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Testing asymmetry in financial time series

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Abstract

This[1] paper examines the problem of evaluating the presence of asymmetry in the marginal distribution of financial returns, by means of a suitable statistical test. After a brief description of existing tests, a bootstrap procedure is proposed. A Monte Carlo study showed that this test works properly and that, in terms of power, it is competitive with existing tests. An application to real financial time series is also presented.

Keywords: Skewness, symmetry test, financial returns, bootstrap

1 Introduction

Financial time series and their statistical modeling have been studied in depth in the last few decades, and the huge amount of work in this area has led to a quite general consensus on some empirical features known as *stylized facts*. Non-normality of financial returns, excess of kurtosis, heavy tails and clustering effects are examples of *stylized facts*. However, there are some statistical characteristics that are still disputed, both because empirical findings are not univocal and because the tools to detect them correctly are relatively recent.

One of the questionable features of financial time series is skewness of the unconditional distribution of returns [2]. Although some authors have found or assumed

[1]I would like to thank Adelchi Azzalini, Silvano Bordignon, Nunzio Cappuccio and Amado Peiró for their help and suggestions and two anonymous referees for useful comments. Thanks are due to Serena Ng who provided me with the software for the Bai and Ng (2005) test. Financial support from the Italian MIUR is also gratefully acknowledged.

[2]Note that here we do not refer to the asymmetrical effects that negative and positive returns may have on volatility, but to the symmetry of the marginal distribution of returns.

relevant asymmetries in return distributions (e.g. Kim and White, 2004; Engle and Patton, 2001; Cont, 2001; Chen *et al.*, 2001), others (e.g. Bera and Premaratne, 2001; Peiró, 2004) are more doubtful about the pervasive presence of skewness in returns and believe that, in many cases, it is due to unsuitable measurement tools. However, relatively little work has been done to detect skewness with respect to other characteristics. This is curious, considering that skewness, besides being important from a statistical point of view, is also relevant from a financial one because it may be considered as a further measure of risk. For example, Kim and White (2003) stress that, if investors prefer right-skewed portfolios, then, for equal variance, one should expect a "skew premium" to reward investors willing to invest in left-skewed portfolios. With respect to optimal portfolio allocation, Chunhachinda *et al.* (1997) showed that it may change considerably if higher-than-second moments are considered in selection. Along the same lines, Jondeau and Rockinger (2004) measured the advantages of using a strategy based on high-order moments. Other examples of the economic and financial importance of asymmetry are given by Peiró (2004).

In view of the importance attributed to symmetry in the literature, we believe it is of interest on one hand to go deeper into it and, on the other, to have available statistical tests that can correctly identify the presence of asymmetry in data.

Over the years, various measures of skewness have been proposed and studied (e.g. Kim and White, 2004; Joanes and Gill, 1998). However, most of the empirical and theoretical works regarding financial markets have used the conventional measure of skewness given by the standardized third moment:

$$S = \frac{\mu_3}{\mu_2^{3/2}}, \tag{1}$$

where μ_j is the $j-$th central moment. It is well-known (e.g. Kendall and Stuart, 1969) that the estimate $\hat{S}$ of S, obtained by replacing the corresponding sample moments in (1), under the hypothesis $S = 0$, has a Gaussian asymptotic distribution which allows symmetry to be tested. However, the variance of this distribution depends crucially on the hypotheses of gaussianity and independence of data. Many authors (e.g. Bai and Ng 2005; Premaratne and Bera 2005; Bera and Premaratne, 2001; Peiró 1999, 2004; Lupi and Ordine 2001) have noted that the assumptions of gaussianity and independence are not realistic in several contexts, including that of financial returns. Some of these authors have also shown how the variance of the asymptotic distribution of the sample skewness coefficient changes when one or more of these assumptions are relaxed.

Within this context, the aim of the present work is to examine the problem of asymmetry in financial time series, starting from a comparative analysis of various existing tests. In order to overcome some of their limitations, a bootstrap test is proposed and its performance is studied by means of Monte Carlo simulations. The tests were then applied to 72 real time series.

2 Testing for skewness

This section briefly reviews some symmetry tests proposed in the literature, based on the standardized third moment, in order to highlight their advantages and disadvantages.

When data are generated by an i.i.d. gaussian process, it is well-known (see, for example Kendall and Stuart, 1969) that, asymptotically,

$$\sqrt{\frac{n}{6}}\hat{S} \xrightarrow{d} N(0,1). \tag{2}$$

Thus, for practical purposes, we can consider the relationship $\hat{S} \sim N(0, \sqrt{\frac{6}{n}})$ for testing symmetry.

Although this limiting distribution has been widely used in different contexts, and often in the analysis of financial data, it is clear that its applicative framework cannot be generalized and that, in particular, it cannot be extended to time series. When data are autocorrelated, Lomnicki (1961) proved that, for gaussian generating processes which may be written in a moving average form such as $y_t = \theta(L)\varepsilon_t$, with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, asymptotically:

$$\sqrt{\frac{n}{6}} \left(\sum_{j=-\infty}^{\infty} \rho_j^3 \right)^{-1/2} \hat{S} \xrightarrow{d} N(0,1), \tag{3}$$

where ρ_j is the autocorrelation coefficient at lag j.

However, for non-gaussian data, and specifically for data the distribution of which is leptokurtic or platikurtic, previous results no longer hold in either dependent or independent cases. In particular, for leptokurtic distributions, the variance of the test statistics is underestimated and this leads to rejection of the null hypothesis of symmetry too often, whereas the opposite occurs for platikurtic distributions, making the test too conservative with respect to the hypothesis of symmetry.

Leaving for the moment normality, Bera and Premaratne (2001), exploiting a result of Godfrey and Orme (1991), derived the distribution of $\hat{S}$ under the hypothesis of symmetry for i.i.d. but not necessarily gaussian data. In particular, assuming the existence of moments up to the sixth, they showed that, asymptotically, $V_1^{-1/2}\hat{S} \xrightarrow{d} N(0,1)$ with:

$$V_1 = \frac{1}{n}\left(9 + \mu_6\mu_2^{-3} - 6\mu_4\mu_2^{-2}\right) \tag{4}$$

and μ_j j-th central moment. In this case, therefore, the variance of the distribution of $\hat{S}$ depends on the second, fourth and sixth moments.

In their work, Bera and Premaratne (2001) showed by Monte Carlo simulations that their test works properly for i.i.d. data but apply it to real time series without any simulation. Recently, Bai and Ng (2005) derived the limiting distribution of $\hat{S}$ in the more general case of dependent data, not necessarily gaussian, and under an arbitrary skewness coefficient S. Assuming the existence of the sixth moment and some mixing conditions which guarantee that the central limit theorem holds for the 4×1 vector series $W_t = [Y_t - \mu, (Y_t - \mu)^2 - \sigma^2, (Y_t - \mu)^3 - \mu_3, (Y_t - \mu)^4 - \mu_4]$, they found that, under the hypothesis $S = 0$, $V_2^{-1/2}\hat{S} \xrightarrow{d} N(0,1)$ with

$$V_2 = \frac{1}{n}\frac{\alpha\Gamma\alpha'}{\sigma^6}, \tag{5}$$

where $\alpha = [1, -3\sigma^2]$ and Γ is the 2×2 matrix defined as $\Gamma = \lim_{n\to\infty} nE(\bar{Z}\bar{Z}')$, with $\bar{Z}$ sample mean of

$$Z_t = \begin{bmatrix} (Y_t - \mu)^3 \\ (Y_t - \mu) \end{bmatrix}. \tag{6}$$

In this framework, the serial dependence in Y_t is explained through Γ, which represents the spectral density matrix of Z_t at frequency 0. It is not difficult to show that, in the independent case, the Bai and Ng test reduces to that of Premaratne and Bera which is, thus, a particular case of the former.

Both Bai and Ng's and Premaratne and Bera's tests have the drawback of requiring the existence of the sixth moment. This means, for example, that they cannot be applied to Student$-t$ distributions, t_ν, with $\nu \leq 6$, because clearly only moments of orders less than the degrees of freedom exist.

This fact, which is not particularly important in some contexts, becomes very important in the case of financial time series, since they have leptokurtic and heavy-tail marginal distribution and, therefore, the existence of high-order moments cannot taken for granted and should generally be verified.

Instead, in real applications, it is quite common to estimate models which do not admit the sixth moment. An example is given by a common GARCH(1,1) model with Student$-t$ innovations. Table 1 lists the results of parameter estimation of such a model for four cases in which the conditional distributions do not have the sixth moment. In addition, if we consider that the marginal distribution has higher kurtosis than the conditional one, this problem is clearly one which can influence several real financial time series. This consideration is in line with the findings of Chen (2001), who in an empirical study investigated the moment conditions of daily excess returns of twelve major stock indices and found that all the returns have finite third moments but not finite sixth moments. Other authors who showed that the existence of the sixth moment is too restrictive for economic and financial data are Jansen and de Vries (1991), Loretan and Phillips (1994) and de Lima (1997).

Series	Period	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\nu}$
Motorola	01/03/95 – 09/02/01	$1.8 * 10^{-4}$	0.055	0.925	5.4
Pepsi	01/03/95 – 09/02/01	$4.0 * 10^{-6}$	0.041	0.943	5.9
3M	10/01/99 – 01/10/04	$2.1 * 10^{-6}$	0.036	0.956	5.2
SEAT pg	22/09/98 – 10/01/04	$6.1 * 10^{-6}$	0.080	0.911	4.9

Table 1: Estimates of a GARCH(1,1) model with Student$-t$ innnovations for some real time series. Parameters α, β and ν are all significant at 5% level. Parameter ν represents degrees of freedom.

3 A bootstrap test of skewness

To bypass the problem of the existence of moments, this section proposes a bootstrap test which only requires the existence of moments up to the third. This is the minimum requirement for the asymmetry coefficient to exist. We assume that the data generating process is stationary and can be described by:

$$y_t = \mu_t + e_t \tag{7}$$

where μ_t models the evolution of the conditional mean and e_t is an uncorrelated, but not necessarily independent or homoscedastic, process. In the following, we assume also that μ_t is linear and can be well described by an ARMA process. In this context, testing the marginal symmetry of y_t is equivalent to testing the marginal symmetry of e_t.

The test we propose works particularly well for uncorrelated and possibly heteroscedastic data, in which case, up to a constant, $y_t = e_t$. When data are correlated the test can again be applied, but a filtering phase through ARMA models is required, in order to account for linear dependence in the data. Instead, the method is not appropriate for models that generate asymmetry through the mean equation, e.g., threshold autoregressive models.

For financial data this framework is very general because most of the financial return time series are uncorrelated (or almost uncorrelated) and heteroscedastic. On the whole, the algorithm can be made completely automatic.

The basic idea of the procedure is to use observed data to obtain a distribution in such a way that it is symmetric, and use it to calculate critical values. The procedure for testing the hypothesis system $H_0 : S_y = 0$, $H_1 : S_y \neq 0$ is the following:

1. Given a time series $\{y_t\}, t = 1, ..., n$, fit a suitable ARMA(p, q) model. Orders p and q can be chosen by automatic criteria, i.e. those of Akaike or Schwarz. Let e_t be the series of the residuals of the model, i.e. $e_t = y_t - \hat{\mu}_t$.

2. Calculate $\hat{S}_e$ for the series e_t.

3. Define $e_t^* =| e_t - me(e_t) |$, where $me(e_t)$ is the median of e_t and $| \cdot |$ denotes the absolute value.

4. Generate the bootstrap series

$$\tilde{e}_t = me(e_t) + e_t^* * z_t \qquad\qquad t = 1, 2, ..., n$$

where e_t^* is sampled with replacement from the empirical distribution of e^*, and z_t is such that $P(z_t = -1) = P(z_t = 1) = 1/2$. The distribution of $\tilde{e}_t$ represents a symmetrized version of that of e_t.

5. Calculate skewness coefficient $\hat{S}_{\tilde{e}}$ for series $\tilde{e}_t$.

6. Repeat steps 4) and 5) M times, with large M, yielding M bootstrap replications $\tilde{e}_t^{(i)}$ and the corresponding estimates $\hat{S}_{\tilde{e}}^{(i)}$ for $i = 1, ..., M$.

7. Consider the bootstrap distribution of $\hat{S}_{\tilde{e}}$ obatined through M estimates $\hat{S}_{\tilde{e}}^{(i)}$ and find quantiles $\hat{S}_{\tilde{e},\alpha/2}$ and $\hat{S}_{\tilde{e},1-\alpha/2}$.

8. Reject H_0 at level α if $\hat{S}_e < \hat{S}_{\tilde{e},\alpha/2}$ or $\hat{S}_e > \hat{S}_{\tilde{e},1-\alpha/2}$.

The procedure has been described in its most general form in order to face serial correlation. Model at step 1) may be estimated by the pseudo-maximum likelihood approach. Under standard regularity conditions (White, 1980; Gourieroux *et al.* 1984), the estimates are consistent even if the underlying distribution is non-gaussian.

However, note that for uncorrelated data $y_t = \mu + e_t$ and, thus, step 1) does not require the estimation of any model but at most to de-mean the data.

In step 4), data were handled as if they were independent, whereas they are generally only uncorrelated. Although bootstrap frameworks for i.i.d. data cannot usually be applied to conditionally heteroscedastic data, in this particular case it can be done because we are not interested in producing time series with the same dynamical structure as that of the original one, but in studying a feature of the marginal distribution of the series.

To understand this point let us suppose, for simplicity, that we have a time series generated by an uncorrelated, zero-mean, but not independent process. In this case, $y_t = e_t$. If all the conditions required by Bai and Ng (2005) are satisfied, then the asymptotic distribution of $\hat{S}_y$ is $N(0, V_2)$, with V_2 defined in (5). After some algebra, it is possible to show that an estimate of matrix Γ in (5) is given by:

$$\hat{\Gamma} = \begin{bmatrix} \hat{\mu}_6 + \sum_{j=1}^{n-1} \hat{\gamma}_{y^3}(j) & \hat{\mu}_4 + \sum_{j=1}^{n-1} \hat{\gamma}_{y^3,y}(j) \\ \\ \hat{\mu}_4 + \sum_{j=1}^{n-1} \hat{\gamma}_{y^3,y}(j) & \hat{\mu}_2 + \sum_{j=1}^{n-1} \hat{\gamma}_{y}(j) \end{bmatrix}$$

where $\hat{\mu}_j$ is an estimate of the $j-$th central moment, $\gamma_{y^r}(j)$ is the autocovariance of y_t^r at lag j, and $\gamma_{y^3,y}(j)$ is the cross-covariance between y_t^3 and y_t. Thus, the components of matrix Γ are $\hat{\mu}_2$, $\hat{\mu}_4$, $\hat{\mu}_6$, $\hat{\gamma}_y$ $\hat{\gamma}_{y^3}$ and $\hat{\gamma}_{y^3,y}$ and V_2 depends only on these ones. Note that V_2 does not depend on $\hat{\gamma}_{y^2}$, the covariance between y_t^2, which characterizes financial returns and conditional heteroscedastic models like, for example, GARCH models. In this case, the proposed procedure neglects only the correlation between third moments and the cross-correlation between first and third moments. When these quantities are negligible or even absent - as for example in GARCH models - the bootstrap distribution leads to a good approximation of the true distribution of $\hat{S}$, with the advantage that it is not based on asymptotic considerations. When, instead, not all the moments up to the sixth exist, the results of Bai and Ng (2005) no longer hold and the final heuristic legitimation of the bootstrap procedure lies in the simulation results of Section 4, which are intended as a sort of "proof", by simulation methods, of the conjecture that, in this particular case, resampling residuals is appropriate.

Note, however, that the proposed procedure is not directly connected to matrix Γ, which in turn is only a component of V_2; in particular, when the conditions required by Bai and Ng (2005) do not hold, it works by approximating *via* bootstrap the true and unknown distribution of the skewness coefficient.

As a final remark, we point out that the proposed bootstrap scheme is not based on pivotal quantities[3].

4 Validation

Having described the procedure for the bootstrap test, we now must validate it, and Monte Carlo simulations were used to study the real level and power of the test and to compare them with those of other tests.

For the bootstrap test, the orders of model at step 1) were chosen by minimizing the Schwarz criterion. The bootstrap distribution of $\hat{S}$ was obtained using $M = 10000$ replications; in some pilot analyses, increasing M to 25000 did not change the results in any particular way. In addition, in all simulations, bilateral tests at level $\alpha = 10\%$, 5% and 1% were carried out.

The data were generated by 20 processes (DGP), unlike the dependence structure and characteristics of marginal distributions. The 20 processes and their coefficients of asymmetry and kurtosis are listed in the Appendix and in Table 2.

For each generator process, the evaluation of the effective level and power of the test was based on 2000 Monte Carlo replications of length $n = 100$, 200 and 500. When working with financial time series, these values correspond to series of very short, short and medium lengths.

Analyses were divided into three parts: i) for independent data, comparison with

[3]This implies that critical values depend on which data generating process is used to determine the distribution under the null hypothesis. A possible way to face this problem, but not pursued here, is described in Beran (1988).

test performance based on (2), called AS (*asymptotic sample skewness*), on the Premaratne and Bera (PB), Bai and Ng (BN) and Bootstrap (BT) tests; 2) the same analyses were conducted on dependent data also including the Lomnicki (LO) test; 3) applications to real time series.

For full comparisons, all the results of the simulations refer to the application of the tests on the same time series.

4.1 Independent data

To study the effective level of the tests in the i.i.d. case, four symmetrical distributions were considered, S1, S2, S3 and S4. They are, respectively: standard normal, Student-t with seven degrees of freedom; Beta(2,2) and a distribution belonging to the Generalized Lambda family, which was also considered by Bai and Ng (2005). This family contains symmetrical and asymmetrical distributions which can be generated in terms of the inverse of the cumulative distribution function $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$, $0 < u < 1$ (see, for example, Karian and Dudewicz (2000)).

To evaluate the adequacy of the nominal (p_0) and effective (p) levels, the following hypothesis system was verified by a binomial test:

$$\begin{aligned} H_0 &: p = p_0 \\ H_1 &: p \neq p_0 \end{aligned} \tag{8}$$

with $p_0 = 0.1,\ 0.05,\ 0.01$, depending on significance level. The results (Table 3) indicate that only AS in the gaussian case gives effective levels statistically equal to the nominal ones for all three values of n. For gaussian data, the other three tests give effective levels not significantly different from the nominal ones only for $n = 500$. For $n = 200$ and 100, the real levels are lower but, on the whole, satisfactory at levels 10% and 5%, slightly less at 1%. For non-gaussian but symmetrical distributions with leptokurtosis, the AS test rejects the hypothesis of symmetry too often. The higher the kurtosis, the higher the effective level. Instead, when the distribution is platikurtic, the test is too conservative and the null hypothesis is almost never rejected. In both these situations, nominal and effective sizes are very different.

With regard to BT, PB and BN tests, the results of Table 3 show that they clearly face leptokurtosis and platikurtosis correctly and have similar effective levels. In addition, although the null hypothesis of system (8) is sometimes rejected, effective levels are comparable with nominal ones, particularly as n grows. The only exception is the 1% level, for which all three tests have much smaller effective levels.

Note also that, in the i.i.d. case, PB and BN are the same and provide almost identical results. Analyses concerning the power of the tests were conducted on five asymmetrical distributions (A1, A2,...,A5) with different degrees of kurtosis. In this case, distributions were Beta(2,1), two Generalized Lambda, Skew-Normal, and Skew-t. The last two distribution families were introduced by Azzalini (1985, 1986).

The general framework of the experiment is identical to that for study of test levels. The results of simulations are given in Table 4. As expected, power grows with series length, intensity of asymmetry and nominal test level.

For the reasons described above, the AS test always has the greatest power, and the largest differences are found between AS and the other three tests. Differences between BT, PB and BN are smaller. To assess their significance, a binomial test was applied to the test powers. In this case, the binomial test concerns couples of powers p_1 and p_2 and the examined hypothesis system was:

$$\begin{aligned} H_0 &: p_1 \leq p_2 \\ H_1 &: p_1 > p_2. \end{aligned} \tag{9}$$

where, conventionally, it was assumed that p_1 is always the larger of the two powers in question. The hypothesis system (9) was verified only for the powers of BT and BN, BP being a particular case of BN. In Table 4, the asterisk means that the null hypothesis is rejected at 5% level and the circle at 1% level. For example, if we consider the power of BT for $A5$, at a nominal level of 5% and for $n = 500$, the circle means that the power 78.8 is significantly greater, at 1% level, than the value of 71.2% reached by the BN test.

This kind of analysis shows that, in two out of the five processes, the power of BT is significantly greater than that of BN; the opposite is true only in a few isolated cases. In more detail, it indicates that, when kurtosis is very high, BT has more power. Analogous conclusions are reached with respect to the size-adjusted powers of the tests (Davidson and MacKinnon, 1999, 2006).

4.2 Dependent data

If we wish to apply the tests to financial time series, we must to study their behaviors and performances under more general assumptions. We therefore concentrate on data with some dependence structure.

As the BP test was based on the independence hypothesis, hereafter only LO, BT and BN are examined and compared. For the same reason, the results of the AS test are reported for the sake of comparison but are not discussed.

When data are serially correlated, the distribution of $\hat{S}$ changes. If autocorrelation is neglected, test performance may be seriously affected. The effect of autocorrelation is shown in Table 5, which lists the effective levels of the tests for a gaussian AR(1), with parameter ϕ. Here, the marginal distribution of the data is gaussian and thus symmetric. When the correlation structure is weak, it has no particular effects on the tests. However, when it becomes stronger, if not explained, it leads to an effective level which is definitely greater than the nominal one and thus deduces asymmetry even where there is none.

In the case of $\phi = 0.9$, reported by Bai and Ng (2005) as problematic, BT again gives quite satisfactory results and significantly better than those of BN.

Thus, in applications it is important to account for dependence and, in particular, for correlation. Conversely, it is also interesting to note that low levels of correlation do not have dramatic consequences on the test performance.

Analyses of the effective levels of dependent data were conducted on data generated from eight models with different dependence structures and degrees of kurtosis (S5, S6,...,S12). In particular, data were generated by AR(1) and ARMA(1,1) models, with gaussian and Student$-t$ innovations, which have a linear dependence structure. They also have marginal distributions which are symmetric but leptokurtic in the non-gaussian case. The family of GARCH processes was then used, because they produce uncorrelated, but not independent, data. Also, they have been extensively used in the financial literature. The marginal distributions of the considered GARCH models are symmetric and, also in the gaussian case, leptokurtic. In order to have distributions with higher kurtosis, we also considered models with Student-t innovations.

To evaluate the effective level of the tests and to compare their powers, hypothesis systems (8) and (9) were again considered within the same framework of Section 4.1. The results are shown in Tables 6, 7 and 8. As in the independent case, the hypothesis of equal nominal and effective levels in some cases is rejected. On average, the effective levels are satisfactory at nominal levels of 10% and 5%, but are much lower at the nominal level of 1%. Note that, for very high levels of kurtosis, as in S10, BT provides effective levels more similar to nominal ones.

For processes $S11$ and $S12$ the conditions required for the application of the BN test are not satisfied. In particular, for $S11$ the fourth moment does not exist, whereas $S12$ does not have finite sixth moment. Even though the BN test does not behave so baldly, the effective size is about a half that of the nominal one at 10% level and about a third at 5% level. Furthermore, for BN, there are no improvements when n grows. For the bootstrap test, instead, effective levels are much closer to the nominal ones and seem to converge with growing n.

With regard to test power, our study was based on a bilinear model and two GARCH models with A4 and A5 innovations, called A6, A7 and A8. They produce uncorrelated data with asymmetric marginal distributions and different levels of kurtosis; all of them have in common not too excessive asymmetry and quite high kurtosis. Since power also depends on asymmetry intensity, it is clear that, if very asymmetrical distributions are chosen, very high power can be reached. In this work, instead, we preferred to consider processes with not too asymmetrical marginal distributions, in order to verify performance in relatively more difficult situations. In this sense, here the main interest lies in comparing the powers of the various tests, more than the powers themselves. Table 8 shows that, in all three analyzed cases, BT has significantly more power than BN. In general, as expected, power grows with n and with level test and asymmetry.

Again, considering the size-adjusted powers (Davidson and MacKinnon, 1999, 2006) leads to the very similar conclusions about the relative performance of the tests.

5 Empirical applications

Having verified that the bootstrap test works properly and compared it with other tests, this section applies and compares the AS, BT and BN tests in 72 real daily financial time series. Again, the Premaratne and Bera test is not considered because it is a particular case of BN. Instead, the AS and LO tests higlight the differing results which may be obtained.

The time series describe the returns of 30 stocks belonging to the Dow-Jones index, 30 belonging to the MIB30 index (the Italian stock index of the most highly capitalized firms) and 12 well-known international stock indexes (Dow-Jones, S&P500, Nasdaq100, Nikkei, FTSE100, SMI, CAC40, DAX, Mibtel, MIB30, Midex, Hang-Seng). The data refer to different periods, but most of them concern the interval January 1999 – October 2004. The lengths of the series range between $n = 575$ and $n = 4982$.

Since some series clearly have outliers, these were removed and replaced with the means of the previous data. Outliers were detected by visual inspection, but all of them were at least 20 times the standard deviation of the data.

Since previous analyses had shown that none of the proposed tests works well at a level of 1%, here we consider only the usual 5% level, which seems more reliable. The comparison are only made in terms rejection or not rejection of the null hypothesis of symmetry.

As expected, AS and LO reject the hypothesis symmetry very often - in 51 cases out of 72 (for 20 MIB30 stocks, 23 Dow-Jones stocks, and 8 stock indexes). The two tests behave similarly for these time series because they are practically uncorrelated. The number of rejections for the bootstrap test and BN is much smaller: the former rejects the symmetry in 8 cases (6 MIB30 stocks, 1 Dow-Jones stock, and 1 index). Instead, BN rejects H_0 in 4 cases of MIB30 stocks, 1 Dow-Jones stock and 1 index; i.e. 6 series out of 72.

Only for 23 series out of 72 BT and AS reach the same conclusions about the presence or otherwise of asymmetry in the data. Table 9 lists some of the most representative cases.

Conversely, there is very good agreement between BT and BN - not surprisingly, as the performance of these two tests does not differ dramatically. However, there are three cases (3M, Seat Pagine Gialle, ST Microlectronics) in which the hypothesis of symmetry is rejected by BT but not by BN (Table 9). It is interesting to note that two of these three cases are precisely those considered in Section 2 as examples of time series whose distributions may not have the sixth moment (see Table 1).

6 Conclusions

This work describes some symmetry tests and connected problems when applied to financial return time series. Since one of these problems is the existence of the sixth moment, a bootstrap test requiring only the existence of the moments up to the third is proposed. The procedure leads to a test that approximates the true distribution of $\hat{S}$, the sample skewness coefficient, neglecting correlations between high-order moments. The test is very simple and intuitive and gives good results for dependent and non-gaussian data. Its performances, in terms of effective level and power, were compared by Monte Carlo simulations considering several generating processes. Results indicate that the test works well.

Regarding asymmetry in returns distribution, first of all it should be noted that results referring to AS and LO are not reliable. Analyses of BT and BN point out that skewness is not pervasive in financial returns and that, when present, it seems to be the exception more than the rule. Thus, we cannot classify marginal asymmetry of return distribution as a stylized fact.

Another practical indication emerging from this study is that all tests provide unsatisfactory results at low levels (e.g., 1%) and that, when leptokurtosis occurs, they tend to be conservative with respect to the null hypothesis. At the standard 5% level, simulations indicate that BT is slightly more powerful than BN.

Lastly, we believe that asymmetry in financial time series is a topic which should be studied in more depth, by means of both tests and models. At the same time, more accurate empirical exploration of series at different frequencies and, in paticular, at intradaily frequencies, would be appropriate.

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Appendix

Symmetric models for i.i.d. data:

S1: $N(0,1)$;

S2: t_7;

S3: $Beta(2,2)$;

S4: $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$ with $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = -0.24$ $\lambda_4 = -0.24$, $u \sim U(0,1)$.

Asymmetric models for i.i.d. data:

A1: $Beta(2,1)$;

A2: $Skew\ Normal(0,1,-2)$;

A3: $Skew\ t(0,1,-2,10)$;

A4: $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$ with $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = -0.0075$ $\lambda_4 = -0.03$, $u \sim U(0,1)$;

A5: $F^{-1}(u) = \lambda_1 + \left[u^{\lambda_3} - (1-u)^{\lambda_4}\right]/\lambda_2$ with $\lambda_1 = 0$, $\lambda_2 = -1$, $\lambda_3 = -0.1$ $\lambda_4 = -0.18$, $u \sim U(0,1)$.

Symmetric models for dependent data:

S5: $y_t = 0.7y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$;

S6: $y_t = 0.7y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim t_7$;

S7: $y_t = 0.7y_{t-1} + \varepsilon_t - 0.6\varepsilon_{t-1}, \quad \varepsilon_t \sim N(0,1)$;

S8: $y_t = 0.7y_{t-1} + \varepsilon_t - 0.6\varepsilon_{t-1}, \quad \varepsilon_t \sim t_7$;

S9: $y_t = \varepsilon_t, \quad \varepsilon_t \mid I_{t-1} \sim N(0,\sigma_t^2), \quad \sigma_t^2 = 0.2 + 0.3\ \varepsilon_{t-1}^2 + 0.6\ \sigma_{t-1}^2$;

S10: $y_t = \varepsilon_t, \quad \varepsilon_t \mid I_{t-1} \sim t_7(0,\sigma_t^2), \quad \sigma_t^2 = 0.2 + 0.3\ \varepsilon_{t-1}^2 + 0.6\ \sigma_{t-1}^2$;

S11: $y_t = \varepsilon_t, \quad \varepsilon_t \mid I_{t-1} \sim N(0,\sigma_t^2), \quad \sigma_t^2 = 0.1 + 0.9\ \varepsilon_{t-1}^2$;

S12: $y_t = \varepsilon_t, \quad \varepsilon_t \mid I_{t-1} \sim t_5(0,\sigma_t^2), \quad \sigma_t^2 = 0.1 + 0.1\ \varepsilon_{t-1}^2 + 0.8\ \sigma_{t-1}^2$.

Asymmetric models for dependent data:

A6: $y_t = 0.6\ y_{t-1}\ \varepsilon_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$;

A7: GARCH$(1,1)$ with $A4$ innovations and $\sigma_t^2 = 0.2 + 0.3\ \varepsilon_{t-1}^2 + 0.6\ \sigma_{t-1}^2$;

A8: GARCH$(1,1)$ with $A5$ innovations and $\sigma_t^2 = 0.2 + 0.3\ \varepsilon_{t-1}^2 + 0.6\ \sigma_{t-1}^2$.

DGP	S	K	DGP	S	K
S1	0.0	3	A1	-0.56	2.4
S2	0.0	5	A2	-0.67	3.3
S3	0.0	2.1	A3	-0.86	5.0
S4	0.0	37.5	A4	1.51	7.4
S5	0.0	3	A5	2.00	19.4
S6	0.0	3.7	A6	1.10	9.6
S7	0.0	3	A7	1.52	7.4
S8	0.0	3.7	A8	2.00	19.6
S9	0.0	10			
S10	0.0	141.0			
S11	0.0	230.1			
S12	0.0	412.7			

Table 2: Asymmetry (S) and kurtosis (K) coefficients of DGPs. Values of S and K are (rounded) means of 5000 coefficient estimates on series of length $n = 10000$ generated by Monte Carlo simulations.

PGD - (S,K)	AS 10%	AS 5%	AS 1%	BT 10%	BT 5%	BT 1%	PB 10%	PB 5%	PB 1%	BN 10%	BN 5%	BN 1%
$S1$ - $(0,3)$												
n=100	8.8	4.1	1.1	7.1°	2.1°	0.2°	8.5	3.8	0.4	8.8	3.5°	0.3°
n=200	9.0	4.1	0.8	9.0	3.1°	0.2°	9.8	3.6°	0.5	9.8	3.8	0.4
n=500	9.8	4.5	0.9	9.8	4.1	0.5	10.0	4.2	0.6	9.8	4.3	0.6
$S2$ - $(0,5)$												
n=100	37.7°	29.5°	18.4°	9.7	2.5°	0.1°	10.6	4.1	0.4	10.5	4.0	0.2○
n=200	42.0°	34.2°	23.4°	9.2	3.2°	0.4	9.6	3.8	0.4	9.7	3.6°	0.2°
n=500	46.3°	38.8°	27.2°	10.1	4.2	0.1°	9.9	4.3	0.2°	9.8	4.5	0.2°
$S3$ - $(0,2.1)$												
n=100	0.6°	0.1°	0.0°	8.7	3.7	0.3°	9.5	4.4	0.5	9.4	4.1	0.4
n=200	0.5°	0.1°	0.0°	8.9	3.8	0.5	9.2	4.2	1.1	9.1	4.1	0.8
n=500	0.4°	0.1°	0.0°	10.2	5.1	0.7	10.4	5.1	0.7	10.2	4.9	0.6
$S4$ - $(0,37.5)$												
n=100	68.5°	63.3°	53.5°	7.5°	2.2°	0.0°	7.8°	3.1°	0.2°	8.0°	3.0°	0.2°
n=200	76.8°	72.2°	64.8°	8.7	2.4°	0.0°	7.3°	2.2°	0.0°	7.0°	2.1°	0.0°
n=500	82.8°	79.5°	73.9°	8.8	2.9	0.2°	6.9°	2.1°	0.1°	6.9°	2.0°	0.1°

Table 3: Independent data: effective test levels based on 2000 Monte Carlo replications. The two numbers under process name are asymmetry (S) and kurtosis (K) coefficients. Asterisk (*): the null hypothesis of system (8) is rejected at 5% level; circle ($^\circ$) denotes the same conclusion at 1%.

PGD (S, K)	AS 10%	AS 5%	AS 1%	BT 10%	BT 5%	BT 1%	PB 10%	PB 5%	PB 1%	BN 10%	BN 5%	BN 1%
A1 - $(-0.56, 2.4)$												
n=100	83.6	68.7	30.8	96.9	91.9	60.2	97.2	93.8	75.2	96.9	92.8	68.3°
	(98.0)	*(95.9)*	*(88.3)*	*(98.5)*	*(95.9)*	*(88.5)*	*(97.1)*	*(94.5)*	*(81.5)*	*(97.0)*	*(94.1)*	*(79.2)*
n=200	99.4	98.0	84.8	100	99.9	98.9	100	99.9	99.3	100	99.8	99.0
	(100)	*(100)*	*(99.8)*	*(100)*	*(100)*	*(99.8)*	*(100)*	*(100)*	*(99.7)*	*(100)*	*(100)*	*(99.5)*
n=500	100	100	100	100	100	100	100	100	100	100	100	100
	(100)	*(100)*	*(100)*	*(100)*	*(100)*	*(100)*	*(100)*	*(100)*	*(100)*	*(100)*	*(100)*	*(100)*
A2 - $(-0.67, 3.3)$												
n=100	50.5	38.5	20.0	46.5	23.9	1.8	48.6	30.5	7.0	48.1	29.6°	5.8°
	(52.5)	*(40.2)*	*(19.1)*	*(52.9)*	*(40.2)*	*(19.1)*	*(49.5)*	*(34.6)*	*(13.5)*	*(48.9)*	*(32.6)*	*(12.2)*
n=200	80.8	70.7	48.4	79.7	63.5	23.9	80.3	66.2	29.1	79.9	65.1	27.1*
	(80.0)	*(71.2)*	*(47.7)*	*(80.0)*	*(71.2)*	*(47.7)*	*(79.4)*	*(67.1)*	*(37.5)*	*(78.5)*	*(66.5)*	*(36.8)*
n=500	99.2	98	92.9	99.2	97.5	85.5	99.2	97.8	87.4	99.2	97.8	86.8
	(99.1)	*(97.5)*	*(92.6)*	*(99.5)*	*(97.5)*	*(92.6)*	*(98.9)*	*(97.5)*	*(88.9)*	*(98.8)*	*(97.5)*	*(88.4)*
A3 - $(-0.86, 5.0)$												
n=100	82.1	75.1	58.7	63.1	36.2	3.5	62.3	40.8	10.7	62.4	38.8	9.4°
	(61.5)	*(41.9)*	*(14.5)*	*(62.1)*	*(41.9)*	*(14.5)*	*(64.5)*	*(49.4)*	*(21.5)*	*(64.7)*	*(48.3)*	*(21.0)*
n=200	97.0	95.0	87.6	87.8	72.1	30.2	85.1	70.5	34.0	85.5	69.8	32.2
	(86.5)	*(72.5)*	*(27.6)*	*(86.8)*	*(72.6)*	*(27.6)*	*(88.0)*	*(78.1)*	*(54.0)*	*(87.9)*	*(76.7)*	*(50.5)*
n=500	100	100	99.7	99.4	96.5	84.5	98.5	95.6	84.0	98.5	95.6	83.6
	(99.5)	*(98.6)*	*(86.4)*	*(99.6)*	*(98.9)*	*(86.5)*	*(98.4)*	*(96.3)*	*(88.4)*	*(98.4)*	*(96.4)*	*(87.5)*
A4 - $(1.51, 7.4)$												
n=100	99.4	98.6	95.0	89.0°	68.7°	13.8	83.6	67.6	30.3	83.0	64.8	26.8°
	(92.2)	*(78.7)*	*(32.8)*	*(90.2)*	*(78.4)*	*(33.0)*	*(85.8)*	*(73.8)*	*(45.1)*	*(85.1)*	*(73.1)*	*(43.5)*
n=200	100	100	100	98.3°	92.0°	62.6°	94.0	85.7	60.0	93.8	84.7	57.3
	(99.7)	*(97.5)*	*(58.5)*	*(98.7)*	*(97.1)*	*(58.3)*	*(95.7)*	*(90.5)*	*(73.4)*	*(95.5)*	*(89.85)*	73.3
n=500	100	100	100	99.9°	98.7°	93.1°	98.9	96.8	89.5	98.9	96.7	89.2
	(100)	*(100)*	*(99.5)*	*(99.8)*	*(98.4)*	*(97.9)*	*(99.2)*	*(98.2)*	*(94.0)*	*(99.2)*	*(98.1)*	*(93.9)*
A5 - $(2, 19.4)$												
n=100	88.5	85.2	78.5	44.2	21.1	1.8	44.8	26.1	5.4	44.5	24.9	4.2
	(59.5)	*(40.8)*	*(12.1)*	*(59.5)*	*(40.8)*	*(12.1)*	*(47.3)*	*(32.5)*	*(11.4)*	*(47.2)*	*(31.5)*	*(11.1)*
n=200	97.2	96.4	94.1	71.8°	51.1°	13.1	66.4	45.5	15.1	66.5	45.2	13.8
	(81.1)	*(66.5)*	*(22.7)*	*(81.1)*	*(66.9)*	*(22.8)*	*(64.9)*	*(50.0)*	*(25.9)*	*(64.3)*	*(50.5)*	*(25.5)*
n=500	99.9	99.8	99.6	91.0°	78.8°	49.2°	84.1	71.5	42.4	83.9	71.2	41.6
	(97.9)	*(94.5)*	*(42.6)*	*(97.5)*	*(94.3)*	*(42.7)*	*(85.8)*	*(77.2)*	*(51.6)*	*(85.5)*	*(77.0)*	*(51.3)*

Table 4: Independent data: test powers based on 2000 Monte Carlo replications. Process name is followed by asymmetry (S) and kurtosis (K) coefficients . Asterisk (*): the null hypothesis of the system (9) is rejected at 5% level; circle (°) denotes the same conclusion at 1%. In brackets: size-adjusted powers.

PGD	AS	AS	AS	LO	LO	LO	BT	BT	BT	PB	PB	PB	BN	BN	BN
n=200	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
$\phi = 0.1$	9.7	4.7	0.9	9.4	5.5	1.4	9.7	4.2	0.2°	10.3	4.3	0.5*	9.9	4.2	0.5*
$\phi = 0.2$	10.2	5.5	1.1	9.6	4.6	0.9	9.5	4.1	0.2°	10.4	4.2	0.4*	10.4	3.9*	0.4*
$\phi = 0.4$	11.4*	6.4°	1.5*	10.4	5.6	1.1	9.0	4.0*	0.3°	11.5*	5.6	0.9	9.2	4.2	0.4*
$\phi = 0.7$	20.4°	13.2°	4.5°	8.9*	4.2	0.9	9.5	3.7*	0.4*	23.9°	15.2°	4.8°	10.2	3.8*	0.2°
$\phi = 0.9$	35.2°	27.2°	15.2°	7.1°	3.2°	0.6	8.9	3.0°	0.2°	47.2°	37.5°	22.6°	6.5°	1.4°	0.0°

Table 5: Effective test levels for process $y_t = \phi y_{t-1} + \varepsilon_t$. Symbols: see Table 3.

PGD (S, K)	AS 10%	AS 5%	AS 1%	LO 10%	LO 5%	LO 1%	BT 10%	BT 5%	BT 1%	BN 10%	BN 5%	BN 1%
$S5$ - $(0, 3)$												
n=100	15.5°	9.6°	3.0°	7.5°	4.0°	0.8°	8.8	3.0°	0.1°	7.4°	2.2°	0.1°
n=200	20.3°	13.5°	4.2°	9.0°	4.2°	0.9°	9.7	3.6°	0.3°	10.3	3.7°	0.2°
n=500	22.1°	14.6°	5.7°	9.1°	4.6°	1.1°	9.9	4.9	0.9	9.3	3.8*	0.4*
$S6$ - $(0, 3.7)$												
n=100	27.8°	20.2°	11.3°	18.5°	11.2°	4.3°	7.0°	2.5°	0.1°	8.4*	2.5°	0.1°
n=200	32.9°	26.2°	14.2°	20.7°	13.9°	7.0°	8.9	2.9°	0.2°	9.6	3.3°	0.2°
n=500	41.1°	32.9°	20.3°	25.1°	17.5°	8.4°	9.7	3.5°	0.4*	10.4	4.7	0.4*
$S7$ - $(0, 3)$												
n=100	22.4°	14.8°	5.7°	7.2°	3.6°	0.5°	9.0	2.8°	0.1°	6.2°	1.4°	0.0°
n=200	25.1°	17.4°	9.4°	8.0°	4.1°	0.9°	9.2	3.4°	0.2°	8.6*	2.6°	0.0°
n=500	30.2°	21.9°	10.7°	9.6°	4.9°	1.1°	8.8	4.1	0.6	10.3	3.8*	0.2°
$S8$ - $(0, 3.7)$												
n=100	30.9°	22.8°	13°	13.7°	8.6°	3.6°	8.1°	2.6°	0.0°	7.0°	1.8°	0.0°
n=200	35.8°	27.6°	16.4°	16.3°	10.8°	3.77°	9.6	3.5°	0.2°	7.1°	2.3°	0.1°
n=500	42.2°	34.1°	21.7°	20.1°	13.5°	5.9°	8.9	3.7*	0.3°	9.3	3.5°	0.2°

Table 6: Dependent data: effective test levels based on 2000 Monte Carlo replications. Asymmetry (S) and kurtosis (K) coefficients follow the process name. Symbols: see Table 3.

PGD (S,K)	AS 10%	AS 5%	AS 1%	LO 10%	LO 5%	LO 1%	BT 10%	BT 5%	BT 1%	BN 10%	BN 5%	BN 1%
$S9$ - $(0,10)$												
n=100	23.5°	16.9°	9.4°	24.4°	17.8°	11.8°	7.7°	2.3°	0.2°	8.2°	2.9°	0.2°
n=200	37.5°	30.4°	19.9°	35.6°	30.8°	19.6°	10.1	3.6∘	0.4*	8.5*	3.4∘	0.4*
n=500	50.3°	43.0°	31.9°	49.7°	42.6°	31.7°	10.1	4.2	0.6°	8.0°	2.8°	0.3°
$S10$ - $(0,141)$												
n=100	47.3°	40.4°	29.9°	49.5°	42.6°	31.4°	7.1°	1.7°	0.0°	7.5°	2.4°	0.2°
n=200	63.5°	58.0°	49.2°	64.5°	60.6°	51.0°	9.6	3.6°	0.2°	7.8°	2.0°	0.1°
n=500	78.8°	75.8°	69.7°	79.1°	75.9°	69.1°	8.4*	3.5°	0.2°	5.8°	1.8°	0.1°
$S11$ - $(0,230.1)$												
n=100	58.3°	51.5°	42.4°	58.3°	51.9°	42.8°	8.5*	2.4°	0.1°	6.7°	2.2°	0.2°
n=200	71.4°	66.3°	57.3°	71.5°	66.8°	57.5°	9.5	3.0°	0.1°	4.6°	1.3°	0.04°
n=500	82.9°	80.7°	74.5°	82.1°	80.5°	74.4°	10.3	4.3	0.4*	5.1°	1.4°	0.05°
$S12$ - $(0,412.7)$												
n=100	66.6°	61.0°	52.4°	67.1°	61.7°	52.7°	6.5°	1.4°	0.1°	6.2°	2.0°	0.2°
n=200	78.7°	74.6°	66.9°	79.5°	75.0°	67.3°	8.1°	3.1°	0.2°	6.1°	1.4°	0.05°
n=500	88.5°	86.5°	81.7°	88.5°	86.4°	81.8°	9.8	4.0*	0.3°	5.6°	1.3°	0.05°

Table 7: Dependent data: effective test levels based on 2000 Monte Carlo replications. Asymmetry (S) and kurtosis (K) coefficients follow the process name. Symbols: see Table 3.

PGD $-(S,K)$	AS 10%	AS 5%	AS 1%	LO 10%	LO 5%	LO 1%	BT 10%	BT 5%	BT 1%	BN 10%	BN 5%	BN 1%
A6 - $(1.1, 9.6)$												
n=100	67.8	61.4	48.6	68.2	61.7	49.5	38.7°	17.3°	0.8	30.2	13.0	1.2
	(47.7)	*(34.1)*	*(12.1)*	*(47.3)*	*(33.3)*	*(11.7)*	*(47.5)*	*(34.1)*	*(12.1)*	*(35.2)*	*(20.0)*	*(5.6)*
n=200	89.9	86.7	77.9	88.8	85.3	75.9	67.0°	45.6°	9.8°	50.8	28.5	4.8
	(70.5)	*(53.1)*	*(17.4)*	*(70.2)*	*(53.0)*	*(16.8)*	*(70.5)*	*(53.1)*	*(17.4)*	*(56.3)*	*(39.2)*	*(12.9)*
n=500	98.8	98.3	97.1	98.5	98.5	97.3	88.5°	78.1°	48.2°	78.2	63.5	28.5
	(93.3)	*(85.5)*	*(35.9)*	*(93.2)*	*(85.4)*	*(34.9)*	*(93.3)*	*(85.5)*	*(35.9)*	*(82.8)*	*(72.3)*	*(47.5)*
A7 - $(1.5, 7.5)$												
n=100	99.2	98.5	95.0	99.5	98.5	0.9575	89.1°	66.5	16.3	83.5	64.1	24.9°
	(94.5)	*(84.9)*	*(47.5)*	*(94.3)*	*(85.1)*	*(47.6)*	*(94.5)*	*(85.0)*	*(47.9)*	*(85.3)*	*(74.1)*	*(41.4)*
n=200	100	100	99.9	100	100	100	98.0°	92.5°	64.6°	93.9	85.6	58.7
	(98.5)	*(97.4)*	*(66.5)*	*(98.5)*	*(97.4)*	*(66.7)*	*(99.5)*	*(98.4)*	*(66.8)*	*(94.5)*	*(89.0)*	*(69.8)*
n=500	100	100	100	100	100	100	99.9	98.9°	93.8°	99.2	96.9	89.6
	(100)	*(100)*	*(94.5)*	*(100)*	*(100)*	*(94.7)*	*(100)*	*(100)*	*(94.5)*	*(99.3)*	*(98.3)*	*(92.6)*
A8 - $(2, 19.6)$												
n=100	86.8	84.2	75.9	87.3	85.2	76.7	44.0*	20.6	1.1	40.1	21.6	5.2°
	(70.2)	*(55.8)*	*(24.7)*	*(70.4)*	*(55.5)*	*(24.6)*	*(71.2)*	*(56.0)*	*(24.7)*	*(45.9)*	*(33.0)*	*(13.4)*
n=200	96.9	96.2	93.9	97.9	97.1	94.9	70.1°	47.7°	12.0	60.3	40.9	12.4
	(82.4)	*(68.0)*	*(31.1)*	*(82.5)*	*(68.0)*	*(31.5)*	*(82.6)*	*(68.3)*	*(31.0)*	*(67.0)*	*(52.4)*	*(25.2)*
n=500	99.8	99.7	99.6	99.9	99.8	99.7	92.0°	81.1°	49.5°	85.5	72.9	42.1
	(94.6)	*(85.9)*	*(34.3)*	*(94.6)*	*(85.8)*	*(34.9)*	*(95.6)*	*(86.9)*	*(34.6)*	*(87.2)*	*(79.3)*	*(57.0)*

Table 8: Dependent data: test powers based on 2000 Monte Carlo replications. Asymmetry (S) and kurtosis (K) coefficients follow the process name. In brackets: size-adjusted powers. Symbols: see Table 4.

Series	$\hat{S}$	AS	BT	BN
3M	0.45	S	S	NS
Intel	-0.41	S	NS	NS
Banca Intesa	0.24	S	S	S
Capitalia	0.50	S	NS	NS
Seat Pagine Gialle	0.78	S	S	NS
ST Microelectronics	0.20	S	S	NS
Tim	0.23	S	S	S
Ftse100	-0.26	S	S	S
Nasdaq100	0.28	S	NS	NS
Mibtel	-0.46	S	NS	NS

Table 9: Results of the tests at the level of 5% on some real time series. S=Significant; NS=Not Significant.

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