# Price competition, business hours, and shopping time flexibility 

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## Price Competition, Business Hours, and Shopping Time Flexibility

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#### Abstract

\title{ Price Competition, Business Hours, and Shopping Time Flexibility* } by Oz Shy and Rune Stenbacka

We analyze differentiated retail industries where shops engage in two-stage competition with respect to opening hours and prices. We explore the effects of consumers' shopping time flexibility by comparing bi-directional consumers with forward- or backward-oriented consumers, who can either postpone or advance their shopping, but not both. We demonstrate that retailers with longer opening hours charge higher prices and that opening hour differentiation softens price competition. We calculate both symmetric and asymmetric subgame perfect equilibria in closing hours and demonstrate how the equilibrium configurations depend on the cost increases associated with extended business hours, as well as the relative densities of day and night shoppers.


Keywords: Business Hours, Delayed or Advanced Shopping, Differentiated Business Hours, Price Competition, Shopping Time Flexibility.

JEL Classification: L1, L81, M2

[^0]
## Preiswettbewerb, Öffnungszeiten und Flexibilität der Einkaufszeit

Wir analysieren unterschiedliche Einzelhandelsindustrien, in denen Läden in einem zweistufigen Wettbewerb bezüglich Öffnungszeiten und Preisen stehen. Wir erforschen die Effekte von Kundenflexibilität, indem wir völlig flexible Kunden mit vor- bzw. rückwärtsorientierten Kunden vergleichen können, welche ihren Einkauf nur vorziehen oder verschieben können, aber nicht beides. Wir zeigen, dass Einzelhändler mit längeren Öffnungszeiten höhere Preise verlangen und dass die Differenzierung der Öffnungszeiten den Preiswettbewerb abschwächt. Wir berechnen sowohl symmetrische als auch unsymmetrische teilspielperfekte Gleichgewichte für die Öffnungszeiten und zeigen, wie die Ausgestaltung der Gleichgewichte von den zusätzlichen Kosten der ausgedehnten Öffnungszeiten und der relativen Dichte von Tag- und Nachteinkäufern abhängt.

## 1. Introduction

During the past decade, issues related to shopping hour restrictions have been the subject of repeated and intense debates in many European countries. Countries like Austria, Denmark, Finland, Germany and Norway still maintain substantial restrictions on shopping hours. For example, in Germany up until very recently stores were required to close by 8 P.M. on weekdays, and by 1 or 2 P.m. on Saturdays, except in city centers where shops typically remained open until 4 P.M. There were some exceptions to these rules during the weekends prior to Christmas. Presently these regulatory boundary conditions for the retail industry have been considerably liberalized, but it is still an issue subject to much political debate to determine how far to proceed with the process of shopping hour liberalization as well as to decide whether shopping hour regulation should be a federal policy issue or not. Still in Europe, on Sundays there are typically no major retail activities at all or very limited retail operation in, for example, all the countries mentioned above. Other European countries, like Sweden and United Kingdom, have taken radical steps towards a more complete liberalization of trading hours.

In this manuscript we analyze a duopolistic differentiated retail industry where shops engage in two-stage competition with respect to opening hours and prices. We demonstrate that the retailer with the longer opening hours tends to charge a higher price in equilibrium, and also has a higher overall market share even though it has a lower market share during the period when both retailers maintain parallel operation. We then calculate the symmetric subgame perfect equilibrium in closing hours and demonstrate how the possible equilibrium configurations depend in a crucial way on the cost increases associated with extended business hours. We find that the equilibrium business hours are asymmetric for an intermediate range of costs for business hour expansions. In particular, we investigate the relationship between the emerging business hour equilibrium and the flexibility of consumers to adjust their shopping times. We carry out this research task by comparing the business hour equilibrium with bi-directional consumers, who are able to both advance and postpone their shopping times, to the equilibrium configuration generated by forward- or backward oriented consumers, who can adjust their shopping times in
only one direction, i.e., either postponing or advancing. Finally, we explore the welfare implications of competition in business hours. Our study suggests that competition does not create incentives for retailers to expand their business hours beyond social optimum. In this respect our model does not justify restrictions on shopping hours. This conclusion holds true irrespectively of the degree of flexibility of consumers to adjust their shopping to the business hours.

The existing literature on the effects of a deregulation of shopping hours has largely focused on exploring the consequences of liberalizing business hour regulations. In that respect the literature has generated ambiguous predictions. Kay and Morris (1987) present conditions under which competition in a retail market with homogenous consumers could induce opening at times when high costs would induce price increases relative to a situation with restricted shopping hours. In light of their empirical evidence they, however, conclude that deregulation of shopping hours would in practice lead to lower costs and prices in the retail sector. Tangay, Vallee and Lanoie (1995) predict that a trade hour deregulation would shift demand from small shops towards large ones and that this shift in demand makes it possible for large shops to increase prices. In empirical tests based on Canadian data they found that the Canadian deregulation of opening hours in 1990 has generated price increases at large stores that tend to maintain extensive business hours.

Also more theoretically oriented studies have presented mixed results. Clemenz (1990) shows that opening hour deregulation may lead to lower retail prices within the framework of a model with consumer search. The mechanism behind this result is that longer shopping hours facilitate more extensive search activity, which, in turn, leads to lower retail prices. In a subsequent study Clemenz (1994) investigates a homogenous market where customers are differentiated with respect to their preferred shopping times. Within such a framework he focuses on the polar cases of monopoly and perfect competition, and he shows that a monopolist will maintain business hours that exceed the socially optimal opening hours.

In contrast to all the studies mentioned above, in the present study we investigate the welfare implications of imperfect competition within the framework of a two-stage model where firms commit to opening hours in the long run, whereas they are engaged in price competition in the short run. Thus, our model concentrates on the strategic aspects of the opening hour decision.

Such a focus we share with the recent study by Inderst and Irmen (forthcoming). Inderst and Irmen demonstrate the incentives of firms to use opening hours as an instrument to achieve product differentiation, and thereby increase the market power. This is a possible prediction from our model as well. However, our study is importantly differentiated from the existing literature as it explores the relationship between the equilibrium business hour configuration and the flexibility of consumers to advance or postpone their shopping. We demonstrate that the costs of expanding shopping hours essentially determine the type of emerging business hour equilibrium, in particular whether the equilibrium is symmetric or asymmetric, and that this characterization is essentially linked to the flexibility of consumers to adjust their shopping activities.

The present study is organized as follows. Section 2 develops a model of a duopolistic retail industry where the stores compete in two dimensions: Closing hours and prices. Sections 3 and 4 explore the pricing equilibria as functions of possible closing hour configurations for bidirectional as well as for forward-or backward-oriented consumers. Section 5 characterizes the subgame perfect closing hour equilibria. In Section 6 we explore the potential implications for the regulation of business hours. Finally, we offer some concluding comments in Section 7.

## 2. A Retail Industry Model of Business Hours

Consider a duopolistic retail industry with two shops, indexed by $i=A, B$, selling a homogeneous product to heterogeneous consumers. Let $p_{A}$ denote the price charged by shop $A$, and $p_{B}$ the price charged by $B$. In line with the Hotelling model of product differentiation, firm $A$ is located at the down end of the unit interval, whereas firm $B$ is located at the upper end of the interval.

Time is indexed continuously on the unit circle. This circle could be interpreted as a single day. More generally, it portrays the time during which it is possible for people to go shopping. We divide the possible shopping period into two equal time intervals. We call the time interval between $t=0$ and $t=\frac{1}{2}$ the day period, and the period between $t=\frac{1}{2}$ and $t=1$ (which is also back to $t=0$ ) as the night period. Figure 1 illustrates how time is indexed on the unit circle.

Representing time on the unit circle is qualitatively very essential, since it formalizes the idea


Figure 1: Time and shoppers on the unit circle.
Remark: Time is measured as the arc-distance from $t=0$.
that there are important spillovers between time periods. For example, the option of consumers to postpone their shopping until early next day serves as an alternative to late night shopping and, of course, this alternative will affect the business hour decisions of a shop with long opening hours as it will limit the possibilities to exploit night-time customers through high prices.

We assume that both stores open at $t=0$. We let $\bar{t}_{i}$ denote the endogenously-determined closing time of store $i$, for $i=A, B$. Thus, $\bar{t}_{i}$ also denotes the proportion of the time during which store $i(i=A, B)$ is open.

### 2.1 Shoppers

The consumers are differentiated along two dimensions: (i) Preferred shopping time and, (ii) distance relative to stores' location. Each point in time $t(0 \leq t \leq 1)$ represents an ideal shopping time for a continuum of potential shoppers, who are further differentiated according to their location relative to stores. Thus, each consumer is represented by a coordinate $(t, x)$, where $t$ is the arc index on the unit circle, and $x \in[0,1]$ captures the customer-specific horizontal differentiation characteristics.

Within the day period, between $t=0$ and $t=\frac{1}{2}$, consumers are uniformly distributed on the half-unit circle with density $n_{1}$, so that the total number of day consumers is $n_{1} / 2$. Similarly the density of shoppers during the night hours, between $t=\frac{1}{2}$ and $t=1$, is $n_{2}$, so that the total number of night shoppers is $n_{2} / 2$. Summing up, the total number of all shoppers is $\left(n_{1}+n_{2}\right) / 2$.

Along the dimension $[0,1]$ the consumers are horizontally differentiated so that consumers with an address close to 0 tend to prefer store $A$, whereas consumers with an address close to 1 tend to prefer store $B$. Figure 2 illustrates how shoppers would be distributed if we "open" the time unit circle (in fact, a sphere) into the two-dimensional time-location space.


Figure 2: "Opened" circle: The distribution of shoppers across locations and ideal shopping time.

The utility of a consumer indexed by the pair $(t, x)$ is ${ }^{1}$

$$
U_{x, t} \stackrel{\text { def }}{=} \begin{cases}\beta-p_{A}-\lambda x & \begin{array}{l}
\text { shopping at } A ; \text { store } A \text { is open } \\
\beta-p_{A}-\lambda x-\tau \min \left\{t-\bar{t}_{A} ; 1-t\right\} \\
\beta-p_{B}-\lambda(1-x)
\end{array}  \tag{1}\\
\beta-p_{B}-\lambda(1-x)-\tau \min \left\{t-\bar{t}_{B} ; 1-t\right\} & \text { shopping at } A ; \text { store } A \text { is closed } \\
\text { shopping at } B ; \text { store } B \text { is open } \\
\text { shoping at } B ; \text { store } B \text { is closed. }\end{cases}
$$

The parameter $\beta$ measures a consumer's basic utility derived from the consumption of the retail service. The parameter $\lambda>0$, which we refer to as the location parameter, formally measures the transportation cost per unit of distance. More generally, this parameter captures the disutility

[^1]an individual experiences from consuming a retail service different from his ideal choice. The parameter $\tau>0$, which we refer to as the value-of-time parameter, measures the per-unit-oftime disutility an individual experiences when he has to adjust the shopping time if the store is closed at the ideal time for this consumer. In terms of interpretation we can think of the parameter $\tau$ as a measure of shoppers' inflexibility to adjust their business transactions to take place while shops are open. The utility function (1) highlights the decision problem faced by shoppers if their ideal time happens to be when a store is closed. The shopper can either advance the shopping to the closing hour $\bar{t}_{i}$ of store $i$, or shopping can be postponed until the store reopens at $t=0$ (same as $t=1$ ). Formally, these shoppers choose the minimal arc distance which we write as $\min \left\{t-\bar{t}_{i} ; 1-t\right\}$, for consumers indexed by $\bar{t}_{i}<t<1$.

### 2.2 Stores

Store $A$ is located at point $x=0$ and shop $B$ at point $x=1$. Each store competes with two strategic instruments: The price, $p_{i}$, and the closing hour, $\bar{t}_{i}, i=A, B$. To keep the model simple we restrict shops' decisions with respect to their closing hours $\bar{t}_{i}(i=A, B)$ according to the following assumption.

## Assumption 1

(a) The density of night shoppers does not exceed the density of day shoppers. Formally, $n_{2} \leq n_{1}$.
(b) Both stores open at $t=0$. Stores must choose whether to open full-time $\left(\bar{t}_{i}=1\right)$, or half-time $\left(\bar{t}_{i}=\frac{1}{2}\right), i=A, B$.
(c) A store's total costs of operating full-time and half-time are $k_{\text {full }}$ and $k_{\text {half }} r$ respectively, where $k_{\text {full }}>k_{\text {half }}$.

Assumption 1(a) represents an analytically convenient way to capture the basic feature of regular fluctuations between phases of high demand and low demand within the period. In Shy and Stenbacka (2004) we explore more complicated classes of consumer distribution functions by sacrificing price competition. Assumption 1(c) represents a very general description of the costs associated with nonstop and part-time opening hours, respectively. This assumption only says that
the operating costs of nonstop shopping hours exceed those associated with part-time opening hours. However, Assumption 1(c) does not postulate whether the per-hourly costs of operating the retail activity are increasing or decreasing as a function of the business hours. If $k_{\text {full }}<2 k_{\text {half }}$ the retail activity exhibits increasing returns to scale with respect to the opening hours, that is, the cost per unit of time of operation is decreasing as a function of the length of the business operation. Conversely, if $k_{\text {full }}>2 k_{\text {half }}$ the per-hourly costs of retailing increase as a function of the opening hours. This may apply to retail markets where labor market regulation mandates that the stores pay overtime compensation or compensation for "uncomfortable" working hours. This could also capture, for example, additional costs for security arrangements during "late hours."

The competition between the stores takes place within the framework of a two-stage interaction:

Stage I: Stores $A$ and $B$ commit to their closing hours $\bar{t}_{A}$ and $\bar{t}_{B}$, simultaneously.
Stage II: Stores take closing hours as given, and simultaneously set their prices $p_{A}$ and $p_{B}$.

Stages I and II are completed before the clock turns $t=0$ when both stores open.

### 2.3 Classification of shoppers

Our investigation will focus on three types of shoppers (analyzed separately).

## Definition 1

We say that shoppers are
(a) Bi-directional if they can either advance or postpone their shopping if the store is closed at their ideal shopping time.
(b) Forward-oriented if they can postpone their shopping beyond their ideal time, but cannot advance their shopping to an earlier time.
(c) Backward-oriented if they can advance their shopping time earlier than their ideal time, but cannot postpone it.

The utility function (1) actually represents bi-directional shoppers as they can either advance or postpone their shopping. This behavior implies that a shopper with an ideal time $t$ bears a time cost of $\tau \epsilon$ regardless of whether she postpones her shopping to $t+\epsilon$ or, whether she advances her shopping to an earlier time $t-\epsilon$. However, in modern economies many institutions impose limitations on individuals' ability to adjust their schedules to short business hours. For example, it may be almost impossible for a teacher to reschedule lectures, whereas critical negotiations on a substantial deal might make it very costly for businessmen to fit shopping into their schedule. These simply examples illustrate the general feature that individuals might have limited flexibility to adjust themselves to highly restricted business hours and that the disutility of advancing shopping activities may very well differ from that associated with postponing these activities. To formally capture these features in an analytically tractable way we next introduce forward- and backward-oriented consumers.

Recall that $\bar{t}_{i}$ is the closing hour of store $i, i=A, B$. Then, the utility function (1) can be modified to capture backward-oriented or forward-oriented consumers by replacing the terms $\tau \min \left\{t-\bar{t}_{i}, 1-t\right\}$ in the utility function by the backward and forward time cost functions,

$$
\begin{equation*}
B\left(t, \bar{t}_{i}\right) \stackrel{\text { def }}{=} \tau\left(t-\bar{t}_{i}\right) \quad \text { and } \quad F\left(t, \bar{t}_{i}\right) \stackrel{\text { def }}{=} \tau(1-t) \quad \text { for all } t>\bar{t}_{i} \text {, } \tag{2}
\end{equation*}
$$

respectively. Thus, if shoppers are forward-oriented, the function $F$ applies, as it indicates that the cost of postponing their shopping time is proportional to the unit time cost parameter, $\tau$, whereas the option of advancing the shopping is eliminated. In contrast, the $B$ function applies if shoppers are backward-oriented as it indicates that the option of postponing the shopping to a later time does not exist. The asymmetry between the functions $B$ and $F$ as defined in (2) follows from the requirement that both stores open at $t=0$ thereby making stores differ only in their closing hours, $\bar{t}_{A}$ and $\bar{t}_{B}$.

Of course, restrictions on consumers to adjust their shopping time to stores' opening hours could be captured in much more general ways. One option could be to capture the disutility associated with adjustments forwards with a parameter $\tau_{f}$, which could very well differ from the
corresponding parameter $\tau_{b}$ measuring the disutility associated with adjustments backwards. ${ }^{2}$ In this respect our introduction of forward- and backward oriented consumers serve merely as a first coarse approximation highlighting the potential asymmetry between the disutilities caused by advancing and postponing the transactions.

## 3. Equilibrium Prices Under Identical Closing Hours

In this section we assume that both stores close at the same time, $\bar{t}=\bar{t}_{A}=\bar{t}_{B}$, where either $\bar{t}=\frac{1}{2}$, or $\bar{t}=1$. Section 3.1 analyzes bi-directional consumers, whereas Section 3.2 focuses on forward- and backward-oriented shoppers.

### 3.1 Bi-directional shoppers

Figure 3 illustrates two possible equilibrium configurations. Under identical closing hours, all


Figure 3: Equilibrium configurations under identical closing hours with bi-directional shoppers. Left: Both open part time. Right: Both open full time.
consumers indexed by $x<\frac{1}{2}$ shop at store $A$ whereas all consumers indexed by $x>\frac{1}{2}$ shop at

[^2]store $B$. In addition, as indicated on Figure 3(left), when both stores open for half time only, shoppers indexed by $\frac{1}{2}<t<\frac{3}{4}$ advance their shopping to the stores' closing hour $\bar{t}=\frac{1}{2}$, whereas shoppers indexed by $\frac{3}{4}<t<1$ postpone their shopping to the next day's opening hour $t=0$. Clearly, as indicated in Figure 3(right), if both stores are open nonstop all buyers conduct their shopping at their ideal times.

Assume that stores $A$ and $B$ charge retail prices $p_{A}$ and $p_{B}$, respectively. From (1), we can conclude that during the time interval when stores maintain parallel operations the equation $\beta-p_{A}-\lambda x=\beta-p_{B}-\lambda(1-x)$ implicitly determines the location of a consumer who is indifferent between shopping at $A$ and $B$. Hence, the location of such an indifferent consumer is given by

$$
\begin{equation*}
\hat{x}=\frac{1}{2}+\frac{p_{B}-p_{A}}{2 \lambda}, \quad t \leq \min \left\{\bar{t}_{A}, \bar{t}_{B}\right\} . \tag{3}
\end{equation*}
$$

Clearly, the ideal time $t$ does not appear in (3) since stores are either both open or both closed, and therefore only location and prices affect consumers' decisions on where to shop. Given $\bar{t}$, each store chooses its price to solve

$$
\begin{equation*}
\max _{p_{A}} \pi_{A}=\left(p_{A}-c\right) \frac{n_{1}+n_{2}}{2} \hat{x}-k \quad \text { and } \quad \max _{p_{B}} \pi_{B}=\left(p_{B}-c\right) \frac{n_{1}+n_{2}}{2}(1-\hat{x})-k \tag{4}
\end{equation*}
$$

where $k=k_{\text {half }}$ if $\bar{t}=\frac{1}{2}$, and $k=k_{\text {full }}$ if $\bar{t}=1$. Substituting (3) into (4), the equilibrium prices and profit levels are given by

$$
\begin{equation*}
p_{A}=p_{B}=c+\lambda, \quad \pi_{A}=\pi_{B}=\frac{\lambda\left(n_{1}+n_{2}\right)}{4}-k, \quad \text { for } k \in\left\{k_{\mathrm{half}}, k_{\text {full }}\right\} \tag{5}
\end{equation*}
$$

Thus, when stores maintain parallel opening hours, their revenue is unaffected by the opening hours. Consequently,

## Proposition 1

With parallel opening hours, both stores earn a higher profit when they both restrict their opening hours to part-time compared with both operating full-time.

In light of Proposition 1 we can conclude that retailers would have a common incentive to coordinate their business hours so as to reach cost reductions through short business hours. Such business hours coordination would take place at the expense of increased inconvenience for consumers.

### 3.2 Forward- and backward-oriented shoppers

We now turn to analyzing forward- and backward-oriented shoppers (see Definition 1) under the assumption that both stores maintain identical business hours. Figure 4 illustrates two possible equilibrium configurations when both stores are open part time only. Comparing Figure 4 with


Figure 4: Equilibrium configurations under part-time operation with forward- or backward-oriented shoppers. Left: Forward-oriented shoppers. Right: Backward-oriented shoppers.

Figure 3 reveals that under symmetric operations there is no difference in shoppers' allocation between the stores. This implies that prices and profits are the same as for bi-directional shoppers. This means that the prices and profit levels given by (4) also hold for the present case.

## 4. Equilibrium Prices Under Different Closing Hours

Suppose now that store $A$ operates part-time, hence closes at $\bar{t}_{A}=\frac{1}{2}$, whereas store $B$ is open nonstop. Technically, this means that $B$ closes at $\bar{t}_{B}=1$ (and reopens immediately).

### 4.1 Bi-directional shoppers

Figure 5 illustrates two possible equilibrium configurations when shoppers are bi-directional. The


Figure 5: Asymmetric closing hours equilibrium configurations: Left: Low value of time or high transportation cost. Right: High value of time or low transportation cost (ruled out by Assumption 2).
equilibrium displayed in Figure 5(left) has some consumers of any ideal shopping time $t$ shopping at $A$. Such a configuration is possible if transportation costs (horizontal differentiation) is sufficiently important relative to value of time for those shoppers indexed around $t=\frac{3}{4}$. In contrast, Figure 5(right) displays closely-located stores (or a high value of time), where all consumers indexed near $t=\frac{3}{4}$ shop at $B$ simply because $B$ is the only store that is open.

As it turned out, the equilibrium displayed in Figure 5(right) does not have a closed-form solution. In the present analysis we will focus on the equilibrium displayed in Figure 5(left) by making the following assumption.

## Assumption 2

The value of time parameter is bounded relative to the transportation cost parameter. Formally,

$$
\tau<\frac{12 \lambda\left(n_{1}+n_{2}\right)}{3 n_{1}+2 n_{2}} .
$$

During the time interval when both stores are open, the shoppers who are indifferent between $A$ and $B$ are indexed by $\hat{x}$ already computed in (3). For the time interval when only store $B$ is open, $A$-shoppers who are indifferent between advancing their shopping to $\bar{t}_{A}=\frac{1}{2}$ and postponing their shopping to $t=1$ are implicitly defined by $\beta-p_{A}-\lambda x-\tau\left(t-\frac{1}{2}\right)=\beta-p_{A}-\lambda x-\tau(1-t)$
yielding $t=\frac{3}{4}$. Define by $\tilde{x}$ the shoppers who are indifferent between adjusting their shopping time to $A$ and shopping at $B$ at their ideal time. Formally $\tilde{x}$ solves

$$
\begin{equation*}
\beta-\lambda(1-\tilde{x})-p_{B}=\beta-\lambda \tilde{x}-p_{A}-\tau\left(\frac{3}{4}-\frac{1}{2}\right), \quad \text { yielding } \quad \tilde{x}=\frac{4\left(p_{b}-p_{A}+\lambda\right)-\tau}{8 \lambda} . \tag{6}
\end{equation*}
$$

Next, Figure 5 (left) implies that the total number of people shopping at $A$ and $B$ are given by

$$
\begin{equation*}
q_{A}=\frac{n_{1} \hat{x}}{2}+\frac{n_{2} \tilde{x}}{2}+\frac{n_{2}(\hat{x}-\tilde{x})}{4} \quad \text { and } \quad q_{B}=\left(\frac{n_{1}}{2}+\frac{n_{2}}{2}\right)(1-\hat{x})+\frac{n_{2}(\hat{x}-\tilde{x})}{4} . \tag{7}
\end{equation*}
$$

Substituting (3) and (6) into (7) yields

$$
\begin{align*}
& q_{A}=\frac{n_{1}\left(p_{B}-p_{A}+\lambda\right)}{4 \lambda}+\frac{n_{2}\left(8 p_{B}-8 p_{A}+8 \lambda-\tau\right)}{32 \lambda}  \tag{8}\\
& q_{B}=\frac{8 n_{1}\left(p_{A}-p_{B}+\lambda\right)+n_{2}\left(8 p_{A}-8 p_{B}+8 \lambda+\tau\right)}{32 \lambda}
\end{align*}
$$

Each store $i$ then chooses its price $p_{i}$ to maximize $\pi_{i}=\left(p_{i}-c\right) q_{i}$. The unique Nash equilibrium with respect to prices is given by

$$
\begin{equation*}
p_{A}^{\mathrm{bi}}=\frac{24 c\left(n_{1}+n_{2}\right)+24 n_{1} \lambda+n_{2}(24 \lambda-\tau)}{24\left(n_{1}+n_{2}\right)} \quad \text { and } \quad p_{B}^{\mathrm{bi}}=\frac{24 c\left(n_{1}+n_{2}\right)+24 n_{1} \lambda+n_{2}(24 \lambda+\tau)}{24\left(n_{1}+n_{2}\right)} \tag{9}
\end{equation*}
$$

The implied equilibrium profits are then

$$
\begin{equation*}
\pi_{A}^{\mathrm{bi}}=\frac{\left[24 n_{1} \lambda+n_{2}(24 \lambda-\tau)\right]^{2}}{2304 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {half }} \quad \text { and } \quad \pi_{B}^{\mathrm{bi}}=\frac{\left[24 n_{1} \lambda+n_{2}(24 \lambda+\tau)\right]^{2}}{2304 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {full }} \tag{10}
\end{equation*}
$$

Substituting the equilibrium prices (9) into (8) yields the number of shoppers

$$
\begin{equation*}
q_{A}^{\mathrm{bi}}=\frac{24 n_{1} \lambda+n_{2}(24 \lambda-\tau)}{96 \lambda} \quad \text { and } \quad q_{B}^{\mathrm{bi}}=\frac{24 n_{1} \lambda+n_{2}(24 \lambda+\tau)}{96 \lambda} . \tag{11}
\end{equation*}
$$

Finally, substituting the equilibrium prices (9) into (3) and (6) yields the cutoff shoppers as illustrated in Figure 5(left).

$$
\begin{equation*}
\hat{x}^{\mathrm{bi}}=\frac{12 n_{1} \lambda+n_{2}(12 \lambda+\tau)}{24 \lambda\left(n_{1}+n_{2}\right)} \quad \text { and } \quad \tilde{x}^{\mathrm{bi}}=\frac{3 n_{1}(4 \lambda-\tau)+2 n_{2}(6(\lambda-\tau)}{24 \lambda\left(n_{1}+n_{2}\right)} \tag{12}
\end{equation*}
$$

In view of Figure 5, we must verify that $\hat{x}-\tilde{x}=\tau / 8 \lambda>0$. We are now ready to summarize the findings of this section.

## Proposition 2

Suppose that store $A$ is open part-time whereas store $B$ operates nonstop. Also, assume that all shoppers are bi-directional. Then, the store with longer business hours charges a higher price $\left(p_{B}>p_{A}\right)$ and serves more customers $\left(q_{B}>q_{A}\right)$. However, during the time period when both stores are open, store $B$ has a lower market share $\left(\hat{x}>\frac{1}{2}\right)$.

Comparing the equilibrium prices (9), equilibrium sales (11), and revenues given by the first terms of (10), reveals that the asymmetry between $A$ 's and $B$ 's equilibrium values disappears when the density of night shoppers, $n_{2}$, approaches zero. Furthermore, the differences between the equilibrium prices $p_{B}-p_{A}$ as well as the equilibrium sales $q_{B}-q_{A}$ increase monotonically with $n_{2}$.

### 4.2 Forward- and backward-oriented shoppers

Suppose again that store $A$ operates part-time ( $\bar{t}_{A}=\frac{1}{2}$ ) whereas store $B$ operates full-time $\left(\bar{t}_{B}=1\right)$. Figure 6 displays two equilibrium configurations for forward- and backward-oriented shoppers. Comparing Figure 6(left) with Figure 5(left) reveals that forward-oriented $A$-shoppers located to the right and near $\bar{t}_{A}=\frac{1}{2}$ have the longest waiting time, as compared with $t=\frac{3}{4}$ for bi-directional consumers. The upward sloping part for $t>\frac{1}{2}$ means that as the time gets closer to $A$ 's reopening hour, more and more shoppers prefer to postpone their shopping to $t=1$.

The dashed lines in Figure 6 display the market division under backward-oriented consumers. In this case, $A$-shoppers located near and to the left of $t=1$ have the longest time needed to advance their shopping to $A$ 's closing hour $\bar{t}_{A}=\frac{1}{2}$.

Forward- and backward-oriented shoppers yield the same equilibrium allocation of the number of shoppers, prices and profits. ${ }^{3}$ For this reason, we sketch only the derivation of the equilibrium with forward-oriented shoppers. In addition, Assumption 2 (needed to obtain the equilibrium illustrated on Figure 6(left)) can be relaxed to $\tau<6 \lambda\left(n_{1}+n_{2}\right) /\left(3 n_{1}+2 n_{2}\right)$. Under this assumption, the indifferent shopper $\hat{x}$ is already given in (3). The shoppers indexed by $\tilde{x}$ in

[^3]

Figure 6: Equilibrium configurations under part-time operation with forward- and backward-oriented shoppers. Left: Low value of time or high transportation cost. Right: High value of time or low transportation cost (not analyzed). Remark: Solid lines (forward), dashed lines (backward).

Figure 6 (left) are found by equating the utility of a $t=\frac{1}{2}$ consumer shopping at $B$ (with no delay) with the utility of shopping at $A$ (with a delay of $1-\frac{1}{2}$ ). Formally, $\beta-\lambda(1-\tilde{x})-p_{B}=$ $\beta-\lambda x-p_{A}-\tau\left(1-\frac{1}{2}\right)$, or $\tilde{x}=\left[2\left(p_{B}-p_{A}+\lambda\right)+\tau\right] /(4 \lambda)$. The number of $A$-shoppers is then given by $q_{A}=n_{1} \hat{x} \frac{1}{2}+n_{2} \tilde{x} \frac{1}{2}+n_{2}(\hat{x}-\tilde{x}) / 4$, and $q_{B}=\left(n_{1}+n_{2}\right)(1-\hat{x}) \frac{1}{2}+n_{2}(\hat{x}-\tilde{x}) / 4$. Each store $i$ chooses $p_{i}$ to maximize $\pi_{i}=\left(p_{i}-c\right) q_{i}$ yielding equilibrium prices given by

$$
\begin{align*}
p_{A}^{\mathrm{f} / \mathrm{b}} & =\frac{12 c\left(n_{1}+n_{2}\right)+12 n_{1} \lambda+n_{2}(12 \lambda-\tau)}{12\left(n_{1}+n_{2}\right)}  \tag{13}\\
p_{B}^{\mathrm{f} / \mathrm{b}} & =\frac{12 c\left(n_{1}+n_{2}\right)+12 n_{1} \lambda+n_{2}(12 \lambda+\tau)}{12\left(n_{1}+n_{2}\right)} .
\end{align*}
$$

The equilibrium numbers of shoppers at each store are

$$
\begin{equation*}
q_{A}^{\mathrm{f} / \mathrm{b}}=\frac{12 n_{1} \lambda+n_{2}(12 \lambda-\tau)}{48 \lambda} \quad \text { and } \quad q_{B}^{\mathrm{f} / \mathrm{b}}=\frac{12 n_{1} \lambda+n_{2}(12 \lambda+\tau)}{48 \lambda} . \tag{14}
\end{equation*}
$$

Then, the equilibrium profit levels are

$$
\begin{equation*}
\pi_{A}^{\mathrm{f} / \mathrm{b}}=\frac{\left[12 n_{1} \lambda+n_{2}(12 \lambda-\tau)\right]^{2}}{576 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {half }} \quad \text { and } \quad \pi_{B}^{\mathrm{f} / \mathrm{b}}=\frac{\left[12 n_{1} \lambda+n_{2}(12 \lambda+\tau)\right]^{2}}{576 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {full }} . \tag{15}
\end{equation*}
$$

Finally, the indifferent shoppers described in Figure 6(left) are indexed by

$$
\begin{equation*}
\hat{x}^{\mathrm{f} / \mathrm{b}}=\frac{6 n_{1} \lambda+n_{2}(6 \lambda+\tau)}{12 \lambda\left(n_{1}+n_{2}\right)} \quad \text { and } \quad \tilde{x}^{\mathrm{f} / \mathrm{b}}=\frac{3 n_{1}(2 \lambda-\tau)+2 n_{2}(3 \lambda-\tau)}{12 \lambda\left(n_{1}+n_{2}\right)} . \tag{16}
\end{equation*}
$$

Proposition 2 can be easily verified for the present case (forward-oriented) by computing that $q_{B}-q_{A}=n_{2} \tau /(24 \lambda)>0$ and $p_{B}-p_{A}=n_{2} \tau /\left[6\left(n_{1}+n_{2}\right)\right]>0$, and that $\hat{x}>1 / 2$. Hence, we do not formally restate this proposition. However, it is interesting to compare how these quantity and price differences vary between bi-directional and forward/backward oriented shoppers. This comparison is summarized in Proposition 3.

## Proposition 3

(a) The store that opens full-time charges a higher price, has more customers, and earns a higher profit under forward/backward oriented shoppers compared with bi-directional shoppers.
Formally, $p_{B}^{\mathrm{f} / \mathrm{b}}>p_{B}^{\mathrm{bi}}, \quad q_{B}^{\mathrm{f} / \mathrm{b}}>q_{B}^{\mathrm{bi}} \quad$, and $\quad \pi_{B}^{\mathrm{f} / \mathrm{b}}>\pi_{B}^{\mathrm{bi}}$.
(b) The store that opens part time charges a lower price, has fewer customers, and earns lower profit under forward/backward oriented shoppers compared with bi-directional shoppers.
Formally, $p_{A}^{\mathrm{f} / \mathrm{b}}<p_{A}^{\mathrm{bi}}, \quad q_{A}^{\mathrm{f} / \mathrm{b}}<q_{A}^{\mathrm{bi}} \quad$, and $\quad \pi_{A}^{\mathrm{f} / \mathrm{b}}<\pi_{A}^{\mathrm{bi}}$.

With forward- or backward-oriented consumers, the advantage of the nonstop-operating store increases compared with bi-directional shoppers. The monopoly power of shop $B$ on the time interval $\left[\frac{1}{2}, 1\right]$ increases as the average waiting time for $A$-shoppers increases.

## 5. Equilibrium Business Hours

In this section we solve for the equilibrium business hours. In particular, we wish to characterize the conditions under which a symmetric retail industry generates an asymmetric equilibrium in business hours.

Table 1 displays stores' profit levels under part-time and full-time operations for bi-directional shoppers.

Store $B$ :

| $A$ |  | Half-time |  | Full-time |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Half |  | $\frac{\lambda\left(n_{1}+n_{2}\right)}{4}-k_{\text {half }}$ | $\frac{\lambda\left(n_{1}+n_{2}\right)}{4}-k_{\text {half }}$ |  |
|  | $\frac{\left[24 n_{1} \lambda+n_{2}(24 \lambda-\tau)\right]^{2}}{2304 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {half }}$ | $\frac{\left[24 n_{1} \lambda+n_{2}(24 \lambda+\tau)\right]^{2}}{2304 \lambda\left(21+n_{2}\right)}-k_{\text {full }}$ |  |  |  |
|  | Full | $\frac{\left[24 n_{1} \lambda+n_{2}(24 \lambda+\tau)\right]^{2}}{2304 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {full }}$ | $\frac{\left[24 n_{1} \lambda+n_{2}(24 \lambda-\tau)\right]^{2}}{2304 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {half }}$ | $\frac{\lambda\left(n_{1}+n_{2}\right)}{4}-k_{\text {full }}$ |  |
|  |  |  |  |  |  |

Table 1: Profit levels under bi-directional shoppers
We will be using the following notation: $\Delta k \stackrel{\text { def }}{=} k_{\text {full }}-k_{\text {half }}$ as well as

We can now state the following proposition (restricting our investigation to pure actions only).

## Proposition 4

Suppose that shoppers are bi-directional. If the difference between the cost of operating full-time and part-time is high $\left(\Delta k>\Delta k_{H}^{\mathrm{bi}}\right)$ then both operating part-time is a unique equilibrium. If this difference is low ( $\Delta k<\Delta k_{L}^{\text {bi }}$ ) then both stores operate nonstop. For intermediate cost differences ( $\Delta k_{L}^{\mathrm{bi}} \leq \Delta k \leq \Delta k_{H}^{\mathrm{bi}}$ ) there are two asymmetric equilibria where one store opens part-time while the other is open nonstop.

Proposition 4 is rather intuitive as it states that if operating full-time is very costly relative to operating part-time, then no store will deviate from part-time operation. The reverse logic holds for a low cost difference between full- and part-time operation, in which case both stores open nonstop. Clearly, the most interesting case is the intermediate cost difference where the extra cost of operating full-time can be borne by one and only one store that enjoys a limited monopoly power during night hours business. In these equilibria, the store that opens part-time has lower revenues and a lower cost of operation.

We next characterize the equilibria with forward- and backward-oriented shoppers. Table 2 displays stores' profit levels under part-time and full-time operations-

Similar to (17), we define

$$
\begin{equation*}
\Delta k_{L}^{\mathrm{f} / \mathrm{b}} \stackrel{\text { def }}{=} \frac{n_{2} \tau\left[24 n_{1} \lambda+n_{2}(24 \lambda-\tau)\right]}{576 \lambda\left(n_{1}+n_{2}\right)} \quad \text { and } \quad \Delta k_{H}^{\mathrm{f} / \mathrm{b}} \stackrel{\text { def }}{=} \frac{n_{2} \tau\left[24 n_{1} \lambda+n_{2}(24 \lambda+\tau)\right]}{576 \lambda\left(n_{1}+n_{2}\right)} . \tag{18}
\end{equation*}
$$

Store $B$ :

| A | Half-time |  | Full-time |  |
| :---: | :---: | :---: | :---: | :---: |
| Half | $\frac{\lambda\left(n_{1}+n_{2}\right)}{4}-k_{\text {half }}$ | $\frac{\lambda\left(n_{1}+n_{2}\right)}{4}-k_{\text {half }}$ | $\frac{\left[12 n_{1} \lambda+n_{2}(12 \lambda-\tau)\right]^{2}}{576 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {half }}$ | $\frac{\left[12 n_{1} \lambda+n_{2}(12 \lambda+\tau)\right]^{2}}{576 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {full }}$ |
| Full | $\frac{\left[12 n_{1} \lambda+n_{2}(12 \lambda+\tau)\right]^{2}}{576 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {full }}$ | $\frac{\left[12 n_{1} \lambda+n_{2}(12 \lambda-\tau)\right]^{2}}{576 \lambda\left(n_{1}+n_{2}\right)}-k_{\text {half }}$ | $\frac{\lambda\left(n_{1}+n_{2}\right)}{4}-k_{\text {full }}$ | $\frac{\lambda\left(n_{1}+n_{2}\right)}{4}-k_{\text {full }}$ |

Table 2: Profit levels under forward- and backward-oriented shoppers

## Proposition 5

Suppose that shoppers are either forward- or backward-oriented. If the difference between the cost of operating full-time and part-time is high $\left(\Delta k>\Delta k_{H}^{\mathrm{f} / \mathrm{b}}\right)$ then both operating part-time is a unique equilibrium. If this difference is low $\left(\Delta k<\Delta k_{L}^{\mathrm{f} / \mathrm{b}}\right)$ then both stores operate nonstop. Otherwise ( $\Delta k_{L}^{\mathrm{f} / \mathrm{b}} \leq \Delta k \leq \Delta k_{H}^{\mathrm{f} / \mathrm{b}}$ ) there are two equilibria where one store opens part-time while the other is opened nonstop.

The intuition behind Proposition 5 is identical to the intuition behind Proposition 4, except for the magnitudes of the cost differences that we analyze below and illustrate in Figure 7.


Figure 7: Equilibrium configurations as functions of operating cost difference: Top: Bi-directional shoppers. Bottom: Forward/backward oriented shoppers.

By directly comparing (17) with (18) we can conclude that $\Delta k_{L}^{\mathrm{bi}}<\Delta k_{L}^{\mathrm{f} / \mathrm{b}}$, thereby confirming Figure 7. This means that a retail industry facing consumers with limited shopping hour flexibility will maintain an equilibrium business hour configuration with nonstop operation for higher cost differentials of operation $\Delta k=k_{\text {full }}-k_{\text {half }}$. This captures the intuition that the strategic benefit of nonstop operation as a response to nonstop operation on behalf of the rival is increased as the
intertemporal flexibility of the consumers is reduced. Analogously, direct comparisons reveal that $\Delta k_{H}^{\mathrm{bi}}<\Delta k_{H}^{\mathrm{f} / \mathrm{b}}$, as also illustrated in Figure 7. This means that the threshold with respect to the differential in the cost of operation $\Delta k=k_{\text {full }}-k_{\text {half }}$ for the retail industry to switch to parallel part-time operation is higher when the consumers have more limited shopping hour flexibility. Again this relationship captures the intuition that the strategic benefit of nonstop operation as a response to part-time operation on behalf of the rival is higher when the consumers have lower shopping time flexibility.

The overall lesson to be learned from these comparisons is that the equilibrium configuration with respect to the business hours survives across environments where the consumers have different shopping hour flexibility. However, when the consumers have reduced shopping hour flexibility the strategic return to extended business hours increase. For that reason our theoretical prediction is that the frequency of part-time business hours is reduced when the limitations on the consumers' adjustments to restricted business hours become more severe.

## 6. Regulation of Business Hours

From an economic perspective business hour regulations should typically be imposed if there is some form of a market failure such that the equilibrium configuration represents a bias relative to the socially optimal business hours. Social welfare is defined as the sum of consumer surplus and industry profits. Within the context of our model prices are merely transfers from shoppers to stores. Therefore, the welfare properties of an equilibrium configuration with respect to business hours have to be evaluated along the following three dimensions:
(a) The social costs of adjusting shopping to the business hours. This aggregate cost, proportional to the value-of-time parameter $\tau$, measures the social costs from consumers advancing their shopping to a store's closing hour or delaying their shopping to a store's opening hour.
(b) The aggregate transportation costs. This aggregate cost, proportional to the parameter $\lambda$, measures the aggregate disutility caused by the horizontal differentiation which requires the consumers to travel to a store.
(c) The aggregate cost of operations. Business hour decisions are sensitive to the cost differential between part-time and nonstop operations $\Delta k=k_{\text {full }}-k_{\text {half }}$.

We initially evaluate the welfare implications of symmetric equilibria with parallel operations, i.e., configurations in which both duopolists operate either part-time or nonstop. For this purpose we first explore the case of bi-directional consumers. In light of Figure 3, the social costs of adjusting shopping to the business hours (Loss of Value of Time) is

$$
\begin{equation*}
L V T_{\frac{1}{2}, \frac{1}{2}}^{\mathrm{bi}}=n_{2} \tau \int_{\frac{1}{2}}^{\frac{3}{4}}\left(t-\frac{1}{2}\right) d t+n_{2} \tau \int_{\frac{3}{4}}^{1}(1-t) d t=\frac{n_{2} \tau}{16} \tag{19}
\end{equation*}
$$

if both stores are open part-time.
For forward-oriented shoppers, comparing Figure 4 with Figure 3 reveals that forward- and backward-oriented shoppers have to advance/postpone their shopping for more hours compared with bi-directional shoppers. In fact, for these consumers, the aggregate loss of time (19) becomes

$$
\begin{equation*}
L V T_{\frac{1}{2}, \frac{1}{2}}^{\mathrm{f} / \mathrm{b}}=n_{2} \tau \int_{\frac{1}{2}}^{1}\left(t-\frac{1}{2}\right) d t=\frac{n_{2} \tau}{8}=2 L V T_{\frac{1}{2}, \frac{1}{2}}^{\mathrm{bi}} \tag{20}
\end{equation*}
$$

Clearly full-day opening hours socially dominates part-time operation of both stores if and only if the extra cost associated with longer hours is lower than the aggregate time loss under part-time operations. Formally,

## Proposition 6

Suppose that shoppers are bi-directional (forward or backward oriented). Then nonstop opening hours dominate part-time opening hours from a social point of view if and only if

$$
\begin{equation*}
\Delta k \leq \Delta \hat{k}^{\mathrm{bi}} \stackrel{\text { def }}{=} \frac{L V T_{\frac{1}{2}, \frac{1}{2}}^{\mathrm{bi}}}{2}=\frac{n_{2} \tau}{32} \quad\left(\Delta k \leq \Delta \hat{k}^{\mathrm{f} / \mathrm{b}} \stackrel{\text { def }}{=} \frac{L V T_{\frac{1}{2}, \frac{1}{2}}^{\mathrm{f} / \mathrm{b}}}{2}=\frac{n_{2} \tau}{16}\right) \tag{21}
\end{equation*}
$$

Thus, from Proposition 6 we can conclude that society is more likely to be better off with nonstop operation rather than part-time operation the (a) higher is the value-of-time parameter $\tau$, (b) higher is $n_{2}$ (i.e., a higher shopper density during the night) and (c) lower is $\Delta k=k_{\text {full }}-k_{\text {half }}$.

We next evaluate whether there are economic reasons to restrict business hours. In other words, can it happen that the duopolistic industry would shift to nonstop operations under circumstances when part-time operation would be socially optimal? By comparing (17) and (18) with (21), respectively, we can conclude that $\Delta \hat{k}^{\text {bi }}>\Delta \hat{k}_{H}^{\text {bi }}$ and $\Delta \hat{k}^{\mathrm{f} / \mathrm{b}}>\Delta \hat{k}_{H}^{\mathrm{f} / \mathrm{b}}$. Consequently, nonstop operation is socially optimal in all those circumstances where the duopolistic business hour equilibrium induces nonstop operation. We formulate this conclusion in

## Proposition 7

With parallel opening hours the duopolistic equilibrium always induces underprovision of business hours.

From Proposition 7 we can immediately conclude that policies that restrict the maximum business hours cannot be justified by reference to our welfare analysis. Such a conclusion seems intuitive, because the source of the market failure is that the duopolistic industry does not internalize the social costs borne by those consumers, who have to adjust their shopping so as to take place during the business hours.

Our welfare analysis so far was restricted to the configurations with parallel opening hours. As shown in Section 5 the business hour equilibrium will exhibit asymmetric closing hours for an intermediate interval of the cost differential between part-time and nonstop operation. Overall, the formal computation of social welfare when stores maintain different business hours is very tedious and might not therefore be very useful for comparisons with equilibrium outcomes for the purpose of identifying market failures. Any welfare evaluation of asymmetric business hours will easily lead to fairly involved comparisons with respect to the consequences in three dimensions: (a) the social costs of adjusting shopping to the business hours, (b) the aggregate transportation costs and (c) the costs of operations. In fact, Figures 5 and 6 demonstrate that any equilibrium with unequal closing hours is associated with higher aggregate transportation costs than any equilibrium with parallel operations. Thus, the welfare evaluations lead to comparisons where the consequences for the social transportation costs have to be traded off relative to the social costs of adjusting shopping to the business hours.

## 7. Conclusion

In this manuscript we analyzed a duopolistic differentiated retail industry where shops engage in two-stage competition with respect to business hours and prices. In the price equilibrium retailers with longer opening hours charge higher prices, but they nevertheless have a higher overall market share when evaluated over the whole time period. We characterized the symmetric subgame perfect equilibrium in closing hours and demonstrated that the cost increases associated with extended business hours determine the possible equilibrium configurations. For example, we demonstrated that the equilibrium business hours are asymmetric for an intermediate range of costs for business hour expansions.

We focused particularly on the relationship between the emerging business hour equilibrium and the flexibility of consumers to adjust their shopping times. We did this by comparing the business hour equilibrium when consumers are bi-directional to the equilibrium configuration generated by forward- or backward oriented consumers. Overall, we found a negative relationship between consumers' shopping hour flexibility and the strategic returns to retailers from extended business hours. Lastly, we conducted a welfare analysis to explore whether there is a need to impose restrictions on the maximal number of business hours. We established that duopolistic competition induces underprovision of business hours. In this respect our model does not lend support for restrictions on shopping hours and this conclusion holds true irrespectively of the degree of flexibility of consumers to adjust their shopping to the business hours.

Our analysis and the potential limitations imposed by the stylized model invite generalizations and extensions along several dimensions. We explored the effects of customers' shopping hour flexibility by comparing bi-directional consumers with forward- or backward-oriented consumers. An analytically more complete way of analyzing these effects would be to consider a whole range of possible values for the value-of-time parameter $\tau$. A more detailed analysis in this respect might be particularly interesting for predicting the plausible consequences for business hours of future anticipated changes in the organization of work with extended possibilities for out-of-the-office working arrangements.

Our analysis was restricted to business hour arrangements involving either nonstop or halftime operations. In this respect the model could be extended to capture more general forms of part-time operations, where the part-time option could capture any proportion of the time period. Such an extension could be particularly interesting if one wants to further explore the nature of the way in which opening hour differentiation softens price competition.

Finally, our way of specifying ideal shopping times might not capture all central aspects of opening hour competition. Namely, extensive opening hours might incorporate a real options value by creating flexibility in the eyes of consumers, who are ex ante uncertain about precisely when they want to shop. Inclusion of such aspects of flexibility seems to obviously reinforce our policy conclusions, but it would be interesting to explore the effect on the market provision of business hours.

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[^0]:    * We are thankful for constructive comments by Pierre Régibeau and Paul Heidhues. Financial support from The Yrjö Jahnsson Foundation and The Hanken Foundation is gratefully acknowledged. Part of this research was conducted while Rune Stenbacka was visiting WZB, whose hospitality is gratefully acknowledged.
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[^1]:    ${ }^{1}$ To simplify our calculations we rule out a reservation utility, which means that all consumers must go shopping in one of the stores.

[^2]:    ${ }^{2}$ Direction-dependent differentiation parameters were proposed earlier in Shy (1996, Ex. 3 on p.165).

[^3]:    ${ }^{3}$ Intuitively, this feature is graphically captured in Figure 6(left) by the fact that the solid and dashed lines are mirror images of each other on the time interval $\frac{1}{2} \leq t \leq 1$.

