of  
19 early exercise of the option at each of the three exercise dates considered. The  
20 number of simulated paths to obtain the present value of each option amounts to  
21 400,000, the result of 200,000 direct runs plus another 200,000 estimations using the  
22 “antithetical variates” technique<sup>16</sup> and  $M$  and  $K$  parameter values equal to 250.  
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#### 27 *IV.1. Valuing the option to expand*

28 Table 2 shows the estimated values of the option to expand for the scenarios within  
29 the parameters established by the pure geometric Brownian process and the mixed  
30 process.<sup>17</sup> These results highlight the significant differences in the value of the option  
31 and, therefore, the impact of possible mistakes made by conventional ROV models,  
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<sup>15</sup> The parameter  $\lambda = 0.2$  implies that, on average, only one discrete jump will occur during the five-year life span of the underlying project. We feel it is more interesting to show the valuation results assuming a highly volatile and low frequency process of random jumps, as opposed to multiple smaller jumps, which may prove hard to distinguish from continuous evolution itself. On the choice of parameters see <http://www.puc-rio.br/marco.ind/stoch-a.html#jump-dif>.

<sup>16</sup> The technique of antithetical variates consists of generating two symmetrical observations at zero for each of the random simulations of the normal distribution.

<sup>17</sup> It should be noted that the 0% level of the volatility of the discontinuous jump corresponds to the pure geometric Brownian process.

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3 particularly when the influence of continuous motion is smaller. Hence, in the  
4  
5 scenario with least volatility and continuous variation ( $\alpha = 0\%$  and  $\sigma = 7\%$ ), the  
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7 value of the option to expand increases ten fold when a jump with a 50% dispersion  
8  
9 is introduced, and is reduced by almost 63% when jump dispersion reaches 500%.

12 **[Insert Table 2]**

15 These results underscore the fact that the sign of the relation between the  
16  
17 value of the option and the volatility of the jump depends on the latter's level. Thus,  
18  
19 the value of the option increases in those scenarios in which jump dispersion is  
20  
21 lowest, as a consequence of the rise in total volatility to which the variable is subject.  
22  
23 These differences are more significant in the case of lower continuous volatility ( $\sigma =$   
24  
25  $7\%$ ), since in this case the increase which the total volatility undergoes when there is  
26  
27 a jump of equal dispersion is much greater, in relative terms, than at levels of  
28  
29 permanently high volatility. Thus, for example, a jump dispersion equal to 100%  
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31 implies a total volatility in the process of 53.9% or 50.5% depending on whether the  
32  
33 standard deviation of the continuous motion is either 20% or 7% respectively.<sup>18</sup>

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39 As jump volatility increases, this relation is inverted and gains in significance  
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41 at higher levels of continuous volatility. This result will remain the same provided  
42  
43 the mean size of the jumps,  $k$ , is zero and does not affect the trend of the process.  
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45 This case implies that the average taken by the distribution of the jump logarithm  
46  
47 inversely depends on the value allocated to the jump volatility ( $\mu_\pi = -\sigma_\pi^2/2$ ) and,  
48  
49 therefore, an increase in these values of over 100% reduces this average and thereby  
50  
51 the value of the underlying project. Moreover, the upper limit of the cash flows  
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53 imposed by the maximum production capacity reinforces the inverse reaction  
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55 between the value of this option and jump volatility.  
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3 As regards the continuous evolution of the state variable, the expected  
4 positive relation between the growth rate and the value of the option for the various  
5 levels of volatility and stochastic processes considered is confirmed. Evidently, the  
6 more favourable the scenario foreseen vis-à-vis the growth in car production the  
7 more likely and profitable will be the expansion of the initially planned investment.  
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15 This is not the case, however, for the continuous volatility of the state  
16 variable, which evidences a negative relation with the value of the option to expand,  
17 which is contrary to the *ceteris paribus* relation established by the theory of financial  
18 options. Yet, this result concurs both with the nature of the investment project and  
19 the discretisation method used for the state variable. On the one hand, the maximum  
20 production capacity of the project leads to an asymmetrical effect of the volatility on  
21 the net cash flow probability distribution. Greater volatility implies greater dispersion  
22 of the lower values, whereas the higher values remain bounded by the maximum  
23 production capacity and, as a result, the increase in volatility will diminish the value  
24 of expanding, particularly at those levels where growth of the state variable is  
25 highest.<sup>19</sup>  
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41 The assumption that the state variable evolves in the continuous field  
42 following a lognormal diffusion process leads to its relative variation being  
43 distributed normally with a tendency reduced 0.5 times the variance of the process.  
44 As a result, the volatility parameter not only affects the deviation of future values,  
45 but also the expected value in its simulation. Hence, the increase in volatility not  
46 only widens the range of possible future values of the underlying, but also reduces its  
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58 <sup>18</sup> The estimation of the total volatility for the mixed process was performed using the expression  
59 obtained in Navas (2003), who amends the one initially obtained in Merton (1976).

60 <sup>19</sup> In fact, the reader can note that this negative relation disappears in cases of zero jump volatility and  $\alpha$   
values equal to 0 and 7%.

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3 average simulated value and, thus, the possibilities of optimal exercise of the option  
4  
5 to expand.  
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8 **[Insert Fig. 2]**  
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10 Finally, Fig. 2 shows the behaviour of the critical values at the three exercise  
11 dates depending on the parameter values of the stochastic process. These values  
12 logically reflect the relation between the value of the option and the previously  
13 described characteristics of the state variable evolution. The figure also highlights the  
14 expected positive trend of the critical values together with their lower dispersion as  
15 the option expiration approaches.<sup>20</sup> This positive trend ceases to be apparent in the  
16 higher levels of jump dispersion, since the joint volatility increase undergone by the  
17 state variable process delays optimal exercise of the option.  
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30 *IV.II. Valuing the option to contract*  
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32 The valuation results of the option to contract are shown in Table 3. Once again,  
33 different scenarios are considered with regard to the parameters of the pure  
34 geometric Brownian motion and the mixed process. The inclusion of discontinuous  
35 jumps impacts the value of the option, although the relative variation which this  
36 option undergoes is noticeably lower than in the option to expand. The lower relative  
37 influence in the option to contract is a consequence of the stochastic process nature.  
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39 Positive growth levels in the state variable mean that the likelihood of exercising the  
40 option hardly varies when there is an increase in the discrete volatility despite the fall  
41 in the value of the underlying.  
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54 **[Insert Table 3]**  
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59 <sup>20</sup> It should be noted that, as the project presents a finite life span, approaching the option expiration  
60 implies less time to compensate for the cost of additional investment through the higher cash flows  
arising from expansion.

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3 It can also be seen that the greater the influence of the continuous component  
4 (high  $\alpha$  and  $\sigma$  values) the lower is the influence of the jump in the option value. By  
5 contrast, when the effect of the continuous motion is barely relevant, the increase in  
6 the jump dispersion significantly reduces the value of the option. This result reflects  
7 the nature of the state variable, whose lower limit is bounded at zero such that the  
8 jump dispersion produces an asymmetric effect on the cash flow probability  
9 distribution. Jump volatility implies greater dispersion in the underlying project  
10 higher values, whereas lower values remain bounded and, thus, it diminishes the  
11 chances of optimal exercise of this option.  
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25 The results also confirm the expected negative relation between the drift of  
26 the state variable and the value of the option for the different levels of volatility  
27 analysed. The less favourable the expected evolution for the state variable, the more  
28 valuable is any action aimed at reducing the size of the initial project. The same  
29 happens with the continuous volatility of the state variable, which displays a positive  
30 relation with the value of the option to contract, resulting not only from the expected  
31 asymmetrical increase of possible gains for the option, but also the “implicit”  
32 reduction of the mean simulated value of the underlying, already mentioned for the  
33 option to expand. Only when the state variable shows a flat trend ( $\alpha = 0\%$ ), does the  
34 increase in volatility favour the value of the underlying asset without affecting the  
35 probability of exercise, once again due to the lower boundary inherent in the nature  
36 of the state variable.  
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53 Finally, Fig. 3 shows the evolution of critical values at exercise dates in terms  
54 of the stochastic process parameters. The figure bears out the expected upward trend  
55 of the optimal exercise frontier and the lower dispersion of the critical values  
56 approaching expiration of the right. Given the finite lifespan of the investment, the  
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3 reduction in the present value of its cash flows is lower as the expiration date of the  
4  
5 right approaches, and as a result, exercising the option will be advisable on a greater  
6  
7 number of occasions. Only when the jump dispersion is extremely high –and thus the  
8  
9 volatility of the joint process– can any reduction be seen in the critical values  
10  
11 corresponding to the point prior to the expiration date indicating deferment in  
12  
13 exercising the right.  
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17 **[Insert Fig. 3]**  
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## 19 **V. Conclusion**

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21 This paper identifies some potential problems in the application of ROV. Traditional  
22  
23 ROV models work well when the state variable follows a Brownian-type continuous  
24  
25 motion. As this may not always be the correct assumption about the stochastic  
26  
27 process followed by the state variable, understanding how sensitive real option  
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29 values are to failures becomes a critical issue.  
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35 We focus on the valuation of American options whose state variable evolves  
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37 following a mixed process, comprising continuous random walks together with  
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39 random discontinuous jumps. The presence of random discontinuities complicates  
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41 valuation of American options in commonly used numerical techniques –binomial  
42  
43 trees and finite differences– thus making simulation the only technique applicable  
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45 with any degree of simplicity and generality. We specifically propose a flexible  
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47 model by merging Monte Carlo simulation and dynamic programming which allows  
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49 us to weigh up the potential errors when omitting the possible occurrence of random  
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51 jumps.  
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56 We have applied the model to value a real investment case, whose valuation  
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58 using a binomial model is outlined in Azofra *et al.* (2004). In general terms, the  
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60 application results confirm that one critical issue in correctly implementing ROV is



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3 correct estimation of the state variable process. We observe that random mean-zero  
4 discontinuities alter the value of the options to expand and to contract, particularly  
5 when the influence of the continuous variation is lower and jump dispersion is high.  
6 Further, the joint effect of the jump together with the real restrictions caused by the  
7 maximum production capacity of the investment or the very nature of the state  
8 variable, magnify the differences identified in the estimations.  
9

10  
11 Since the influence of random jumps is not quantified in the evaluation  
12 obtained using conventional numerical techniques, estimating the expanded NPV  
13 may lead to errors similar to those which occur when using the much criticised DCF  
14 approach. In our case study, leaving out discontinuities entails the appearance of  
15 biases –downward when the jump dispersion is low and upward in the opposite case–  
16 which may lead to inefficient investment decisions being taken.  
17

18  
19 At lower discrete volatility levels, undervaluing the option to expand may  
20 lead to sub-optimal deferral of exercising the right and even the rejection of  
21 profitable projects. Over-valuation, might could appear in traditional ROV models in  
22 the case of high jump dispersion may explain the naïve selection of negative  
23 expanded NPV projects. Finally, the probability of early non-optimal exercise of the  
24 option to contract increases with the relative influence of discrete volatility, leading  
25 to the possible rejection of subsequent positive cash flows.  
26

27  
28 Although in this work we only assess the influence of random jumps with a  
29 zero mean value, our results show that the influence of discontinuities on option  
30 values will be even greater and thus errors arising from conventional ROV models  
31 even more evident. Summing up, the differences observed allow us to justify the  
32 effort required to flexibilise ROV models when the investment opportunities  
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3 available to the firm suggest considering exercise at more than one future date and  
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5 the inclusion of stochastic processes other than pure geometric Brownian.  
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For Peer Review

Figures

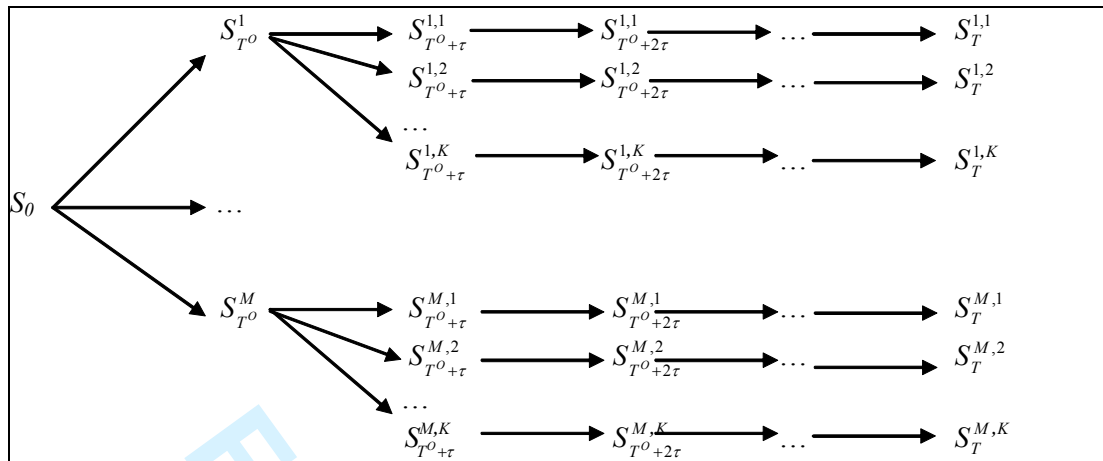


Fig. 1. Simulation paths for the uncertain state variable

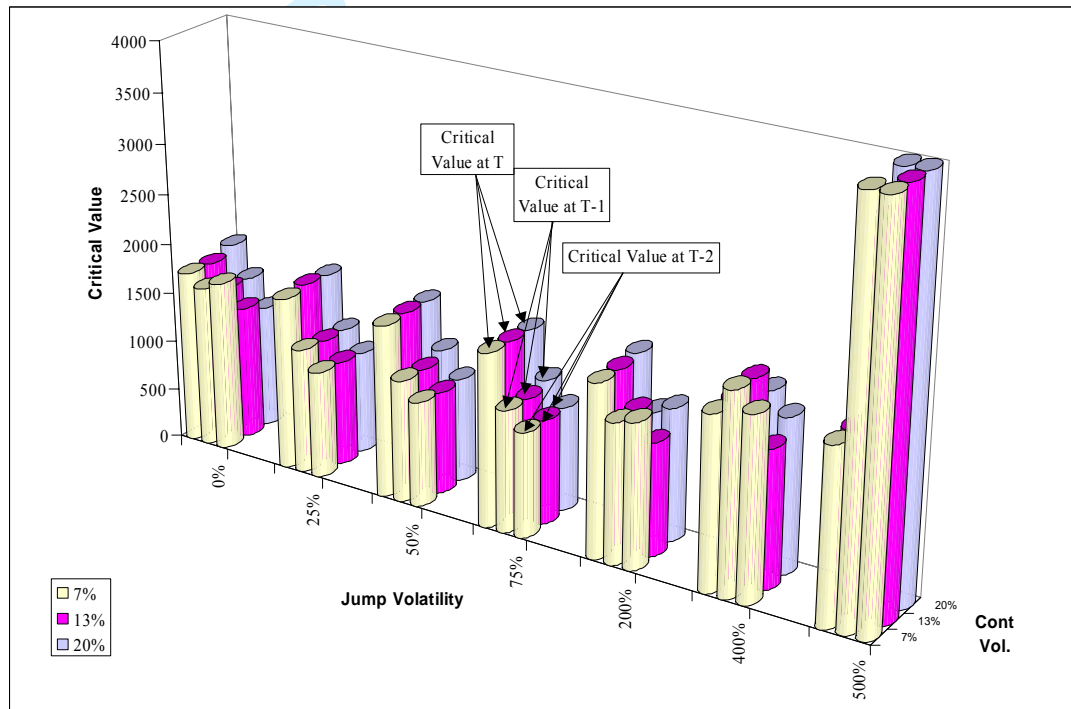
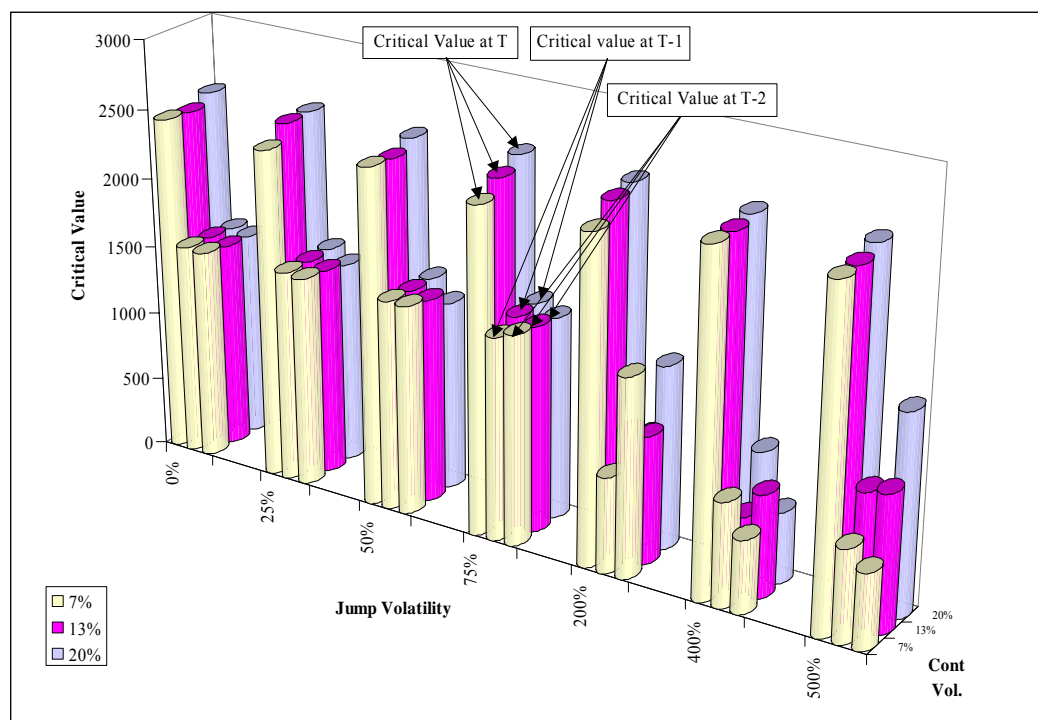


Fig. 2. Critical values for the option to expand.

Notes: Growth rate of state variable is 7%. All values are in thousands USD.



**Fig. 3. Critical values for the option to contract**

Notes: Growth rate of state variable is 7%. All values are in thousands USD.

## Tables

**Table 1. Predicted Cash Flows**

|                                     | Year 1    | Year 2    | Year 3    | Year 4    | Year 5    |
|-------------------------------------|-----------|-----------|-----------|-----------|-----------|
| Revenues                            | 31,009.74 | 64,419.80 | 64,024.38 | 64,042.57 | 64,061.30 |
| Cost of Goods Sold                  | 18,084.20 | 37,115.88 | 38,229.35 | 39,376.23 | 40,557.52 |
| Personnel Costs                     | 3,254.57  | 4,080.05  | 4,202.45  | 4,328.52  | 4,458.38  |
| Other Operating Costs               | 705.89    | 1,243.18  | 1,280.47  | 1,318.88  | 1,358.45  |
| General and Administrative Expenses | 4,565.59  | 6,031.87  | 6,212.82  | 6,399.21  | 6,591.18  |
| Taxes                               | 815.30    | 3,378.46  | 3,345.60  | 4,102.07  | 3,970.55  |
| Total Payments                      | 27,425.55 | 51,849.43 | 53,270.69 | 55,524.91 | 56,936.08 |
| Net Free Cash Flow                  | 3,584.20  | 12,570.38 | 10,753.69 | 8,517.65  | 7,125.22  |

Notes: Values are in thousands USD. Source: Azofra *et al.* (2004).

**Table 2. Present value of the option to expand**

| Growth rate = 0%      |     | Jump Volatility |          |          |          |          |        |
|-----------------------|-----|-----------------|----------|----------|----------|----------|--------|
|                       |     | 0%              | 25%      | 50%      | 200%     | 400%     | 500%   |
| Continuous Volatility | 7%  | 312.87          | 3,407.67 | 3,437.74 | 3,176.89 | 858.29   | 1.16   |
|                       | 13% | 2,929.65        | 3,324.56 | 3,262.76 | 3,081.87 | 506.82   | 55.09  |
|                       | 20% | 2,982.51        | 3,133.02 | 3,082.54 | 3,069.44 | 1,320.62 | 135.52 |
| Growth rate = 7%      |     |                 |          |          |          |          |        |
| Continuous Volatility | 7%  | 5,540.52        | 6,267.29 | 6,269.84 | 6,205.98 | 1,282.93 | 234.40 |
|                       | 13% | 6,049.64        | 6,057.93 | 6,187.02 | 6,076.66 | 6,013.01 | 297.03 |
|                       | 20% | 5,711.13        | 5,798.84 | 5,765.69 | 5,754.64 | 5,376.63 | 450.16 |

| <b>Growth rate = 15%</b>     |            |           |           |           |           |           |          |
|------------------------------|------------|-----------|-----------|-----------|-----------|-----------|----------|
| <b>Continuous Volatility</b> | <b>7%</b>  | 10,407.63 | 10,514.03 | 10,483.70 | 10,458.74 | 2,991.41  | 2,410.86 |
|                              | <b>13%</b> | 9,875.19  | 10,044.93 | 10,016.15 | 10,022.05 | 10,035.88 | 2,312.44 |
|                              | <b>20%</b> | 9,390.97  | 9,331.24  | 9,263.66  | 9,424.31  | 9,147.26  | 5,315.77 |

Note: Values are in thousands of USD.

**Table 3. Present value of the option to contract**

| <b>Growth rate = 0%</b>      |            | <b>Jump Volatility</b> |            |            |             |             |             |
|------------------------------|------------|------------------------|------------|------------|-------------|-------------|-------------|
|                              |            | <b>0%</b>              | <b>25%</b> | <b>50%</b> | <b>200%</b> | <b>400%</b> | <b>500%</b> |
| <b>Continuous Volatility</b> | <b>7%</b>  | 7,452.35               | 7,485.04   | 7,518.70   | 7,451.86    | 7,520.56    | 6,547.17    |
|                              | <b>13%</b> | 7,327.73               | 7,207.16   | 7,301.42   | 7,273.81    | 6,704.01    | 6,466.43    |
|                              | <b>20%</b> | 7,104.75               | 7,086.80   | 7,004.11   | 6,898.37    | 6,907.27    | 6,057.61    |
| <b>Growth rate = 7%</b>      |            |                        |            |            |             |             |             |
| <b>Continuous Volatility</b> | <b>7%</b>  | 4,122.86               | 4,083.16   | 4,099.23   | 4,040.44    | 4,257.15    | 4,000.61    |
|                              | <b>13%</b> | 4,166.67               | 4,184.72   | 3,905.42   | 4,066.78    | 4,135.14    | 3,835.34    |
|                              | <b>20%</b> | 4,365.83               | 4,263.87   | 4,352.07   | 4,312.32    | 4,228.64    | 4,056.90    |
| <b>Growth rate = 15%</b>     |            |                        |            |            |             |             |             |
| <b>Continuous Volatility</b> | <b>7%</b>  | 213.86                 | 286.00     | 255.16     | 333.05      | 266.64      | 242.14      |
|                              | <b>13%</b> | 607.83                 | 611.86     | 608.74     | 624.18      | 747.00      | 671.86      |
|                              | <b>20%</b> | 1,164.20               | 1,110.30   | 1,104.35   | 1,236.70    | 1,370.97    | 1,166.68    |

Note: Values are in thousands of USD.