The internal market and hospitals efficiency: a stochastic distance function approach.
Ferrari, Alessandra

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Abstract
The UK internal market was one of the first European attempts to introduce a competitive mechanism in the provision of hospital services. The assumption was that competition would have led hospitals to increase efficiency in the use of their resources. The aim of this paper is to analyse the effectiveness of this kind of reforms by measuring the changes in technical efficiency of a panel of 52 acute Scottish hospitals observed from 1991/92 to 1996/97. The time period covers the whole duration of the internal market and the sample contains a different mix of both trusts and non-trusts, where the former embed the proper working of the reform. The selected model is a stochastic output distance function that includes an interaction dummy variable to allow for parameters to change over time. The results show a structural break after which hospitals change not only the way in which they provide their services, but also the kind of services they provide, favouring the quicker treatment of patients on a day basis. No significant improvement in technical efficiency is detected instead over time, nor any significant difference in efficiency between trusts and non trusts.
1. Introduction

The UK was one of the first European countries to reform its health services. As regards hospital services, a clear distinction was introduced between purchasers (District Health Authorities and GP fund-holders) and providers (hospital trusts). By becoming trusts hospitals were given more autonomy in the management of their resources and had to sell their services to the purchasers via contracts on the so-called internal market (Bartlett and Le Grand, 1994). Competition for contracts was thought to give hospitals incentives to efficient behaviour. Even though the “quasi-market” was formally eliminated in 1997, its main features remained in place. Other countries are now following in the same direction, and the debate about the efficiency and effectiveness of this kind of reform is a major policy issue (see for example Gerdtham and Lothgren, 2001).

Not surprisingly the reform generated scientific interest, a lot of which of a theoretical nature, on the efficiency properties of different contracts or the credibility of competitive conditions on the market. The empirical work has been instead more restricted, especially when it comes to the analysis of efficiency changes (Glennerster, 1998). Among the main contributions, Propper (1996) and Propper and Soderlund (1998) studied the effects of competition on hospital prices and costs, and found some evidence of price reduction eventually mirrored in the costs, but mainly a great variability in prices that bore no respect to the average cost pricing rule advocated by the Government. The effects of competition on the quality of hospital services were the focus of Propper et al. (2004) who found evidence of a decrease in quality associated to the working of competition. Studies of hospitals efficiency on more recent, post-reform data sets can be found for example in Jacobs (2001) and Street and Jacobs (2002), who
focused on the (lack of) significance of the differences in inefficiency between hospitals. Jacobs and Dawson (2003) showed the non-effectiveness of hospitals efficiency targets; these have become increasingly popular with policy makers, and their feasibility, as well as that of general efficiency analysis of public services, have been thoroughly analysed by Smith and Street (2003).

Not many contributions exist instead about the effects on hospitals efficiency during the years of the reform, when competition was actively promoted as the key feature of the system. Among these studies, Soderlund et al. (1997) estimated a classical linear regression model on a sample of NHS hospitals in England for the years 1992 to 1994 which revealed a general productivity improvement however not associated to trust status. Due to its easier availability, others have used the acute Scottish hospitals data set used in this paper 2. Scott and Parkin (1995) estimated a translog cost function for the years 1992/93 which highlighted the prevalence of constant returns to scale and economies of scope between different kinds of outputs. Maniadakis et al. (1999, 2000) used Data Envelopment Analysis (DEA) to calculate Malmquist indexes of total factor productivity (TFP) for the period 1991/92-1995/96, finding a general improvement in TFP mainly attributable to shifts of the frontier, and a worsening of the quality level 3.

The aim of this paper is to analyse the changes in technical efficiency and performance of hospitals during the years of the reform by means of a stochastic distance function. As opposed to DEA, the stochastic parametric approach allows the statistical testing of hypotheses, making the results more reliable 4. It also allows analysing the characteristics of the production process that a non-parametric method by definition does not identify. Furthermore the chosen model
is a stochastic distance function (Coelli and Perelman, 1996) for technical
efficiency, which is a frontier model as opposed to the classical linear regression
one; as will be seen in Section 2, the frontier model specifically separates the
noise in the data from the estimation of inefficiency, which is the aim of the
exercise. The choice of a distance function form is due to the multiple output
nature of the production process that rules out the direct estimation of a
production function\(^5\).

Unfortunately, data for the whole of the NHS are not available; the analysis is
performed on a sample of 52 acute hospitals in Scotland observed between
1991/92 and 1996/97, thus covering the whole duration of the reforms.

The rest of the paper is structured as follows. The estimation of stochastic
frontiers and the distance function model are discussed in Section 2. Section 3
describes the data set. The model selection process and the results are in Section
4, and general conclusions in Section 5. A final appendix details the additional
statistical analysis performed to check on the robustness of the model.

2. The stochastic frontier and the distance function.

Following Debreu and Farrell seminal papers, the efficiency of a firm can be
defined and measured as the distance of its actual performance from a frontier\(^6\).

If this frontier is the production function, i.e. the maximum attainable output
from a given set of inputs, the distance will measure technical inefficiency.

Following Shephard, this can be measured as a radial expansion \(D_o\) for the output
vector(s) in order to reach the frontier, where \(0 < D_o \leq 1\). If \(D_o =1\) the
observation is efficient as it lies on the frontier, if \(D_o <1\), the observation is
inefficient as it lies below it.
Econometrically, in the single equation - cross sectional case, a production frontier is usually estimated as:

\[ \ln y_i = \alpha + \beta^* \ln x_i + \varepsilon_i \]  

(1)

where

\[ \varepsilon_i = \nu_i - \mu_i \]

is a composite error term in which

\[ \nu_i \sim N(0, \sigma^2) \]

is the stochastic component and

\[ \mu_i = -\ln D_0 \]

is the efficiency measure.

The efficiency measure \( \mu_i \) must come from a positively skewed distribution; for instance if this is a half normal (which will be used later), then

\[ \mu_i = |U_i| \]

\[ U_i \sim N(0, \sigma^2_{u}) \]

Due to the presence of a composite error term OLS gives consistent but inefficient slope parameters’ estimates, and the use of ML is to be preferred if the distribution of \( \mu_i \) is known or an assumption can be made about it.

Following Battese and Corra (1997) the influence of the inefficiency component can be measured by a parameter \( \gamma = \sigma_u^2 / \sigma^2 \), where \( \sigma^2 = \sigma_u^2 + \sigma^2 \). The significance of \( \gamma \) can be tested with an LR test which, if the null hypothesis \( H_0: \gamma = 0 \) is true, follows a mixed \( \chi^2 \) distribution. If the null hypothesis is true and inefficiency is not significant, the model is equivalent to a standard "average" production function, and its log-likelihood is the same as that of the linear model estimated by OLS.

As (1) can be estimated only for the single output case, an alternative model has been proposed by Coelli and Perelman (1996, 2000) to deal with the multiple
outputs case. The idea is to directly express $D_o$ as a function of the $K$ inputs and $M$ outputs of each of the $N$ firms. Using a log-linear translog function specification this is:

$$\ln D_{oi} = \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y_{m_i} + \frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{M} \alpha_{mn} \ln y_{m_i} \ln y_{n_i} + \sum_{k=1}^{K} \beta_k \ln x_{k_i} + \frac{1}{2} \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{kl} \ln x_{k_i} \ln x_{l_i} + \sum_{k=1}^{K} \sum_{m=1}^{M} \delta_{km} \ln x_{k_i} \ln y_{m_i}$$

$$i = 1, \ldots, N$$

(2)

Using the fact that $D_o$ is linearly homogeneous in outputs, adding the stochastic component $v_i$ and setting $\ln D_{oi} = -u_i$, (2) can be rearranged as

$$\ln y_{m_i} = \alpha_0 + \sum_{m=1}^{M-1} \alpha_m \ln y^*_{m_i} + \frac{1}{2} \sum_{m=1}^{M-1} \sum_{n=1}^{M-1} \alpha_{mn} \ln y^*_{m_i} \ln y^*_{n_i} + \sum_{k=1}^{K} \beta_k \ln x_{k_i} + \frac{1}{2} \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{kl} \ln x_{k_i} \ln x_{l_i} + \sum_{k=1}^{K} \sum_{m=1}^{M-1} \delta_{km} \ln x_{k_i} \ln y^*_{m_i} + v_i - u_i$$

(3)

where $y_M$ is the M-th output and $y^*_{m_i} = y_{m_i}/y_M$. Equation (3) can be now estimated as a usual production frontier, by regressing (the log of) one output on (the logs of) the inputs and (the logs of) the outputs ratios. Note that the coefficients of a production frontier correspond to the negative of the coefficients of a distance function.

3. The data

The data are a sample of 52 acute hospitals in Scotland in the years 1991/92 to 1996/97 (from now on referred to as 1992 and 1997 respectively), making a
panel data set of 312 observations\textsuperscript{10}. The data set was actually reduced from the originally available one after a detailed analysis, motivated also by others’ concerns (Scott and Parkin, 1995), revealed that the information on some of the cross sectional units was not reliable. Furthermore, as suggested also in Smith and Street (2005), additional statistical analysis of the residuals has been performed to check for the reliability of the results and the robustness of the model.

Coming to the definition of the inputs and outputs variables, the following choices have been made\textsuperscript{11}. Output is usually measured as the total number of cases treated. Since cases are very heterogeneous they are usually divided into various specialty (or casemix) categories, which qualify the hospital as a multiple-output unit. The use of index numbers can overcome the problem of the trade-off between heterogeneity and degrees of freedom as long as one can define weights that correctly represent the differences between cases. On the assumption that more difficult illnesses are more input demanding than the less serious ones, measures of their reference average cost are produced by the NHS and customarily used by the literature.

This paper follows this approach and the weights reflect the average costliness of a case in each category, calculated for the whole of Scotland in order not to bias them with some measure of inefficiency of each hospital. Furthermore, to avoid average cost changes over time due to possible inefficiency changes, they have been normalised each year to sum to 1. In detail, if

$q_{jit}$ is the total number of cases treated in category $j$ by hospital $i$ at time $t$, and $c_{jit}$ is the average cost per case in category $j$ at hospital $i$ at time $t$; then
\[ c_{jt} = \frac{1}{N} \sum_{i=1}^{N} c_{ijt} \] is the average cost (across hospitals) per case in category \( j \) at time \( t \). The weight for each category is calculated as

\[ w_{jt} = \frac{c_{jt}}{\sum_{j=1}^{J} c_{jt}} \]

and so finally the index for each hospital in each year is

\[ y_{jt} = \sum_{j=1}^{J} w_{jt} q_{jt} \]

The many output categories have then been summarised in two indexes: one for the inpatients and one for the outpatients, day patients and day cases. The main difference between the two categories is that in the former patients spend several days in the hospital and in the latter no more than one day, sometimes without even using a bed (and the staff associated to it). As a substitution between the two kinds of services could have taken place, it was preferred to keep them separated, so the two final output indexes are \( y_1 \) (index of inpatients) and \( y_2 \) (index of outpatients, day patients and day cases).

Finally, 5 variables identify the inputs:

\( x_1 = \) total capital charges (£000)

\( x_2 = \) medical staff FTE (full time equivalent);

\( x_3 = \) nursing staff FTE;

\( x_4 = \) other staff FTE;

\( x_5 = \) total number of beds.

The capital measure\(^{12}\) was deflated using the “Hospital and Community Health Services pay and price inflation values”. The “other staff” input includes
professional, technical, administrative, clerical and all other staff. The descriptive
statistics of the data are reported in Table 1.

[Table 1 here]

4. The model

The main problem encountered in the estimation of the translog output distance
function was the modelling of time effects. A series of LR tests was performed to
choose among alternative time specifications; this initially led to the estimation
of model with intercept dummy variables for each year in the panel, like the
following:\footnote{13}

\[
\ln y_{2it} = \alpha_0 + \alpha_1 \ln y^* + \alpha_{11} (\ln y^*)^2 + \sum_{k=1}^{5} \beta_k \ln x_{kit} +
\]

\[
+ \sum_{k=1}^{5} \sum_{l=1}^{4} \beta_{kl} \ln x_{kit} \ln x_{lit} + \sum_{k=1}^{5} \delta_k \ln x_{kit} \ln y^* + \sum_{l=1}^{5} \zeta_l D_{it} + \xi D_{each} + \nu_{it} - u_{it}
\]

(4)

In the above, (the log of) the index of outpatients, day patients and day cases \((y_2)\)
is regressed over (the log of) the five inputs \((x)\) and the outputs ratio \((y^* =
y_1/y_2)\), plus five dummy variables to allow for a different intercept each year,
and a dummy variable for teaching hospitals. A dummy variable for trust status
could not be introduced because of the implicitly assumed correlation with the
inefficiency component (the issue is discussed more in detail later). Following
Battese and Coelli (1992), the inefficiency component was modelled as a
function of time as

\[
u_{it} = u_i \exp[-\eta(t-T)]
\]

(5)

so that a value of \(\eta > 0 \quad (<0)\) implies increasing (decreasing) efficiency over time.
A value of \(\eta = 0\) implies no time effect, and the hypothesis can be tested by
means of an LR test. Equation (4) however produced unsatisfactory results; in particular, even though the model with the intercept dummies had been statistically selected against other time specifications, not all the dummies were significant, their overall pattern was counterintuitive and contradicting existing evidence (see for example Maniadakis et al., 1999, 2000, and Ferrari, 2006) and finally one of the inputs elasticities was negative. All this raised the suspicion that the effect of time had not been adequately captured, namely that a pooled model might not be appropriate and that all the parameters of the distance function, not just the intercepts, might have changed over time.

As the use of Chow tests for parameters stability was ruled out for lack of degrees of freedom an alternative approach was used instead. This consists of estimating several times the distance function with a time interaction dummy instead of the intercept dummies. In particular, a time dummy \( d \) is introduced, which takes a value of 1 for a particular year(s), and 0 else, and this is multiplied to all the variables in the translog distance function, as:

\[
\ln y_{it} = \alpha_0 + \alpha_1 \ln y^* + \alpha_{11} (\ln y^*)^2 + \sum_{k=1}^{5} \beta_k \ln x_{kit} + \\
+ \sum_{k=1}^{5} \sum_{l=1}^{4} \beta_{kl} \ln x_{kit} \ln x_{lit} + \sum_{k=1}^{5} \delta_k \ln x_{kit} \ln y^* + \lambda_0 d + \lambda_1 \ln y^* d + \lambda_{11} (\ln y^*)^2 d + \\
+ \sum_{k=1}^{5} \rho_k \ln x_{ktd} + \sum_{k=1}^{5} \sum_{l=1}^{4} \rho_{kl} \ln x_{kit} \ln x_{kit} d + \sum_{k=1}^{5} \zeta_k \ln x_{kit} \ln y^* d + \xi D_{teach} + v_{it} - u_{it}
\]

(6)

The inputs and outputs of (6) are the same as those of (4), \( D_{teach} \) is a dummy variable for teaching hospitals and \( d \) is the time interaction dummy. When \( d=0 \) the parameters of the function are the \( \alpha s, \beta s \) and \( \delta s \); when \( d=1 \) they are the respective \( (\alpha s + \lambda s), (\beta s + \rho s) \) and \( (\delta s + \zeta s) \). The inefficiency component \( u_{it} \) varies
over time as in (5) and is modelled as a half normal distribution, since the LR test on the 0-mean of a truncated normal distribution leads not to reject the null hypothesis in all cases.

The interaction dummy is first set equal to 1 for 1992 (and 0 else), then for 1992 and 1993 (and 0 else) and so on. In this way 5 different distance functions are estimated, each with a different time effect which is captured by the parameters of the interaction dummy. The likelihood results of the five estimations of (6) are reported in Table 2.

[Table 2 here]

In each case the significance of the time interaction parameters is tested for by means of LR tests against a restricted model with no time effects and as expected the null hypothesis is always rejected. On the grounds of the Akaike information criterion the model as specified in (6) is also to be preferred to the pooled one in (4).

Since the models are not nested in one another but they all have the same number of parameters, selection by minimisation of any standard information criterion is equivalent to selection of the model with the greatest maximised log likelihood. This happens when separating 1992 and 1993 from all other years, as the model has about 27 points of difference in the log-likelihood from its closest alternative. This therefore points to the fact that the parameters of the distance (and production) function might have changed after 1993.

The model in which 1992 and 1993 are separated from the following years is therefore analysed. The main results are shown in Table 3, which reports in order: the ML and OLS log-likelihood values, the estimated values of $\gamma$ and $\eta$, the inputs elasticities and then the elasticity of $y_2$ with respect to the outputs ratio
\( y^* (e_{y^*}) \), the total input elasticity (or elasticity of scale) and finally the elasticity of substitution between \( y_2 \) and \( y_1 \). All elasticities are calculated at the sample mean and their significance is tested by means of an LR test that in all cases leads to reject the null hypothesis (i.e. all are significant).

[Table 3 here]

The results of the estimation can be summarised as follows. The \( \gamma \) parameter is significant (LR test is 337.62) meaning that so is inefficiency. Very interestingly however \( \eta \) is not: the result of the LR test (2.46) leads not to reject the null hypothesis that \( \eta = 0 \), so that no significant difference in (in)efficiency appears to have taken place over time.

The most notable difference between the two time periods is the change in the elasticity of substitution between \( y_2 \) and \( y_1 \): the absolute value increases by 60%, implying that the opportunity cost of treating someone as an inpatient over time has increased considerably. A pattern has emerged towards treating patients more and more as day patients/cases or directly as outpatients. This is confirmed when looking at Table 1, that shows a very big rate of increase in the value of \( y_2 \) as opposed to a relatively small increase in that of \( y_1 \).

As regards the inputs, two variables improve their elasticity after 1994 (namely the nursing staff and the beds), and the other three lower it. Whether this is due to a change in the productivity of an input or just to a change in its levels can be revealed by testing for the significance of the relevant dummy variable parameters. This reveals that the increased elasticity of the beds input is a consequence of the reduction in the levels of the variable, well known also to the
general public via the news. This is also consistent with the aforementioned trend towards day-based treatment, as beds would in that case be used more intensively.

The reduced elasticities of capital and of other staff are taken to be a direct consequence of the reforms. The increase in capital levels could be due to the investment in information technology that hospitals made in order to deal with the new contracting issues (Fattore, 1999). As this activity is not directly linked to the treatment of patients (the output variable) this might explain the reduced elasticity of the input. However, increased capital levels are also the consequence of accountancy changes related to the change to trust status, which made the hospitals owners of their assets, so concluding that there was overcapitalisation would be misleading. The data did not offer any other measure of capital but the one used, which has its limitations although its entire removal from the equation was rejected from the data; keeping this in mind, since no further detail is available, the result has to be taken with caution.

Similarly, the “other staff” variable increases in level and its elasticity decreases from 0.33 to 0.12. One reasonable explanation is that administrative staff increased to deal with the new contracting issues. Another possibility is that the lower increase in nursing staff might have led to the transfer of some of their duties to cheaper but less qualified (and therefore less efficient) staff. A pattern towards the use of cheaper labour inputs in Scottish hospitals was revealed by Gray et al. (1986), and this might have been reinforced by the financial concerns of the reforms.
Finally, the results also show a positive difference in the intercepts indicating a possible improvement in average productivity, that is a shift upwards of the frontier. Considering that the shape of the frontier has changed, the higher intercept could indicate that the improvement is mainly in the production of the dependent variable, i.e. again $y_2$.

The fact that 1994 is the first year in which some hospitals start to change to trust status naturally leads as to think that is the reason behind the structural break. The relevance of trust status in explaining changes in technology and inefficiency is therefore analysed more in detail. One approach to do so is the estimation of a “one-step model” (Wang and Schmidt, 2002), which however failed to converge to a maximum. More sophisticated models are currently discussed by the literature and will be worth exploring for future research. For this paper the interaction dummy specification remains the preferred one. Using the estimates of (6), the elasticities of trusts and non trusts are calculated and compared to one another, as reported in Table 4. This shows that the pattern revealed by Table 3 after 1994 seems to be more marked for the trusts sample than for the other hospitals, which confirms the hypothesis that the change in technology is related to the change in status. However, no significant link between trust status and efficiency can be detected: a t-test on the equality of the mean inefficiency score, computed as $E[u_i | \hat{\xi}_i]$ of trusts and non trusts is performed and the null hypothesis cannot be rejected (the p-value is 0.34).

[Table 4 here]

As mentioned in Section 3, to check on the reliability of the results and the robustness of the model specification, two additional pieces of analysis have
been performed and are reported in detail in the appendix. First we compare the theoretical density function of $\varepsilon_{it}$ with that of the estimated residuals; this shows that the model fits the data well and rules out the presence of outliers. Secondly for each hospital we computed the 2.5%, 50% and 97.5% quantiles corresponding to the conditional density function $f(u|\hat{\varepsilon}_{it},...\hat{\varepsilon}_{it})$. We find that there is very little overlap and thus that inefficiency is not only significant overall but also significantly different between hospitals.


This paper estimated a stochastic distance function to analyse the changes in (technical) efficiency and performance of acute hospitals in Scotland during the years of the reform. A structural break was detected in 1994, the year of the first trust wave, after which hospitals change not only the way in which they provide their services, but also the mix of services they provide. The opportunity cost of inpatients increases as hospitals tend to treat patients more and more on a day basis. This view is supported by the fact that both the number and the costliness of outpatients, day patients and day cases increase quite significantly, indicating a possible “swap” between the two categories of output considered. This could be the result of an attempt of hospitals at reducing their costs by reducing the length of stay, especially if the contracts constrained them to provide minimum levels of treatment (as it was the case especially with the widely used ‘block contracts’). The involvement in the new contracting activity, and the financial concerns that it brings with itself, appears also to translate into reduced inputs productivity. This at least seems to be true for the capital and other staff variables.
(whose levels increase over time), and for the medical staff. Nurses and beds instead improve their productivity, and are also associated with the lowest increases in levels (with the latter strongly negative in fact).

This increase in the day-basis treatment is also behind the shift upwards of the frontier. However, although technical inefficiency remains significant, it does not show any significant improvement.

There are clearly limitations to this work, among which the non-availability of a specific quality measure and the relatively small sample size. With these in mind, the general conclusion that can be drawn from this work is that the reform did not unambiguously produce the expected efficiency improvements. The move towards day-based care might be going hand in hand with the reduction in the quality of treatment found elsewhere in the literature, and this makes the overall scenario not particularly optimistic.

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References


Appendix.

Analysis of the residuals of equation (4).

In this appendix we provide a graphical comparison of the theoretical distribution of the composed error term, with an estimate of the density of the actual residuals. The theoretical distribution, resulting from a convolution, may be found in Greene (1997). For the density estimator we use a kernel method (see for example Silverman, 1986), with the standard plug-in bandwidth. For the relevance of this kind of analysis see for example Smith and Street (2005) and Street and Jacobs (2002) for a similar application to hospitals efficiency.

Figure A1 shows in order: the theoretical density of the composed error term \( \varepsilon_i = v_i - u_i \) (with \( v_i \sim N(0, \sigma_v^2) \), \( u_i = |U_i| \) and \( U_i \sim N(0, \sigma_u^2) \)), the estimated density of the residuals from the estimation of equation (4), and the distribution of the 312 residuals.

The residuals seem to be in broad agreement with their theoretical distribution and there are no serious outliers, indicating that the results are robust.

[Figure A1 here]

Figure A2 for shows, for the 52 hospitals, the quantiles corresponding to \( f(u_i | \hat{\varepsilon}_1, ..., \hat{\varepsilon}_r) \), and the quantiles corresponding to \( f(D_{\alpha} | \hat{\varepsilon}_1, ..., \hat{\varepsilon}_r) \), where \( D_{\alpha} = 100e^{-\alpha} \). The very little overlap indicates that inefficiency is significantly different between hospitals.

[Figure A2 here]
Tables and Figures

Table 1: Descriptive statistics of the inputs and outputs variables (standard deviation into brackets)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Average 92-97</th>
<th>Yearly rate of growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inpatients index</td>
<td>141746</td>
<td>0.3</td>
</tr>
<tr>
<td>Outpatients et al. index</td>
<td>30342</td>
<td>9.0</td>
</tr>
<tr>
<td>Capital (£000)</td>
<td>1513</td>
<td>1.6</td>
</tr>
<tr>
<td>Medical staff (WTE)</td>
<td>88</td>
<td>4.1</td>
</tr>
<tr>
<td>Nursing staff (WTE)</td>
<td>457</td>
<td>0.7</td>
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<tr>
<td>Other staff (WTE)</td>
<td>302</td>
<td>4.0</td>
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<tr>
<td>Beds</td>
<td>357</td>
<td>-2.7</td>
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</tbody>
</table>

Table 2: Log-likelihood of the translog distance function with time interaction dummy.

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<th></th>
<th>92</th>
<th>92 - 93</th>
<th>92 - 94</th>
<th>92 - 95</th>
<th>92 - 96</th>
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<tr>
<td>$\mathcal{L}$</td>
<td>177.99</td>
<td>218.54</td>
<td>190.93</td>
<td>191.68</td>
<td>170.19</td>
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Table 3: results from the estimation of equation (4).

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<td>OLS $\mathcal{L}$</td>
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<td>$\gamma$</td>
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<td>$\eta$</td>
<td>0.02</td>
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<table>
<thead>
<tr>
<th>Year</th>
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<th>$\text{emed}$</th>
<th>$e_{\text{nurs}}$</th>
<th>$e_{\text{oth}}$</th>
<th>$e_{\text{bed}}$</th>
<th>$e_{\gamma}$</th>
<th>$e_{\text{tot}}$</th>
<th>$e_{y1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992-1993</td>
<td>0.028*</td>
<td>1.049*</td>
<td>0.156*</td>
<td>0.334*</td>
<td>0.043*</td>
<td>-0.493*</td>
<td>1.611</td>
<td>-0.973</td>
</tr>
<tr>
<td>1994-1997</td>
<td>0.005*</td>
<td>0.672*</td>
<td>0.254*</td>
<td>0.121*</td>
<td>0.289*</td>
<td>-0.613*</td>
<td>1.341</td>
<td>-1.582</td>
</tr>
</tbody>
</table>

* = significant at 5% (or less) ** = significant 1 at 10%.

Table 4: partial elasticities of trusts and non trusts hospitals.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Trusts</th>
<th>Non trusts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\text{cap}}$</td>
<td>-0.001</td>
<td>0.022</td>
</tr>
<tr>
<td>$e_{\text{med}}$</td>
<td>0.667</td>
<td>0.687</td>
</tr>
<tr>
<td>$e_{\text{nurs}}$</td>
<td>0.252</td>
<td>0.259</td>
</tr>
<tr>
<td>$e_{\text{oth}}$</td>
<td>0.121</td>
<td>0.128</td>
</tr>
<tr>
<td>$e_{\text{bed}}$</td>
<td>0.308</td>
<td>0.207</td>
</tr>
<tr>
<td>$e_{\gamma}$</td>
<td>-1.594</td>
<td>-1.506</td>
</tr>
</tbody>
</table>
Figure A1: comparison of the theoretical density function of $\varepsilon_i$ with that of the estimated residuals.

Figure A2: quantiles corresponding to $f(u_i | \hat{\varepsilon}_i, \ldots, \hat{\varepsilon}_{iT})$, and to $f(D_{qi} | \hat{\varepsilon}_i, \ldots, \hat{\varepsilon}_{iT})$. 
2 ISD Scotland, Scottish Health Service Costs, NHS in Scotland.
3 Measured as the survival rate 30 days after discharge.
4 See Jacobs (2001) for a discussion of the relative advantages of the two methodologies for the hospital sector, and Parkin and Hollingsworth for an application to the Scottish data set.
5 This problem arises when estimating a production function as opposed to a cost function. See for example Conrad and Strauss (1983) and Wagstaff (1989).
6 For an overview of the many techniques of frontier estimation, including more recent developments, see for example Sena (2003).
7 Many comprehensive reviews of the stochastic frontier model exist in the literature; see for example Greene (1997) and Kumbhakar and Lovell (2000).
8 A more parsimonious specification like the Cobb-Douglas should not be used because, apart from other restrictions, it is not concave in the output dimensions.
9 The detailed explanation is omitted for reasons of space and can be found in Coelli and Perelman (1996, 2000).
10 The data are from the Scottish Health Service Costs statistics.
12 This was the only available capital measure in the data, and it comprises: a) depreciation on fixed assets; b) interest paid on money borrowed to finance any of the projects in a); c) a 6% return on capital (trusts only). Even though not ideal its entire removal from the estimations was rejected by the data.
13 Since this first model was eventually discarded, for reasons of space the details are omitted, but they are available in a previous longer version of this work (Ferrari 2004).
14 The Akaike information criterion is used to compare models that are non nested in one another. It is specified as $AIC = -2\mathcal{L} + 2n$ where $\mathcal{L}$ is the value of the maximised log-likelihood and $n$ is the number of parameters. The preferred model is the one with the lowest AIC value.
15 The details of the calculation of the elasticity of substitution are in Ferrari 2004.
16 The presence of the squared and interaction terms makes the translog prone to multicollinearity. As a consequence it is usually advisable to test for joint parameters’ significance rather than relying on their individual ones.
17 In particular these are calculated at the average sample values in 1994 and 1995, which are the years where a reasonable mix of trusts and non trusts exists. 1992 and 1997 in fact have 0 in one category, and 1993 and 1996 have 7 or less in one category.
18 See Greene (1997) for a discussion.