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The presence of target zone nonlinearities when narrower bands exist within official zones

The presence of target zone nonlinearities when narrower bands exist within official zones

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Abstract

The presence of target zone nonlinearities is generally refuted in empirical research. We argue that this may be due to estimation being performed vis-à-vis official limits when monetary authorities in fact are targeting a narrower band. Estimation results for the Belgian and French franc confirm that nonlinearities are present when narrower zones are accounted for.

Keywords: Brownian motion, conditional probability density function, exchange rates, nonlinearities, target zones.

JEL classification: F31, F33.

[*]University of Amsterdam, Department of Economics, Roetersstraat 11, 1018 WB Amsterdam, the Netherlands. Tel.: +31-20-5254194, fax: +31-20-5254254, e-mail: Dirk.Veestraeten@uva.nl. I am indebted to Roel Beetsma, Giuseppe Cavaliere, Paul De Grauwe, Casper de Vries, Hans Dewachter, Henk Jager, Eiko Kenjoh, Franc Klaassen, Piet Sercu, Marno Verbeek and Koen Vermeylen for stimulating discussions. Of course, the usual disclaimer applies.

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1   Introduction

Empirical research on nonlinearities and implicit bands in exchange rate target zones has largely evolved independently. The literature on nonlinearities in the relationship between fundamentals and exchange rates found only sparse evidence of departures from linearity (see, for instance, Flood, Rose and Mathieson, 1991; de Jong, 1994; Lindberg and Söderlind, 1994; Garratt, Psaradakis and Sola, 2001). On the other hand, the empirical literature recorded strong evidence of narrower or implicit zones within official limits in the Exchange Rate Mechanism (see, amongst others, Bartolini and Prati, 1999; Anthony and MacDonald, 1999; Chung and Tauchen, 2001). However, connecting these two distinct lines of the literature could be a worthwhile exercise since the presence of narrower zones implies that expectation formation is to be evaluated with respect to these actual limits rather than vis-à-vis the wider, officially announced zone. This is of the utmost importance for the search for target zone nonlinearities since they are likely to be overlooked when inspection is performed with respect to limits that are too wide. Indeed, the Krugman (1991) model shows that nonlinearities primarily are to be found near the limits. Using official limits in estimation when actually a narrower zone is in place will therefore tend to underestimate nonlinearities.

This paper estimates the Krugman (1991) target zone model in which asymmetric implicit fluctuation zones are allowed for. The sample consists of the Belgian and French francs and covers the period 1994-1997 in which official band limits were at $\pm 15\%$. Using the analytical characterisation of the likelihood function for reflected Brownian motion, that was recently derived in Veestraeten (2004), we are able to employ maximum likelihood estimation.

2   The target zone model of Krugman (1991)

The model of Krugman (1991) starts from the log-linear asset pricing equation that expresses the (log of the) exchange rate, s, as the sum of the (log of the) fundamental, f, and its own expected change:

$$s = f + \alpha \frac{E[ds]}{dt}, \tag{1}$$

with $\alpha > 0$ and E denoting the expectations operator.[1]

Krugman (1991) specified the dynamics of f as a Brownian motion with drift between two limits. The lower and upper band limits, $\underline{f}$ and $\overline{f}$, result from the intervention obligations within the target zone arrangement. Monetary authorities thus decrease (increase) the control variable, i.e. the fundamental, whenever it reaches its upper (lower) band limit which gives rise to regulated or reflected Brownian motion:

$$df = \mu dt + \sigma dz\,(t) + dL - dU, \tag{2}$$

where dL and dU are the infinitesimal regulators.

Expressing the exchange rate as an explicit function of the fundamental, i.e. $s = s\,(f)$, applying Itô's lemma to the expectations term and solving Equation (1) then yields the well-known S-shaped exchange rate function, see Krugman (1991):

$$s\,(f) = f + \alpha\mu + A_1 \exp\,(\lambda_1 f) + A_2 \exp\,(\lambda_2 f). \tag{3}$$

The two exponential terms cause the renowned bending of the exchange rate function and thus generate the target zone nonlinearities. The no-expected-arbitrage-profits condition then leads to the so-called smooth-pasting conditions that require the exchange rate function to be flat at the boundaries such that intervention does not lead to discontinuities in the exchange rate path. The resulting boundary conditions then allow for the determination of the two constants of integration A_1 and A_2 which completely solves the model.

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3 The conditional density function of the exchange rate

The exchange rate is a one-to-one transform of the fundamental since its first derivative to the fundamental can be shown to be always non-negative. Moreover, the first derivative of the inverted exchange rate function to the exchange rate is continuous. These monotone mapping and continuously differentiability properties allow us to express the conditional density of the exchange rate in terms of the conditional density of the fundamental through a simple change of variables:

$$p_s\left(s, t; s_0, t_0\right) = \frac{p_f\left(s^{-1}\left(s\right), t; s^{-1}\left(s_0\right), t_0\right)}{\left|\dfrac{ds^{-1}\left(s\right)}{df}\right|}, \tag{4}$$

where $p_s\left(\cdot\right)$ and $p_f\left(\cdot\right)$ denote the conditional density functions of the exchange rate and the fundamental, respectively. The future and the present point of time are denoted by t and t_0 and the function $s^{-1}\left(s\right)$ is the inverse function of $s\left(f\right)$. The conditional density function of the fundamental, i.e. of a reflected Brownian motion with drift, was recently derived in Veestraeten (2004) and in the present notation reads as follows:

$$
\begin{aligned}
p_f\left(f, t; f_0, t_0\right) = & \sum_{n=-\infty}^{+\infty}\left\{\frac{1}{\sigma\sqrt{2\pi(t-t_0)}}\exp\left(\frac{2\mu n\left(\underline{f}-\overline{f}\right)}{\sigma^2}\right)\exp\left(-\frac{\left(f+2n(\overline{f}-\underline{f})-f_0-\mu\left(t-t_0\right)\right)^2}{2\sigma^2\left(t-t_0\right)}\right)\right\} \\
& +\sum_{n=-\infty}^{+\infty}\left\{\frac{1}{\sigma\sqrt{2\pi(t-t_0)}}\exp\left(-\frac{2\mu\left(n\overline{f}-(n+1)\underline{f}+f_0\right)}{\sigma^2}\right)\right. \\
& \qquad \left. \exp\left(-\frac{\left(2n\overline{f}-2\left(n+1\right)\underline{f}+f_0+f-\mu\left(t-t_0\right)\right)^2}{2\sigma^2\left(t-t_0\right)}\right)\right\} \\
& -\frac{2\mu}{\sigma^2}\sum_{n=0}^{+\infty}\left\{\exp\left(\frac{2\mu\left(n\overline{f}-(n+1)\underline{f}+f\right)}{\sigma^2}\right)\left[1-\Phi\left(\frac{\mu\left(t-t_0\right)+2n\overline{f}-2\left(n+1\right)\underline{f}+f_0+f}{\sigma\sqrt{t-t_0}}\right)\right]\right\} \\
& +\frac{2\mu}{\sigma^2}\sum_{n=0}^{+\infty}\left\{\exp\left(\frac{2\mu\left(n\underline{f}-(n+1)\overline{f}+f\right)}{\sigma^2}\right)\Phi\left(\frac{\mu\left(t-t_0\right)-2\left(n+1\right)\overline{f}+2n\underline{f}+f_0+f}{\sigma\sqrt{t-t_0}}\right)\right\},
\end{aligned}
\tag{5}
$$

where $\Phi\left(z\right) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{z}\exp\left(-\frac{1}{2}y^2\right)dy$ and denotes the cumulative standard normal distribution function.

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4 Maximum likelihood estimation

The Markov property of the fundamental carries over to the exchange rate given their one-to-one relationship such that the joint distribution of $\mathbf{s}$, $\mathbf{P}(\mathbf{s})$, can be written as:

$$\mathbf{P}(\mathbf{s}) = \prod_{t=0}^{T-1} p_s(s_{t+1}, t+1; s_t, t). \tag{6}$$

The loglikelihood function for the observed exchange rate series $\mathbf{s}$, in terms of the parameter vector $\mathbf{\Theta} = \{\alpha, \ \mu, \ \sigma, \ \underline{f}, \ \overline{f}\}$, then emerges as:

$$LogLikF(\mathbf{s}, \mathbf{\Theta}) = \sum_{t=0}^{T-1} \ln\left(p_s(s_{t+1}, t+1; s_t, t) \mid \mathbf{\Theta}\right). \tag{7}$$

The parameters in the maximisation process consist of the four parameters that fully specify the stochastic process of the fundamental, namely its instantaneous drift and standard deviation, μ and σ, and the upper and lower band limits, $\overline{f}$ and $\underline{f}$. The fifth parameter is the sensitivity of the exchange rate to its own expected change, α, which governs the extent of nonlinearities, see Equations (1) and (3). Earlier research, such as de Jong (1994) and Lindberg and Söderlind (1994), indicated that estimation of target zone models is rendered more complex by the flatness of the likelihood surface such that obtaining standard errors for all five coefficients is not sensible. We therefore calculate asymptotic standard errors for four parameters whilst keeping α at its estimated value. However, model specification and especially the significance of α will be formally tested via Likelihood Ratio (LR) tests.

The exchange rates of the Belgian and French currencies are Wednesday prices and are quoted against the German mark. The sample spans the period March 1994 until February 1997. The starting point was chosen in function of the theoretical prerequisite of credibility since the turmoil in the Exchange Rate Mechanism (ERM) of 1993 for these two currencies only faded away towards the end of that year. The endpoint reflects doubts on the continued appropriateness of the target zone model in view of the prospect of the European Monetary Union (EMU). In fact, De Grauwe,

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Dewachter and Veestraeten (1999) argued that the start of EMU on 1 January 1999 had to be seen as a regime switch that already more than one year earlier drastically altered exchange rate dynamics.

Table 1 presents estimation results for the Krugman (1991) model. The first two rows present results when estimating with respect to the official target zone limits. The subsequent rows relate to the case in which implicit bands are allowed for. The estimation results and especially the loglikelihood values strongly suggest that implicit zones were present such that estimation with respect to official band limits could have obscured nonlinearities. This will be formally tested via the LR specification tests in Table 2.

Tables 1 and 2: around here.

The first LR-test in Table 2 examines whether α significantly differs from zero, i.e. whether nonlinearities are present, when estimation is pursued with respect to the official zone. Constraining α to be zero causes s to equal f, see Equation (1), such that the exchange rate in the constrained model follows reflected Brownian motion. The restriction on α is non-standard since α is constrained to be nonnegative and, moreover, the restriction lies on the boundary of its domain. Chernoff (1954) showed that the resulting LR-statistic is distributed $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$. We follow de Jong (1994) and approximate this distribution as χ_1^2. The second LR-test pursues the same strategy but now estimation allows for implicit bands. The findings reveal that estimation with respect to official bands obscures nonlinearities in the case of the Belgian franc since the constrained model cannot be rejected (LR-test 1) whereas it can safely be rejected when implicit limits are allowed for. For the French franc, nonlinearities are present in both cases but the p-value is far lower when analysing them vis-à-vis an implicit zone. The third LR test evaluates whether the implicit zones are significantly different from the official zones. Hereto, the two models in Table 1 are compared in which the official-zone model is the constrained model and the test statistic thus is distributed as χ_2^2. Unsurprisingly, implicit zones significantly differ

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from officially announced fluctuation ranges in line with the findings reported in Bartolini and Prati (1999), Anthony and MacDonald (1999) and Chung and Tauchen (2001).

5   Conclusion

This paper examined exchange rate target zone nonlinearities by explicitly relating their presence to the potential pursuit of implicit, narrower fluctuation ranges within the official band. Since nonlinearities are typically to be detected near the band limits, estimation should devote careful attention to the precise location of the fluctuation limits. Estimation with respect to official limits when in fact monetary authorities are pursuing a narrower zone will inevitably bias results towards rejecting the presence of nonlinearities. The maximum likelihood estimation results confirmed the existence of an underestimation bias when official limits are used. In fact, the presence of nonlinearities for the Belgian franc between 1994 and 1997 had to be discarded when estimating with respect to official limits, whereas they were clearly present when gauged with respect to narrower, implicit zones. Or, nonlinearities can be overlooked if estimation fails to account for implicit limits. The underestimation bias was not present in the case of the French currency although rejection of the hypothesis of no nonlinearities was much stronger when examined with respect to implicit limits.

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References

[1] Anthony, M. and R. MacDonald, 1999, The width of the band and exchange rate mean-reversion: some further ERM-based results, Journal of International Money and Finance 18, 411-428.

[2] Bartolini, L. and A. Prati, 1999, Soft exchange rate bands and speculative attacks: theory, and evidence from the ERM since August 1993, Journal of International Economics 49, 1-29.

[3] Chernoff, H., 1954, On the distribution of the likelihood ratio, Annals of Mathematical Statistics 25, 573-578.

[4] Chung, C.-S. and G. Tauchen, 2001, Testing target zone models using Efficient Method of Moments, Journal of Business & Economic Statistics 19, 255-269.

[5] De Grauwe, P., H. Dewachter and D. Veestraeten, 1999, Price dynamics under stochastic process switching: some extensions and an application to EMU, Journal of International Money and Finance 18, 195-224.

[6] de Jong, F., 1994, A univariate analysis of EMS exchange rates using a target zone model, Journal of Applied Econometrics 9, 31-45.

[7] Flood, R. P., A. K. Rose and D. J. Mathieson, 1991, An empirical exploration of exchange rate target zones, Carnegie-Rochester Series on Public Policy 35, 7-66.

[8] Garratt, A., Z. Psaradakis and M. Sola, 2001, An empirical reassessment of target-zone nonlinearities, Journal of International Money and Finance 20, 533-548.

[9] Krugman, P. R., 1991, Target zones and exchange rate dynamics, Quarterly Journal of Economics 106, 669-682.

[10] Lindberg, H. and P. Söderlind, 1994, Testing the basic target zone model on Swedish data 1982-1990, European Economic Review 38, 1441-1469.

[11] Veestraeten, D., 2004, The conditional probability density function for a reflected Brownian motion, Computational Economics 24, 185-207.

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Table 1: Maximum likelihood estimates of the Krugman (1991) model with either official band limits or implicit band limits, sample period: 2 March 1994 - 29 January 1997.

Currency	α	μ	σ	$\underline{f}$	$\overline{f}$	Loglikelihood
Official zone						
Belgian franc	21.00281 [na]	-0.00203 [0.00477]	0.00778 [0.00089]	-	-	827.841
French franc	14.33483 [na]	-0.04479 [0.04595]	0.05125 [0.01845]	-	-	627.822
Implicit zone						
Belgian franc	5.60832 [na]	-0.01351 [0.03580]	0.03725 [0.03990]	2.98805 [0.01099]	3.15616 [0.31373]	865.629
French franc	8.99944 [na]	-0.03573 [0.07973]	0.08792 [0.07462]	1.12590 [0.03699]	1.60526 [0.23268]	651.102

Note: Asymptotic standard errors are given within brackets.

Table 2: Likelihood Ratio tests, sample period: 2 March 1994 - 29 January 1997.

	LR-test 1	p-value[1]	LR-test 2	p-value[1]	LR-test 3	p-value[2]
Belgian franc	0.03564	0.8503	75.61152	3.5E-18	75.57589	3.9E-17
French franc	7.68802	0.0056	54.24664	1.8E-13	46.55863	7.8E-11

[1] Under χ_1^2.
[2] Under χ_2^2.

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Response to the referee report

I would like to thank professor Mark P. Taylor for his positive appraisal of my paper and for giving me the opportunity to revise the paper under consideration. The discussion of his remark allowed me to better clarify the rationale of the paper as well as its positioning within the target zone literature. In this "Response to the referee report", I will first include the relevant part of the referee report (section 1). In the second section, I will specify the answer to the question raised in the referee report. Section 3 proceeds by listing the textual changes that have been undertaken in the revised version of the paper.

1   Referee report

In particular, I would like you to think about how your work relates to two papers that I wrote on exactly this topic (listed below). In it, I demonstrate that the failure to find nonlinearity is not surprising if the target zone model is parameterised properly. Does the fact that you can detect nonlinearity therefore mean (as I suspect) that your results are all the more impressive since they reject linearity even when the test has low power? Please have a look at the papers and let me know how you square your results. They are:

"On the Mean-Reverting Properties of Target Zone Exchange Rates: A Cautionary Note" Authors: Taylor, Mark P. and Matteo Iannizzotto, Economics Letters v71, n1 (April 2001): pp. 117-29

"The Target Zone Model, Non-linearity and Mean Reversion: Is the Honeymoon Really Over?" Authors: Matteo Iannizzotto and Mark P. Taylor, Economic Journal v109, n454 (March 1999): C96-110

2   Reply to comments

Iannizzotto and Taylor (1999) and Taylor and Iannizzotto (2001) estimate the Krugman (1991) model via the method of simulated moments and detect evidence of small but significant degrees of target zone nonlinearities. Subsequently, they examine standard mean reversion tests that in the existing literature are frequently employed as a testing procedure for the presence of target zone nonlinearities. Their Monte Carlo simulations convincingly show that (Augmented) Dickey-Fuller and variance ratio

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tests possess low power and fail to reject the absence of nonlinearities even when the data generating process in fact is that of a credible target zone arrangement. Phrased differently, standard mean reversion tests tend to overlook target zone nonlinearities.

In my paper, the conditional density function of regulated Brownian motion between two reflecting boundaries for the fundamental is directly embedded into the Krugman (1991) exchange rate target zone model. This specification is subsequently estimated within a maximum likelihood framework. Likelihood ratio tests then indicate that target zone nonlinearities are present when estimation is pursued vis-à-vis implicit, narrower zones. In other words, target zone nonlinearities can be overlooked when estimation is pursued with respect to official limits when in fact an implicit zone is present.

Both approaches have in common that they point to the potential of target zone nonlinearities being overlooked in existing research. Iannizzotto and Taylor (1999) and Taylor and Iannizzotto (2001) build their argument upon the power properties of mean reversion tests, whereas my paper addresses the question as to which the nonlinearities may be obscured by narrower zones within official zones. The two approaches thus focus on different reasonings that can account for overlooking of nonlinearities. However, they take different routes in establishing this result. Personally, I see the two approaches as complementary. This is the focus of the changes to the original version of the paper.

3   Textual changes

I have included a reference to the papers of Iannizzotto and Taylor (1999) and Taylor and Iannizzotto (2001) in the introduction to the revised paper. The last sentence of the first paragraph in the introduction will be extended by a footnote according to which the power argument can be thought of as a valid, additional reason for actually detecting few indications for the presence of nonlinearities. The reference list has been extended to include Iannizzotto and Taylor (1999) and Taylor and Iannizzotto (2001) and the acknowledgements have been changed in order to thank the editor.

1.1. Alteration #1: Original phrasing

The literature on nonlinearities in the relationship between fundamentals and exchange rates found only sparse evidence of departures from linearity (see, for instance, Flood, Rose and Mathieson, 1991; de Jong, 1994; Lindberg and Söderlind, 1994; Garratt, Psaradakis and Sola, 2001).

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Using official limits in estimation when actually a narrower zone is in place will therefore tend to underestimate nonlinearities.

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Using official limits in estimation when actually a narrower zone is in place will therefore tend to underestimate nonlinearities.[1]

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$$+ \sum_{n=-\infty}^{+\infty}\left\{\frac{1}{\sigma\sqrt{2\pi(t-t_0)}}\exp\left(-\frac{2\mu\left(n\overline{f} - (n+1)\underline{f} + f_0\right)}{\sigma^2}\right)\right.$$

$$\left.\exp\left(-\frac{\left(2n\overline{f} - 2(n+1)\underline{f} + f_0 + f - \mu\left(t - t_0\right)\right)^2}{2\sigma^2\left(t - t_0\right)}\right)\right\}$$

$$- \frac{2\mu}{\sigma^2}\sum_{n=0}^{+\infty}\left\{\exp\left(\frac{2\mu\left(n\overline{f} - (n+1)\underline{f} + f\right)}{\sigma^2}\right)\left[1 - \Phi\left(\frac{\mu\left(t - t_0\right) + 2n\overline{f} - 2(n+1)\underline{f} + f_0 + f}{\sigma\sqrt{t - t_0}}\right)\right]\right\}$$

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The loglikelihood function for the observed exchange rate series $\mathbf{s}$, in terms of the parameter vector $\mathbf{\Theta} = \{\alpha, \mu, \sigma, \underline{f}, \overline{f}\}$, then emerges as:

$$LogLikF(\mathbf{s}, \mathbf{\Theta}) = \sum_{t=0}^{T-1} \ln\left(p_s(s_{t+1}, t+1; s_t, t) \mid \mathbf{\Theta}\right). \tag{7}$$

The parameters in the maximisation process consist of the four parameters that fully specify the stochastic process of the fundamental, namely its instantaneous drift and standard deviation, μ and σ, and the upper and lower band limits, $\overline{f}$ and $\underline{f}$. The fifth parameter is the sensitivity of the exchange rate to its own expected change, α, which governs the extent of nonlinearities, see Equations (1) and (3). Earlier research, such as de Jong (1994) and Lindberg and Söderlind (1994), indicated that estimation of target zone models is rendered more complex by the flatness of the likelihood surface such that obtaining standard errors for all five coefficients is not sensible. We therefore calculate asymptotic standard errors for four parameters whilst keeping α at its estimated value. However, model specification and especially the significance of α will be formally tested via Likelihood Ratio (LR) tests.

The exchange rates of the Belgian and French currencies are Wednesday prices and are quoted against the German mark. The sample spans the period March 1994 until February 1997. The starting point was chosen in function of the theoretical prerequisite of credibility since the turmoil in the Exchange Rate Mechanism (ERM) of 1993 for these two currencies only faded away towards the end of that year. The endpoint reflects doubts on the continued appropriateness of the target zone model in view of the prospect of the European Monetary Union (EMU). In fact, De Grauwe,

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Dewachter and Veestraeten (1999) argued that the start of EMU on 1 January 1999 had to be seen as a regime switch that already more than one year earlier drastically altered exchange rate dynamics.

Table 1 presents estimation results for the Krugman (1991) model. The first two rows present results when estimating with respect to the official target zone limits. The subsequent rows relate to the case in which implicit bands are allowed for. The estimation results and especially the loglikelihood values strongly suggest that implicit zones were present such that estimation with respect to official band limits could have obscured nonlinearities. This will be formally tested via the LR specification tests in Table 2.

Tables 1 and 2: around here.

The first LR-test in Table 2 examines whether α significantly differs from zero, i.e. whether nonlinearities are present, when estimation is pursued with respect to the official zone. Constraining α to be zero causes s to equal f, see Equation (1), such that the exchange rate in the constrained model follows reflected Brownian motion. The restriction on α is non-standard since α is constrained to be nonnegative and, moreover, the restriction lies on the boundary of its domain. Chernoff (1954) showed that the resulting LR-statistic is distributed $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$. We follow de Jong (1994) and approximate this distribution as χ_1^2. The second LR-test pursues the same strategy but now estimation allows for implicit bands. The findings reveal that estimation with respect to official bands obscures nonlinearities in the case of the Belgian franc since the constrained model cannot be rejected (LR-test 1) whereas it can safely be rejected when implicit limits are allowed for. For the French franc, nonlinearities are present in both cases but the p-value is far lower when analysing them vis-à-vis an implicit zone. The third LR test evaluates whether the implicit zones are significantly different from the official zones. Hereto, the two models in Table 1 are compared in which the official-zone model is the constrained model and the test statistic thus is distributed as χ_2^2. Unsurprisingly, implicit zones significantly differ

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from officially announced fluctuation ranges in line with the findings reported in Bartolini and Prati

(1999), Anthony and MacDonald (1999) and Chung and Tauchen (2001).

5   Conclusion

This paper examined exchange rate target zone nonlinearities by explicitly relating their presence to the

potential pursuit of implicit, narrower fluctuation ranges within the official band. Since nonlinearities

are typically to be detected near the band limits, estimation should devote careful attention to the

precise location of the fluctuation limits. Estimation with respect to official limits when in fact

monetary authorities are pursuing a narrower zone will inevitably bias results towards rejecting the

presence of nonlinearities. The maximum likelihood estimation results confirmed the existence of

an underestimation bias when official limits are used. In fact, the presence of nonlinearities for the

Belgian franc between 1994 and 1997 had to be discarded when estimating with respect to official

limits, whereas they were clearly present when gauged with respect to narrower, implicit zones. Or,

nonlinearities can be overlooked if estimation fails to account for implicit limits. The underestimation

bias was not present in the case of the French currency although rejection of the hypothesis of no

nonlinearities was much stronger when examined with respect to implicit limits.

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References

[1] Anthony, M. and R. MacDonald, 1999, The width of the band and exchange rate mean-reversion: some further ERM-based results, Journal of International Money and Finance 18, 411-428.

[2] Bartolini, L. and A. Prati, 1999, Soft exchange rate bands and speculative attacks: theory, and evidence from the ERM since August 1993, Journal of International Economics 49, 1-29.

[3] Chernoff, H., 1954, On the distribution of the likelihood ratio, Annals of Mathematical Statistics 25, 573-578.

[4] Chung, C.-S. and G. Tauchen, 2001, Testing target zone models using Efficient Method of Moments, Journal of Business & Economic Statistics 19, 255-269.

[5] De Grauwe, P., H. Dewachter and D. Veestraeten, 1999, Price dynamics under stochastic process switching: some extensions and an application to EMU, Journal of International Money and Finance 18, 195-224.

[6] de Jong, F., 1994, A univariate analysis of EMS exchange rates using a target zone model, Journal of Applied Econometrics 9, 31-45.

[7] Flood, R. P., A. K. Rose and D. J. Mathieson, 1991, An empirical exploration of exchange rate target zones, Carnegie-Rochester Series on Public Policy 35, 7-66.

[8] Garratt, A., Z. Psaradakis and M. Sola, 2001, An empirical reassessment of target-zone nonlinearities, Journal of International Money and Finance 20, 533-548.

[9] Iannizzotto, M. and M.P. Taylor, 1999, The target zone model, non-linearity and mean reversion: is the honeymoon really over?, Economic Journal 109, C96-C110.

[10] Krugman, P. R., 1991, Target zones and exchange rate dynamics, Quarterly Journal of Economics 106, 669-682.

[11] Lindberg, H. and P. Söderlind, 1994, Testing the basic target zone model on Swedish data 1982-1990, European Economic Review 38, 1441-1469.

[12] Taylor, M.P. and M. Iannizzotto, 2001, On the mean-reverting properties of target zone exchange rates: a cautionary note, Economics Letters 71, 117-129.

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[13] Veestraeten, D., 2004, The conditional probability density function for a reflected Brownian motion, Computational Economics 24, 185-207.

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Table 1: Maximum likelihood estimates of the Krugman (1991) model with either official band limits or implicit band limits, sample period: 2 March 1994 - 29 January 1997.

Currency	α	μ	σ	$\underline{f}$	$\overline{f}$	Loglikelihood
Official zone						
Belgian franc	21.00281 [na]	-0.00203 [0.00477]	0.00778 [0.00089]	-	-	827.841
French franc	14.33483 [na]	-0.04479 [0.04595]	0.05125 [0.01845]	-	-	627.822
Implicit zone						
Belgian franc	5.60832 [na]	-0.01351 [0.03580]	0.03725 [0.03990]	2.98805 [0.01099]	3.15616 [0.31373]	865.629
French franc	8.99944 [na]	-0.03573 [0.07973]	0.08792 [0.07462]	1.12590 [0.03699]	1.60526 [0.23268]	651.102

Note: Asymptotic standard errors are given within brackets.

Table 2: Likelihood Ratio tests, sample period: 2 March 1994 - 29 January 1997.

	LR-test 1	p-value[1]	LR-test 2	p-value[1]	LR-test 3	p-value[2]
Belgian franc	0.03564	0.8503	75.61152	3.5E-18	75.57589	3.9E-17
French franc	7.68802	0.0056	54.24664	1.8E-13	46.55863	7.8E-11

[1] Under χ_1^2.
[2] Under χ_2^2.

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