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Rolling-sampled parameters of ARCH and Levy-stable models

Short Title: ARCH & Levy-stable rolling samples

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Abstract

In this paper an asymmetric autoregressive conditional heteroskedasticity (ARCH) model is applied to some well-known financial indices (DAX30, FTSE20, FTSE100 and SP500), using a rolling sample of constant size, in order to investigate whether the values of the estimated parameters of the model change over time. Although, there are changes in the estimated parameters reflecting that structural properties and trading behaviour alter over time, the ARCH model adequately forecasts the one-day-ahead volatility. A simulation study is run to investigate whether the time variant attitude holds in the case of a generated ARCH data process revealing that either in that case the rolling-sampled parameters are time-varying. The rolling analysis is also applied to estimate the parameters of a Levy-stable distribution. The empirical findings support that the stable parameters are also time-variant.

Keywords: ARCH model, GED distribution, Leverage effect, Levy-stable distribution, Rolling sample, Spill over, Value at risk

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The usual disclaimer applies.

1. Introduction

In the recent literature, regarding the description of the characteristics of financial markets, one can find a vast number of specifications of both ARCH and Stochastic Volatility (SV) processes that have been considered for. However, the SV models[1] are not as popular as the ARCH processes in applied studies. The purpose of the present study is to apply an asymmetric ARCH model to some well known financial indices, using a rolling sample of constant size, in order to observe the changes over time in the values of the estimated parameters. A thorough investigation is conducted by comparing the parameters of the full-sampled estimated model to the parameters of the rolling sub-sample estimated models. We conclude that the values of the estimated parameters change over time, indicating a data set that alters across time reflecting the information that financial markets reveal.

In ARCH modelling, the distribution of stock returns has fat tails with finite or infinite unconditional variance and time dependent conditional variance. Estimation of stable distributions is an alternative approach in modelling the unconditional distribution of returns. Thus, we adopt the estimation procedure of McCulloch (1986) and the parameters of the Levy-stable distribution are estimated at each of a sequence of points in time, using a rolling sample of constant size. The empirical findings suggest that the parameters of the unconditional distribution are also not constant over time.

The data set used consists of the DAX30, FTSE20, FTSE100 and SP500 continuously compound rate of daily returns. The period covered for the DAX30 is from January 14th 1992, for the FTSE20 from January 3rd 1996, for the FTSE100 from January 9th 1992 and for the SP500 from January 7th 1992 to July 5th 2002, respectively. A thorough investigation is conducted by comparing the parameters of the full-sampled estimated model to the parameters of the rolling sub-sample estimated models.

The paper is divided in eight sections. Section 2 lays out the asymmetric ARCH model that is applied in the Greek stock market. In section 3 the estimated parameters

[1] The reader who is interested in SV models is referred to Barndorff-Nielsen et al. (2001), Chib et al. (1998), Ghysels et al. (1996), Harvey and Shephard (1993), Jacquier et al. (1994, 1999), Shephard (1996, 2004), Taylor (1994).

of the rolling sub-samples are presented, while the rolling-sampled parameters of the asymmetric ARCH model for the DAX30, FTSE100 and SP500 stock indices are discussed in section 4. In section 5, we examine whether the changes in the rolling-sampled estimated parameters are related with i) the specific structure of the applied asymmetric ARCH model, ii) the sample size, iii) the maximum likelihood estimation method, or iv) the initial values of the likelihood algorithm. Also, in section 6, a simulation study examines whether the parameters are time-varying in the case of a generated ARCH process. In section 7 the unconditional distribution of returns is estimated and the phenomenon of time-variant parameters is observed in the Levy-stable distribution. Finally, in section 8 we summarize the main conclusions.

2. *An asymmetric ARCH model for the Greek stock market*

A wide range of proposed ARCH models is covered in surveys such as Bera and Higgins (1993), Bollerslev et al. (1992), Bollerslev et al. (1994), Degiannakis and Xekalaki (2004), Gouriéroux (1997), Hamilton (1994), Li et al. (2001), Palm (1996) and Poon and Granger (2003). Unambiguously, ARCH models provide accurate volatility forecasts. The Nobel price award to R.F. Engle for ARCH volatility modeling is the uncontested proof of the contribution of ARCH models in time series and econometric modelling (Diebold 2003, 2004). A plethora of studies applied ARCH models to predict future volatility by updating the available information set at each of a sequence of points in time. Among others, Brooks and Persand (2003), Angelidis et al. (2004), Giot and Laurent (2003) predict Value-at-Risk (VaR) measures. Andersen and Bollerslev (1997a, 1997b, 1998), Andersen et al. (1999a), Andersen et al. (1999b), Andersen et al. (2000a), Andersen et al. (2000b), Andersen et al. (2003), Andersen et al. (2004), Barndorff-Nielsen and Shephard (1998), Bollerslev and Wright (2000) and Ebens (1999) demonstrate that for empirically relevant ARCH specifications the forecasts correlate closely with the intra-day realized volatility, while Degiannakis and Xekalaki (2001), Engle et al. (1997) and Noh et al. (1994) use ARCH processes to forecast volatility of options.

An ARCH process, $\varepsilon_t(\theta)$, can be presented as

$$\varepsilon_t(\theta) = z_t \sigma_t(\theta)$$
$$z_t \overset{i.i.d.}{\sim} f\left(E(z_t) = 0, V(z_t) = 1\right) \tag{1}$$
$$\sigma_t^2(\theta) = g\left(\sigma_{t-1}, \sigma_{t-2}, \ldots; \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots; \upsilon_{t-1}, \upsilon_{t-2}, \ldots\right),$$

where θ is a vector of unknown parameters, $f(.)$ is the density function of z_t, $g(.)$ is a linear or non-linear functional form and υ_t is a vector of predetermined variables included in information set I at time t. By definition, $\varepsilon_t(\theta)$ is serially uncorrelated with mean zero, but with a time varying conditional variance equal to $\sigma_t^2(\theta)$, or $\varepsilon_t(\theta) \mid I_{t-1} \sim f\left(0, \sigma_t^2(\theta)\right)$. Engle (1982) in his seminal study assumed that z_t are normally distributed, whereas Bollerslev (1987) and Nelson (1991) introduced the student t and the generalized error distributions, respectively, in order to model the leptokurtosis of the conditionally distributed $\varepsilon_t(\theta)$. In the case of modeling a leptokurtotic and asymmetric conditional distribution of $\varepsilon_t(\theta)$, the generalized t (Bollerslev et al. 1994), the skewed generalized t (Theodossiou 1998), the skewed student t (Lambert and Laurent 2000) and the skewed generalized error (Bali 2005 and Theodossiou 2002) distributions were utilized. Since very few financial time series have a constant conditional mean of zero, an ARCH model can be presented in a regression form by letting ε_t be the unpredictable component of the conditional mean

$$y_t = E(y_t \mid I_{t-1}) + \varepsilon_t, \tag{2}$$

where $y_t = \ln(P_t / P_{t-1})$ denotes the continuously compound rate of return from time $t-1$ to t, and P_t is the asset price at time t.

In order to investigate the characteristics of the Athens Stock Exchange (ASE) market, we apply an ARCH model of the following form:

$$y_t = \mu_0 + \mu_1 \sigma_t^2 + \left(\mu_2 + \mu_3 e^{-\sigma_t^2/\mu_4}\right) y_{t-1} + \varepsilon_t, \tag{3}$$
$$\varepsilon_t = z_t \sigma_t,$$

$$z_t \overset{i.i.d.}{\sim} GED(0,1;v),$$

$$\ln(\sigma_t^2) = a_0 + \ln(1 + N_t\delta_0) + \frac{1}{(1-\Delta_1 L)}\left(\Psi_1 L\left(\left|\frac{\varepsilon_t}{\sigma_t}\right| - E\left|\frac{\varepsilon_t}{\sigma_t}\right|\right) + \gamma L\frac{\varepsilon_t}{\sigma_t}\right),$$

where $GED(0,1;v)$ denotes the generalized error distribution (GED), v is the tail thickness parameter of the GED, L is the lag operator and N_t is the number of non-trading days preceding the t^{th} day. The density function of a GED random variable is given by

$$f(z_t) = \frac{ve^{-2^{-1}\left|\frac{z_t}{\lambda}\right|^v}}{\lambda 2^{v+1/v}\Gamma(1/v)}, \quad (4)$$

for $-\infty < z < \infty$, $0 < v \le \infty$, where $\Gamma(.)$ denotes the gamma function and

$$\lambda \equiv \left(\frac{2^{-2v^{-1}}\Gamma(1/v)}{\Gamma(3/v)}\right)^{1/2}. \quad (5)$$

The conditional variance specification has the form of the exponential GARCH, or EGARCH model, which is suggested by Nelson (1991). The EGARCH model captures the asymmetric effect exhibited in financial markets, as the conditional variance, σ_t^2, depends on both the magnitude and the sign of lagged innovations. Assuming GED distributed innovations with EGARCH specification for the conditional variance we take into account that i) the unconditional distribution of innovations is symmetric but with excess kurtosis and ii) their conditional distribution is asymmetric and leptokurtotic. Parameter γ allows for the leverage effect. The leverage effect, first noted by Black (1976), refers to the tendency of changes in stock returns to be negatively correlated with changes in returns volatility, i.e. volatility tends to rise in response to 'bad news' and to fall in response to 'good news'. If $\gamma = 0$ then a positive surprise, $(\varepsilon_t > 0)$, has the same effect on volatility as a negative surprise, $(\varepsilon_t < 0)$. If $-1 < \gamma < 0$, a positive surprise increases volatility less than a negative surprise. If $\gamma < -1$, a positive surprise actually reduces volatility while a negative surprise increases volatility. Moreover, the logarithmic transformation ensures that the forecasts of the variance are non-negative. Parameter δ_0 allows us to explore the contribution of non-trading days to volatility. According to Fama (1965) and French and Roll (1986) information that accumulates when financial markets are closed is reflected in prices after the markets reopen. The conditional mean is modeled such as to capture the relationship between investors' expected return and risk[2] (μ_1), the non-synchronous trading effect[3] (μ_2), and the inverse relation between volatility and serial correlation[4] (μ_3).

Model (3) is expanded in order to take into account the phenomenon of volatility spill over from one market to the other[5]. For y_t denoting the daily return of the FTSE20 index, the conditional variance is modeled in the following form

$$\ln\left(\sigma_t^2\right) = a_0 + \ln\left(1 + N_t \delta_0\right) + \frac{1}{\left(1-\Delta_1 L\right)}\left(\Psi_1 L\left(\left|\frac{\varepsilon_t}{\sigma_t}\right| - E\left|\frac{\varepsilon_t}{\sigma_t}\right|\right) + \gamma L \frac{\varepsilon_t}{\sigma_t}\right) + \delta_1 \ln\left(\sigma^2_{SP500,t-1}\right) + \delta_2 \ln\left(\sigma^2_{DAX30,t-1}\right), \tag{6}$$

where the parameters δ_1 and δ_2 account for the volatility spill over from U.S. and German stock markets to the ASE market, respectively. The ASE in co-operation with the London-based Financial Times Stock Exchange has introduced the FTSE20 index, which includes the 20 largest companies ranked by their capitalization and trading volume. The daily conditional volatilities of the SP500 and DAX30 index returns at time

[2] The relationship between investors' expected return and risk was presented in an ARCH framework, by Engle et al. (1987). They introduced the ARCH in mean model where the conditional mean is an explicit function of the conditional variance.

[3] According to Campbell et al. (1997), 'The non-synchronous trading or non-trading effect arises when time series, usually asset prices, are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other, possible irregular lengths.'

[4] LeBaron (1992) found a strong inverse relation between volatility and serial correlation for SP500, CRSP and Dow Jones returns. As LeBaron stated, it is difficult to estimate μ_4 in conjunction with μ_3 when using a gradient type of algorithm. So, μ_4 is set to the sample variance of the series.

[5] Engle et al. (1990) evaluated the role of the information arrival process in the determination of volatility in a multivariate framework providing a test of two hypotheses: heat waves and meteor showers. Using meteorological analogies, they supposed that information follows a process like a heat wave so that a hot day in New York is likely to be followed by another hot day in New York but not typically by a hot day in Tokyo. On the other hand, a meteor shower in New York, which rains down on the earth as it turns, will almost surely be followed by one in Tokyo. Thus, the heat wave hypothesis is that the volatility has only country specific autocorrelation, while the meteor shower hypothesis states that volatility in one market spills over to the next. See also Kanas (1998).

t are shown by $\sigma^2_{SP500,t}$ and $\sigma^2_{DAX30,t}$. These are regarded as exogenous variables that have been estimated according to framework (3).

The data set consists of the FTSE20 index daily returns in the period from January 3rd, 1996 to July 5th, 2002 and the conditional variance of the DAX30 and SP500 returns from January 2nd, 1996 to July 5th, 2002. In order to estimate the conditional variance of the DAX30 and SP500 indices, their daily returns are used for the periods of January 14th, 1992 to July 5th, 2002, and January 7th, 1992 to July 5th, 2002, respectively. Figure 1a plots the FTSE20 daily returns.

INSERT FIGURE 1 ABOUT HERE

Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt (1963) algorithm that is computed as

$$\theta^{(i+1)} = \theta^{(i)} + \left(\sum_{t=1}^{T} \frac{\partial l(\theta)_t^{(i)}}{\partial \theta} \frac{\partial l(\theta)_t^{(i)}}{\partial \theta'} - \eta \mathrm{I} \right)^{-1} \sum_{t=1}^{T} \frac{\partial l_t^{(i)}}{\partial \theta}, \tag{7}$$

where $\theta = \left(\mu_0, \mu_1, \mu_2, \mu_3, a_0, \delta_0, \Psi_1, \Delta_1, \gamma, \delta_1, \delta_2, v\right)$ is the parameter vector to be estimated, $l(\theta)_t = \ln\left(f\left(z_t(\theta)\right)\right) - \frac{1}{2}\ln\left(\sigma_t^2(\theta)\right)$ is the log likelihood contribution for each observation t, I is the identity matrix and η is a positive number chosen by the algorithm. The process is repeated until the maximum of the percentage changes in the coefficients is smaller than 0.001%.

INSERT TABLE 1 ABOUT HERE

Table 1 presents the estimated parameters of model (3). The estimated risk premium is positively, though weakly related with the conditional variance (coefficient μ_1). The coefficient μ_2, which allows for first order autocorrelation, is insignificant. Daily serial correlation is inversely related to the conditional volatility of the FTSE20 index, which is consistent with the results of LeBaron (1992) (coefficient μ_3). The estimated value of coefficient γ is –0.064 and statistically insignificant, which implies that surprises of same magnitude but opposite signs have the same effect on volatility. The estimated parameters δ_1 and δ_2 are insignificant, indicating that there is no evidence of volatility spillover from Frankfurt and Chicago indices to ASE market. The estimated coefficient v is 1.335 with a standard error of 0.043, so the distribution of the standardized innovations is significantly thicker tailed than the normal distribution. The estimated value of δ_0 is about 0.187 and statistically significant. Thus, a non-trading day contributes less than a fifth as much to volatility as a trading day.

3. *Rolling-sampled parameters of the asymmetric ARCH model*

Our purpose is to examine if the estimated parameters of the asymmetric ARCH model change over time and whether there is any impact of time-varying estimated parameters on volatility forecasting accuracy. The ARCH process is estimated, at each of a sequence of points in time, using a rolling sample of constant size equal to 1000 trading days, a sample size that is preferred[6] by the majority of applied studies.

We produce one-day-ahead conditional volatility predictions for the trading days of 11th January 2000 to 5th July 2002. The daily conditional volatility of the SP500 and DAX30 returns, were estimated by framework (3), using, also, a rolling sample of constant size equal to 1000. Since the ARCH model is estimated at each point in time, we use the maximum likelihood estimates at time $t-1$ as starting values for the iterative maximization algorithm at time t. Figure 2 depicts the rolling-sampled estimated parameters of the model as well as the ± 2.06 times the conditional standard deviation confidence interval of the parameters estimated using the full data sample. From visual inspection, the estimated rolling parameters are, clearly, out of the confidence interval bounds in many cases. Table 2 presents the percentage of rolling-sampled estimations, which are outside of the 95% confidence interval. That is, the rolling estimations of coefficient Δ_1 are outside the 95% confidence interval of the full-sampled Δ_1 estimation in the 54.40% of the cases. An important characteristic, which is extracted from the

[6] Engle et al. (1993), Engle et al. (1997), Noh et al. (1994), Angelidis et al. (2004) note that the size of the rolling sample turns out to be rather important while Frey and Michaud (1997), Hoppe (1998) and Degiannakis and Xekalaki (2006) comment that the use of short sample sizes generates more accurate volatility forecasts, since it incorporates changes in trading behaviour more efficiently.

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rolling-sampled estimated parameters, is the fact that the estimated values do not fluctuate in a mean reverting form but they change gradually. Sudden changes of the values of the rolling estimated parameters, which are characterized by a mean reverting form, should indicate an improperly maximum likelihood estimation procedure. On the other hand, gradual changes of the estimated coefficients indicate a data set that alters from time to time, forcing us to believe that the values of the estimated parameters reflect the information that financial markets reveal.

INSERT FIGURE 2 ABOUT HERE

INSERT TABLE 2 ABOUT HERE

The percentage of estimated rolling parameters that are statistically different from the parameter values estimated using the full data sample, as presented in Table 3, is also indicative for the changes of the estimated values across time. There are four parameters, whose rolling-sampled estimators differ statistically significant from their full-sampled estimators in more than 10% of the trading days.

INSERT TABLE 3 ABOUT HERE

The values of the rolling parameters indicate that the characteristics of the ASE market change during the examined period. Table 4 presents the percentage of the trading days for which the estimated rolling parameters are statistically insignificant. The coefficient δ_0, which accounts for the contribution of non-trading days to volatility, is not statistically different to zero for the 57.07% of the rolling cases, although it is significant in the full sample, at 1% level of significance. Therefore, the statistical inference based on the estimated values of parameter δ_0 would lead to an insignificant contribution of non-trading days to volatility in about the half number of trading days. Moreover, in the full sample, FTSE20 index is characterized by an inverse relation between volatility and serial correlation. On the contrary, the values of rolling μ_3 are not different to zero in most of the cases. However, there are parameters, such as Ψ_1, Δ_1 and v, whose estimations are statistically significant in both full and rolling sample. On the other hand, the values of the coefficients μ_2, γ and δ_1 are statistically insignificant in both rolling and full sampled estimations. Hence, we may infer that the values of the estimated parameters change across time, reflecting the individual features of particular periods that characterize financial markets.

INSERT TABLE 4 ABOUT HERE

In Figure 3.a, the 95% in-sample confidence interval of the FTSE20 index of daily returns is plotted from 11th January 2000 to 5th July 2002. There are 31 (4.99%) violations of the confidence interval, which reflect a correctly specified model that fits data satisfactory. However, a model that uses a large number of parameters may exhibit an excellent in-sample fit but a poor out-of-sample performance. Studies such as Heynen and Kat (1994), Hol and Koopman (2000) and Pagan and Schwert (1990) examined a variety of volatility prediction models with in-sample and out-of-sample data sets. We investigate the possibility that model over-fitting can be occurred and evaluate the performance of the estimated ARCH model by computing the out-of-sample forecasts. In the sequel, the one-day-ahead 95% prediction intervals are constructed. Let us compute the one-day-ahead conditional mean, $y_{t+1|t} \equiv E\left(y_{t+1}\left(\theta^{(t)}\right) | I_t\right)$, and conditional variance, $\sigma^2_{t+1|t} \equiv E\left(\varepsilon^2_{t+1}\left(\theta^{(t)}\right) | I_t\right)$, using the following formulas:

$$y_{t+1|t} = \mu_0{}^{(t)} + \mu_1{}^{(t)}\sigma^2_{t+1|t} + \left(\mu_2{}^{(t)} + \mu_3{}^{(t)} e^{-\sigma^2_{t+1|t} / \mu_4{}^{(t)}}\right) y_t\,,$$

$$\ln\left(\sigma^2_{t+1|t}\right) = a_0^{(t)} + \ln\left(1 + N_{t+1}\delta_0^{(t)}\right) + \frac{1}{\left(1 - \Delta_1^{(t)} L\right)}\left(\Psi_1^{(t)}\left(\left|\frac{\varepsilon_{t|t}}{\sigma_{t|t}}\right| - E\left|\frac{\varepsilon_t}{\sigma_t}\right|\right) + \gamma^{(t)}\frac{\varepsilon_{t|t}}{\sigma_{t|t}}\right) + \delta_1^{(t)}\ln\left(\sigma^2_{SP500,t|t}\right) + \delta_2^{(t)}\ln\left(\sigma^2_{DAX30,t|t}\right), \tag{8}$$

where $\theta^{(t)} \equiv \left(\mu_0^{(t)}, \mu_1^{(t)}, \mu_2^{(t)}, \mu_3^{(t)}, a_0^{(t)}, \delta_0^{(t)}, \Psi_1^{(t)}, \Delta_1^{(t)}, \gamma^{(t)}, \delta_1^{(t)}, \delta_2^{(t)}, v^{(t)}\right)$ is the parameter vector that is estimated using the sample data set which is available at time t, $\varepsilon_{t|t} \equiv E\left(\varepsilon_t | I_t\right)$ denotes the prediction error conditional on the information set that is available at time t, and $\sigma_{t|t} \equiv \sqrt{E\left(\varepsilon_t^2 | I_t\right)}$ is the conditional standard deviation which is computed by the ARCH model, in equation (6), using the information set available at time t. Note that for $z_t \sim GED(0,1;v)$, the expected value of its absolute price is equal to $E\left|\varepsilon_t \sigma_t^{-1}\right| = \Gamma\left(2/v^{(t)}\right)\left(\Gamma\left(1/v^{(t)}\right)\Gamma\left(3/v^{(t)}\right)\right)^{-1/2}$.

Figure 3.b plots the one-day-ahead 95% prediction interval, which is constructed as the one-day-ahead conditional mean ± 2.06 times the conditional standard deviation, both measurable to I_t information set, or $y_{t+1|t} \pm GED\left(0,1;v^{(t)},0.025\right)\sigma_{t+1|t}$, where $GED\left(0,1;v^{(t)},a\right)$ is the $100(1-a)$ quantile of the GED distribution. Hence, each trading day, (t), the next trading day's, ($t+1$), prediction intervals are constructed, using only information available at current trading day, t. There are 29 observations (4.67%) outside the 95% prediction intervals.

INSERT FIGURE 3 ABOUT HERE

For a more formal method of evaluating forecasting adequacy, we apply two hypotheses tests that measure the forecasting accuracy in a VaR framework. One-day-ahead VaR at a given probability level, a, is the next trading day's predicted amount of financial loss of a portfolio, or $VaR_{t+1|t}(1-a) = GED\left(0,1;v^{(t)},a\right)\sigma_{t+1|t}$. Kupiec (1995) introduced a likelihood ratio statistic for testing the null hypothesis that the proportion of confidence interval violations is not larger than the VaR forecast. The test statistic, which is asymptotically X_1^2 distributed, is computed as $LR_K = 2[\ln((n/N)^n(1-n/N)^{N-n})$ $-\ln(p^n(1-p)^{N-n}]$, where $n \equiv \sum_{i=1}^{N} d\left(y_{t+1} < VaR_{t+1|t}(a/2)\right) + d\left(y_{t+1} > VaR_{t+1|t}(1-a/2)\right)$, for $d\left(y_{t+1} < VaR_{t+1|t}(a/2)\right) = 1$ if $y_{t+1} < VaR_{t+1|t}$ and $d\left(y_{t+1} < VaR_{t+1|t}(a/2)\right) = 0$ otherwise, is the number of trading days over the out-of-sample period N that a violation has occurred, and p is the expected frequency of violations. Christoffersen (1998) developed a likelihood ratio statistic that jointly investigates whether i) the proportion of violations is not larger than the VaR forecast and ii) the violations are independently distributed. The statistic is computed as $LR_C = -2\ln((1-p)^{N-n}p^n)$ $+2\ln((1-\pi_{01})^{n_{00}}\pi_{01}{}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}{}^{n_{11}}))$, where $\pi_{ij} = n_{ij}\Big/\sum_j n_{ij}$ and n_{ij} is the number of observations with value i followed by j, for $i,j=0,1$. The values $i,j=1$ denote that a violation has been made, while $i,j=0$ indicates the opposite. Under the null hypothesis, the LR_C is asymptotically chi-squared distributed with two degrees of freedom. The main advantage of Christoffersen's test is that it can reject a VaR model that generates either too many or too few clustered violations. The p-values in testing the null hypothesis of correct proportion of 95% and 99% confidence interval violations are 70.28% and 8.15%, respectively, whereas in the case of Christoffersen's test the p-values are 40.03% and 17.98% for 95%-VaR and 99%-VaR violations, respectively.

Despite the fact that the values of the estimated coefficients change over time, the model adequately forecasts the one-day-ahead volatility. Thus, at least in the case of ASE market, changes in the values of the estimated parameters do not indicate inadequacy of the model in describing the data. On the contrary, model's parameters should be re-estimated on a daily base in order to reflect any changes that have been occurred in the stock market and have been incorporated in the prices of assets.

4. The asymmetric ARCH model for other stock markets

In this section we investigate whether the values of the estimated parameters change over time in other stock markets as well. The asymmetric ARCH model in framework (6) is applied for the DAX30, FTSE100 and SP500 stock indices:

$$y_{A,t}=\mu_0+\mu_1\sigma^2_{A,t}+\left(\mu_2+\mu_3 e^{-\sigma^2_{A,t}/\mu_4}\right)y_{A,t-1}+\varepsilon_t,$$

$$\ln\left(\sigma^2_{A,t}\right)=a_0+\ln\left(1+N_t\delta_0\right)+\frac{1}{\left(1-\Delta_1 L\right)}\left(\Psi_1 L\left(\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right|-E\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right|\right)+\gamma L\frac{\varepsilon_t}{\sigma_{A,t}}\right.$$
$$\left.+\delta_1\ln\left(\sigma^2_{B,t-1}\right)+\delta_2\ln\left(\sigma^2_{C,t-1}\right)\right). \qquad (9)$$

In order to account for the volatility spill over effect from one market to the others, when (9) is estimated for stock market A (for instance SP500), the daily conditional volatilities of stock markets B and C (that is FTSE100 and DAX30 respectively) are regarded as exogenous variables.

Table 5 presents the estimated parameters of the above model (9) for each market separately. The data set used is from January 3rd, 1996 to July 5th, 2002. Briefly discussing the values of the parameters, we note that i) the relation of the conditional variance with the risk premium, although positive, is statistically insignificant (coefficient μ_1), ii) the non-synchronous trading effect is not present in the estimated models (coefficient μ_2) and iii) concerning the case of the SP500 stock index, the daily serial correlation is inversely related to its conditional volatility (coefficient μ_3). Moreover, the leverage effect is not present in the German stock market. On the contrary, for the SP500 and FTSE100 stock indices, the estimated value of parameter γ is statistically significant at 1% level of significance. The volatility spill over effect is statistically significant for the U.K. stock market. Regarding the SP500 index daily returns, there is evidence that volatility spillovers from Frankfurt to Chicago stock market. Finally, for the DAX30 and SP500 cases, parameter v is statistically different to the value of 2 at any level of significance, justifying the use of a thick-tailed distribution. The continuously compounded returns of the underlying indices are plotted in Figures 1.b to 1.d.

INSERT TABLE 5 ABOUT HERE

Following the approach presented in section 3, the rolling parameters of the ARCH models are estimated using a rolling sample of 1000 observations[7]. As in the case of the ASE market, the values of the rolling parameters differ from their full-sampled estimations. Table 6 presents the percentage of rolling parameters, which are outside the 95% confidence interval of the full-sampled parameters. Characteristic examples of the change in the parameter values are Ψ_1 and v for DAX30 as well as Δ_1 for SP500. However, there are rolling parameters which do not change significantly across time, such as γ (leverage effect), and δ_0 (contribution of non-trading days to volatility).

INSERT TABLE 6 ABOUT HERE

[7] Figures of the estimated rolling parameters for the DAX30, FTSE100 and SP500 indices, similar to Figure 2, are available upon request.

Table 7 presents the percentage of rolling-sampled parameters that are statistically different from the parameter values estimated using the full data sample. Although, in the case of the FTSE100 index, only the rolling estimators of Δ_1 parameter differ statistically from their full data sample estimator, in the case of the SP500 index the there are four parameters, which show a statistically significant difference from their full-sampled estimators in more than 20% of the trading days.

INSERT TABLE 7 ABOUT HERE

According to Table 8, which presents the percentage of trading days that the rolling parameters are statistically insignificant, there are parameters whose rolling-sampled estimations are statistically insignificant while their full-sampled estimations are significant. For example, parameters μ_3 and δ_1 for the SP500 index, as well as parameter γ for FTSE100 index, although they appear to be significant in the full sample, almost all their rolling-sampled estimations are insignificant at 5% level of significance. Of course, there are parameters whose estimations are statistically different to zero in both the full sample and the rolling samples (i.e. the parameter Δ_1 for the DAX30 and SP500 indices).

INSERT TABLE 8 ABOUT HERE

Hence, a change in the values of the estimated parameters of the asymmetric ARCH model does not characterize only the Greek stock market. However, although the estimated parameters are time varying, the in-sample and out-of-sample forecasting ability of the model is accurate. There are 19, 17 and 29 cases, or 2.99%, 2.66% and 4.57%, observed returns outside the 95% confidence intervals for the DAX30, FTSE100 and SP500 indices, respectively. On the other hand, the one-day-ahead 95% prediction intervals exclude 22, 21 and 32 observations, or 3.46%, 3.29% and 5.04% for the DAX30, FTSE100 and SP500 indices, respectively[8]. As far as the adequacy in one-day-ahead VaR forecasting is concerned, both Kupiec's and Christoffersen's tests do not reject the null hypothesis of correct proportion of violations in all the cases, except for

[8] Figures, similar to Figure 3, that depict the in-sample 95% confidence interval and the one-day-ahead 95% prediction intervals for the DAX30, FTSE100 and SP500 indices are also available upon request.

the 95%-VaR of the FTSE100 index. In the case of Kupiec's test the p-values are 6.08%, 3.45% and 96.37% for 95%-VaR and 13.63%, 56.56% and 52.70% for 99%-VaR, for the DAX30, FTSE100 and SP500 indices, respectively. Testing the null hypothesis of whether the violations are equal to the expected ones as well as if they are independent, we observe that the relative p-values are 16.42%, 0.15% and 95.19% in the 95%-VaR case and 32.51%, 7.10% and 73.92% in the 99%-VaR case, for the DAX30, FTSE100 and SP500 indices, respectively. So, in most of the cases, the examined model produces adequate VaR forecasts.

5. *Extensions*

In order to investigate whether the phenomenon of time-variant values of estimated parameters, in the four stock markets considered, is related to a specific structural characteristic of the model specification, we estimate another ARCH specification. Degiannakis (2004) and Giot and Laurent (2003) used an ARCH model with the APARCH volatility specification of Ding et al. (1993) and the skewed student-t distribution for the standardized innovations. We estimated such a model for our datasets and found similar qualitative results. The estimated parameters are time varying.

We have also re-estimated model (9) using alternatively i) larger sample sizes of rolling parameters, ii) the BHHH algorithm (Berndt et al. 1974) instead of the Marquardt algorithm in estimating the maximum likelihood parameters and iii) the same starting values at each point in time, instead of the estimates at time $t-1$ as starting values for the likelihood algorithm at time t. Despite the slight changes occurred in each case, the rolling parameters are time-variant for all cases.

6. *Rolling-sampled parameters from simulated processes*

A simulation study could shed light in rolling-sampled estimated parameters behaviour. A series of simulations is run in order to investigate if the time-variant attitude holds even in the case of an ARCH data generating process. We generate a series of 32000 values from the standard normal distribution, $z_t \overset{i.i.d.}{\sim} N(0,1)$. Then an AR(1)GARCH(1,1) process is created, $\{y_t\}_{t=1}^{32000}$, where $y_t = 0.0005 + 0.15 y_{t-1} + \varepsilon_t$, by multiplying the i.i.d. process with a specific conditional variance form $\varepsilon_t = z_t \sqrt{\sigma_t^2}$, for $\sigma_t^2 = 0.0005 + 0.05\varepsilon_{t-1}^2 + 0.90\sigma_{t-1}^2$. The AR(1)GARCH(1,1) model is applied on the $\{y_t\}_{t=1002}^{32000}$ generated data. Dropping out the first 1001 data, maximum likelihood rolling-sampled estimates of the parameters are obtained by numerical maximization of the log-likelihood function, using a rolling sample of constant size equal to 1000. According to Table 9, about 58% of the 30000 conditional variance rolling-sampled parameters are outside the 95% confidence interval of the parameters estimated using the whole sample set of the 30000 simulated data. The procedure is repeated for an AR(1)EGARCH(1,1) conditional variance form, $\ln\left(\sigma_t^2\right) = a_0 + a_1 \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln\left(\sigma_{t-1}^2\right)$, but the results are robust to the choice of the conditional variance specification.

A series of 32000 values from the first order autoregressive process are also produced. The AR(1) process is created as $y_t = 0.0001 + 0.12 y_{t-1} + z_t$, for $z_t \overset{i.i.d.}{\sim} N(0,1)$. Dropping out the first 1001 data, 30000 maximum likelihood rolling-sampled estimates of the parameters are also obtained. As far as the case of the AR(1) process is concerned, we infer that the rolling estimated parameters are time-invariant, as on average the 5% of the estimated rolling parameters are outside the 95% confidence levels.

Both the AR(1)GARCH(1,1) and the AR(1) processes were simulated for various sets of parameters, but there are no qualitative differences to the fore mentioned conclusions. Moreover, a series of simulations were repeated i) for ARCH volatility forms without any conditional mean specification, ii) based on estimation procedures of the most well known packages, EVIEWS® 4.1 and OX-G@ARCH® 3.4, iii) for larger rolling samples of 5000 values, iv) for non-overlapping data samples, but there are no qualitative differences in any of these cases[9].

So, the simulation study provides evidence that the time-variant attitude of rolling-sampled parameters estimations characterizes not only the examined data sets but the ARCH data generating process itself as well.

INSERT TABLE 9 ABOUT HERE

7. Rolling-sampled parameters from a Levy-stable distribution

The study of the shape of stock returns can be dated back to 1900 where Louis Bachelier in his Ph.D. Thesis, 'The Theory of Speculation', first presented it – see Bachelier (1900). Mandelbrot (1963) and Fama (1965) made the first re-examination of the unconditional distribution of stock returns. Mandelbrot (1963) concluded that price changes can be characterized by a stable Paretian distribution with a characteristic exponent, a, less than two, thus exhibiting fat tails and infinite variance. Fama (1965) examined the distribution of thirty stocks of the Dow Jones Industrial Average; his results were consistent with Mandelbrot's. Thereafter, it has been accepted that the stock returns distributions are fat-tailed and peaked. In an attempt to model the unconditional distribution of stock returns several researchers have considered alternative approaches. See for example, Blattberg and Gonedes (1974), Bradley and Taqqu (2002), Clark (1973), Kon (1984), McDonald (1996), Mittnik and Rachev (1993), Rachev and Mittnik (2000). De Vries (1991), Ghose and Kroner (1995) and Groenendijk et al. (1995) demonstrate that ARCH models share many of the properties of Levy-stable distribution but the true data generating process for an examined set of financial data is more likely ARCH than Levy-stable. A number of studies, such as Liu and Brorsen (1995), Mittnik et al. (1999), Panorska et al. (1995), Tsionas (2002), examined the properties of ARCH models with Levy-stable distributed innovations.

[9] All the simulation studies are available to the readers upon request.

The probability density function of a stable distribution cannot be described in a closed mathematical form. By definition, a univariate distribution function is stable if and only if its characteristic function has the form

$$\varphi(t) = \exp\left\{ i\delta t - \gamma |t|^{a} \left(1 - i\beta \left(\frac{t}{|t|} \right) \omega(t,a) \right) \right\}, \tag{10}$$

where $i = \sqrt{-1}$, $t \in R$ with

$$\omega(t,a) = \begin{cases} \tan\left(\dfrac{\pi\alpha}{2}\right), & a \neq 1 \\ \dfrac{-2}{\pi}\log|t|, & a = 1. \end{cases}$$

The particular distribution represented by its characteristic function is determined by the values of four parameters: a, β, γ and δ. The parameter a, $0 < \alpha \leq 2$, is called the characteristic exponent. It measures the thickness of the tails of a stable distribution. The smaller the value a, the higher the probability in the distribution tails. If $a < 2$ then we have thicker tails than the tails of normal distribution. Thus, stable distributions have thick tails and consequently increase the likelihood of the occurrence of large shocks. The skewness parameter β, $-1 \leq \beta \leq 1$, is a measure of the asymmetry of the distribution. The distribution is symmetric, if $\beta = 0$. For $\beta > 0$, the distribution is skewed to the right and for $\beta < 0$, the distribution is skewed to the left. As $|\beta|$ approaches one, the degree of skewness increases. The scale parameter γ, $\gamma > 0$, is a measure of the spread of the distribution. It is similar to the variance of the normal distribution, $\gamma = \sigma/\sqrt{2}$. However, the scale parameter γ is finite for all stable distributions, despite the fact that the variance is infinite for all $a < 2$. The location parameter δ, $-\infty < \delta < +\infty$, is the mean of the distribution, when $a > 1$, and the median for $0 < a \leq 1$. The case of $a = 2$, $\beta = 0$ corresponds to the normal distribution, while $a = 1$, $\beta = 0$ corresponds to the Cauchy distribution.

In estimating the parameters of the stable distribution of index returns, we adopt the estimation procedure suggested by McCulloch (1986). The estimation procedure is a quantile method and works for $0.6 \leq a \leq 2$ and any value of the other parameters. Essentially, McCulloch suggests that if we have a random variable x, which follows a stable distribution and denote the p^{th} quantile of this distribution by $x(p)$, then the population quantile can be estimated by the sample quantile $\hat{x}(p)$. McCulloch's estimator uses five quantiles to estimate a and β as follows:

$$\hat{v}(\alpha) = \frac{\hat{x}(0.95) - \hat{x}(0.05)}{\hat{x}(0.75) - \hat{x}(0.25)} \tag{11}$$

$$\hat{v}(\beta) = \frac{\hat{x}(0.95) + \hat{x}(0.05) - 2\hat{x}(0.50)}{\hat{x}(0.95) - \hat{x}(0.05)} . \tag{12}$$

Since $v(a)$ is monotonic in a and $v(\beta)$ is monotonic in β, we are able to find a and β by inverting $v(a)$ and $v(\beta)$:

$$\hat{a} = g_1(\hat{v}(a), \hat{v}(\beta)), \tag{13}$$

$$\hat{\beta} = g_2(\hat{v}(a), \hat{v}(\beta)). \tag{14}$$

McCulloch tabulated g_1 and g_2 for various values of $v(a)$ and $v(\beta)$. A similar procedure is also applied for the scale and location parameters. An alternative procedure to estimate the parameters of the stable distribution is the regression method proposed by Koutrouvelis (1980).

Following a procedure similar to that of ARCH modelling, the parameters of the stable distribution are estimated, at each of a sequence of points in time, using a rolling sample of constant size equal to 1000 trading days. Thus, the rolling-sampled parameters are estimated for the trading days of January 1996 to July 2002 for the DAX30, FTSE100 and SP500 indices and from January 2000 to July 2002 for the FTSE20 index.

The empirical findings, for the case of the Greek stock market, are graphically summarized in Figure 4, which plots the rolling-sampled estimates of parameters along with the 95% confidence interval of the parameters estimated using the full data sample. Inspection of Figure 4 shows that the estimates of a are less than two. The case of FTSE20 reveals that 92% of the a's rolling-sampled estimates are between 1.44 and

1.55. The parameter β is greater than zero, which implies skewness to the right. The rolling values of β are positive and range from 0.003 to 0.22 but there are not outside the 95% confidence interval for any case[10].

INSERT FIGURE 4 ABOUT HERE

In Table 10, we present the estimates of the parameters of stable distribution based on all data available as well as the standard deviation of the rolling-sampled estimated parameters. The estimates of a do not approach two in any of the examined indices. However, there are estimated rolling parameters that are statistically different from the parameter values estimated using the full data sample. For example, the rolling-sampled estimates of the tail index (a) are statistically different to the full sample estimated parameter in the 51.46% of the trading days for the case of the SP500 index. In 9.59% and 9.42% of the trading days the rolling estimates of parameter β are statistically different to the relevant full-sampled values for the DAX30 and FTSE100 indices, respectively, whereas the location (δ) parameters are time-variant in none of the cases. Another important parameter of the stable distribution, from the point of view of portfolio theory, is the scale parameter, γ. As far as the FTSE20 index is concerned, the rolling-sampled estimates of the scale parameter differ statistically from its full-sampled value in the 56.48% of the trading days. Hence, the parameter estimates, using the full data sample are statistically different from the parameter values estimated using the rolling samples of constant size for one parameter in each index.

INSERT TABLE 10 ABOUT HERE

8. *Discussion*

We estimated an asymmetric ARCH model using daily returns of the FTSE20, DAX30, FTSE100 and SP500 indices and concluded that although the estimated parameters of the model change over time, the model does not lose its ability to forecast the one-day-ahead volatility accurately. Gallant et al. (1991), Stock (1988), Lamoureux and

[10] Figures depicting the rolling-sampled estimates of the parameters for the DAX30, FTSE100 and SP500 indices are available upon request.

Lastrapes (1990) and Schwert (1989) among others have aimed at explaining the economic interpretation of the ARCH process. As Engle et al. (1990) and Lamoureux and Lastrapes (1990) have noted, the explanation of the ARCH process must lie either in the arrival process of news or in market dynamics in response to the news. Based on some earlier work by Clark (1973) and Tauchen and Pitts (1983), Gallant et al. (1991) provided a theoretical interpretation of the ARCH effect. They assumed that the asset returns are defined by a stochastic number of intra-period price revisions and information flows into the market in an unknown rate. As the daily information does not come to the stock market in a constant and known rate, the estimation of the ARCH stochastic process that explains the dynamics of the stock market could be revised at regular time intervals. In our case the ARCH process is estimated using daily returns. Thus, the parameters of the model may be revised on a daily base, because of the observed phenomenon of changes in the estimated parameters. If we used data of higher frequency, i.e. ten-minutes intra-daily returns, the estimated model may be revised more frequent than on a daily base. The change of the estimated values of the parameters has to be further examined in intra-daily high-frequency data sets, on a future research.

Furthermore, the rolling parameter analysis was applied to the unconditional distribution of returns. The empirical results indicate that in all cases - DAX30, FTSE20, FTSE100 and SP500 - the parameters of the asymmetric stable distribution of stock returns change across time. We observed the phenomenon of parameter changing across time for both the conditional (ARCH process) and the unconditional (Levy-stable) distribution of returns.

Altering the method of model estimation, the rolling-sampled parameters remain time-variant. Even in the case of a simulated ARCH process, the property of time varying rolling-sampled parameters holds. To the best of authors' knowledge, this is the first study that investigates the phenomenon of time varying estimated parameters either i) in real-world financial data or ii) in a simulated data generating process. However, the theoretical interpretation of this phenomenon has to be further investigated.

References

Andersen, T. and Bollerslev, T. (1997a). Answering the Critics: Yes, Arch Models Do Provide Good Volatility Forecasts. NBER Working Paper, No. 6023.

Andersen, T. and Bollerslev, T. (1997b). Intraday Periodicity and Volatility Persistence in Financial Markets. *Journal of Empirical Finance*, **4**, 115-158.

Andersen, T. and Bollerslev, T. (1998). DM-Dollar Volatility: Intraday Activity Patterns, Macroeconomic Announcements, and Longer-Run Dependencies. *Journal of Finance*, **53**, 219-265.

Andersen, T., Bollerslev, T., Diebold, F.X. and Labys, P. (1999a). Exchange Rate Returns Standardized by Realized Volatility are (Nearly) Gaussian. NBER Working Paper, No. 7488.

Andersen, T., Bollerslev, T. and Lange, S. (1999b). Forecasting Financial Market Volatility: Sample Frequency vis-à-vis Forecast Horizon. *Journal of Empirical Finance*, **6**, 457-477.

Andersen, T., Bollerslev, T. and Cai, J. (2000a). Intraday and Interday Volatility in the Japanese Stock Market. *Journal of International Financial Markets, Institutions and Money*, **10**, 107-130.

Andersen, T., Bollerslev, T., Diebold, F.X. and Ebens, H. (2000b). The Distribution of Stock Return Volatility. NBER Working Paper, No. 7933.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P. (2003). Modeling and Forecasting Realized Volatility. *Econometrica*, **71**, 529-626.

Andersen, T.G., Bollerslev, T. and Diebold, F.X. (2005). Parametric and Nonparametric Volatility Measurement. In: Yacine Aït-Sahalia and Lars Peter Hansen (Eds.), *Handbook of Financial Econometrics*, Amsterdam, North Holland.

Angelidis, T., Benos, A. and Degiannakis, S. (2004). The Use of GARCH Models in VaR Estimation. *Statistical Methodology*, **1(2)**, 105-128.

Bachelier, L. (1900). Theorie de la Speculation, Gauthier-Villars, Paris, reprinted in 1995, Jaques Gubay (Ed.), Paris. English translation in P.H. Cootner, *The Random Character of Stock Market Prices*, MIT press.

Bali, T.G. (2005). Modeling the Dynamics of Interest Rate Volatility with Skewed Fat-Tailed Distributions. *Annals of Operations Research*, forthcoming.

Barndorff-Nielsen, O.E. and Shephard, N. (1998). Aggregation and Model Construction for Volatility Models. University of Aarhus and Nuffield College, Oxford, Department of Mathematical Sciences, Manuscript.

Barndorff-Nielsen, O.E., Nicolato, E. and Shephard, N. (2002). Some Recent Developments in Stochastic Volatility Modelling. *Quantitative Finance*, 2, 11-23.

Bera, A.K. and Higgins, M.L. (1993). ARCH Models: Properties, Estimation and Testing. *Journal of Economic Surveys*, **7**, 305-366.

Berndt, E., Hall, B., Hall, R. and Hausman, J. (1974). Estimation and Inference in Nonlinear Structural Models. *Annals of Economic and Social Measurement*, **3**, 653-665.

Black, F. (1976). Studies of Stock Market Volatility Changes. *Proceedings of the American Statistical Association, Business and Economic Statistics Section*, 177-181.

Blattberg, R.C. and Gonedes, N.J. (1974). A comparison of the stable and student distributions as statistical models for stock prices, *Journal of Business*, **47**, 244-80.

Bollerslev, T. (1987). A Conditional Heteroskedastic Time Series Model for Speculative Prices and Rates of Return. *Review of Economics and Statistics*, **69**, 542-547.

Bollerslev, T. and Wright, J.H. (2000). Semiparametric Estimation of Long-Memory Volatility Dependencies: The Role of High Frequency Data. *Journal of Econometrics*, **98**, 81-106.

Bollerslev, T., Chou, R. and Kroner, K.F. (1992). ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence. *Journal of Econometrics*, **52**, 5-59.

Bollerslev, T., Engle, R.F. and Nelson, D. (1994). ARCH Models. In: R.F. Engle and D. McFadden (Eds.), *Handbook of Econometrics*, Volume 4, , Elsevier Science, Amsterdam, 2959-3038.

Bradley, B. and Taqqu, M. (2002). Financial Risk and Heavy Tails. In: S. Rachev (Ed.), *Heavy-tailed distributions in Finance*, North-Holland.

Brooks, C. and Persand, G. (2003). The Effect of Asymmetries on Stock Index Return Value-at-Risk Estimates. *Journal of Risk Finance*, **4(2)**, 29-42.

Campbell, J., Lo, A. and MacKinlay, A.C. (1997). *The Econometrics of Financial Markets*. New Jersey. Princeton University Press.

Chib, S., Kim, S. and Shephard, N. (1998). Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models. *Review of Economic Studies*, **65**, 361-393.

Christoffersen, P. (1998). Evaluating interval forecasts. *International Economic Review*, **39**, 841-62.

Clark, P.K. (1973). A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices. *Econometrica*, **41**, 135-156.

Degiannakis, S. (2004). Volatility Forecasting: Evidence from a Fractional Integrated Asymmetric Power ARCH Skewed-t Model. *Applied Financial Economics*, **14**, 1333-1342.

Degiannakis, S. and Xekalaki, E. (2001). Using a Prediction Error Criterion for Model Selection in Forecasting Option Prices. Athens University of Economics and Business, Department of Statistics, Technical Report, 131. Available online at: http://stat-athens.aueb.gr/~exek/papers/Xekalaki-TechnRep131(2001a)ft.pdf.

Degiannakis, S. and Xekalaki, E. (2004). Autoregressive Conditional Heteroscedasticity Models: A Review. *Quality Technology and Quantitative Management*, **1(2)**, 271-324.

Degiannakis, S. and Xekalaki, E. (2006). Assessing the Performance of a Prediction Error Criterion Model Selection Algorithm in the Context of ARCH Models, *Applied Financial Economics*, forthcoming.

De Vries, C.G. (1991). On the Relation Between GARCH and Stable Processes. *Journal of Econometrics*, **48**, 313-324.

Diebold, F.X. (2003). The ET Interview: Professor Robert F. Engle. *Econometric Theory*, **19**, 1159-1193.

Diebold, F.X. (2004). The Nobel Memorial Prize for Robert F. Engle, prepared at the invitation of the Scandinavian Journal of Economics.

Ding, Z., Granger, C.W.J. and Engle, R.F. (1993). A Long Memory Property of Stock Market Returns and a New Model. *Journal of Empirical Finance*, **1**, 83-106.

Ebens, H. (1999). Realized Stock Volatility. Johns Hopkins University, Department of Economics, Working Paper, No. 420.

Engle, R.F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation. *Econometrica*, **50**, 987–1008.

Engle, R.F., Lilien, D.M. and Robins, R.P. (1987). Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model. *Econometrica*, **55**, 391–407.

Engle, R.F., Ito, T. and Lin, W.L. (1990). Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market. *Econometrica*, **58**, 525-542.

Engle, R.F., Hong, C., Kane, A. and Noh, J. (1993). Arbitrage Valuation of Variance Forecasts with Simulated Options, *Advances in Futures and Options Research*, **6**, 393-415.

Engle, R.F., Kane, A. and Noh, J. (1997). Index-Option Pricing with Stochastic Volatility and the Value of Accurate Variance Forecasts. *Review of Derivatives Research*, **1**, 120-144.

Fama, E.F. (1965). The Behaviour of Stock Market Prices. *Journal of Business*, **38**, 34-105.

French, K.R. and Roll, R. (1986). Stock Return Variances: The Arrival of Information and the Reaction of Traders. *Journal of Financial Economics*, **17**, 5-26.

Frey, R. and Michaud, P. (1997). The Effect of GARCH-type Volatilities on Prices and Payoff-Distributions of Derivative Assets - a Simulation Study, ETH Zurich, Working Paper.

Gallant, A.R., Hsieh, D.A. and Tauchen, G. (1991). On Fitting a Recalcitrant Series: The Pound/Dollar Exchange Rate 1974-83. In: W.A. Barnett, J. Powell, and G. Tauchen (Eds.), *Nonparametric and Semiparametric Methods in Econometrics and Statistics*, Cambridge University Press, Cambridge.

Ghose, D. and Kroner, K.F. (1995). The relationship between GARCH and symmetric stable processes: Finding the source of fat tails in financial data. *Journal of Empirical Finance*, **2**, 225–251.

Groenendijk, P.A., Lucas, A. and de Vries, C.G. (1995). A note on the relationship between GARCH and symmetric stable processes. *Journal of Empirical Finance*, **2**, 253–264.

Ghysels, E., Harvey, A. and Renault, E. (1996). Stochastic Volatility. In: G.S. Maddala (Ed.), *Handbook of Statistics*, Vol. 14, Statistical Methods in Finance, 119-191, Amsterdam, North Holland.

Giot, P. and Laurent, S. (2003). Value-at-Risk for Long and Short Trading Positions. *Journal of Applied Econometrics*, **18**, 641-664.

Gourieroux, C. (1997). *ARCH models and Financial Applications*. Springer-Verlag, New York.

Hamilton, J.D. (1994). *Time Series Analysis*. New Jersey, Princeton University Press.

Harvey, A.C. and Shephard, N. (1993). The Econometrics of Stochastic Volatility. London School of Economics, Discussion Paper, 166.

Heynen, R. and Kat, H. (1994). Volatility Prediction: A Comparison of the Stochastic Volatility, GARCH(1,1), and EGARCH(1,1) Models, *Journal of Derivatives*, Winter, **94**, 50-65.

Hol, E. and Koopman, S. (2000). Forecasting the Variability of Stock Index Returns with Stochastic Volatility Models and Implied Volatility, Tinbergen Institute, Discussion Paper No. 104, 4.

Hoppe, R. (1998). VAR and the Unreal World, *Risk*, **11**, 45-50.

Jacquier, E., Polson, N. and Rossi, P. (1994). Bayesian Analysis of Stochastic Volatility Models. *Journal of Business and Economic Statistics*, **12**, 4, 371-417.

Jacquier, E., Polson, N. and Rossi, P. (1999). Stochastic Volatility: Univariate and Mulrivariate Extensions. CIRANO, Scientific Series, 99s, 26.

Kanas, A. (1998). Volatility Spillovers Across Equity Markets: European Evidence. *Applied Financial Economics*, **8**, 245-256.

Kupiec, P.H. (1995). Techniques for verifying the accuracy of risk measurement models, *Journal of Derivatives*, **3**, 73-84.

Kon, S.J. (1984). Models of stock returns – A comparison. *Journal of Business*, **39**, 147-165.

Koutrouvelis, I.A. (1980). Regression-Type Estimation of the Parameter of Stable Laws. *Journal of the American Statistical Association*, **75**, 918-928.

Lambert, P. and Laurent, S. (2000). Modeling Skewness Dynamics in Series of Financial Data. Institut de Statistique, Louvain-la-Neuve, Discussion Paper.

Lamoureux, G.C. and Lastrapes, W.D. (1990). Heteroskedasticity in Stock Return Data: Volume Versus GARCH Effects. *Journal of Finance*, **45**, 221-229.

LeBaron, B. (1992). Some Relations Between Volatility and Serial Correlations in Stock Market Returns. *Journal of Business*, **65**, 199-219.

Li, W.K., Ling, S. and McAleer, M. (2001). A Survey of Recent Theoretical Results for Time Series Models With GARCH Errors. Institute of Social and Economic Research, Osaka University, Japan, Discussion Paper, 545.

Liu, S.M. and Brorsen, B.W. (1995). Maximum likelihood estimation of a GARCH-stable model. *Journal of Applied Econometrics*, **10**, 273–285.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices, *Journal of Business*, **36**, 299-319.

Marquardt, D.W. (1963). An Algorithm for Least Squares Estimation of Nonlinear Parameters. *Journal of the Society for Industrial and Applied Mathematics*, **11**, 431-441.

McCulloch, J.H. (1986). Simple Consistent Estimates of Stable Distribution Parameters. *Communications in Statistics: Simulation and Computation*, **15**, 1109-1136.

McDonald, J.B. (1996). Probability Distributions for Financial Models. In: G.S. Maddala and C.R. Rao (Eds.), *Statistical Methods in Finance*, vol. 14, *Handbook of Statistics*, Elsevier, Amsterdam.

Mittnik, S. and Rachev, S.T. (1993). Modelling Asset Returns with Alternative Stable Distributions, *Economic Review*, **12**, 261-330.

Mittnik, S., Rachev, S.T., Doganoglu, T. and Chenyao, D. (1999). Maximum Likelihood Estimation of Stable Paretian Models. *Mathematical and Computer Modelling*, **29**, 275-293.

Nelson, D. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, **59**, 347-370.

Noh, K., Engle, R.F. and Kane, A. (1994). Forecasting Volatility and Option Prices of the S&P 500 Index. *Journal of Derivatives*, **(2)**, 17-30.

Pagan, A. and Schwert, G. (1990). Alternative Models for Conditional Stock Volatility, *Journal of Econometrics*, **45**, 267-290.

Palm, F. (1996). GARCH Models of Volatility. In: G.S. Maddala and C.R. Rao (Eds.), *Handbook of Statistics*, vol. 14, Elsevier, Amsterdam, 209-240.

Panorska, A., Mittnik, S. and Rachev, S.T. (1995). Stable GARCH Models for Financial Time Series. *Applied Mathematics Letters*, **8**, 33-37.

Poon, S.H. and Granger, C.W.J. (2003). Forecasting Volatility in Financial Markets: A Review. *Journal of Economic Literature*, **XLI**, 478-539.

Rachev, S. and Mittnik, S. (2000). *Stable Paretian Models in Finance*, John Wiley.

Schwert, G.W. (1989). Why Does Stock Market Volatility Changes Over Time. *Journal of Finance*, **44**, 1115-1153.

Shephard, N. (1996). Statistical Aspects of ARCH and Stochastic Volatility Models. In: D.R. Cox, D.V. Hinkley and O.E. Barndorff-Nielsen (Eds.), *Time Series Models in Econometrics, Finance and Other Fields*, Chapman & Hall, London, 1-67.

Shephard, N. (2004). *Stochastic Volatility: Selected Readings*, Oxford University Press.

Stock, J.H. (1988). Estimating Continuous Time Process Subject to Time Deformation. *Journal of the American Statistical Association*, **83**, 77-85.

Tauchen, G. and Pitts, M. (1983). The Price Variability-Volume Relationship on Speculative Markets. *Econometrica*, **51**, 485-505.

Taylor, S.J. (1994). Modelling Stochastic Volatility. *Mathematical Finance*, **4(2)**, 183-204.

Theodossiou, P. (1998). Financial Data and the Skewed Generalized t Distribution. *Management Science*, **44(12)**, 1650-1661.

Theodossiou, P. (2002). Skewed Generalized Error Distribution of Financial Assets and Option Pricing. Working Paper, School of Business, Rutgers University.

Tsionas, E.G. (2002). Likelihood-Based Comparison of Stable Paretian and Competing Models: Evidence from Daily Exchange Rates. *Journal of Statistical Computation and Simulation*, **72(4)**, 341-353.

Tables and Figures

Table 1. Parameter estimates for the FTSE20 index returns using data from January 3rd, 1996 to July 5th, 2002.

Parameter	Coefficient	Standard error	Coefficient / Standard error
μ_0	-0.000980	0.000516	-1.898440
μ_1	2.852553	1.744956	1.634742
μ_2	0.053237	0.048253	1.103283
μ_3	0.317119	0.112867	2.809660
a_0	-6.832800	1.077622	-6.340627
δ_0	0.187064	0.055310	3.382094
Ψ_1	0.394402	0.019925	19.79393
Δ_1	0.919999	0.023994	38.34286
γ	-0.064062	0.061404	-1.043284
δ_1	0.010323	0.024886	0.414812
δ_2	0.002381	0.023214	0.102560
v	1.335436	0.042741	-15.54980

Notes: With v=1.335, the 97.5% point of the generalized error distribution is 2.06. With v=1.335, the 99.5% point of the generalized error distribution is 2.94.

Table 2. Percentage of rolling-sampled estimated parameters that are outside the 95% confidence interval.

Parameter	Lower/Upper Bound	Percent of estimations below the lower limit	Percent of estimations above the upper limit
μ_0	(-0.002 0.000)	56.48%	0.00%
μ_1	(-1.780 7.485)	0.00%	7.04%
μ_2	(-0.075 0.181)	0.00%	0.00%
μ_3	(0.017 0.617)	0.00%	0.32%
a_0	(-9.694 -3.972)	14.40%	0.48%
δ_0	(0.040 0.334)	0.48%	0.64%
Ψ_1	(0.342 0.447)	12.80%	0.32%
Δ_1	(0.856 0.984)	54.40%	0.00%
γ	(-0.227 0.099)	0.00%	0.00%
δ_1	(-0.056 0.076)	0.00%	5.12%
δ_2	(-0.059 0.064)	32.16%	0.00%
v	(1.222 1.449)	0.48%	26.40%

Table 3. Percentage of rolling-sampled estimated parameters that are statistically different from the parameter values estimated using the full data sample.

Parameter	5% sign. Level	1% sign. Level	Parameter	5% sign. Level	1% sign. Level
μ_0	21.86%	1.29%	Ψ_1	7.40%	0.00%
μ_1	0.96%	0.00%	Δ_1	18.97%	10.13%
μ_2	0.00%	0.00%	γ	0.00%	0.00%
μ_3	0.00%	0.00%	δ_1	0.00%	0.00%
a_0	17.20%	3.86%	δ_2	12.54%	0.16%
δ_0	0.00%	0.00%	v	1.29%	0.00%

Table 4. Percentage of rolling-sampled estimated parameters that are statistically insignificant at 5% and 1% levels of significance.

Parameter	5% sign. Level	1% sign. Level	Parameter	5% sign. Level	1% sign. Level
μ_0	30.06%	76.21%	Ψ_1	0.00%	0.00%
μ_1	32.80%	97.11%	Δ_1	0.00%	0.00%
μ_2	99.84%	100%	γ	100%	100%
μ_3	65.11%	87.78%	δ_1	100%	100%
a_0	0.00%	0.48%	δ_2	89.55%	99.84%
δ_0	27.65%	57.07%	v	0.00%	0.00%

Table 5. Parameter estimates for the DAX30, FTSE100 and SP500 index daily returns (January 3rd, 1996 to July 5th, 2002).

Parameter	Coefficient			Standard error			Coefficient / Standard error		
	DAX30	FTSE100	SP500	DAX30	FTSE100	SP500	DAX30	FTSE100	SP500
μ_0	0.000	-0.000	-0.001	0.001	0.000	0.001	0.596	-0.067	-1.027
μ_1	1.995	1.251	4.297	2.709	3.924	4.013	0.736	0.319	1.071
μ_2	0.024	0.005	-0.100	0.061	0.060	0.062	0.398	0.078	-1.620
μ_3	-0.075	0.144	0.333	0.138	0.126	0.121	-0.544	1.140	2.745
a_0	-9.858	-1.326	-4.059	0.919	0.522	0.685	-10.727	-2.538	-5.929
δ_0	0.095	0.012	0.039	0.050	0.036	0.040	1.880	0.342	0.956
Ψ_1	0.190	0.056	0.060	0.007	0.062	0.044	27.847	0.892	1.378
Δ_1	0.973	-0.001	0.785	0.013	0.185	0.028	73.455	-0.003	28.040
γ	-0.068	-0.108	-0.236	0.079	0.036	0.079	-0.856	-2.969	-2.975
δ_1	-0.008	0.694	0.081	0.012	0.144	0.035	-0.688	4.822	2.295
δ_2	0.004	0.201	0.041	0.012	0.095	0.031	0.386	2.116	1.314
v	1.735	1.858	1.689	0.029	0.095	0.038	-9.137	-1.495	-8.184

Notes: With v=1.735, v=1.858, v=1.689, the 97.5% point of the generalized error distribution are 2.00, 1.98 and 2.00, respectively. With v=1.735, v=1.858, v=1.689, the 99.5% point of the generalized error distribution are 2.70, 2.65 and 2.72, respectively. For the DAX30 index, parameters δ_1 and δ_2 present the volatility spillover from the FTSE100 and SP500 indices, respectively. For the FTSE100 index, parameters δ_1 and δ_2 present the volatility spillover from the DAX30 and SP500 indices, respectively. For the SP500 index, parameters δ_1 and δ_2 present the volatility spillover from the DAX30 and FTSE100 indices, respectively.

Table 6. Percentage of rolling-sampled estimated parameters that are outside the 95% confidence interval. (Values in parenthesis present the lower and upper bounds of the 95% confidence interval).

	DAX30		FTSE100		SP500	
μ_0	(-0.001 0.002)	33.18%	(-0.001 0.001)	24.11%	(-0.002 0.001)	20.66%
μ_1	(-4.989 8.978)	0.00%	(-8.762 11.263)	0.80%	(-5.978 14.572)	16.48%
μ_2	(-0.133 0.182)	0.00%	(-0.148 0.157)	1.28%	(-0.258 0.058)	0.00%
μ_3	(-0.431 0.281)	0.00%	(-0.178 0.465)	12.32 %	(0.022 0.644)	0.48%
a_0	(-12.227 -7.489)	3.20%	(-2.659 -0.007)	16.64%	(-5.812 -2.306)	24.00%
δ_0	(-0.035 0.224)	0.00%	(-0.080 0.105)	0.00%	(-0.065 0.142)	0.00%
Ψ_1	(0.172 0.207)	62.24%	(-0.104 0.215)	0.00%	(-0.052 0.173)	20.96%
Δ_1	(0.939 1.007)	22.08%	(-0.472 0.471)	1.12%	(0.713 0.857)	60.48%
γ	(-0.271 0.136)	0.00%	(-0.201 -0.015)	1.12%	(-0.439 -0.033)	0.48%
δ_1	(-0.038 0.022)	3.04%	(0.327 1.062)	0.48%	(-0.009 0.171)	0.00%
δ_2	(-0.025 0.034)	1.60%	(-0.041 0.444)	0.00%	(-0.039 0.121)	35.36%
v	(1.660 1.811)	46.72%	(1.616 2.100)	0.48%	(1.591 1.787)	9.44%

Table 7. Percentage of rolling-sampled estimated parameters that are statistically different from the parameter values estimated using the full data sample.

	DAX30		FTSE100		SP500	
Parameter	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level
μ_0	13.67%	0.80%	4.02%	0.00%	14.15%	4.34%
μ_1	0.00%	0.00%	0.16%	0.00%	8.52%	0.64%
μ_2	0.00%	0.00%	1.13%	0.00%	0.00%	0.00%
μ_3	0.00%	0.00%	3.22%	0.64%	0.00%	0.00%
a_0	16.72%	7.40%	0.48%	0.00%	24.28%	6.59%
δ_0	0.00%	0.00%	0.00%	0.00%	2.73%	0.00%
Ψ_1	0.00%	0.00%	0.00%	0.00%	7.56%	0.00%
Δ_1	2.57%	0.00%	14.47%	5.79%	31.67%	3.54%
γ	5.14%	0.00%	4.50%	0.00%	36.17%	10.13%
δ_1	0.00%	0.00%	0.80%	0.32%	0.00%	0.00%
δ_2	0.00%	0.00%	0.16%	0.00%	24.92%	0.00%
v	16.72%	0.32%	0.00%	0.00%	0.00%	0.00%

Table 8. Percentage of the rolling-sampled estimated parameters that are statistically insignificant at 5% and 1% levels of significance.

	DAX30		FTSE100		SP500	
Parameter	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level
μ_0	88.36%	99.37%	94.69%	100.00%	66.35%	84.28%
μ_1	93.87%	100.00%	99.22%	100.00%	57.08%	87.26%
μ_2	100.00%	100.00%	99.22%	100.00%	100.00%	100.00%
μ_3	100.00%	100.00%	79.69%	96.56%	92.77%	100.00%
a_0	0.00%	0.00%	17.81%	40.78%	1.57%	18.08%
δ_0	81.45%	100.00%	100.00%	100.00%	100.00%	100.00%
Ψ_1	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%
Δ_1	0.00%	0.00%	31.25%	38.91%	0.00%	0.00%
γ	100.00%	100.00%	100.00%	100.00%	0.00%	44.18%
δ_1	100.00%	100.00%	0.00%	0.16%	94.03%	100.00%
δ_2	100.00%	100.00%	67.97%	96.56%	59.91%	91.19%
v	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 9. AR(1)GARCH(1,1) simulated process. Percentage of rolling-sampled estimated parameters that are outside the 95% confidence interval.

$$y_t = \mu_0 + \mu_1 y_{t-1} + \varepsilon_t$$

$$\varepsilon_t = z_t \sqrt{\sigma_t^2}\ ,\ z_t \overset{i.i.d.}{\sim} N(0,1)$$

$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + a_2 \sigma_{t-1}^2$$

	μ_0	μ_1	a_0	a_1	a_2
Simulated Values	0.005	0.150	0.040	0.0500	0.900
Estimated Values (Full Data Sample)	-0.003	0.158	0.037	0.0138	0.895
Rolling parameters outside the 95% c.i.	11.70%	3.32%	73.17%	30.88%	72.17%

Table 10. Stable parameter estimates, using the full data sample, of the FTSE20, DAX30, FTSE100 and SP500 index daily returns, their standard errors and the percentage of rolling-sampled estimated parameters that are statistically different from the parameter values estimated using the full data sample at 5% level of significance.

	Tail index a	Skewness β	Location δ	Scale γ
	FTSE20			
Coefficient	1.48303	0.07799	-0.00033	0.01005
Standard error	0.05606	0.07965	0.00143	0.00081
5% sign. Level	0.32%	0.00%	0.00%	56.48%
	DAX30			
Coefficient	1.58306	-0.14798	0.00101	0.00754
Standard error	0.15725	0.18828	0.00069	0.00217
5% sign. Level	1.53%	9.59%	0.12%	0.00%
	FTSE100			
Coefficient	1.68238	-0.06489	0.00046	0.00591
Standard error	0.10944	0.25581	0.00039	0.00165
5% sign. Level	2.13%	9.42%	0.49%	0.00%
	SP500			
Coefficient	1.49172	-0.11841	0.0005	0.00525
Standard error	0.07160	0.09609	0.00052	0.00218
5% sign. Level	51.46%	5.00%	0.00%	0.00%

Notes: The standard error of parameter a is computed as the standard deviation of the rolling-sampled estimated parameters, $\hat{a}^{(t)}$, for $t = 1,\ldots,T$ trading days, i.e. $\sqrt{(T-1)^{-1}\sum_{t=1}^{T}\left(\hat{a}^{(t)} - \bar{a}^{(T)}\right)^2}$, where $\bar{a}^{(T)} = T^{-1}\sum_{t=1}^{T}\hat{a}^{(t)}$.

Figure 1. FTSE20, DAX30, FTSE100 and SP500 continuously compounded daily returns from January 3rd, 1996 to July 5th, 2002.

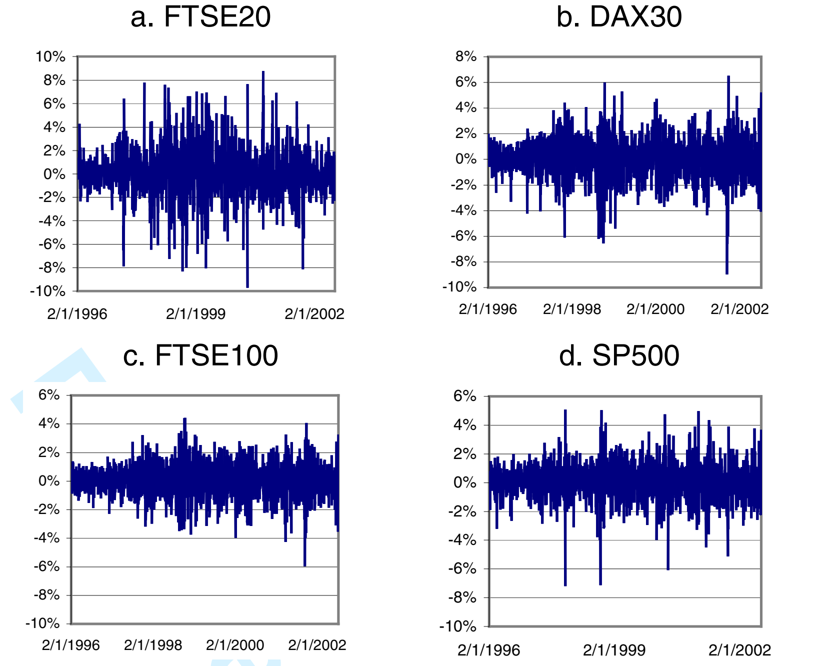
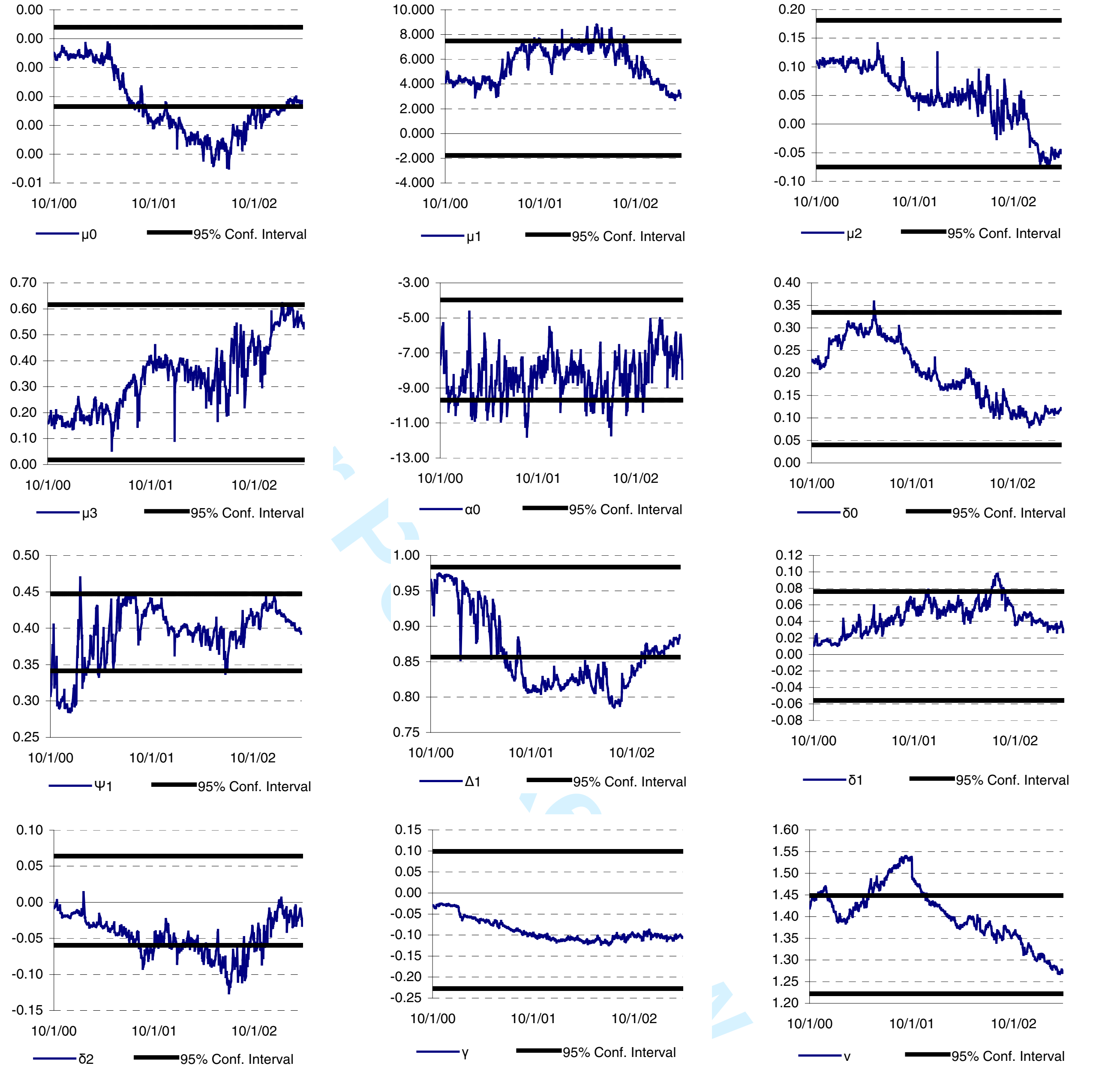


Figure 2. The rolling-sampled estimated parameters of the ARCH model and the 95% confidence interval of the parameters estimated using the full data sample.



Notes: The 95% confidence interval is constructed as $\hat{\theta} \pm GED(0,1;1.335,0.025)\hat{S}_{\theta}\sqrt{1621/1000}$, where $\hat{\theta}$ denotes the parameter vector estimated using the full data sample, $\hat{S}_{\theta}$ is the standard deviation of $\hat{\theta}$ and $GED(0,1;v,a)$ is the $(1-a)$ percentile of the GED distribution, with v denoting the tail thickness parameter.

Figure 3.a. In-sample 95% confidence interval of the FTSE20 index daily returns for the ARCH model (11th January 2000 to 5th July 2002).

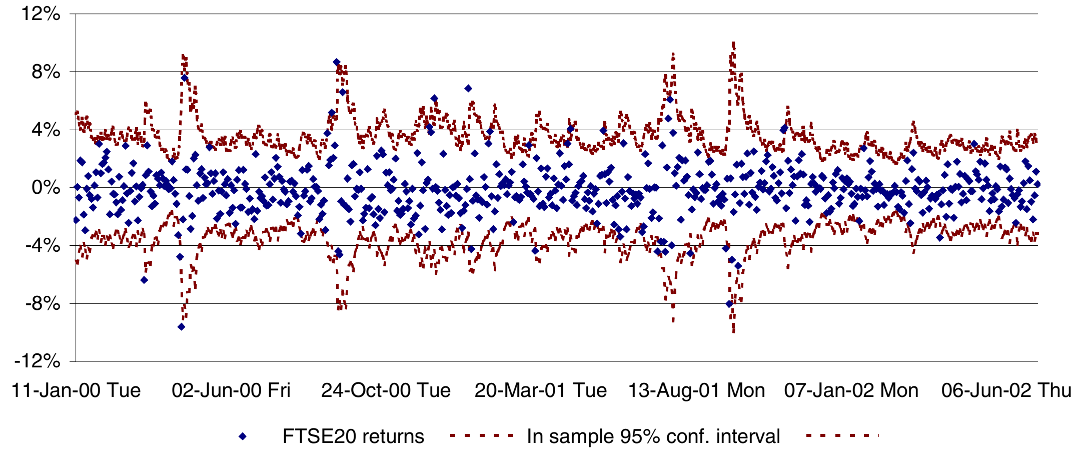


Figure 3.b. One-step-ahead 95% prediction interval of the FTSE20 index daily returns for the ARCH model (11th January 2000 to 5th July 2002).

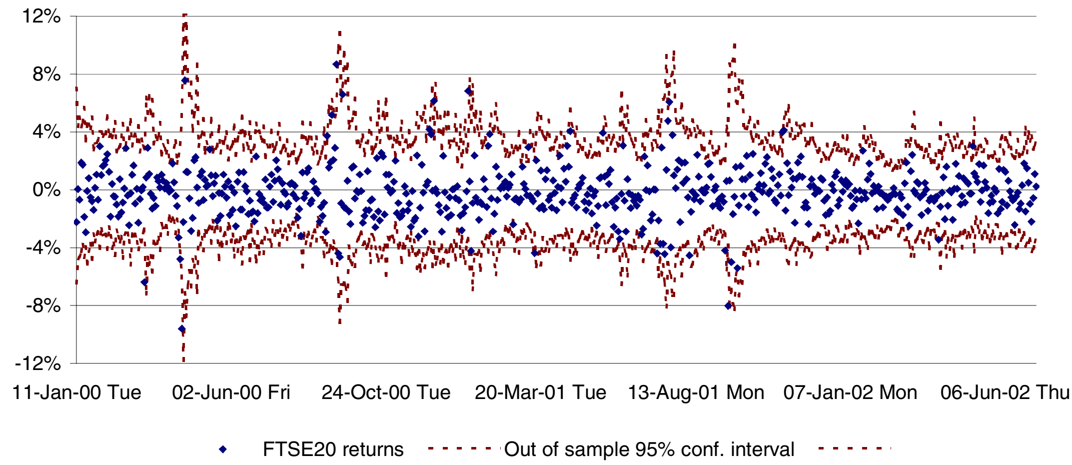
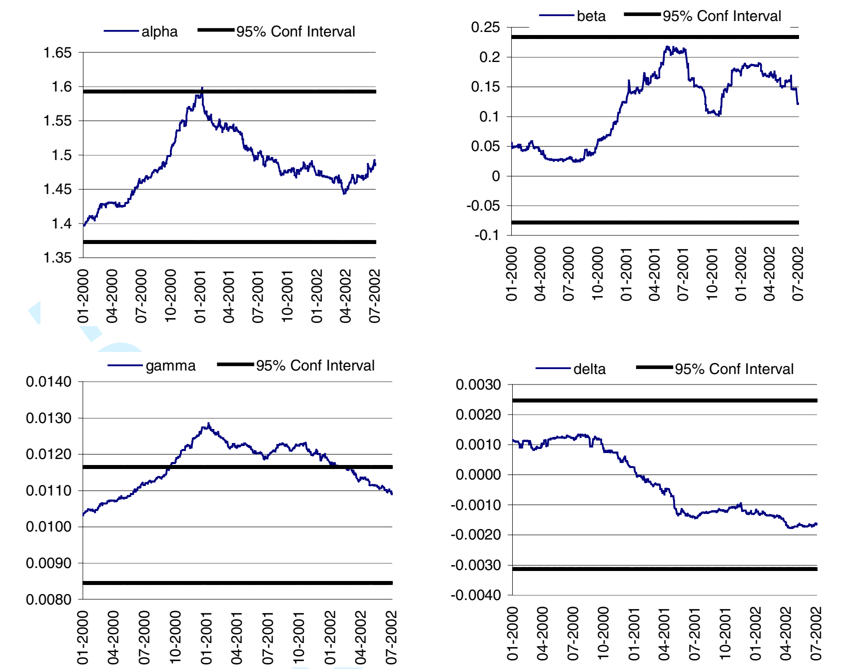


Figure 4. FTSE20 index daily returns. The rolling-sampled estimated parameters of the stable distribution and the 95% confidence interval of the parameters estimated using the full data sample.



Rolling-sampled parameters of ARCH and Levy-stable models

Short Title: ARCH & Levy-stable rolling samples

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Abstract

In this paper an asymmetric autoregressive conditional heteroskedasticity (ARCH) model and a Levy-stable distribution are applied to some well-known financial indices (DAX30, FTSE20, FTSE100 and SP500), using a rolling sample of constant size, in order to investigate whether the values of the estimated parameters of the models change over time. Although, there are changes in the estimated parameters reflecting that structural properties and trading behaviour alter over time, the ARCH model adequately forecasts the one-day-ahead volatility. A simulation study is run to investigate whether the time variant attitude holds in the case of a generated ARCH data process revealing that even in that case the rolling-sampled parameters are time-varying.

Keywords: ARCH model, GED distribution, Leverage effect, Levy-stable distribution, Rolling sample, Spill over, Value at risk.

JEL codes: C32, C52, C53, G15.

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The usual disclaimer applies.

1. Introduction

In the recent literature, regarding the description of the characteristics of financial markets, one can find a vast number of specifications of both ARCH and Stochastic Volatility (SV) processes that have been considered for. However, the SV models[1] are not as popular as the ARCH processes in applied studies. The purpose of the present study is to apply an asymmetric ARCH model to some well known financial indices, using a rolling sample of constant size, in order to observe the changes over time in the values of the estimated parameters. A thorough investigation is conducted by comparing the parameters of the full-sampled estimated model to the parameters of the rolling sub-sample estimated models. We conclude that the values of the estimated parameters change over time, indicating a data set that alters across time reflecting the information that financial markets reveal. The analysis is extended to simulated time series indicating that the time-varying estimated coefficients characterize the ARCH data generating process itself.

In ARCH modelling, the distribution of stock returns has fat tails with finite or infinite unconditional variance and time dependent conditional variance. Estimation of stable distributions is an alternative approach in modelling the unconditional distribution of returns. Thus, we adopt the estimation procedure of McCulloch (1986) and the parameters of the Levy-stable distribution are estimated at each of a sequence of points in time, using a rolling sample of constant size. The empirical findings suggest that the parameters of the unconditional distribution are also not constant over time.

Reviewing the relevant literature we notice absence of studies showing that although the parameters of a well-specified model vary significantly over time, their time varying attitude does not influence model's forecasting ability. The main object of our study is to provide evidence that model's parameters should be re-estimated on a frequent base in order to reflect any changes that have been occurred in the stock market and have been incorporated in the prices of assets.

[1] The reader who is interested in SV models is referred to Barndorff-Nielsen et al. (2002), Chib et al. (1998), Ghysels et al. (1996), Jacquier et al. (1999), Shephard (2004), Taylor (1994).

The paper is divided in six sections. Section 2 lays out the asymmetric ARCH model that is applied in the FTSE20, DAX30, FTSE100 and SP500 stock indices. In section 3, the estimated rolling-sampled parameters of the asymmetric ARCH model are discussed. In section 4, a simulation study examines whether the parameters are time-varying in the case of a generated ARCH process. In section 5, the unconditional distribution of returns is estimated and the phenomenon of time-variant parameters is investigated in the Levy-stable distribution. Finally, in section 6 we summarize the main conclusions.

2. An asymmetric ARCH model

A wide range of proposed ARCH models is covered in surveys such as Andersen and Bollerslev (1998), Bera and Higgins (1993), Bollerslev et al. (1992), Bollerslev et al. (1994), Degiannakis and Xekalaki (2004) and Poon and Granger (2003). The Nobel price award to R.F. Engle for ARCH volatility modeling is the uncontested proof of the contribution of ARCH models in time series and econometric modelling (Diebold 2003). A plethora of studies applied ARCH models to predict future volatility by updating the available information set at each of a sequence of points in time. Among others, Balaban and Bayar (2005) tested in 14 countries the relationship between stock market returns and their forecast volatility, Blair et al. (2001) compared the information content of implied volatilities and intraday returns in the context of forecasting S&P100 volatility, Wei (2002) forecast China's weekly stock market volatility and Yu (2002) predicted stock price volatility using daily New Zealand data. Angelidis et al. (2004), Degiannakis (2004), Brooks and Persand (2003) and Giot and Laurent (2003) predicted Value-at-Risk (VaR) measures, while Degiannakis and Xekalaki (2001), Engle et al. (1997) and Noh et al. (1994) used rolling ARCH models to forecast volatility of options.

An ARCH process, $\varepsilon_t(\theta)$, can be presented as

$$\begin{gathered}\varepsilon_t(\theta)=z_t\sigma_t(\theta)\\ z_t \overset{i.i.d.}{\sim} f\left(E(z_t)=0,V(z_t)=1\right)\\ \sigma_t^2(\theta)=g\left(\sigma_{t-1},\sigma_{t-2},\ldots;\varepsilon_{t-1},\varepsilon_{t-2},\ldots;\upsilon_{t-1},\upsilon_{t-2},\ldots\right),\end{gathered} \tag{1}$$

where θ is a vector of unknown parameters, $f(.)$ is the density function of z_t, $g(.)$ is a linear or non-linear functional form and υ_t is a vector of predetermined variables included in information set I at time t. Since very few financial time series have a constant conditional mean of zero, an ARCH model can be presented in a regression form by letting ε_t be the unpredictable component of the conditional mean

$$y_{A,t} = E(y_{A,t} \mid I_{t-1}) + \varepsilon_t\,, \tag{2}$$

where $y_{A,t} = \ln(P_{A,t}/P_{A,t-1})$ denotes the continuously compound rate of return from time $t-1$ to t, and $P_{A,t}$ is the asset price A at time t. In order to investigate the characteristics of stock market A, we apply an ARCH model of the following form:

$$\begin{gathered} y_{A,t} = \mu_0 + \mu_1\sigma^2_{A,t} + \left(\mu_2 + \mu_3 e^{-\sigma^2_{A,t}/\mu_4}\right) y_{A,t-1} + \varepsilon_t\,, \\ \varepsilon_t = z_t\sigma_{A,t}\,, \\ z_t \overset{i.i.d.}{\sim} GED(0,1;v)\,, \\ \ln(\sigma^2_{A,t}) = a_0 + \ln(1+N_t\delta_0) + \frac{1}{(1-\Delta_1 L)}\left(\Psi_1 L\left(\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right| - E\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right|\right) + \gamma L\frac{\varepsilon_t}{\sigma_{A,t}}\right), \end{gathered} \tag{3}$$

where $GED(0,1;v)$ denotes the generalized error distribution (GED), v is the tail thickness parameter of the GED, L is the lag operator and N_t is the number of non-trading days preceding the t^{th} day. The density function of a GED random variable is given by

$$f(z_t) = \frac{v e^{-2^{-1}\left|\frac{z_t}{\lambda}\right|^v}}{\lambda 2^{v+1/v}\,\Gamma(1/v)}, \tag{4}$$

for $-\infty < z < \infty$, $0 < v \le \infty$, where $\Gamma(.)$ denotes the gamma function and

$$\lambda \equiv \left(\frac{2^{-2v^{-1}}\Gamma(1/v)}{\Gamma(3/v)}\right)^{1/2}. \tag{5}$$

The conditional variance specification has the form of the exponential GARCH, or EGARCH model, which is suggested by Nelson (1991). The EGARCH model captures the asymmetric effect exhibited in financial markets, as the conditional variance, σ_t^2, depends on both the magnitude and the sign of lagged innovations. Assuming GED distributed innovations with EGARCH specification for the conditional variance we take into account that i) the unconditional distribution of innovations is symmetric but with excess kurtosis and ii) their conditional distribution is asymmetric and leptokurtotic. Parameter γ allows for the leverage effect. The leverage effect, first noted by Black (1976), refers to the tendency of changes in stock returns to be negatively correlated with changes in returns volatility, i.e. volatility tends to rise in response to 'bad news' and to fall in response to 'good news'. Moreover, the logarithmic transformation ensures that the forecasts of the variance are non-negative. Parameter δ_0 allows us to explore the contribution of non-trading days to volatility. According to Fama (1965) and French and Roll (1986) information that accumulates when financial markets are closed is reflected in prices after the markets reopen. The conditional mean is modeled such as to capture the relationship between investors' expected return and risk[2] (μ_1), the non-synchronous trading effect[3] (μ_2), and the inverse relation between volatility and serial correlation[4] (μ_3).

Model (3) is expanded in order to take into account the phenomenon of volatility spill over from one market to the other[5]:

$$\ln\left(\sigma_{A,t}^2\right) = a_0 + \ln\left(1 + N_t\delta_0\right) + \frac{1}{\left(1 - \Delta_1 L\right)}\left(\Psi_1 L\left(\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right| - E\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right|\right) + \gamma L\frac{\varepsilon_t}{\sigma_{A,t}}\right) + \delta_1 \ln\left(\sigma_{B,t-1}^2\right) + \delta_2 \ln\left(\sigma_{C,t-1}^2\right). \tag{6}$$

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[2] The relationship between investors' expected return and risk was presented in an ARCH framework, by Engle et al. (1987). They introduced the ARCH in mean model where the conditional mean is an explicit function of the conditional variance.

[3] According to Campbell et al. (1997), 'The non-synchronous trading or non-trading effect arises when time series, usually asset prices, are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other, possible irregular lengths.'

[4] LeBaron (1992) found a strong inverse relation between volatility and serial correlation for SP500, CRSP and Dow Jones returns. As LeBaron stated, it is difficult to estimate μ_4 in conjunction with μ_3 when using a gradient type of algorithm. So, μ_4 is set to the sample variance of the series.

[5] Engle et al. (1990) evaluated the role of the information arrival process in the determination of volatility in a multivariate framework providing a test of two hypotheses: heat waves and meteor showers. Using meteorological analogies, they supposed that information follows a process like a heat wave so that a hot day in New York is likely to be followed by another hot day in New York but not typically by a hot day in Tokyo. On the other hand, a meteor shower in New York, which rains down on the earth as it turns, will almost surely be followed by one in Tokyo. Thus, the heat wave hypothesis is that the volatility has only country specific autocorrelation, while the meteor shower hypothesis states that volatility in one market spills over to the next. See also Kanas (1998).

where the parameters δ_1 and δ_2 account for the volatility spill over from B and C stock markets to the A stock market, respectively. In order to account for the volatility spill over effect from one market to the others, when (6) is estimated for stock market A, the daily conditional volatilities of stock markets B and C are regarded as exogenous variables that have been estimated according to framework (3)[6].

The data set used in this paper consists of the Financial Times Stock Exchange 20 (FTSE20) index for Greece, the Deutscher Aktien Index 30 (DAX30) for Germany, the Financial Times Stock Exchange 100 (FTSE100) index for U.K. and the Standard & Poor's 500 (SP500) index for U.S.A. The period covered for the FTSE20, DAX30, FTSE100 and SP500 is from January 3rd 1996, January 14th 1992, January 9th 1992 and January 7th 1992 to July 5th 2002, respectively. A thorough investigation is conducted by comparing the parameters of the full-sampled estimated model to the parameters of the rolling sub-sample estimated models. Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt (1963) algorithm.

INSERT TABLE 1 ABOUT HERE

Table 1 presents the estimated parameters of model (6) for each market separately. The standardized residuals, $\varepsilon_t\sigma_{A,t}^{-1}$, and their squared values, $\varepsilon_t^2\sigma_{A,t}^{-2}$, from all models obey the standard assumptions of autocorrelation and heteroskedasticity absence. Indicatively, we present the Ljung-Box *Q*-statistic for the null hypothesis that there is not autocorrelation up to 20th order computed on $\varepsilon_t\sigma_{A,t}^{-1}$ and $\varepsilon_t^2\sigma_{A,t}^{-2}$. Briefly discussing the values of the parameters, we note that i) the relation of the conditional variance with the risk premium, although positive, is statistically insignificant (coefficient μ_1), ii) the non-synchronous trading effect is not present in the estimated models (coefficient μ_2) and iii) concerning the cases of the FTSE20 and SP500 stock indices, the daily serial correlation is inversely related to its conditional volatility (coefficient μ_3). Moreover, the leverage effect is not present in the Greek and German stock

[6] For example, in the case of the FTSE20 index daily returns, the conditional variance of the DAX30 and SP500 returns were regarded as exogenous variables. In order to estimate the conditional variance of the DAX30 and SP500 indices, their daily returns were used for the period of January 1992 to July 2002, or 1000 trading days prior January 3rd, 1996.

markets. On the contrary, for the SP500 and FTSE100 stock indices, the estimated value of parameter γ is statistically significant at 1% level of significance. The volatility spill over effect is statistically significant for the U.K. stock market. Regarding the SP500 index daily returns, there is evidence that volatility spillovers from Frankfurt to Chicago stock market. Finally, for the FTSE20, DAX30 and SP500 cases, parameter v is statistically different to the value of 2 at any level of significance, justifying the use of a thick-tailed distribution. The estimated value of δ_0 is about 0.187 and statistically significant only in the case of the Greek market indicating that a non-trading day contributes less than a fifth as much to volatility as a trading day.

3. *Rolling-sampled parameters of the asymmetric ARCH model*

Our purpose is to examine if the estimated parameters of the asymmetric ARCH model change over time and whether there is any impact of time-varying estimated parameters on volatility forecasting accuracy. The ARCH process is estimated, at each of a sequence of points in time, using a rolling sample of constant size equal to 1000 trading days, a sample size that is preferred[7] by the majority of applied studies.

We produce one-day-ahead conditional volatility predictions for the trading days of 11th January 2000 to 5th July 2002. Since the ARCH model is estimated at each point in time, we use the maximum likelihood estimates at time $t-1$ as starting values for the iterative maximization algorithm at time t. Figure 1 depicts the rolling-sampled estimated parameters for the FTSE20 index as well as the ± 2.06 times the conditional standard deviation confidence interval of the parameters estimated using the full data sample[8]. From visual inspection, the estimated rolling parameters are, clearly, out of the confidence interval bounds in many cases. Table 2 presents the percentage of rolling-sampled estimations, which are outside of the 95% confidence interval of the full-sampled parameters. Characteristic examples of the change in the parameter values are Ψ_1 and v for DAX30 as well as Δ_1 for FTSE20 and SP500. However, there are rolling parameters which do not change significantly

[7] Engle et al. (1993), Engle et al. (1997), Noh et al. (1994), Angelidis et al. (2004) note that the size of the rolling sample turns out to be rather important while Frey and Michaud (1997), Hoppe (1998) and Degiannakis and Xekalaki (2006) comment that the use of short sample sizes generates more accurate volatility forecasts, since it incorporates changes in trading behaviour more efficiently.

[8] Figures of the estimated rolling parameters for the DAX30, FTSE100 and SP500 indices, similar to Figure 1, are available upon request.

across time, such as γ (leverage effect), and δ_0 (contribution of non-trading days to volatility). An important characteristic, which is extracted from the rolling-sampled estimated parameters, is the fact that the estimated values do not fluctuate in a mean reverting form but they change gradually. Sudden changes of the values of the rolling estimated parameters, which are characterized by a mean reverting form, should indicate an improperly maximum likelihood estimation procedure. On the other hand, gradual changes of the estimated coefficients indicate a data set that alters from time to time, forcing us to believe that the values of the estimated parameters reflect the information that financial markets reveal.

INSERT FIGURE 1 ABOUT HERE

INSERT TABLE 2 ABOUT HERE

The percentage of estimated rolling parameters that are statistically different from the parameter values estimated using the full data sample, as presented in Table 3, is also indicative for the changes of the estimated values across time. There are four parameters, in the case of the Greek market, whose rolling-sampled estimators differ statistically significant from their full-sampled estimators in more than 10% of the trading days. Although, in the case of the FTSE100 index, only the rolling estimators of Δ_1 parameter differ statistically from their full data sample estimator, in the case of the SP500 index there are four parameters, which show a statistically significant difference from their full-sampled estimators in more than 20% of the trading days.

INSERT TABLE 3 ABOUT HERE

The values of the rolling parameters indicate that the characteristics of the markets change during the examined period. According to Table 4, which presents the percentage of trading days that the rolling parameters are statistically insignificant, there are parameters whose rolling-sampled estimations are statistically insignificant while their full-sampled estimations are significant. For example, parameters μ_3 and δ_1 for the SP500 index, as well as parameter γ for FTSE100 index, although they appear to be significant in the full sample, almost all their rolling-sampled estimations are insignificant at 5% level of significance. Therefore, in the full sample, an inverse relation between volatility and serial correlation characterizes FTSE20 index, but the values of rolling μ_3 are not different to zero in most of the cases. Of course, there are parameters whose estimations are statistically different to zero in both the full sample and the rolling samples (i.e. the parameter Δ_1 for the FTSE20, DAX30 and SP500 indices). Hence, we may infer that the values of the estimated parameters change across time, reflecting the individual features of particular periods that characterize financial markets.

INSERT TABLE 4 ABOUT HERE

However, although the estimated parameters are time varying, the in-sample and out-of-sample forecasting ability of the model is accurate. There are 31, 19, 17 and 29 cases, or 4.99%, 2.99%, 2.66% and 4.57%, observed returns outside the 95% confidence intervals for the FTSE20, DAX30, FTSE100 and SP500 indices, respectively. In Figure 2.a, the 95% in-sample confidence interval of the FTSE20 index of daily returns is plotted from 11th January 2000 to 5th July 2002. However, a model that uses a large number of parameters may exhibit an excellent in-sample fit but a poor out-of-sample performance. Studies such as Heynen and Kat (1994), Hol and Koopman (2000) and Pagan and Schwert (1990) examined a variety of volatility prediction models with in-sample and out-of-sample data sets. We investigate the possibility that model over-fitting can be occurred and evaluate the performance of the estimated ARCH model by computing the out-of-sample forecasts. In the sequel, the one-day-ahead 95% prediction intervals are constructed. Let us compute the one-day-ahead conditional mean, $y_{t+1|t} \equiv E\left(y_{t+1}\left(\theta^{(t)}\right) | I_t\right)$, and conditional variance, $\sigma^2_{t+1|t} \equiv E\left(\varepsilon^2_{t+1}\left(\theta^{(t)}\right) | I_t\right)$, using the following formulas:

$$y_{A,t+1|t} = \mu_0^{(t)} + \mu_1^{(t)}\sigma^2_{A,t+1|t} + \left(\mu_2^{(t)} + \mu_3^{(t)} e^{-\sigma^2_{A,t+1|t} / \mu_4^{(t)}}\right) y_{A,t},$$

$$\ln\left(\sigma^2_{A,t+1|t}\right) = a_0^{(t)} + \ln\left(1 + N_{t+1}\delta_0^{(t)}\right) + \frac{1}{\left(1-\Delta_1^{(t)}L\right)}\left(\Psi_1^{(t)}\left(\left|\frac{\varepsilon_{t|t}}{\sigma_{A,t|t}}\right| - E\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right|\right) + \gamma^{(t)}\frac{\varepsilon_{t|t}}{\sigma_{A,t|t}}\right) + \delta_1^{(t)}\ln\left(\sigma^2_{B,t|t}\right) + \delta_2^{(t)}\ln\left(\sigma^2_{C,t|t}\right), \quad (7)$$

where $\theta^{(t)} \equiv \left(\mu_0^{(t)}, \mu_1^{(t)}, \mu_2^{(t)}, \mu_3^{(t)}, a_0^{(t)}, \delta_0^{(t)}, \Psi_1^{(t)}, \Delta_1^{(t)}, \gamma^{(t)}, \delta_1^{(t)}, \delta_2^{(t)}, v^{(t)}\right)$ is the parameter vector that is estimated using the sample data set which is available at time t, $\varepsilon_{t|t} \equiv E(\varepsilon_t \mid I_t)$ denotes the prediction error conditional on the information set that is available at time t, and $\sigma_{A,t|t} \equiv \sqrt{E\left(\varepsilon_t^2 \mid I_t\right)}$ is the conditional standard deviation which is computed by the ARCH model, in equation (6), using the information set available at time t. Note that for $z_t \sim GED(0,1;v)$, the expected value of its absolute price is equal to $E\left|\varepsilon_t \sigma_{A,t}^{-1}\right| = \Gamma\left(2/v^{(t)}\right)\left(\Gamma\left(1/v^{(t)}\right)\Gamma\left(3/v^{(t)}\right)\right)^{-1/2}$.

Figure 2.b plots the one-day-ahead 95% prediction interval, which is constructed as the one-day-ahead conditional mean ± 2.06 times the conditional standard deviation, both measurable to I_t information set, or $y_{A,t+1|t} \pm GED\left(0,1;v^{(t)},0.025\right)\sigma_{A,t+1|t}$, where $GED\left(0,1;v^{(t)},a\right)$ is the $100(1-a)$ quantile of the GED distribution. Hence, each trading day, (t), the next trading day's, ($t+1$), prediction intervals are constructed, using only information available at current trading day, t. There are 29, 22, 21 and 32 observations or 4.67%, 3.46%, 3.29% and 5.04% for the FTSE20, DAX30, FTSE100 and SP500 indices, respectively, outside the 95% prediction intervals[9].

INSERT FIGURE 2 ABOUT HERE

For a more formal method of evaluating forecasting adequacy, we apply two hypotheses tests that measure the forecasting accuracy in a VaR framework. One-day-ahead VaR at a given probability level, a, is the next trading day's predicted amount of financial loss of a portfolio, or $VaR_{t+1|1}(1-a) = GED\left(0,1;v^{(t)},a\right)\sigma_{A,t+1|t}$. Kupiec (1995) introduced a likelihood ratio statistic for testing the null hypothesis that the proportion of confidence interval violations is not larger than the VaR forecast. The test statistic, which is asymptotically X_1^2 distributed, is computed as $LR_K = 2[\ln((n/N)^n(1-n/N)^{N-n}) - \ln(p^n(1-p)^{N-n}]$, where $n \equiv \sum_{i=1}^{N} d\left(y_{t+1} < VaR_{t+1|t}(a/2)\right) + d\left(y_{t+1} > VaR_{t+1|t}(1-a/2)\right)$ is the number of trading days over

[9] Figures, similar to Figure 2, that depict the in-sample 95% confidence interval and the one-day-ahead 95% prediction intervals for the DAX30, FTSE100 and SP500 indices are also available upon request.

the out-of-sample period N that a violation has occurred, for $d\left(y_{t+1} < VaR_{t+1|t}\left(a/2\right)\right) = 1$ if $y_{t+1} < VaR_{t+1|t}$ and $d\left(y_{t+1} < VaR_{t+1|t}\left(a/2\right)\right) = 0$ otherwise, and p is the expected frequency of violations. Christoffersen (1998) developed a likelihood ratio statistic that jointly investigates whether i) the proportion of violations is not larger than the VaR forecast and ii) the violations are independently distributed. The statistic is computed as $LR_C = -2\ln((1-p)^{N-n}p^n)$ $+2\ln((1-\pi_{01})^{n_{00}}\pi_{01}{}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}{}^{n_{11}}))$, where $\pi_{ij} = n_{ij} \Big/ \sum_j n_{ij}$ and n_{ij} is the number of observations with value i followed by j, for $i, j = 0,1$. The values $i, j = 1$ denote that a violation has been made, while $i, j = 0$ indicate the opposite. Under the null hypothesis, the LR_C is asymptotically chi-squared distributed with two degrees of freedom. The main advantage of Christoffersen's test is that it can reject a VaR model that generates either too many or too few clustered violations. Both tests do not reject the null hypothesis of correct proportion of violations in all the cases, except for the 95%-VaR of the FTSE100 index. In the case of Kupiec's test the p-values are 70.28%, 6.08%, 3.45% and 96.37% for 95%-VaR and 8,15%, 13.63%, 56.56% and 52.70% for 99%-VaR, for the FTSE20, DAX30, FTSE100 and SP500 indices, respectively. Testing the null hypothesis of whether the violations are equal to the expected ones as well as if they are independent, we observe that the relative p-values are 40.03%, 16.42%, 0.15% and 95.19% in the 95%-VaR case and 17.98%, 32.51%, 7.10% and 73.92% in the 99%-VaR case, for the FTSE20, DAX30, FTSE100 and SP500 indices, respectively.

Despite the fact that the values of the estimated coefficients change over time, the model adequately forecasts the one-day-ahead volatility. Thus, changes in the values of the estimated parameters do not indicate inadequacy of the model in describing the data. On the contrary, model's parameters should be re-estimated on a daily base in order to reflect any changes that have been occurred in the stock market and have been incorporated in the prices of assets[10].

[10] In order to investigate whether the phenomenon of time-variant values of estimated parameters is related to a specific structural characteristic of the model specification, we estimate another ARCH specification. Degiannakis

4. Rolling-sampled parameters from simulated processes

A simulation study could shed light in rolling-sampled estimated parameters' behaviour. A series of simulations is run in order to investigate if the time-variant attitude holds even in the case of an ARCH data generating process. We generate a series of 32000 values from the standard normal distribution, $z_t \overset{i.i.d.}{\sim} N(0,1)$. Then an AR(1)GARCH(1,1) process is created, $\{y_t\}_{t=1}^{32000}$, where $y_t = 0.0005 + 0.15 y_{t-1} + \varepsilon_t$, by multiplying the i.i.d. process with a specific conditional variance form $\varepsilon_t = z_t \sqrt{\sigma_t^2}$, for $\sigma_t^2 = 0.0005 + 0.05\varepsilon_{t-1}^2 + 0.90\sigma_{t-1}^2$. The AR(1)GARCH(1,1) model is applied on the $\{y_t\}_{t=1002}^{32000}$ generated data. Dropping out the first 1001 data, maximum likelihood rolling-sampled estimates of the parameters are obtained by numerical maximization of the log-likelihood function, using a rolling sample of constant size equal to 1000. According to Table 5, about 58% of the 30000 conditional variance rolling-sampled parameters are outside the 95% confidence interval of the parameters estimated using the whole sample set of the 30000 simulated data. The procedure is repeated for an AR(1)EGARCH(1,1) conditional variance form, $\ln(\sigma_t^2) = a_0 + a_1 \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln(\sigma_{t-1}^2)$, but the results are robust to the choice of the conditional variance specification.

A series of 32000 values from the first order autoregressive process are also produced. The AR(1) process is created as $y_t = 0.0001 + 0.12 y_{t-1} + z_t$, for $z_t \overset{i.i.d.}{\sim} N(0,1)$. Dropping out the first 1001 data, 30000 maximum likelihood rolling-sampled estimates of the parameters are also obtained. As far as the case of the AR(1) process is concerned, we infer that the rolling estimated parameters are time-invariant, as on average 5% of the estimated rolling parameters are outside the 95% confidence levels.

(2004) and Giot and Laurent (2003) used an ARCH model with the APARCH volatility specification of Ding et al. (1993) and the skewed student-t distribution for the standardized innovations. We estimated such a model for our datasets and found similar qualitative results. The estimated parameters are time varying. We have also re-estimated model (6) using alternatively i) larger sample sizes of rolling parameters, ii) the BHHH algorithm (Berndt et al. 1974) instead of the Marquardt algorithm in estimating the maximum likelihood parameters and iii) the same starting values at each point in time, instead of the estimates at time $t-1$ as starting values for the likelihood algorithm at time t. Despite the slight changes occurred in each case, the rolling parameters are time-variant for all cases.

Both the AR(1)GARCH(1,1) and the AR(1) processes were simulated for various sets of parameters, but there are no qualitative differences to the fore mentioned conclusions. Moreover, a series of simulations were repeated i) for ARCH volatility forms without any conditional mean specification, ii) based on estimation procedures of the most well known packages, EVIEWS® 4.1 and OX-G@ARCH® 3.4, iii) for larger rolling samples of 5000 values, iv) for non-overlapping data samples, but there were no qualitative differences in any of these cases[11].

So, the simulation study provides evidence that the time-variant attitude of rolling-sampled parameters estimations characterizes not only the examined data sets but the ARCH data generating process itself as well.

INSERT TABLE 5 ABOUT HERE

5. *Rolling-sampled parameters from a Levy-stable distribution*

In this section, we investigate whether the phenomenon of parameter changing across time is related with the unconditional distribution of returns also. Mandelbrot (1963) and Fama (1965) made the first re-examination of the unconditional distribution of stock returns. Mandelbrot (1963) concluded that price changes can be characterized by a stable Paretian distribution with a characteristic exponent, a, less than two, thus exhibiting fat tails and infinite variance. Fama (1965) examined the distribution of thirty stocks of the Dow Jones Industrial Average; his results were consistent with Mandelbrot's. Thereafter, it has been accepted that the stock returns distributions are fat-tailed and peaked. In an attempt to model the unconditional distribution of stock returns several researchers have considered alternative approaches. See for example, Blattberg and Gonedes (1974), Bradley and Taqqu (2002), Clark (1973), Kon (1984), McDonald (1996), Mittnik and Rachev (1993), Panas (2001), Rachev and Mittnik (2000).[12]

[11] All the simulation studies are available to the readers upon request.
[12] De Vries (1991), Ghose and Kroner (1995) and Groenendijk et al. (1995) demonstrate that ARCH models share many of the properties of Levy-stable distribution but the true data generating process for an examined set of financial data is more likely ARCH than Levy-stable. A number of studies, such as Liu and Brorsen (1995), Mittnik et al. (1999), Panorska et al. (1995), Tsionas (2002), examined the properties of ARCH models with Levy-stable distributed innovations.

The probability density function of a stable distribution cannot be described in a closed mathematical form. By definition, a univariate distribution function is stable if and only if its characteristic function has the form

$$\varphi(t) = \exp\left\{ i\delta t - \gamma |t|^{a} \left(1 - i\beta \left(\frac{t}{|t|} \right) \omega(t,a) \right) \right\}, \tag{8}$$

where $i = \sqrt{-1}$, $t \in R$, $\omega(t,a) = \tan\left(\frac{\pi\alpha}{2}\right)$ if $a \neq 1$ and $\omega(t,a) = \frac{-2}{\pi}\log|t|$ if $a = 1$. The particular distribution represented by its characteristic function is determined by the values of four parameters: a, β, γ and δ. The parameter a, $0 < \alpha \leq 2$, is called the characteristic exponent. It measures the thickness of the tails of a stable distribution. The smaller the value a, the higher the probability in the distribution tails. If $a < 2$ then we have thicker tails than the tails of normal distribution. Thus, stable distributions have thick tails and consequently increase the likelihood of the occurrence of large shocks. The skewness parameter β, $-1 \leq \beta \leq 1$, is a measure of the asymmetry of the distribution. The distribution is symmetric, if $\beta = 0$. As $|\beta|$ approaches one, the degree of skewness increases. The scale parameter γ, $\gamma > 0$, is a measure of the spread of the distribution. It is similar to the variance of the normal distribution, $\gamma = \sigma / \sqrt{2}$. However, the scale parameter γ is finite for all stable distributions, despite the fact that the variance is infinite for all $a < 2$. The location parameter δ, $-\infty < \delta < +\infty$, is the mean of the distribution, for $a > 1$, and the median for $0 < a \leq 1$. The case of $a = 2$, $\beta = 0$ corresponds to the normal distribution, while $a = 1$, $\beta = 0$ corresponds to the Cauchy distribution.

In estimating the parameters of the stable distribution of index returns, we adopt the estimation procedure suggested by McCulloch (1986). The estimation procedure is a quantile method and works for $0.6 \leq a \leq 2$ and any value of the other parameters. Essentially, McCulloch suggests that if we have a random variable x, which follows a stable distribution and denotes the p^{th} quantile of this distribution by $x(p)$, then the population quantile can be estimated by the sample quantile $\hat{x}(p)$. McCulloch's estimator uses five quantiles to estimate

a and β as $\hat{v}(\alpha) = \dfrac{\hat{x}(0.95) - \hat{x}(0.05)}{\hat{x}(0.75) - \hat{x}(0.25)}$ and $\hat{v}(\beta) = \dfrac{\hat{x}(0.95) + \hat{x}(0.05) - 2\hat{x}(0.50)}{\hat{x}(0.95) - \hat{x}(0.05)}$. Since $v(a)$ is monotonic in a and $v(\beta)$ is monotonic in β, we are able to find a and β by inverting $v(a)$ and $v(\beta)$, thus $\hat{a} = g_1(\hat{v}(a), \hat{v}(\beta))$ and $\hat{\beta} = g_2(\hat{v}(a), \hat{v}(\beta))$. McCulloch tabulated g_1 and g_2 for various values of $v(a)$ and $v(\beta)$. A similar procedure is also applied for the scale and location parameters. An alternative procedure to estimate the parameters of the stable distribution is the regression method proposed by Koutrouvelis (1980).

Following a procedure similar to that of ARCH modelling, the parameters of the stable distribution are estimated, at each of a sequence of points in time, using a rolling sample of constant size equal to 1000 trading days. The empirical findings, for the case of the Greek stock market, are graphically summarized in Figure 3, which plots the rolling-sampled estimates of parameters along with the 95% confidence interval of the parameters estimated using the full data sample. Inspection of Figure 3 shows that the estimates of a are less than two. The case of FTSE20 reveals that 92% of the a's rolling-sampled estimates are between 1.44 and 1.55. The parameter β is greater than zero, which implies skewness to the right. The rolling values of β are positive and range from 0.003 to 0.22 but there are not outside the 95% confidence interval for any case[13].

INSERT FIGURE 3 ABOUT HERE

In Table 6, we present the estimates of the parameters of stable distribution based on all data available as well as the standard deviation of the rolling-sampled estimated parameters. The estimates of a do not approach the value of two in any of the examined indices. However, there are estimated rolling parameters that are statistically different from the parameter values estimated using the full data sample. For example, the rolling-sampled estimates of the tail index (a) are statistically different to the full sample estimated parameter in the 51.46% of the trading days for the case of the SP500 index. The rolling estimates of parameter β are statistically different to the relevant full-sampled values in 9.59% and 9.42%

[13] Figures depicting the rolling-sampled estimates of the parameters for the DAX30, FTSE100 and SP500 indices are available upon request.

of the trading days for the DAX30 and FTSE100 indices, respectively, whereas the location (δ) parameters are time-variant in none of the cases. Another important parameter of the stable distribution, from the point of view of portfolio theory, is the scale parameter, γ. As far as the FTSE20 index is concerned, the rolling-sampled estimates of the scale parameter differ statistically from its full-sampled value in the 56.48% of the trading days. Hence, the parameter estimates, using the full data sample are statistically different from the parameter values estimated using the rolling samples of constant size for one parameter in each index.

INSERT TABLE 6 ABOUT HERE

6. *Discussion*

We estimated an asymmetric ARCH model using daily returns of the FTSE20, DAX30, FTSE100 and SP500 indices and concluded that although the estimated parameters of the model change over time, the model does not lose its ability to forecast the one-day-ahead volatility accurately. Furthermore, the rolling parameter analysis was applied to the unconditional distribution of returns. We observed the phenomenon of parameter changing across time for both the conditional (ARCH process) and the unconditional (Levy-stable) distribution of returns. Even in the case of a simulated ARCH process, the property of time varying rolling-sampled parameters holds. One possible reason for parameter instability might be that the behaviour of the market participants has undergone fundamental changes. Parameters instability indicates a change in market behavior but we can not determine the source of that change. The term *'a data set that alters'*, could incorporate a wide range of possible sources, i.e. financial legislation, market microstructure, market participants' perspective, technological revolution or even macroeconomic policy.

Gallant et al. (1991), Stock (1988), Lamoureux and Lastrapes (1990) and Schwert (1989) among others have aimed at explaining the economic interpretation of the ARCH process. As Engle et al. (1990) and Lamoureux and Lastrapes (1990) have noted, the explanation of the ARCH process must lie either in the arrival process of news or in market dynamics in response to the news. Based on some earlier work by Clark (1973) and Tauchen and Pitts (1983), Gallant et al. (1991) provided a theoretical interpretation of the ARCH effect. They assumed that the asset returns are defined by a stochastic number of intra-period price revisions and information flows into the market in an unknown rate. As the daily information does not come to the stock market in a constant and known rate, the estimation of the ARCH stochastic process that explains the dynamics of the stock market could be revised at regular time intervals. In our case the ARCH process is estimated using daily returns. Thus, the parameters of the model may be revised on a daily base, because of the observed phenomenon of changes in the estimated parameters. If we used data of higher frequency, i.e. ten-minutes intra-daily returns, the estimated model might be revised more frequent than on a daily base.

To the best of authors' knowledge, this is the first study that investigates the phenomenon of time varying estimated parameters either i) in real-world financial data or ii) in a simulated data generating process. A natural extension of this study would be to analyse the change and the relative economic interpretation of the estimated values of the parameters in intra-daily high-frequency data sets.

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References

Andersen, T. and Bollerslev, T. (1998). ARCH and GARCH Models, *Encyclopedia of Statistical Sciences Vol.II*, (eds.) Samuel Kotz, Campbell B. Read and David L. Banks, New York, John Wiley and Sons Inc.

Angelidis, T., Benos, A. and Degiannakis, S. (2004). The Use of GARCH Models in VaR Estimation. *Statistical Methodology*, **1(2)**, 105-128.

Balaban, E. and Bayar, A. (2005). Stock returns and volatility: empirical evidence from fourteen countries. *Applied Economics Letters*, **12**, 603-611.

Barndorff-Nielsen, O.E., Nicolato, E. and Shephard, N. (2002). Some Recent Developments in Stochastic Volatility Modelling. *Quantitative Finance*, 2, 11-23.

Bera, A.K. and Higgins, M.L. (1993). ARCH Models: Properties, Estimation and Testing. *Journal of Economic Surveys*, **7**, 305-366.

Berndt, E., Hall, B., Hall, R. and Hausman, J. (1974). Estimation and Inference in Nonlinear Structural Models. *Annals of Economic and Social Measurement*, **3**, 653-665.

Black, F. (1976). Studies of Stock Market Volatility Changes. *Proceedings of the American Statistical Association, Business and Economic Statistics Section*, 177-181.

Blattberg, R.C. and Gonedes, N.J. (1974). A comparison of the stable and student distributions as statistical models for stock prices. *Journal of Business*, **47**, 244-80.

Blair, B.J., Poon, S.-H. and Taylor, S.J. (2001). Forecasting S&P 100 volatility: the incremental information content of implied volatilities and high frequency index returns. *Journal of Econometrics*, **105**, 5-26.

Bollerslev, T., Chou, R. and Kroner, K.F. (1992). ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence. *Journal of Econometrics*, **52**, 5-59.

Bollerslev, T., Engle, R.F. and Nelson, D. (1994). ARCH Models. In: R.F. Engle and D. McFadden (Eds.), *Handbook of Econometrics*, Volume 4, Elsevier Science, Amsterdam, 2959-3038.

Bradley, B. and Taqqu, M. (2002). Financial Risk and Heavy Tails. In: S. Rachev (Ed.), *Heavy-tailed distributions in Finance*, North-Holland.

Brooks, C. and Persand, G. (2003). The Effect of Asymmetries on Stock Index Return Value-at-Risk Estimates. *Journal of Risk Finance*, **4(2)**, 29-42.

Campbell, J., Lo, A. and MacKinlay, A.C. (1997). *The Econometrics of Financial Markets*. New Jersey. Princeton University Press.

Chib, S., Kim, S. and Shephard, N. (1998). Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models. *Review of Economic Studies*, **65**, 361-393.

Christoffersen, P. (1998). Evaluating interval forecasts. *International Economic Review*, **39**, 841-62.

Clark, P.K. (1973). A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices. *Econometrica*, **41**, 135-156.

Degiannakis, S. (2004). Volatility Forecasting: Evidence from a Fractional Integrated Asymmetric Power ARCH Skewed-t Model. *Applied Financial Economics*, **14**, 1333-1342.

Degiannakis, S. and Xekalaki, E. (2001). Using a Prediction Error Criterion for Model Selection in Forecasting Option Prices. Athens University of Economics and Business, Department of Statistics, Technical Report, 131. Available online at: http://stat-athens.aueb.gr/~exek/papers/Xekalaki-TechnRep131(2001a)ft.pdf.

Degiannakis, S. and Xekalaki, E. (2004). Autoregressive Conditional Heteroscedasticity Models: A Review. *Quality Technology and Quantitative Management*, **1(2)**, 271-324.

Degiannakis, S. and Xekalaki, E. (2006). Assessing the Performance of a Prediction Error Criterion Model Selection Algorithm in the Context of ARCH Models, *Applied Financial Economics*, forthcoming.

De Vries, C.G. (1991). On the Relation Between GARCH and Stable Processes. *Journal of Econometrics*, **48**, 313-324.

Diebold, F.X. (2003). The ET Interview: Professor Robert F. Engle. *Econometric Theory*, **19**, 1159-1193.

Ding, Z., Granger, C.W.J. and Engle, R.F. (1993). A Long Memory Property of Stock Market Returns and a New Model. *Journal of Empirical Finance*, **1**, 83-106.

Engle, R.F., Lilien, D.M. and Robins, R.P. (1987). Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model. *Econometrica*, **55**, 391–407.

Engle, R.F., Ito, T. and Lin, W.L. (1990). Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market. *Econometrica*, **58**, 525-542.

Engle, R.F., Hong, C., Kane, A. and Noh, J. (1993). Arbitrage Valuation of Variance Forecasts with Simulated Options, *Advances in Futures and Options Research*, **6**, 393-415.

Engle, R.F., Kane, A. and Noh, J. (1997). Index-Option Pricing with Stochastic Volatility and the Value of Accurate Variance Forecasts. *Review of Derivatives Research*, **1**, 120-144.

Fama, E.F. (1965). The Behaviour of Stock Market Prices. *Journal of Business*, **38**, 34-105.

French, K.R. and Roll, R. (1986). Stock Return Variances: The Arrival of Information and the Reaction of Traders. *Journal of Financial Economics*, **17**, 5-26.

Frey, R. and Michaud, P. (1997). The Effect of GARCH-type Volatilities on Prices and Payoff-Distributions of Derivative Assets - a Simulation Study, ETH Zurich, Working Paper.

Gallant, A.R., Hsieh, D.A. and Tauchen, G. (1991). On Fitting a Recalcitrant Series: The Pound/Dollar Exchange Rate 1974-83. In: W.A. Barnett, J. Powell, and G. Tauchen (Eds.), *Nonparametric and Semiparametric Methods in Econometrics and Statistics*, Cambridge University Press, Cambridge.

Ghose, D. and Kroner, K.F. (1995). The relationship between GARCH and symmetric stable processes: Finding the source of fat tails in financial data. *Journal of Empirical Finance*, **2**, 225–251.

Groenendijk, P.A., Lucas, A. and de Vries, C.G. (1995). A note on the relationship between GARCH and symmetric stable processes. *Journal of Empirical Finance*, **2**, 253–264.

Ghysels, E., Harvey, A. and Renault, E. (1996). Stochastic Volatility. In: G.S. Maddala (Ed.), *Handbook of Statistics*, Vol. 14, Statistical Methods in Finance, 119-191, Amsterdam, North Holland.

Giot, P. and Laurent, S. (2003). Value-at-Risk for Long and Short Trading Positions. *Journal of Applied Econometrics*, **18**, 641-664.

Heynen, R. and Kat, H. (1994). Volatility Prediction: A Comparison of the Stochastic Volatility, GARCH(1,1), and EGARCH(1,1) Models, *Journal of Derivatives*, Winter, **94**, 50-65.

Hol, E. and Koopman, S. (2000). Forecasting the Variability of Stock Index Returns with Stochastic Volatility Models and Implied Volatility, Tinbergen Institute, Discussion Paper No. 104, 4.

Hoppe, R. (1998). VAR and the Unreal World, *Risk*, **11**, 45-50.

Jacquier, E., Polson, N. and Rossi, P. (1999). Stochastic Volatility: Univariate and Multivariate Extensions. CIRANO, Scientific Series, 99s, 26.

Kanas, A. (1998). Volatility Spillovers Across Equity Markets: European Evidence. *Applied Financial Economics*, **8**, 245-256.

Kupiec, P.H. (1995). Techniques for verifying the accuracy of risk measurement models, *Journal of Derivatives*, **3**, 73-84.

Kon, S.J. (1984). Models of stock returns – A comparison. *Journal of Business*, **39**, 147-165.

Koutrouvelis, I.A. (1980). Regression-Type Estimation of the Parameter of Stable Laws. *Journal of the American Statistical Association*, **75**, 918-928.

Lamoureux, G.C. and Lastrapes, W.D. (1990). Heteroskedasticity in Stock Return Data: Volume Versus GARCH Effects. *Journal of Finance*, **45**, 221-229.

LeBaron, B. (1992). Some Relations Between Volatility and Serial Correlations in Stock Market Returns. *Journal of Business*, **65**, 199-219.

Liu, S.M. and Brorsen, B.W. (1995). Maximum likelihood estimation of a GARCH-stable model. *Journal of Applied Econometrics*, **10**, 273–285.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices, *Journal of Business*, **36**, 299-319.

Marquardt, D.W. (1963). An Algorithm for Least Squares Estimation of Nonlinear Parameters. *Journal of the Society for Industrial and Applied Mathematics*, **11**, 431-441.

McCulloch, J.H. (1986). Simple Consistent Estimates of Stable Distribution Parameters. *Communications in Statistics: Simulation and Computation*, **15**, 1109-1136.

McDonald, J.B. (1996). Probability Distributions for Financial Models. In: G.S. Maddala and C.R. Rao (Eds.), *Statistical Methods in Finance*, vol. 14, *Handbook of Statistics*, Elsevier, Amsterdam.

Mittnik, S. and Rachev, S.T. (1993). Modelling Asset Returns with Alternative Stable Distributions, *Economic Review*, **12**, 261-330.

Mittnik, S., Rachev, S.T., Doganoglu, T. and Chenyao, D. (1999). Maximum Likelihood Estimation of Stable Paretian Models. *Mathematical and Computer Modelling*, **29**, 275-293.

Nelson, D. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, **59**, 347-370.

Noh, K., Engle, R.F. and Kane, A. (1994). Forecasting Volatility and Option Prices of the S&P 500 Index. *Journal of Derivatives*, **2**, 17-30.

Pagan, A. and Schwert, G. (1990). Alternative Models for Conditional Stock Volatility, *Journal of Econometrics*, **45**, 267-290.

Panas, E. (2001). Estimating Fractal Dimension Using Stable Distributions and Exploring Long Memory Through ARFIMA Models in Athens Stock Exchange. *Applied Financial Economics*, **11**, 395-402.

Panorska, A., Mittnik, S. and Rachev, S.T. (1995). Stable GARCH Models for Financial Time Series. *Applied Mathematics Letters*, **8**, 33-37.

Poon, S.H. and Granger, C.W.J. (2003). Forecasting Volatility in Financial Markets: A Review. *Journal of Economic Literature*, **XLI**, 478-539.

Rachev, S. and Mittnik, S. (2000). *Stable Paretian Models in Finance*, John Wiley.

Schwert, G.W. (1989). Why Does Stock Market Volatility Changes Over Time. *Journal of Finance*, **44**, 1115-1153.

Shephard, N. (2004). *Stochastic Volatility: Selected Readings*, Oxford University Press.

Stock, J.H. (1988). Estimating Continuous Time Process Subject to Time Deformation. *Journal of the American Statistical Association*, **83**, 77-85.

Tauchen, G. and Pitts, M. (1983). The Price Variability-Volume Relationship on Speculative Markets. *Econometrica*, **51**, 485-505.

Taylor, S.J. (1994). Modelling Stochastic Volatility. *Mathematical Finance*, **4(2)**, 183-204.

Tsionas, E.G. (2002). Likelihood-Based Comparison of Stable Paretian and Competing Models: Evidence from Daily Exchange Rates. *Journal of Statistical Computation and Simulation*, **72(4)**, 341-353.

Wei, W. (2002). Forecasting stock market volatility with non-linear GARCH models: a case for China. *Applied Economics Letters*, **9**, 163-166.

Yu, J. (2002). Forecasting volatility in the New Zealand stock market. *Applied Financial Economics*, **12**, 193-202.

Tables and Figures

Table 1. Parameter estimates for the FTSE20, DAX30, FTSE100 and SP500 index daily returns (January 3rd, 1996 to July 5th, 2002).

Parameter	Coefficient				Coefficient / Standard error			
	FTSE20	DAX30	FTSE100	SP500	FTSE20	DAX30	FTSE100	SP500
μ_0	-0.001	0.000	-0.000	-0.001	-1.898	0.596	-0.067	-1.027
μ_1	2.853	1.995	1.251	4.297	1.634	0.736	0.319	1.071
μ_2	0.053	0.024	0.005	-0.100	1.103	0.398	0.078	-1.620
μ_3	0.317[b]	-0.075	0.144	0.333[a]	2.809	-0.544	1.140	2.745
a_0	-6.833[a]	-9.858[a]	-1.326[b]	-4.059[a]	-6.341	-10.727	-2.538	-5.929
δ_0	0.187[a]	0.095	0.012	0.039	3.382	1.880	0.342	0.956
Ψ_1	0.394[a]	0.190[a]	0.056	0.060	19.79	27.847	0.892	1.378
Δ_1	0.920[a]	0.973[a]	-0.001	0.785[a]	38.34	73.455	-0.003	28.040
γ	-0.064	-0.068	-0.108[a]	-0.236[a]	-1.043	-0.856	-2.969	-2.975
δ_1	0.010	-0.008	0.694[a]	0.081[b]	0.415	-0.688	4.822	2.295
δ_2	0.002	0.004	0.201[b]	0.041	0.103	0.386	2.116	1.314
v	1.335[a]	1.735[a]	1.858	1.689	-15.540	-9.137	-1.495	-8.184
Q_{20}	20.065	22.597	23.913	24.090	[0.391]	[0.256]	[0.200]	[0.193]
Q^2_{20}	16.663	23.747	24.696	13.003	[0.615]	[0.206]	[0.171]	[0.838]

Notes: With v=1.335, v=1.735, v=1.858, v=1.689, the 97.5% point of the generalized error distribution are 2.06, 2.00, 1.98 and 2.00, respectively. With v=1.335, v=1.735, v=1.858, v=1.689, the 99.5% point of the generalized error distribution are 2.94, 2.70, 2.65 and 2.72, respectively. For the FTSE20 index, parameters δ_1 and δ_2 present the volatility spillover from the SP500 and DAX30 indices, respectively. For the DAX30 index, parameters δ_1 and δ_2 present the volatility spillover from the FTSE100 and SP500 indices, respectively. For the FTSE100 index, parameters δ_1 and δ_2 present the volatility spillover from the DAX30 and SP500 indices, respectively. For the SP500 index, parameters δ_1 and δ_2 present the volatility spillover from the DAX30 and FTSE100 indices, respectively. Q_{20} and Q^2_{20} are the Q-statistics of order 20 computed on the standardized residuals and their squared values, respectively. The relative p-values are presented in brackets.

[a] Indicates that the coefficient is statistically significant at 1% level of significance.

[b] Indicates that the coefficient is statistically significant at 5% level of significance.

Table 2. Percentage of rolling-sampled estimated parameters that are outside the 95% confidence interval. (Values in parenthesis present the lower and upper bounds of the 95% confidence interval).

	FTSE20		DAX30	
μ_0	(-0.002 0.000)	56.48%	(-0.001 0.002)	33.18%
μ_1	(-1.780 7.485)	7.04%	(-4.989 8.978)	0.00%
μ_2	(-0.075 0.181)	0.00%	(-0.133 0.182)	0.00%
μ_3	(0.017 0.617)	0.32%	(-0.431 0.281)	0.00%
a_0	(-9.694 -3.972)	14.88%	(-12.227 -7.489)	3.20%
δ_0	(0.040 0.334)	1.12%	(-0.035 0.224)	0.00%
Ψ_1	(0.342 0.447)	13.12%	(0.172 0.207)	62.24%
Δ_1	(0.856 0.984)	54.40%	(0.939 1.007)	22.08%
γ	(-0.227 0.099)	0.00%	(-0.271 0.136)	0.00%
δ_1	(-0.056 0.076)	5.12%	(-0.038 0.022)	3.04%
δ_2	(-0.059 0.064)	32.16%	(-0.025 0.034)	1.60%
v	(1.222 1.449)	26.88%	(1.660 1.811)	46.72%
	FTSE100		SP500	
μ_0	(-0.001 0.001)	24.11%	(-0.002 0.001)	20.66%
μ_1	(-8.762 11.263)	0.80%	(-5.978 14.572)	16.48%
μ_2	(-0.148 0.157)	1.28%	(-0.258 0.058)	0.00%
μ_3	(-0.178 0.465)	12.32%	(0.022 0.644)	0.48%
a_0	(-2.659 -0.007)	16.64%	(-5.812 -2.306)	24.00%
δ_0	(-0.080 0.105)	0.00%	(-0.065 0.142)	0.00%
Ψ_1	(-0.104 0.215)	0.00%	(-0.052 0.173)	20.96%
Δ_1	(-0.472 0.471)	1.12%	(0.713 0.857)	60.48%
γ	(-0.201 -0.015)	1.12%	(-0.439 -0.033)	0.48%
δ_1	(0.327 1.062)	0.48%	(-0.009 0.171)	0.00%
δ_2	(-0.041 0.444)	0.00%	(-0.039 0.121)	35.36%
v	(1.616 2.100)	0.48%	(1.591 1.787)	9.44%

Table 3. Percentage of rolling-sampled estimated parameters that are statistically different from the parameter values estimated using the full data sample.

	FTSE20		DAX30		FTSE100		SP500	
Parameter	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level
μ_0	21.86%	1.29%	13.67%	0.80%	4.02%	0.00%	14.15%	4.34%
μ_1	0.96%	0.00%	0.00%	0.00%	0.16%	0.00%	8.52%	0.64%
μ_2	0.00%	0.00%	0.00%	0.00%	1.13%	0.00%	0.00%	0.00%
μ_3	0.00%	0.00%	0.00%	0.00%	3.22%	0.64%	0.00%	0.00%
a_0	17.20%	3.86%	16.72%	7.40%	0.48%	0.00%	24.28%	6.59%
δ_0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.73%	0.00%
Ψ_1	7.40%	0.00%	0.00%	0.00%	0.00%	0.00%	7.56%	0.00%
Δ_1	18.97%	10.13%	2.57%	0.00%	14.47%	5.79%	31.67%	3.54%
γ	0.00%	0.00%	5.14%	0.00%	4.50%	0.00%	36.17%	10.13%
δ_1	0.00%	0.00%	0.00%	0.00%	0.80%	0.32%	0.00%	0.00%
δ_2	12.54%	0.16%	0.00%	0.00%	0.16%	0.00%	24.92%	0.00%
v	1.29%	0.00%	16.72%	0.32%	0.00%	0.00%	0.00%	0.00%

Table 4. Percentage of the rolling-sampled estimated parameters that are statistically insignificant at 5% and 1% levels of significance.

	FTSE20		DAX30		FTSE100		SP500	
Parameter	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level
μ_0	30.06%	76.21%	88.36%	99.37%	94.69%	100%	66.35%	84.28%
μ_1	32.80%	97.11%	93.87%	100%	99.22%	100%	57.08%	87.26%
μ_2	99.84%	100%	100%	100%	99.22%	100%	100%	100%
μ_3	65.11%	87.78%	100%	100%	79.69%	96.56%	92.77%	100%
a_0	0.00%	0.48%	0.00%	0.00%	17.81%	40.78%	1.57%	18.08%
δ_0	27.65%	57.07%	81.45%	100%	100%	100%	100%	100%
Ψ_1	0.00%	0.00%	0.00%	0.00%	100%	100%	100	100
Δ_1	0.00%	0.00%	0.00%	0.00%	31.25%	38.91%	0.00%	0.00%
γ	100%	100%	100%	100%	100%	100%	0.00%	44.18%
δ_1	100%	100%	100%	100%	0.00%	0.16%	94.03%	100%
δ_2	89.55%	99.84%	100%	100%	67.97%	96.56%	59.91%	91.19%
v	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 5. AR(1)GARCH(1,1) simulated process. Percentage of rolling-sampled estimated parameters that are outside the 95% confidence interval.

$$y_t = \mu_0 + \mu_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t = z_t \sqrt{\sigma_t^2}\ ,\ z_t \overset{i.i.d.}{\sim} N(0,1)$$
$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + a_2 \sigma_{t-1}^2$$

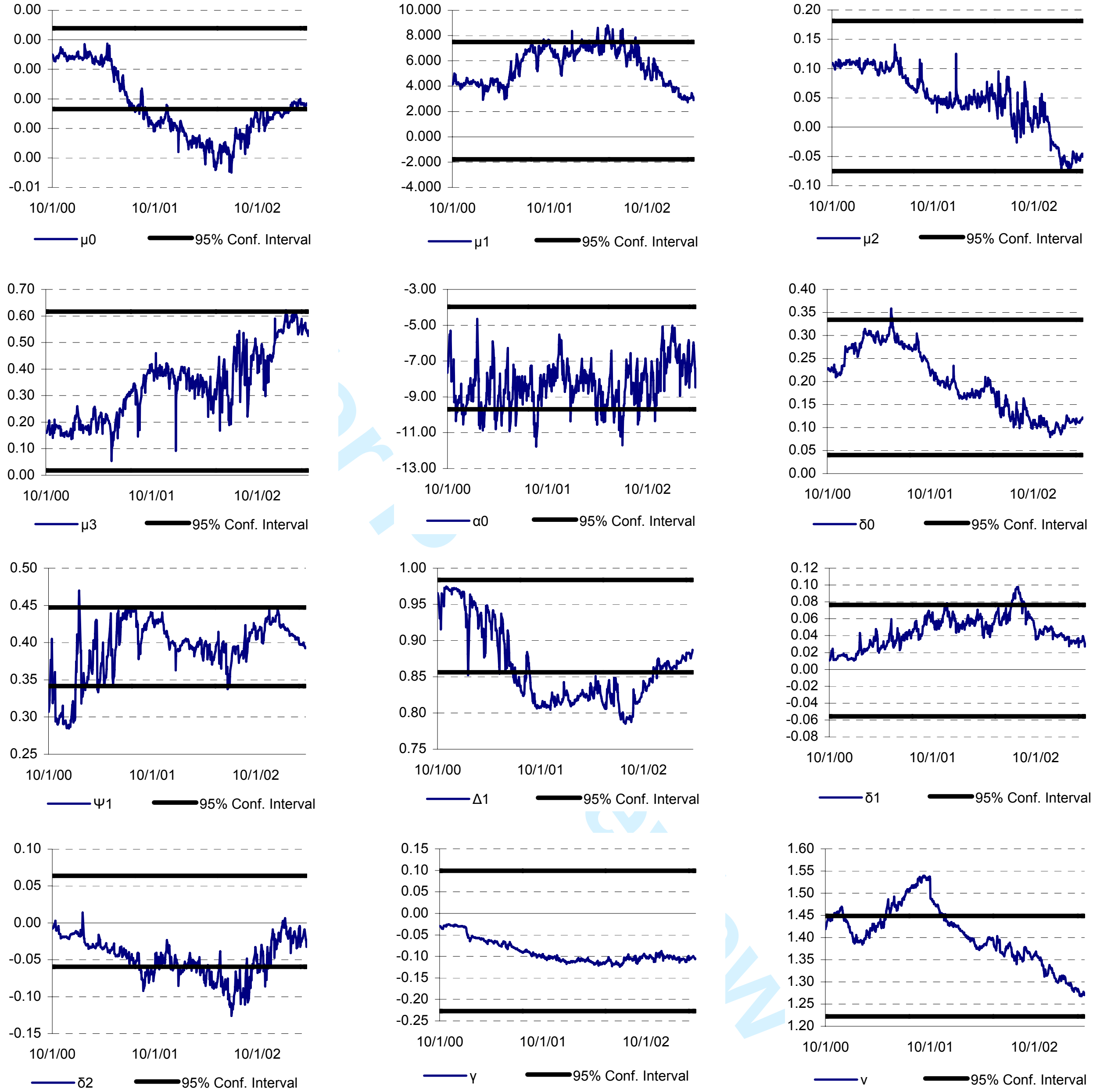
	μ_0	μ_1	a_0	a_1	a_2
Simulated Values	0.005	0.150	0.040	0.0500	0.900
Estimated Values (Full Data Sample)	-0.003	0.158	0.037	0.0138	0.895
Rolling parameters outside the 95% c.i.	11.70%	3.32%	73.17%	30.88%	72.17%

Table 6. Stable parameter estimates, using the full data sample, of the FTSE20, DAX30, FTSE100 and SP500 index daily returns, their standard errors and the percentage of rolling-sampled estimated parameters that are statistically different from the parameter values estimated using the full data sample at 5% level of significance.

	Tail index a	Skewness β	Location δ	Scale γ
	FTSE20			
Coefficient	1.48303	0.07799	-0.00033	0.01005
Standard error	0.05606	0.07965	0.00143	0.00081
5% sign. Level	0.32%	0.00%	0.00%	56.48%
	DAX30			
Coefficient	1.58306	-0.14798	0.00101	0.00754
Standard error	0.15725	0.18828	0.00069	0.00217
5% sign. Level	1.53%	9.59%	0.12%	0.00%
	FTSE100			
Coefficient	1.68238	-0.06489	0.00046	0.00591
Standard error	0.10944	0.25581	0.00039	0.00165
5% sign. Level	2.13%	9.42%	0.49%	0.00%
	SP500			
Coefficient	1.49172	-0.11841	0.0005	0.00525
Standard error	0.07160	0.09609	0.00052	0.00218
5% sign. Level	51.46%	5.00%	0.00%	0.00%

Notes: The standard error of parameter a is computed as the standard deviation of the rolling-sampled estimated parameters, $\hat{a}^{(t)}$, for $t = 1,\ldots,T$ trading days, i.e. $\sqrt{(T-1)^{-1}\sum_{t=1}^{T}\left(\hat{a}^{(t)} - \bar{a}^{(T)}\right)^2}$, where $\bar{a}^{(T)} = T^{-1}\sum_{t=1}^{T}\hat{a}^{(t)}$.

Figure 1. The rolling-sampled estimated parameters of the ARCH model and the 95% confidence interval of the parameters estimated using the full data sample.



Notes: The 95% confidence interval is constructed as $\hat{\theta} \pm GED(0,1;1.335,0.025)\hat{S}_{\theta}\sqrt{1621/1000}$, where $\hat{\theta}$ denotes the parameter vector estimated using the full data sample, $\hat{S}_{\theta}$ is the standard deviation of $\hat{\theta}$ and $GED(0,1;v,a)$ is the $(1-a)$ percentile of the GED distribution, with v denoting the tail thickness parameter.

Figure 2.a. In-sample 95% confidence interval of the FTSE20 index daily returns for the ARCH model (11th January 2000 to 5th July 2002).

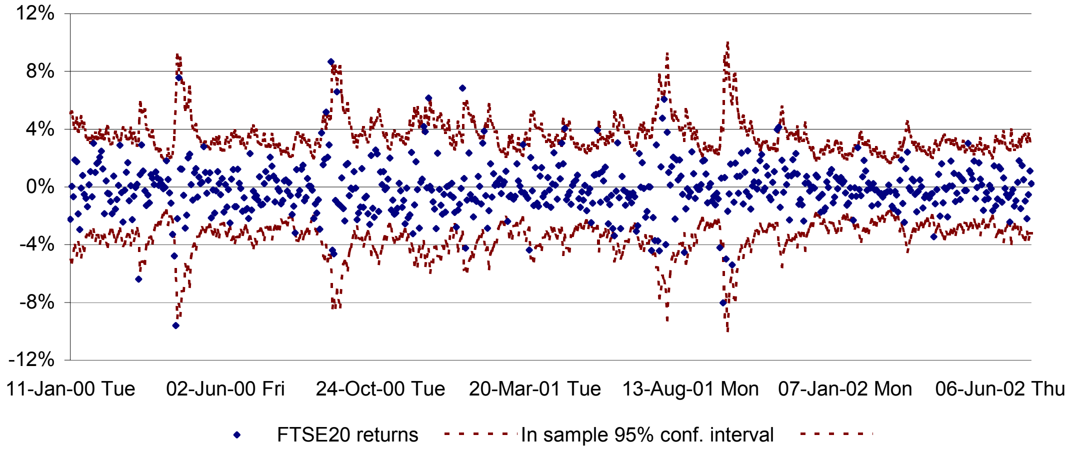


Figure 2.b. One-step-ahead 95% prediction interval of the FTSE20 index daily returns for the ARCH model (11th January 2000 to 5th July 2002).

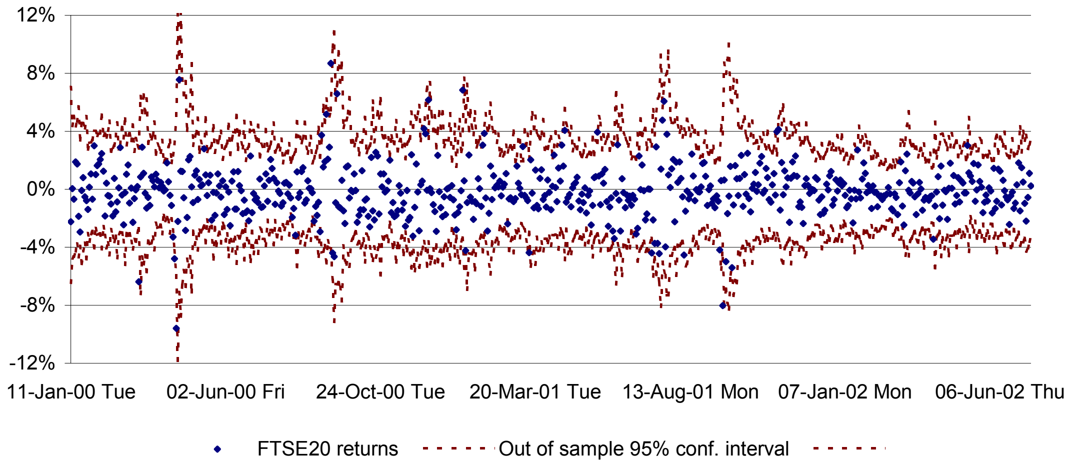


Figure 3. FTSE20 index daily returns. The rolling-sampled estimated parameters of the stable distribution and the 95% confidence interval of the parameters estimated using the full data sample.