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A Simple Multivariate Test for Asymmetry*

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Abstract

Since many macroeconomic models are linear, it is not desirable to use them with an asymmetric dependent variable. In this paper we formulate a univariate test for symmetry, based on the third central moment, and extend it to a multivariate test; the test does not require modelling and it is robust against serial correlation, autoregressive conditional heteroscedasticity and non-normality. In the empirical application of the test it is found that orthodox theory seem to be supported; consumption expenditure on durable goods is found to be symmetric while consumption expenditure on nondurable goods is asymmetric for the US and UK, with peaks being higher than troughs are deep. Also, the empirical importance of the choice between the univariate and the multivariate test for possibly correlated series is underscored; the results from the two approaches clearly differ. Given the widespread practice of using consumption expenditure on nondurable goods as the dependent variable in linear models for the US and the UK, our results might be noteworthy.

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1 Introduction

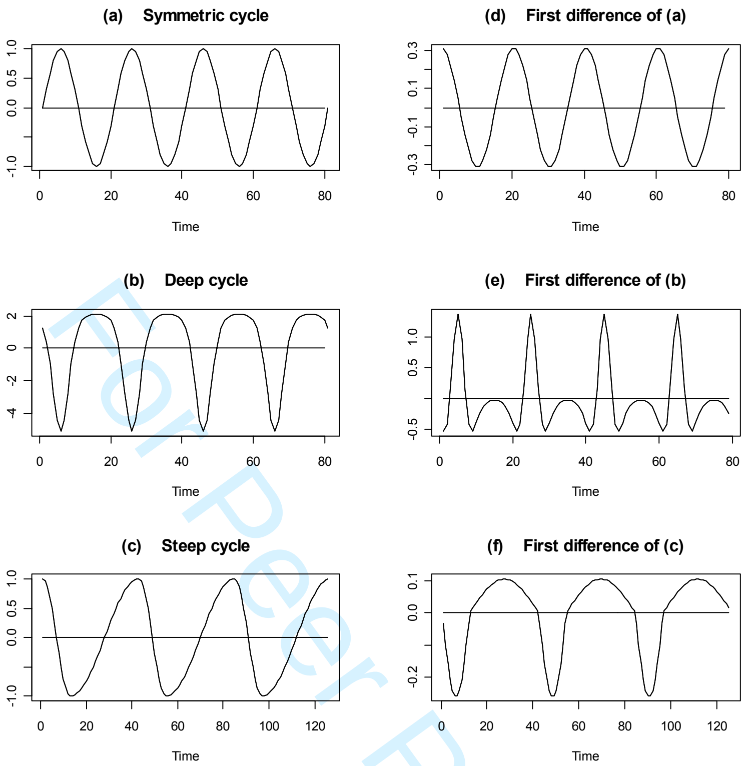
There has long been recognition that business cycles can be asymmetric, see e.g. Neftci (1984) and Potter (1995). Asymmetry of a macroeconomic times series is not a problem, though, unless it is modelled or treated as a symmetric series. However, since many macroeconomic models are linear, i.e. symmetric, we run the risk of trying to square the circle if we use them with an asymmetric dependent variable.

In this paper we propose a simple multivariate test for asymmetry, based on the third central moment. The test allow us to test for, using the terminology of Sichel (1993), two types of asymmetry; *steepness* and *deepness*. Deepness considers peaks and troughs distance above and below trend, while steepness considers the speed at which the peaks and troughs are approached. Positive deepness is present when peaks are taller than troughs are deep and negative deepness is present when the converse is true. Positive steepness is when the peaks are approached faster than are the troughs and negative steepness is present when the converse is true. In order to obtain an intuitive idea of the meaning of these patterns, we present one symmetric cycle, one cycle with negative deepness and one cycle with negative steepness; see Figures 1(a)-1(c) below.[1]

In applications where we test the dependent variable in a symmetric model for symmetry, our test can be viewed as a model specification test. Although in an economic setting in this paper, the proposed test is general in the sense that it can be applied to any stationary series. Also, the test does not require modeling and is robust against serial correlation, autoregressive conditional heteroscedasticity and non-normality. There are numerous proposed tests in the literature (using STAR-models for instance), that to a greater extent determine the finer characteristics of the asymmetry while modeling it, treatments that is

[1] These stylized cycles are only intended to give an intuitive understanding of the two concepts; deepness and steepness. In reality, the series under investigation is usually much less well-behaved. For a more thorough presentation of the data series used in this paper, see the Appendix.

Figure 1: Three different cycles in levels and first differences.



beyond the scope of this paper.[2]

Turning to empirical applications, one objection to this kind of diagnostic testing could be that such testing is superfluous since the question we are asking should already be answered by economic theory. However, it is not obvious to what degree a supposed asymmetry should be present. Moreover, the proposed diagnostic test could then be viewed as a way of testing theory regarding the

[2] Examples of such models are found in Bradley and Jansen (2000), for instance, where they fit non-linear time series models to output data for 26 countries. The result is a great deal of heterogeneity concerning the different countries' business cycles as the characteristics of the series are not easily classified into common groups as there is little unanimity concerning the non-linearity. In our context, however, this kind of precision is not needed as we are interested in the overall question of whether or not there are asymmetries. While it is of interest in other circumstances to model the possible nonlinearity with great precision, the verdict of a significant rejection (or not) of symmetry, is sufficient for our purposes.

behavior of the dependent variable. Speaking in favor of our approach is also the fact that not all economic theory is unanimous. Regarding consumption as well as business cycle theory, to mention but a few, competing views do not agree on whether growth and consumption is symmetric or not.

The theoretical aim of the paper is twofold. First, we want to create a univariate test for asymmetry for which the statistical properties are clearly derived. Second, we want to develop the univariate test into a multivariate, thus being able to take into account the possible interdependence between two or more of the series to be tested.

The empirical aim is to test theories of consumption suggesting symmetric or asymmetric behavior, as well as testing the validity of common practices in empirical studies on aggregate consumption. Lastly, the empirical section contains a test of the difference between the univariate and the multivariate test. In the empirical application we use data from the United States, the United Kingdom, Canada and Sweden.

The disposition of the paper is as follows. Section 2 starts with a brief discussion on asymmetry, followed by the derivation of the univariate test as well as the multivariate test. Section 3 first looks at the data and methods of detrending, followed by the empirical testing and a summary of the results. Section 4 comments and concludes. A more exhaustive graphical presentation of the data if found in the appendix.

2 Asymmetry

There are several ways in which a cycle may deviate from symmetry. Sichel (1993) suggests two types of asymmetry of prime importance, as presented in the previous section, which he refers to as deepness and steepness, respectively. Our main objective here is to detect deviations from symmetry of business cycles (i.e. to detect deviations from the pattern exemplified in Figure 1(a)). The series displayed in Figure 1, p.3, suggest that possible steepness (Figure 1(b)) or deepness (Figure 1(c)) may be analyzed with trigonometric models. However, this is not to be recommended as the cycles are stylized, seldom occurring in economic data. We will therefore aim to find techniques for evaluation of asymmetry that do not rely on a parametric model.

2.1 The univariate test for asymmetry

Consider a variable $Y_t \equiv (X_t - \mu_X)^3$ and define $\mu_X \equiv E[X_t]$, $\mu_Y \equiv E[Y_t]$ and $\gamma_j \equiv E\left[(Y_t - \mu_Y)(Y_{t-j} - \mu_Y)\right]$.[3] Then consider a set of constants $\{\psi_j\}_{j=0}^{\infty}$ such that $\sum_{j=0}^{\infty} |\psi_j| < \infty$ by assumption. By Wold's decomposition theorem (Hamilton (1994) p. 108), the variable Y_t can be written in the form

$$Y_t = \mu_Y + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} \tag{1}$$

where $\psi_0 = 1$ and $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables. The linear model in eq.(1) is frequently referred to as the $MA(\infty)$ representation of Y_t. Our interest lies in assessing whether μ_Y is zero or not. As μ_Y is unknown, we need an estimate of it along with a known asymptotic null distribution. Assuming that $E[\varepsilon_t^2] = \sigma_\varepsilon^2 < \infty$, it may be shown that

$$\hat{\theta} = \frac{\sqrt{T}(\bar{Y} - \mu_Y)}{\sqrt{v^2}} \xrightarrow{l} N(0,1) \tag{2}$$

where $v \equiv \sum_{j=-\infty}^{\infty} \gamma_j$ (see e.g. Hamilton (1994), p. 195). Eq.(2) does not lead to a feasible test regarding μ_Y since we assumed that v is known, which is usually not true. However, we may estimate v by

$$\hat{v}^2 = \hat{\gamma}_0 + 2\sum_{i=1}^{q} \left[1 - \frac{i}{q+1}\right] \hat{\gamma}_i. \tag{3}$$

[3] Note that in our case, we know from our method of detrending that $\mu_X = 0$; see section 3.2, p.8.

Then, if q is chosen such that $\left(q/T^{1/3}\right) \to 0$, it follows that $\hat{v} \xrightarrow{p} v$ (Hamilton (1994) p. 283). Hence,

$$\frac{\sqrt{T}(\bar{Y} - \mu_Y)}{\sqrt{\hat{v}^2}} \xrightarrow{l} N(0,1). \tag{4}$$

Definition 1 *A test of* $H_0 : E(X_t - \mu_X)^3 = 0$ *against* $H_A : E(X_t - \mu_X)^3 \neq 0$ *is defined by the rejection area* $|\hat{\theta}| > q_\alpha$ *where* q_α *is the* $(1-\alpha) \times 100^{th}$ *percentile of the standard normal distribution. That is,* H_0 *is rejected for values of* $|\hat{\theta}| > q_\alpha$.

2.2 The multivariate test for asymmetry

A multivariate version of the above may readily be constructed, the approach being analogous to the univariate case. Consider the P-dimensional random vector

$$\mathbf{Y}_t \equiv (\mathbf{X}_t - \boldsymbol{\mu}_\mathbf{x})^3$$

and let $\boldsymbol{\mu}_{\mathbf{X}_t} \equiv E[\mathbf{X}_t]$ and $\boldsymbol{\mu}_{\mathbf{Y}_t} \equiv E[\mathbf{Y}_t]$ and $\boldsymbol{\Gamma}_j \equiv E\left[(\mathbf{Y}_t - \boldsymbol{\mu}_\mathbf{Y})(\mathbf{Y}_{t-j} - \boldsymbol{\mu}_\mathbf{Y})'\right]$. Furthermore, consider a set of constants $\{\boldsymbol{\Psi}_j\}_{j=0}^{\infty}$ such that $\sum_{j=0}^{\infty} |\boldsymbol{\Psi}_j| < \infty$ by assumption and $\boldsymbol{\Psi}_0 = \mathbf{I}$. Then, by Wold's decomposition theorem (Lütkepohl (1991), p. 20), the variable $\mathbf{Y}_t$ can be written in the form

$$\mathbf{Y}_t = \boldsymbol{\mu}_\mathbf{Y} + \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j \boldsymbol{\varepsilon}_{t-j}.$$

Again, our interest lies in assessing whether $\boldsymbol{\mu}_\mathbf{Y}$ is equal to zero or not. As $\boldsymbol{\mu}_\mathbf{Y}$ is unknown, we need an estimate of it along with an asymptotically known null distribution. Assuming that $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'] = \boldsymbol{\Sigma}_\varepsilon < \infty$, it may be shown that

$$\sqrt{T}\mathbf{v}^{-1/2}(\bar{\mathbf{Y}} - \boldsymbol{\mu}_\mathbf{Y}) \xrightarrow{l} N(\mathbf{0}, \mathbf{I})$$

where $\mathbf{v} \equiv \mathbf{\Gamma}_0 + \sum_{j=1}^{\infty} \left(\mathbf{\Gamma}_j + \mathbf{\Gamma}_j'\right)$ (see, e.g., Brockwell and Davis (1991)). A consistent estimate of $\mathbf{v}$ may be obtained by

$$\hat{\mathbf{v}} = \hat{\mathbf{\Gamma}}_0 + \sum_{i=1}^{q} \left[1 - \frac{i}{q+1}\right] \left(\hat{\mathbf{\Gamma}} + \hat{\mathbf{\Gamma}}'\right) \tag{5}$$

and hence

$$\hat{\mathbf{\Theta}} = \sqrt{T}\hat{\mathbf{v}}^{-1/2} \left(\bar{\mathbf{Y}} - \boldsymbol{\mu}_{\mathbf{Y}}\right) \xrightarrow{l} N(\mathbf{0}, \mathbf{I}).$$

Definition 2 *A one-sided test of the hypothesis* $H_0 : E(\mathbf{X}_t - \boldsymbol{\mu}_{\mathbf{x}})^3 = \mathbf{0}$ *against* $H_A : E(\mathbf{X}_t - \boldsymbol{\mu}_{\mathbf{x}})^3 \neq \mathbf{0}$ *is defined by the rejection area* $\hat{\mathbf{\Theta}}'\hat{\mathbf{\Theta}} > Q_\alpha$ *where* Q_α *is the* $(1-\alpha) \times 100^{th}$ *percentile from the* $\chi^2_{(P)}$ *distribution. That is,* H_0 *is rejected for values of* $\hat{\mathbf{\Theta}}'\hat{\mathbf{\Theta}} > Q_\alpha$*.*

3 Empirical application

3.1 Data

The data set comprises quarterly aggregate consumption data for four countries; the US, the UK, Canada and Sweden.[4] For the US, the three series are total personal consumption expenditure, personal expenditure on non-durable goods and services and personal expenditure on durable goods for 1959:1-2002:4. All three series are seasonally adjusted, are in 1996 USD and have been obtained from the US Department of Commerce. For the UK, the three series are total domestic household consumption expenditure, households' domestic expenditure on non-durable goods and services and households' domestic expenditure on durable goods for 1963:1-2002:3. All three series are seasonally adjusted, are in 1995 GBP and have been obtained from the Office for National Statistics. For Canada, the three series are total personal consumption expenditure, personal expenditure on non-durable goods and services and personal expenditure on durable goods for 1961:1-2002:3. All three series are seasonally adjusted, are in 1997 CAD and have been obtained from Statistics Canada. For Sweden, the

[4] For a graphical presentation of the data, see the appendix, p.19.

three series are total household final consumption expenditure, household final consumption expenditure on non-durable goods and services and household final consumption expenditure on durable goods for 1963:1-2001:4. All three series are in 1991 SEK and have been obtained from Statistics Sweden and the National Institute for Economic Research. The Swedish series have been seasonally adjusted using the X12 ARIMA program. Henceforth, we will use c_t^i for total consumption expenditure, cd_t^i for consumption expenditure on durable goods and cnd_t^i for consumption expenditure on non-durable goods and services where the superscript $i = US, UK, CAN$ or SWE for the US, the UK, Canada or Sweden respectively.

Since our empirical application of the test for asymmetry stems from an interest in whether the dependent variable in a linear model is linear, the starting point of our choice of data is the type of data used in such models. When estimating an aggregate consumption function, the dependent variable chosen is usually both seasonally adjusted and in natural logarithmic form. Thus, besides the seasonal adjustment, we have taken the natural logarithm of all series. From a statistical point of view, tampering with the original data as mentioned above might seem out of place. However, since the objective of, and motivation for, our application of the test is a macro consumption model and its empirical specification, such tampering is both motivated and necessary.

3.2 Detrending - obtaining a stationary series

A series that exhibits growth over time, as most macroeconomic time series, is asymmetric a priori which necessitates detrending of the variable of interest. This is obviously controversial as we are tampering with the original series. However, since we are only interested in one specific characteristic, we can adjust our requirements accordingly. As long as the choice of detrending process does not influence the proposed test for asymmetry, any possible concerns for undesired properties that might follow from the detrending process are a non-issue in the current context. Next, we consider a simple way of detrending a time series.

3.2.1 First differencing

We previously defined $X_t \equiv Z_t - \tau_t$ without paying any explicit attention to the trend component τ_t. Now, in our case, since c_t and ξ_t have zero-means by construction, it follows that $\mu_{x_t} = [X_t] = 0$, and hence our variable Y_t is defined by $Y_t = X_t^3$. However, we have not yet discussed how to obtain X_t, i.e. how to remove the trend from Z_t. In general, we assume the trend component to be defined by $\tau_t = Z_{t-1}$ or by $\tau_t = \beta t$. Then, if $\tau_t = Z_{t-1}$, by taking the first difference of Z_t, the trend is removed. But if $\tau_t = \beta t$, the first difference removes the trend but at the same time introduces a constant (since $\beta t - \beta(t-1) = \beta$) which in turn may be removed through differencing the data once more; i.e. $E[\Delta^2 X_t] = 0$.

First differencing is appropriate for the steepness test but since the deepness test requires a series in levels, another method must be adopted. A description of the filtering process of Hodrick and Prescott (1997) follows next.

3.2.2 The Hodrick-Prescott filter

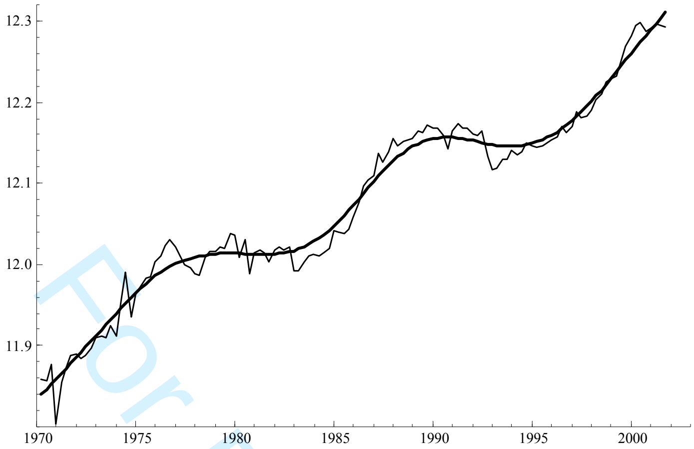
Formally, the Hodrick-Prescott (HP hereafter) filter proposes a trend τ_t for an original series Z_t which is the solution to the problem

$$\min \sum_{t=1}^{T} \{(Z_t - \tau_t)^2 + \lambda[(1-L)^2 \tau_t]^2\} \tag{6}$$

where L is the lag operator. λ is known as the smoothing parameter and the choice of it will determine the characteristics of the filtered series. As an illustrative example, a choice of $\lambda = 0$ leads to $\tau_t = Z_t$ while a choice of $\lambda = \infty$ leads to a linear trend. Drawing upon earlier work in general, for instance Canova (1994 and 1998) and Ravn and Uhlig (2002), and the original in particular, we let $\lambda = 1600$ as we have quarterly data. A graphical illustration of the two variables, τ_t and Z_t, is found in Figure 2 below in an application to Swedish total private consumption for the period 1970:2-2001:4.

The HP filter has been criticized for amplifying fluctuations in the cycle

Figure 2: Swedish total private consumption 1970:2-2001:4, original series and estimated HP-trend with $\lambda = 1600$.



under investigation, not a particularly desirable quality in several respects. This feature however, when combined with the test proposed here, is actually not negative but rather the contrary. If anything, it makes detection of asymmetry easier and is thus very much in line with the aim of the test.[5]

3.2.3 The choice of detrending method and the series to be tested

Thus, the variable X_t that is the starting point in Section 2.1 and 2.2 where we derive the tests, is obtained in different ways for the two tests.[6] For the deepness test, we get the desired variable by subtracting the HP estimated trend, τ_t, from the original series Z_t. That is, for the deepness test $X_t = Z_t - \tau_t$. For the steepness test we instead employ first differencing for detrending, followed by taking the first difference once more, this to get a series describing the rate of

[5] The advantages and drawbacks of various methods of detrending have been an issue for quite some time, and Canova (1998), for instance, provides a good overview of the field. The HP filter is, for instance, found to perform best when it comes to identifying the turning point of business cycles (Canova (1994)). Also, none of the HP filter's drawbacks presented in Canova (1998) poses a problem in the current context. All calculations for the HP filter have been done using PCGive 10.0.

[6] For a graphical presentation of all series, see the appendix p.19.

change in the detrended series. That is, for the steepness test $X_t = \Delta^2 Z_t$. In our application, we have that Z_t is equal to c_t^i, cd_t^i or cnd_t^i for both tests.

3.3 Testing

Theories on consumption as well as empirical practices, imply symmetric or asymmetric properties of different measures of aggregate consumption, properties that will be tested for in this section.

3.3.1 Does cd_t exhibit steepness?

It is fair to say that there is an established consensus that durable goods are to be viewed as investment goods and modelled accordingly, see e.g. Leahy and Zeira (2000). Furthermore, several theories agree on that cd_t exhibit steepness: Leahy and Zeira (2000) propose a model in which consumption expenditure on durable goods exhibits negative steepness while Gregorio et al (1998) presents a model where the steepness is positive.[7] Using our univariate test, we can try these hypotheses using data for the four countries in question. Looking at Table 7 below, we find the results from the tests.

Table 7: p-values from univariate testing of cd_t

H_0 :	cd_t^{US}	cd_t^{UK}	cd_t^{CAN}	cd_t^{SWE}
Steepness	0.37	0.64	0.47	0.88

(7)

As is evident from the table above, we cannot find any clear evidence of steepness for any of the four countries. Even applying significance levels that would be more generous than usual, does not help us since the lowest p-value is 0.37. Next, we to test if the widespread practice of using cnd_t as the dependent variable in linear consumption functions is recommendable.

[7] Gale (1996) proposes a model for investment goods that share the characteristic of negative steepness with the model in Leahy and Zeira (2000).

3.3.2 Is cnd_t asymmetric?

If consumption was shown to be asymmetric, it would be at odds with mainstream consumption theory, be it the Permanent Income Hypothesis or alternatives to it such as liquidity constraints, rule-of-thumb behavior or habit formation. A recent theoretical alternative is a loss aversion model in Bowman et al (1999), a model "...of consumption and saving based on Kahneman and Tversky's Prospect Theory that implies a fundamental asymmetry in consumption behavior inconsistent with other models of consumption." (Abstract). Applying the test for asymmetry to cnd_t is thus a test of mainstream theory, although rejection of symmetry not automatically equals support of the model proposed in Bowman et al (1999).[8] The results can be found in Table 8 below.

Table 8: p-values from univariate testing of cnd_t

	cnd_t^{US}	cnd_t^{UK}	cnd_t^{CAN}	cnd_t^{SWE}
Deepness	0.08*	0.09*	0.80	0.89
Steepness	0.80	0.80	0.43	0.96

(8)

* denotes significance on the 10% level

Here we see that for the two larger economies, the UK and the US, cnd_t display deepness while we cannot reject the hypothesis of normality for Canada and Sweden. The well-known similarities between the two Anglo-Saxon business cycles, are further supported by our findings since the two t-values are both positive, meaning that we have positive deepness; i.e. peaks are higher than troughs are deep.[9]

3.3.3 Are all consumption series symmetric?

Using c_t as the dependent variable in a linear consumption function is a practice that has grown more common over time; see e.g. Bjellerup (2005). In our

[8] Bowman et al (1999) also use c_t, as mentioned in note 14, p.166. Their main measure of consumption, however, is cnd_t.

[9] Although not as reliable as the test of course, this result can be viewed against the background of the number of positive and negative observations in the detrended series. In the two graphs on the left in figure 9, p.22 in the Appendix, we can see that the number of negative observations clearly outnumber the number of positive observations.

context, such use of c_t carries the implicit assumption of c_t, cd_t and cnd_t having the same symmetric properties. Using the univariate test we can test for the symmetry of c_t, and using the multivariate (or in this case bivariate) we can test for the symmetry of cd_t and cnd_t. The bivariate test is in contrast to the previous section where no interdependence was possible given the setting. Here, however, cd_t and cnd_t are of interest given the use of them as an aggregate; c_t.

In Table 9 below, we see the results from the univariate testing of c_t, that is the p-values from finite sample testing of eq.(4). The table reveals that, with a high probability, total consumption in the UK exhibits deepness.[10]

Table 9: p-values from univariate testing of c_t

	c_t^{US}	c_t^{UK}	c_t^{CAN}	c_t^{SWE}
Deepness	0.58	0.07*	0.17	0.87
Steepness	0.36	0.40	0.32	0.97

(9)

* denotes significance on the 10% level

Apart from the deepness test for the UK, all the other tests stay well clear of any conventional significance levels. Turning to the two components of c_t, cd_t and cnd_t, we can look at Table 10 where we find the p-values from the χ^2-distributed bivariate test.

Table 10: p-values from bivariate testing of cd_t and cnd_t

	US	UK	CAN	SWE
Deepness	0.245	0.408	0.315	0.923
Steepness	0.446	0.780	0.291	0.952

(10)

Here we can see that we are not able to reject the null hypothesis of symmetry in any of the cases. Before commenting on the results, we also want to investigate whether it matters, in the current context, if the series are modeled as correlated or not.

[10] Although this type of classification not is the main aim of the paper, we can conclude that that it is a positive deepness, by looking at the t-statistic which is positive; i.e. the peaks are higher than the troughs are deep for c_t^{UK}.

3.3.4 The impact of correlation - the univariate vs. the multivariate test

From a theoretical point of view, given some correlation between the series, it is obvious that the univariate and the multivariate tests will yield different results. However, from an empirical point of view, these differences can of course vary in magnitude therefore a closer look at the two approaches is warranted. More specifically, we are interested in whether the verdict of rejection differs or not between approaches. Before proceeding, it is important to stress that this section is intended only to highlight the possible difference between the two approaches and in that way underscore the importance of having chosen the right approach. The choice itself, however, is of course made on a theoretical basis depending on the situation and the data at hand. Here, we merely demonstrate the difference between the two options given the current context.

The novelty of the multivariate approach is that $\mathbf{v}$ in eq.(5) captures the covariances of the series, something of course lacking in the univariate approach where the series are tested one at a time, independently of one another. In Table 11 below, the importance of the covariance is demonstrated in the current empirical application through a comparison between the two approaches.

Table 11: p-values from the bivariate ('Joint') and the univariate tests (cd_t and cnd_t)

		Deepness	Steepness
US	Joint	0.25	0.45
	cd_t	0.11	0.37
	cnd_t	0.08*	0.78
UK	Joint	0.41	0.71
	cd_t	0.13	0.64
	cnd_t	0.09*	0.80
CAN	Joint	0.32	0.29
	cd_t	0.04**	0.47
	cnd_t	0.80	0.43
SWE	Joint	0.92	0.95
	cd_t	0.67	0.88
	cnd_t	0.89	0.62

(11)

** denotes significance on the 5% level
* denotes significance on the 10% level

As we know from the previous section, we are not able to reject the null hypothesis of symmetry in any of the eight bivariate tests. If we instead turn to the disaggregated approach in the univariate setting, we find that we are able to reject the null hypothesis in three cases; the deepness test for cd_t^{CAN}, cnd_t^{UK} and cnd_t^{US}, corresponding to three out of the eight cases in the bivariate test.

3.4 Results

The models in Gregorio et al (1998) and Leahy and Zeira (2000) imply that cd_t exhibits positive and negative steepness, respectively, implications that not found support in our four-country sample. The null hypothesis of the test of symmetry was not even close of being rejected given that the lowest p-value was 0.37.

As for the alleged symmetry of cnd_t, the results are different. For the two large Anglo-Saxon economies, the US and the UK, we can reject the null as both series exhibits positive deepness. Given the widespread use of cnd_t in linear consumption models, we believe our results should be a cause of some concern. Specifically, our results suggest that any linear consumption model of the UK or the US will consistently underestimate the height of the peaks in the business cycle. Thus, the results here are broadly in line with previous findings in model based tests of asymmetry in Shea (1995a), Shea (1995b) and Bowman et al (1999), yielding tentative evidence in favor of the loss-aversion model of Bowman et al (1999).

The next hypothesis, that the three consumption measures, c_t, cd_t and cnd_t, have the same symmetric properties, was difficult to reject. Again, we highlight that cd_t and cnd_t were tested jointly, in contrast to the previous hypothesis test where we tested cnd_t in a univariate setting. Here, we could only reject the null hypothesis in one out of the sixteen tests, and then at the less conventional 10% significance level. Put differently, we could not reject it for three out four countries and in the case of the UK we could reject it, based on a rejection of the deepness test of c_t^{UK} at a 7% significance level.

Turning to the question of the importance of correlation between the series to be tested, we find it to be very important in the current context. In the bivariate case we could not reject the null hypothesis of symmetry in any of the eight cases. However, in 3 out of the 8 corresponding cases for the univariate test, we found that symmetry for one of the two series was rejected. Of course, the results for the difference between the two approaches will vary with the data, but the results here at least indicate that the potential for very different conclusions not is negligible. Again, we want to stress that this only is intended as a comment on the differences between the two alternatives, not the choice between them.

We also want to point out that the question of aggregation is highly non-trivial. In the current setting, we can take the example of the UK and the US. For both economies, cnd_t exhibits positive deepness while the aggregate c_t only does for the UK. To address the possible explanation for such issues, quite a different type of analysis would have to be employed, an analysis beyond the scope of this paper.

4 Comments and conclusions

In this paper we formulate a univariate test for symmetry based on the third central moment and extended it to a multivariate test. The test is robust in several aspects. First, it does not require modeling. Second, it is robust against serial correlation, autoregressive conditional heteroscedasticity as well as non-normality.

In the empirical application of the test we focus on aggregate consumption. Theory, as well as its empirical applications, have several implications which the test allows us to test for. First, significant deviations from symmetry for cd_t can not be found. Second, the alleged symmetry of the consumption of nondurable goods (cnd_t), does not find the expected support since the US and the UK series both exhibited positive deepness, i.e. the peaks are higher than the troughs are deep. For total consumption (c_t), no such asymmetry is found. Given the widespread use of cnd_t in linear models, we believe our results should be a cause of some concern. Specifically, our results suggest that any linear consumption model of the UK or the US will consistently underestimate the height of the peaks in the business cycle. Third, the univariate and multivariate tests are used to test the hypothesis that c_t, cd_t and cnd_t have the same symmetric properties. Out of the four countries in the sample (the US, the UK, Canada and Sweden), it is only for the UK that the hypothesis is rejected. Thus, from the point of symmetry, the use of c_t as the dependent variable in a consumption regression is given some support, although not unanimous. Fourth, the empirical importance of choosing whether to test possibly correlated series in a univariate or a multivariate setting is underscored. The results from the two approaches differs significantly, emphasizing why the importance of this choice should not be underestimated. However, such a choice is guided by theoretical considerations; the results here merely highlight the importance of that choice in the current empirical context.

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Appendix

Graphical illustration of the data

In the graphical presentation of the various series as used in the paper given below, we have chosen not to include the HP-trend in the diagrams in Figure 3, since the graphs have to be of considerable size to be meaningful. The example given in Figure 2, p.9, serves well as a general illustration of what the estimated HP-trend looks like. Since the test can be viewed as a test for the number of positive and negative observations we have included two numbers (#xx) in each of the graphs depicting the deepness and steepness series. The number above the zero line indicates the number of positive observations while the number below the zero line indicates the number of negative observations.

Figure 3: The original series in logs for the three measures of consumption for all four countries.

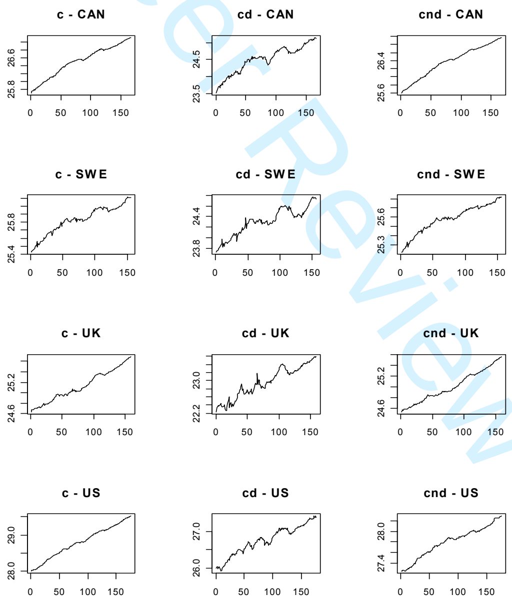

Figure 4: Deepness and steepness series for c_t^{SWE} and c_t^{CAN}.

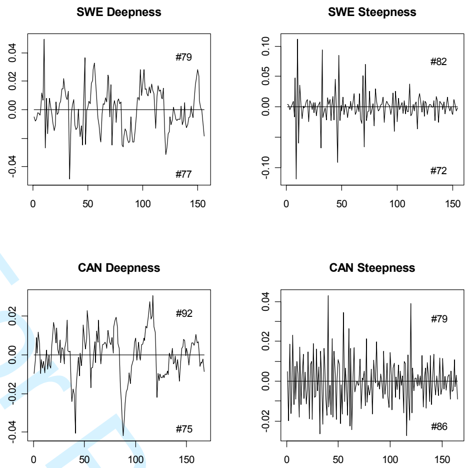

Figure 5: Deepness and steepness series for c_t^{UK} and c_t^{US}.

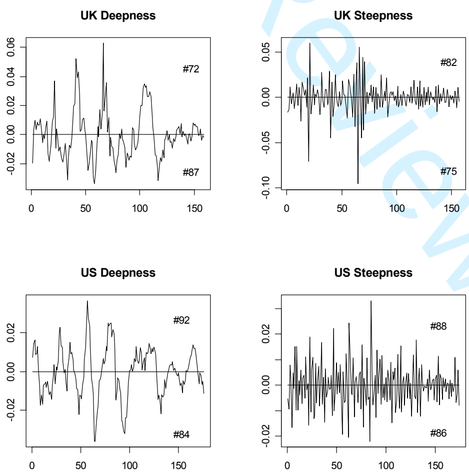

Figure 6: Deepness and steepness series for cd_t^{SWE} and cd_t^{CAN}.

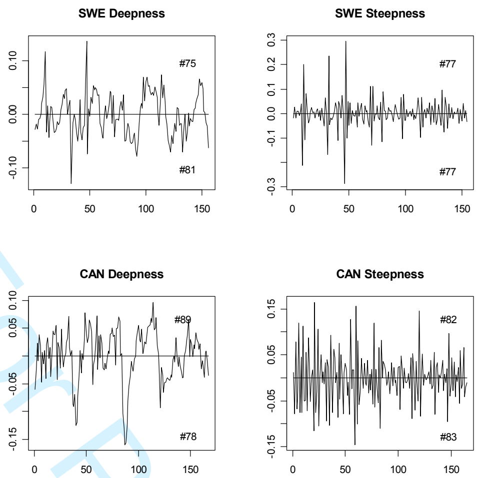

Figure 7: Deepness and steepness series for cd_t^{UK} and cd_t^{US}.

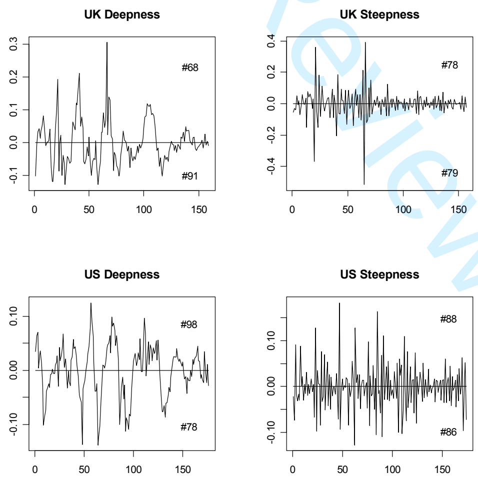

Figure 8: Deepness and steepness series for cnd_t^{SWE} and cnd_t^{CAN}.

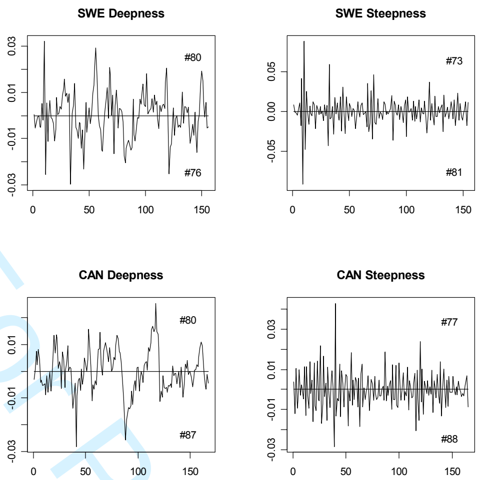

Figure 9: Deepness and steepness series for cnd_t^{UK} and cnd_t^{US}.