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The Time-Series Properties of Norwegian Inflation and Nominal Interest Rate

Pär Österholm

Department of Economics, Uppsala University

Abstract

This paper investigates the time-series properties of Norwegian inflation and nominal interest rate using annual data from 1850 to 2004. A number of different univariate unit-root tests are employed to examine whether the time series are mean reverting or generated by unit-root processes. Results show very strong evidence in favour of mean reversion in inflation but a unit root in the nominal interest rate. This implies that there exists no long-run relationship between these two variables, a conclusion which is further supported by cointegration tests and estimated vector error correction models. The cointegration analysis also points to an important potential pitfall when using cointegration techniques on systems where some variables are stationary processes.

JEL Classification: C22, C32, E31, E43

Keywords: Unit root, Mean reversion, Fisher hypothesis

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1. Introduction

The time-series properties of nominal interest rates and inflation have been a field of much empirical research. This focus is not surprising since the time-series properties in many cases can provide important information regarding the relevance of different economic hypotheses. For example, with a foundation in the efficient market hypothesis, it can be argued that the nominal interest rate should be generated by a unit-root process. A potential unit root in the nominal interest rate also raises the question of a unit root in inflation. As it is commonly assumed in the literature – see, for example, Fama (1975) and Crowder and Hoffman (1996) – that the real interest rate is either constant or a stationary process, the nominal interest rate and inflation need to be integrated of the same order for the Fisher hypothesis to be empirically relevant. However, the time-series properties of these variables are not only interesting for distinguishing between different economic hypotheses, they can also determine which methodological approaches could be employed to test a particular hypothesis. For example, cointegration techniques – which are associated with a number of advantages – can be used when variables are generated by unit-root processes. Accordingly, knowledge about the presence or absence of unit roots in inflation and interest rates are important from several perspectives.

This paper aims to establish the time-series properties of Norwegian inflation and nominal interest rate. There is little previous research addressing this question but given the interesting historical features of the Norwegian economy – for example, the low rate of unemployment during the 1970s and 1980s when making international comparisons and the development of a substantial oil sector since the early 1970s – there are reasons to investigate the behaviour of these key variables more closely. In particular, we will focus on whether inflation and the nominal interest rate are mean reverting or unit-root processes. This will be done by employing a range of univariate unit-root tests to the two series. Relying on unit-root tests for inference, this paper is similar to a reasonably large literature in this field; see, for example, Campbell and Shiller (1991), Culver and Papell (1997), Wu and Chen (2001), Charemza et al. (2005) and Basher and Westerlund (2006). A novelty of this paper is the dataset used; we apply the unit-root tests to a very long

\[1\] Whilst a constant or mean-reverting real interest rate has found plenty of support in the literature and is a common modelling choice, this assumption has also been questioned; see, for example, Mills and Stephenson (1985) and Rose (1988).
sample of annual data from 1850 to 2004 of the variables in question. Usage of such a
long sample has benefits from an econometric point of view. It is well-known that for the
sample sizes typically available in macroeconomics, the power of Dickey-Fuller type unit-
root tests is generally low for highly persistent, but stationary, time series; see, for
example, Froot and Rogoff (1995). By using longer series, we should hence improve the
power of the tests. This point is particularly interesting when considering the commonly
stated claim that it is the span of the data – rather than sampling frequency – that is most
important for being able to distinguish if a time series has a unit root or is stationary.\(^2\)

Apart from establishing the univariate time-series properties of the two variables, this
paper also aims to relate the two variables to each other. The main question in this aspect
will be the relevance of the Fisher hypothesis. Both the unit-root tests and cointegration
analysis will be used for this purpose.

Results from the unit-root tests unambiguously support the conclusion that the nominal
interest rate is generated by a unit-root process, whereas inflation is stationary. These
results are further confirmed by the cointegration analysis and estimated vector error
correction models. This implies that the traditional Fisher hypothesis is not supported by
the Norwegian data.\(^3\) Moreover, the analysis based on vector error correction models also
points to an important potential pitfall: Using cointegration techniques on systems where
some variables may be stationary processes, both the number of cointegrating vectors and
restrictions on these vectors should be carefully investigated.

The rest of this paper is organised as follows. In Section two, data are presented and unit-
root tests and cointegration analysis are conducted. Section three concludes.

2. Empirical analysis

The empirical analysis uses annual data on Norwegian nominal interest rate and CPI
inflation – \(i_t\) and \(\pi_t\) respectively – from 1850 to 2004. The nominal interest rate is given
by the yield on the most actively traded maturities of long-term government bonds;

\(^2\) An early reference pointing this out is Shiller and Perron (1985)
\(^3\) As a comparison to the results based on the Norwegian dataset, we also conduct the same analysis using
monthly US data ranging from April 1953 to July 2005.
numbers are averages of monthly observations. Data were provided by Norges Bank and are thoroughly discussed in Eitrheim et al. (2004). Time-series plots of the two variables are presented in Figure 1.

In addition to using the novel Norwegian dataset, we will also – in order to facilitate comparisons – use some much more traditional data. These consist of monthly observations of US nominal interest rate – given by the yield on the five-year government bond – and year-ended CPI inflation from April 1953 to July 2005. The US data were taken from the FRED database of the Federal Reserve Bank of St Louis and are shown in Figure 2.

*Figure 1. Time-series plots of Norwegian interest rate and inflation 1850 to 2004.*
Figure 2. Time-series plots of US interest rate and inflation April 1953 to July 2005.

2.1 Unit roots or mean reversion

Turning to the empirical analysis of the data, we initially investigate the time series using five different unit-root tests: The Augmented Dickey-Fuller test (Said and Dickey, 1984), the Phillips and Perron (1988) (PP) test, the Kapetanios et al. (2003) test, the Zivot and Andrews (1992) test and the KPSS (Kwiatkowski et al., 1992) test. The number of unit-root tests available for an empirical analysis of this kind is very large and the choice of which to use is clearly arbitrary. However, we believe that the five tests chosen here provide a sensible range as they address three different issues that could be important.
First, the first four tests test whether the time series in question has a unit root versus the alternative hypothesis of stationarity; the KPSS test on the other hand reverses the burden of proof as it has mean reversion under the null hypothesis. Second, the KSS test offers a method to test for potential non-linear mean reversion, a data generating process which standard unit root tests tend to have poor power properties against; see, for example, Michael et al. (1997). Third, by employing the Zivot and Andrews test – where we allow for an endogenously determined breakpoint in the intercept – the poor power of Dickey-Fuller type tests when structural breaks are present is addressed.\(^4\) As a general principle, we do believe that structural breaks too often are used with the benefit of hindsight to control for something that was difficult to anticipate and that it therefore should be used very restrictively in empirical work.\(^5\) We will, however, nevertheless allow for a break in the intercept of the time series in question to ensure that our conclusions are robust to that possibility. Taken together, we conclude that the tests actually employed in this paper appear well-suited for our purpose.

As all tests – except the Kapetanios et al. (2003) (KSS) test – have been standard tools in the macroeconomic literature for more than a decade now, they will not be presented to the reader. We will, however, describe the KSS test a little more closely. This tests the null hypothesis of a unit root against the alternative hypothesis of a globally stationary exponential smooth-transition autoregressive (ESTAR) process.\(^6\) The test is based on estimation of

\[
\Delta x_t = \Delta x_{t-1} + \sum_{j=1}^r \theta_j \Delta x_{t-j} + \psi_t,
\]

\(^4\) See, for example, Perron (1989).
\(^5\) Moreover, unit-root tests that allow for structural breaks – such as Perron (1989), Zivot and Andrews (1992) and Vogelsang and Perron (1998) – have some well-documented shortcomings. One well-known problem is the risk of spurious rejection of the null hypothesis when the breakpoint is chosen endogenously; see, for example, Nunes et al. (1997) and Lee and Strazicich (2001).
\(^6\) In an ESTAR model, the speed of mean reversion is not constant. Instead, the process can display unit root behaviour in the region close to its equilibrium but strong mean reversion when the process is far from its mean.
where $x_i$ refers to demeaned values of the original time series, $\bar{x}_i$, and $\psi_i \sim iid(0, \sigma^2)$. For all time series under investigation, the null hypothesis $H_0 : \lambda = 0$ is then tested versus $H_1 : \lambda < 0$ using the $t$-statistic on $\hat{\lambda}$. The null hypothesis of a unit root is rejected for large negative values of the test statistic; critical values can be found in Kapetanios et al. (2003).

In the ADF, Zivot and Andrews and KSS tests, we must first determine lag length in the test equations. For the ADF and Zivot and Andrews tests, we employed the Akaike (1974) information criterion for this purpose. Regarding lag length in the KSS test regressions, this was set equal to that of the ADF test. Kapetanios et al. (2003) pointed out that linear dynamics can be seen as a first-order approximation if the true augmentations are non-linear in nature. For the PP and KPSS tests, a Newey-West estimator was employed to correct for serial correlation in the residuals. All tests are conducted allowing for mean reversion around a constant level different from zero. We do not allow for any time trends, as we argue that linear time trends in nominal interest rates or inflation are inconsistent with economic theory. Results from the unit-root tests can be found in Table 1.

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<th></th>
<th>Norway</th>
<th>United States</th>
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<tbody>
<tr>
<td>ADF</td>
<td>$-2.406$</td>
<td>$-6.298^*$</td>
</tr>
<tr>
<td>PP</td>
<td>$-1.988$</td>
<td>$-6.259^*$</td>
</tr>
<tr>
<td>KSS</td>
<td>$-2.873$</td>
<td>$-6.446^*$</td>
</tr>
<tr>
<td>ZA</td>
<td>$-2.300$</td>
<td>$-6.913^*$</td>
</tr>
<tr>
<td>KPSS</td>
<td>$0.553^*$</td>
<td>$0.395$</td>
</tr>
</tbody>
</table>

ADF, PP, KSS, ZA and KPSS are the test statistics from the Augmented Dickey-Fuller, Phillips and Perron, Kapetanios et al., Zivot and Andrews and KPSS tests respectively. *significant at the 5% level

Turning to the results for Norway first, Table 1 presents what must be described as overwhelming evidence in favour of a unit root in the nominal interest rate and mean reversion in inflation. It is interesting to note that all five tests reach the same conclusion regarding each time series; such conclusive evidence is only rarely presented when different tests are applied to a macroeconomic time series. The finding that the nominal

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7 The value of the delay parameter has been set to $d = 1$. Just like Taylor et al. (2001), we believe that that the delay parameter should be small.
interest rate and inflation appear to be integrated of different orders is interesting as we relate it to the Fisher hypothesis. As pointed out by, for example, Crowder and Hoffman (1996, p. 103), such a finding implies that “…the textbook representation of the Fisher hypothesis may be rejected out of hand”.

The results for the United States are equally conclusive regarding the nominal interest rate – all five tests conclude that this is a unit root process. For the inflation rate though, we find that the US data have very different properties to the Norwegian. Whilst the results are a touch less conclusive, since only four out of five tests agree, we still believe that there is fairly strong evidence in favour of a unit root in the US inflation rate. Given that the nominal interest rate and inflation appear to be integrated of the same order, there is clearly a possibility that the Fisher hypothesis is valid in the United States. With these findings in mind we next turn to a closer inspection of how nominal interest rates and inflation relate to each other, with a particular focus on the Fisher hypothesis.

2.2 The Fisher hypothesis

A traditional formulation of the Fisher hypothesis is that the expected nominal interest rate over the period is equal to the expected real interest rate plus expected inflation. Relying on the commonly made assumption of a constant or mean-reverting real interest rate, an empirical version of the Fisher hypothesis can be written as

\[ i_t = r^* + \gamma \pi_t + \nu_t, \quad (2) \]

where the constant \( r^* \) has the interpretation of the equilibrium real interest rate, the error term \( \nu_t \) is assumed to be a stationary ARMA process and \( \gamma \) should be equal to unity; see, for example, MacDonald and Murphy (1989).\(^9\)

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\(^8\) This finding would be consistent with the viewpoint of, for example, Cogley and Sargent (2001) and Stock and Watson (2006) that US inflation is a unit root process.

\(^9\) Note that even though the constant is interpreted as the equilibrium real interest rate in this setting, this does not mean that this variable needs to be stationary in practice. A unit root in the equilibrium real interest rate could accordingly be one reason for failing to find cointegration between the nominal interest rate and inflation.
Given the high persistence of nominal interest rates and inflation in many countries, a popular approach to test the Fisher hypothesis in more recent years has been to employ cointegration techniques; see, for example, Atkins (1989), MacDonald and Murphy (1989), Mishkin (1992), Wallace and Warner (1993), Evans and Lewis (1995), Crowder and Hoffman (1996), Payne and Ewing (1997), Junttila (2001) and Granville and Mallick (2004). Usage of cointegration techniques makes sense to some extent as it has been pointed out that the Fisher hypothesis is better interpreted as a long-run equilibrium condition (Summers, 1983). As stated above, a necessary condition for this modelling choice to be relevant is that both inflation and the nominal interest rate are unit root processes. The results presented above indicate that this appears to be true for the United States, but for Norway there is extremely strong evidence that the nominal interest rate and inflation have different orders of integration. Let us nevertheless turn to cointegration analysis to see if this can provide us with further information.

A cointegrated vector autoregression (VAR) can be written as

\[
\Delta y_t = \delta + \Pi y_{t-1} + \sum_{j=1}^{p-1} F_j \Delta y_{t-j} + \varepsilon_t, \tag{3}
\]

where \( y_t \) is an \( n \times 1 \) vector of variables that are assumed to be integrated of order one and \( \varepsilon_t \) is a \( n \times 1 \) vector of innovations. Particular attention is paid to the rank of the matrix \( \Pi \) as this matrix will have reduced rank if the variables of the system are cointegrated; the rank will be equal to the number of cointegrating vectors. It should also be noted that \( \Pi \) only can have full rank if all variables in the system are stationary; see Taylor and Sarno (1998) and Österholm (2004).

Turning to our particular application, we can – if the nominal interest rate and inflation both are integrated of order one, and moreover, cointegrated – write the system as

\[
\begin{pmatrix}
\Delta i_t \\
\Delta \pi_t
\end{pmatrix} = \delta + \alpha \beta \begin{pmatrix}
i_{t-1} \\
\pi_{t-1}
\end{pmatrix} + \sum_{j=1}^{p-1} f_{ij} \begin{pmatrix}
i_{t-j} \\
\pi_{t-j}
\end{pmatrix} + \begin{pmatrix}
\Delta i_{t-j} \\
\Delta \pi_{t-j}
\end{pmatrix} + \begin{pmatrix}
\varepsilon_t^i \\
\varepsilon_t^\pi
\end{pmatrix} \tag{4}
\]
where \( \alpha = (\alpha_1, \alpha_2)' \) and the cointegrating vector is given by \( \beta' = (\beta_1, \beta_2) \). Regarding the constant, \( \delta \), a restriction has been imposed such that we do allow for an intercept in the cointegrating relationship but no drift in the variables.

Before estimating a cointegrated VAR though, we must first establish \( i) \) lag length for the VAR and \( ii) \) the number of cointegrating vectors. Lag length was determined by employing the Akaike (1974) information criterion to the bivariate VAR in levels. \( p = 2 \) and \( p = 8 \) were thereby established for Norway and the United States respectively. Testing for cointegration using the Johansen (1988, 1991) trace and maximum eigenvalue tests, we find that both the trace and maximum eigenvalue tests conclude that there is one cointegrating vector for Norway as well as the United States. Test statistics are reported in Table 2. Note also that these cointegration tests show that both variables cannot be stationary in either country as the rank then would be equal to two.

### Table 2. Results from cointegration tests.

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<th>Norway</th>
<th>United States</th>
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<tr>
<td></td>
<td>( J_{\text{trace}} )</td>
<td>( J_{\text{max}} )</td>
</tr>
<tr>
<td>( H_0 : r = 0 )</td>
<td>41.296*</td>
<td>38.015*</td>
</tr>
<tr>
<td>( H_0 : r = 1 )</td>
<td>3.281</td>
<td>3.281</td>
</tr>
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</table>

\( J_{\text{trace}} \) is the test statistic from Johansen’s trace test for cointegrating rank.  
\( J_{\text{max}} \) is the test statistic from Johansen’s maximum eigenvalue test for cointegrating rank.  
* significant at the 5% level

We next estimate the model in equation (4) for Norway, with one cointegrating vector and lag length set to \( p - 1 = 1 \). This yields an estimate of the cointegrating vector\(^{10}\) of \( \hat{\beta}' = (1 - 1.466) \). This estimate appears encouraging, as it is not very far from the most traditional interpretation of the Fisher hypothesis which suggests the cointegrating vector \( \beta' = (1 - 1) \). We therefore next impose this restriction on the cointegrating vector and test whether the restriction is rejected by the data using a likelihood ratio test. Doing this, we find that the test statistic is 0.708; since the test statistic follows a \( \chi^2 \)-distribution with one degree of freedom, the restriction is not rejected at the five percent level.

\(^{10}\) The constant term has been omitted for notational convenience.
So far it seems that the cointegration analysis has lent strong support for the Fisher hypothesis on the Norwegian data. One cointegrating vector was found in the system and we could not reject the hypothesis that this vector is $\mathbf{\beta}' = (1 \ -1)$. However, in this particular application we have – based on the unit-root tests – very strong reason to suspect that inflation is not a unit-root process. As stated above, this leads us to question the above finding that there is cointegration between inflation and the nominal interest rate above.

In order to establish whether the above finding of cointegration between Norwegian nominal interest rate and inflation is for real, we suggest that a final restriction should be tested. If the nominal interest rate is a unit-root process and inflation truly is stationary, we should not only find – which has been done above – that the matrix $\Omega$ has rank one but also be unable to reject the restriction $\mathbf{\beta}' = (0 \ 1)$. This restriction implies that inflation alone provides a cointegrating vector and the interpretation accordingly that it is stationary. Estimating the model with the restriction $\mathbf{\beta}' = (0 \ 1)$ imposed, we find a test statistic of 3.006. As the test statistic follows a $\chi^2$-distribution with one degree of freedom, the null hypothesis can not be rejected at the five percent level.\footnote{We could – despite the overwhelming evidence from the unit root tests – also test the hypothesis $\mathbf{\beta}' = (1 \ 0)$ to investigate whether the nominal interest rate is judged stationary. Doing this, we get a test statistic of 34.728; this null hypothesis is hence forcefully rejected by the data.} Our conclusion is hence that the above result of cointegration between the nominal interest rate and inflation in Norway is not a relevant finding. All evidence support that the nominal interest rate and inflation are integrated of different orders; no long-run equilibrium relationship can therefore exist between the two.

Finally, the model in equation (4), with one cointegrating vector and a lag length of $p-1=7$, is estimated on the US data. The estimated cointegrating vector is $\hat{\mathbf{\beta}}' = (1 \ -1.352)$ and when the restriction $\mathbf{\beta}' = (1 \ -1)$ is tested, the null hypothesis cannot be rejected. Recalling that we found some evidence of stationarity of the US inflation rate though, we should obviously scrutinise these results further. However, unlike what we found using the Norwegian data, stationarity of both US nominal interest rate and
inflation is clearly rejected; the test statistics associated with the likelihood ratio tests of the restrictions $\mathbf{b}' = (1 \ 0)$ and $\mathbf{b}' = (0 \ 1)$, which are shown in Table 3, are both clearly larger than the critical value from a $\chi^2$-distribution with one degree of freedom.

Summing up, it hence seems that cointegration techniques provide an appropriate tool for the empirical analysis of the US data, since both the nominal interest rate and inflation appear to be I(1). In addition to cointegration between US nominal interest rate and inflation providing support for the Fisher hypothesis, we also find that the most traditional interpretation cannot be rejected by the data.

### Table 3. Results from testing restrictions on the cointegrating vector.

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<tbody>
<tr>
<td>$\mathbf{b}' = (1 \ -1)$</td>
<td>0.708</td>
<td>1.555</td>
</tr>
<tr>
<td>$\mathbf{b}' = (1 \ 0)$</td>
<td>34.728*</td>
<td>13.155*</td>
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<tr>
<td>$\mathbf{b}' = (0 \ 1)$</td>
<td>3.006</td>
<td>6.441*</td>
</tr>
</tbody>
</table>

* significant at the 5% level

### 3. Conclusions

This paper has investigated the time-series properties of Norwegian inflation and nominal interest rate using annual data from 1850 to 2004, employing both univariate unit-root tests and cointegration analysis. Our results unequivocally support the conclusion that Norwegian inflation is mean reverting whereas the interest rate is a unit-root process. This implies that the real interest rate is not a mean-reverting process – a finding in line with those of Rose (1988). The findings in this paper also raise questions regarding previous studies that have relied on cointegrating methods despite having found evidence of stationarity of the included variables; see, for example Crowder and Hoffman (1996) and Granville and Mallick (2004). We have shown that great care should be taken when cointegration techniques are applied to systems where some variables may be stationary processes. In particular, the cointegrating rank of the system, estimates of the cointegrating vectors and potential restrictions on these vectors are issues that all should be carefully scrutinised.
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