Asset Allocation Using Flexible Dynamic Correlation Models with Regime Switching

Otranto, Edoardo

QUANTITATIVE FINANCE

Asset Allocation Using Flexible Dynamic Correlation Models with Regime Switching

Journal:	*Quantitative Finance*
Manuscript ID:	RQUF-2007-0232.R1
Manuscript Category:	Research Paper
Date Submitted by the Author:	10-Jul-2008
Complete List of Authors:	Otranto, Edoardo; DEIR
Keywords:	Markov Processes, GARCH models, Time Series Analysis, Volatility Modelling
JEL Code:	C10 - General < C1 - Econometric and Statistical Methods: General < C - Mathematical and Quantitative Methods, C32 - Time-Series Models < C3 - Econometric Methods: Multiple/Simultaneous Equation Models < C - Mathematical and Quantitative Methods, C51 - Model Construction and Estimation < C5 - Econometric Modeling < C - Mathematical and Quantitative Methods
Note: The following files were submitted by the author for peer review, but cannot be converted to PDF. You must view these files (e.g. movies) online.	
Otranto Files.zip	

Asset Allocation Using Flexible Dynamic Correlation Models with Regime Switching*

Edoardo Otranto
Dipartimento di Economia, Impresa e Regolamentazione and CRENoS,
University of Sassari,
Via Torre Tonda 34, 07100 Sassari, Italy;
E-mail: eotranto@uniss.it, Tel. +39 0792017317, Fax +39 0792017312

Abstract

The asset allocation decision is often considered as a trade-off between maximizing the expected return of a portfolio and minimizing the portfolio risk. The riskiness is evaluated in terms of variance of the portfolio return, so that it is fundamental to consider correctly the variance of its components and their correlations. The evidence of the heteroskedastic behavior of the returns and the time-varying relationships among the portfolio components have recently shifted attention to the multivariate GARCH models with time varying correlation. In this work we insert a particular Markov Switching dynamics in some Dynamic Correlation models to consider the abrupt changes in correlations affecting the assets in different ways. This class of models is very general and provides several specifications, constraining some coefficients. The models are applied to solve a sectorial asset allocation problem and are compared with alternative models.

Keywords: Markov Chain, Multivariate GARCH, portfolio performance, switching parameters, volatility.

*I am very grateful to two anonymous referees for their important suggestions that have provided considerable and useful changes with respect to the first version of the paper. I am grateful to Luca Deidda, Giampiero M. Gallo and Alessandro Trudda for the useful comments. Financial support from Italian MIUR under grant 2006137221_001 is gratefully acknowledged.

1 Introduction

The problem of choosing the weights of single assets of a financial portfolio depends on the hypotheses underlying the economic model adopted. The simplest approach is the mean-variance analysis (Markowitz, 1959), in which the asset allocation is derived solving a constrained maximization problem; in this case the optimal allocation depends on the expected returns and the covariance matrix of the same returns. This framework requires strong hypotheses, but the solution is very simple and for this reason it is the most utilized approach. Other approaches are based on the Capital Asset Pricing Model (Sharpe, 1964), which introduces the hypothesis of equilibrium between supply and demand; the Arbitrage Pricing Theory (Ross, 1976), in which the risk of the assets is generated from multiple sources; the Bayesian approach of Black and Litterman (1991 and 1992), in which the Markovitz approach is integrated by priors representing the views of the operators.

An optimal allocation requires the forecast of the return and the evaluation of the risk of the portfolio; the risk is generally represented by the portfolio variance, hence the correct specification of the covariance matrix is of paramount importance.

Because of the large number of assets that could be potentially considered, the analysis were formerly conducted using simple models (Arnott and Fabozzi, 1988) and the variances and covariances were considered constant along the entire period under study (see, for example, French and Poterba, 1991). Recent studies emphasize the empirical evidence in favor of time varying variances and time varying correlation between assets. In particular, a number of studies point out that correlations between assets are higher during turmoil periods than during quiet periods (see, for example, Clare et al., 1998, and Longin and Solnik, 2001). For this reason and in absence of transaction costs, a diversified portfolio is potentially re-balanced each period, changing the risk of the returns (the covariance matrix is time-varying) and making new information available (the expected returns are compared with the realised returns to adjust the asset allocation).

The introduction of multivariate GARCH models has provided the possibility to solve such problems. In particular the diffusion of the Dynamic Conditional Correlation (DCC) model of Engle (2002) has provided the possibility to use a simple class of models with few unknown parameters, which consider both variances and correlations as time varying.

One of the limits of this model is to hypothesize the same dynamics for the correlations of all the assets. To avoid this problem, Billio et al. (2006) propose the Flexible DCC (FDCC) model, in which k groups of assets follow k different DCC models, providing flexible dynamics. This model was experimented on a sectorial asset allocation problem, using the Italian sectors, and it outperformed the DCC model and the Conditional Constant Correlation (CCC) model of Bollerslev (1990).

As noted by Pelletier (2006), the correlation matrix of financial time series is often subject to regime switching. In general, the shocks affecting the markets provide an abrupt yet persistent increase in the correlations. For example, the dramatic event of 11 September 2001 has produced an increase in the correlation of the markets (Drakos, 2004), but the models with constant coefficients can not capture this fact. For this purpose Pelletier (2006) introduces the Regime Switching Dynamic Correlation (RSDC) model, in which the unconditional level of correlation changes over time, following a Markov

Switching dynamics.

In this work we suggest to insert a particular Markovian dynamics in a general FDCC model, in which the unconditional correlation matrix and the dynamics of the conditional correlation can switch from one regime to another. The correlations between assets follow a Markov Switching model (MS hereafter), allowing the possibility that groups of homogeneous assets stay in different states at time t. We call this model the Regime Switching Flexible Dynamic Correlation (RSFDC) model. The introduction of some constraints provides different specifications; the case in which the unconditional correlation is not switching can be considered as an extension of the FDCC model; the case in which the dynamic coefficients of the conditional correlation are zero is similar to the RSDC model, the only difference being constituted by the different Markov chain driving the switching.

The new models are applied to a portfolio of Italian sectors during a time period which includes September 2001, and they are compared with CCC, DCC, FDCC and RSDC models. The procedures are evaluated in terms of portfolio performance through classical performance measures, and in terms of covariance performance, following the approach of Engle and Colacito (2006). The results show some evidence in favor of the presence of MS dynamics and good results for the RSFDC models.

In the next section we briefly review the dynamic correlation models, whereas in section 3 we develop the new models. In section 4 we illustrate the empirical application. Some remarks conclude the paper.

2 Dynamic Correlation Models

Let us assume an n-variate process $\boldsymbol{y}_t$:

$$\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{H}_t^{1/2}\boldsymbol{u}_t \qquad t = 1, ...T \tag{2.1}$$

where $\boldsymbol{\mu}$ is a constant vector, $\boldsymbol{H}_t$ is a positive definite matrix and $\boldsymbol{u}_t$ is a Normal i.i.d. process with mean $\mathbf{0}$ and covariance matrix given by the $n \times n$ identity matrix $\boldsymbol{I}_n$. $\boldsymbol{H}_t$ can be decomposed into:

$$\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t \tag{2.2}$$

where $\boldsymbol{D}_t$ is a diagonal matrix containing the standard deviations of $\boldsymbol{y}_t$ and $\boldsymbol{R}_t$ is the correlation matrix of $\boldsymbol{y}_t$. Each squared element of $\boldsymbol{D}_t$ follows a univariate GARCH model (Bollerslev, 1986). In our framework the vector $\boldsymbol{y}_t$ contains the data on the n assets included in a portfolio.

In a CCC specification (Bollerslev, 1990) $\boldsymbol{R}_t = \boldsymbol{R}$; in other words, the correlation between assets is considered constant along time. As said in the introduction, this hypothesis is not realistic, as is confirmed by several empirical studies.

A more realistic representation was proposed by Engle (2002), introducing the DCC model. In this case the $\boldsymbol{R}_t$ matrix is obtained as:

$$\begin{aligned} \boldsymbol{R}_t &= \tilde{\boldsymbol{Q}}_t^{-1}\boldsymbol{Q}_t\tilde{\boldsymbol{Q}}_t^{-1} \\ \boldsymbol{Q}_t &= (1-a-b)\boldsymbol{R} + a\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}' + b\boldsymbol{Q}_{t-1} \\ \tilde{\boldsymbol{Q}}_t &= \text{diag}(\boldsymbol{Q}_t) \end{aligned} \tag{2.3}$$

where a and b are unknown scalar coefficients ($a + b < 1$), $\boldsymbol{R}$ is the unconditional correlation matrix. Note that we need to rescale the matrix $\boldsymbol{Q}_t$ as in the first equation of (2.3) because it does not directly provide a correlation matrix; in fact the elements on the diagonal of $\boldsymbol{Q}_t$ are not constrained to be equal to one. Let us note also that the CCC model is a particular case of (2.3), where a and b are equal to zero.

The DCC representation is very useful and has had a lot of success because it provides a time varying conditional correlation involving few parameters to be estimated. It could be easily extended to consider q lags of $\boldsymbol{u}_t$ and p lags of $\boldsymbol{Q}_t$ in the second equation of (2.3). On the other hand it imposes a strong restriction: the correlations of all the assets included in $\boldsymbol{y}_t$ follow the same dynamics. Engle (2002) generalizes the model (2.3) providing a different formulation for each element of $\boldsymbol{Q}_t$:

$$\boldsymbol{Q}_t = (\boldsymbol{\iota}_n\boldsymbol{\iota}_n' - \boldsymbol{A} - \boldsymbol{B}) \odot \boldsymbol{R} + \boldsymbol{A} \odot \boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}' + \boldsymbol{B} \odot \boldsymbol{Q}_{t-1} \tag{2.4}$$

where $\odot$ indicates the Hadamard product, $\boldsymbol{\iota}_n$ is a $n \times 1$ vector of ones and $\boldsymbol{A}$ and $\boldsymbol{B}$ are matrices of unknown coefficients. This is a particular specification of the class of MARCH models of Ding and Engle (2001), which requires the estimation of a lot of parameters, in the unconstrained case. Ding and Engle (2001) show that $\boldsymbol{Q}_t$ is positive definite if $(\boldsymbol{\iota}_n\boldsymbol{\iota}_n' - \boldsymbol{A} - \boldsymbol{B})$, $\boldsymbol{A}$ and $\boldsymbol{B}$ are positive definite.

A solution to the trade-off between the estimation of few parameters and a more flexible representation of the correlation dynamics was suggested by Billio et al. (2006) with the FDCC model, introducing a block-diagonal structure in the second equation of (2.3). They define k groups of assets; each group follows a proper DCC model substituting the second equation of (2.3) as:

$$\boldsymbol{Q}_t = \boldsymbol{c}\boldsymbol{c}' + \boldsymbol{a}\boldsymbol{a}' \odot \boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}' + \boldsymbol{b}\boldsymbol{b}' \odot \boldsymbol{Q}_{t-1} \tag{2.5}$$

where

$$\boldsymbol{c} = [c_1\boldsymbol{\iota}_{n_1}, ..., c_k\boldsymbol{\iota}_{n_k}]'$$

and n_i ($i = 1, ...k$) is the number of assets in the group i (similarly for $\boldsymbol{a}$ and $\boldsymbol{b}$). To avoid explosive patterns, the set of constraints $a_ia_j + b_ib_j < 1$ ($i, j = 1, ...k$) is added. Note that the unconditional correlation matrix $\boldsymbol{R}$ is not included in the FDCC model, whereas the additional vector of parameters $\boldsymbol{c}$ is introduced.

A different approach in modelling the correlations was proposed by Pelletier (2006), introducing the RSDC model. In this case, the unconditional correlation matrix follows a regime switching model and it is constant within a regime but different across regimes. In practice the CCC model of Bollerslev (1990) is a special case of the RSDC model with only one regime. Formally, the covariance matrix in the RSDC model is given by (2.2), with:

$$\boldsymbol{R}_t = \boldsymbol{R}_{s_t} \tag{2.6}$$

where the suffix s_t is a discrete unobservable variable representing the regime at time t, which follows a Markov chain assuming h possible states. The probability to switch from state i to state j is indicated by:

$$p_{ij} = Pr\left[s_t = j|s_{t-1} = i\right] \qquad (i, j = 1, ..., h)$$

Pelletier (2006) provides a simple algorithm based on the Hamilton (1990) filter and smoother to obtain the maximum likelihood estimators of model (2.6).

Another approach in a MS framework is due to Billio and Caporin (2005), who extend the DCC model (2.3) by allowing both the unconditional correlation and the parameters to be driven by an unobservable Markov chain.

A strong advantage of the previous representations is that, under the assumption of normality of $\boldsymbol{u}_t$, it is possible to split the log-likelihood into the sum of the volatility part and the correlation part.[1] Let L be the log-likelihood of the full model, L_V the volatility part of the log-likelihood, L_C the correlation part, $\boldsymbol{\theta}_V$ the parameters present in the univariate GARCH models and $\boldsymbol{\theta}_C$ the parameters present in the correlation model. Engle (2002) has shown that:

$$L_V(\boldsymbol{\theta}_V) = -\frac{1}{2}\sum_{t=1}^{T}\left[nlog(2\pi) + 2log(|\boldsymbol{D}_t|) + \boldsymbol{u}_t{}'\boldsymbol{u}_t\right] \tag{2.7}$$

$$L_C(\boldsymbol{\theta}_C|\boldsymbol{\theta}_V) = -\frac{1}{2}\sum_{t=1}^{T}\left[nlog(2\pi) + log(|\boldsymbol{R}_t|) + \boldsymbol{u}_t{}'\boldsymbol{R}_t^{-1}\boldsymbol{u}_t\right] \tag{2.8}$$

$$L(\boldsymbol{\theta}_V, \boldsymbol{\theta}_C) = L_V(\boldsymbol{\theta}_V) + L_C(\boldsymbol{\theta}_C|\boldsymbol{\theta}_V) \tag{2.9}$$

L_V is the sum of n univariate log-likelihoods and its maximization is equivalent to maximizing each univariate log-likelihood. The maximization of (2.9) consists of two steps: in the first step the function (2.7) is maximized, obtaining the estimates of $\boldsymbol{\theta}_V$; in the second step (2.8) is maximized conditional on the estimates of the first step.

3 The Regime Switching Flexible Dynamic Correlation Model

We propose a model that introduces a particular MS dynamics implying changes in the parameters of groups of homogeneous variables. As said in the previous section, the introduction of MS dynamics in the correlation matrix has been already made by Pelletier (2006), who uses as basic model the CCC model, and Billio and Caporin (2005), who introduce such dynamics in the DCC model (2.3). Our basic model can be considered a combination of these models and the FDCC model (2.5), with the dynamic coefficients also allowed to switch. The more general form of the model is:

$$\boldsymbol{Q}_{t,s_t} = (\boldsymbol{\iota}_n\boldsymbol{\iota}_n' - \boldsymbol{a}_{s_t}\boldsymbol{a}_{s_t}' - \boldsymbol{b}_{s_t}\boldsymbol{b}_{s_t}') \odot \boldsymbol{R}_{s_t} + \boldsymbol{a}_{s_t}\boldsymbol{a}_{s_t}{}' \odot \boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}' + \boldsymbol{b}_{s_t}\boldsymbol{b}_{s_t}' \odot \boldsymbol{Q}_{t-1,s_{t-1}} \tag{3.1}$$

We call this model RSFDC-F (where the final F stays for full and indicates that we allow both the unconditional correlation matrix and the dynamic coefficients to switch from one regime to another). As in the RSDC model, the suffix s_t is a discrete unobservable variable representing the regime at time t, which follows a Markov chain assuming

[1] If the assumption of normality is not valid, we can use the same approach and the estimator is a Quasi-Maximum Likelihood estimator (Bollerslev and Wooldridge, 1992).

h possible states, but the form of the transition probability matrix is different. To stress the fact that the assets considered in the model are homogeneous within the k groups but different among the groups, we give a particular interpretation of the states. We suppose that each group can switch between 2 states (call them 0 and 1). At time t, the group g_1 is in state s_{1t}, the group g_2 in state s_{2t},..., the group g_k in state s_{kt} (s_{1t}, s_{2t}, ..., s_{kt} can assume value 0 or 1). In practice, the state s_t is constituted by a combination of k unobservable dichotomic variables, so that the Markov Chain can assume $h = 2^k$ possible regimes.[2] The parameters of the model (3.1) can assume two possible values, depending on the state of the group at time t. Formally, the vector of coefficients $\boldsymbol{a}_{s_t}$ is given by:

$$\boldsymbol{a}_{s_t} = \left[a_{1,s_{1t}}\boldsymbol{\iota}_{n_1}, ..., a_{k,s_{kt}}\boldsymbol{\iota}_{n_k}\right]';$$

(similarly for $\boldsymbol{b}_{s_t}$). For example, in the case of two groups ($k = 2$), constituted respectively by 2 and 3 assets, if $s_{1t} = 0$ and $s_{2t} = 1$, then $s_t = 01$ and the vector of coefficients $\boldsymbol{a}$ will be:

$$\boldsymbol{a}_{01} = \left[a_{1,0}, a_{1,0}, a_{2,1}, a_{2,1}, a_{2,1}\right]'.$$

Similarly the 5×5 unconditional correlation matrix R_{01} will be composed by two blocks on the diagonal; the first block will contain the 2×2 sub-matrix relative to the first group, with correlations in state 0, whereas the second block will be composed by the 3×3 sub-matrix relative to the assets of the second group, with correlations in state 1. The out-of-diagonal blocks will represent the cross-correlations between the assets of group 1 and the assets of group 2 in the state 01.

It is interesting to note that, constraining the dynamic coefficients equal to zero, we obtain a model similar to the RSDC model (2.6), but the changes in regime are relative to the single blocks of $\boldsymbol{R}_{s_t}$ and not to the overall matrix, as in Pelletier (2006). We call this specification RSFDC-C (where the final C indicates the the switch is relative only to the correlation matrix).

Another specification is obtained constraining the unconditional correlation matrix to remain constant over time and allowing only the dynamic coefficients to be switching. This case can be viewed as a FDCC model with MS, or, more correctly, as a general DCC (2.4) with MS because we keep the dependence on the unconditional correlation matrix to include the correlation targeting property (differently from the specification (2.5) of Billio et al., 2006). We call this model RSFDC-D (the final D indicates that the switch is relative only to the dynamic parameters).

The estimation of model (3.1) can make use of the split of the likelihood into two parts, as in (2.7)-(2.9), the switching coefficients being present only in (2.8). After the estimation of $\boldsymbol{\theta}_V$, by maximizing (2.7), we obtain the residuals $\hat{\boldsymbol{u}}_t$, which substitute $\boldsymbol{u}_t$ in (2.8). For the second part (maximization of (2.8)), we can use the following procedure:[3]

1. for a given value of the parameters $\boldsymbol{a}_{s_t}$, $\boldsymbol{b}_{s_t}$ and p_{ij}, run the Hamilton (1990) filter and smoother, obtaining the probability $Pr[s_t = i|\boldsymbol{\Psi}_T]$, for each $i = 1, ..., h$, where $\boldsymbol{\Psi}_T$ represents the full information available;

[2] A similar interpretation was given by Edwards and Susmel (2001), studying the contagion in emerging financial markets.

[3] We are very grateful to an anonymous referee who has suggested this procedure.

2. using these probabilities, compute the sample correlation matrix for each regime ($\boldsymbol{R}_{s_t}$) by weighting each $\hat{\boldsymbol{u}}_t\hat{\boldsymbol{u}}_t'$ by the probability of being in that regime, obtained in the previous step. This procedure is very similar to the approach used by Pelletier (2006) to estimate $\boldsymbol{R}_{s_t}$;

3. given the estimates of $\boldsymbol{R}_{s_t}$, estimate the parameters $\boldsymbol{a}_{s_t}$, $\boldsymbol{b}_{s_t}$, p_{ij};

4. iterate these three steps until convergence.

The procedure can be used to estimate the RSFDC-F model, though it becomes simpler using the constrained models RSFDC-C (in step 1 and 3 $\boldsymbol{a}_{s_t}$ and $\boldsymbol{b}_{s_t}$ vanish) and RSFDC-D (the classical Hamilton filter is ran to explicit the likelihood).

A problem arises in the estimation algorithm considering that in (3.1) the matrix $\boldsymbol{Q}_{t,s_t}$ depends on which regime it was at time $t-1$. This means that evaluating the likelihood recursively, we need to keep track of all the possible paths that the regimes might take from $t = 1$ to $t = T$, involving a non tractable model. To avoid this problem, we use the solution proposed by Kim (1994), dealing with a similar problem with a state-space MS model. After each step of the Hamilton filter, at time t we collapses the $h \times h$ possible matrices $\boldsymbol{Q}_{t,s_{t-1},s_t,}$ into h matrices by an average over the probabilities at time $t-1$:

$$\hat{\boldsymbol{Q}}_{t,s_t} = \frac{\sum_{i=1}^{h} Pr[s_{t-1} = i, s_t = j|\boldsymbol{\Psi}_t]\boldsymbol{Q}_{t,s_{t-1},s_t}}{Pr[s_t = j|\boldsymbol{\Psi}_t]} \tag{3.2}$$

where $\boldsymbol{\Psi}_t$ represents the information available at time t and the probabilities present in (3.2) are obtained by the Hamilton filter. In practice, we can not use model (3.1) directly, but we have to replace $\boldsymbol{Q}_{t-1,s_{t-1}}$ with $\hat{\boldsymbol{Q}}_{t-1,s_{t-1}}$, calculated as in (3.2); in other words we change slightly the model so that it becomes tractable. This is the solution often used to deal with non tractable MS models (see, for example, many examples showed in Kim and Nelson, 1999, and Billio and Caporin, 2005, for a similar problem with dynamic conditional correlations).

It is important to notice that the potential large dimension of the transition probabilities matrix could imply computational problems in the maximization algorithm. This case is frequent when some combinations of states are not verified in the data. For example, considering three groups, if the state $s_t = 011$ is not found or it occurs rarely, it is likely that the maximization algorithm will not converge. In this case it is preferable to set the corresponding transition probabilities equal to 0 (as in Hamilton and Susmel, 1994). In the following section we will show how to proceed in a real case.

4 An Application to the Sectorial Asset Allocation

In this section we consider a hypothetical portfolio constituted by indices relative to the two main sectors of the Italian Mibtel general index: banks (Ban), insurance (Ins) and finance holdings (Hol) relative to the finance sector; minerals metals (Min), chemicals (Che) and textile clothing (Tex) relative to the industrial sector. Billio et al. (2006) have analyzed all the sectorial indices of the Mibtel, noticing similar correlation dynamics among the indices belonging to the same sector. For this reason we consider two groups of homogenous variables ($g_1 = F$ constituted by Ban, Ins and Hol, and $g_2 = I$ constituted by Min, Che and Tex). The data considered cover the period 4 January 2000 - 29 August 2003 (daily data; 954 observations)[4] and in Figure 1 we show the 6 series of the returns of the indices. We notice that the time series start with a turmoil period that seems to end at the beginning of 2001; the volatility increases dramatically after 11 September 2001 and remains high in the rest of the series. This behavior is less evident for the industrial indices, in particular for Min.

Following the approach proposed, we consider a 6-variate GARCH(1,1) model for the volatility part of the model, whereas we estimate a CCC, a DCC, an FDCC,[5] an RSDC and the models proposed here (the general model RSFDC-F and the sub-cases RSFDC-C and RSFDC-D) for the correlation part. The model considered is (2.1)-(2.2), in which the squared elements on the diagonal of $\boldsymbol{D}_t$ are six univariate GARCH(1,1) models as:

$$h_{it} = \gamma_i + \alpha_i \epsilon_{it}^2 + \beta_i h_{it-1}, \quad i = Ban, Ins, Hol, Min, Che, Tex$$

where $\epsilon_{it} = y_{it} - \mu_i$ are obtained from (2.1). In Table 1 we show the estimation results of the GARCH models.

The second estimation step needs the standardized disturbances $\boldsymbol{u}_t$; they are obtained from the first step, by standardizing the disturbances ϵ_{it} contained in the $(T \times 6)$ matrix $\boldsymbol{E}$ by $\boldsymbol{H}_t^{1/2}\boldsymbol{E}$. In Table 2 we show the estimation results of each dynamic correlation model and the corresponding log-likelihood.[6] As it is well known, these likelihoods cannot be directly used within a likelihood ratio testing approach for the presence of nuisance parameters present only under the alternative hypothesis (see, for example, Hansen, 1992), but we show them for a first comparison among models.

Let us observe the estimates of the models without switching (CCC, DCC and FDCC). The unconditional correlation matrix $\boldsymbol{R}$ is equal for all these models (it is the only estimated part of the CCC model). It is given by:

$$\boldsymbol{R} = \begin{bmatrix} 1.000 & 0.764 & 0.662 & 0.358 & 0.606 & 0.543 \\ 0.764 & 1.000 & 0.586 & 0.395 & 0.532 & 0.503 \\ 0.662 & 0.586 & 1.000 & 0.280 & 0.703 & 0.539 \\ 0.358 & 0.395 & 0.280 & 1.000 & 0.176 & 0.226 \\ 0.606 & 0.532 & 0.703 & 0.176 & 1.000 & 0.514 \\ 0.543 & 0.503 & 0.539 & 0.226 & 0.514 & 1.000 \end{bmatrix} \tag{4.1}$$

The matrix shows a higher correlation among the financial indices with respect to the industrial indices. The DCC model shows a large value of the parameter b, which implies a strong persistence of the correlation. Distinguishing between sectors in the FDCC model,

[4]We choose this period so that the data relative to the terroristic attack of 11 September 2001 is in the middle of the series.

[5]We specify the FDCC model as in (3.1) without switching parameters to compare similar models with the correlation targeting.

[6]We have estimated also a DCC model with MS, as in Billio and Caporin (2005), considering the same change of state for all the variables. In this case there is no evidence for the presence of regimes: the probability of permanence in a state is 1 and in the other is 0. This result seems to support the idea that, for the time series studied, the presence of regimes in a dynamic conditional correlation model has to be analyzed separately for each sector.

this persistence decreases, in particular for the industrial sector, whereas the value of the a coefficients increases (the likelihood seems to favor the FDCC model). This effect is due to our different specification with respect to (2.5) to take into account the correlation targeting. In fact, in our specification, the unconditional correlation matrix $\boldsymbol{R}$ enters the equation in different ways with respect to the DCC model; in the latter it enters with a very small coefficient equal to $(1 - a - b)$; in the FDCC model it enters with different coefficients, in correspondence of the different blocks. In practice, in the FDCC model with correlation targeting, $\boldsymbol{R}$ is pre-multiplied (element by element) by the matrix $(\boldsymbol{\iota}_n\boldsymbol{\iota}_n' - \boldsymbol{a}\boldsymbol{a}' - \boldsymbol{b}\boldsymbol{b}')$, so that its effect is evaluated differently in the different blocks. As a matter of fact, part of the correlation persistence captured by the coefficient b in the DCC model is moved to the constant part of the FDCC model. To support this idea we have estimated also a model, like (2.5), avoiding the constraint of the correlation targeting; we obtain results very similar to DCC, with coefficients c_F and c_I near to zero and coefficients b_F and b_I more than 0.9.

The introduction of regime switching increases considerably the likelihood, particularly in the case of the RSDC model, in which the change in regime is contemporaneous for all the assets. We have two different correlation matrices in correspondence of regime 0 and regime 1 for the RSDC model; they are given by:

$$\boldsymbol{R}_0 = \begin{bmatrix} 1.000 & 0.635 & 0.527 & 0.095 & 0.401 & 0.344 \\ 0.635 & 1.000 & 0.380 & 0.204 & 0.277 & 0.274 \\ 0.527 & 0.380 & 1.000 & 0.064 & 0.631 & 0.371 \\ 0.095 & 0.204 & 0.064 & 1.000 & -0.160 & -0.044 \\ 0.401 & 0.277 & 0.631 & -0.160 & 1.000 & 0.373 \\ 0.344 & 0.274 & 0.371 & -0.044 & 0.373 & 1.000 \end{bmatrix}$$

$$\boldsymbol{R}_1 = \begin{bmatrix} 1.000 & 0.899 & 0.823 & 0.661 & 0.836 & 0.763 \\ 0.899 & 1.000 & 0.812 & 0.624 & 0.805 & 0.737 \\ 0.823 & 0.812 & 1.000 & 0.545 & 0.800 & 0.737 \\ 0.661 & 0.624 & 0.545 & 1.000 & 0.556 & 0.515 \\ 0.836 & 0.805 & 0.800 & 0.556 & 1.000 & 0.686 \\ 0.763 & 0.737 & 0.737 & 0.515 & 0.686 & 1.000 \end{bmatrix}$$

The states 0 and 1 can be considered respectively as a regime of low correlation and a regime of high correlation. In this case the persistence of a regime is a function of the transition probabilities ($\frac{1}{1-p_{ii}}$, $i = 0, 1$, see Hamilton, 1989); the persistence of the state of low and high correlation is very similar (around 7 days).

The introduction of the RSFDC models involves some computational difficulties. The transition probabilities matrix has dimension 4×4 and the element $p_{ij,lm}$ indicates the probability that at time t the group F is in state l and the group I in state m, given that at time $t-1$ the group F was in state i and the group I in state j $(i, j, r, s = 0, 1)$. The maximization algorithm described in section 3 does not converge, because the covariance matrix of the parameters becomes singular, from whatever starting point.[7] To understand why such situation arises, we have tried to apply the filtering and smoothing algorithm (Hamilton, 1990 and Kim, 1994) to the series using the values of the parameters where

[7]This happens not only for the RSFDC-F model, but also for the RSFDC-C and RSFDC-D models.

the algorithm stops (this is possible because the filtering and smoothing algorithms do not require the inversion of the covariance matrix). The problem seems related to the identification of the number of states in the Markov chain. In fact the smoothing algorithm provides the probabilities $Pr\left[s_t = lm|\mathbf{\Psi}_T\right]$, which are the probabilities that at time t the state of the group of financial indices is l and the state of the group of industrial indices is m, given the full information available. The behavior of the smoothing probabilities shows evidence of the presence of states 0 and 1 for group F and only one state for group I. In other words only the financial group has a switching correlation, whereas the industrial group follows a DCC model without switching. This result is consistent with the estimation of the FDCC model, where the coefficient b_I is small, so that the block of the correlation matrix relative to the industrial sector changes slightly over time.

In other words, we have to adopt a 2×2 MS model with some constraints on the RSFDC specification. Now, the transition probabilities matrix is:

$$\boldsymbol{P} = \begin{bmatrix} p_{0\cdot,0\cdot} & p_{0\cdot,1\cdot} \\ p_{1\cdot,0\cdot} & p_{1\cdot,1\cdot} \end{bmatrix}$$

where $p_{0\cdot,1\cdot} = 1 - p_{0\cdot,0\cdot}$ and $p_{1\cdot,0\cdot} = 1 - p_{1\cdot,1\cdot}$ and the dot indicates that we have always the same regime for group I. In Table 2 we can observe the estimated coefficients for the three RSFDC models. In the RSFDC-F case the model identified does not contain the coefficients $\boldsymbol{b}_F$ and $\boldsymbol{b}_I$; in practice the persistence of the correlations, which in FDCC model was partially captured by the constant part, now is captured by the presence of two correlation matrices corresponding to the two regimes and the dynamic part is only represented by the lagged squared disturbances. This is more evident when we consider the RSFDC-D model, in which the correlation matrix is constant and equal to (4.1); in this case the coefficients b are present in state 0 and the persistence of the regimes is very high (272 days for state $0\cdot$ and 216 days for state $1\cdot$, against 3 days in state $0\cdot$ and 30 days in state $0\cdot$ in model RSFDC-F)).

The two estimated correlation matrices of model RSFDC-F are given by:

$$\boldsymbol{R}_{0\cdot} = \begin{bmatrix} 1.000 & 0.412 & 0.136 & 0.042 & 0.042 & -0.035 \\ 0.412 & 1.000 & 0.013 & 0.064 & 0.010 & -0.036 \\ 0.136 & 0.013 & 1.000 & 0.027 & 0.186 & 0.046 \\ 0.042 & 0.064 & 0.027 & 1.000 & 0.177 & 0.226 \\ 0.042 & 0.010 & 0.186 & 0.177 & 1.000 & 0.514 \\ -0.035 & -0.036 & 0.046 & 0.226 & 0.514 & 1.000 \end{bmatrix}$$

$$\boldsymbol{R}_{1\cdot} = \begin{bmatrix} 1.000 & 0.822 & 0.749 & 0.380 & 0.652 & 0.611 \\ 0.822 & 1.000 & 0.678 & 0.411 & 0.577 & 0.561 \\ 0.749 & 0.678 & 1.000 & 0.300 & 0.709 & 0.577 \\ 0.380 & 0.411 & 0.300 & 1.000 & 0.177 & 0.226 \\ 0.652 & 0.577 & 0.709 & 0.177 & 1.000 & 0.514 \\ 0.611 & 0.561 & 0.577 & 0.226 & 0.514 & 1.000 \end{bmatrix}$$

The differences between states are sharper than the RSDC case. Notice also that the correlation between the series of sector F and the series of sector I are almost zero in state

$0\cdot$. Note also the characteristic of this model, in which the lower block on the diagonal is equal in state $0\cdot$ and state $1\cdot$ and also equal to the corresponding block in (4.1).

Changing specification and considering RSFDC-C, in which the dynamic coefficients are not considered, the inference on the regime changes again. The two correlation matrices are:

$$\boldsymbol{R}_{0\cdot} = \begin{bmatrix} 1.000 & 0.755 & 0.646 & 0.352 & 0.597 & 0.556 \\ 0.755 & 1.000 & 0.569 & 0.390 & 0.521 & 0.510 \\ 0.646 & 0.569 & 1.000 & 0.272 & 0.695 & 0.551 \\ 0.352 & 0.390 & 0.272 & 1.000 & 0.177 & 0.226 \\ 0.597 & 0.521 & 0.695 & 0.177 & 1.000 & 0.514 \\ 0.556 & 0.510 & 0.551 & 0.226 & 0.514 & 1.000 \end{bmatrix}$$

$$\boldsymbol{R}_{1\cdot} = \begin{bmatrix} 1.000 & 0.961 & 0.986 & 0.126 & 0.190 & -0.002 \\ 0.961 & 1.000 & 0.950 & 0.133 & 0.197 & 0.037 \\ 0.986 & 0.950 & 1.000 & 0.126 & 0.196 & 0.007 \\ 0.126 & 0.133 & 0.126 & 1.000 & 0.177 & 0.226 \\ 0.190 & 0.197 & 0.196 & 0.177 & 1.000 & 0.514 \\ -0.002 & 0.037 & 0.007 & 0.226 & 0.514 & 1.000 \end{bmatrix}$$

In this case, in state $1\cdot$, where we have an increase in the correlations among the series of the financial sectors, we can observe a decrease of correlations between the series of the financial sector and the series of the industrial sector. The state $1\cdot$ is not persistent (2 days), whereas the persistence of state $0\cdot$ is around 27 days.

As for likelihood functions, the RSFDC-F model has the highest one among the RSFDC models.

In Figure 2 we show the smoothed probabilities of the state 1, identified as high correlation for the model RSDC, and the smoothed probabilities of states $1\cdot$ for the RSFDC models, identified as high correlation for the finance sector in models RSFDC-F and RSFDC-C; in the RSFDC-D case it represents the state in which the coefficient a_F increases changing the regime. The graphs show four different behaviors. The change in regime derived by model RSDC seems to have a different behavior before and after 9/11: before this date the periods of high correlation are infrequent and short; after 9/11 they are the majority. In the case of RSFDC-F model we have a similar break in correspondence of 9/11, but in the first span the periods of high correlation are frequent and short; after 9/11 the high correlation is dominant with only four short periods of low correlation. It is likely that the presence of switching dynamic coefficients adjusts the correlation without the need for a change in regime. The inference derived by the RSFDC-C model is not easy to explain; in the full period the switch to regime $1\cdot$ is infrequent and short. A purely constant correlation matrix without any dynamics for the industrial sector seems inadequate and, given also the value of the likelihood ratio with respect to the RSFDC-F model,[8] we will not consider this model any further. Finally, the RSFDC-D model shows a clear behavior; in the first period the probability that the state is $1\cdot$ is around 1; it decreases at the end of October 2000 and it is around zero (where the state is almost surely $0\cdot$) from April 2001 until the terrorist attack of 11 September 2001. After this date the

[8]In this case model RSFDC-C is nested in RSFDC-F without nuisance parameters under the alternative hypothesis, so the use of the likelihood ratio test makes sense.

probability of state $1\cdot$ has an abrupt increase and it remains around 1 until the end of the series, excluding the period October-December 2002 and August 2003, in which the graph shows two slight troughs.

Despite the differences, comparing these graphs with the returns of the time series in Figure 1 we notice a certain coherence between periods of high volatility and the state 1 for model RSDC and $1\cdot$ for models RSFDC-F and RSFDC-D; this result seems to confirm the idea that periods of turmoil are associated with periods of high correlation. On the other side, it is difficult to establish from these results what model performs the best. We will carry on with comparisons based on the performance of a hypothetical portfolio and with the comparison of their realized volatility and the relative performance of the covariance matrices.[9]

4.1 Evaluating the portfolio performance

We expect the covariance model which estimates the risk in an appropriate way to provide a good portfolio allocation. To evaluate this property we have performed a historical simulation. We start considering the time series at the end of July 2001 and perform the optimal asset allocation solving the simple mean-variance problem (Markovitz, 1959), but using different estimations for the covariance matrix. The portfolio weights are computed under the hypothesis of short selling constraints (positive weights) and no transaction costs. Under these hypotheses and considering the absence of risk-free assets, the optimal portfolio allocation is given by the following weights:

$$\frac{\boldsymbol{\Sigma}^{-1}_{t+1|t}\boldsymbol{\mu}_{t+1|t}}{\boldsymbol{\iota}'_n\boldsymbol{\Sigma}^{-1}_{t+1|t}\boldsymbol{\mu}_{t+1|t}}$$

where $\boldsymbol{\mu}_{t+1|t}$ and $\boldsymbol{\Sigma}_{t+1|t}$ represent the expected vector of returns and their expected covariance matrix at time $t+1$ given the information until time t. We repeat the same experiment several times, using the previous formula to calculate the weights, adding one observation and estimating a new (2.1)-(2.2) model with six different specifications for the covariance matrix (CCC, DCC, FDCC, RSDC, RSFDC-F and RSFDC-D). We make this simulation until the end of the period, estimating recursively the six models (543 replications).

In Table 3 we show the two first unconditional moments of the portfolio weights for the various models and the coefficient of an AR(1) model estimated for each series of weights; this last indicator would represent a sort of turnover index of portfolio weights: values around 1 will indicate a certain persistence of the weights and hence a small turnover (viceversa for values around 0). On average, all the models seem to distribute the investment among the six indices, except RSFDC-F, which suggests more investments in the industrial sector. In general it is possible to note a high degree of turnover; the index with the least turnover is Min for all the models, but it is less persistent for RSFDC-F than all other models. Note that the volatility of Min is the most regular among the six indices, as can be seen on Figure 1 and Table 1, where the constant parameter of the GARCH model is very high with respect to the constants of the other indices and the coefficient β is the lowest.

[9]We are grateful to an anonymous referee who has suggested many of these analyses.

In Figure 3 we show a graphical *horse-race* among the models to gain an idea of the results of the six allocation strategies. We suppose to have a starting investment of 100 and then we apply the returns obtained by the previous historical simulation along the period considered. We note that the date of 11/9 is that one which discriminates the different behaviors of the models. All the models show a trough after this date, which is deeper for the no switching regime models. After this date the portfolio created by RSFDC-F reacts better than others and at the end of 2001 its value has increased more than the others. This gap lingers until July 2002, when a new trough characterizes the graph; after this date the increase of RSFDC-F is very large compared to the others; the other portfolios show a similar behavior, except for the one derived by FDCC, which has the worst performance during 2002. In 2003 the portfolios behave in similar ways and the distances are approximately constant.

The evaluation of the performance can be made through a portfolio benchmark; the purpose of the optimal allocation is to obtain a portfolio which has a higher return and a lower volatility than the portfolio benchmark (Philips et al., 1996). In our case the global market index is a natural benchmark (in this case the Mibtel). The comparison of the six simulated portfolios with the benchmark is made using four classical measures based on returns. The first one is the Jensen α (Jensen 1968 and 1969), which is the constant of a regression model where the portfolio return is the dependent variable and the return of the portfolio benchmark is the independent variable. The second is the *ratio* proposed by Treynor (1965), obtained as the ratio between the expected portfolio return and the slope of the same regression (often used to evaluate portfolio without risk-free assets). Then we consider the *appraisal ratio* of Treynor and Black (1973), obtained as the ratio between the Jensen α (which represents the systematic risk) and the standard deviation of the disturbances of the same regression (which represents the idiosyncratic risk). Finally we show the Sharpe ratio (mean of returns divided by their standard deviation) of each portfolio. The performance increases when the measures increase. In Table 4 we show the unconditional moments of the portfolio returns for all the models and the four measures. All the portfolios show a higher mean of returns and a lower variance than the benchmark. In particular the RSFDC models show the highest mean and the lowest variance (RSFDC-F better than RSFDC-D). Considering the measures of performance, we note that in all cases the RSFDC-F model has the best performance for the period considered and that all the portfolios have better performance than the benchmark (which shows a negative Sharpe ratio). The good performance of the RSFDC-F model during the periods of turmoil with sudden shocks is confirmed by calculating the Sharpe ratio separately for each year of the historical simulation. These results are shown in the bottom part of Table 4; we note that in 2001 and 2002 the Sharpe ratio of the RSFDC-F model is almost twice the amount of the others, whereas in 2003, when the benchmark also has a positive Sharpe ratio, the portfolios have similar performances, except for FDCC.

The differences among the models in terms of Sharpe ratio are small, so we have compared the Sharpe ratios across the models and with respect to the benchmark, following the contributions of Memmel (2003) and Ledoit and Wolf (2008). More accurately we have tested the null hypothesis of the equality of two Sharpe ratios against the alternative that their difference is not zero. Following Ledoit and Wolf (2008), we denote with μ_i the mean of returns of the portfolio obtained by model i and δ_i the uncentered second moment of the returns of portfolio derived by model i; in addition, let $\boldsymbol{v}$ be the vector containing the elements $(\mu_i, \mu_j, \delta_i, \delta_j)$. Ledoit and Wolf (2008) verify the equality of the Sharpe ratios derived by models i and j, by testing the null hypothesis:

$$\frac{\mu_i}{(\delta_i - \mu_i^2)^{1/2}} - \frac{\mu_j}{(\delta_j - \mu_j^2)^{1/2}} = 0 \tag{4.2}$$

assuming that, under the null, $(T)^{1/2}\boldsymbol{v}$ is asymptotically Normal with mean 0 and covariance matrix $\boldsymbol{\Sigma}$. In our application, we estimate $\boldsymbol{\Sigma}$ taking into account the serial correlation and heteroskedasticity of returns (HAC inference, Newey and West, 1987, Andrews, 1991); then the standard error of distribution of (4.2) is easily obtained applying the delta method. In Table 5 we show the p-values of the absolute value of (4.2) for each pair of model and for Mibtel. All the portfolios display a significantly higher Sharpe ratio than Mibtel, but there is not significant difference in performance among the portfolios. This result is not surprising; in fact the asset allocations of each portfolio differ only for the correlation matrices and not for returns and variances. In general, the effect of returns is more important than the one of covariance matrices in the allocation strategies (see, for example, Chopra and Ziemba, 1993 and Engle and Colacito, 2006). To evaluate the goodness of the correlation matrices adopted we need to isolate the effect of covariance information from the effect of returns, which we do in the following sub-section.

4.2 Evaluating the Correlation Matrices

The analysis carried out in the previous section is useful to evaluate the performance of the different portfolios and to compare them with a benchmark. To evaluate only the goodness of correlation matrices we adopt the approach proposed by Engle and Colacito (2006); they show that the realized volatility is smallest for the correctly specified covariance matrix for any vector of expected returns. They suggest to select arbitrary vectors of expected returns, then construct optimal portfolio weights with the alternative covariance models and to calculate the sample variance of each portfolio. The strategy with the smallest covariance for each vector of expected returns will be the best strategy. The key problem here is the choice of the vectors of expected returns; the main experiments of Engle and Colacito (2006) only regard two assets and many alternatives can be used. In the case of high order asset allocation the choice of an appropriate vector of expected returns is not easy. For example, Engle and Colacito (2006), considering a portfolio composed by 21 stocks and 13 bonds in a framework with a free-risk asset and tangency portfolio, select only hedging portfolios, obtained putting one entry equal to 1 and the other equal to zero; in this way the asset with 1 is hedged against all other assets. Our framework is different from that of Engle and Colacito (2006), given the absence of free-risk assets and the constraints of positivity of the weights summing up to one; we consider the following 22 alternative vectors of expected returns:

- a vector in which the expected return of each asset is equal to $1/6$ (we call it ER);
- 6 vectors in which the expected return of one asset is equal to $4/6$ and the others equal to $1/15$; we denote each case with the name of the asset with highest return;

- 15 vectors obtained setting the return of two assets equal to $1/2$ and the others equal to 0; we denote each case with the names of the assets with nonzero return.

In Table 6 we show the sample standard deviations of each portfolio for the 22 cases, setting the lowest standard deviation equal to 100; in this way a number like $(100+x)$ means that, knowing the true covariance matrix, an $x\%$ higher return could be required. We notice that in 10 cases the RSFDC-F model is the best one, in 3 cases it is the DCC, in 4 the FDCC, in 2 the RSDC and in 3 the RSFDC-D model. Anyway, in many cases where the RSFDC-F model is not the best one, the difference with respect to the others is very small (the same for DCC and RSFDC-D). The CCC model has not a bad performance, but there are no cases where it has the minimum variance, confirming the widespread idea that the correlation of assets is not constant along the time.

The subsequent step is to compare the six approaches at 22 different expected returns. Following Engle and Colacito (2006), this can be obtained using a Diebold and Mariano (1995) procedure to test differences between each pair of covariance estimators jointly for the 22 expected returns. We call $d_t^{ij,k}$ the difference at time t between the squared return of portfolio i and the squared return of portfolio j for the $k-th$ hypothesized vector of expected returns $\boldsymbol{\mu}^k$. Let $\boldsymbol{D}_t^{ij}=(d_t^{ij,1},...,d_t^{ij,22})'$; the approach consists in estimating the model:

$$\boldsymbol{D}_t^{ij}=\beta\boldsymbol{\iota}_{22}+\boldsymbol{\epsilon}_{d,t}$$

using the generalized method of moments with vector HAC covariance and then verifying the null hypothesis:

$$\beta=0 \tag{4.3}$$

If the null is accepted, we can consider the two covariance estimators $\boldsymbol{H}_t^i$ and $\boldsymbol{H}_t^j$ equal.[10] In Table 7 we show the values of the t statistic to verify (4.3) in a pair wise comparison. It is useful to consider the sign of the t-value when the null is rejected; in fact $d_t^{ij,k}$ is constructed as the difference of the squared realized returns of the methods indicated in row i and column j: a positive number is evidence in favor of the method in the column. Notice that RSFDC-F performs better than all the alternative estimators. From Table 7 we can deduce that the models with dynamic conditional correlation performs better than the models that only consider the unconditional correlation (switching or not). In fact, the second best is represented by RSFDC-D, but the differences with respect to DCC and FDCC are not large, whereas they are significant with respect to CCC and RSDC.

5 Concluding Remarks

We propose a new class of models to represent the time-varying correlations between assets in a DCC framework. The use of the DCC family as basic model is particularly appealing because of the small number of parameters involved, thus bypassing the problems of large number of coefficients of other multivariate models, such as that proposed by Engle and Kroner (1995) or Kroner and Ng (1998).

[10] It is possible to improve the sampling properties of the test adjusting $d_t^{ij,k}$ by the geometric mean of the two variance estimators $\boldsymbol{H}_t^i$ and $\boldsymbol{H}_t^j$. In this application the results are very similar to those obtained testing (4.3), so we do not show them.

The models proposed possess a particular MS dynamics, which provides different changes in regime for different groups of variables. Formally the models are classical MS models with a potentially high number of states. In fact, the particular structure in groups is obtained allowing the parameters to switch in an appropriate way. This idea brings some theoretical problems because it is not possible to estimate exactly the model parameters for the dependence on all the previous regimes. Anyway the models become tractable using the Kim (1994) approach, which is generally employed to solve this kind of problems in a MS framework.

The computational problems are greater than the classical MS models, due to the possible high number of states involved by the Markov chain. In our case, dealing with two groups, the problem is bypassed observing that the computational problems are due to the lack of evidence for the presence of four regimes. It is likely that, with more than two groups, as the number of possible states increases, the problem of not observing some regimes is even more serious. This is an open problem which needs a more accurate analysis, maybe looking for alternative solutions and computational algorithms.

In the application proposed, our model RSFDC-F seems to perform better than other models: it has a higher Sharpe ratio; it seems appropriate in cases of abrupt changes in correlations, due, for example, to unexpected events, such as a terroristic attack; it outperforms the other models in terms of Engle and Colacito (2006) tests.

The example was conducted in a simple framework, under the hypotheses of Markovitz (1959), and the hypotheses of positivity restrictions and no transaction costs. The exercise can be easily extended to include other cases. We have preferred to work in this framework because it is simple and more frequent in literature.

References

[1] Andrews, D.W.K. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation, *Econometrica*, 59, 817-858.

[2] Arnott, R., Fabozzi, F.J. (1988). *Asset Allocation: a Handbook of Portfolio Policies, Strategies and Tactics*. Probus Publishing Company: Chicago, Illinois.

[3] Billio, M., Caporin, M. (2005). Multivariate Markov switching dynamic conditional correlation GARCH representations for contagion analysis, *Statistical Methods and Applications*, 14, 145-161.

[4] Billio, M., Caporin, M., Gobbo, M. (2006). Flexible Dynamic Conditional Correlation Multivariate GARCH models for asset allocation, *Applied Financial Economics Letters*, 2, 123-130.

[5] Black , F., Litterman, R. (1991). Asset allocation: combining investors views with market equilibrium, *Journal of Fixed Income*, 718.

[6] Black , F., Litterman, R. (1992). Global portfolio optimization, *Financial Analysts Journal*, 2843.

[7] Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 31, 307-327.

[8] Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH approach, *Review of Economics and Statistics*, 72, 498-505.

[9] Bollerslev, T., Wooldridge, J.M. (1992). Quasi-Maximum Likelihood estimation and inference in dynamic models with time varying covariances, *Econometric Reviews*, 11, 143-172.

[10] Chopra, V., Ziemba, W. (1993). The effect of errors in means, variances and covariances on optimal portfolio choice, *Journal of Portfolio Management*, 19, 6-11.

[11] Clare, A.D., O'Brien, S.H., Thomas, S.H., Wickens, M.R. (1998). Macroeconomics shocks and the domestic CAPM: evidence from the UK stock market, *International Journal of Finance and Economics*, 3, 111-126.

[12] Diebold, F.X., Mariano, R.S. (1995). Comparing predictive accuracy, *Journal of Business and Economic Statistics*, 13, 253-263.

[13] Ding, Z., Engle, R. (2001). Large scale conditional covariance m atrix modeling, estimation and testing, *Academia Economic Papers*, 29, 157-184.

[14] Drakos, K. (2004). Terrorism-induced structural shifts in financial risk: airline stocks in the aftermath of the September 11th terror attacks, *European Journal of Political Economy*, 20, 436-446.

[15] Edwards, S., and Susmel, R. (2001). Volatility dependence and contagion in emerging equity markets, *Journal of Development Economics*, 66, 505-532.

[16] Engle, R. (2002). Dynamic Conditional Correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models, *Journal of Business and Economic Statistics*, 20, 339-350.

[17] Engle, R., Colacito, R. (2006). Testing and valuing dynamic correlations for asset allocation, *Journal of Business and Economic Statistics*, 24, 238-253.

[18] Engle, R., Kroner, K.F. (1995). Multivariate simultaneous GARCH, *Econometric Theory*, 11, 122-150.

[19] French, K. R., Poterba, J.M. (1991). Investor diversification and international equity markets, *American Economic Review*, 81, 222-226.

[20] Hamilton, J.D. (1989). A new approach to the economic analysis of time series in the business cycle, *Econometrica* 57, 357-384.

[21] Hamilton, J.D. (1990). Analysis of time series subject to changes in regime, *Journal of Econometrics* 45, 39-70.

[22] Hamilton, J.D., Susmel, R. (1994). Autoregressive conditional heteroskedasticity and change in regime, *Journal of Econometrics* 64, 307-333.

[23] Hansen, B.E. (1992): The Likelihood Ratio Test under nonstandard conditions: testing the Markov Switching Model of GNP, *Journal of Applied Econometrics* 7, S61:S82.

[24] Jensen, M. (1968). The performance of mutual funds in the period 1945-1964, *Journal of Finance* 23, 389-416.

[25] Jensen, M. (1969). Risk, the price of capital assets, and the evaluation of investment portfolios, *Journal of Business* 42, 167-247.

[26] Kim, C.J. (1994). Dynamic linear models with Markov-switching, *Journal of Econometrics*, 60 1-22.

[27] Kim, C.J., Nelson, C.R. (1999). *State-Space Models with Regime Switching*. MIT Press, Cambridge, Massachusetts.

[28] Kroner, K.F., Ng, V.K. (1998). Modeling asymmetric comovements of asset returns, *Review of Financial Studies*, 11, 817-844.

[29] Ledoit, O., Wolf, M. (2008). Robust performance hypothesis testing with the Sharpe Ratio, *Working Paper* No. 320, Institute for Empirical Research in Economics, University of Zurich.

[30] Longin, F., Solnik, B. (2001). Correlation structure of international equity markets during extremely volatile periods, *Journal of Finance*, 56, 649-676.

[31] Markowitz, H. (1959). *Portfolio Selection: Efficient Diversification of Investments*. John Wiley.

[32] Memmel, C. (2003). Performance hypothesis testing with the Sharpe Ratio, *Finance Letters*, 1, 21-23.

[33] Newey, W.K., West, K.D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica*, 55, 703-708.

[34] Pelletier, D. (2006). Regime switching for dynamic correlations, *Journal of Econometrics* 131, 445-473.

[35] Philips, T.K., Rogers, G.T., Capaldi, R.E. (1996). Tactical asset allocation: 1977-1994, *Journal of Portfolio Management*, 23, 57-64.

[36] Ross, S.A. (1976). The arbitrage theory of capital asset pricing, *Journal of Economic Theory*, 13, 341-360.

[37] Sharpe, W. F. (1964). Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.

[38] Treynor, J. (1965). How to rate management of investment funds, *Harvard Business Review* 43, 63-75.

[39] Treynor, J., Black, F. (1973). How to use security analysis to improve portfolio selection, *Journal of Business* 46, 66-86.

Table 1: Estimation results of the volatility part (standard errors in parentheses).

Index	μ	γ	α	β
Ban	-0.008	0.046	0.084	0.892
	(0.028)	(0.017)	(0.014)	(0.018)
Ins	0.009	0.054	0.122	0.855
	(0.052)	(0.019)	(0.021)	(0.023)
Hol	-0.019	0.054	0.125	0.851
	(0.036)	(0.019)	(0.023)	(0.025)
Min	0.061	0.234	0.100	0.800
	(0.045)	(0.073)	(0.026)	(0.049)
Che	-0.015	0.026	0.078	0.918
	(0.049)	(0.017)	(0.015)	(0.016)
Tex	0.033	0.068	0.096	0.870
	(0.034)	(0.028)	(0.020)	(0.030)

Table 2: Estimation results for the dynamic correlation models (standard errors in parentheses) and Log-Likelihood.

			DCC				
a	b						Log-Lik
0.036	0.922						-1489.36
(0.006)	(0.018)						
			FDCC				
a_F	a_I	b_F	b_I				Log-Lik
0.305	0.276	0.556	0.298				-1416.34
(0.025)	(0.030)	(0.077)	(0.043)				
			RSDC				
					p_{00}	p_{11}	Log-Lik
					0.857	0.870	-1282.00
					(0.024)	(0.028)	
			RSFDC-F				
$a_{F,0}$	$a_{F,1}$	$a_{I,0}$			$p_{0.,0.}$	$p_{1.,1.}$	Log-Lik
0.042	0.310	0.264			0.713	0.967	-1350.61
(0.171)	(0.033)	(0.039)			(0.087)	(0.010)	
			RSFDC-C				
					$p_{0.,0.}$	$p_{1.,1.}$	Log-Lik
					0.963	0.438	-1452.95
					(0.013)	(0.116)	
			RSFDC-D				
$a_{F,0}$	$a_{F,1}$	$a_{I,0}$	$b_{F,0}$	$b_{I,0}$	$p_{0.,0.}$	$p_{1.,1.}$	Log-Lik
0.346	0.120	0.277	0.618	0.249	0.996	0.995	-1408.73
(0.025)	(0.045)	(0.039)	(0.026)	(0.019)	(0.002)	(0.005)	

Table 3: Unconditional moments and turnover index of portfolio weights for several correlation models (for each case the first row indicates the mean, the second row the variance and the third the AR(1) coefficient).

	Ban	Ins	Hol	Min	Che	Tex
CCC	0.200	0.160	0.192	0.138	0.156	0.155
	0.070	0.053	0.067	0.035	0.057	0.049
	0.175	0.194	0.169	0.347	0.172	0.224
DCC	0.198	0.163	0.182	0.136	0.165	0.157
	0.069	0.054	0.063	0.033	0.059	0.049
	0.182	0.192	0.148	0.335	0.156	0.216
FDCC	0.197	0.159	0.195	0.139	0.154	0.156
	0.068	0.052	0.069	0.035	0.055	0.048
	0.155	0.140	0.117	0.325	0.145	0.194
RSDC	0.200	0.156	0.195	0.139	0.157	0.154
	0.070	0.051	0.069	0.035	0.057	0.049
	0.177	0.194	0.172	0.351	0.172	0.226
RSFDC-F	0.151	0.131	0.145	0.180	0.206	0.187
	0.044	0.035	0.039	0.064	0.096	0.066
	0.239	0.162	0.207	0.266	0.188	0.190
RSFDC-D	0.198	0.159	0.185	0.135	0.172	0.151
	0.069	0.053	0.066	0.034	0.061	0.047
	0.162	0.190	0.137	0.359	0.143	0.213

Table 4: Descriptive statistics and performance measures of portfolio with respect to the Mibtel index for several correlation models.

	CCC	DCC	FDCC	RSDC	RSFDC-F	RSFDC-D	Mibtel
Mean	0.088	0.091	0.058	0.089	0.117	0.090	-0.058
Variance	2.048	2.064	2.140	2.044	1.966	2.011	2.340
Jensen α	0.123	0.127	0.095	0.124	0.150	0.124	
Treynor ratio	0.146	0.147	0.092	0.147	0.204	0.153	
Appraisal ratio	0.086	0.089	0.065	0.087	0.107	0.088	
Sharpe ratio	0.061	0.064	0.040	0.062	0.084	0.064	-0.038
Sharpe R. 2001	0.027	0.032	0.019	0.029	0.053	0.032	-0.058
Sharpe R. 2002	0.036	0.034	0.005	0.036	0.067	0.034	-0.067
Sharpe R. 2003	0.148	0.152	0.127	0.147	0.147	0.157	0.036

Table 5: p-values relative to statistic (4.2) to verify the equality of a pair of Sharpe ratios.

	DCC	FDCC	RSDC	RSFDC-F	RSFDC-D	Mibtel
CCC	0.480	0.298	0.495	0.287	0.480	0.008
DCC		0.279	0.485	0.302	0.500	0.006
FDCC			0.293	0.133	0.281	0.030
RSDC				0.292	0.485	0.008
RSFDC-F					0.305	0.001
RSFDC-D						0.007

Table 6: Comparison of volatilities with the Engle-Colacito approach.

	CCC	DCC	FDCC	RSDC	RSFDC-F	RSFDC-D
ER	101.955	100.229	100.000	101.188	105.392	102.159
Ban	103.654	102.497	101.887	103.007	100.000	103.268
Ins	107.365	105.815	105.383	106.977	100.000	105.950
Hol	101.897	100.537	100.000	101.322	100.291	101.685
Min	105.993	101.708	100.000	104.505	107.040	104.372
Che	100.048	100.830	100.811	101.364	102.698	100.000
Tex	102.880	101.295	100.000	102.642	100.946	102.760
Ban-Ins	100.164	100.226	100.037	100.066	100.000	100.096
Ban-Hol	100.221	100.911	100.977	102.156	100.000	100.095
Ban-Min	101.956	102.535	103.342	103.530	100.000	101.661
Ban-Che	102.388	103.260	102.992	103.494	100.000	101.997
Ban-Tex	100.873	101.182	101.594	101.131	100.000	100.616
Ins-Hol	100.141	100.955	101.109	101.810	100.777	100.000
Ins-Min	102.326	102.813	103.780	103.818	100.000	101.902
Ins-Che	101.950	103.361	103.806	103.059	100.000	101.989
Ins-Tex	102.405	103.300	103.725	103.120	100.000	102.480
Hol-Min	100.063	100.000	101.122	100.207	101.104	100.018
Hol-Che	100.966	100.000	100.075	100.715	100.340	100.950
Hol-Tex	100.208	100.719	100.503	100.000	103.251	100.062
Min-Che	100.231	100.649	102.091	100.000	100.948	100.070
Min-Tex	100.978	100.000	102.167	101.034	100.596	100.230
Che-Tex	100.318	101.415	101.144	100.668	103.734	100.000

Table 7: t-values to verify the equality of a pair of covariance matrices (Diebold-Mariano test).

	CCC	DCC	FDCC	RSDC	RSFDC-F	RSFDC-D
CCC		1.581	0.993	-2.569	4.191	5.266
DCC	-1.581		-0.742	-4.122	2.584	0.776
FDCC	-0.993	0.742		-3.306	2.850	1.269
RSDC	2.569	4.122	3.306		4.680	5.153
RSFDC-F	-4.191	-2.584	-2.850	-4.680		-2.631
RSFDC-D	-5.266	-0.776	-1.269	-5.153	2.631	

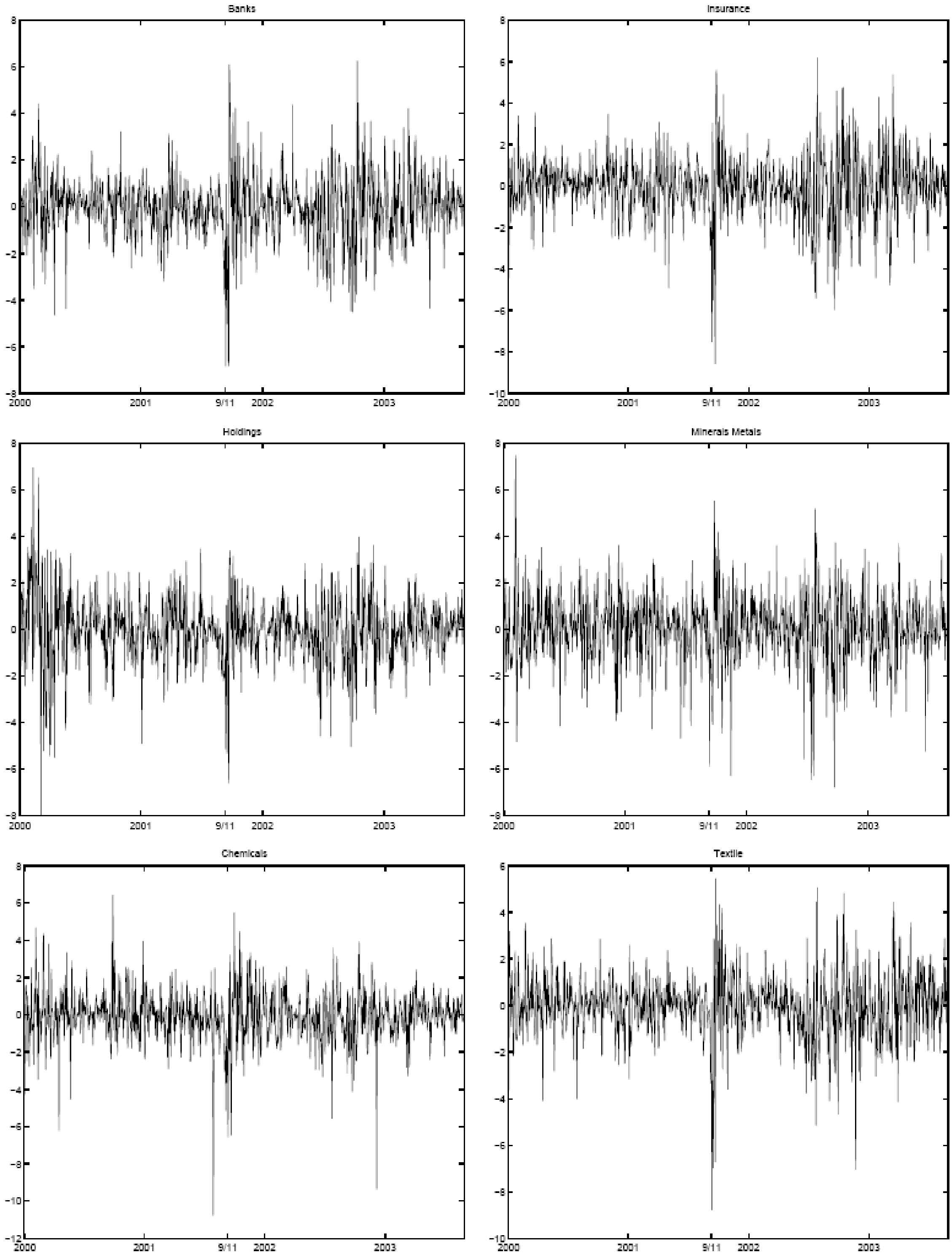

Figure 1: Returns of six sectorial indices (4 January 2001-29 August 2003).

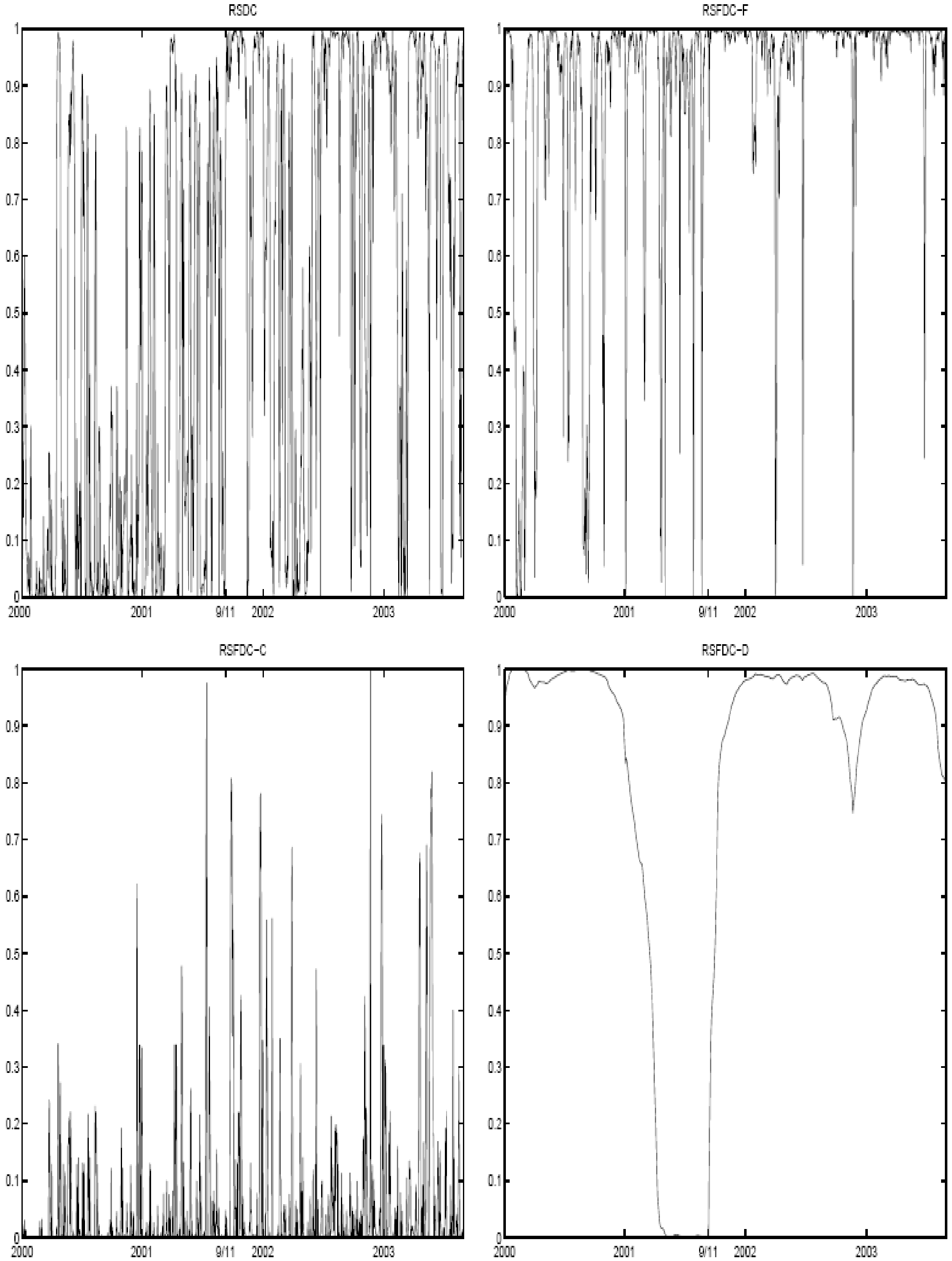

Figure 2: Smoothed Probabilities of state 1 for RSDC model and state 1· for RSFDC models.

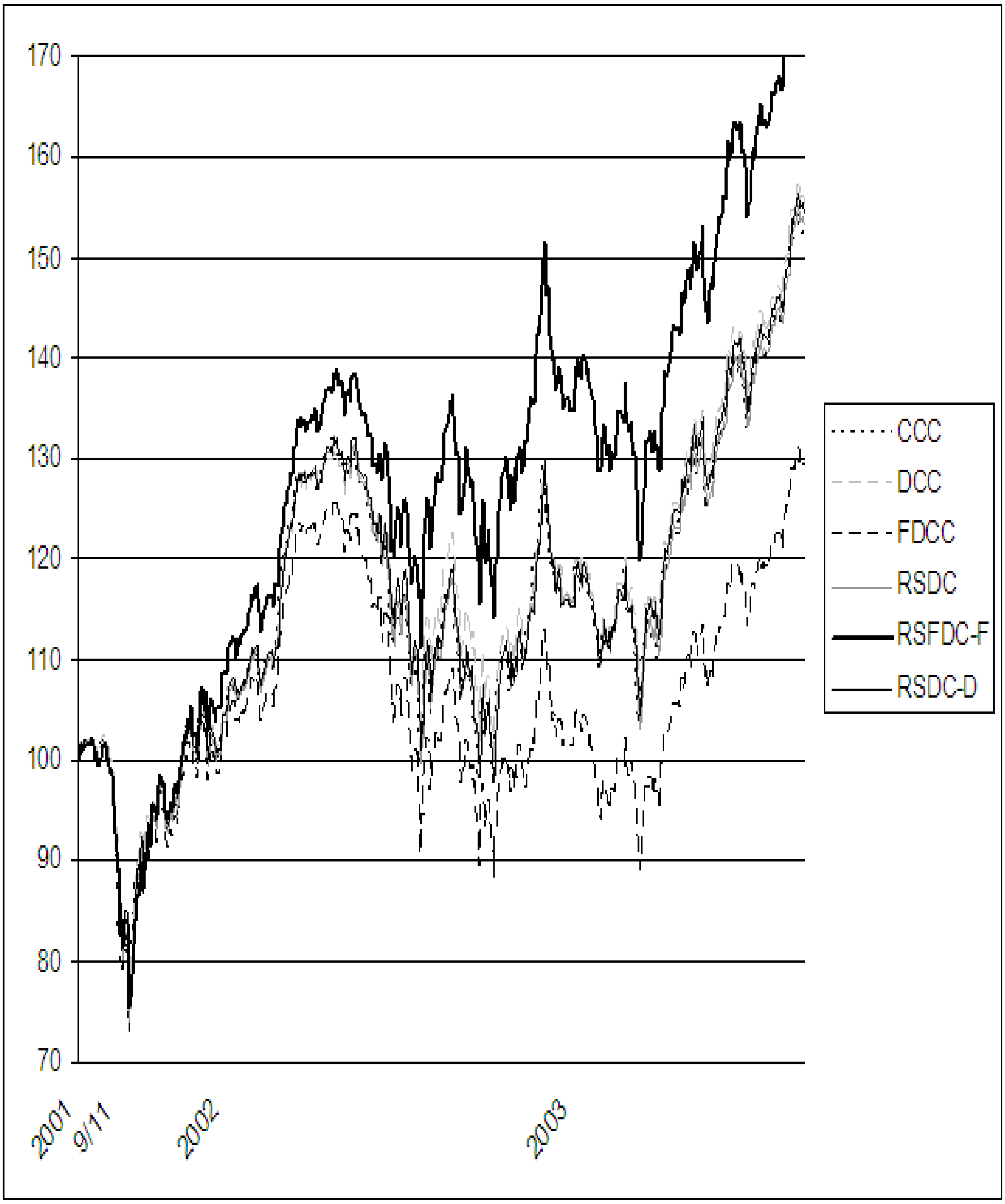

Figure 3: Portfolio value using different correlation models (starting investment=100).