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ABSTRACT

A Comparison of Multiple-Unit All-Pay and Winner-Pay Auctions under Incomplete Information

by Yasar Barut, Dan Kovenock, and Charles Noussair*

This paper examines the properties of independent-private-value all-pay and winner-pay auctions when there are multiple units sold. We study bidding behavior, efficiency and revenue in a set of nine experimental sessions, each with six bidders. All-pay auctions were played in six of the sessions, three sessions with four units and three sessions with two units auctioned. A four-unit winner-pay auction was played in three of the sessions. Our data show that the all-pay auction and the winner-pay auction are empirically revenue equivalent and yield higher revenue than the risk neutral Bayesian equilibrium. Revenue is higher in the all-pay auction when K=2 than when K=4, despite the fact that Bayesian equilibrium revenues are identical for the two cases. Our evidence also suggests that the winner-pay auction is more likely than the all-pay auction to lead to a Pareto-efficient allocation.

ZUSAMMENFASSUNG

Ein Vergleich von All-Pay- und Winner-Pay-Auktionen mehrerer Einheiten bei unvollständiger Information

In diesem Aufsatz werden die Eigenschaften von independent-private-value all-pay- und winner-pay-Auktionen untersucht, bei denen jeweils mehrere Einheiten verkauft werden. Es wird das Gebots-Verhalten, die Effizienz und der Erlös in einem Set von neun Experimenten mit je sechs Teilnehmern betrachtet. In sechs der Experimente werden all-pay-Auktionen durchgeführt, wovon in drei Durchgängen je vier Einheiten und in den anderen drei je zwei Einheiten versteigert werden. In den drei restlichen Experimenten werden winner-pay-Auktionen durchgeführt, wobei je vier Einheiten versteigert werden. Das Experiment zeigt, daß all-pay- und winner-pay-Auktionen ähnliche Erlöse erzielen, die jedoch höher sind als der im Bayesianischem Gleichgewicht mit risikoneutralen Bietern spieltheoretisch vorausgesagte Gewinn. Bei all-pay-Auktionen ist der Erlös höher, wenn K=2 als im Fall K=4, während die theoretisch berechneten Gleichgewichtserlöse in beiden Fällen identisch sind. Diese Ergebnisse legen nahe, daß winner-pay-Auktionen eher zu einer Pareto-effizienten Allokation führen als all-pay-Auktionen.

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1 Introduction

Since the classic paper by Vickrey (1961) economists have used game-theoretic methods to analyze the economic properties of auctions. Vickrey derived the Bayesian equilibrium of a game in which N risk neutral agents simultaneously bid for a single object. It was common knowledge to all agents that each agent's valuation of the object was an independent draw from a uniform distribution. Vickrey restricted his attention to winner-pay mechanisms, in which a bidder was required to make a payment only if he obtained an object. He focused on the determination of equilibrium bidding strategies and the revenue generated by the auctions. Since Vickrey's pioneering work, auction theory has become perhaps the greatest success of game-theoretic analysis. Equilibrium bidding strategies and their revenue and efficiency properties have been derived for a wide array of auction environments, extending Vickrey's work to cover bidder risk-aversion, correlated values, asymmetric bidders, and different winner-pay institutions. As it has evolved, the theory has generated both innovative theoretical techniques and relevant policy prescriptions.

One of the most important extensions of auction theory is the analysis of multiple-object auctions, which is the topic of the research described in this paper. One reason for the emphasis on multiple-object auctions is the fact that they are widely observed in the field.[1] Vickrey (1961, 1962) himself first generalized his original work to cover the case of multiple-unit winner-pay auctions, where N risk neutral bidders simultaneously bid to receive one of K, $1 < K < N$, identical units of the object auctioned. A later extension by Harris and Raviv (1981) handled the case of non-uniform distributions and identical concave utility functions. Maskin and Riley (1992) recently analyzed optimal multiple object winner-pay auctions.

The behavioral properties of different types of auctions have been extensively explored by experimental economists (see Kagel, 1995, for a survey). Indeed, the literature on auctions has

[1] Better-known examples of multiple-unit auctions include the auctioning of government debt, initial issues of common stocks, import quotas, and produce.

been one of the most active in experimental economics. It has generated a considerable body of empirical results which have, in turn, led to new theory (see for example Cox et al., 1982), and have provided the context for general methodological debate (Harrison, 1989). A subset of this research has considered auctions of multiple identical units. Early experimental studies of multiple-unit auctions (Smith, 1967; Belovicz, 1979; Miller and Plott, 1985; Burns, 1985) focused on the policy relevant issue of the revenue generating properties of auctions. As the game-theoretic literature on multiple-object auctions progressed, experiments were conducted to test equilibrium predictions of models of bidding under incomplete information (see for example Cox et al., 1984, 1985; McCabe et al. 1990, 1991 and Kagel and Levin (1998).

The point of departure of this paper is that, in principle, the rules of an auction need not have the property that bidders do not make payments when they fail to obtain an object. An alternative class of auctions that has evolved separately in the literatures on theoretical biology and public choice, involves winners and losers alike forfeiting their bids. In the single-prize all-pay auction, for instance, each player submits a non-refundable bid and only the highest bidder receives the prize. The war-of-attrition is a second-price all-pay auction in which the high-bidder wins the prize and pays the amount bid by the second-highest bidder; all other players forfeit their bids. While these latter two types of contests have received much attention outside the confines of auction theory, it is only very recently that their properties as auction mechanisms have been examined with the rigor of the analysis applied to winner-pay mechanisms (see, for instance, Krishna and Morgan (1996) or Amann and Leininger (1995)). The analysis of multiple-object all-pay auctions has received even less attention. Holt (1979) and Holt and Sherman (1982) provide the first theoretical characterization of equilibrium in multiple-unit all-pay auctions. The first paper characterizes the risk-neutral equilibrium in the context of research grant competition and the second applies this to equilibrium queuing for goods. Glazer and Hassin (1988) analyse a specific example of a multiple-unit all-pay auction.

Several recent experimental studies have considered bidding behavior in single-unit all-pay auctions under complete information and have found overdissipation. That is, the bids tend to be more aggressive than in the Nash equilibria (Davis and Reilly, (1998); Potters, de Vries and van Winden (1998); and Gneezy and Smorodinsky (1999)). Amann and Leininger (1997) examined the properties of single-unit all-pay auctions under incomplete information and also found substantial overbidding relative to the Bayesian-Nash equilibrium. Total revenue generated varied between 1.6 and 2.5 times the maximal valuation drawn. To our knowledge, to date no experimental work has been carried out to empirically examine the properties of all-pay auctions when multiple units are sold. This paper fills this void by examining multiple unit all pay auctions in a simple independent private values setting. We compare bidding behavior, efficiency and revenue in a set of nine experimental sessions, each consisting of 21 separate auction trials. We use a uniform distribution of valuations and an independent private values information structure as formulated by Vickrey and described in the first paragraph of this section.

In each of the nine sessions of our experiment, six subjects were given the role of bidders. Multiple-unit sealed-bid all-pay auctions were played in six of the sessions: three with four units auctioned and three with two units auctioned. A four-unit winner-pay sealed-bid auction was played in three of the sessions. In the sections to follow we lay out the theoretical framework, the experimental design, and the results of our experiments. Section 2 describes the all-pay and winner-pay mechanisms examined and derives the equilibrium bid functions for the parameters that we use. An expression for expected revenue is derived from the Revenue Equivalence Theorem. Section 3 lays out the experimental procedures. Section 4 reviews the experimental data and lists five conclusions as "results" and one conjecture, strongly suggested by the data, but requiring further experimental work to verify.

Our main results are consistent with past experimental evidence, when such evidence exists, and provide interesting new insights upon which future experimental work may be based. First, both the all-pay auction and the winner-pay auction yield higher revenue than the risk-neutral Bayesian equilibrium. Hence, we replicate previous work by Cox, Smith, and Walker

(1984) showing overbidding by inexperienced bidders for the case of independent-private-value multiple-prize winner-pay auctions. We also show that previous results on the single-prize all-pay auction under complete information, carry over to the independent-private-values multiple-prize environment in a natural way. Bidders on average bid more than is dictated by the risk-neutral Bayesian equilibrium.

Interestingly, the overdissipation that arises relative to the Bayesian equilibrium is not evenly distributed across different underlying values in the two auctions. In the winner-pay auction, bidders uniformly over-dissipate for all realizations of unit values; bidders tend to bid above the Bayesian equilibrium level for values throughout the support of the distribution of valuations. On the other hand, bidders with low values in the all-pay auction tend to underbid, with a high incidence of zero bids. Overbidding appears to occur mainly in ranges of values that are likely to be close to the relevant order statistic corresponding to the margin between receiving and not receiving a prize. These tend to be for higher values in the case of two units ($K = 2$) than in the case of four units ($K = 4$). Despite the fact that both mechanisms overdissipate relative to the Bayesian equilibrium, our second result states that the all-pay auction and the winner-pay auction yield revenue no different from each other; the two auctions are empirically revenue equivalent. Moreover, in a manner consistent with previous work on winner-pay auctions (Cox et al., 1984), our third result shows that the revenue is higher in the all-pay auction when K=2 than when K=4. This arises despite the fact that the Bayesian equilibrium revenues under the two auction procedures are identical.

Our fourth result compares the efficiency of the two auction mechanisms. Our evidence suggests that the winner-pay auction is more likely to lead to a Pareto-efficient allocation than the all-pay auction. That is, the winner-pay auction is more likely to allocate the units to the agents who value the units the most. This arises because in an intermediate range of values the all-pay auction exhibits a high variance in bidding behavior, with bidders either bidding extremely aggressively, by placing very high bids with low margins, or extremely passively, by bidding at or near zero. In this range, the rank order of bids often differs from the rank order of valuations.

Our fifth and final result compares the revenue in the two auctions studied in this paper with those studied by Alsemgeest et al. (1998). There, the parameters $N = 6$ and $K = 4$ were used to compare revenues in a Uniform-Price Sealed Bid Auction with Lowest-Accepted-Bid Pricing and an English Clock Auction. From the Revenue Equivalence Theorem we know that the expected revenue generated by a symmetric Bayesian equilibrium of these two mechanisms is identical to that of the winner-pay and all-pay auctions that we analyze below. However, while our experimental results indicate that both the winner-pay and all-pay auctions yield the same revenue as the Uniform-Price Sealed Bid Auction with Lowest-Accepted-Bid Pricing, they yield higher revenue than the English Clock Auction.

Of course, the time horizon in our experiments, 20 periods (with one extra practice period that was not tabulated), is quite likely to be too short to accurately measure long run behavior. We end our results section by formulating a conjecture concerning the long run evolution of revenue as the auctions are repeated. We postulate that there is a convergence process governing the evolution of the revenue over time. For the convergence process specified, revenue converges to the Bayesian equilibrium level for $K = 4$ for both the winner-pay and all-pay auctions, though it converges to a level higher than the Bayesian equilibrium for the case where $K = 2$. This is consistent with an individual level analysis (reported in section 4.2) which indicates that individual bidding behavior in all-pay resembles the Bayesian equilibrium more closely when $K = 4$ than when $K = 2$.

2 Theoretical Predictions

2.1 The Environment

In our experiment, we study an independent-private-value environment with the following structure. There are N bidders, indexed by i, each of whom draws a value v_i independently from a common distribution $F(v)$ with support on the interval $[0, \overline{v}]$. v_i is the value to bidder i of obtaining one of K identical goods to be sold in an auction. At the time of bidding, each bidder i knows N, K, v_i and $F(v)$. Bidders do not know the values which other bidders have, but do know that every other player draws a valuation from $F(v)$. The above is common knowledge to all players.

2.2 Equilibrium in the All-Pay Auction

There are two games that we analyze. The first game is the *all-pay* auction. Each bidder, after drawing his valuation, simultaneously submits a bid, where i's bid is denoted by b_i. The auctioneer orders the N bids from highest to lowest. Each of the K highest bidders receives one unit and thus obtains a payoff equal to his valuation. If there is a tie for the Kth highest bid the unit is given to the each of the tied demanders with equal probability. Each of the N bidders pays the amount of his bid, regardless of whether or not he obtains a unit and thus incurs a loss equal to his bid. Therefore the final payoff to bidder i is $v_i - b_i$ if he obtains a unit, and $-b_i$ if he does not obtain a unit.

The expected payoff to bidder i is given by:

$$E\pi_i = v_i Z(b_i, N, K, F(v)) - b_i \tag{1}$$

where Z equals the probability that i obtains a unit, which equals the probability that b_i is one of the K highest bids in the auction.

Suppose that there is a common Bayesian equilibrium bidding function $B(v)$ used by all bidders, such that $\frac{dB(v)}{dv} > 0, \forall v \in [0, \overline{v}]$. That is, consider a symmetric strictly monotonic Bayesian equilibrium in this game. The assumptions of symmetry and strict monotonicity seem natural assumptions to use when providing a benchmark to laboratory data. The symmetry of the players' position in the bidding process and the simultaneous nature of the game would make it difficult to coordinate on asymmetric equilibria. Monotonicity is a simple assumption that bids are increasing in valuations, a concept which is likely to be understood by the subjects.

Since $B(v)$ is strictly monotonic it is invertible. Let $V(b) = B^{-1}(b)$. Under the assumptions of symmetry and strict monotonicity of the bidding strategies, we can write $Z(b_i, N, K, F(v))$ as:

$$Z = \sum_{j=N-K}^{N-1} \left(\frac{(N-1)!}{(N-j-1)!j!}\right) F(V(b_i))^j (1 - F(V(b_i)))^{N-j-1} \tag{2}$$

Since the bidding function is symmetric and monotonic, the probability a given bidder other than i bids less than b_i, is equal to the probability that a given bidder has a valuation less than v_i. Thus, Z equals the probability that at least $N - K$ out of $N - 1$ bidders other than i have valuations less than v_i.

In all treatments of the experiment $N = 6$ and $F(v)$ is uniform on the interval $[0, \overline{v}]$. K equals either 2 or 4 depending on the treatment. In the case in which $K = 2$,

$$Z = \left(\frac{V(b_i)}{\overline{v}}\right)^5 + 5\left(\frac{V(b_i)}{\overline{v}}\right)^4 \left(1 - \frac{V(b_i)}{\overline{v}}\right) \tag{3}$$

In the above equation $(\frac{V(b_i)}{\overline{v}})^5$ is the probability that exactly 5 out of 5 bidders other than i have valuations less than v_i and $5(\frac{V(b_i)}{\overline{v}})^4(1 - \frac{V(b_i)}{\overline{v}})$ is the probability that exactly 4 out of 5 bidders other than i have valuations less than v_i. In, equilibrium, bidder i must be choosing b_i to maximize profit and thus maximizing (1). The first order condition for the case of $N = 6$

and $K = 2$ is:

$$\frac{dE\pi}{db_i} = 0 = \frac{20v_i}{\overline{v}}(\frac{V(b_i)}{\overline{v}})^3(1 - \frac{V(b_i)}{\overline{v}})\frac{dV(b_i)}{db_i} - 1 \quad (4)$$

Because the strategy profile is symmetric, $V(b_i) = v_i$. Using symmetry and the invertibility of $V(b_i)$ we can solve for $\frac{dB(v_i)}{dv_i}$:

$$\frac{dB(v_i)}{dv_i} = 20(\frac{v_i}{\overline{v}})^4(1 - \frac{v_i}{\overline{v}}) \quad (5)$$

Since it is a dominated strategy for a player to bid an amount greater than his valuation, it must be the case that $B(0) = 0$. If we impose the boundary condition that $B(0) = 0$, we obtain:

$$B(v_i) = \overline{v}[4(\frac{v_i}{\overline{v}})^5 - \frac{10}{3}(\frac{v_i}{\overline{v}})^6] \quad (6)$$

Equation (6) defines a symmetric strictly monotonic Bayesian equilibrium bidding function for $N = 6$ and $K = 2$. In the case where $K = 4$,

$$Z = 1 - (1 - \frac{V(b_i)}{\overline{v}})^5 - 5\frac{V(b_i)}{\overline{v}}(1 - \frac{V(b_i)}{\overline{v}})^4 \quad (7)$$

The above expression equals 1 minus the probability that all bidders other than i have valuations higher than v_i minus the probability that 4 out of 5 of the other bidders have valuations greater than i. The first order necessary condition is:

$$\frac{dE\pi}{db_i} = 0 = 20\frac{v_i}{\overline{v}}(1 - \frac{V(b_i)}{\overline{v}})^3\frac{V(b_i)}{\overline{v}}\frac{dV(b_i)}{db_i} - 1 \quad (8)$$

Solving for $\frac{dB(v_i)}{dv_i}$, we find:

$$\frac{dB(v_i)}{dv_i} = 20(\frac{v_i}{\overline{v}})^2 - 60(\frac{v_i}{\overline{v}})^3 + 60(\frac{v_i}{\overline{v}})^4 - 20(\frac{v_i}{\overline{v}})^5 \quad (9)$$

which, together with the boundary condition $B(0) = 0$ implies:

$$B(v_i) = \overline{v}[\frac{20}{3}(\frac{v_i}{\overline{v}})^3 - 15(\frac{v_i}{\overline{v}})^4 + 12(\frac{v_i}{\overline{v}})^5 - \frac{10}{3}(\frac{v_i}{\overline{v}})^6] \tag{10}$$

Equation (10) defines a symmetric, strictly monotonic Bayesian equilibrium to the all-pay auction in the case of $N = 6$ and $K = 4$.

2.3 Equilibrium in the Winner-Pay Auction

The only difference between the rules of the winner-pay auction and the all-pay auction is that in the winner-pay auction, bidders who fail to obtain units are not required to make payments to the seller. More precisely, the rules of the winner-pay auction are the following. Each bidder, after drawing his valuation, simultaneously submits a bid, where i's bid is denoted by b_i. The auctioneer orders the N bids from highest to lowest. The K highest bidders each receive one unit and obtain a payoff equal to their valuation for the object they purchase. If there is a tie for the Kth highest bidder the unit is given to the each of the tied demanders with equal probability. Each of the K bidders who obtains a unit pays the amount of his bid. The $N - K$ bidders who do not obtain units do not make any payments. Therefore the final payoff to bidder i is $v_i - b_i$ if he obtains a unit and 0 if he does not obtain a unit. The winner-pay auction is also commonly referred to as the discriminatory or discriminative sealed-bid auction.

The expected payoff to each bidder in the winner-pay auction is given by:

$$(v_i - b_i)Z(b_i, N, K, F(v)) \tag{11}$$

where Z is the probability that the bidder wins a unit in the auction, or alternatively the probability that at least $N - K$ other bidders have a value less than that that of i. For the uniform distribution with $N = 6$ and $K = 4$, Z is the same expression as in equation (7). The first order condition is:

$$\frac{dE\pi}{db_i} = 0 = 20(v_i - b_i)(1-(\frac{V(b_i)}{\overline{v}}))^3 \frac{V(b_i)}{\overline{v}} \frac{dV(b_i)}{db_i} - 1 + (1-\frac{V(b_i)}{\overline{v}})^5 + 5\frac{V(b_i)}{\overline{v}}(1-\frac{V(b_i)}{\overline{v}})^4 \quad (12)$$

Inverting $V(b_i)$, imposing the symmetry condition $V(b_i) = v_i$, and solving for $\frac{dB}{dv_i}$, we have:

$$\frac{dB}{dv_i} = \frac{20(v_i - b_i)(1-(\frac{V(b_i)}{\overline{v}}))^3 \frac{V(b_i)}{\overline{v}}}{1-(1-\frac{V(b_i)}{\overline{v}})^5 - 5\frac{V(b_i)}{\overline{v}}(1-\frac{V(b_i)}{\overline{v}})^4} \quad (13)$$

A solution to (13) is given by:

$$B(v_i) = \overline{v}[\frac{-20\frac{v_i}{\overline{v}} + 45(\frac{v_i}{\overline{v}})^2 - 36(\frac{v_i}{\overline{v}})^3 + 10(\frac{v_i}{\overline{v}})^4}{-30 + 60\frac{v_i}{\overline{v}} - 45(\frac{v_i}{\overline{v}})^2 + 12(\frac{v_i}{\overline{v}})^3}] \quad (14)$$

There are two appropriate boundary conditions in specifying equation (14). As in the all-pay auction, it is a dominated strategy for a player to bid an amount greater than his valuation, so that in equilibrium it must be the case that $B(0) = 0$. However, the function $\frac{dB}{dv_i}$ has a singularity at $v_i = 0$, due to division by 0, which does not allow the specification of a unique solution. To obtain an alternative boundary condition we apply the revenue equivalence theorem, which is discussed in more detail in the next subsection. One consequence of the Revenue Equivalence Theorem (Myerson, 1981; Bulow and Klemperer, 1992), is that any two auctions, in which in equilibrium (a) the item is sold to the bidders with the highest valuations and (b) the expected payoff of a bidder with valuation 0 equals 0, must generate the same expected payoff to bidders of the same type. Consider a bidder with valuation $\overline{v}$. Due to the assumption of monotonicity of the bidding functions, this bidder obtains a unit with probability 1 in both the winner-pay and the all-pay auctions. Since he pays $\frac{\overline{v}}{3}$ in the all-pay auction, he must be paying the same amount in the winner-pay auction, in order for his expected payoff to be equal in the two auctions. Therefore $B(\overline{v}) = \overline{v}/3$ in the winner-pay auction as well.

[Figure 1: About Here]

Figure 1 shows the equilibrium bidding functions for the three treatments of our experiment. Notice that the bidding function for the winner-pay auction is concave and that for the all-pay auction is convexoconcave. Each of the auctions has the property that $B(\overline{v}) = \frac{(N-K)\overline{v}}{N}$.

2.4 Expected Revenue

The equilibrium expected revenue of both the winner-pay and the all-pay auctions is equal to:

$$ER = K \int_0^{\overline{v}} v_{K+1} dF_{K+1}(v_{K+1}) \tag{15}$$

where v_{K+1} equals the $K+1$st highest valuation held by the N bidders and $F_{K+1}(v_{K+1})$ is the cumulative distribution function of v_{K+1}. To see that equation (15) is the correct expression for the expected revenue, consider the Revenue Equivalence Theorem. The theorem states that in an independent-private-value environment such as that studied here, any two auctions in which the Bayesian equilibria have properties (a) and (b), described in section 2.3, must yield the same expected revenue in equilibrium.

Both of the auctions studied in this paper have properties (a) and (b). Note that there also exist demand revealing multi-unit auctions, in which each bidder bids an amount equal to her valuation. One such auction is the *Uniform-Price Sealed-Bid (winner-pay) Auction with Highest-Rejected-Bid Pricing*, introduced by Vickrey (1961), which also satisfies (a) and (b). In this demand revealing auction, each bidder submits a bid simultaneously, and the K highest bidders receive units and pay an amount equal to the $K + 1$st highest bid. In the dominant strategy equilibrium, each winning bidder pays an amount equal to the highest rejected bid, which is equal to the $K + 1$st highest value, implying that the allocation and payment rules satisfy properties (a) and (b). Because the demand revealing auction generates an equilibrium outcome in which each of the K highest bidders pays an amount equal to the $K + 1$st highest valuation, by the Revenue Equivalence Theorem, the expected revenue of any auction satisfying (a) and (b) must equal K times the expected value of the $K+1$st highest valuation. For the uniform distribution the equilibrium expected revenue equals:[2]

$$ER = \frac{\overline{v}K(N-K)}{N+1} \tag{16}$$

In each of the three treatments of our experiment, $\overline{v} = 1,000$ and the equilibrium expected revenue is equal to 1,143 in each period. Of course, for particular realizations of valuations, the actual revenue resulting from equilibrium play will typically differ from 1,143.

3 Procedures in the Experiment

3.1 The Design

We conducted nine experimental sessions, each consisting of one practice period and twenty periods that counted toward subjects' earnings. In each session, six subjects were given the role of bidders in an auction. No subject participated in more than one session of the study. There were three different treatments, and there were three sessions conducted under each treatment. The auction type used in six of the nine sessions was the discriminative all-pay auction described in section 2.2 and in the remaining three sessions, the discriminative winner-pay auction described in section 2.3 was used. In three of the sessions under each auction type, there were four identical units sold and in the remaining three sessions of all-pay, there were two units sold each period. We will refer to the all-pay auction with four units sold as the A4 treatment, the all-pay auction with two units sold as the A2 treatment, and the winner-pay auction with four units sold as the W4 treatment.

In each period, bidders were assigned independent valuations for obtaining any one of the

[2]See Cox et al. (1985) for another derivation of the same expression for expected revenue.

identical units for sale. The valuations were denominated in terms of the experimental currency and were integers drawn from the discrete uniform distribution with support on the $[1, 1000]$ interval. New valuations were drawn each period so that an individual's valuation differed from period to period. We used each of three different sets of 120 valuations, six valuations for each of the 20 periods for three sessions, once in each of the three treatments. We will call the three sets of valuations R, V, and P, so that, for example, session A4R indicated the session during which the all-pay auction was used to sell 4 units to demanders who had the set of valuations R. This structure provided us with 60 paired observations to compare any two of the three treatments. A given period under valuation set R had the exact same assignment of valuations; for example, the valuation of bidder i in period j of A4R, W4R, and A2R were identical across the three treatments.

Sessions A4P, W4P and A2P were conducted at Purdue University and the other 6 sessions were run at Rice University. All of the auctions were run "by hand" and were not computerized. All subjects in both locations were undergraduate students. All subjects received a participation fee, which they could use to pay off any losses they incurred. In the session in which $K = 4$ the fee was \$10, and when $K = 2$ the fee was \$14 in the two sessions run at Rice and \$10 in the session run at Purdue. In A4 and W4, 400 units of the experimental currency corresponded to \$1. In A2 a conversion rate of 200 to \$1 was used at Purdue and a conversion rate of 250 to \$1 was used at Rice.[3] In all treatments, subjects also received profits during each period equal to the valuation for any unit they purchased minus the amount they spent to acquire the unit. In the all-pay auctions, this meant that subjects also lost an amount equal to their bids when they did not receive a unit in the auction.[4]

[3]The different conversion rates for these auctions were intended to equalize the expected payoff to subjects across treatments since the all-pay auction with two prizes was expected to yield lower payoffs to the subjects compared to the other two auctions with four prizes. We paid subjects a higher participation fee and used a lower conversion rate at Rice because we observed that the subjects in A2P at Purdue, which had been conducted earlier, lost a substantial amount of money before making any profit and received rather low final earnings.

[4]We decided to pay the subjects according to their loss or profit for each period rather than paying according to the outcome of one or more randomly chosen periods. The second approach would in theory eliminate the income effect that potentially could result from treating the accumulated earnings of a subject as a state variable.

3.2 Information Structure

Communication between subjects was not allowed during the experiment. It was common knowledge that each subject knew his own valuation and the distribution from which each player's valuation was drawn, but not the actual valuations of his competitors, at the time of bidding. Each subject knew the number of prizes and the number of competitors. During the course of the session, additional information became available. During the three sessions conducted at Purdue, and in the "R" sessions at Rice, the bids submitted during a period were displayed on the blackboard at the end of the period. The winning bids were indicated without publicly revealing the identity of the individuals who submitted the winning bids. In sessions A4V, W4V and A2V, we also posted the valuations along with the bids at the end of the period, without publicly revealing the identity of the individuals corresponding to the posted bids and valuations. Thus, in the V sessions, after the end of the period, subjects could associate any player's bid with her valuations, but could not associate the bids or values with the identification number or identity of the bidder. The history of bids (and corresponding valuations in the V sessions) remained on the blackboard for the rest of the session. Table 1 summarizes some relevant characteristics of the nine sessions.

However in the all-pay auctions, especially with only two prizes and six subjects, the second approach could be expected to result in very low earnings for some of the subjects who failed to win a prize, if earnings are made salient.

Table 1: Characteristics of the Sessions Conducted

Session	Type of Auction	Number of Units	Location	Information Displayed	Participation Fee	Conversion Rate
A4P	All-Pay	4	Purdue	–	$10	400/$1
A4V	All-Pay	4	Rice	Valuations	$10	400/$1
A4R	All-Pay	4	Rice	–	$10	400/$1
A2P	All-Pay	2	Purdue	–	$10	200/$1
A2V	All-Pay	2	Rice	Valuations	$14	250/$1
A2R	All-Pay	2	Rice	–	$14	250/$1
W4P	Winner-Pay	4	Purdue	–	$10	400/$1
W4V	Winner-Pay	4	Rice	Valuations	$10	400/$1
W4R	Winner-Pay	4	Rice	–	$10	400/$1

4 Results

4.1 Market-Level Analysis

Figures 2a - 4c illustrate the data from the experiment. In the figures, the bids of each of the six subjects in a session are given by a different symbol and the equilibrium bidding function is indicated by the smooth series of crosses. Figures 2a - c show data from periods 11-20 from all three sessions of all-pay with $K = 4$. Several patterns are evident from the data shown in the figures. The first pattern is the widespread incidence of 0 bidding by demanders with valuations of 300 or less. The second is the convexoconcave pattern of the bids as a function of the valuations, which is the pattern that occurs in the symmetric, monotonic, risk-neutral, Bayesian equilibrium (BE) derived in section 2. The third pattern is that for values between 400 and 800 when $K = 4$, the great majority of the observed bids are in excess of the Bayesian equilibrium level.

[Figures 2-4: About Here]

Similar patterns are observed in figures 3a-c, which graph the observed bids and the equilibrium strategy in the all-pay auction when $K = 2$. The figures illustrate the tendency to submit bids equal to zero for units valued at less than 600. In sessions A2P and A2R, there are very few bids submitted for amounts between 100 and 500. In these two sessions most bidders with valuations greater than 650 submitted bids greater than 600, and the vast majority of bidders with valuations greater than 600 submitted bids greater than than the Bayesian equilibrium prediction. In session A2V, many bids between 100 and 500 are observed and overbidding is less prevalent at higher values.

Figures 4a-4c display data from periods 11-20 of the winner-pay sessions. Several stark differences between the data in figure 4 and those in figures 2 and 3 are evident. In the winner-pay data there is almost no zero bidding, even for very low valuations. Throughout the range of valuations, the bids are a concave function of valuations, as is the case in the Bayesian equilibrium, as well as in the data of Cox et al. (1984, 1985). In session W4R the data track the BE fairly closely on average, whereas in session W4V bidders with high valuations are bidding less than the Bayesian Equilibrium and those with low valuations are bidding higher than the BE level. In session W4P, bidding is higher than the BE throughout the range of valuations.

The first result reported below concerns the overall revenue in the two auctions. The complete data from periods 1-20 of all sessions is used in establishing the result. We observe an overall tendency at the market level for revenue to be higher than the Bayesian equilibrium prediction under both treatments.

Result 1: Both the all-pay auction and the winner-pay auction yield higher revenue than the risk-neutral Bayesian equilibrium.

Support for Result 1: As can be seen in the data in Table 2, the average observed revenue is higher than the BE level of 1,143 in five of the six sessions of all-pay and in two of the three sessions of winner-pay. At the level of the individual period, in 44 of the 60 periods of all-pay with $K = 4$ and in 40 of the 60 periods of winner-pay with $K = 4$, revenue is higher than the BE level. In $K = 2$, the revenue is greater than the BE level in 46 of 60 periods. Using a sign-test,[5] we can reject the hypothesis that the median revenue is equal to the BE level at the 1 percent level of significance in each of the three treatments. □

The data in figures 2-4 suggest that the sources of the excess revenues are different between the two auctions and depend on the number of units sold. Table 3 indicates the percentage of the bids that exceeded the BE level for different ranges of valuations for the pooled data from all sessions. In the winner-pay auction, the higher than predicted revenue in the two treatments indicates a general tendency to bid higher than the prediction. However, in the all-pay auction it is bidders who have valuations in a narrow subset of the support of the distribution of valuations that cause the revenue to exceed the BE level. This range is for intermediate values in the $K = 4$ case and for higher values in the $K = 2$ case.

[5]We consider the variable y_{jt} where $y_{jt} = 0$ if the revenue of period t in session j is less than the BE level and $y_{jt} = 1$ the revenue exceeds the BE. We then test whether or not the variable y_{jt} is binomial with .5 probability that $y_{jt} = 1$.

Table 2: Observed Revenue and Efficiency: All Treatments[6]

Session	Avg. Rev.	Rev > BE	Avg. Eff.	Eff. = 1
A4R	1,167.2	18/20	.959	13/20
A4V	1,314.7	13/20	.979	14/20
A4P	1,191.5	13/20	.958	10/20
Pooled AP K = 4	1,224.5	44/60	.965	37/60
W4R	1,266.1	17/20	.990	16/20
W4V	1,003.1	5/20	.988	13/20
W4P	1,349	18/20	.983	17/20
Pooled WP K = 4	1,192.7	40/60	.987	46/60
A2R	1,201.8	12/20	.953	11/20
A2V	1,089	15/20	.963	10/20
A2P	1,790	19/20	.959	10/20
Pooled AP K = 2	1,360.3	46/60	.958	31/60

Table 3: Percentage of Bids Greater than the Bayesian Equilibrium Prediction: All Treatments

Range of Values	AP, K = 4	AP, K = 2	WP, K = 4
0 - 199	45.1	36.7	66.7
200 - 399	57.6	25.0	67.0
400 - 599	78.9	27.5	57.9
600 - 799	68.1	72.1	61.2
800 - 1000	44.4	87.8	68.8

[6]The variable *Avg. Rev.* indicates the average observed revenue per period over the 20 periods of the session. The practice period is omitted in all of the data analysis reported in this paper. *Rev > BE* indicates the number of periods of the session in which observed revenue was greater than the BE prediction for that period for that auction (the BE prediction depends on the actual valuations drawn that period), expressed as a fraction of the total number of periods, which was always 20, during the session. *Avg. Eff.* indicates the sum of the valuations of bidders who received units during the auction process, divided by the sum of the K highest valuations held by bidders during that period. If Avg. Eff. equals 1, the K highest-valued bidders received the units. The variable *Eff. = 1* indicates the number of periods during the session, in which the realized efficiency equaled 1, expressed as a fraction of the total number of periods during the session.

In the winner-pay auction the majority of bids is higher than the BE level throughout the range of valuations, and the fraction of bids that exceed the BE level is fairly constant throughout the range. In the all-pay auction, for $K = 4$, a great majority of the bids in the interval between 200-800, and especially between 400 and 600, are higher than the BE level, and the differences are large enough to more than completely offset the differences at low values (most of the bids for low-valued units equal 0, but the equilibrium predictions are also very low, so that the differences are small), and at very high values (where only a slight majority are less than the BE prediction). In the case of $K = 2$ most players with valuation below 600 bid less than the BE level but the vast majority of those with values above 600 bid above the BE level.

The fact that the revenue of both auctions is greater than the risk-neutral Bayesian equilibrium level raises the possibility that revenue differs between the two auctions. The 60 paired observations of $K = 4$ data allow us to compare the revenue of the two auctions. However, despite the departures from the theoretical prediction, we are not able to detect a systematic difference in the revenue generated by the two auctions.

Result 2: The all-pay auction and the winner-pay auction yield revenue no different from each other.

Support for Result 2: Under K = 4, the all-pay auction yielded higher revenue than winner-pay in 31 of the 60 periods, and winner-pay yielded higher revenue in 29 of the 60. We cannot reject the hypothesis that the auctions yield the same revenue at any conventional level of significance.[7] □

[7] We can make a rough comparison of the revenue in our A2 treatment with that in the winner-pay auctions of Cox et al. (1984), who report two sessions in which valuations were drawn from a uniform distribution, $N = 6$, and $K = 2$. On average they observe revenue 7.2% greater than the BE. In our all-pay data with the same parameters, we observe revenue 19.1% higher than the BE level. It may be the case that some of the difference is due to the fact that there were 20 periods in our sessions and 30 periods in the sessions of Cox et al. As we discuss later in this section, it appears that revenue is declining as the auction process is repeated.

The next result is a comparison of revenue between the different parameters of the all-pay auction. The parameters were chosen so that the expected revenue in the BE would be identical in the $K = 4$ and $K = 2$ treatments. However, we find that actual observed revenue is greater when $K = 2$ than when $K = 4$.

Result 3: In the all-pay auction revenue is higher when $K = 2$ than when $K = 4$.

Support for Result 3: Since the realization of market demand is identical in the two treatments, we are able to compare each pair of 60 observations from the two treatments. The $K = 2$ treatment generated higher revenue than the $K = 4$ treatment in 37 out of 60 periods. At the 5 percent level of significance we can reject the hypothesis that revenue is equal in the two treatments. □

Result four considers the efficiency of the allocations of the two auctions. The allocation resulting from the auction is Pareto-efficient if the K units sold are obtained by the K bidders with the highest valuations. As reported in result four, Pareto-efficient allocation occurred more frequently under winner-pay rules than under all-pay rules. We can also calculate a measure called *efficiency*, to evaluate the welfare properties of outcomes of the auction processes. This measure, commonly used in experimental research, equals the total realized gains from exchange in a market period, divided by the maximum possible gains from trade (which in our auctions would result if the K units for sale were sold to the demanders with the highest valuations).

Result 4: The winner-pay auction is more likely to lead to a Pareto-efficient allocation than the all-pay auction. The losses from inefficiency are greater in the all-pay auction than in the winner-pay auction.

Support for Result 4: The data appear in Table 2 above. Comparison between the three winner-pay and the three all-pay sessions in which $K = 4$, indicates that the four highest-valued demanders received units in 37 out of 60 all-pay auctions and in 46 out of 60 winner-pay auctions. As for the percentage efficiency, in the three sessions of all-pay the averages were 95.9, 97.9 and 95.8. In the three sessions of winner-pay the averages were 99.0, 98.8, and 98.3. All three sessions of winner-pay generate higher average efficiency than any of the sessions of all-pay. □

Thus efficiency is somewhat higher under winner-pay than all-pay. The nature of the inefficiency in all-pay is suggested by studying Figures 2 and 4. Bidders in the intermediate range of valuations in A4 exhibit heterogeneity in bidding behavior, and therefore the rank ordering of bids often differs from the rank ordering of valuations. In session A4P, there are bidders with valuations between 400 and 600 who bid less than 100 and fail to win units which are instead purchased by demanders with valuations between 200 and 400 who bid more than 100. In session A4V, there is similar inefficient low bidding by some demanders with valuations between 200-400. In the winner-pay auction, the bidding strategies tend to be monotonic, and not differ greatly between bidders, so that only very infrequently is a demander likely to fail to win a unit, that will instead be purchased by a demander with a much lower valuation.

The design of our experiment also allows a comparison between the revenue generated by the two auctions studied here and two (winner-pay) auctions using different rules, which were studied by Alsemgeest et al. (1998). The parameters $N = 6$ and $K = 4$ were used in their study of the *uniform-price sealed-bid auction with lowest-accepted-bid pricing* (abbreviated here to SB) and *the English clock auction* (abbreviated to EC). They drew valuations from a distribution identical to that used here, enabling a comparison of average revenue levels between our study and theirs. They did not use the same realizations of demand draws as those used here so we are not able to pair our individual observations with theirs, and our inferences rely on comparisons of the sample means of revenues.

The English clock is an auction that begins with all bidders publicly and simultaneously announcing their initial quantity demanded at price zero. If there is excess demand, the price is increased by a small increment, and the bidders then announce new quantities demanded. These new quantities must be less than or equal to the previous quantity announced, and therefore exit is irrevocable. The procedure continues until quantity demanded and quantity supplied are equal. The market-clearing price becomes the uniform per-unit price charged to demanders. Each bidder's profit on each item equals the difference between his valuation for the item and the price he pays to obtain it. In the English clock, each bidder has a dominant strategy to declare a quantity demanded of one unit at all prices less than his valuation and then to declare a quantity demanded of zero units at all prices above his valuation.[8] If all bidders use their dominant strategy, the price equals the $K+1$st highest valuation, and the revenue to the auctioneer equals K times the $K+1$st valuation, identical in expectation to the equilibrium revenue in AP and WP.

The rules of the uniform price sealed bid auction with lowest accepted bid pricing (SB) are the following. Each bidder, after drawing his valuation, simultaneously submits a bid. The K highest bidders each receive one unit and obtain a payoff equal to their valuation for the object they purchase. If there is a tie for the Kth highest bidder, the unit is given to the each of the tied demanders with equal probability. Each of the K bidders who obtains a unit pays an amount equal to Kth highest bid submitted overall. The $N-K$ bidders who do not obtain units do not make any payments. In the SB auction, if demanders' valuations are distributed uniformly there is a symmetric, monotonic Bayesian equilibrium in which each bidder i uses a bidding strategy

$$B(v_i) = \frac{N-K}{N-K+1} v_i \tag{17}$$

[8]If individual demanders have value for and are allowed to purchase multiple units, the dominant strategy property does not hold. On all units except for their most highly-valued unit, bidders have an incentive to reduce their quantity demanded at prices lower than their marginal willingness to pay.

See appendix A for a derivation of equation (17). In this equilibrium, the expected revenue is also identical to that of the two auctions studied in this paper. As the support of result 5 argues, both the AP and the WP auction yield higher revenue than the English clock, but do not yield revenue significantly different from SB.

Result 5: Both the winner-pay and the all-pay auctions yield higher revenue than the English Clock. Both the winner-pay and all-pay auctions yield revenue no different than the SB auction.

Support for Result 5: Consider the variable $R_{jt} - R*$, which represents observed revenue minus Bayesian equilibrium revenue. In the data of Alsemgeest et al. (1998), in the first 10 periods of two sessions of the English clock, revenue is on average 34 less than the BE level per period.[9] Average revenue is 201 more than the equilibrium level for SB in the first 10 periods over two sessions.[10] In the first ten periods of the three sessions of all-pay with $K = 4$, the average revenue is 136.5 more than the BE level. In the first ten periods of the three sessions of winner-pay with K = 4, the average revenue is 152.2 more than the BE level. Calculating the standard error for the difference in sample means[11] and conducting a two-sided hypothesis test, we find that the mean difference from the BE level in EC is different from (lower) than that in AP as well as different from that in WP at the .005 level of significance (t = 2.97 and 3.67 in the two tests with 48 degrees of freedom).[12] The mean difference from the BE in SB is not different than that in

[9]McCabe et al. (1990, 1991) also found that the English Clock generated revenue close to the dominant strategy prediction and that individual bidders had a strong tendency to use the dominant strategy.

[10]We use the first 10 periods of the Alsemgeest et al. design to make our comparison because of the crossover design used in that study. Beginning in period 11, a second treatment was used in each session. Only the first 10 periods are free of the possibility of hysteresis effects, which often exist in auction experiments.

[11]See for example Hildebrand (1986), chapter 11, for a discussion of pooled-variance t methods for testing the significance of differences between two sample means.

[12]Smith (1967), Belovicz (1979) and Miller and Plott (1985) also compared discriminative and uniform-price sealed bid winner-pay auctions. Both Smith and Belovicz study environments where the auction is for a security which has a common value to all bidders and therefore the independent private values structure is not present. Smith (1967) finds that the difference in revenue of the two auctions depends on the amount of excess demand. When there is a large amount of excess demand SB generates more revenue, and when there is a small amount of excess demand the discriminative auction generates more revenue. Belovicz (1979) finds that greater excess

either AP or WP at any conventional level of significance (t = .124 and .135 for the two tests). □

The time horizon in our experiment is by necessity short. However, suppose that we postulate that there is a convergence process governing the evolution of revenue in the auction as the auction is repeated. Suppose further that the level to which the revenue converges as the number of repetitions approaches infinity is common to all of the sessions of a treatment. Then, we can make a conjecture about how revenue would evolve in our auction, if the auction process were repeated many more (possibly infinitely many) times, by extrapolating from the data in our sessions. As indicated in Conjecture 1, it appears that revenue is converging to the Bayesian equilibrium level in the A4 and W4 treatments.

Conjecture 1: Over a sufficiently long time horizon, revenue will converge to the BE level under A4 and W4

Support for Conjecture 1: Consider the following regression equation:[13]

$$R_{jt} - R^{*} = \beta_{1R}\frac{D_1}{t} + \beta_{1V}\frac{D_2}{t} + \beta_{1P}\frac{D_3}{t} + \beta_2\frac{(t-1)}{t} \tag{18}$$

where t equals the period number ($t = 1$ means the first period of a particular session, $t = 2$, the second period, etc...), R_{jt} is the revenue generated in period t of session j and R^{*} is the BE revenue. D_j is a dummy variable that equals 1 if the observation is from session j and equals 0 otherwise. The coefficients of the model have a straightforward interpretation. β_{1j} is an estimate of the revenue in period 1 of session j. β_2 is the estimated value to which the

demand leads to higher prices in SB. In a private values environment, Miller and Plott (1985) report that when demand is inelastic at the competitive equilibrium, the discriminative auction generates higher revenue than SB; this result is reversed when demand is elastic. They also find that SB generates approximately the competitive equilibrium revenue. Under their particular configuration of demand, which was stationary from period to period, rather than independently drawn each period, each bidder submitting a bid equal to her valuation constituted a Nash equilibrium to SB.

[13]The particular specification used for the estimation was first used by Noussair et al. (1995).

time series of revenue is converging as $t \to \infty$. The model allows the initial value of the time series to differ between sessions, but requires the revenue to converge to a common asymptote. A question addressed by the regression model is the following. If the revenue is converging to a common level across sessions, is that level equal to the revenue predicted by the Bayesian equilibrium derived in section 2? The estimates from the regression model are given in Table 4. In the table, the estimated standard errors are given in parentheses.

Table 4: Estimated Convergence of Observed Minus Equilibrium Revenue Over Time: All Treatments

Treatment	β_{1R}	β_{1V}	β_{1P}	β_2	R^2
A4	287.7	787.6	496.8	37.4	.25
	(177.6)	(177.6)	(177.6)	(36.3)	
A2	28.1	-53.7	725.1	193.8	.07
	(307.4)	(307.4)	(307.4)	(62.9)	
W4	551.5	75.7	462.6	41.2	.25
	(133.6)	(133.6)	(133.6)	(27.3)	

In the A4 and W4 treatments, the asymptotic estimate of revenue (β_2) is not different from the Bayesian equilibrium prediction, suggesting that the excess of revenues over equilibrium levels may be a short-run phenomenon. All six of the β_{1j} coefficients in A4 and W4 are greater than 0 and in four of the sessions they are significantly greater at the 5 percent level of significance, indicating that convergence is taking place from above. In the A2 treatment, revenue appears to be converging to a level higher then the BE level, if indeed there is convergence at all. The R^2 is very low in the K = 2 treatment. □

Estimating the model separately for each session in the A2 treatment separately isolates the source of the lack of convergence. The model, estimated separately for each session j, is:

$$R_{jt} = \beta_1 \frac{1}{t} + \beta_2 \frac{t-1}{t} \tag{19}$$

and the estimated coefficients are given in Table 5. The estimates show very low R^2's. However, the estimated value of β_2 is different from 0 only in the A2P session.

Table 5: Estimated Convergence of Observed Minus Equilibrium Revenue Over Time, Individual Sessions of the A2 treatment

Session	β_1	β_2	R^2
A2R	291.4	-16.2	.07
	(221.0)	(73.6)	
A2V	94.3	75.8	.00
	(332.7)	(110.8)	
A2P	313.7	521.9	.02
	(300.2)	(100.0)	

An illustrative example of convergence toward the BE is the evolution of bidding strategies in session A2V. Figure 5 shows the strategies used in periods 1-10 and figure 3b shows the bidding behavior in periods 11-20. The contrast between the early and late periods is striking. In the early data shown in figure 5 all bidders seem to follow a binary rule. If they receive a valuation that is sufficiently high, they bid between 600 and 700. If the valuation is too low, they bid close to 0. Over time, it appears that subjects come to realize that when they receive low valuations, they can increase their probability of winning at low cost by raising their bids from very low levels. Other bidders suffer large losses by bidding over 600 and failing to receive a unit, and in response lower their bids over time when they receive high valuations. This process pushes bidding toward the BE level over time. It is possible that the process is enhanced when the valuation information is revealed publicly, as in the V sessions.

[Figure 5: About Here]

In contrast, we reject the hypothesis that the session-specific revenue is moving to the BE level for session A2P. To explore this session further, consider the data from periods 1-10 of session A2P given in figure 6, and compare it to the data from periods 11-20 which are given in figure 3c. The trend indicates that over the course of the session, behavior is evolving toward a binary strategy of submitting bids less than 100 when valuations are below a cut-off point of roughly 600-700, and submitting bids greater than 600, when valuations are greater than the cut-off point. Convergence toward the BE is not taking place in A2P.

[Figure 6: About Here]

4.2 Individual Level Analysis

At the level of the individual subject, we can consider whether individual bidding behavior is in accordance with the predictions of the theoretical model presented in section 2. We focus on all-pay since winner-pay has already extensively been studied. For each of the 18 subjects in the $K = 4$ all-pay data we estimate

$$b_i = \beta_1 \frac{v_i^3}{1000^2} + \beta_2 \frac{v_i^4}{1000^3} + \beta_3 \frac{v_i^5}{1000^4} + \beta_4 \frac{v_i^6}{1000^5} \tag{20}$$

and test the hypotheses that $\beta_1 = \frac{20}{3}, \beta_2 = -15, \beta_3 = 12$, and $\beta_4 = -\frac{10}{3}$, the Bayesian equilibrium values. For each of the 18 bidders, none of the four estimated coefficients is significantly different from the equilibrium level. However, the standard errors are quite large at the level of the individual subject. For the 18 subjects in the $K = 2$ all-pay data we estimate

$$b_i = \beta_1 \frac{v_i^5}{1000^4} + \beta_2 \frac{v_i^6}{1000^5} \tag{21}$$

and test the hypotheses that $\beta_1 = 4$ and that $\beta_2 = -\frac{10}{3}$, the BE values. Both coefficients are not significantly different from the BE prediction for 7 of the 18 bidders. This is further indication that the BE more accurately describes behavior when $K = 4$ than when $K = 2$. The estimates from the pooled data from all subjects, in each of the six sessions of all-pay, are also given in table 6.

Table 6: Estimated Bidding Strategies in the All-Pay Auction

Session	β_1	β_2	β_3	β_4	R^2
A4R	14.06	-41.15	42.07	-14.67	.62
	(2.43)	(10.21)	(13.94)	(6.20)	
A4V	7.98	-14.74	5.83	1.57	.83
	(2.11)	(8.96)	(12.36)	(5.57)	
A4P	10.74	-29.75	29.64	-10.33	.62
	(2.82)	(11.79)	(16.07)	(7.15)	
A4 All	11.94	-33.47	33.10	-11.26	.66
Sessions	(1.51)	(6.31)	(8.63)	(3.86)	
A4 BE	$\frac{20}{3}$	-15	12	$\frac{-10}{3}$	
A2R	5.72	-5.21			.56
	(.56)	(.51)			
A2V	4.36	-3.69			.63
	(.57)	(.63)			
A2P	7.78	-7.21			.80
	(.41)	(.44)			
A2 All	6.06	-5.52			.76
Sessions	(.30)	(.32)			
A2 BE	4	$\frac{-10}{3}$			

At the 5 percent level of significance, we cannot reject the hypothesis that any one of the coefficients differs from the BE prediction for A4V and A2V, the two sessions in which valuation information was posted along with the bid information, as well as for A4P. In session A4R, three of four coefficients are significantly different from the BE prediction. In sessions A2P and

A2R as well as the pooled data from both treatments, all coefficients are different from the BE prediction at the 5 percent level of significance.

By comparing the estimates in table 5, with analogous equations which allow each of the coefficients to differ for each subject, and by using F-tests of the restrictions, we can test for symmetry of the bidding strategies in a given session. We are able to reject the hypothesis that bidders are using a common bidding strategy in two of the three A4 sessions as well as in two of the three A2 sessions. F = 1.48 in A4R, 1.86 in A4V and 4.36 in A4P. In the A2 sessions F = 9.70 in A2V, 1.43 in A2P, and 2.27 in A2R. The critical levels of F are 1.68 for 5 percent and 2.06 for 1 percent significance. Thus we find evidence of heterogeneity between the subjects in four of the six sessions.[14]

An example of heterogeneity, from session A2R, is shown in figure 3a above. In the data in the figure, the heterogeneity is largely due to the fact that one of the subjects, whose bids are shown with x's, bid 50 whenever his valuation was greater than 600 and bid 10 or less whenever his valuation was less than 600. By adopting this strategy the subject avoided losses of more than 50 and would occasionally win units at low prices. This strategy was very successful and that particular subject had by far the highest earnings of any subject in the entire $K = 2$ treatment. Each of the other five bidders always bid 500 or more whenever he had a value of 700 or more, yielding small profits in the event of obtaining a unit, and large losses in the event of failing to obtain a unit.

[14] Cox et al. (1984) also report considerable heterogeneity in their winner-pay data. In only 4 of their 28 sessions did they fail to reject the hypothesis that bidders were using the same strategy. To reject homogeneity, they employed a Kruskal-Wallis test on the deviations of individuals' observed bids from the risk neutral bids. Applying the same Kruskal-Wallis test to our data, we calculate KW = 11.2, 4.75, and 37.9 in sessions A4R, A4V, and A4P respectively. KW = 9.59, 13.1, and 2.1 in A2R, A2V and A2P, and KW = 8.9, 3.7 and 13.0 in W4R, W4V and W4P. The critical value of KW is 11.07 for 5 percent significance and 15.09 for 1 percent significance. By the KW measure, we reject homogeneity of bidding behavior in four of our nine sessions at the 5 percent level. The greater level of heterogeneity found by Cox et al. may be due to the fact there are more than six bidders in most of their sessions or the fact that most of their sessions have more than 20 periods.

5 Conclusion

Our main result, that sealed bid all-pay auctions generate higher revenue on average than the Bayesian equilibrium prediction, can be viewed as an extension of previous experimental results on all-pay auctions. The overdissipation of rents relative to the Nash equilibrium level that occurs when there is a single unit sold (Davis and Reilly, 1998; Potters et al., 1998, Gneezy and Smorodinsky, 1999; and Amann and Leininger, 1999) appears to extend to our environment with multiple units and incomplete information. Thus the phenomenon of overdissipation extends to a more general setting than those studied previously.

Revenue is very similar in the winner-pay and all-pay auctions. However, it is difficult to interpret this result as support for the general notion of revenue equivalence for a class of mechanisms, because there are many violations of the Revenue Equivalence Theorem that have been documented by previous studies. Examples include Coppinger et al. (1980), Cox et al. (1982) and Kagel and Levin (1993) for the case of single-unit supply and McCabe et al. (1990) and Alsemgeest et al. (1998) for the case of multiple-unit supply.

Though the revenue is not different between the two auctions, the realized gains from trade are lower in the all-pay auction than in the winner-pay auction. Since the revenue to the seller is no different between the two auctions, buyers are worse off under the all-pay than under the winner-pay auction. The lower payoffs to bidders do not translate into higher earnings for the seller, but rather translate into a dead-weight loss. This observation provides one rationale for why all-pay auctions are rarely, if ever, used in the field to sell goods. All-pay auctions generate no additional revenue over winner-pay auctions but, since winner-pay auctions allocate the units more efficiently, they would be preferred by bidders, who would be more likely to participate.[15]

We observe higher revenue in the all-pay auction when $K = 2$, than when $K = 4$, even

[15] There are other important reasons why all-pay auction may not be commonplace. One obvious reason is their susceptibility to collusion between the auctioneer and one or more bidders. Another reason is the possible existence of loss aversion on the part of bidders, who may have to make payments, without receiving any units.

though the Bayesian equilibrium revenue is the same in the two treatments. This can be viewed as an extension of the observation of Cox et al. (1984) that when the ratio of N to K is high in discriminative winner-pay auctions, revenue is higher relative to the Nash equilibrium than when the ratio is low. One explanation they offer is that if K is close to N, cooperative behavior among bidders is more likely. They report experiments with $N = 6$ and $K = 2$ in which revenue is substantially higher than the BE, as it is in our $K = 2$ sessions of all-pay.

In the $K = 4$ treatment, revenue appears to be converging toward the Bayesian equilibrium level over time in both all-pay and winner-pay. This is also true in two of three session of all-pay when $K = 2$. This convergence conjecture is a prediction about the long-run behavior of the same subjects if the sessions of our experiment were extended for many more periods. At first glance the convergence result for winner-pay seems at odds with results reported by Cox et al. (1984, 1985), who report bidding less than the BE level for some levels of N and K. However, the apparent differences between our data and theirs can be easily reconciled. They do not report results for auctions in which $N = 6$ and $K = 4$; the closest multi-unit auctions they report in terms of the ratio of bidders to units sold are sessions in which $N = 10$ and $K = 7$. They find that in three of four sessions with inexperienced subjects, revenue is greater than the BE level. The two sessions they report in which the discriminative winner-pay auction was the first treatment in a sequence of treatments, in which hysteresis effects from previous auctions are not present, are most similar (in terms of previous subject experience) to our sessions. In both of these sessions, revenue was significantly greater than the BE level, as it is in our data. They also observe, as we do, that revenue is decreasing over time as the process is repeated. However, when subjects are experienced in the sense of having previously participated in an auction experiment, they generate revenue below the BE level. All of these results are completely consistent with our findings and our interpretation of our data. We make no claims concerning expected behavior if subjects were to return for another session with another group of subjects or behavior in discriminative winner-pay auctions after previous experience in a different type of auction.

A Derivation of the Bayesian equilibrium of the SB auction

Suppose that there are N bidders, indexed by i where $(i = 1, ...N)$, and K identical objects to be sold. Each bidders draws a valuation for obtaining one of the units being sold, $(v_1, ..., v_N)$, independently from a uniform distribution on [0,1].[16] Each bidder knows N, K, her own valuation and the distribution of valuations, but does not know the actual valuation drawn by any other demander. Assume $N > K > 0$.

The auction proceeds as follows. Each bidder simultaneously submits a sealed-bid b_i. The K highest bidders each receive a unit and pay a uniform per-unit price equal to the Kth highest bid. If a tie occurs the unit(s) are randomly assigned to the tied demanders. A strategy for bidder i is a function $B_i(v_i) : [0,1] \to [0,\infty]$ which assigns a bid for any valuation that i might draw.

In the proof of the following proposition, we derive a symmetric Bayes-Nash equilibrium to the above game in the above environment.

Proposition: The strategy profile $B^*(v) = (B_1^*(v_1), ..., B_N^*(v_N))$, where:

$$B_i^*(v_i) = \frac{N-K}{N-K+1} v_i; \forall i, \tag{22}$$

constitutes a symmetric Bayes-Nash equilibrium.

Proof: Clearly $B^*(v)$ is symmetric. Now suppose each bidder j $(j \neq i)$ is using $B_j^*(v_j)$. Bidder i chooses b_i to maximize expected profit which is given by:

[16] We use the interval [0,1] rather than $[0, \overline{v}]$ for notational convenience. The proposition proven in this appendix remains true if the valuations are drawn from a uniform distribution on $[0, \overline{v}]$.

$$E\pi_i = (v_i - x_{N-K+1})(\text{Prob[at least N - K + 1 other bidders bid} \leq b_i])$$
$$+ (v_i - b_i)(\text{Prob[exactly N - K other bidders bid} \leq b_i]),$$

where x_z equals the zth lowest of the other players' (other than i) bids and $z \in \{1, ..., N-1\}$. The first term in the above expression indicates the payoff when i wins a unit but her bid is not the Kth highest bid overall, multiplied by the probability of the occurrence of that event. The second term gives the profit to i when her bid is the Kth highest overall, multiplied by the probability of the occurrence of that event.

Let $F_z(y)$ denote the probability that x_z is less than or equal to y if all bidders besides i are using $B_j^*(v_j)$ and let $f_z(y)$ denote the corresponding density function. Rewriting the expression for expected profit,

$$E\pi_i(b_i) = \int_0^{b_i} (v - x_{N-K+1}) f_{N-K+1}(x_{N-K+1}) dx_{N-K+1} + (v_i - b_i)(F_{N-K}(b_i) - F_{N-K+1}(b_i)). \tag{23}$$

The first order condition is:

$$\frac{dE\pi_i}{db_i} = (v_i - b_i) f_{N-K}(b_i) - (F_{N-K}(b_i) - F_{N-K+1}(b_i)) = 0. \tag{24}$$

Since $B_j^*(v_j)$ is strictly monotonic in v_j and thus invertible for all $j \neq i$:

$$Prob[B_j^*(v_j) \leq b_i] = Prob[v_j \leq B_j^{*(-1)}(b_i).] \tag{25}$$

Using the last equation, the fact that each bidder besides i is using the strategy B_j^*, and the fact that the distribution of valuations is uniform on $[0, 1]$ we see that:

$$(F_{N-K}(b_i) - F_{N-K+1}(b_i)) = \frac{(N-1)!}{(N-K)!(K-1)!}(B_j^{*(-1)}(b_i))^{N-K}(1 - B_j^{*(-1)}(b_i))^{K-1}. \tag{26}$$

The last equation is equal to the probability that exactly $N - K$ of the $N - 1$ bidders have a valuation lower that v_i. One can also calculate:

$$F_{N-K}(b_i) = \sum_{\ell=N-K}^{N-1} \frac{(N-1)!}{\ell!(N-\ell-1)!}(B_j^{*(-1)}(b_i))^{\ell}(1-B_j^{*(-1)}(b_i))^{N-\ell-1} \quad (27)$$

and the corresponding density function:

$$f_{N-K}(b_i) = \frac{(N-1)!}{(N-K-1)!(K-1)!}(B_j^{*(-1)}(b_i))^{N-K-1}(1-B_j^{*(-1)}(b_i))^{K-1}(\frac{dB_j^{*(-1)}(b_i)}{db_i}) \quad (28)$$

Equation (24) becomes:

$$0 = (v_i - b_i)\frac{(N-1)!}{(N-K-1)!(K-1)!}(B_j^{*(-1)}(b_i))^{N-K-1}(1-B_j^{*(-1)}(b_i))^{K-1}(\frac{dB_j^{*(-1)}(b_i)}{db_i}) \quad (29)$$

$$-\frac{(N-1)!}{(N-K)!(K-1)!}(B_j^{*(-1)}(b_i))^{N-K}(1-B_j^{*(-1)}(b_i))^{K-1}.$$

A solution to the last equation can be found by setting $b_i = B_j^*(v_i) = \frac{N-K}{N-K+1}v_i$, and noting that $B_j^{*(-1)}(b_i) = \frac{N-K+1}{N-K}b_i$ and therefore $\frac{dB_j^{*(-1)}(b_i)}{db_i} = \frac{N-K+1}{N-K}$. Therefore B_j^* satisfies the first order necessary conditions to be a best response to itself. A second order sufficient condition is:

$$\frac{d^2E\pi_i}{db_i^2} = (v_i - b_i)f'_{N-K}(b_i) - 2f_{N-K}(b_i) + f_{N-K+1}(b_i) < 0, \forall b_i \in [0, B_j^*(1)] \quad (30)$$

or equivalently:

$$(v_i - b_i)\frac{(N-1)!}{(N-K-1)!(K-1)!}[(N-K-1)(B_j^{*(-1)}(b_i))^{N-K-2}(1-B_j^{*(-1)}(b_i))^{K-1}(\frac{dB_j^{*(-1)}(b_i)}{db_i})^2$$

$$(31)$$

$$-(K-1)(B_j^{*(-1)}(b_i))^{N-K-1}(1-B_j^{*(-1)}(b_i))^{K-2}(\frac{dB_j^{*(-1)}(b_i)}{db_i})^2$$

$$+\frac{d^2B_j^{*(-1)}(b_i)}{db_i^2}(B_j^{*(-1)}(b_i))^{N-K-1}(1-B_j^{*(-1)}(b_i))^{K-1}]$$

$$-2\frac{(N-1)!}{(N-K-1)!(K-1)!}(B_j^{*(-1)}(b_i))^{N-K-1}(1-B_j^{*(-1)}(b_i))^{K-1}(\frac{dB_j^{*(-1)}(b_i)}{db_i})$$

$$+\frac{(N-1)!}{(N-K)!(K-2)!}(B_j^{*(-1)}(b_i))^{N-K}(1-B_j^{*(-1)}(b_i))^{K-2}(\frac{dB_j^{*(-1)}(b_i)}{db_i}) < 0.$$

Substituting $v_i = \frac{N-K+1}{N-K}b_i = B_j^{*(-1)}(b_i)$, $\frac{N-K+1}{N-K} = \frac{dB_j^{*(-1)}(b_i)}{db_i}$, $0 = \frac{d^2B_j^{*(-1)}(b_i)}{db_i^2}$, and cancelling terms, it can be shown that the inequality in (31) holds if and only if:

$$K < N + 1. \tag{32}$$

Since $N > K$ the second order conditions are satisfied by $B_j^*(v_j)$, which therefore is a best response to itself. Therefore, the strategy profile $B_i^*(v_i) = \frac{N-K}{N-K+1}v_i$ for all i is a Bayesian equilibrium.

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Figure 1: Equilibrium Bidding Functions: All Treatments

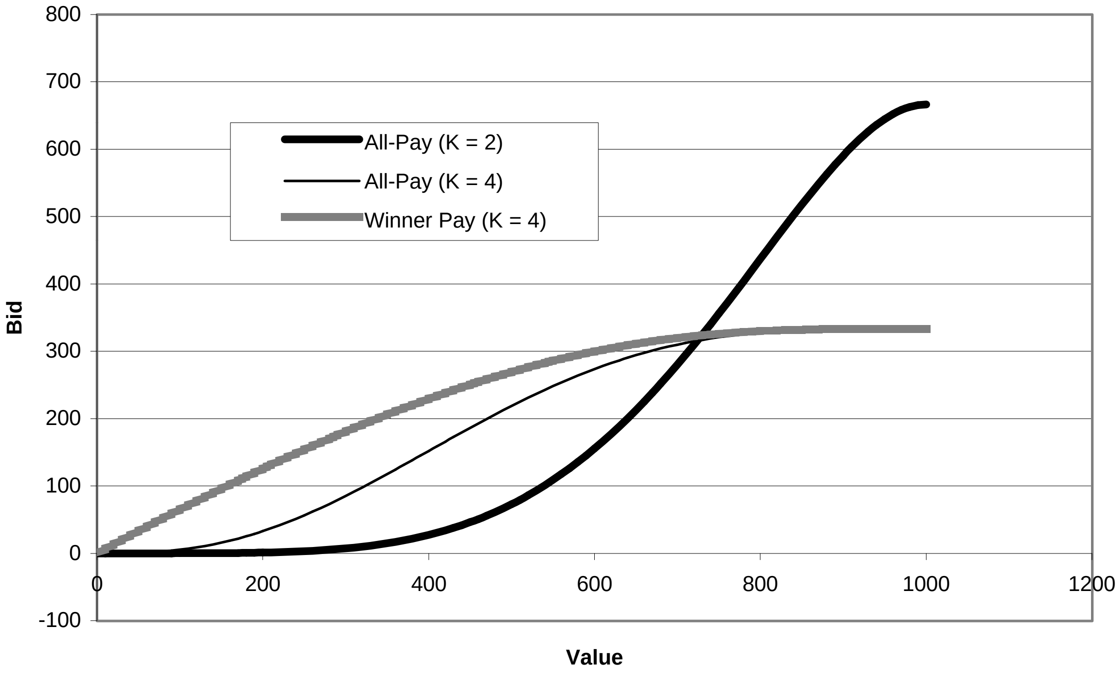


Figure 2a: Observed and Equilibrium Bids: Session A4R: Periods 11-20

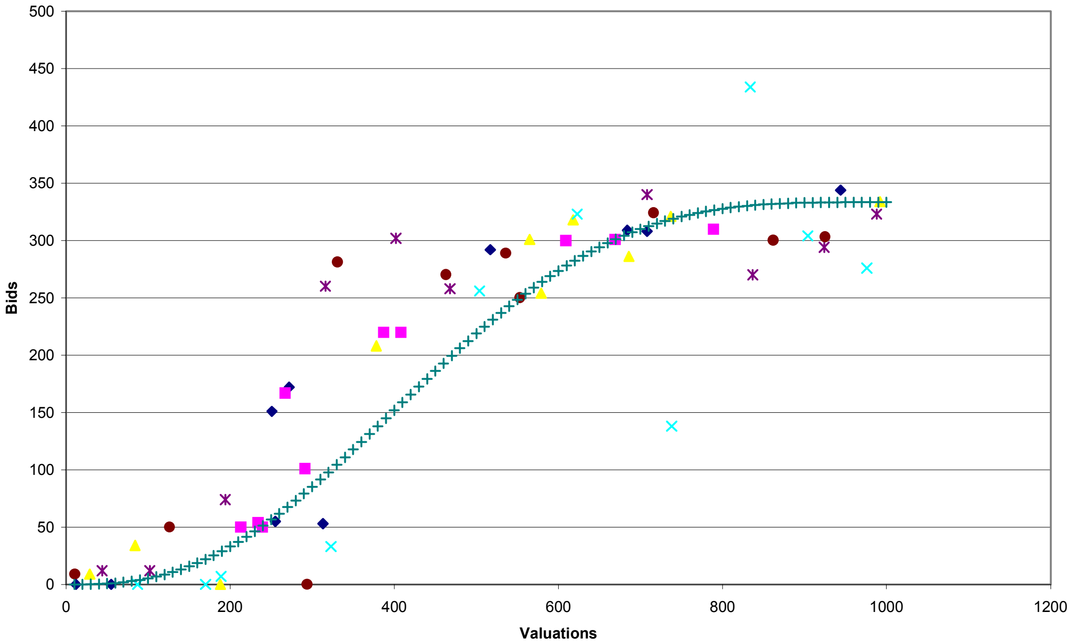


Figure 2b: Observed and Equilibrium Bids in Session A4V: Periods 11-20

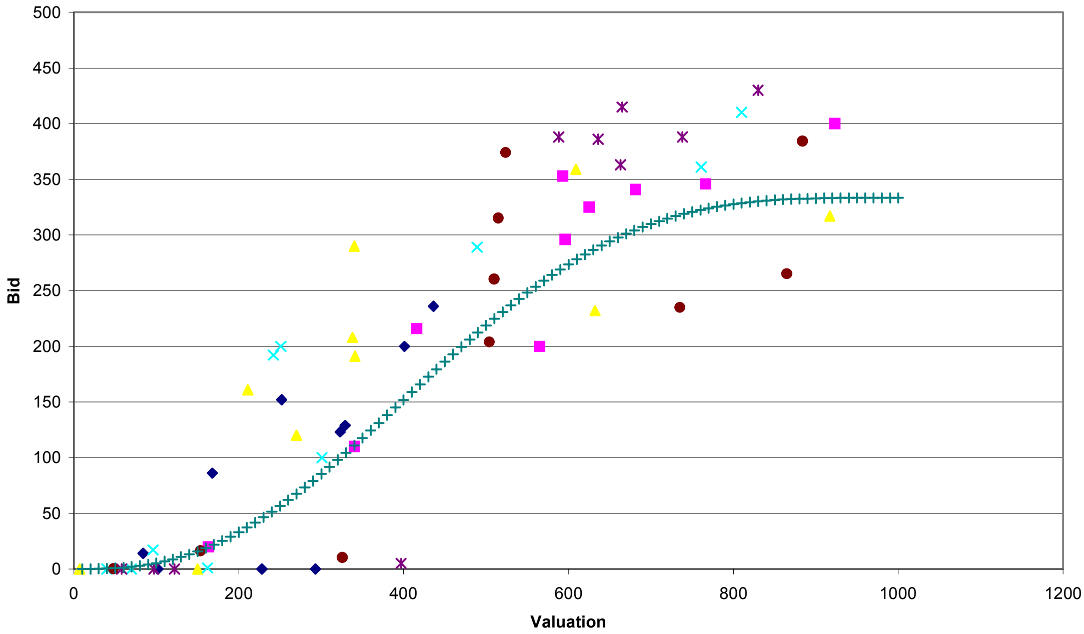


Figure 2c: Observed and Equilibrium Bids: Session A4P: Periods 11-20

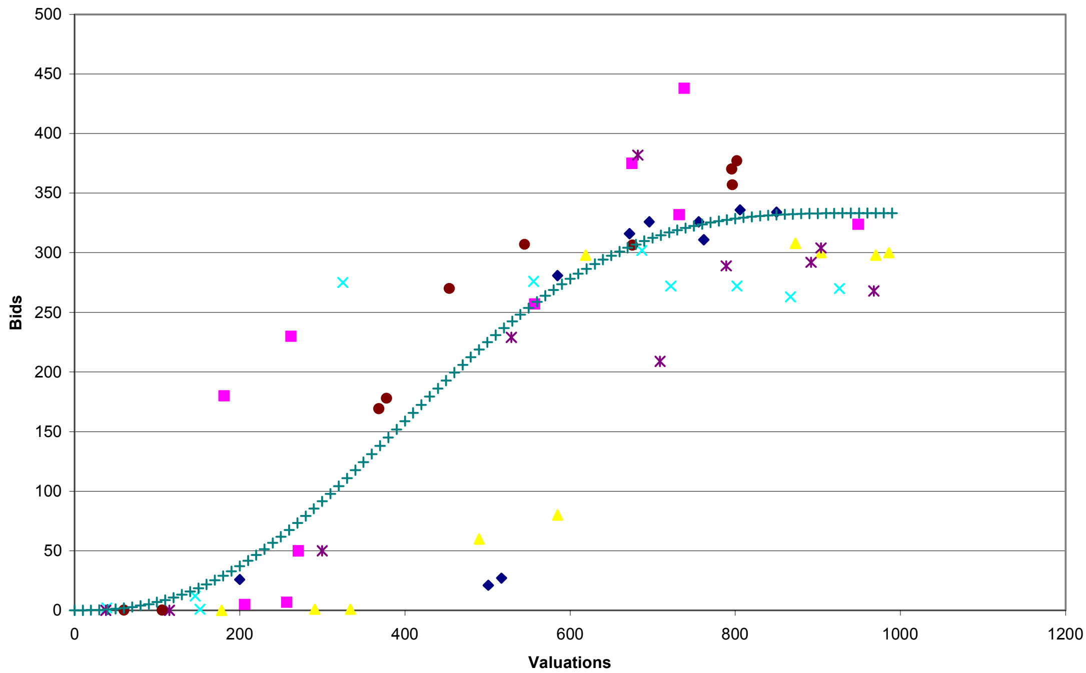


Figure 3a: Observed and Equilibrium Bids: Session A2R: Periods 11-20

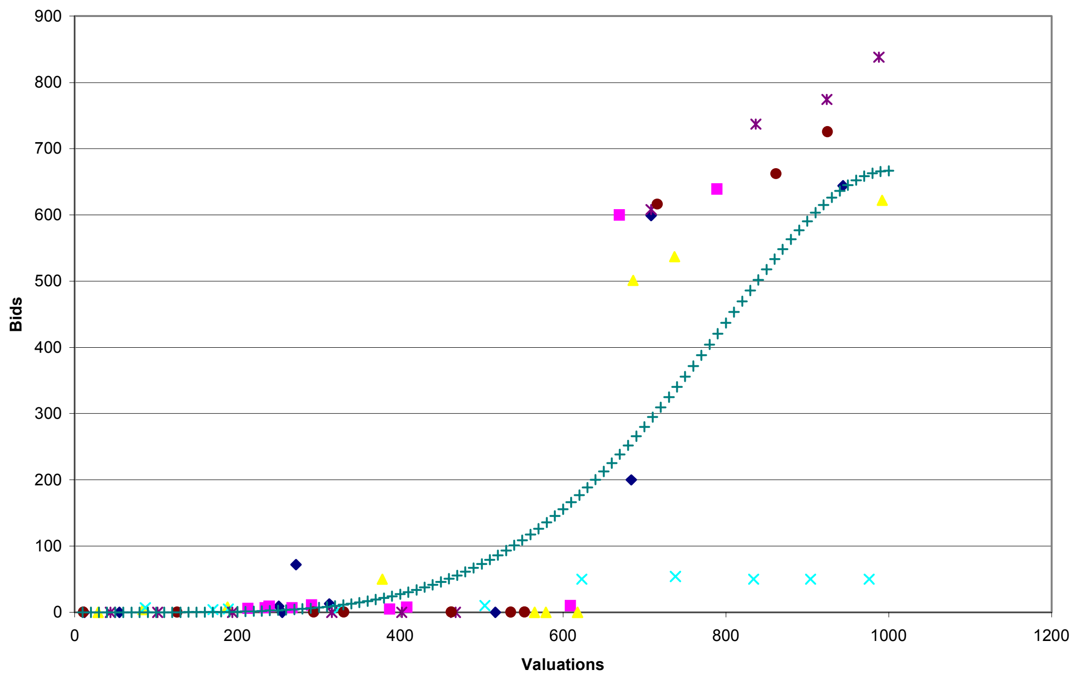


Figure 3b: Observed and Equilibrium Bids: Session A2V: Periods 11-20

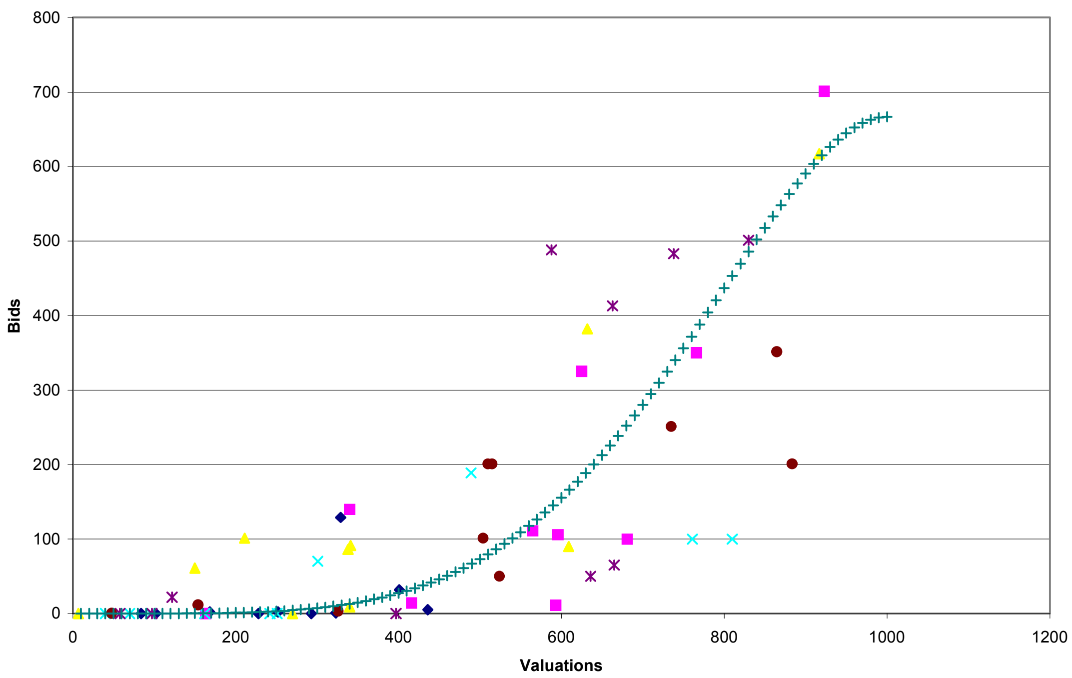


Figure 3c: Observed and Equilibrium Bids: Session A2P: Periods 11-20

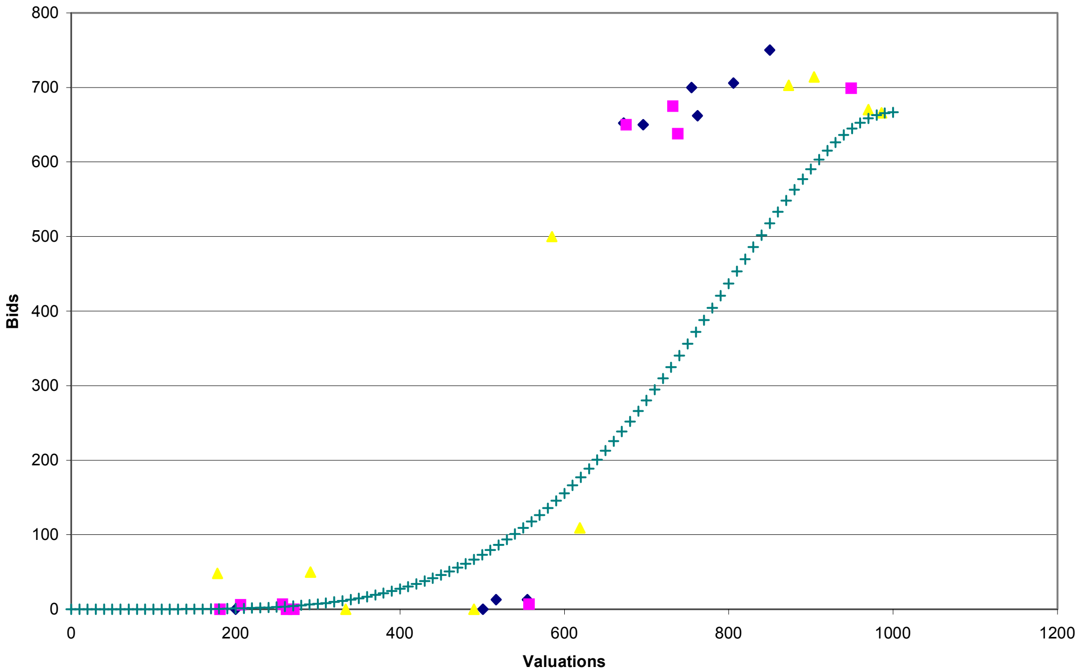


Figure 4a: Observed and Equilibrium Bids: Session W4R: Periods 11-20

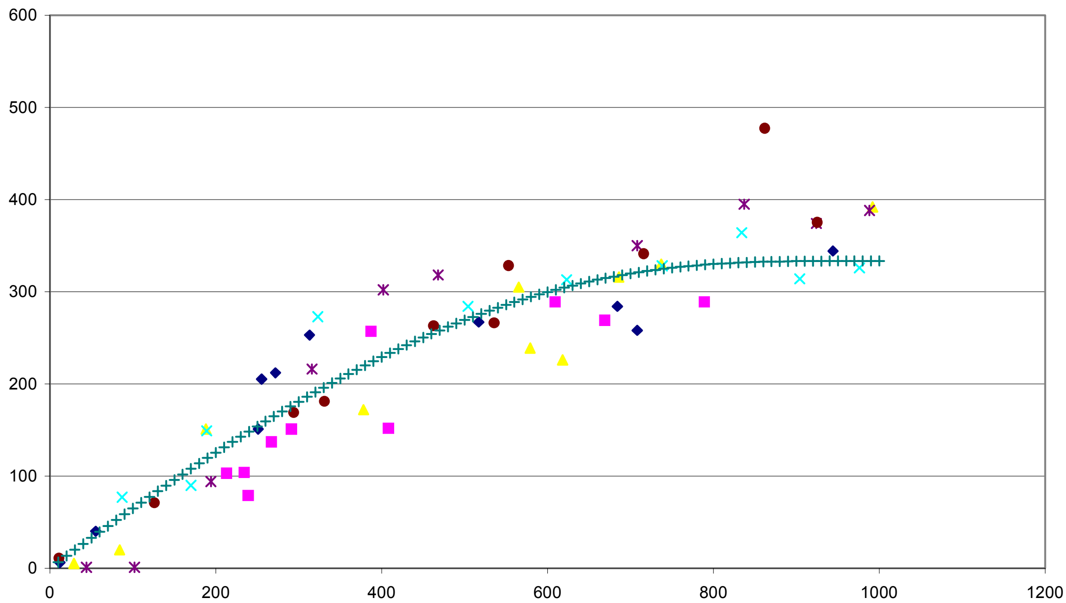


Figure 4b: Observed and Equilibrium Bids: Session W4V: Periods 11-20

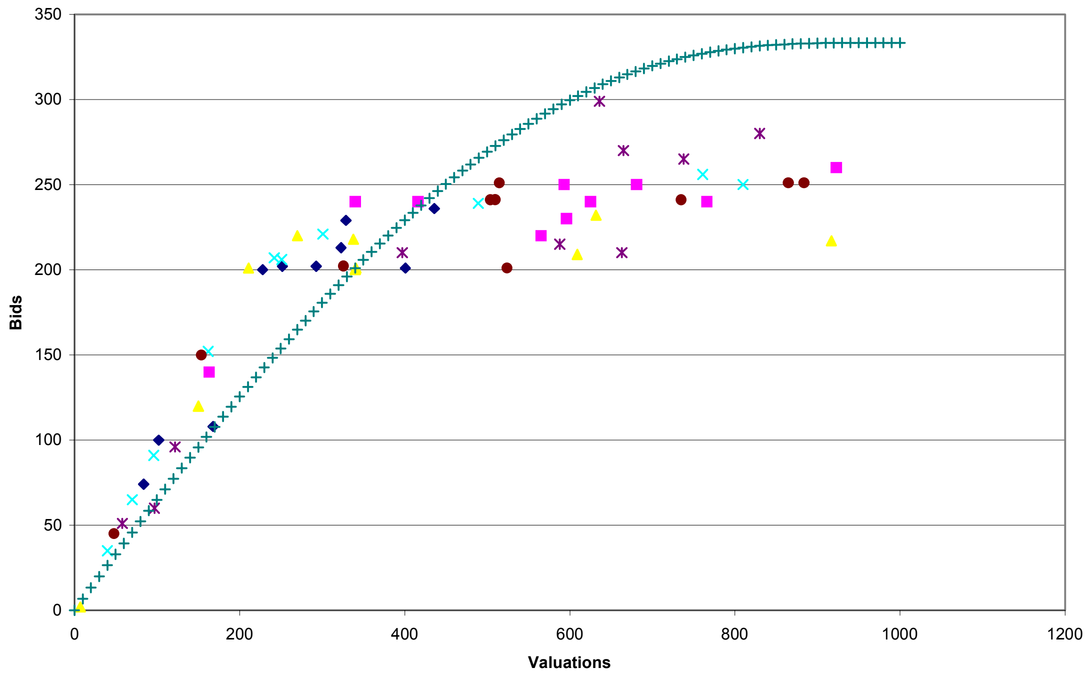


Figure 4c: Observed and Equilibrium Bids: Session W4P: Periods 11-20

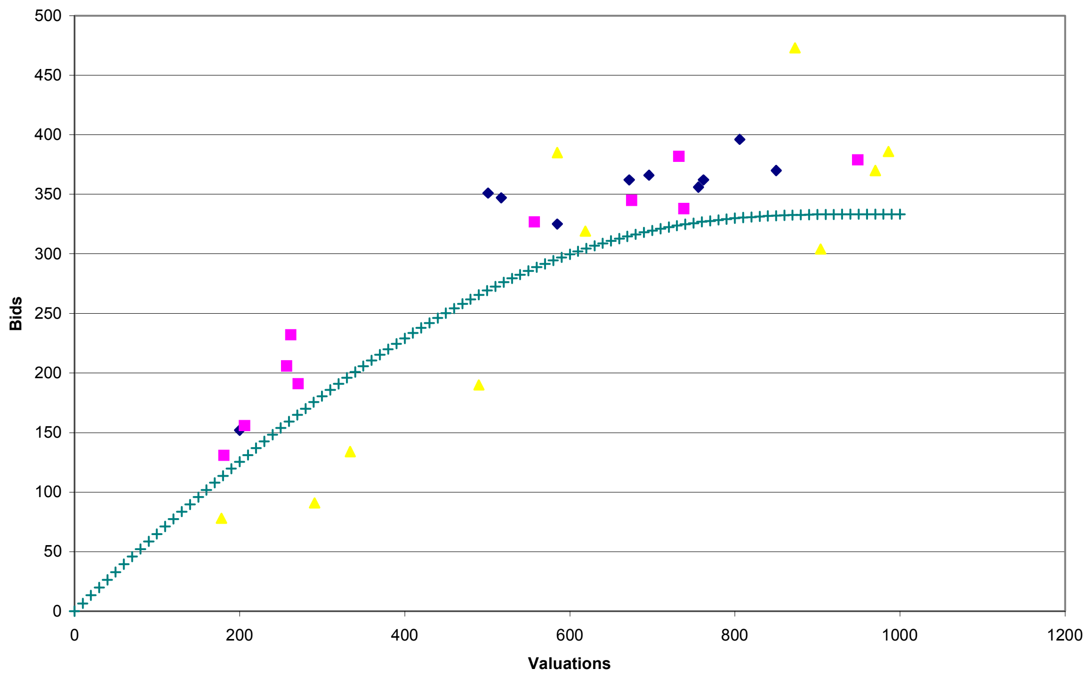


Figure 5: Observed and Equilibrium Bids: Session A2V: Periods 1-10

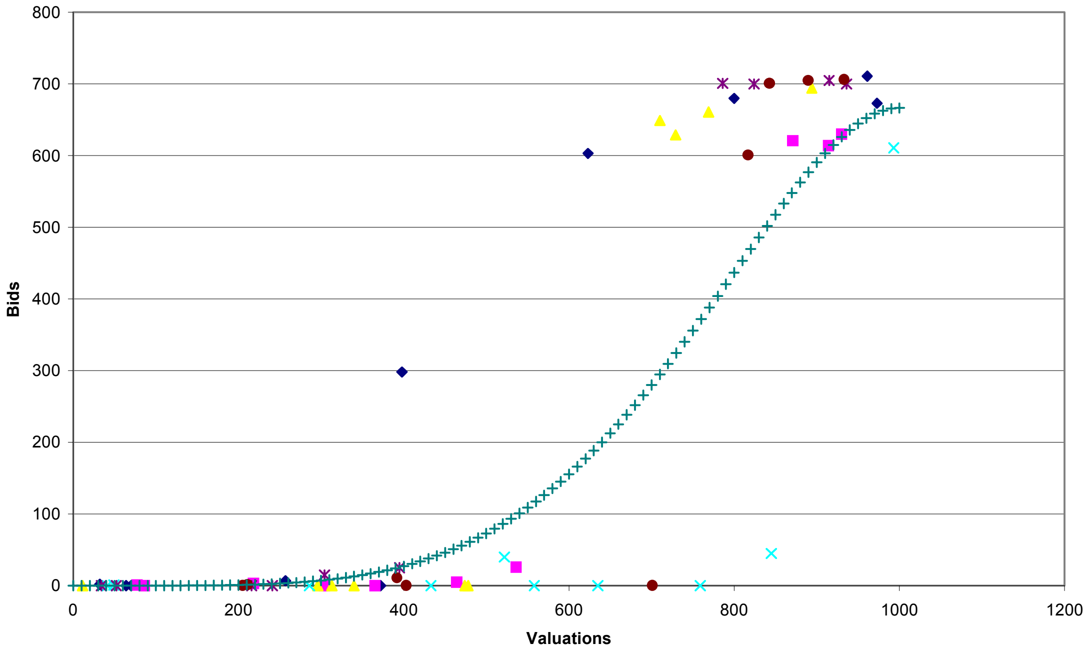
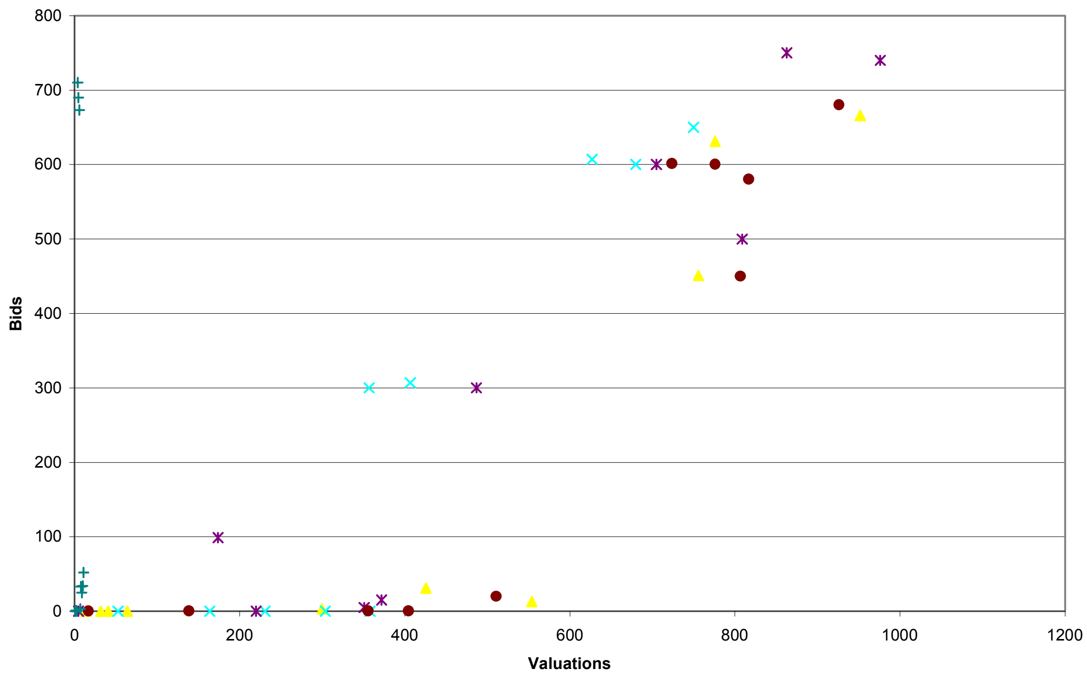


Figure 6: Observed and Equilibrium Bids in Session A2P: Periods 1-10



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