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ABSTRACT

New Product Introduction by Incumbent Firms

by Ralph Siebert[*]

In this study we analyze the incentives of incumbent firms to introduce a new product in different quality areas and investigate the variety of products offered in the market. We consider a duopoly where initially each firm offers one product of different quality, and assume that the production technology of one of the firms (the “potential innovator“) changes which allows the introduction of a new product. The innovator also has the opportunity to keep or withdraw the original product from the market. We find that innovation occurs depending on the production costs for quality and the firms‘ original product qualities. The innovator always introduces a new product with higher quality in order to concentrate sales on high income consumers. Moreover, the innovator is better off to withdraw its original product in order to reduce price competition and to avoid cannibalizing its own product demand. As a result, only two products remain in the market.

ZUSAMMENFASSUNG

Neue Produkteinführung etablierter Unternehmen

In dieser Studie untersuchen wir die Anreize etablierter Unternehmen, neue Produkte mit unterschiedlicher Qualität in den Markt einzuführen. Wir betrachten ein Duopol, in dem jedes Unternehmen ein Produkt mit unterschiedlicher Qualität anbietet und nehmen an, daß ein technologischer Fortschritt die Produktionstechnologie eines Unternehmens (der potentielle Innovator) beeinflußt, um ein neues Produkt in den Markt einzuführen. Der Innovator hat die Möglichkeit das alte Produkt vom Markt abzuziehen. Wir zeigen, daß die Entscheidung, ein neues Produkt in den Markt einzuführen, von den Produktionskosten für Qualität, und den vorigen Produktqualitäten abhängt. Der Innovator führt ein neues Produkt mit höherer Qualität ein und zieht das alte Produkt aus dem Markt, um einen höheren Preiswettbewerb und einen Nachfrageverdrängungseffekt zu vermeiden. Folglich werden nur zwei Produkte im Markt angeboten.

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1 Introduction

Many industries are characterized by oligopolistic competition in vertically differentiated markets. When innovation occurs we often observe that incumbents introduce new products of higher quality. One example is the electronics industry, especially the Personal Computer and the mobile phone market where technological progress motivates product innovation. For instance, new PCs enter the market with faster processors or new mobile phones with longer 'stand by time' are introduced. Furthermore, we observe that original products are frequently withdrawn from the market after innovation occurs.

Why do firms introduce a new product of higher quality and why are firms better off to withdraw their original product from the market?

Up to this point we do not find any explanation in the existing literature why incumbent firms often introduce a new product of higher quality and why firms are better off to withdraw their original product from the market.

In order to provide some insight we analyze the following setting: Suppose that there are two firms, one offers a product with low quality (the low quality firm) whereas the other firm offers a product with high quality (the high quality firm). The products are produced at the same marginal costs. Firms set prices in the product market and no entry occurs. Suppose that one firm (the potential innovator) benefits by an unexpected technological progress which enables the firm to produce a new product. Note that we allow either of the firms to be an innovator. The new product is allowed to be lower or higher in quality, but a higher product quality requires higher R&D investments. Beyond that, the innovator is allowed to keep or withdraw the original product when innovation occurs.

Introducing a new product may attract new consumers and increases the firm's profit. But in which quality area should the firm locate its new product? We can distinguish between three quality areas where the innovator may introduce its new product: a *low quality area* (new product quality is smaller than the original low quality product), an *intermediate quality area* (new product quality is located between both existing products), and a *high quality area* (new product quality is higher than the original high quality product). At first glance, we expect that when R&D costs are slightly increasing in quality, offering a new product in the high quality area might be more profitable. Whereas, when quality costs are rapidly increasing, offering a new product in the intermediate quality area might be better.

Suppose that the R&D costs are sufficiently low, such that the high quality area is more attractive for the innovator. The relevant literature for this innovation scenario includes Shaked and Sutton (1982), who analyze a model of vertical product differentiation.[1] They show that a higher quality yields higher revenues

[1] In models characterized by vertical product differentiation all consumers are supposed to

in a vertical differentiation setting. For this reason, we expect that the innovator has incentives to introduce a new product in the high quality area when the R&D production costs for quality is sufficiently low. As mentioned above, the innovator has the choice to withdraw or to keep the original product in the market. There are only few studies which focus on firms which may withdraw their original product. All of them are using locational models in contrast to our setting which is a vertical differentiation model.[2] An early study by Schmalensee (1978) shows that a multiproduct firm has the opportunity to proliferate the product space in order to prevent the rival (entrant) from introducing a new product. Judd (1985) shows that the proliferation strategy by the multiproduct firm may not be credible, once the firm is allowed to withdraw the original product from the market. He analyzes the decision of a firm to either keep the original product close to the rival in the market or to withdraw it. The firm is confronted by a trade-off: on the one hand, it would like to keep the original product in the market which increases sales but decreases prices. On the other hand, withdrawing the first product increases prices of its existing products but decreases sales. Judd (1985) shows that a firm better withdraws its products close to the rival since it softens price competition towards the rival's product which also affects its original product price. As a result, the firm yields higher profits despite a smaller variety of goods. According to this study we may expect the innovator in our setting to withdraw the original product which is next to the rival's product, since it softens price competition towards the rival's product.

Recall, the above scenario assumes R&D costs to be sufficiently low that innovation takes place in the high quality area. However, when R&D costs are relatively high the innovator may introduce a new product in the intermediate quality area. The innovator's decision to keep or withdraw the original product from the market is not as intuitive as before because now the innovator's original product is closest to its own new product and not to the rival's product.[3] Let us consider the decision of the high quality firm. As we know from Shaked and

have identical taste and rank qualities in the same order. But consumers differ in their income. Gabsewicz and Thisse (1979 and 1980) and Shaked and Sutton (1982) are the first studies which focused on vertical product differentiation. Choi and Shin (1992) modify the vertical differentiation model by Shaked and Sutton (1982) assuming that the market is not covered. Hence, some consumers in the lower quality area do not buy any product.

[2]In locational models consumers' preferences are distributed over a spectrum of products where each consumer chooses the closest product. Dixit and Stiglitz (1977), Salop (1979), and Brander and Eaton (1984) are some pioneering studies on horizontal differentiation. Shaked and Sutton (1983) show that in vertical product differentiation models an upper bound of firms exists, in contrast to the horizontal models where the market can support an arbitrarily large number of firms. For more details on the distinction between horizontal and vertical product differentiation, see Chaumpsaur and Rochet (1989), Constantatos and Perrakis (1997), and Cremer and Thisse (1992).

[3]Chaumpsaur and Rochet (1989) analyze firms' incentives to offer different intervals of product qualities. But in contrast to Chaumpsaur and Rochet (1989) our model focuses on pure vertical differentiation and firms are allowed to withdraw former products.

Sutton (1982) the high quality firm earns higher profits with the original product because it is of higher quality than the new product. Consequently, we may expect the high quality firm to keep the original product in the market when it offers a new product in the intermediate quality area. But it is still unclear where the high quality firm locates its new product. The high quality firm has the choice to locate the new product quality close to the rival's product quality or close to its own product quality. According to the model from Shaked and Sutton (1982) two countervailing effects, the *demand* and the *strategic effect,* explain its decision on product quality. The *demand effect* indicates that more consumers are captured from the low quality firm's product the more the new product quality approaches the product quality of the low quality provider. The *strategic effect* indicates that introducing a product closer to the low quality firm's quality increases price competition in the market. The principle of 'maximal product differentiation' by Shaked and Sutton (1982) describes that the benefit to firms by moving product qualities apart from each other in order to soften price competition (*strategic effect*) outweighs the market share gained by moving qualities closer to each other (*demand effect*). Their results show that firms engage in 'maximal product differentiation' where one firm offers the highest feasible product quality and the other firm offers the lowest.

According to this principle, we may expect that innovation by the high quality firm takes place close to its original product quality where the effect of softening price competition (*strategic effect*) outweighs the effect of gaining the market share (*demand effect*). However, the high quality firm also has to account for the impact on its original product when it keeps the original product in the market, e.g. the *cannibalization effect.*[4] The *cannibalization effect* indicates that more consumers are captured from the high quality firm's original product the more the new product quality approaches its original product quality. The high quality firm's decision to offer a new product in the intermediate quality area is then determined by the following trade-off. Introducing a new product close to its own product quality softens price competition in the market which increases product prices (*strategic effect).* However, the new product attracts only few consumers from the low quality firm's product (*demand effect)* but many consumers from its own original product (*cannibalization effect).* On the other hand, introducing a new product similar to the low quality firm's product lowers product prices (*strategic effect),* but attracts many consumers from the rival's product (*demand effect)* and *cannibalizes* its own product demand only to a low extent.

Combining the principle of 'maximal product differentiation' with the *cannibalization effect*, we find that the high quality firm's incentives to offer a new product close to the original product are reduced in order to avoid *cannibalizing*

[4]Since firms set prices in the product market, the innovator will internalize the strategic effect towards the former product. However, the innovator will impact its former product demand when it offers a new product in the intermediate quality area.

its own high quality product demand.

Turning to the decision of the low quality firm when it offers a new product in the intermediate quality area, we would expect the low quality firm to withdraw the original low quality product from the market since it offers a new product of higher quality, which yields higher profits. Withdrawing the original product reduces price competition in the market (*strategic effect*) and gains customers which were buying the original product and switch to buy the new product (*cannibalization effect*).

Finally, the innovator has the choice to offer a new product in the low quality area. According to Judd (1985) we may expect the innovator to withdraw the original product (which is close to the rival's product in this scenario) from the market in order to reduce price competition in the market. But when the innovator withdraws the original product it finally offers a new product of lower quality which yields lower profits in a vertical differentiation setting. Consequently, we may expect the innovator to keep the original product in the market. However, in case the low quality firm stays in the market it may loose profits since its new product (with lower quality) *cannibalizes* demand of its original product. For the high quality firm it is not obvious if *cannibalization* occurs because its products are not located next to each other and it is unclear whether the *cannibalization effect* impacts only neighboring products, or not.

In the following analysis we will investigate the different innovation scenarios, in more detail. This study presents a first insight into the innovation incentives of incumbent firms to introduce new products in vertically differentiated markets. The aim of this study is to explain firms' incentives for introducing new products in different quality areas whereby firms have the choice of either keeping or withdrawing the original product from the market.[5] We analyze in which quality areas innovation occurs and derive the variety of products that firms offer in the market. By decomposing the total derivatives of firms' profits into several components we make the model computationally tractable.

We find four types of equilibria depending on who the innovator is, on the production costs for quality, and on the original product qualities. Assuming innovation occurs in equilibrium, all equilibria are characterized by two facts: innovators always introduce a new product of higher quality, and innovators always withdraw their original product from the market. The equilibria are as follows.

1) When the high quality firm is the innovator, it introduces a new product in the high quality area if the production costs for quality is small and the original product qualities are small. The high quality firm withdraws the original product from the market after innovation occurred (case a).

[5] Rosenkranz (1996) assumes that firms always withdraw their first product from the market when they introduce a new product into the market.

2) When the low quality firm is the innovator, it introduces a new product in the high quality area if the quality costs are very small, the high quality firm's product quality is small, and the own original product quality is small, but relatively higher than in the intermediate innovation case. The low quality firm withdraws the original product from the market after innovation occurred (case d).

3) When the low quality firm is the innovator, it introduces a new product in the intermediate quality area if the production costs for quality is small, its own original product quality is very small, and the high quality firm's product quality is large. The low quality firm withdraws the original product from the market after innovation occurred (case e).

4) No innovation occurs, if the production costs for quality and the low quality firm's original product quality are high.

The remainder of this paper is organized as follows. Section 2 describes our model of vertical product differentiation and analyzes firms' innovation incentives. We conclude in Section 3.

2 The Model

Let us consider an outset in which two firms ($i = 1, 2$) offer one product with quality $s_1^0 \leqslant \frac{4}{7}s_2^0$ in the market.[6] Thus, firm 1 is the low quality and firm 2 the high quality firm. We model a two-stage duopoly game. One firm, chosen by nature, benefits from a technological progress which improves its production technology and enables the firm to introduce a new product into the market. We distinguish between two scenarios depending on which firm is subject to technological progress: the high quality firm may introduce a new product, and the low quality firm may introduce a new product.

In the first stage, the potential innovator (firm i) decides whether to introduce a new product and whether to withdraw its original product. When the firm introduces a new product it chooses its new product quality $s_i^1 \in [0, \infty]$. The new product quality is allowed to be lower or higher than the original product quality. We can distinguish between three quality areas which depend on where the innovator locates its new product quality: a low quality area, $s_i^1 < s_1^0$, an

[6]The outset is based on the model by Choi and Shin (1992) which is a modification of Shaked and Sutton (1982) where the version of Tirole (1992) is used. The results are shown in Appendix 1. The subscript refers to the firm, whereas the superscript '0' indicates the outset. Note that the assumption on product qualities is relatively unrestricted. The only restriction is given by $s_1^0 \leqslant \frac{4}{7}s_2^0$, for the following reason: If the low quality provider offers a higher quality than $\frac{4}{7}s_2^0$, it could earn higher profits by simply decreasing its product quality, see Appendix 1, equation (33).

intermediate quality area, $s_1^0 < s_i^1 < s_2^0$, and a high quality area, $s_i^1 > s_2^0$. The innovator has to invest in R&D when it produces a higher quality than its original product but does not have to invest in R&D when it offers a new product with lower quality. The quality costs for the innovating firm i, which already offers product s_i^0, is given by the following costs function

$$F\left(s_i^0, s_i^1(\gamma), \gamma\right) = \begin{cases} 0 \text{ for } s_i^1 \leqq s_i^0 \\ \gamma\left(s_i^1 - s_i^0\right)^2 \text{ for } s_i^1 > s_i^0 \end{cases}$$

where $\frac{\partial F\left(s_i^0, s_i^1(\gamma), \gamma\right)}{\partial s_i^1} > 0$ and $\frac{\partial^2 F\left(s_i^0, s_i^1(\gamma), \gamma\right)}{\partial s_i^{1^2}} > 0$, for $s_i^1 > s_i^0$. The parameter γ describes the convexity of the costs curve, or how costly it is for the firm to produce quality. After choosing the new product quality the innovator decides whether to keep or withdraw its first product from the market. In terms of the number of products the following cases may occur: the innovator keeps the first product in the market and three products are offered, the innovator withdraws the first product and two products are offered, or no innovation occurs and the original two products are offered in the market.

Tables 2.1 and 2.2 show all the different cases when innovation occurs. In order to get a better understanding of the different cases, we use the following notation: the number refers to the firm which offers the product. The products are ranked in increasing quality order, that is, a number at the bottom indicates the lowest product quality and a number at the top the highest. Bold numbers indicate the new product of each firm and a number in brackets indicates the option to either keep or withdraw the original product from the market. When no innovation occurs the outcome is shown by the outset.

New Product Introduction by the High Quality Firm

a	b	c
2	(2)	(2)
(2)	**2**	1
1	1	**2**

Table 2.1: The innovation cases when the high quality firm is the innovator

New Product Introduction by the Low Quality Firm

d	**e**	**f**
1	2	2
2	**1**	(1)
(1)	(1)	**1**

Table 2.2: The innovation cases when the low quality firm is the innovator

In the second stage, firms maximize profits by simultaneously choosing prices in the product market having observed the product qualities and the number of products in the market. When the innovator keeps its first product in the market it is allowed to internalize price competition among its own products. More precisely, it takes into account that a price change of one of its products has an impact on its other product. No entry is assumed to occur. Production costs do not depend on quality and are set to 0.

Consumers' preferences are given by $U = \theta s - p$ if they buy a good and zero otherwise. Each consumer has the same ranking of qualities and prefers higher quality for a given price (p). Consumers differ in their income. Their income parameter θ is uniformly distributed over the interval $[0, 1]$. The assumption on the income parameter induces that the market is not covered, which means that some consumers do not buy any one of these products. Every consumer is allowed to buy at most one of the products.

We look for pure strategies and solve the game by applying backward induction. We begin with investigating the product market competition (stage 2) and derive prices, demand, and profits for the different innovation cases. Next we investigate the choice to introduce a new product and analyze the decision on product quality and the number of products (stage 1).

2.1 Product Market Competition - Stage 2

In this section prices, demand, and profits in the product market for the different innovation cases are derived. We first examine case a from Table 2.1, where the high quality firm may introduce a new product in the high quality area. The cases b to f are analogous to case a. We present the results for these cases in the Appendix. Moreover, we focus on the case where the original product is kept in the market. When the innovator withdraws its original product each firm offers one product and results are again analogous (see Appendix 1, adjusted for the corresponding product qualities).

High Quality Innovation by the High Quality Firm (Case a)

When the high quality firm introduces a new product in the high quality area and keeps its original product, three products are offered in the market. Consequently, three indifferent consumers exist in the market. One of them is indifferent between buying the product with highest quality s_2^1 or with second highest quality s_2^0 from the high quality firm. The income parameter of this consumer is given by $\theta_3 = \frac{(p_2^1 - p_2^0)}{(s_2^1 - s_2^0)}$. The consumer who is indifferent between buying the high quality firm's original product with quality s_2^0 and the low quality firm's product with quality s_1^0 is described by the income parameter $\theta_2 = \frac{(p_2^0 - p_1^0)}{(s_2^0 - s_1^0)}$, whereas the income parameter $\theta_1 = \frac{p_1^0}{s_1^0}$ represents the consumer who is indifferent between buying the product with lowest quality from the low quality firm and not buying at all. For the demand functions, we get

$$D_2^1\left(p_2^0, p_2^1, s_2^0, s_2^1\right) = \int_{\theta_3}^{\theta=1} f(\theta)\, d\theta = 1 - \frac{(p_2^1 - p_2^0)}{(s_2^1 - s_2^0)}, \tag{1}$$

$$D_2^0\left(p_1^0, p_2^0, p_2^1, s_1^0, s_2^0, s_2^1\right) = \int_{\theta_2}^{\theta_3} f(\theta)\, d\theta = \frac{(p_2^1 - p_2^0)}{(s_2^1 - s_2^0)} - \frac{(p_2^0 - p_1^0)}{(s_2^0 - s_1^0)}, \tag{2}$$

and

$$D_1^0\left(p_1^0, p_2^0, s_1^0, s_2^0\right) = \int_{\theta_1}^{\theta_2} f(\theta)\, d\theta = \frac{(p_2^0 - p_1^0)}{(s_2^0 - s_1^0)} - \frac{p_1^0}{s_1^0}. \tag{3}$$

Firms' objective functions in stage 2 are given by[7]

$$\pi_1^0(p_1^0, D_1^0) = p_1^0 D_1^0(\cdot), \text{ and}$$

$$\pi_2^{0,1}\left(p_2^0, D_2^0, p_2^1, D_2^1\right) = p_2^0 D_2^0(\cdot) + p_2^1 D_2^1(\cdot).$$

Each firm maximizes its objective function with respect to its own product price. The first order condition for the low quality firm, is given by

$$\frac{\partial \pi_1^0(p_1^0, D_1^0)}{\partial p_1^0} \equiv 0 \Longrightarrow p_1^0\left(p_2^0\right) = \frac{p_2^0 s_1^0}{2 s_2^0}.$$

[7] The variable $\pi_i^{k,l}$, for $k, l = 0, 1$ and $k \neq l$ refers to firm i's profits in stage two. The presence of both superscripts k and l indicates that the firm offers both products in the market. Whereas one superscript (e.g. π_i^k) indicates that firm i offers only one product in the market. Moreover, $\pi_i^{(k),l}$ for $k, l = 0, 1$ and $k \neq l$ indicates that firm i has the opportunity to either keep or withdraw the corresponding product with index k from the market.

The first order condition for the high quality firm with respect to the price of the high quality product, is as follows

$$\frac{\partial \pi_2^{0,1}(p_2^0, D_2^0, p_2^1, D_2^1)}{\partial p_2^1} \equiv 0 \Longrightarrow p_2^1\left(p_2^0\right) = \frac{2p_2^0 - s_2^0 + s_2^1}{2},$$

and with respect to its original product price,

$$\frac{\partial \pi_2^{0,1}(p_2^0, D_2^0, p_2^1\left(p_2^0\right), D_2^1)}{\partial p_2^0} \equiv 0 \Longrightarrow p_2^0\left(p_1^0\right) = \frac{p_1^0 - s_1^0 + s_2^0}{2s_2^0}.$$

Note that the innovator (high quality firm) is allowed to internalize the price effect of its low quality product price on its high quality product price.
The reaction functions are strictly monotone and have a unique Nash equilibrium. Solving the first order conditions yields the corresponding equilibrium prices

$$p_1^0(s_1^0, s_2^0) = \frac{s_1^0\left(s_2^0 - s_1^0\right)}{4\left(s_2^0 - s_1^0\right)},\ p_2^0(s_1^0, s_2^0) = \frac{2s_2^0\left(s_2^0 - s_1^0\right)}{4s_2^0 - 4s_1^0}, \text{ and}$$

$$p_2^1(s_1^0, s_2^0, s_2^1) = \frac{4s_2^0 s_2^1 - s_1^0\left(s_2^1 + 3s_2^0\right)}{2\left(4s_2^0 - 4s_1^0\right)}.$$

Substituting these into equations (1), (2), and (3) gives us the equivalent demand

$$D_1^0(s_1^0, s_2^0) = \frac{s_2^0}{4s_2^0 - s_1^0},\ D_2^0(s_1^0, s_2^0) = \frac{s_1^0}{2\left(4s_2^0 - s_1^0\right)}, \text{ and } D_2^1 = \frac{1}{2}.$$

Similarly, firms' profits in the product market are

$$\pi_1^0(s_1^0, s_2^0) = \frac{s_1^0 s_2^0\left(s_2^0 - s_1^0\right)}{\left(4s_2^0 - s_1^0\right)^2}, \text{ and}$$

$$\pi_2^{0,1}(s_1^0, s_2^0, s_2^1) = \frac{s_1^0 s_2^0\left(s_1^0 + s_2^0\right)}{\left(4s_2^0 - s_1^0\right)^2} + \frac{4s_2^0 s_2^1 - s_1^0\left(3s_2^0 + s_2^1\right)}{4\left(4s_2^0 - s_1^0\right)}. \tag{4}$$

The derivative of the high quality firm's second-stage profit function with respect to its original product quality is given by

$$\frac{\partial \pi_2^{0,1}(s_1^0, s_2^0, s_2^1)}{\partial s_2^0} = \frac{s_1^{0^2}\left(s_1^0 + 20s_2^0\right)}{4\left(4s_2^0 - s_1^0\right)^3} > 0. \tag{5}$$

As we see from case a (as well as from cases b to f shown in the Appendix) firms' profits (stage 2) depend on the product qualities and the number of products in the market.

In the next section, we investigate the innovator's decision to introduce a new product as well as the incentive to withdraw the original product.

2.2 R&D Market - Stage 1

In stage 1, the innovator has to draw two decisions: whether to introduce a new product and whether to withdraw the original product. Firm i's profits in stage 1 (in the following also called first-stage profits) are firm i's profits in the product market (stage 2) minus its R&D costs, i.e.

$$\Pi_i^{(0),1}\left(s_1^0, s_2^0, s_i^1(\gamma), \gamma\right) = \pi_i^{(0),1}\left(\cdot, s_i^1(\gamma)\right) - F\left(\cdot, s_i^1(\gamma), \gamma\right). \tag{6}$$

Firms' profits from a new product introduction is then given by,

$$\Phi\left(s_1^0, s_2^0, \gamma\right) = \pi_i^{(0),1}\left(\cdot, s_i^1(\gamma)\right) - F\left(\cdot, s_i^1(\gamma), \gamma\right) - \Omega_i^0\left(s_1^0, s_2^0\right) > 0 \tag{7}$$

where $\Omega_i^0\left(s_1^0, s_2^0\right)$ indicates firm i's profits when no innovation occurs.

Furthermore, the innovator decides whether to keep the original product in the market which will be optimal whenever

$$\pi_i^{0,1}\left(s_1^0, s_2^0, s_i^1\right) - \pi_i^1\left(s_j^0, s_i^1\right) > 0, \tag{8}$$

with $i, j = 1, 2$ and $i \neq j$.

We first investigate the innovator's decision to keep or withdraw the original product, as of equation (8). We then investigate the innovator's incentive to introduce a new product in a certain quality area, using equation (6). Finally, we will derive the innovation incentives by comparing the first-stage profits to when no innovation occurs as per equation (7).

We begin with analyzing the innovation cases where the high quality firm is the potential innovator.

New Product Introduction by the High Quality Firm

In this innovation scenario we analyze the case where the high quality firm introduces a new product in the high quality area (case a), before we turn to the cases when it may introduce a new product in the intermediate quality area (case b), or in the low quality area (case c), see also Table 2.1.

High Quality Innovation (Case a)

We begin by analyzing the innovator's incentives to keep the original product in the market. In principle, we need to solve the innovator's first order condition (6) with respect to the new product quality. However, polynomials of high degree prevent us from explicitly solving the innovator's first order condition. As a result, we cannot compare the innovator's profits as shown in equation (8). For that reason, we investigate the high quality firm's marginal profits at stage 2 after innovation has occurred with respect to its original product quality.

For analyzing and explaining how the choice to keep or withdraw affects the innovator's profits, we decompose the total derivative of the reduced-form profit function (stage 2) into several effects.[8] The total derivative with respect to its original product quality s_2^0, is given by[9]

$$\frac{d\pi_2^{0,1}}{ds_2^0} = \underbrace{\overbrace{\frac{\partial \pi_2^{0,1}}{\partial D_2^0}}^{+}\overbrace{\frac{\partial D_2^0}{\partial p_1^0}}^{+}\overbrace{\frac{dp_1^0}{ds_2^0}}^{+}}_{\textit{strategic effect}} + \underbrace{\overbrace{\frac{\partial \pi_2^{0,1}}{\partial D_2^0}}^{+}\overbrace{\frac{\partial D_2^0}{\partial s_2^0}}^{+}}_{\textit{demand effect}} + \underbrace{\overbrace{\frac{\partial \pi_2^{0,1}}{\partial D_2^1}}^{+}\overbrace{\frac{\partial D_2^1}{\partial s_2^0}}^{-}}_{\textit{cannibalization effect}} > 0. \tag{9}$$

The incentive for the high quality firm to withdraw its original product with quality s_2^0, is determined by the *strategic effect,* the *demand effect,* and the *cannibalization effect.*[10] The *demand effect* shows that increasing the original product in the market increases the innovator's profits through higher demand. The *cannibalization effect* indicates that keeping the original product cannibalizes the new product demand which lowers the innovator's profits. Since the *cannibalization effect* dominates the *demand effect* the original product will attract less consumers than it *cannibalizes* its new product demand, see equation (9). Furthermore, a *strategic effect* indicates tougher price competition in the market.

The high quality firm earns higher profits by increasing the product quality towards s_2^1 and is therefore better off withdrawing its original product from the market. In this case, two products are offered in the market: the original product by the low quality firm with quality s_1^0 and the new product by the high quality firm with quality s_2^1. The same results as in Appendix 1, setting $s_2^1 = s_2^0$ apply.

Next, we investigate the innovator's incentive to introduce a new product in the high quality area given it withdraws the original product from the market. Equation (6) shows the innovator's objective function, which is concave in the high quality firm's new product quality s_2^1, because the profit function is concave (see Appendix 1, equation (37), setting $s_2^1 = s_2^0$) and the costs function is convex. An unique solution for s_2^1 exists. Note that a boundary solution may exist where s_2^1 is equal to s_2^0. This solution is equivalent to the case where no innovation occurs. Taking the first order condition of equation (6) with respect to its new product quality s_2^1, gives us

$$\frac{\partial \Pi_2^1\left(s_1^0, s_2^1, \gamma\right)}{\partial s_2^1} = \frac{4s_2^1\left(2s_1^{0^2} - 3s_1^0 s_2^1 + 4s_2^{1^2}\right)}{\left(4s_2^1 - s_1^0\right)^3} - 2\gamma\left(s_2^1 - s_2^0\right) = 0. \tag{10}$$

[8] Decomposing the derivative in several effects will be necessary in later scenarios in order to show the sign of the derivative.

[9] Second-stage optimization, implies $\frac{\partial \pi_2^{0,1}}{\partial p_2^1} = 0$ and $\frac{\partial \pi_2^{0,1}}{\partial p_2^0} = 0$. Thus, the effect of s_2^0 on $\pi_2^{0,1}$ through the high quality firm's price change can be ignored by applying the envelope theorem. Equation (5) shows the derivative of the innovator's profits.

[10] A *strategic effect* towards the innovator's new product price does not occur since the innovator internalizes price competition among its own products.

As we see in equation (10) marginal profits (first term) are similar to the outset (see Appendix 1, equation (34)) and are determined by a demand and a strategic effect, which are both positive.

We want to compare first-stage profits (stage 1) when innovation occurs in the high quality area with the profits when no innovation occurs. The innovator's objective function is given by (7). As mentioned above, solving the innovator's first order condition of equation (6) for s_2^1 given by (10) is not possible. However, by implicitly differentiating the objective function (7) we are able to derive the conditions on costs and the original product qualities which have to hold for the high quality firm to introduce a new product in the high quality area. We use the objective function shown in equation (7) for the case when the high quality firm introduces a new product in the high quality area, given by

$$\Phi_0\left(s_1^0, s_2^0, s_2^1(\gamma), \gamma\right) = \pi_2^1\left(s_1^0, s_2^1(\gamma)\right) - F\left(s_2^0, s_2^1(\gamma), \gamma\right) - \Omega_2^0\left(s_1^0, s_2^0\right) \tag{11}$$

with $s_2^1 > s_2^0$. We begin with investigating the costs conditions using the total derivative of the objective function with respect to the costs parameter γ. Rearranging yields

$$\begin{aligned}\frac{d\Phi_0\left(s_1^0, s_2^0, s_2^1(\gamma), \gamma\right)}{d\gamma} &= \frac{\partial s_2^1(\gamma)}{\partial\gamma}\left[\frac{\partial\pi_2^1\left(s_1^0, s_2^1(\gamma)\right)}{\partial s_2^1} - \frac{\partial F\left(s_2^0, s_2^1(\gamma), \gamma\right)}{\partial s_2^1}\right] \\ &\quad - \frac{\partial F\left(s_2^0, s_2^1(\gamma), \gamma\right)}{\partial\gamma}\end{aligned} \tag{12}$$

where

$$\frac{\partial\pi_2^1\left(s_1^0, s_2^1(\gamma)\right)}{\partial s_2^1} - \frac{\partial F\left(s_2^0, s_2^1(\gamma)\right)}{\partial s_2^1} = \frac{\partial\Pi_2^1\left(s_1^0, s_2^1(\gamma), \gamma\right)}{\partial s_2^1}.$$

Since $s_2^1(\gamma)$ is optimally chosen, such that it maximizes the innovator's profits we make use of the envelope theorem, given by

$$\frac{\partial\Pi_2^1\left(s_1^0, s_2^1(\gamma), \gamma\right)}{\partial s_2^1} = 0.$$

Substituting into equation (12), gives

$$\frac{d\Phi_0\left(s_1^0, s_2^0, s_2^1(\gamma), \gamma\right)}{d\gamma} = -\frac{\partial F\left(s_2^0, s_2^1(\gamma), \gamma\right)}{\partial\gamma} = -\left(s_2^1 - s_2^0\right)^2 < 0. \tag{13}$$

As we see in equation (13), the total derivative is equal to the partial derivative evaluated at the optimal choice of s_2^1. Finally, we only have to take into account the direct effect of an increase of γ on costs, but not the indirect effect via the choice of s_2^1.

Equation (13) shows that the innovator's objective function is continuously decreasing in γ. In other words, the innovator earns less profits the higher the production costs for quality.

In a next step we have to show that there exists a γ^0 which fulfills

$$\Phi_0\left(s_1^0, s_2^0, s_2^1(\gamma), \gamma\right)\Big|_{\gamma=\gamma^0} = 0.$$

The argument is as follows: Setting $\gamma = 0$ and inserting into equation (11), we get

$$\Phi_0\left(s_1^0, s_2^0, s_2^1(\gamma), \gamma\right)\Big|_{\gamma=0} = \frac{4{s_2^1}^2\left(s_2^1 - s_1^0\right)}{\left(4s_2^1 - s_1^0\right)^2} - \frac{4{s_2^0}^2\left(s_2^0 - s_1^0\right)}{\left(4s_2^0 - s_1^0\right)^2} > 0, \tag{14}$$

see equation (34). From (13) and (14) the existence of an unique $\gamma = \gamma^0 > 0$ follows, where $\Phi_0\left(s_1^0, s_2^0, \gamma\right)\big|_{\gamma=\gamma^0} = 0$ holds. We can summarize, when γ is relatively small ($\gamma < \gamma^0$, saying that the production of quality is not too costly) the high quality firm introduces a new product in the high quality area and withdraws the original product from the market.

However, the objective function (11) indicates that the high quality firm's innovation also depends on the original product qualities s_1^0 and s_2^0. Differentiating the innovator's objective function with respect to the low quality firm's product quality, taking into account the envelope theorem, gives

$$\frac{d\Phi_0\left(s_1^0, s_2^0, s_2^1\left(s_1^0\right)\right)}{ds_1^0} = \frac{\partial\pi_2^1\left(s_1^0, s_2^1\left(s_1^0\right)\right)}{\partial s_1^0} - \frac{\partial\Omega_2^0\left(s_1^0, s_2^0\right)}{\partial s_1^0} < 0.$$

As we see, the high quality firm earns higher profits the smaller the rival's product quality. Similarly, differentiating equation (11) with respect to the high quality firm's original product quality, is as follows

$$\frac{d\Phi_0\left(s_1^0, s_2^0, s_2^1\left(s_2^0\right)\right)}{ds_2^0} = -\left[\frac{\partial F\left(s_2^0, s_2^1\left(s_2^0\right)\right)}{\partial s_2^0} + \frac{\partial\Omega_2^0\left(s_1^0, s_2^0\right)}{\partial s_2^0}\right] < 0.$$

The high quality firm's profits are higher the lower its own original product quality.

We turn to investigate the high quality firm's innovation incentives when it offers a new product in the intermediate quality area.

Intermediate Quality Innovation (Case b)

The high quality firm's objective function for this case is given by equation (8). For the same reasons as in case a we are not able to compare the innovator's profits explicitly. We therefore investigate how the choice to keep or withdraw affects the high quality firm's second-stage marginal profits. Taking the total derivative of the high quality firm's reduced-form profit function at stage 2 with respect to its original product quality s_2^0, gives us[11]

$$\frac{d\pi_2^{1,0}}{ds_2^0} = \underbrace{\overbrace{\frac{\partial \pi_2^{1,0}}{\partial D_2^0}}^{+}\overbrace{\frac{\partial D_2^0}{\partial s_2^0}}^{+}}_{\text{demand effect}} + \underbrace{\overbrace{\frac{\partial \pi_2^{1,0}}{\partial D_2^1}}^{+}\overbrace{\frac{\partial D_2^1}{\partial s_2^0}}^{-}}_{\text{cannibalization effect}} > 0. \tag{15}$$

As we see in equation (15) marginal profits are determined by the *demand effect* and the *cannibalization effect.* The *demand effect* shows that an increase in the original product quality attracts more consumers and increases profits. The *cannibalization effect* shows that the original product *cannibalizes* demand of its new product.[12] Equation (15) indicates that the *demand effect* dominates the *cannibalization effect.* Therefore, keeping the original product in the market *cannibalizes* demand for the new product to a lower extent than the original product's ability to attract customers. The high quality firm benefits by keeping the original product in the market, although *cannibalization* towards its new product demand occurs because the original product is of higher quality and gives higher profits. Three products are offered in the market, the low quality firm's product and both high quality firm's products.

Next, we analyze whether the high quality firm has an incentive to introduce a new product in the intermediate quality area given the original product stays in the market. Equation (6) shows the high quality firm's objective function. We investigate the total derivative of the high quality firm's first-stage profit function (6) with respect to the new product quality s_2^1, which is given by[13]

$$\frac{d\Pi_2^{1,0}}{ds_2^1} = \underbrace{\overbrace{\frac{\partial \pi_2^{1,0}}{\partial D_2^1}}^{+}\overbrace{\frac{\partial D_2^1}{\partial p_1^0}}^{+}\overbrace{\frac{dp_1^0}{ds_2^1}}^{+}}_{\text{strategic effect}} + \underbrace{\overbrace{\frac{\partial \pi_2^{1,0}}{\partial D_2^1}}^{+}\overbrace{\frac{\partial D_2^1}{\partial s_2^1}}^{+}}_{\text{demand effect}} + \underbrace{\overbrace{\frac{\partial \pi_2^{1,0}}{\partial D_2^0}}^{+}\overbrace{\frac{\partial D_2^0}{\partial s_2^1}}^{-}}_{\text{cannibalization effect}} + \overbrace{\frac{\partial F}{\partial s_2^1}}^{=0} > 0. \tag{16}$$

As we see in equation (16), one *demand effect,* one *cannibalization effect,* and one *strategic effect* influence marginal first-stage profits. The *demand effect* indicates that an increase in the high quality firm's new product quality increases demand.

[11] See also Appendix 2, equation (38).

[12] It may seem surprising at first glance that the high quality firm is cannibalizing its new product demand by moving further apart with its former product quality. But since the high quality firm internalizes price competition towards its own products it sacrifices some of its new product demand by pricing relatively high in order to attract more consumers buying the former product. The former product is of higher quality and earns higher profits.

[13] The high quality firm's R&D production cost for quality is zero because a new product with lower quality is introduced into the market. Equation (5) shows the derivative of the high quality firm's profit function (stage 2), setting $s_2^1 = s_2^0$, and vice versa.

The *cannibalization effect* indicates that the product introduction lowers its original product demand. The *demand effect* is dominated by the *cannibalization effect.* Therefore, the introduction of a new product attracts fewer consumers than it *cannibalizes* its original product demand. Furthermore, a *strategic effect* increases price competition towards the rival's product price which reduces the new product demand. The high quality firm earns higher profits by letting the quality of its new product approach its original product quality in order to relax price competition and to avoid cannibalizing its original product demand. Consequently, the high quality firm will not introduce a new product in the intermediate quality area.

Low Quality Innovation (Case c)

The innovator's objective function is given by equation (8). In case the high quality firm withdraws, it offers one product in the low quality area. It is easy to see from equation (32) in Appendix 1 that the low quality firm earns less profits than the high quality provider. Hence, the quality leader is worse off introducing a new product in the low quality area. We can conclude that the high quality firm will keep the original product in the market when it introduces a new product in the low quality area.

Next, we analyze whether the high quality firm has an incentive to introduce a new product in the low quality area given the original product stays in the market. We investigate the total derivative of the high quality firm's reduced-form first-stage profit function (6) with respect to the new product quality s_2^1, i.e.[14]

$$\frac{d\Pi_2^{1,0}}{ds_2^1} = \underbrace{\overbrace{\frac{\partial \pi_2^{1,0}}{\partial D_2^0}}^{+}\overbrace{\frac{\partial D_2^0}{\partial p_1^0}}^{+}\overbrace{\frac{dp_1^0}{ds_2^1}}^{-}}_{\textit{first strategic effect}} + \underbrace{\overbrace{\frac{\partial \pi_2^{1,0}}{\partial D_2^1}}^{+}\overbrace{\frac{\partial D_2^1}{\partial p_1^0}}^{+}\overbrace{\frac{dp_1^0}{ds_2^1}}^{-}}_{\textit{second strategic effect}} + \underbrace{\overbrace{\frac{\partial \pi_2^{1,0}}{\partial D_2^1}}^{+}\overbrace{\frac{\partial D_2^1}{\partial s_2^1}}^{+}}_{\textit{demand effect}} + \overbrace{\frac{\partial F}{\partial s_2^1}}^{=0} < 0. \qquad (17)$$

Equation (17) shows that the high quality firm's marginal profits are determined by two *negative strategic effects* and one *positive demand effect.* Both *strategic effects* indicate that price competition towards the low quality firm's product is softened by decreasing the new product quality. The relaxed price competition has a positive impact on both the high quality firm's product demands. The *demand effect* indicates that an increase in quality determines a higher demand. In this scenario no *cannibalization* occurs because the high quality firm's new product does not directly influence the demand of its original product. Equation (17) indicates that the *second strategic effect* is larger than the *demand effect* which implies a total negative effect. It follows that the high quality firm earns higher profits by softening price competition when it decreases the new product

[14] See also Appendix 3, equation (40).

quality. For this reason, the high quality firm has no incentive to introduce a new product in the low quality area.

We can summarize the different cases when the high quality firm may introduce a new product with the following proposition.

Proposition 1 *The high quality firm offers a new product in the high quality area and withdraws the original product from the market when the production costs for quality is relatively small ($\gamma < \gamma^0$) and the original product qualities are small. Otherwise, the high quality firm does not introduce a new product in the market.*

In the next section we investigate the innovation cases when the low quality firm is the innovator, see Table 2.2.

New Product Introduction by the Low Quality Firm

In the following we analyze the cases when the low quality firm offers a new product in the high quality area (case d), in the intermediate quality area (case e), or in the low quality area (case f).

High Quality Innovation (Case d)

When the low quality firm introduces a new product in the high quality area, it has the choice to keep or withdraw the original product from the market. The low quality firm's objective function is given by equation (8). Again, we are not able to compare the low quality firm's profits (stage 2) explicitly. Therefore, we investigate the total derivative of the low quality firm's profit function with respect to s_1^0, given by[15]

$$\frac{d\pi_1^{0,1}}{ds_1^0} = \underbrace{\overbrace{\frac{\partial\pi_1^{0,1}}{\partial D_1^0}}^{+}\overbrace{\frac{\partial D_1^0}{\partial p_2^0}}^{+}\overbrace{\frac{dp_2^0}{ds_1^0}}^{-}}_{\text{first strategic effect}} + \underbrace{\overbrace{\frac{\partial\pi_1^{0,1}}{\partial D_1^1}}^{+}\overbrace{\frac{\partial D_1^1}{\partial p_2^0}}^{+}\overbrace{\frac{dp_2^0}{ds_1^0}}^{-}}_{\text{second strategic effect}} + \underbrace{\overbrace{\frac{\partial\pi_1^{0,1}}{\partial D_1^0}}^{+}\overbrace{\frac{\partial D_1^0}{\partial s_1^0}}^{+}}_{\text{demand effect}} < 0. \tag{18}$$

Two *negative strategic effects* and one *positive demand effect* determine the low quality firm's marginal profits. Both *strategic effects* indicate that price competition is softened towards the rival's price by decreasing the quality of the original product. The relaxed price competition has a positive impact on both the low quality firm's product demands. The *demand effect* shows that increasing the original product quality attracts more consumers. No *cannibalization* occurs in this scenario because the new product does not directly impact the demand of the original product; only neighboring products do so. Equation (18) indicates

[15] See also Appendix 3, equation (40).

that the *second strategic effect* dominates the *demand effect* resulting in a total negative effect. The low quality firm earns higher profits by withdrawing the original product from the market in order to soften price competition. As a result, two products are offered in the market when the low quality firm introduces a new product in the high quality area. The same results as in Appendix 1 apply, setting $s_1^1 = s_2^0$, and $s_2^0 = s_1^0$.

Next, we investigate the low quality firm's incentive to introduce a new product in the high quality area given it withdraws the original product from the market. The low quality firm's objective function is given by equation (6).[16] We analyze the first order condition of the low quality firm's first-stage profit function (6) with respect to its new product quality s_1^1. The first order condition, is given by

$$\frac{\partial \Pi_1^1\left(s_2^0, s_1^1\right)}{\partial s_1^1} = \frac{4s_1^1\left(2s_2^{0^2} - 3s_2^0 s_1^1 + 4s_1^{1^2}\right)}{\left(4s_1^1 - s_2^0\right)^3} - 2\gamma\left(s_1^1 - s_1^0\right) = 0. \tag{19}$$

Equation (19) shows that the low quality firm's profits increase in the new product quality.

Next, we want to compare the profits (stage 1) when the low quality firm introduces a new product in the high quality area with the profits when no innovation occurs. For analyzing the low quality firm's innovation incentives we apply the same procedure as for case a, because solving the innovator's first order condition (19) for s_1^1 is not possible. We use the objective function (7) for the innovation case where the low quality firm introduces a new product in the high quality area, given by

$$\Phi_1\left(s_1^0, s_2^0, s_1^1(\gamma), \gamma\right) = \pi_1^1\left(s_2^0, s_1^1(\gamma)\right) - F\left(s_1^0, s_1^1(\gamma), \gamma\right) - \Omega_1^0\left(s_1^0, s_2^0\right). \tag{20}$$

with $s_1^1 > s_2^0$. Implicitly differentiating the objective function (20) we are able to derive the conditions on costs and the original product qualities for this innovation case.

We begin with investigating the costs conditions. Differentiating both sides of equation (20) with respect to γ, applying the envelope theorem, and rearranging yields

$$\frac{d\Phi_1\left(s_1^0, s_2^0, s_1^1(\gamma), \gamma\right)}{d\gamma} = -\frac{\partial F\left(s_1^0, s_1^1(\gamma), \gamma\right)}{\partial \gamma} = -\left(s_1^1 - s_1^0\right)^2 < 0. \tag{21}$$

It is shown that the innovator's objective function decreases in γ. Setting $\gamma = 0$ and inserting into equation (20) yields

$$\Phi_1\left(s_1^0, s_2^0, s_1^1(\gamma), \gamma\right)\Big|_{\gamma=0} = \frac{4s_1^{1^2}\left(s_1^1 - s_2^0\right)}{\left(4s_1^1 - s_2^0\right)^2} - \frac{s_1^0 s_2^0\left(s_2^0 - s_1^0\right)}{\left(4s_2^0 - s_1^0\right)^2} > 0. \tag{22}$$

[16] Analogously to the former cases, the concavity of the low quality firm's objective function follows from the properties of the profit function (stage 2) as well as the cost function.

From equations (21)and (22) the existence of an unique $\gamma = \gamma^{'} > 0$ follows where $\Phi_1 \left(s_1^0, s_2^0, \gamma\right)|_{\gamma=\gamma'} = 0$ holds. When γ is small ($\gamma < \gamma^{'}$, or the production of quality is not too costly) the low quality firm will introduce a new product in the high quality area and withdraws the original product from the market.

However, the low quality firm's objective function (20) indicates that the low quality firm's innovation also depends on the original product qualities. Differentiating the objective function (20) with respect to the high quality firm's product quality, taking into account the envelope theorem, gives

$$\frac{\partial \Phi_1 \left(s_1^0, s_2^0, s_1^1 \left(s_2^0\right)\right)}{\partial s_2^0} = \frac{\partial \pi_1^1 \left(s_2^0, s_1^1 \left(s_2^0\right)\right)}{\partial s_2^0} - \frac{\partial \Omega_1^0 \left(s_1^0, s_2^0\right)}{\partial s_2^0} < 0.$$

When the high quality firm's product quality is small the low quality firm offers a new product in the high quality area. Differentiating equation (20) with respect to the low quality firm's original product quality, yields

$$\frac{\partial \Phi_1 \left(s_1^0, s_2^0, s_1^1 \left(s_1^0\right)\right)}{\partial s_1^0} = - \left[\frac{\partial F \left(s_1^0, s_1^1 \left(s_1^0\right)\right)}{\partial s_1^0} + \frac{\partial \Omega_1^0 \left(s_1^0, s_2^0\right)}{\partial s_1^0}\right] < 0.$$

The low quality firm offers a new product in the high quality area when its own original product quality is small. The following lemma summarizes this innovation case.

Lemma 2 *The low quality firm introduces a new product in the high quality area and withdraws the original product from the market when the production costs for quality is small ($\gamma < \gamma^{'}$) and the original product qualities are small.*

Let us turn to analyze the innovation case e from Table 2.2.

Intermediate Quality Innovation (Case e)

In order to determine the low quality firm's decision to withdraw the former product from the market, we investigate the total derivative of the low quality firm's profit function (stage 2) with respect to product quality s_1^0, given by[17]

$$\frac{d\pi_1^{0,1}}{ds_1^0} = \underbrace{\overbrace{\frac{\partial \pi_1^{0,1}}{\partial D_1^0}}^{+} \overbrace{\frac{\partial D_1^0}{\partial s_1^0}}^{+}}_{demand\ effect} + \underbrace{\overbrace{\frac{\partial \pi_1^{0,1}}{\partial D_1^1}}^{+} \overbrace{\frac{\partial D_1^1}{\partial s_1^0}}^{-}}_{cannibalization\ effect} < 0. \tag{23}$$

As we see in equation (23) marginal profits are determined by one *demand*, and one *cannibalization effect*. The *demand effect* shows that increasing the original

[17]The derivative of the low quality firm's profit function is shown in Appendix 4, equation (42).

product quality attracts more consumers. The *cannibalization effect* shows that some consumers switch to the new product. Because the *cannibalization effect* dominates the *demand effect* the low quality firm is better off to withdraw the first product from the market. Two products are offered in the market. The same results as in Appendix 1 apply, setting $s_1^1 = s_1^0$.

In a next step, we investigate the low quality firm's incentive to introduce a new product in the intermediate quality area given it withdraws the original product from the market. The first order condition of the first-stage profit function (6), is given by[18]

$$\frac{\partial \Pi_1^1 \left(s_1^1, s_2^0\right)}{\partial s_1^1} = \frac{s_2^{0^2}\left(4s_2^0 - 7s_1^1\right)}{\left(4s_2^0 - s_1^1\right)^3} - 2\gamma\left(s_1^1 - s_1^0\right) = 0. \tag{24}$$

Marginal profits (stage 2) are similar to the outset, see Appendix 1, equation (33) setting $s_1^1 = s_1^0$. The low quality firm benefits by a higher *demand effect* as long as the new quality is smaller than $\frac{4}{7}s_2^0$, and suffers by a higher *strategic effect* when the new quality is higher than $\frac{4}{7}s_2^0$.

Because solving the low quality firm's first order condition (24) for s_1^1 is not possible, we apply the same procedure as for case a in order to analyze the low quality firm's innovation incentives. We apply the objective function shown in equation (7) to this case and compare the low quality firm's profit after introducing a new product in the intermediate quality area with the case when no innovation occurs, given by

$$\Phi_2\left(s_1^0, s_1^1(\gamma), s_2^0, \gamma\right) = \pi_1^1\left(s_1^1(\gamma), s_2^0\right) - F\left(s_1^0, s_1^1(\gamma), \gamma\right) - \Omega_1^0\left(s_1^0, s_2^0\right), \tag{25}$$

with $s_2^0 > s_1^1 > s_1^0$. Differentiating both sides of equation (25) with respect to γ, applying the envelope theorem, and rearranging, gives us

$$\frac{d\Phi_2\left(s_1^0, s_1^1(\gamma), s_2^0, \gamma\right)}{d\gamma} = -\frac{\partial F\left(s_1^0, s_1^1(\gamma), \gamma\right)}{\partial \gamma} = -\left(s_1^1 - s_1^0\right)^2 < 0. \tag{26}$$

It is shown that the low quality firm's profits decrease in γ. Setting $\gamma = 0$ and inserting into equation (25), gives

$$\Phi_2\left(s_1^0, s_1^1(\gamma), s_2^0, \gamma\right)\Big|_{\gamma=0} = \frac{s_1^1 s_2^0\left(s_2^0 - s_1^1\right)}{\left(4s_2^0 - s_1^1\right)^2} - \frac{s_1^0 s_2^0\left(s_2^0 - s_1^0\right)}{\left(4s_2^0 - s_1^0\right)^2} > 0. \tag{27}$$

From equation (26) and (27) follows that an unique $\gamma = \gamma'' > 0$ exists, where $\Phi_2\left(s_1^1, s_2^0, \gamma\right)\big|_{\gamma=\gamma''} = 0$ applies.

[18] The concavity of the first-stage profit function (6) follows from the properties of the profit function (stage 2) as well as the cost function.

When γ is small (the production of quality is not too costly) the low quality firm introduces a new product in the intermediate quality area and withdraws the original product from the market.

For investigating how the incentives to innovate depend on the original product qualities we differentiate the low quality firm's objective function (25) with respect to the high quality firm's product quality. Taking into account the envelope theorem, gives us

$$\frac{\partial \Phi_2\left(s_1^0, s_2^0, s_1^1\left(s_2^0\right)\right)}{\partial s_2^0} = \frac{\partial \pi_1^1\left(s_1^1\left(s_2^0\right), s_2^0\right)}{\partial s_2^0} - \frac{\partial \Omega_1^0\left(s_1^0, s_2^0\right)}{\partial s_2^0} > 0.$$

The low quality firm's profits after innovation increase the larger the rival's product quality. Differentiating equation (25) with respect to the low quality firm's original product quality, is given by

$$\frac{\partial \Phi_2\left(s_1^0, s_2^0, s_1^1\left(s_1^0\right)\right)}{\partial s_1^0} = -\left[\frac{\partial F\left(s_1^0, s_1^1\left(s_1^0\right)\right)}{\partial s_1^0} + \frac{\partial \Omega_1^0\left(s_1^0, s_2^0\right)}{\partial s_1^0}\right] < 0.$$

The low quality firm's profits after innovation are higher the lower its own original product quality. After investigating the innovation case where the low quality firm introduces a new product in the intermediate quality area we get the following lemma.

Lemma 3 *The low quality firm introduces a new product in the intermediate quality area and withdraws the original product from the market when the production costs for quality is small ($\gamma < \gamma''$), the low quality firm's original product quality is small and the high quality firm's original product quality is high.*

Next, we investigate the conditions concerning costs and original product qualities when the low quality firm introduces a new product in the high quality area (case d) or in the intermediate quality area (case e). The low quality firm introduces a new product in the high quality area, when[19]

$$\begin{aligned}\Phi_3\left(s_1^0, s_1^1(\gamma), s_2^0, \hat{s}_1^1(\gamma), \gamma\right) &= \overset{\wedge}{\pi}_1^1\left(s_2^0, \hat{s}_1^1(\gamma)\right) - F\left(s_1^0, \hat{s}_1^1(\gamma), \gamma\right) \\ &\quad - \left(\pi_1^1\left(s_1^1(\gamma), s_2^0\right) - F\left(s_1^1(\gamma), s_1^0, \gamma\right)\right) > 0,\end{aligned} \tag{28}$$

with $\hat{s}_1^1 > s_2^0 > s_1^1$. Differentiating equation (28) with respect to γ, applying the envelope theorem, and rearranging, gives us

[19] In order to distinguish between the two cases d and e we change the notation of the low quality firm's new product quality in case d to $\hat{s}_1^1$.

$$
\begin{aligned}
\frac{d\Phi_3\left(s_1^0, s_1^1(\gamma), s_2^0, \hat{s}{}_1^1(\gamma), \gamma\right)}{d\gamma} &= -\frac{\partial F\left(s_1^0, \hat{s}{}_1^1(\gamma), \gamma\right)}{\partial \gamma} + \frac{\partial F\left(s_1^1(\gamma), s_1^0, \gamma\right)}{\partial \gamma} \\
&= -\left(\hat{s}{}_1^1 - s_1^0\right)^2 + \left(s_1^1 - s_1^0\right)^2 < 0, \qquad (29)
\end{aligned}
$$

The low quality firm's incentive to offer a new product in the high quality area declines as the production costs for quality increases. Setting $\gamma = 0$ and inserting into equation (28), is

$$
\Phi_3\left(s_1^0, s_1^1(\gamma), s_2^0, \hat{s}{}_1^1(\gamma), \gamma\right)\Big|_{\gamma=0} = \frac{4\,\hat{s}{}_1^{1^2}\left(\hat{s}{}_1^1 - s_2^0\right)}{\left(4\,\hat{s}{}_1^1 - s_2^0\right)^2} - \frac{s_1^1 s_2^0\left(s_2^0 - s_1^1\right)}{\left(4s_2^0 - s_1^1\right)^2} > 0, \tag{30}
$$

From equation (29) and (30) follows that an unique $\gamma = \gamma^{'''} > 0$ exists, where $\Phi_3\left(s_1^0, s_2^0, \gamma\right)\big|_{\gamma=\gamma^{'''}} = 0$ applies.

When the production of quality is relatively cheap ($\gamma < \gamma^{'''}$) the low quality firm introduces a new product in the high quality area.

For investigating how the innovation incentives depend on the original product qualities we first differentiate the low quality firm's objective function (28) with respect to the high quality firm's original product quality, taking into account the envelope theorem, which gives

$$
\frac{\partial\Phi_3\left(s_1^0, s_1^1\left(s_2^0\right), s_2^0, \hat{s}{}_1^1\left(s_2^0\right), \gamma\right)}{\partial s_2^0} = \frac{\partial\pi_1^1\left(s_2^0, \hat{s}{}_1^1\left(s_2^0\right)\right)}{\partial s_2^0} - \frac{\partial\pi_1^1\left(s_1^1\left(s_2^0\right), s_2^0\right)}{\partial s_2^0} < 0,
$$

see also Appendix 1. The low quality firm introduces a new product in the high quality area when the high quality firm's original product quality is small. Differentiating equation (28) with respect to the low quality firm's original product quality, gives

$$
\begin{aligned}
\frac{\partial\Phi_3\left(s_1^0, s_1^1\left(s_1^0\right), s_2^0, \hat{s}{}_1^1\left(s_1^0\right), \gamma\right)}{\partial s_1^0} &= \frac{\partial F\left(s_1^0, s_1^1\left(s_1^0\right)\right)}{\partial s_1^0} - \frac{\partial F\left(s_1^0, \hat{s}{}_1^1\left(s_1^0\right)\right)}{\partial s_1^0} \\
&= 2\gamma\left(\hat{s}{}_1^1 - s_1^1\right) > 0.
\end{aligned}
$$

The low quality firm introduces a new product in the high quality area when its original product quality is high. After comparing both innovation scenarios we can conclude with the following lemma.

Lemma 4 *The low quality firm introduces a new product in the high quality area compared to offering a new product in the intermediate quality area when the production costs for quality and the high quality firm's product quality are relatively smaller* ($\gamma < \gamma'''$)*, and the own original product quality is relatively higher.*

Finally, we investigate the case when the low quality firm offers a new product in the low quality area.

Low Quality Innovation (Case f)

According to the innovator's objective function (8), we investigate how the choice to keep or withdraw affects the low quality firm's marginal profits (stage 2). The total derivative of the low quality firm's reduced-form profit function with respect to its original product quality s_1^0, is given by[20]

$$\frac{d\pi_1^{1,0}}{ds_1^0} = \underbrace{\overbrace{\frac{\partial\pi_1^{1,0}}{\partial D_1^0}}^{+}\overbrace{\frac{\partial D_1^0}{\partial p_2^0}}^{+}\overbrace{\frac{dp_2^0}{ds_1^0}}^{-}}_{\textit{strategic effect}} + \underbrace{\overbrace{\frac{\partial\pi_1^{1,0}}{\partial D_1^0}}^{+}\overbrace{\frac{\partial D_1^0}{\partial s_1^0}}^{+}}_{\textit{demand effect}} + \underbrace{\overbrace{\frac{\partial\pi_1^{1,0}}{\partial D_1^1}}^{+}\overbrace{\frac{\partial D_1^1}{\partial s_1^0}}^{-}}_{\textit{cannibalization effect}} \gtreqless 0 \text{ for } s_1^0 \lesseqgtr \frac{4}{7}s_2^0.$$

When the low quality firm keeps the original product in the market, one *demand effect,* one *strategic effect,* and one *cannibalization effect* influence marginal profits, where the *demand effect* dominates the *cannibalization effect.* However, a *strategic effect* increases price competition towards the high quality firm's product and reduces own product demand. In fact, the low quality firm's marginal profits are analogous to the marginal profits in the outset, see equation (33). Thereby, the total effect is negative, if the low quality firm's original product quality is higher than $\frac{4}{7}s_2^0$, whereas it is positive when the quality is smaller than $\frac{4}{7}s_2^0$. However, by definition, the low quality firm's original product quality is supposed to be smaller or equal to $\frac{4}{7}s_2^0$. It follows that the low quality firm is better off to keep the original product in the market. Three products are offered in the market for which the results are shown in Appendix 4.

For analyzing the low quality firm's incentives to introduce a new product in the low quality area we investigate the total derivative of its reduced-form

[20] The marginal profit function (stage 2) is shown in Appendix 1, equation (33).

first-stage profit function (6) with respect to product quality s_1^1, which is given by[21]

$$\frac{d\Pi_1^{1,0}}{ds_1^1} = \underbrace{\overbrace{\frac{\partial \pi_1^{1,0}}{\partial D_1^1}}^{+}\overbrace{\frac{\partial D_1^1}{\partial s_1^1}}^{+}}_{demand\ effect} + \underbrace{\overbrace{\frac{\partial \pi_1^{1,0}}{\partial D_1^0}}^{+}\overbrace{\frac{\partial D_1^0}{\partial s_1^1}}^{-}}_{cannibalization\ effect} - \overbrace{\frac{\partial F}{\partial s_1^1}}^{=0} < 0. \tag{31}$$

As we see from equation (31) marginal profits are determined by one *demand effect* and one *cannibalization effect*. The *demand effect* indicates that an increase in the new product quality attracts more consumers. The *cannibalization effect* shows that an increase in the new product quality reduces demand of the low quality firm's first product. The negative sign in equation (31) indicates that the *cannibalization effect* dominates the *demand effect*. As a result, the low quality firm does not introduce a new product in the low quality area.

After analyzing all innovation scenarios where the low quality firm introduces a new product in the market (cases d, e, and f), we derive the following proposition.

Proposition 5 *The low quality firm introduces a new product in the intermediate quality area when the production costs for quality is small ($\gamma < \gamma^{''}$), the low quality firm's original product quality is small and the high quality firm's original product quality is high. The low quality firm introduces a new product in the high quality area when the production costs for quality and the high quality firm's product quality are very small, and the own original product quality is small, but relatively higher than in the intermediate innovation case. In both innovation cases the low quality firm withdraws the original product from the market.*

Finally, we can derive four types of equilibria depending on who the innovator is, on the production costs for quality, and on the original product qualities.

1) When the high quality firm is the innovator, it introduces a new product in the high quality area if the production costs for quality is small ($\gamma < \gamma^{0}$) and the original product qualities are small. The high quality firm withdraws the original product from the market after innovation occurred (case a).

2) When the low quality firm is the innovator, it introduces a new product in the high quality area if the quality costs are very small, the high quality firm's product quality is small, and the own original product quality is small, but relatively higher than in the intermediate innovation case. The low quality firm withdraws the original product from the market after innovation occurred (case d).

[21]The derivative of the low quality firm's profit function is shown in Appendix 4, equation (42).

3) When the low quality firm is the innovator, it introduces a new product in the intermediate quality area if the production costs for quality is small, its own original product quality is very small, and the high quality firm's product quality is large. The low quality firm withdraws the original product from the market after innovation occurred (case e).

4) No innovation occurs, if the production costs for quality and the low quality firm's original product quality are high.

As we can see from above, all innovation equilibria have two characteristics in common:

(i) innovators always introduce a new product of higher quality into the market, and

(ii) innovators are better off to withdraw their original product from the market in order to avoid a *cannibalization effect* and to keep price competition towards the rival's product soft.

3 Conclusion

This study extends the literature on innovation in markets characterized by vertical product differentiation. The focus of this study is to analyze firms' incentives to introduce a new product in different quality areas and to investigate the variety of products offered in the market. Various effects in different innovation scenarios are examined.

We find that innovation occurs depending on the production costs for quality and the firms' original product qualities. The innovator always introduces a new product of higher quality in order to concentrate sales towards high income consumers. Moreover, the innovator always withdraws the original product quality from the market. By withdrawing the first product, price competition towards the rival's product is softened and a *cannibalization effect* towards its own product demand is avoided. As a result, only two products remain in the market.

This study presents a first insight into the innovation incentives of incumbent firms in a vertically differentiated market. We provide some fundamental results and effects which are important for the introduction of new products in a vertically differentiated product environment, in a sequential or simultaneous setting.

4 APPENDIX

Appendix 1: The Outset

Let us present the prices, demand, and profits for the outset $(k = 0)$ when firms offer one product, each. The outset is based on the model by Choi and Shin (1992) which is a modification of Shaked and Sutton (1982) where we use the version of Tirole (1992). The model is a noncooperative two-stage game where two firms $(i = 1, 2)$ simultaneously choose their qualities in the first stage and given their qualities they compete in the second stage with prices in the product market.

Product qualities are chosen from the following set of qualities defined as $s_i^k \in [0, \bar{s}]$ where $\bar{s}$ is any finite number. Production costs do not depend on quality and are set to 0. Since undifferentiated firms make no profit the qualities are assumed to be different, given by $s_1^0 \leq \frac{4}{7} s_2^0$, indicating that firm 1 is the low quality firm and firm 2 is the high quality firm. We focus on pure strategies. Consumers' preferences are the same as described in the model section above. After deriving the corresponding demand functions, we get for the corresponding equilibrium prices

$$p_1^0\left(s_1^0, s_2^0\right) = \frac{s_1^0\left(s_2^0 - s_1^0\right)}{4s_2^0 - s_1^0}, \text{ and } p_2^0\left(s_1^0, s_2^0\right) = \frac{2s_2^0\left(s_2^0 - s_1^0\right)}{4s_2^0 - s_1^0}.$$

For demand, we get

$$D_1^0\left(s_1^0, s_2^0\right) = \frac{s_2^0}{4s_2^0 - s_1^0}, \text{ and } D_2^0\left(s_1^0, s_2^0\right) = \frac{2s_2^0}{4s_2^0 - s_1^0}.$$

Profits are

$$\Omega_1^0\left(s_1^0, s_2^0\right) = \frac{s_1^0 s_2^0\left(s_2^0 - s_1^0\right)}{\left(4s_2^0 - s_1^0\right)^2}, \text{ and } \Omega_2^0\left(s_1^0, s_2^0\right) = \frac{4s_2^{0^2}\left(s_2^0 - s_1^0\right)}{\left(4s_2^0 - s_1^0\right)^2}. \tag{32}$$

Reduced-form profit functions are continuous and differentiable, given by

$$\frac{\partial \Omega_1^0\left(s_1^0, s_2^0\right)}{\partial s_1^0} = \frac{s_2^{0^2}\left(4s_2^0 - 7s_1^0\right)}{\left(4s_2^0 - s_1^0\right)^3} \gtreqless 0 \text{ for } s_1^0 \lesseqgtr \frac{4}{7}s_2^0, \text{ and} \tag{33}$$

$$\frac{\partial \Omega_2^0\left(s_1^0, s_2^0\right)}{\partial s_2^0} = \frac{4s_2^0\left(2s_1^{0^2} - 3s_1^0 s_2^0 + 4s_2^{0^2}\right)}{\left(4s_2^0 - s_1^0\right)^3} > 0. \tag{34}$$

$$\frac{\partial \Omega_1^0\left(s_1^0, s_2^0\right)}{\partial s_2^0} = -\frac{s_1^{0^2}\left(s_1^0 + 2s_2^0\right)}{\left(s_1^0 - 4s_2^0\right)^3} > 0, \text{ and} \tag{35}$$

$$\frac{\partial \Omega_2^0\left(s_1^0, s_2^0\right)}{\partial s_1^0} = \frac{4s_2^{0^2}\left(s_1^0 + 2s_2^0\right)}{\left(s_1^0 - 4s_2^0\right)^3} < 0. \tag{36}$$

$$\frac{\partial^2 \Omega_1^0\left(s_1^0, s_2^0\right)}{\partial s_1^{0^2}} < 0, \text{ and } \frac{\partial^2 \Omega_2^0\left(s_1^0, s_2^0\right)}{\partial s_2^{0^2}} < 0. \tag{37}$$

From equation (33) we see that the low quality firm's profits first increase in quality since more consumers buy the new product (*demand effect*). But the closer the product quality is moved towards the competitor's product the higher is the price competition (*strategic effect*) which decreases the low quality firm's profits. When both product qualities are identical Bertrand competition drives firms' profits to zero. The low quality provider's optimal distance to the high quality product is given by the point where the *demand effect* and the *strategic effect* are balancing each other. The high quality firm increases profits by offering a higher product quality. We get the result of 'maximal product differentiation' where in equilibrium firms maximally differentiate their products. The low quality firm offers the lowest feasible product quality and the high quality firm offers the highest feasible product quality.

Appendix 2: Intermediate Quality Innovation by the High Quality Firm (Case *b*)

When the high quality firm offers a new product in the intermediate quality area the sequence of qualities offered in the market is given by $s_1^0 < s_2^1 < s_2^0$. Compared to case a, the qualities of the high quality firm's products are in reverse order. Hence, the same results as for case a apply, setting $s_2^0 = s_2^1$, and vice versa.
The total derivative of the high quality firm's reduced-form profit function with respect to its original product quality is given by

$$\frac{d\pi_2^{1,0}(s_1^0, s_2^1, s_2^0)}{ds_2^0} = \frac{s_2^1\left(s_1^0 - s_2^1\right)}{\left(s_1^0 - 4s_2^1\right)\left(s_2^1 - s_1^0\right)} + \frac{4s_2^1 s_2^0 - s_1^0\left(3s_2^1 + s_2^0\right)}{4\left(s_1^0 - 4s_2^1\right)\left(s_2^1 - s_2^0\right)} = \frac{1}{4}. \tag{38}$$

Appendix 3: Low Quality Innovation by the High Quality Firm (Case *c*), or High Quality Innovation by the Low Quality Firm (Case *d*)

In case c, the high quality firm (firm i) introduces a new product in the low quality area $s_2^1 < s_1^0$ and keeps the first product in the market. When the low quality firm (firm i) offers a new product in the high quality area (case d), the results are identical to case c, setting $s_1^0 = s_2^1$, $s_2^0 = s_1^0$, and $s_1^1 = s_2^0$. Focusing on case c, firms' objective functions are given by

$$\pi_1^0(p_1^0, D_1^0) = p_1^0 D_1^0\left(\cdot\right), \text{ and}$$

$$\pi_2^{1,0}\left(p_2^1, D_2^1, p_2^0, D_2^0\right) = p_2^1 D_2^1\left(\cdot\right) + p_2^0 D_2^0\left(\cdot\right) - F\left(s_i^1\right).$$

Each firm maximizes its objective function with respect to its own product price. The first order condition for the low quality firm, is given by

$$\frac{\partial \pi_1^0(p_1^0, D_1^0)}{\partial p_1^0} \equiv 0 \Longrightarrow p_1^0\left(p_2^1\right) = \frac{2s_1^0 p_2^1\left(s_2^1 - s_1^0\right)}{s_2^1\left(3s_1^0 + s_2^0\right) - 4s_1^0 s_2^0}.$$

The first order condition for the high quality firm with respect to the new product price

$$\frac{\partial \pi_2^{1,0}\left(p_2^1, D_2^1, p_2^0, D_2^0\right)}{\partial p_2^1} \equiv 0 \Longrightarrow p_2^1\left(p_1^0\right) = \frac{p_1^0 s_2^1}{s_1^0},$$

and with respect to its original product price,

$$\frac{\partial \pi_2^{1,0}\left(p_2^1\left(p_1^0\right), D_2^1, p_2^0, D_2^0\right)}{\partial p_2^0} \equiv 0 \Longrightarrow p_2^0\left(p_1^0\right) = \frac{p_1^0 - s_1^0 + s_2^0}{2}.$$

The reaction functions are strictly monotone and have a unique Nash equilibrium. Solving the first order conditions yields the corresponding equilibrium prices

$$p_2^1(s_2^1, s_1^0, s_2^0) = \frac{s_2^1\left(s_1^0 - s_2^1\right)\left(s_1^0 - s_2^0\right)}{2\Psi},\ p_1^0(s_2^1, s_1^0, s_2^0) = \frac{s_1^0\left(s_1^0 - s_2^1\right)\left(s_1^0 - s_2^0\right)}{\Psi},$$

$$p_2^0(s_2^1, s_1^0, s_2^0) = \frac{\left(s_2^0 - s_1^0\right)}{2\left(1 + \frac{s_1^0\left(s_1^0 - s_2^1\right)}{3s_2^1 s_1^0 + s_2^1 s_2^0 - 4s_1^0 s_2^0}\right)}.$$

Substituting these gives us the equivalent demand

$$D_2^1(s_2^1, s_1^0, s_2^0) = \frac{s_1^0\left(s_1^0 - s_2^0\right)}{2\Psi},\ D_1^0(s_2^1, s_1^0, s_2^0) = \frac{s_1^0\left(s_2^1 - s_2^0\right)}{\Psi}, \text{ and}$$

$$D_2^0(s_2^1, s_1^0, s_2^0) = \frac{\left(-4s_1^0 s_2^0 + s_2^1\left(3s_1^0 + s_2^0\right)\right)}{2\Psi}.$$

Similarly, firms' profits in the product market are

$$\pi_2^{1,0}(s_2^1, s_1^0, s_2^0) = \frac{s_2^1\left(s_1^0 - s_2^1\right)s_1^0\left(s_1^0 - s_2^0\right)^2}{4\Psi^2} + \frac{\left(s_2^0 - s_1^0\right)\left(-4s_1^0 s_2^0 + s_2^1\left(3s_1^0 + s_2^0\right)\right)}{4\Psi\left(1 + \frac{s_1^0\left(s_1^0 - s_2^1\right)}{\left(3s_2^1 s_1^0 + s_2^1 s_2^0 - 4s_1^0 s_2^0\right)}\right)} - \gamma\left(s_i^1 - s_i^0\right)^2,$$

and

$$\pi_1^0(s_2^1, s_1^0, s_2^0) = \frac{s_1^{0^2}\left(s_1^0 - s_2^1\right)\left(s_2^1 - s_2^0\right)\left(s_1^0 - s_2^0\right)}{\Psi^2}, \tag{39}$$

The derivative of the high quality firm's reduced-form profit function with respect to its new product quality is given by

$$\frac{\partial \pi_2^{1,0}(s_2^1, s_1^0, s_2^0)}{\partial s_2^1} = \frac{s_1^{0^2}\left(s_1^0 - s_2^0\right)^2\left(s_2^1\left(-22s_1^0 + s_2^0\right) + s_1^0\left(s_1^0 + 20s_2^0\right)\right)}{4\left(2s_2^1 s_1^0 + s_1^{0^2} + s_2^1 s_2^0 - 4s_1^0 s_2^0\right)^3} < 0. \tag{40}$$

where $\Psi = \left(2s_2^1 s_1^0 + s_1^{0^2} + s_2^1 s_2^0 - 4s_1^0 s_2^0\right)$.

Appendix 4: Intermediate or Low Quality Innovation by the Low Quality Firm (Case e or f)

In case e, the low quality firm offers a new product in the intermediate quality area ($s_1^0 < s_1^1 < s_2^0$) and keeps the first product in the market. The results for case f are identical to case e, setting $s_1^1 = s_1^0$, and vice versa. Focusing on case e, firms' objective functions are given by

$$\pi_1^{0,1}\left(p_1^0, D_1^0, p_1^1, D_1^1\right) = p_1^0 D_1^0(\cdot) + p_1^1 D_1^1(\cdot),$$
$$\pi_2^0(p_2^0, D_2^0) = p_2^0 D_2^0(\cdot).$$

Each firm maximizes its objective function with respect to its own product price. The first order condition for the low quality firm, with respect to its original product price is given by

$$\frac{\partial \pi_1^{0,1}(p_1^0, D_1^0, p_1^1, D_1^1)}{\partial p_1^0} \equiv 0 \Longrightarrow p_1^0\left(p_1^1\right) = \frac{p_1^1 s_1^0}{s_1^1}$$

and with respect to its new product price, internalizing the price effect of its new product price on its original product price is given by

$$\frac{\partial \pi_1^{0,1}(p_1^0\left(p_1^1\right), D_1^0, p_1^1, D_1^1)}{\partial p_1^1} \equiv 0 \Longrightarrow p_1^1\left(p_2^0\right) = \frac{p_2^0 s_1^1}{2 s_2^0}.$$

The first order condition for the high quality firm, is

$$\frac{\partial \pi_2^0(p_2^0, D_2^0)}{\partial p_2^0} \equiv 0 \Longrightarrow p_2^0\left(p_1^1\right) = \frac{p_1^1 - s_1^1 + s_2^0}{2}.$$

The reaction functions are strictly monotone and have a unique Nash equilibrium. Solving the first order conditions yields the corresponding equilibrium prices

$$p_1^0(s_1^0, s_1^1, s_2^0) = \frac{s_1^0\left(s_1^1 - s_2^0\right)}{s_1^1 - 4s_2^0},\ p_1^1(s_1^1, s_2^0) = \frac{s_1^1\left(s_1^1 - s_2^0\right)}{s_1^1 - 4s_2^0},$$
$$\text{and } p_2^0(s_1^1, s_2^0) = \frac{2s_2^0\left(s_1^1 - s_2^0\right)}{s_1^1 - 4s_2^0}.$$

The demand is

$$D_1^0(\cdot) = 0, D_1^1\left(s_1^1, s_2^0\right) = \frac{s_2^0}{4s_2^0 - s_1^1}, \text{ and } D_2^1 = \frac{2s_2^0}{4s_2^0 - s_1^1}.$$

Firms' profits are as follows

$$\pi_1^0(\cdot) = 0,\ \pi_1^1\left(s_1^1, s_2^0\right) = \frac{s_1^1 s_2^0\left(s_2^0 - s_1^1\right)}{\left(s_1^1 - 4s_2^0\right)^2}, \text{ and}$$

$$\pi_2^0\left(s_1^1, s_2^0\right) = \frac{4s_2^{0^2}\left(s_2^0 - s_1^1\right)}{\left(s_1^1 - 4s_2^0\right)^2}. \tag{41}$$

The derivative of the low quality firm's reduced-form profit function with respect to the original product quality is given by

$$\frac{\partial \pi_1^{0,1}\left(s_1^0, s_1^1, s_2^0\right)}{\partial s_1^0} = \frac{2s_1^1\left(s_2^0 - s_1^1\right)^3}{\left(s_1^0 - s_1^1\right)\left(s_1^1 - 4s_2^0\right)^2} < 0. \tag{42}$$

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