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THE DISTRIBUTION OF DAIRY FARM SIZE IN POLAND: A MARKOV APPROACH BASED ON INFORMATION THEORY

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ABSTRACT

This paper sets out to analyse the evolution of the dairy farm structure of Poland during the post-socialist period. After focusing on how the farm structure has changed over time, an instrumental variable generalized cross entropy estimator is used to develop and estimate a Markov model in order to explore how farm structure will probably develop in the coming decade. The estimator exploits both sample data and prior information, including general and plausible information on farm mobility and structural adjustments based on independent literature. Next, several statistical indicators are computed for farm mobility and for which farms are likely to survive. Finally, milk projections are made and related to policy scenarios. The projections show that the number of dairy farms will continue to decline, but the number of medium and large farms will increase. In the coming decade, subsistence dairy farms are expected to leave the sector slowly. Milk projections show that under the status quo, milk quotas will be binding and overrun, whereas under the 'soft landing' scenario they appear to be only binding after 2010.

Keywords: dairy, farm size, Poland, Markov chain, generalized cross entropy.

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1 INTRODUCTION

In this paper we set out to analyse the evolution of the dairy farm structure of Poland during the post-socialist period. This analysis is of interest to policy makers, in providing insight into how the farm structure is likely to evolve; a particularly relevant issue is what will happen to the subsistence and semi-subsistence farms in the restructuring process. The analysis is also of interest to the upstream and downstream industries that have to decide on investments in dairy processing capacity, milk collection schemes, and providing farm input supplies.

We have four objectives: to examine how the farm structure has changed over time and what path it is likely to follow in the coming decade by making several projections; to test whether the evolution of farm size is explained by non-stationary effects; to compute several statistical indicators of farm mobility and of which farms are likely to survive; and finally, to make milk projections for the coming decade, based on the projected number of dairy farms and to compare them with two possible policy scenarios: 1) status quo milk quota and 2) a gradual phasing out of the milk quota.

We use a Markov probability model (Lee *et al*., 1970) of farm size distribution which is able to analyse movements of individuals between different states when only aggregate data on finite size categories are available for a given time period. A generalized cross entropy (GCE) estimator is used (see Golan *et al*., 1996; Mittelhammer *et al*., 2000). Entropy estimators are particularly suitable when dealing with limited data, as is often the case for empirical applications on Central Eastern European Countries (CEECs). Our paper further extends the approaches of Golan and Vogel (2000), Courchane *et al*. (2000), Karantininis (2002) and Jongeneel *et al*. (2005) by allowing for a heteroscedastic version of the set of Markov equations and for seemingly unrelated regressions (SUR) estimation. Assuming a common and constant variance matrix across the different Markov states, as done, for example, in Karantininis (2002) and Jongeneel *et al*. (2005), could easily create bias on the estimated Markov transition probabilities affecting related indicators as well as projections.

The remainder of this paper is organized as follows. Section 2 describes the farm structure of Poland, with a focus on dairy farming. Section 3 specifies the Markov chain entropy formalism. Section 4 discusses the sample data as well as prior information. Section 5 discusses results. Section 6 presents the associated milk projections and relates these two policy scenarios. In Section 7, the conclusions are presented.

2 Farm structure in Poland, with a focus on dairy farming

Poland is one of the most important dairy producers in the European Union (EU). In 2006 it accounted for about 8 per cent of the total EU-27 cow milk production, being the fourth EU milk producer after Germany, France and United Kingdom. In the last five years, dairy cow numbers have declined by 9.4 per cent and milk yields have improved by 15.1 per cent (FAOSTAT, 2006). Since the demise of the socialist regime, the Polish dairy sector has presented a highly fragmented dairy farm structure, with a large number of small private family farms, just as in other sectors of agriculture. In 1987, about 67 per cent of the dairy farms had only 1-2 cows and these accounted for 41 per cent of the national herd. The number of private dairy farms had already shrunk greatly before transition: by about 25 per cent from 1981 to 1987. Dairy cow numbers declined concomitantly. At the beginning of transition, about 80 per cent of the national milk production was being produced from farms with 10 cows or less (Sznajder, 2002, pp. 242-244).

In Poland, dairy producers after the transition reform can be classified into three main categories: farmers with 1-2 cows, producing milk mostly for the farm household (i.e. subsistence dairy farms); farmers with more than 3-4 cows, who produce milk for sale in local markets and for their own needs (i.e. semi-subsistence dairy farms); and farmers with more than 10 cows, who produce almost exclusively for the dairy industry (Sznajder, 2002, p. 248). In 1996, about one quarter of Polish milk was produced by almost 1 million individual farms keeping 1 to 3 cows, while half was produced by farms with 3 to 9 cows (European Commission, 1998, p. 36). This underscores the great fragmentation of Polish milk production even after transition. In 2005 there were about 700 000 dairy farms: a decline of about 51 per cent as compared with the number of farms in 1995. Also in 2005, about 65 per cent of the farms with dairy cows were subsistence farms with 1-2 cows (Figure 1) and about 53 per cent of the dairy cow stock was concentrated in farms with 1-9 cows. The Polish Ministry of Agriculture expects the number of total farms to fall by 76 per cent between 1996 and 2010 (AgraEurope, 2000, pp. 18-19). At first sight, Figure 1 suggests that the evolution of Polish dairy farms has proceeded without being affected by the EU milk quota system which was announced in 2004 and effectively introduced in 2006. In addition, it appears that the size class with 3 to 9 cows is the ‘switch’ class: farms with smaller herds (i.e. 1-2 cows) show a tendency to decline, whereas for farms with larger herds (i.e. more than 10 cows) the opposite holds. This suggests that some of the dairy farms in the size class with 3 to 9 cows will go out of business, or scale down, or scale up.

Figure 1: Dairy farms in Poland, 1995-2006

3 AN INSTRUMENTAL VARIABLE GENERALIZED CROSS ENTROPY MARKOV CHAIN

The Markov chain approach is very suitable when the only data available are count data in the form of observable proportions or aggregates rather than data at the level of micro units. Movements from state to state are represented by a stochastic process and are typically modelled by estimating the so-called Markov transition probabilities. It is often the case that the proportions/count data are only available for the total aggregate and not for the net shifts, so that the number of unknowns in terms of transition probabilities to be estimated might exceed the number of available data points (i.e. ill-posed problem). In addition, the proportions/count data may be potentially correlated (i.e. ill-conditioned problem). In this context, the maximum entropy (ME) algorithm developed in Golan *et al*. (1996), Fomby and Carter Hill (1997) and Mittelhammer *et al*. (2000) is a suitable candidate for extracting the maximal signal from an initial 'out-of-focus' problem. Fraser (2000) used maximum entropy estimators to estimate the demand for meat in the United Kingdom under severe multicollinearity problems. He showed that maximum entropy estimators relying on minimal underlying distributional assumptions perform well where traditional econometric approaches are unsatisfactory.

Our paper is based on a GCE formalism which is founded on the directed divergence or minimal discriminability principles of Kullback (1959) and Good (1963). GCE is suitable when some 'educated' guesstimates based on previous data, experiments or economic theory are available (i.e. prior estimates). As discussed by Golan (2002), GCE is an information theory distance measure of the information contained in the posterior estimates as compared to the information contained in the prior estimates. Out of all the feasible solutions, GCE selects the one that minimizes the divergence between the data and the priors, the final solution being the closest to the data and priors. Considering the dynamic farm growth process in a Markov problem, it seems likely that farm growth can be explained by non-stationary effects. Several economic variables are then expected to affect the unknown transition probabilities[1]. Applying the formulation as developed in Golan and Vogel (2000) and Courchane *et al*. (2000)[2], it is possible to assess the impact of key variables on the Markov transition probabilities, therewith potentially improving the explanatory power of the model. In formalizing the problem, the non-stationary GCE Markov problem can be formulated as follows:

$$\min I\left(p_{lk}, q_{lk}, w_{tkh}, u_{tkh}\right) = \sum_{l}\sum_{k} p_{lk} \ln\left(p_{lk} / q_{lk}\right) + \sum_{t}\sum_{k}\sum_{h} w_{tkh} \ln\left(w_{tkh} / u_{tkh}\right) \qquad (1)$$

subject to the following constraints:

$$\sum_{t} z_{tn} y_{tk} = \sum_{t}\sum_{l} z_{tn} x_{tl} p_{lk} + \sum_{t} z_{tn} e_{tk}\ , \quad \forall n = 1,\ldots,N, \text{ and } \forall k = 1,\ldots,K \qquad (2)$$

with

$$e_{tk} = \sum_{h} V_{tkh} w_{tkh} \qquad (3)$$

and

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1 For example, a literature review suggests that out of all possible covariates the following appear likely to affect the transition probabilities of dairy farms: technological shift, milk price, feed price, dairy cow stock price (see Goddard *et al*., 1993; Zepeda, 1995b; Karantininis, 2002).

2 One limitation of this approach is that the type of covariates cannot differ across the different Markov states.

$$\sum_k p_{lk} = 1 \tag{4}$$

$$\sum_h w_{tkh} = 1 \tag{5}$$

Equation (1) represents the GCE criterion which minimizes the divergence between the data in the form of posterior transition probabilities p_{lk} and the transition priors q_{lk}[3]; p_{lk} denotes the probability a farm in size class *l* at time *t* will move to size class *k* at time t+1. Probabilities p_{lk} are elements of a $L \times K$ squared matrix of transition probabilities where *l, k* =1,..., *K* and q_{lk} are the counterpart prior elements; w_{tkh} are the elements of a $TKH \times 1$ vector of error posterior probabilities and u_{tkh} are the counterpart prior elements. Equation (2) represents the Markov data consistency constraints, where y_{tk} are the elements of a $TK \times 1$ vector of known proportions falling in the *k*-th Markov states in time (t+1), x_{tl} are the elements of a $TL \times 1$ vector of known proportions falling in the *l*-th Markov states in time (t). The covariates z_{tn}, which operate like instrumental variables, form a $T \times N$ matrix, explaining the non-stationarity effects[4].

The error term e_{tk}, included in equation (2), is reparameterized as given by equation (3), following the classical maximum entropy formalism (Golan *et al.*, 1996, pp. 107-110), where $\mathbf{V}_{tk}$ is an *H*-dimensional vector of support points and $\mathbf{w}_{tk}$

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[3] By analogy, the GCE criterion also minimizes the divergence between the error in the form of posterior probabilities w_{tkh} and the priors u_{tkh} where u_{tkh} are taken to be uniform since no prior information is available on the error term.

[4] The alternative simpler Markov stationary problem can be obtained by simply withdrawing the covariates z_{tn} from equation (2).

is an H-dimensional vector of proper probabilities with $H \geq 2$[5]. Given that each Markov state can be characterized by a different variance, a specific definition of support bounds for each Markov size class is desired. In such a case, specification of a common and constant variance for each Markov states can lead to relatively large support bounds being specified for size classes where the variance is relatively small. The consequence is that the estimates of the transition probabilities for these size classes are likely to converge to the prior estimates and underutilize the information present in the sample data. To avoid this, variances are specified per size class, following the statistical model presented in Golan *et al*. (1996, pp. 182-185). By so doing, different error support bounds are specified for each Markov state relying on the 'three sigma' rule of Pukelsheim (1994) based on the empirical standard deviation of y_k. Equation (4) represents the set of additivity constraints for the required Markov row constraint, while equation (5) does so for the proper probabilities of the reparameterized error. All proper probabilities of signal and noise are required to be non-negative $(\mathbf{p}, \mathbf{w}) >> 0$. The minimization of (1) subject to (2) - (5) yields the following solutions for the estimated values of $\tilde{p}_{lk}$ and $\tilde{w}_{tkh}$ (Golan and Vogel, 2000, pp. 458-459):

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[5] When defining the $\mathbf{v}_{tk}$ vector, there are several options. One is to set $\mathbf{V}_{tk} = [-1, \ldots, 0, \ldots, 1]$ given that the Markov states are expressed in terms of proportions/shares and y_{tk} and x_{tl} follow in a range between zero and one. A second option is to set $\mathbf{v}_{tk} = [-1/K\sqrt{T}, \ldots, 0, \ldots, 1/K\sqrt{T}]$ where K is the number of states and T number of years as suggested in Golan and Vogel (2000), Courchane *et al*. (1991), Karantininis (2002). Both options, although empirically plausible, assume a common and constant variance matrix across the different Markov states.

$$\tilde{p}_{lk} = \frac{q_{lk}\exp\left[\sum_t\sum_n \tilde{\lambda}_{nk} z_{tn} x_{tl}\right]}{\sum_k q_{lk}\exp\left[\sum_t\sum_n \tilde{\lambda}_{nk} z_{tn} x_{tl}\right]} = \frac{q_{lk}\exp\left[\sum_t\sum_n \tilde{\lambda}_{nk} z_{tn} x_{tl}\right]}{\Omega_l\left(\tilde{\lambda}_n\right)} \tag{6.a}$$

and

$$\tilde{w}_{tkh} = \frac{u_{tkh}\exp\left[\sum_n \tilde{\lambda}_{nk} z_{tn} V_{tkh}\right]}{\sum_h u_{tkh}\exp\left[\sum_n \tilde{\lambda}_{nk} z_{tn} V_{tkh}\right]} = \frac{u_{tkh}\exp\left[\sum_n \tilde{\lambda}_{nk} z_{tn} V_{tkh}\right]}{\Psi_k\left(\tilde{\lambda}_n\right)} \tag{6.b}$$

where u_{tkh} are taken to be uniform with $u_{tkh} = 1/H$. A condensed version of the Lagrange problem for the IV-GCE estimator is provided in Appendix A.

The estimation procedure allows for the possibility of non-zero covariances following the one-step GCE-SUR as described by Golan *et al*. (1996, p. 186). In contrast to the two-stage estimation procedure usually applied in conventional estimation procedures, the unknown elements of the covariance matrix are now jointly estimated with the unknown Markov transition probabilities. The one-step GCE-SUR requires the following additional consistency constraints to be added during the estimation:

$$\frac{1}{T}\sum_{t=1}^{T} e_{tk} e_{tg} = \delta_{kg}\left[\left(\frac{1}{T}\sum_{t=1}^{T} e_{tk} e_{tk}\right)\left(\frac{1}{T}\sum_{t=1}^{T} e_{tg} e_{tg}\right)\right]^{1/2}, \text{ for } k \neq g \tag{7}$$

where $\delta_{kg}^2 = \sigma_{kg}^2 / \sigma_{kk}\sigma_{gg}$. The unknown covariance correlation coefficient δ_{kg} is simultaneously estimated without the need to be reparameterized with the rest of the unknowns for each pair $k \neq g$, and $k, g = 1,\ldots, K$.

The relative information content of the estimated parameters can be evaluated through the normalized entropy measure described in Golan *et al*. (1996, p.93). The measure is defined for values between zero and one, with values approaching zero in the case of no uncertainty and values approaching one in the case of perfect uncertainty (i.e. uniform distribution). Additional entropy statistics used in the paper are the so-called entropy ratio and an analogous entropy Chi-square measure, both described in Golan and Vogel (2000, pp. 454-455).

In an instrumental variable GCE (IV GCE) Markov approach, non-stationary effects can be determined by the following elasticity that determines the cumulative effects of a unit change in each covariate z_{tn} on y_{tk}, the vector of proportion falling in the *k*-th Markov state in time (t+1), as given by Karantininis (2002, p. 10):

$$\eta_{kn}^{y} = \frac{\partial y_{kt}}{\partial z_{tn}} \frac{\bar{z}_{tn}}{\bar{y}_k} = \frac{\bar{z}_n}{\bar{y}_k} \sum_l \left[\tilde{p}_{lk} \bar{x}_l^2 \left(\tilde{\lambda}_{nk} - \sum_k \tilde{p}_{lk} \tilde{\lambda}_{nk} \right) \right] \quad (8)$$

Appendix B recovers the probability elasticities for the IV-GCE problem from which the composite elasticity in equation (8) is derived.

Following the Markov formalism based on the Markov equilibrium distribution and absorbing states notions (Judge and Swanson, 1962, pp. 58-59), it is possible to compute several indicators such as the mean number of years for a farm being in a transient Markov state before it is absorbed in an absorbing state, as well as the probability that a transient Markov state will end up in an absorbing state. The projections of farm numbers were obtained in two steps. In the first step, the Markov transition probability matrix was multiplied by itself *n* times in order to obtain the transition probability matrix during *n* time periods. In the second step, individual elements of the transition probability matrix were multiplied by the number of farms present in their respective size class in the base year used for projections.

4 DATA AND PRIOR INFORMATION

We used aggregate data on the size distribution of private farms with dairy cows in Poland. The farms had been classified according to their herd size classes. The data cover the period from 1995 to 2006 and allow the recovery of the number of dairy farms in eight[6] farm size classes: 1 cow, 2 cows, 3-9 cows, 10-29 cows, 30-49 cows, 50-99 cows, 100-199 cows, > 200 cows (Krawiecka, 2006). In order to account for exit and entry, an additional size class was defined, containing the 'inactive farms' and 'potential entrants' ($l, k = 0$). Data were normalized by a common scalar equal to the maximum number of farms contained in the aggregate transition counts. In order to capture potential non-stationary effects on the Markov transition probabilities, several explanatory variables (such as raw milk price, feeding cost, etc.) were used, but because of parsimony and the limited number of observations in the data finally only the trend variable z_{t1} was kept.

The researcher may follow several principles in order to best approximate the farm size growth and to guess or estimate the probability of a farm being in a given size class. In order to avoid data mining and ensure efficiency in estimation, wherever possible the prior information should be derived from sources independent from the sample data. In this study, previous research was examined and the lessons (general patterns) drawn from this formed the basis of the prior information used (see Table 1)[7]. The prior information on Markov transition probability estimates may be one of three types: the probability of a farm persisting in the same farm size class

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6 Nine farm size classes if the artificial entry and exit class is included.

7 A recent example neglecting this independence requirement is Stokes (2006).

(i.e. persistence), the probability of a farm entering and/or exiting the sector (i.e. entry/exit), and the probability of moving to another farm size class (i.e. net shifts).

Persistence

- Table 1 provides an overview of the estimated persistence probabilities reported in dairy sector and other agricultural sector studies. Although the studies found in the literature are not directly comparable (different countries, different sectors, different definitions of size class, and different time span) it appears that on average about 82.5 per cent of dairy farms persist in the same size class from one period to another. More detailed analysis of these studies revealed that persistence is generally lower for small farm size classes as compared to large farm size classes. Based on these findings in the literature, the priors on the diagonal transitional probabilities were set, moving from the top left corner to the lower right corner of the transition probability matrix from 0.80 to 0.90 (i.e. $p_{lk} = 0.80 \quad l = k$ for $l, k = 2,3,4$ and $p_{lk} = 0.90 \quad l = k$ for $l, k = 5,\ldots 8$).

Table 1: Transition probability estimates: Literature overview

Entry/Exit:

- As regards exit, the literature shows two basic results: small farms are more likely to exit than large farms (see also earlier comment), and the smaller the farm, the higher the probability of exit. Combining this with the already specified priors on persistence (which was set to 0.8 for small farms) the priors on the exit probabilities p_{10}, p_{20} and p_{30} were set to 0.20, 0.15 and 0.10 respectively.

- With respect to entry, in all the studies shown in Table 1, the total number of enterprises shows a clear tendency to decline over time. Generally, very little information was known about entering farms, let alone about the probabilities of entrance in different size classes. Given this finding and the character of our data, which required us to focus on net transitions (net entry), it was decided to specify no positive priors on any entry probabilities $(p_{0k} = 0, \forall k \neq 0)$. Since by definition $\sum_k p_{0k} = 1$, these priors on entry also imply that once a farm has gone out of business it will stay out of business (see previous remark about the Entry/Exit size class as an absorbing state and the prior estimate $p_{00} = 1$).

Net Shifts:

- As regarding the net shifts, one pattern observed from the literature is that farms show a tendency to develop gradually. This implies that the probability a farm will move from its current size class to an adjacent size class is generally higher than the probability it will move to more distant size classes. A second finding is that there is usually a 'switch' size class, below which farms show a tendency to decline and ultimately go out of business, whereas above this size class, farms expand their business. This finding is probably to do with the farms being predominantly family businesses and therefore with farm succession being tied to the family cycle (e.g. ageing farmers with no successors are likely to gradually downsize their business). Another explanatory factor might be that farms need to be a certain critical size in order to be considered 'viable', i.e. be able to finance expansion relying on internally generated savings and also be able to acquire external credit (see Swinnen and Mathijs, 1997; Tonini and Jongeneel, 2002). Reviewing previous studies it appeared that which size class is the tipping-point size class is generally country- and case-specific (depending, for example, also on the specified number and width of size classes). Our prior estimate of the 'switch' size class is therefore based on the particular sample considered and set equal to the size class with 3 to 9 cows (see also Figure 1). Our prior for the farms in this size class is that they have a fifty–fifty probability of moving up or down a class ($p_{32} = p_{34} = 0.05$, i.e. uninformative priors). Farms in larger size classes are assumed to have a 0.10 probability of moving up to the adjacent size class, whereas farms in size classes under the 'switch' class are assumed to have the same probability of moving down to the next size class (conditional on prior assumptions previously made about exit for the lower size classes). The prior assumptions made so far imply that most of the lower and upper off-diagonal elements of the transition probability matrix have prior expectations equal to zero (see Disney *et al.* (1988), Zepeda (1995) for a similar approach).

5 ESTIMATION RESULTS AND DISCUSSION

The IV GCE Markov model was estimated including a trend capturing for structural change. The normalized signal entropy $S(\tilde{\mathbf{p}})$ for the system was 0.663 whereas the normalized noise entropy $S(\tilde{\mathbf{w}})$ for the system was 0.971. The information index $I(\tilde{\mathbf{p}})$ or pseudo-R^2 for the signal was 0.337. The estimated $\chi^2_{\sim\,(K\text{-}1)}$ statistic was 0.416, indicating that the estimated transition probabilities did not statistically differ from the priors at five per cent significance level. A similar result was obtained when computing the signal entropy ratio (i.e. only considering the signal distribution) which was equal to 2.324. The Jarque-Bera test revealed that at five percent significance level the hypothesis of normally distributed errors could not be rejected (Verbeek, 2004, p. 185). Statistical testing, at least for the signal part, was done under negative degrees of freedom, given that Kx(K-1) independent[8] transition probabilities had to be estimated, which only had K total aggregate data of finite size categories for T transitions. However, the estimates were fairly robust to changes in the prior magnitude[9].

Even though the power of statistical tests can be weakened when there are negative degrees of freedom, several facts can be drawn from the above results. The computed statistics suggest that the data did not push the final estimates too far from the prior, which indicates either that the data signal is poor, or that the prior estimates conform to the data. This finding is also related to the negative number of degrees of freedom. Table 2 presents the estimated IV GCE Markov model (i.e. non-stationary model).

The estimated transition probability matrix itself already provides insight into the dynamic adjustment of dairy farms. For example, during the period considered there was a strong tendency for farms to persist in the same size class from one year to the next (see transition probabilities on the diagonal containing elements p_{kk}). The off-diagonal elements of the transition matrix provide information on the extent to which dairy farms are going to scale up or down. For example, from one period to the next, about 2 per cent of all farms with 10-29 cows will probably grow into dairy farms with 30-49 cows. In Table 2 the cumulative effects of the trend z_{t1} on the

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8 This is obtained by subtracting from the KxK transition probability matrix the K row adding-up condition in equation (4).

9 For a given prior configuration we carried out several estimations by changing the prior magnitude by only one digit each time. This did not change the final estimates appreciably. To save space, results are not reported here, but they are available upon request from the authors.

number of dairy farms y_{tk} in terms of elasticity is presented in the last row. The trend impact found implies that over time there is a contraction in the farms with 1-9 cows and an increase in the remaining farms. The trend also has a positive impact on the number of farms in the inactive size class (Exit). Our results fit in with Sznajder (2002, p. 253) who shows that in order to have full return from the engaged capital, including rent of the land, a Polish dairy farm needs to have a herd of at least 10-15 dairy cows. This suggests that the minimum efficient size of dairy farms, minimizing the per unit costs, or the minimum locus on the long-run average costs level for farms is a herd size of 10 or more cows.

Table 2: IV GCE-SUR Markov transition probabilities and non-stationary effects

Table 3 reports the estimated mean number of years in each transient state for each non-absorbing state (i.e. transient periods) as well as the probabilities of absorption for each non-absorbing state into the two absorbing states (i.e. absorption probabilities). These estimates provide an additional indicator of the rate of change in the number of dairy farms by herd size class. Thus for a dairy farm with 10-29 dairy cows, the mean number of years before absorption is about 50, whereas for a dairy farms with 2 cows the mean number of years before absorption is about 6. This suggests that the rate of change is faster for the small dairy farms than for the medium and large dairy farms. From the last two columns of Table 3 it also appears that in equilibrium the majority of the dairy farms with 1 and 9 cows will leave the sector, whereas the dairy farms belonging to the remaining size states will continue in dairying. More precisely, only 16 per cent of the dairy farms with 3-9 cows will persist in the dairy sector, whereas 84 per cent are expected to leave the sector.

Table 3: Estimated transient periods and absorption probabilities

Finally, the estimated Markov transition probability matrixes were used to make several projections of the number of dairy farms in the coming decade. In order to assess the predictive power of the estimated Markov models, projected values and actual values were first compared for the most recent available year (i.e. 2006). We compared two types of models: the IV GCE Markov model estimated with SUR, hereinafter called IV GCE-SUR (i.e. non-stationary model) and the similar model without the inclusion of the trend (i.e. stationary model). In addition, for each type of model we compared the model with the priors as defined in Section 4 with a model estimated using uniform (i.e. non-informative) priors. In terms of projections, the best performance was obtained for the IV GCE-SUR model with non uniform priors. In addition, from our results it appears useful to impose some sort of prior information on the estimated Markov transition probabilities, given the relatively low projection power of the models estimated with uniform priors.

Table 4: Dairy farm size distribution: projected versus actual numbers for 2006

The estimated IV GCE-SUR model predicts the total aggregate number of dairy farms reasonably well, although the model tends to overestimate the number of farms in most of the size classes – except for the farms with 2, 30-49 or 100-199 cows, where the model underestimates the total number of farms. This is mainly attributable to the effect of net shifts from one size class to the adjacent size class. Table 5 provides the projections associated with the IV GCE-SUR model. As can be seen it is predicted that by 2013 about 47 per cent of the number of dairy farms active in 2007 will have left the sector (*ceteris paribus*).

Table 5: Projected dairy farm size distribution (IV GCE-SUR)

6 Milk projections and milk quotas in Poland

Based on the estimated projected dairy farm size distribution, the associated aggregate Polish milk supply was calculated. In order to do so, several simplifying assumptions were made on the average number of cows per farm of a certain size class, as well as the autonomous growth of milk yield. In addition, it was assumed that milk was being delivered by farms with more than 10 cows as well as by a proportion of the farms with 3 to 9 cows. Similarly it was assumed that the remaining milk produced from farms with 3 to 9 dairy cows was allocated to direct sales and home consumption. Milk projections were calibrated for the base year 2006. In order to compare the supply with the quota, the milk supply was corrected for the actual fat content. For a more detailed summary of the assumptions, see Appendix C. The milk projections are presented in Figure 2, which shows that direct sales will decline over time and also that milk deliveries are expected to grow slightly. This growth is attributable to restructuring in the sector as well as to genetic improvements in milk yields.

Figure 2: Milk production projections in Poland (2006=100)

When Poland joined the EU in May 2004, its milk production became subject to a milk quota system (following Council Regulation (EC) No 1788/2003), which was effectively implemented in 2006. Reference quantities were determined for deliveries to dairies and for direct sales; they amounted 8.8 and 0.2 million tons respectively. In addition, the CEECs which joined the EU in 2004 were granted a special restructuring reserve in order to take into account the restructuring process in dairy production, in particular the shift from direct sales to deliveries. According to the

Commission Regulation (EC) No 607/2007, a restructuring reserve of about 416 thousand tons was granted to Poland in June 2007, thereby increasing the delivery quota.

Our supply projections were related to two milk quota scenarios. The first scenario represents the status quo milk deliveries and direct sales quota allocation. The second scenario considers a gradual phasing-out of milk quotas, which could be part of a 'soft-landing strategy' before the expected removal of milk quotas in 2015 (e.g. Fischer-Boel, 2007). Phasing-out is assumed to take place by a 2 percent per annum quota increase, starting from 2008 and continuing until 2015. Although hypothetical, such a scenario might well be considered in next year's 'Health Check' evaluation of the Common Agricultural Policy. Figure 3 provides the percentage overrun for milk deliveries under the two different scenarios. Whereas under the status quo the milk quotas are expected to be binding and overrun from 2008 onwards, with the 'soft-landing' scenario they appear to be only binding after 2010. In addition under this 'soft-landing' scenario the percentage of overrun on milk deliveries is less than 10 percent at maximum, and about one third of the percentage of overrun under the status quo.

Figure 3: Percentage overrun for direct sales of milk

7 Conclusions

The projections showed that the number of dairy farms will continue to decline in the coming decade, although with an increase in the number of medium and large farms. The size class with the largest average annual growth rate will be farms with 50-99 cows. The small dairy farms (i.e. semi-subsistence farms) will continue to exit from the sector although their relative share in the total number of dairy farms will tend to persist. It is estimated that on average, a subsistence dairy farms with 1-2 cows will persist for 7 years before absorption. In addition, only dairy farms with at least 10-29 cows and about 16 per cent of the dairy farms with 3-9 cows are expected to survive at the Markov equilibrium. Overall, our findings show that Poland is likely to be characterized by a polarized dairy farm structure with at one extreme a persistent fringe of subsistence and semi-subsistence self-employed small dairy farms and at the other extreme a growing fringe of commercially-oriented dairy farms.

The aggregated milk supply associated with the farm size distribution projections shows a slight increase of about 2 per cent per annum. Looking at the disaggregated figures for delivered and direct sales, it appears that the quantities delivered are increasing and at the same time the direct sales are decreasing. This is attributable to the restructuring of Polish dairy farms, in which there are a declining number of semi-subsistence farms producing for their own consumption and direct sales and simultaneously there is an increase in the number and scale of commercial farms focusing on deliveries. As regards the status quo scenario, the overrun of milk production makes clear that the current quota provision is likely to impede the farm size restructuring[10]. This will particularly affect the size classes with herd sizes of 10 or more dairy cows. In contrast, gradual phasing-out of the milk quota, as analysed in

[10] If milk quotas are made tradable the impact might be limited or even go the other way. The value of the quota might then also act as an exit payment, inducing some farmers to leave the sector even earlier than initially planned. Moreover, evidence from Dawson and White (1990) on the dairy sector in England and Wales shows that even in the case of binding quota, quasi-fixed factors (i.e. labour, land, machinery, and the herd) go on to adjust, be it more sluggishly than if there are no quotas. As such, the 'temporary' quota constraint faced by Polish farmers might not have a big impact on farm restructuring.

the alternative scenario, could facilitate the current restructuring process. Our findings suggest that in the latter case, an appropriate distribution scheme which allocates additional quota to the larger farms that are likely to expand might be relevant. As quota increases are likely to be accompanied by declines in milk prices, they could limit the funds available for investments and modernisation and thus slow down the speed of adjustment, although the direction of adjustment is unlikely to change.

Although the Markov chain approach appears to be flexible for handling a wide scope of dynamic factors, the predicted evolution of the Polish dairy sector might also be affected by other factors, which are not explicitly included or not sufficiently accounted for in the model. Examples are poorly functioning factor markets (hidden unemployment, dis-functioning land market) and the (vertical) integration with the downstream dairy industry (e.g. Petrick and Weingarten, 2004, p. 6 and Latruffe *et al*. 2004). For these reasons, the actual evolution might be different from the one projected in this paper, in particular for the subsistence sector.

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EU Legislation: Council Regulation (EC) 1788/2003, Commission Regulation (EC) 607/2007.

APPENDIX A: THE LAGRANGE PROBLEM FOR THE IV-GCE ESTIMATOR

For simplicity, scalar notation is used. The corresponding Lagrangian for the IV-GCE estimator as discussed in the main part of the text is given by:

$$\mathbf{L} = -\sum_l \sum_k p_{lk} \ln(p_{lk}/q_{lk}) - \sum_t \sum_k \sum_h w_{tkh} \ln(w_{tkh}/u_{tkh}) + \tag{A.1}$$

$$+\sum_n \sum_k \tilde{\lambda}_{nk} \left[\sum_t z_{tn} y_{tk} - \sum_t \sum_l z_{tn} x_{tl} p_{lk} + \sum_t \sum_h z_{tn} V_{tkh} w_{tkh} \right] +$$

$$+\sum_l \tilde{\mu}_l \left[1 - \sum_k p_{lk} \right] +$$

$$+\sum_t \sum_k \tilde{\rho}_{tk} \left[1 - \sum_h w_{tkh} \right]$$

Through the gradient of the Lagrange function with respect to the unknown to be estimated, the optimal first order conditions are given by:

$$\frac{\partial \mathbf{L}}{\partial p_{lk}} = \ln(p_{lk}/q_{lk}) + 1 - \sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl} - \tilde{\mu}_l = 0 \tag{A.2}$$

$$\frac{\partial \mathbf{L}}{\partial w_{tkh}} = \ln(w_{tkh}/u_{tkh}) + 1 - \sum_n \tilde{\lambda}_{nk} z_{tn} V_{tkh} - \tilde{\rho}_{tk} = 0 \tag{A.3}$$

$$\frac{\partial \mathbf{L}}{\partial \lambda_{nk}} = \sum_t z_{tn} y_{tk} - \sum_t \sum_l z_{tn} x_{tl} p_{lk} + \sum_t \sum_h z_{tn} V_{tkh} w_{tkh} = 0 \tag{A.4}$$

$$\frac{\partial \mathbf{L}}{\partial \mu_l} = 1 - \sum_k p_{lk} = 0 \tag{A.5}$$

$$\frac{\partial \mathbf{L}}{\partial \rho_{tk}} = 1 - \sum_h w_{tkh} = 0 \tag{A.6}$$

Taking the first order condition (A.2) and bringing terms to the right hand side as a function of $\ln(p_{lk}/q_{lk})$ yields:

$$\ln(p_{lk}/q_{lk}) = -1 + \sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl} + \tilde{\mu}_l \quad \text{(A.7)}$$

Taking the exponent of the terms on the left and right hand side yields:

$$p_{lk} = q_{lk} \exp\left(-1 + \sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl} + \tilde{\mu}_l\right) \quad \text{(A.8)}$$

From the Markov problem regularities conditions $\sum_k p_{lk} = 1$ is required, which yields:

$$\sum_k q_{lk} \exp\left(-1 + \sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl} + \tilde{\mu}_l\right) = 1 \quad \text{(A.9)}$$

Through this normalization the $\tilde{\mu}_l$ Lagrange multiplier is lost and the IV-GCE Markov transition probabilities are finally recovered:

$$\tilde{p}_{lk} = \frac{q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)}{\sum_k q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)} \quad \text{(A.10)}$$

Since over all λ_{nk} Lagrange multipliers and corresponding restrictions one is redundant it is therefore convenient to normalize the expression in (A.10) by $\tilde{\lambda}_{nk} = 0$ for each covariate $n = 1,\ldots, N$. This provides the following scaled solutions:

$$\tilde{p}_{lk} = \frac{q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)}{q_{l1} + \sum_{k=2}^{K} q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)} \quad \text{(A.11)}$$

In a similar way it is possible to recover the proper probabilities related to the error term. Taking the first order condition (A.3) and bringing terms to the right hand side as a function $\ln(w_{tkh}/u_{tkh})$ yields:

$$\ln(w_{tkh}/u_{tkh}) = -1 + \sum_{n} \widetilde{\lambda}_{nk} z_{tn} V_{tkh} + \widetilde{\rho}_{tk} \qquad \text{(A.12)}$$

Taking the exponent of the terms on the left and right hand side yields:

$$w_{tkh} = u_{tkh} \exp\left(-1 + \sum_{n} \widetilde{\lambda}_{nk} z_{tn} V_{tkh} + \widetilde{\rho}_{tk}\right) \qquad \text{(A.13)}$$

From the entropy proper probabilities it is required that $\sum_{h} w_{tkh} = 1$, which yields:

$$\sum_{h} u_{tkh} \exp\left(-1 + \sum_{n} \widetilde{\lambda}_{nk} z_{tn} V_{tkh} + \widetilde{\rho}_{tk}\right) = 1 \qquad \text{(A.14)}$$

Again through the normalization one constraint is lost and the IV-GCE error proper probabilities are finally recovered:

$$w_{tkh} = \frac{u_{tkh} \exp\left(\sum_{n} \widetilde{\lambda}_{nk} z_{tn} V_{tkh}\right)}{\sum_{h} u_{tkh} \exp\left(\sum_{n} \widetilde{\lambda}_{nk} z_{tn} V_{tkh}\right)} \qquad \text{(A.15)}$$

APPENDIX B: PROBABILITY ELASTICITIES FOR THE IV-GCE PROBLEM

Here the probability elasticities for the IV-GCE estimator are derived. Three types of impact elasticity are derived: the probability elasticity for an increase in x_{lt}, the probability elasticity for increase in the z_{tn} covariates, the cumulated probability elasticities on the total round count y_{kt} for an increase in the z_{tn} covariates.

- The marginal effect on p_{lk} for a change in x_{tl} is given by:

$$\frac{\partial \tilde{p}_{lk}}{\partial x_{lt}} = \frac{\left(q_{l1} + \sum_k q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)\right) \cdot q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right) \sum_n \tilde{\lambda}_{nk} z_{tn}}{\left(q_{l1} + \sum_k q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)\right)^2} + \tag{B.1}$$

$$- \frac{\sum_k q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right) \cdot \sum_n \tilde{\lambda}_{nk} z_{tn} \cdot q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)}{\left(q_{l1} + \sum_k q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)\right)^2}$$

$$\frac{\partial \tilde{p}_{lk}}{\partial x_{lt}} = \tilde{p}_{lk} \sum_n \tilde{\lambda}_{nk} z_{tn} - \tilde{p}_{lk} \sum_k \sum_n \tilde{p}_{lk} \tilde{\lambda}_{nk} z_{tn} = \tilde{p}_{lk} \left[\sum_n \tilde{\lambda}_{nk} z_{tn} - \sum_k \sum_n \tilde{p}_{lk} \tilde{\lambda}_{nk} z_{tn}\right]$$

Expressing the effect on p_{lk} for a change in x_{tl} in terms of elasticity at sample average yields:

$$\frac{\partial \tilde{p}_{lk}}{\partial x_{tl}} \cdot \frac{\bar{x}_{tl}}{\tilde{p}_{lk}} = \bar{x}_{tl} \left[\sum_n \tilde{\lambda}_{nk} \bar{z}_{tn} - \sum_k \sum_n \tilde{p}_{lk} \tilde{\lambda}_{nk} \bar{z}_{tn}\right] \tag{B.2}$$

- The marginal effect on p_{lk} for a change in z_{tn} is given by:

$$\frac{\partial \tilde{p}_{lk}}{\partial z_{tn}} = \frac{\left(q_{l1} + \sum_k q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)\right) \cdot q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{lt}\right) \cdot \tilde{\lambda}_{nk} x_{tl}}{\left(q_{l1} + \sum_k q_{lk} \exp\left(\sum_n \sum_t \tilde{\lambda}_{nk} z_{tn} x_{tl}\right)\right)^2} + \tag{B.3}$$

$$-\frac{q_{lk}\exp\left(\sum_n\sum_t\tilde{\lambda}_{nk}z_{tn}x_{tl}\right)\cdot\sum_k q_{lk}\exp\left(\sum_n\sum_t\tilde{\lambda}_{nk}z_{tn}x_{tl}\right)\cdot\tilde{\lambda}_{nk}x_{tl}}{\left(q_{l1}+\sum_k q_{lk}\exp\left(\sum_n\sum_t\tilde{\lambda}_{nk}z_{tn}x_{tl}\right)\right)^2}$$

$$\frac{\partial\tilde{p}_{lk}}{\partial z_{tn}}=\tilde{p}_{lk}\tilde{\lambda}_{nk}x_{tl}-\tilde{p}_{lk}\sum_k\tilde{p}_{lk}\tilde{\lambda}_{nk}x_{tl}=\tilde{p}_{lk}x_{tl}\left[\tilde{\lambda}_{nk}-\sum_k\tilde{p}_{lk}\tilde{\lambda}_{nk}\right]$$

Expressing the effect on p_{lk} for a change in x_{lt} in terms of elasticity at sample average yields:

$$\frac{\partial\tilde{p}_{lk}}{\partial z_{tn}}\cdot\frac{\bar{z}_{tn}}{\tilde{p}_{lk}}=\bar{z}_{tn}\bar{x}_{tl}\left[\tilde{\lambda}_{nk}-\sum_k\tilde{p}_{lk}\tilde{\lambda}_{nk}\right] \tag{B.4}$$

- The cumulated effect of each covariate z_{tn} on the total round count y_{tk} is given by

$$\sum_l\frac{\partial\tilde{p}_{lk}}{\partial z_{tn}}\cdot\bar{x}_{tl}=\sum_l\tilde{p}_{lk}\bar{x}_{tl}^2\left[\tilde{\lambda}_{nk}-\sum_k\tilde{p}_{lk}\tilde{\lambda}_{nk}\right] \tag{B.5}$$

That in terms of elasticities translates into:

$$\left(\sum_l\frac{\partial\tilde{p}_{lk}}{\partial z_{tn}}\cdot\bar{x}_{tl}\right)\frac{\bar{z}_{tn}}{\bar{y}_{tk}}=\frac{\partial y_{kt}}{\partial z_{tn}}\frac{\bar{z}_{tn}}{\bar{y}_k}=\frac{\bar{z}_{tn}}{\bar{y}_{tk}}\left[\sum_l\tilde{p}_{lk}\bar{x}_{tl}^2\left[\tilde{\lambda}_{nk}-\sum_k\tilde{p}_{lk}\tilde{\lambda}_{nk}\right]\right] \tag{B.6}$$

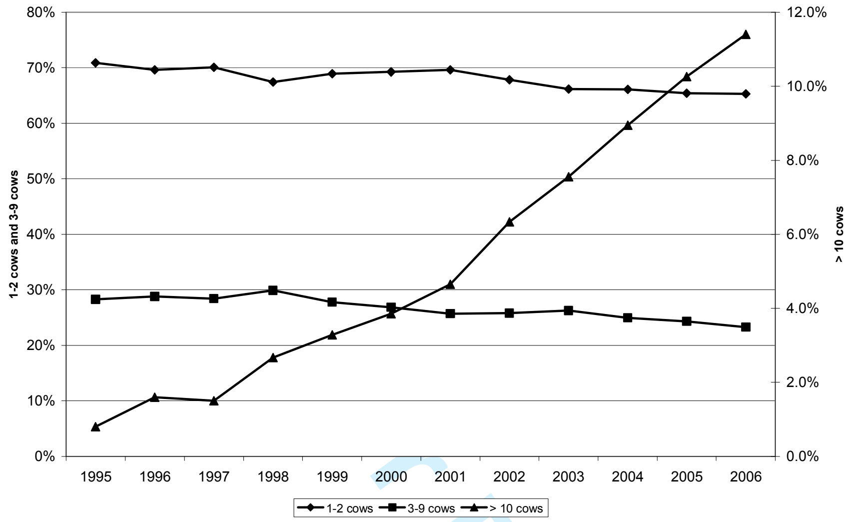
APPENDIX C: MILK PROJECTIONS – MAIN ASSUMPTIONS

Table C.1: Projection assumptions

Year	1	2	3-9	10-29	30-49	50-99	100-199	> 200	Average
Milk Yield Annual growth (%)	0.00	0.00	0.25	0.50	0.75	1.00	1.25	1.50	0.66
Milk Yield/Dairy Cow (Kg/Dairy Cow)	3650	3750	3850	3950	4050	4150	4250	4350	4000
Average Number of Dairy Cows/Farm with Dairy Cows (Hd/Farm)	1	2	6	20	40	75	150	300	74

Figures and Tables in the Text

Figure 2: Dairy farms in Poland, 1995-2006



Note: Percentages are expressed relative to the total number of active dairy farms.

Source: Our calculations based on KRAWIECKA (2006).

Table 6: Transition probability estimates: Literature overview

Authors	Year	Average Estimates	Smallest Class Estimates	Largest Class Estimates	Number of Classes	Transition
			Dairy Studies			
Padberg	1962	0.691	0.733	0.960	4	5 years
Hallberg	1969	0.879	0.768	0.961	5	annual
Keane	1991	0.756	0.360	0.945	7	6 years
Zepeda	1995	0.901	0.877	0.944	3	annual
Stokes	2006	0.898	0.805	0.999	6	annual
			Other Studies			
Judge and Swanson	1962	0.511	0.412	0.672	6	annual
Krenz	1964	0.862	0.804	1.000	6	5 years
Lee et al.	1965	0.650	0.473	0.572	4	annual
Ethridge et al.	1985	0.957	0.919	0.986	5	annual
Edwards et al.	1985	0.687	0.781	0.813	8	4 years
Garcia et al.	1987	0.836	0.930	0.929	11	annual
Disney et al.	1988	0.605	0.400	0.732	4	5 years
Karantininis	2002	0.531	0.386	0.768	18	annual

Note: Estimates may reflect different transition period lengths, as indicated by the last column.

Source: Our calculations, based on estimates from the literature.

Table 7: IV GCE-SUR Markov transition probabilities and non-stationary effects

Class	Exit	1	2	3-9	10-29	30-49	50-99	100-199	> 200	$S(\mathbf{p}_i)$
Entry	1.000									1.000
1	0.118	0.882								0.727
2	0.116	0.054	0.829							0.919
3-9	0.063		0.044	0.872	0.021					0.722
10-29					0.980	0.020				0.302
30-49						0.919	0.081			0.862
50-99							0.984	0.016		0.254
100-199								0.989	0.011	0.183
> 200									1.000	1.000
z_{t1}	0.011	-0.007	-0.002	-0.007	0.011	0.047	0.003	0.132	2.524	

Note: $S(\mathbf{p}_i)$ is the normalized entropy measure for the signal part of the estimated parameters.

Source: Our estimates.

Table 8: Estimated transient periods and absorption probabilities

Class	1	2	3-9	10-29	30-49	50-99	100-199	0	> 200
1	8.447							1.000	0.000
2	2.689	5.865						1.000	0.000
3-9	0.919	2.005	7.825	8.182	2.030	10.164	15.240	0.836	0.164
10-29				49.980	12.402	62.087	93.091	0.001	0.999
30-49					12.403	62.089	93.094	0.001	0.999
50-99						62.089	93.094	0.001	0.999
100-199							93.098	0.001	0.999

Note: The last two columns of the table report the absorption probabilities.

Source: Our estimates.

Table 9: Dairy farm size distribution: projected versus actual numbers for 2006

1	2	3-9	10-29	30-49	50-99	100-199	> 200	Total
				IV GCE-SUR				
286690	124949	148573	68203	5591	1155	140	42	635343
2.47	*-5.37*	*1.15*	*5.99*	*-6.43*	*3.34*	*-7.19*	*21.05*	*0.74*
				IV GCE-SUR (Uniform Prior)				
183155	111209	120992	37372	4275	1184	253	69	458508
-34.54	*-15.77*	*-17.63*	*-41.92*	*-28.46*	*-15.88*	*51.34*	*82.05*	*-27.30*
				GCE-SUR				
292110	126837	153170	67985	5564	1146	127	41	646979
-4.40	*-3.94*	*4.28*	*5.65*	*-6.88*	*-18.63*	*-24.15*	*8.85*	*2.59*
				GCE-SUR (Uniform Prior)				
252441	154765	167159	22858	1779	1286	105	22	600415
-9.78	*17.21*	*13.80*	*-64.48*	*-70.23*	*-8.67*	*-37.21*	*-41.48*	*-4.79*
				Actual 2006				
279791	132037	146887	64350	5975	1408	167	38	630653

Note: Percentage deviations are reported in italics.

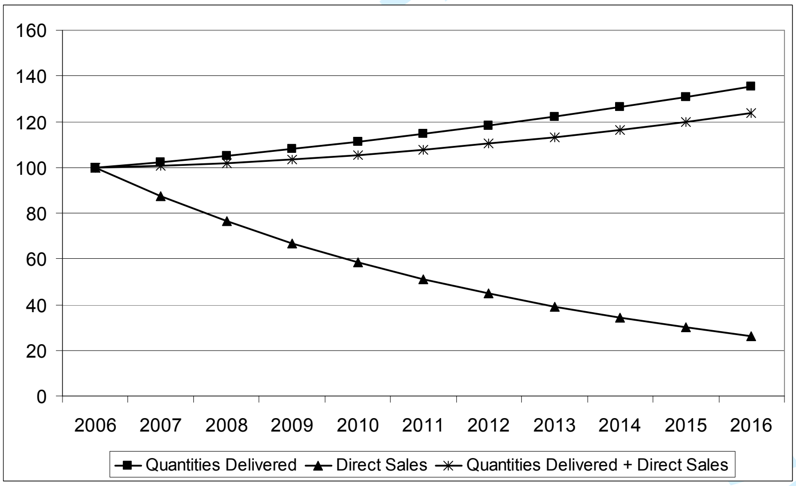
Source: Our estimates.

Table 10: Projected dairy farm size distribution (IV GCE-SUR)

Year	1	2	3-9	10-29	30-49	50-99	100-199	> 200	Total
2007	253833	115943	128116	66135	6781	1867	188	40	572902
2008	230074	101772	111744	67492	7557	2384	216	42	521281
2009	208359	89303	97464	68480	8298	2955	252	44	475155
2010	188538	78335	85009	69149	8999	3576	297	47	433950
2011	170468	68693	74146	69544	9657	4244	351	50	397153
2012	154015	60221	64671	69703	10270	4955	415	54	364303
2013	139049	52779	56406	69662	10837	5703	489	58	334982
2014	125452	46245	49198	69448	11357	6485	573	62	308819
2015	113109	40510	42911	69088	11830	7296	668	68	285480
2016	101917	35478	37428	68603	12259	8132	774	74	264664
Average Annual Growth Rates (%)									
	-10.1	-13.23	-13.7	0.4	6.6	16.3	15.7	6.8	-8.6

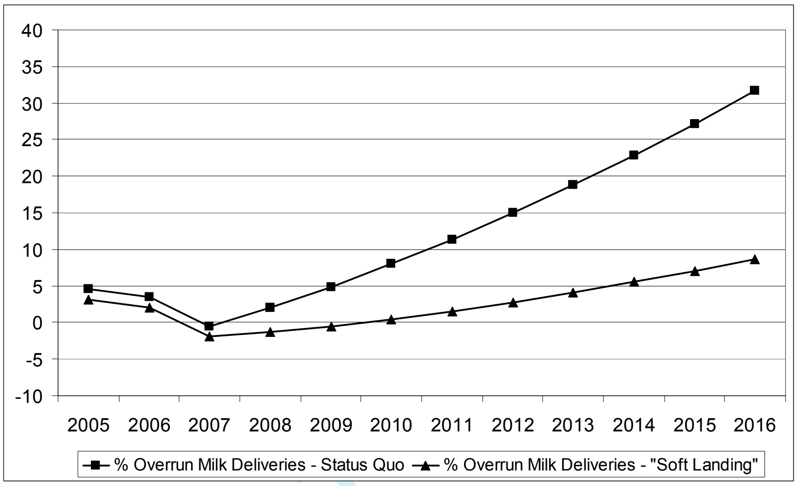
Source: Our estimates.

Figure 2: Milk production projections in Poland (2006=100)



Source: Our projections based on projected dairy farm size distribution.

Figure 3: Percentage overrun for direct sales of milk



Source: Our projections based on projected dairy farm size distribution.