Tax progression under collective wage bargaining and individual effort determination

Koskela, Erkki; Schöb, Ronnie

WISSENSCHAFTSZENTRUM BERLIN
FÜR SOZIALFORSCHUNG

SOCIAL SCIENCE RESEARCH
CENTER BERLIN

Erkki Koskela *
Ronnie Schöb **

Tax Progression under Collective Wage Bargaining and Individual Effort Determination

* University of Helsinki
** Otto-von-Guericke-University Magdeburg

SP II 2006 – 13

August 2006

ISSN Nr. 0722 – 6748

Research Area
Markets and Politics

Research Unit
Market Processes and Governance

Schwerpunkt
Märkte und Politik

Abteilung
Marktprozesse und Steuerung

Zitierweise/Citation:

Erkki Koskela and Ronnie Schöb, **Tax Progression under Collective Wage Bargaining and Individual Effort Determination**, Discussion Paper SP II 2006 – 13, Wissenschaftszentrum Berlin, 2006.

Wissenschaftszentrum Berlin für Sozialforschung gGmbH,
Reichpietschufer 50, 10785 Berlin, Germany, Tel. (030) 2 54 91 – 0
Internet: www.wz-berlin.de

ABSTRACT

Tax Progression under Collective Wage Bargaining and Individual Effort Determination

by Erkki Koskela and Ronnie Schöb *

In this paper, we study the impact of tax policy on wage negotiations, workers' effort, and employment when effort is only imperfectly observable. We show that the different wage-setting motives – rent sharing and effort incentives – reinforce the effects of partial tax policy measures but not necessarily those of more fundamental tax reforms. We show that a higher degree of tax progression always leads to wage moderation, but the well-established result from the wage bargaining literature that a revenue-neutral increase in the degree of tax progression is good for employment does not carry over to the case with wage negotiations and imperfectly observable effort. While it remains true that introducing tax progression increases employment, we cannot rule out negative employment effects from an increase in tax progression when tax progression is already very high.

Keywords: Wage bargaining, effort determination, tax progression

JEL Classification: J41, J51, H22

* Koskela thanks Otto-von-Guericke-University Magdeburg for its great hospitality as well as the Research Unit of Economic Structures and Growth at the University of Helsinki, the Yrjö Jahnsson Foundation and the Academy of Finland (grant No. 1109089) for financial support. Schöb thanks the WZB in Berlin for great hospitality.

ZUSAMMENFASSUNG

Die Wirkung progressiver Besteuerung bei kollektiven Lohnverhandlungen und unbeobachtbarem individuellen Arbeitseinsatz

In einem Modell mit Lohnverhandlungen und unvollständiger Beobachtbarkeit individueller Arbeitsanstrengungen zeigen wir, dass sich die verschiedenen Motive bei der Lohnfindung – Verteilung von Renten zwischen Arbeitgeber und Arbeitnehmern, Effizienzlohnerwägungen – sich in ihren Wirkungen gegenseitig verstärken. Für die Auswirkungen der Steuerpolitik auf die Beschäftigung bedeutet dies, dass eine progressivere Ausgestaltung des Steuersystems grundsätzlich zu mehr Lohnmoderation führt. Da eine höhere Steuerprogression jedoch zugleich die individuellen Anstrengungsanreize verringert, ist der Beschäftigungseffekt einer Steuerreform, die die Progression erhöht, a priori nicht eindeutig. Das aus der Lohnverhandlungsliteratur bekannte Ergebnis, dass Steuerprogression gut für die Beschäftigung sei, lässt sich somit in einem allgemeineren Modellrahmen nicht bestätigen. Zwar ist die Beschäftigung bei einem moderat progressiven Steuersystem generell höher als bei einem proportionalen Steuersystem, doch lassen sich negative Beschäftigungseffekte bei einer weiteren Erhöhung der Steuerprogression nicht ausschließen.

1. Introduction

Tax progression leads to wage moderation and is thus good for employment. This result has been derived for different assumptions about the wage-setting motives such as rent sharing in wage bargaining models (see, e.g., Holm and Koskela 1996, Koskela and Vilmunen 1996, Koskela and Schöb 1999) or effort incentives in efficiency wage models, where firms unilaterally decide upon both the wage rate and the employment level (see, e.g., Pisauro 1991, Rasmussen 2002).

The effect of tax progression, however, has not yet been analyzed in a uniform framework that combines these different wage-setting motives. So far, only very few papers have combined wage bargaining and effort considerations at all. Early contributions by Lindbeck and Snower (1991) and Sanfey (1993) do not provide a uniform answer to the question as to how far different wage-setting motives analyzed in efficiency wage and union bargaining models reinforce or weaken each other. Later, Bulkley and Myles (1996) show that with imperfect monitoring of workers' effort, monopoly trade unions will set a higher wage than the pure efficiency wage set by the firms. This provides a higher bonus for non-shirking and results in a higher level of effort than we would observe in a competitive labor market. Garino and Martin (2000), on the other hand, show that efficiency wages offset the cost of higher wages and thus induce firms to make more concessions in wage negotiations. Thus there is theoretical evidence that the different wage-setting motives reinforce each other.

Within such a framework, Altenburg and Straub (1998) analyze variations of the benefit-replacement ratio. They find that, in contrast to the standard result in both efficiency wage and union bargaining models, the effect of a higher reservation utility on wages, employment, and effort is ambiguous when benefits are financed through lump-sum taxes. A higher replacement ratio may then reduce the wage rate and raise employment. A higher reservation utility of workers will induce firms to reduce their demand for effective labor. If, as a consequence, the labor share decreases, firms experience a higher relative reduction in profits from a wage increase. This explains why the wage may actually fall and – in the end – employment will rise.

To our knowledge, only one paper analyzes the impact of taxes in this framework. Garcia and Rios (2004) adopt the Altenburg and Straub (2002) model to analyze revenue-neutral tax reforms numerically. Their numerical calculations suggest that a revenue-neutral increase in the tax exemption that is financed by an increase in the wage tax increases employment. This indicates that the result by Koskela and Schöb (1999), according to which a revenue-neutral shift from payroll taxes to wage taxes raises employment when there is a higher tax exemption for the latter, also applies when effort is unobservable. Furthermore, they argue that it is better for employment in the case of constant fiscal revenues to compensate higher tax exemption through increases in wage taxes rather than payroll taxes. Since Garcia and Rios (2004) only provide numerical, rather than analytical, results, we first present an analytical framework to elaborate the way in which tax policy affects wage negotiations and employment when effort is only imperfectly observable and trade unions and firms negotiate on wages.

Our comparative statics analysis indicates that the standard results from the trade union literature must be modified in the case of imperfect monitoring of individual effort determination. In these standard models, tax policy only affects wages by altering the size of the labor surplus. When both wage-setting motives are present, however, tax policy also affects the strength with which tax policy parameters affect the negotiated wage and employment. When effort is not observable, tax policy affects the wage elasticity of effort, which in turn affects the wage elasticity of labor demand. Since these alter the scope with which workers can attract labor rents, this constitutes an additional channel by which tax policy can influence the wage negotiation. As it turns out, this additional impact reinforces the effects of partial tax policy measures that we observe in the standard bargaining and efficiency models.

Table 1: Labor taxation in the OECD countries

Country	(1) Average wage tax	(2) Marginal wage tax	(3) average wage tax rate progression *ARP*	(4) Calculated relative tax exemption *a/w*
Australia	28.6	35.4	6.8	22.9
Austria	44.9	55.5	10.6	56.1
Belgium	54.2	66.4	12.2	34.8
Canada	32.3	33.9	1.6	26.4
Czech Republic	43.6	48.1	4.5	34.9
Denmark	41.5	49.2	7.7	20.7
Finland	43.8	55.1	11.3	36.6
France	47.4	66.6	19.2	30.3
Germany	50.7	64.0	13.3	44.9
Greece	34.9	44.2	9.3	95.2
Hungary	45.8	54.7	8.9	52.3
Iceland	29.7	40.4	10.7	30.7
Ireland	23.8	33.2	9.4	49.5
Italy	45.7	58.0	12.3	46.7
Japan	26.6	31.5	4.9	47.8
Korea	16.6	24.8	8.2	80.0
Luxembourg	31.9	45.9	14.0	64.5
Mexico	15.4	23.4	8.0	78.1
Netherlands	43.6	50.7	7.1	56.6
New Zealand	20.7	33.0	12.3	37.3
Norway	36.9	43.2	6.3	25.4
Poland	43.1	45.7	2.6	33.7
Portugal	32.6	39.4	6.8	60.0
Slovak Republic	42.0	48.3	6.3	52.1
Spain	38.0	45.5	7.5	43.3
Sweden	48.0	51.7	3.7	17.0
Switzerland	28.8	36.5	7.7	46.7
Turkey	42.7	44.5	1.8	12.5
United Kingdom	31.2	40.6	9.4	35.1
United States	29.6	34.1	4.5	22.5

Source: OECD (2004)

Legend: Tax rates are for the year 2004 for a single person with 100% of average wage, relative to the gross wage including the social security contributions paid by employees. Column (3) shows the difference between marginal and average rate of income tax. As an approximation it is assumed that for each country the tax schedule consists of a tax exemption and a constant marginal tax rate. The exchange rate between US dollar and euro was assumed to be unity. Social assistance level does not include housing costs. Numbers for social assistance are from 2002 and taken from OECD (2004), Benefits and Wages, OECD Indicators.

In the second main part of the paper, we then analyze revenue-neutral tax reforms that change the degree of tax progression, and derive the qualitative effects such tax reforms have on the negotiated wage, individual effort, and aggregate employment. Table 1 highlights the importance of such an analysis. The labor tax systems in all the OECD countries are progressive and show significant differences in the degree of tax progression. We measure tax progression by the difference between marginal and average tax rates that are shown in the first and second columns.[1] This difference, reported in the third column, is known as the *average wage tax progression ARP* (see Lambert 2001 and our section 5). The higher this difference, the more progressive wage taxation is. The highest difference is for France, with 19.2 percentage points, and the lowest one for Canada, with only 1.4 percentage points.

Our first main result shows that an increase in wage tax progression always leads to wage moderation. In this respect, our model shows that the wage moderation effect of higher tax progression that is present in both the efficiency wage model and the bargaining model carries over to the more general case when both wage-setting motives are at work. The effect on effort and, consequently, on labor demand, however, is ambiguous. Although it remains true that introducing tax progression raises employment, it turns out that the claim “tax progression is good for employment” (Koskela and Vilmunen 1996) only applies to moderate degrees of tax progression.

In section 2 below, we present the basic structure of the model and describe the time sequence of decisions with respect to wage bargaining, labor demand, and individual effort determination. The workers’ individual effort determination and the firms’ labor demand are elaborated in section 3. Section 4 uses the Nash bargaining approach to analyze wage negotiations subject to firms’ labor demand and workers’ effort determination and presents the essential comparative static results. Section 5 applies the analysis to revenue-neutral changes in the labor tax structure and explores the effects of tax progression on the negotiated wage, individual effort, and employment. The main findings are summarized in section 6.

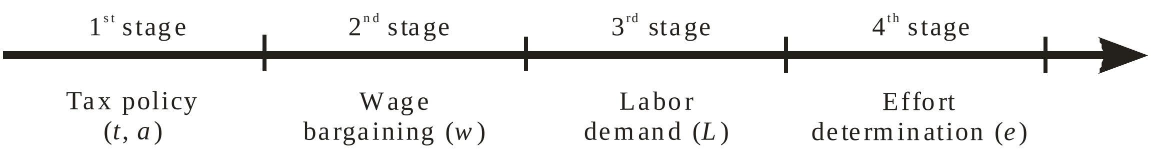
[1] To make these figures comparable with our stylized model framework below, all tax rates are with reference to the gross wage, including payroll taxes paid by the employer.

2. Basic framework

Concerning the time sequence of decisions, we assume that the government behaves as a Stackelberg leader who fixes the tax parameters in the first stage. To raise revenues, the government can employ a wage tax t, which is levied on the gross wage w minus a tax exemption a. Thus the tax base for the wage tax t equals $(w-a)L$, where L denotes total employment. In the presence of a positive tax exemption a, the marginal tax rate t exceeds the average tax rate $t^a \equiv t(1-a/w)$ so that we have a linearly progressive tax system. The net-of-tax wage workers receive is given by $w^n = (1-t)w + ta$. We abstract away from payroll taxes.

At stage 2, firms and trade unions bargain with respect to the gross wage.[2] They take the tax parameters as given and anticipate the consequences that the negotiated gross wage has for labor demand by firms and that the resulting net labor income has for individual effort determination by workers. After the wage negotiations are settled, the firms decide at stage 3 about their labor demand. Since firms cannot perfectly observe effort, the firms have to anticipate the workers' individual effort decisions. At the final stage, stage 4, workers make their individual effort choice. The time sequence of decisions is summarized in Figure 1. In the subsequent sections we derive the decisions taking place at different stages by using backward induction.

Figure 1: Time sequence of decisions



2 Since tax parameters are given from the viewpoint of firms and trade unions, it does not matter whether they bargain over gross or net-of-tax wages (see Koskela and Schöb 2002).

3. Individual effort determination and labor demand

We start by analyzing the 4th stage, where workers decide about their working effort, taking the tax policy, the negotiated wage, and aggregate employment as given. Then we analyze stage 3, where firms determine employment.

3.1. Individual effort determination

We focus on the choice that a single worker faces when employed by a representative firm in a static framework. Effort cannot be fully controlled by firms. They can set a standard effort that we normalize to one. If workers meet this standard, their jobs are secure. If they shirk by providing less effort, however, firms can fire them. The probability of detection depends positively on monitoring effort. Following Bental and Demougin (2006), we consider an isoelastic probability function of employment e^d where $d \in [0; 1]$ denotes the (constant) probability elasticity of effort. The probability of being laid off is thus $1-e^d$. Assuming a representative risk-neutral worker and applying a specific utility function V that is additively separable and quasi-linear, we obtain

$$(1) \qquad V^w = e^d [w^n - g(e)] + (1-e^d)b,$$

where b denotes the workers' outside option, which equals some exogenous unemployment income, and $g(e)$ denotes the disutility of effort e as a convex function, i.e. $g'(e), \; g''(e) > 0$. Working time per worker is fixed and normalized to unity.

For the following, it is convenient to define the workers' surplus as the difference $s \equiv w^n - g(e) - b$. This allows us to rewrite the utility function as $V^w = e^d s + b$, which splits the utility into the expected surplus when working with effort e and the basic income b, which the household receives in any case. The optimal individual effort level can be derived from the first-order condition $V_e^w = de^{d-1}s - e^d g'(e) = 0$. The worker chooses an effort level at which the expected utility loss of working harder, which occurs with probability e^d, equals the expected utility gain from an increased probability of staying in employment and receiving the surplus *s*. Using the parameterization $g(e) = e^{\theta} / \theta, \theta > 1$, the effort function becomes:

(2) $$e = \left(\frac{d\theta}{d+\theta}\right)^{\frac{1}{\theta}} \left(w^n - b\right)^{\frac{1}{\theta}} \equiv A^{\frac{1}{\theta}} \left(w^n - b\right)^{\frac{1}{\theta}}.$$

It is straightforward to show that individual effort is increasing in the net-of-tax wage rate, and decreasing in the outside option. This implies that we have $e_t < 0$, because this lowers the net-of-tax wage and thus reduces the penalty when caught shirking. Accordingly, we observe $e_w > 0$ and $e_a > 0$. In fact, we have $e_t = -\frac{(w-a)}{t} e_a$, a property we will employ later on. The wage elasticity of effort is

(3) $$\varepsilon \equiv \frac{e_w w}{e} = \frac{w(1-t)}{\theta(w^n - b)} > 0.$$

The respective partial derivatives with respect to the outside option *b*, the tax exemption *a*, and the tax rate *t* are

(4) $$\varepsilon_a = -\frac{w(1-t)t}{\theta(w^n - b)^2} < 0,$$

(5) $$\varepsilon_t = \frac{w(b-a)}{\theta(w^n - b)^2}.$$

The partial derivatives (4) and (5) depend on the effects the respective parameters have on the net-of-tax wage relative to the income surplus of working. With respect to an increase in the tax rate, this effect is ambiguous since a rise in the wage tax lowers $w(1-t)$ but at the same time raises the effective tax credit ta. A higher tax rate always increases the difference between the net-of-tax rate in absolute terms, but it may lower the relative difference, which is decisive for the elasticity if the tax exemption *a* is very generous. If $b = a$, the wage elasticity of effort is unaffected by *t* since in this case we have $(w^n - b) = (1-t)(w-b)$. A higher tax exemption *a* implies that a wage rate increase has a lower relative impact on the net-of-tax wage and thus implies a lower wage elasticity of effort. Only if $b > a$ does a rise in the tax rate increase the impact a rise in the wage rate has on effort: the higher *t* is, the stronger the relative increase of $w^n - b$ due to a wage increase is and thus the stronger the relative effect on individual effort.

The direct effect of a change in the tax exemption is unambiguous. An increase in the tax exemption implies that a marginal wage increase now has a lower relative impact.

3.2. Labor demand

In the 3rd stage of the game, each firm takes the tax parameters and the negotiated wage as given and decides about the labor demand L by taking into account how the representative worker will adjust effort. To derive an explicit solution, we postulate a decreasing returns-to-scale Cobb-Douglas production function in terms of labor and effort:

$$(6) \qquad f(eL) = \frac{\delta}{\delta - 1}(eL)^{\frac{\delta-1}{\delta}}, \quad \delta > 1.$$

Profit is given by $\pi = f(eL) - wL$. Since firms anticipate the effort level, workers will provide ($V_e = 0$), and the first order profit maximization condition is $\pi_L = 0 = f'(eL)e - w$. Using this specification, we obtain the following labor demand function:

$$(7) \qquad L = w^{-\delta}e^{\delta-1}.$$

The partial derivative of labor demand with respect to the tax parameters and the negotiated wage rate are

$$L_t = L\,\frac{(\delta-1)\,e_t}{e} < 0\,,\; L_a = L\,\frac{(\delta-1)\,e_a}{e} > 0\,,$$

$$L_w = -\delta\, w^{(-\delta-1)}e^{\delta-1} + w^{-\delta}e^{\delta-2}(\delta-1)e_w = -\frac{L}{w}(\varepsilon(1-\delta)+\delta) < 0\,.$$

Since the wage tax and the tax exemption are levied on workers, they only affect labor demand via the workers' individual effort, which depends on the net-of-tax wage rate. The wage rate w affects labor demand in two different ways. Note that the standard assumption that profit decreases with increases in the wage rate implies that the wage elasticity of effort is smaller than one, i.e. $\varepsilon < 1$. For the concave production function (6), the wage elasticity of labor demand depends on both the technological parameter δ and the wage elasticity of individual effort ε as defined in (3):

$$-\frac{L_w w}{L} \equiv \delta^* = \varepsilon(1-\delta) + \delta . \tag{8}$$

The wage elasticity of labor demand is lower compared to the case where wages do not affect effort. It now depends negatively on the wage elasticity of effort. For $0 \le \varepsilon < 1$ we have $1 < \delta^* \le \delta$. Hence, in the presence of unobservable individual effort determination the wage elasticity of labor demand depends on the tax structure and thus tax policy. If, for instance, a tax reform increases the wage elasticity of effort, labor demand would become less elastic. A wage rise would then be less costly for a trade union since the firm would then lay off fewer workers.

The firm's indirect profit function, which we will use in the next section, can be obtained by inserting labor demand (7) into the profit function:

$$\pi^*(w,e) = f(w^{-\delta} e^{\delta}) - w^{1-\delta} e^{\delta-1} = \frac{w^{1-\delta} e^{\delta-1}}{(\delta-1)} . \tag{9}$$

Having analyzed workers' and firms' behavior with respect to effort and labor demand, we can now turn to the collective wage bargaining of stage 2.

4. Collective wage bargaining

To derive the negotiated wage, we apply the Nash bargaining solution within a 'right-to-manage' model according to which employment is unilaterally determined by the firms.

The wage bargaining takes place in anticipation of the optimal employment decision by the firms (8) and the optimal individual effort decision by workers (2).

The trade union maximizes the sum of the workers utility V^w, and the utility of the unemployed. Since those caught shirking and fired are replaced by unemployed workers, the expected utility of an unemployed worker is

(10) $$V^u = \left(1-(1-e^d)\frac{L}{N-L}\right)b + (1-e^d)\frac{L}{N-L}(w^n - g(e^*)) .$$

While we assume that a single worker who is caught shirking will become and remain unemployed as well as receive b, from the viewpoint of the trade union, an unemployed member will replace a laid-off worker with the lay-off probability, which is $1-e^d$ times the employment share. We can rewrite the linear utilitarian objective function of the trade union as

(11) $$\hat{U} = V^w L + V^u (N-L) = s(e^*)\, L^* \;\; + b\; N ,$$

where the first term captures the workers' surplus from employment and the second term captures the exogenously given minimum income for all N members. L^* denotes optimal employment and e^* optimal effort in the s term. We denote the relative bargaining power of the union by β, and that of the firm by $(1-\beta)$, and assume that the threat points of the trade union and the firm are described by $U^0 = Nb$ and $\pi^0 = 0$, respectively. Applying the Nash bargaining solution, the negotiating parties decide on the wage w in order to solve

(12) $$\underbrace{Max}_{(w)}\, \Omega(w) = U^{\beta} \pi^{*\,1-\beta} , \text{ s.t. } V_e = \pi_L = 0 ,$$

where $U = \hat{U} - U^0 = s(e^*)\, L^*$ is the bargaining surplus to the trade union by including the disutility of effort and π^* is the indirect profit, presented in equation (9). The Nash bargaining solution satisfies the following first-order condition:

(13) $$\Omega_w = \beta \frac{U_w}{U} + (1-\beta)\frac{\pi_w^*}{\pi^*} = 0 .$$

As shown in appendix A, we can solve the first-order condition (13) to find the following implicit Nash bargaining solution for the wage rate in the presence of individual effort determination:

(14) $$w = \left(\frac{\beta + (\delta-1)(1-\varepsilon)}{(\delta-1)(1-\varepsilon) + \beta \dfrac{d}{(d+\theta)}} \right) \left[\frac{g(e^*) + b - ta}{1-t} \right] \equiv M \left[\frac{g(e^*) + b - ta}{1-t} \right],$$

where $d/(d+\theta) < 1$ and thus $M > 1$ for $\varepsilon \leq 1$. The negotiated gross wage rate depends on the exogenous income b when unemployed, the wage tax t and the tax exemption a. Furthermore, it also depends on the disutility from providing effort $g(e^*)$ and the term *M*, which we can interpret as the mark-up. Apart from exogenous parameters, this mark-up also depends on the wage elasticity of effort.

Before we discuss the general case, we will first briefly discuss several special cases, which can be analyzed within the framework developed here.

A. Observable effort

When effort is observable and verifiable, it can become part of the wage contract. If the contract specifies some fixed effort level $\bar{e}$, we obtain the standard right-to-manage model of union bargaining, where the wage depends on the bargaining power of the trade union and the (constant) wage elasticity of labor demand in the case of a Cobb-Douglas production function. Since a constant individual effort $\bar{e}$ implies $\varepsilon = 0$ and a zero probability of being caught shirking, $d = 0$, we have

(15) $$w = \left(\frac{\beta + (\delta-1)}{(\delta-1)} \right) \left[\frac{g(\bar{e}) + b - ta}{1-t} \right] \equiv M\big|_{\varepsilon=d=0} \left[\frac{g(\bar{e}) + b - ta}{1-t} \right],$$

which implies a surplus of $s = \left(\dfrac{\beta}{(\delta-1)} \right) \left(g(\bar{e}) + b - ta \right)$. From (16), we can easily derive the special cases of a monopoly union

$$w\big|_{\varepsilon=d=0,\beta=1} = \left(\frac{\delta}{\delta-1}\right)\left[\frac{g(\bar{e})+b-ta}{1-t}\right]$$

and the competitive labor market outcome where unions have no bargaining power and the gross wage only compensates for the disutility of working

$$w\big|_{\varepsilon=d=\beta=0} = \left[\frac{g(\bar{e})+b-ta}{1-t}\right],$$

in which case the firm exploits the complete workers' surplus, i.e. $s=0$.

B. Unobservable effort without bargaining

When $\beta=0$, the firm unilaterally sets the wage. From the first-order condition $\pi_w^*=0$, it follows immediately that the firm acts according to the well-known Solow-condition (Solow 1979), i.e. we have $\varepsilon=1$ and thus

(16) $$w = \left(\frac{\theta}{\theta-1}\right)\left[\frac{b-ta}{1-t}\right] \equiv M\big|_{\pi_w=0}\left[\frac{b-ta}{1-t}\right].$$

The model therefore also captures the essence of the efficiency models with a mark-up over the total outside option.

C. Unobservable effort with bargaining: comparative statics

For the general case, we have $(1-\varepsilon)>0$ and the mark-up is larger than one when the trade union has some bargaining power, $\beta>0$. It increases with the relative bargaining power of the trade union β, and depends negatively on the direct wage elasticity of labor demand δ. The wage rate now depends on several new terms that, in addition to the relative bargaining power, the wage elasticity of labor demand, the exogenous income, and the tax parameters, enter the formula: (i) the exogenously given probability of monitoring workers d, (ii) the indirect effect $g(e^*)$ via effort provision, and (iii) the elasticity of effort determination ε. Furthermore, unlike in the case of observable effort, the exogenous income b when unemployed, the wage tax rate t, and the tax exemption a will also affect the wage rate via the mark-up M.

The impact of a better monitoring of workers on the negotiated wage is zero as the wage elasticity of effort is not affected by monitoring. We can thus focus on the comparative statics of the tax parameters and the outside option in what follows. In doing so, we will call the term $(g(e^*)+b-ta)/(1-t)$ *the total outside option*.

The tax exemption affects the negotiated wage positively both via the mark-up and the total outside option as follows (see appendix B)

(17) $$w_a = \frac{1}{\Delta}\underbrace{\left(\underbrace{M_\varepsilon}_{+}\underbrace{\varepsilon_a}_{-}\right)}_{-}\left(\frac{g(e^*)+b-ta}{1-t}\right)+\frac{M}{\Delta}\underbrace{\left(\frac{g'(e^*)e_a-t}{1-t}\right)}_{-}<0 .$$

with $\Delta = 1-M_\varepsilon \varepsilon_w M^{-1}w - Mg'(e^*)e_w(1-t)^{-1} > 0$. In the Nash bargaining with observable effort (15), the mark-up is independent of a. With unobservable effort, however, workers will increase effort when the tax exemption rises. This, *ceteris parabus* lowers the mark-up because a lower wage elasticity of effort implies a higher wage elasticity of labor demand (see equation (8)). A higher wage then induces less effort, which makes the worker less productive. As a consequence more layoffs result from a wage increase.

The effect of the wage tax rate can be expressed as

(18) $$w_t = \frac{1}{\Delta}\underbrace{\left(\underbrace{M_\varepsilon}_{+}\underbrace{\varepsilon_t}_{+}\right)}_{+}\left(\frac{g(e^*)+b-ta}{1-t}\right)+\frac{M}{\Delta}\underbrace{\left(\frac{g(e^*)+b-a+(1-t)g'(e^*)e_t}{(1-t)^2}\right)}_{+}>0$$

(see Appendix B). The total effect of a higher wage tax rate on the negotiated wage is *a priori* ambiguous. When we assume $b \geq a$, both the effect on the mark-up and the effect on the total outside option with the given mark-up are unambiguously positive.

Hence, tax parameters in our model with both Nash wage bargaining and individual effort determination affect both of these via a change in the difference between the net-of-tax wage income and the outside option as well as via a change in the mark-up.

We summarize our new characterization of the negotiated wage under individual effort determination in

Proposition 1: Unobservable individual effort determination strengthens the effects tax policy measures have on the negotiated wage compared to the case where effort is observable. Decreasing the tax exemption lowers the negotiated wage. An increase in the wage tax rate increases the negotiated wage when $b \geq a$.

We can easily verify that the effects indeed reinforce each other. If we take the partial derivative of (15), we obtain the comparative statics effect for the standard bargaining model with

$$w_a = M\big|_{\varepsilon=d=0} \left(\frac{-t}{1-t}\right) < 0\,,\; w_t = M\big|_{\varepsilon=d=0} \left(\frac{g(\bar{e}) + b - a}{(1-t)^2}\right).$$

For $b \geq a$, the effects tax parameter changes have on the negotiated wage when effort is observable are always reinforced when effort is not observable. The partial derivative of equation (16) with respect to a shows the same result for the efficiency wage model: the different wage-setting motives thus reinforce the partial tax policy effects on gross wages. We should note, however, that in the case where $b \leq a$ and $g(\bar{e}) + b - a > 0$ we would obtain opposite partial effects for changes in the wage tax rate. An increase in the wage tax will then increase the gross wage when effort is observable but will lower the gross wage when effort is unobservable.

5. Tax-revenue-neutral change in tax progression in terms of wage formation, employment, and individual effort

We are now ready to analyze the impact a revenue-neutral restructuring of the labor tax, i.e. the degree of wage tax progression, has on wage formation, individual effort determination, and employment. The effect of wage tax progression, which keeps the tax revenue $G = [t(w-a)]L$ constant, can be written in the following way:

$dG = 0 = (w-a)Ldt - tLda + [tL + t(w-a)L_w]\,dw$. Recalling the definition of the average tax as $t^a \equiv t(1 - a/w)$, this can be expressed as

(19) $$da\big|_{dG=0} = \frac{(w-a)}{t}dt + \frac{(t - t^a\delta^*)}{t}dw\,.$$

An appropriate and intuitive way to define tax progression is to look at the average tax rate progression (*ARP*), which is given by the difference between the marginal tax rate t and the average tax rate t^a, $ARP = t - t^a$. The tax system is progressive if *ARP* is positive, and tax progression is increased if the difference increases (at a given income level, see Lambert 2001, chapters 7 and 8). The term $t - t^a\delta^*$ indicates the marginal tax revenue per worker when the gross wage increases. It can be decomposed in such a way that we have a *tax progression effect* and a *tax level effect*: $ARP + t^a(1-\delta^*)$. The total effect is non-positive for a linear tax system with $ARP = 0$ since $(1-\delta^*) \le 0$, but may eventually become positive if the tax system is sufficiently progressive since the employment effect is weighted by the average tax rate only. As we will see later on, the degree of tax progression is decisive for the way in which a revenue-neutral change in tax progression affects both employment and individual effort.

5.1 Revenue-neutral tax progression and the negotiated wage

The total effect of changes in the tax parameters *t* and *a* on the negotiated wage is

(20) $$dw = w_t dt + w_a da\,,$$

with the partial derivatives derived in section 4. Substituting (19) into the RHS of (20) for da gives

(21) $$dw = w_t dt + \frac{(w-a)}{t}w_a dt + w_a\left[\frac{t - t^a\delta^*}{t}\right]dw\,,$$

and, thus, the total effect of a revenue-neutral increase in the wage tax rate is

$$(22) \qquad \left.\frac{dw}{dt}\right|_{dG=0} = \frac{w_t + \frac{(w-a)}{t} w_a}{1 - w_a (t - t^a \delta^*) t^{-1}} \,.$$

In what follows, we assume Laffer-efficiency in the sense that a higher wage tax increases tax revenues while a higher tax exemption leads to lower tax revenues even when we take account of the indirect effects via changes in *w*. With respect to the tax exemption, we then have

$$\hat{G}_a = -tL + G_w w_a = -tL\left(1 - w_a \frac{t - t^a \delta^*}{t}\right) < 0 \,\mathbf{.}$$

Substituting the partial derivatives w_a from (17) and w_t from (18) into the numerator of (22) shows that the numerator is unambiguously positive (see appendix C). Hence, we have the following:

> **Proposition 2 (wage moderation):** A revenue-neutral increase in wage tax progression will moderate the negotiated wage in the presence of individual effort determination**.**

The interpretation is straightforward as it turns out that the numerator in equation (24) denotes the compensated effect an increase in the tax rate has on the wage, keeping the value of the Nash maximand constant (see appendix D). The revenue-neutral increase in the tax exemption fully offsets the income effect of the higher wage tax so that only the substitution effect of this progression-enhancing tax reform remains. This finding shows that the result from conventional 'right-to-manage' models in the absence of effort considerations (see, e.g., Koskela and Vilmunen 1996) also applies when we allow for unobservable individual effort determination.

5.2 Revenue-neutral tax progression and individual effort determination

The total effect of changes in the tax parameters t and a and the negotiated wage on effort determination is $de = e_t dt + e_a da + e_w dw$. Substituting the RHS of the tax-revenue neutrality (19) for da gives

$$\text{(23)} \quad \begin{aligned} \left.\frac{de}{dt}\right|_{dG=0} &= \underbrace{e_t + \frac{(w-a)}{t} e_a}_{=0} + \left(e_w + \frac{1}{t}\left(t - t^a \delta^*\right) e_a \right) \underbrace{\left.\frac{dw}{dt}\right|_{dG=0}}_{-} \\ &= \underbrace{\frac{e_a}{t}}_{+} \left((1-t) + t - t^a \delta^*\right) \underbrace{\left.\frac{dw}{dt}\right|_{dG=0}}_{-} . \end{aligned}$$

It is only the induced wage-moderation that affects individual effort decisions. The term e_a/t measures the impact one additional euro has on individual effort. A wage reduction of one euro reduces the net-of-tax wage by $(1-t)$ so that effort falls by $e_a(1-t)/t$. Wage-moderation also affects the amount by which the tax exemption can be raised. It will be lower than the neutral effect of raising a by $(w-a)/t$ if $t - t^a\delta^* < 0$. This always holds in a linear tax system but if the tax system becomes very progressive, i.e. $1 - t^a\delta^* > 0$, individual effort eventually will fall. This case is all the more likely, the smaller the wage elasticity of labor demand and the average tax burden are. If we assume a labor share of 2/3, we have $\delta = 3$, and an average tax below 1/3 would suffice to let effort fall when progression rises. Formally, we have

$$\text{(24)} \quad \left.\frac{de}{dt}\right|_{dG=0} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad \Leftrightarrow \quad 1 \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} t^a \delta^* .$$

A sufficient, but not a necessary, condition for individual effort to fall is $t\delta < 1$ since we have $\delta^* < \delta$ and $t_a < t$. These findings can be summarized in

> **Proposition 3 (individual effort determination):** A revenue-neutral increase in wage tax progression will lower individual effort if (i) the wage elasticity

of labor demand and/or (ii) the marginal tax rate are sufficiently low. A sufficient condition is $t\delta < 1$.

5.3 Revenue-neutral tax progression and employment

Finally, we consider the employment effect. The total effect of changes in the tax parameters *t* and *a* and the negotiated wage on employment is $dL = L_t dt + L_a da + L_w dw$. Substituting the RHS of (19) for da gives

$$\left.\frac{dL}{dt}\right|_{dG=0} = L^* \frac{(\delta-1)}{e^*} \underbrace{(e_t^* + \frac{(w-a)}{t} e_a^*)}_{=0} + \left(L_w^* + \frac{1}{t}\left(t - t^a \delta^*\right) L_a^* \right) \left.\frac{dw}{dt}\right|_{dG=0}$$

(25) $$= \left[L^* \frac{(\delta-1)}{e^*} \left(\frac{1}{t}\left(t - t^a \delta^*\right) e_a \right) - L^* \frac{\delta^*}{w} \right] \left.\frac{dw}{dt}\right|_{dG=0}$$

$$= L^* \frac{(\delta-1)}{e^*} \left.\frac{de}{dt}\right|_{dG=0} - L^* \frac{\delta^*}{w} \left.\frac{dw}{dt}\right|_{dG=0}.$$

The first two terms cancel out since they cover the change in t and a that *ceteris parabus.* would leave the average tax burden, and thus the net-of-tax wage, constant. Hence, we are left with two effects. As we have seen in section 5.2, the tax reform affects individual effort. If – as is likely – effort decreases, labor productivity falls and *ceteris parabus* employment. On the other hand, the wage-moderating effect increases labor demand for any given effort level. The total effect thus becomes ambiguous. From proposition 3 we can immediately infer

> **Proposition 4 (rising employment):** A sufficient, but not necessary, condition that a revenue-neutral increase in wage tax progression will increase employment is $t^a \delta^* \geq 1$.

Substituting the RHS of (23) for $\left.\frac{de}{dt}\right|_{dG=0}$ in (25) we obtain

(26) $$\left.\frac{dL}{dt}\right|_{dG=0} = \frac{-L^*}{w}\left[\delta^* - \frac{(1-t^a\delta^*)(\delta-1)e_a}{te}\right]\left.\frac{dw}{dt}\right|_{dG=0}.$$

From equation (25), it follows immediately that starting from a linear tax system, employment will definitely rise. This leads to

> **Proposition 5 (rising employment):** Introducing tax progression is good for employment when wages are negotiated and effort is determined individually.

Although we have seen that different wage-setting motives reinforce tax policy effects on gross wages, this is not no longer true with respect to employment. With observable and verifiable effort, employment is always decreasing when tax progression rises. When effort is unobservable and not verifiable, we find a countervailing effect via the adverse effect a rise in tax progression has on individual effort.

6. Conclusions

We provide an extended framework to study the implications of the imperfectly observable individual effort of workers on the negotiated wage and the impact of a revenue-neutral change in the wage tax progression on wage negotiations, effort, and employment. The first, and most important, result is that a higher degree of tax progression always leads to wage moderation. Our model confirms this result for the case of observable effort and wage bargaining as well as for the case where firms set efficiency wages unilaterally: the different wage-setting motives reinforce partial tax policy effects present in each model. However, when effort is not observable and verifiable, the clear-cut effect well-known from the wage bargaining literature that tax progression is good for employment does not carry over to the case of imperfectly observable effort. In the general case, it remains true that *introducing* tax progression is good for employment, but if the adverse effect on effort becomes sufficiently large due to too high a degree of tax progression, we cannot rule out the case where employment falls as a consequence of a progressivity-enhancing tax reform.

References

Altenburg, L. and M. Straub (1998): Efficiency wages, trade unions, and employment, *Oxford Economic Papers*, 50, 726-746.

Bental, B. and D. Demougin (2006): *Institutions, bargaining power and labor shares,* SFP 649 Discussion Papers Series, Berlin, 9/January.

Bulkley, G. and G. Myles (1997): Bargaining over effort, *European Journal of Political Economy*, 13, 375-384.

Bulkley, G. and G. Myles (1996): Trade unions, efficiency wages and shirking, *Oxford Economic Papers*, 48, 75-88.

Diamond, P. and M. Yaari (1972): Implications of the theory of rationing for consumer choice under uncertainty, *American Economic Review*, 62, 333-343.

Garcia, J. R. and J.C. Rios (2004): *Effects of Tax Reforms in a Shirking Model with Union Bargaining*, WP-AD2004-42.

Garino, G. and C. Martin (2000): Efficiency Wages and Union-Firm Bargaining, *Economics Letters*, 69, 181-185.

Holm, P. and E. Koskela (1996): Tax progression, structure of labour taxation and employment, *FinanzArchiv*, 53, 28-46.

Koskela, E. and J. Vilmunen (1996): Tax progression is good for employment in popular models of trade union behaviour, *Labour Economics*, 3, 65-80.

Koskela, E. and R. Schöb (1999): Does the composition of wage and payroll taxes matter under Nash bargaining?, *Economics Letters*,64, 343-349.

Koskela, E. and R. Schöb (2002): Optimal Factor Income Taxation in the Presence of Unemployment, *Journal of Public Economic Theory* 4.2002, 387-404.

Lambert, P.J. (2001): *The distribution and redistribution of income*,3rd edition, Manchester University Press.

Lindbeck, A. and D. J. Snower (1991): Interactions between the efficiency wage and insider-outsider theories, *Economics Letters*, 37, 193-196.

Pisauro, G. (1998): The effect of taxes on labour in efficiency wage models, *Journal of Public Economics*, 46, 329-345.

Rasmussen, B.S. (2002): Efficiency wages and the long-run incidence of progressive taxation, *Journal of Economics*, 76, 155-175.

Sanfey, P.J. (1993): On the interaction between efficiency wages and union-firm bargaining models, *Economics Letters*, 41, 319-324.

Shapiro, C. and J.E. Stiglitz (1984): Equilibrium unemployment as a worker discipline device, *American Economic Review*, 74, 433-444.

Solow, R.M. (1979): Another possible source of wage stickiness, *Journal of Macroeconomics*, 1, 79-82.

Appendix A: the negotiated wage

This appendix develops the expressions for the terms π_w^*/π^* and U_w/U in the first-order condition (13) that determines the Nash bargaining solution. We start by looking at the profit response of the firm to a change in the wage rate. The indirect profit function was presented in equation (9). By applying the envelope theorem, according to which the effect which takes place through the labor demand vanishes at the optimum, we find that

(A1)
$$\begin{aligned}\pi_w^* &= \frac{1}{(\delta-1)}\left[(\delta-1)(w)^{1-\delta}e^{\delta-2}e_w-(\delta-1)(w)^{-\delta}e^{\delta-1}\right]\\ &=(w)^{1-\delta}e^{\delta-1}w^{-1}\left[\frac{e_w w}{e}-1\right]=-\frac{(w)^{1-\delta}e^{\delta-1}}{w}\left[1-\varepsilon\right]<0,\end{aligned}$$

from which it follows that

(A2)
$$\frac{\pi_w^*}{\pi^*}=-\frac{(\delta-1)(1-\varepsilon)}{w}<0$$

as $\varepsilon<1.$ With respect to the trade union's utility, we find that

(A3)
$$U_w=\frac{L^*}{w}\left[w(1-t)-g'e_w w-\delta^*(w^n-g(e^*)-b)\right],$$

where $\delta^*=\delta(1-\varepsilon)+\varepsilon=-\frac{L_w^* w}{L^*}$. Thus, it follows that

(A4)
$$\frac{U_w}{U}=\frac{1}{w}\left[\frac{w(1-t)-g'e_w w-\delta^*(w^n-g(e^*)-b)}{w^n-g(e^*)-b}\right].$$

Substituting (A4) and (A2) into (14) yields

(A5)
$$\frac{\beta\left[w(1-t)-g'e_w w-\delta^*(w^n-g(e^*)-b)\right]}{\left[w^n-g(e^*)-b\right]}=(1-\beta)(\delta-1)(1-\varepsilon)\,.$$

This can be rewritten as:

$$
\text{(A6)} \quad
\begin{aligned}
&\beta\left[w(1-t)-g'e_w w-\delta^*(w^n-g(e^*)-b)\right]-w^n(1-\beta)(\delta-1)(1-\varepsilon)\\
&=-(g(e^*)+b)(1-\beta)(\delta-1)(1-\varepsilon) \ \Leftrightarrow\\
&\beta\left[w(1-t)-g'e_w w-\delta^* w^n\right]-w^n(1-\beta)(\delta-1)(1-\varepsilon)\\
&=-(g(e^*)+b)\left[(1-\beta)(\delta-1)(1-\varepsilon)+\beta\delta*\right] \ \Leftrightarrow\\
&\beta\left[w(1-t)-g'e_w w-\delta^* w(1-t)\right]-w(1-t)(1-\beta)(\delta-1)(1-\varepsilon)\\
&=-\left[g(e^*)+b-ta\right]\left[(1-\beta)(\delta-1)(1-\varepsilon)+\beta\delta^*\right]
\end{aligned}
.
$$

Using the definition of the total wage elasticity of labor demand δ^*, we obtain

$$
\text{(A7)} \qquad w(1-t)\left[(\delta-1)(1-\varepsilon)+\beta\frac{g'e_w w}{w(1-t)}\right]=\left[g(e^*)+b-ta\right]\left[\beta+(\delta-1)(1-\varepsilon)\right].
$$

$$
\text{(A8)} \qquad w(1-t)\left[(\delta-1)(1-\varepsilon)+\beta\frac{d}{(\theta+d)}\right]=\left[g(e^*)+b-ta\right]\left[\beta+(\delta-1)(1-\varepsilon)\right].
$$

Appendix B: comparative statics of the negotiated wage in terms of outside option, wage tax, and tax exemption

To see the effect the parameters have on the mark-up, it is convenient to change notation slightly:

$$
M=\frac{\beta+(\delta-1)(1-\varepsilon)}{(\delta-1)(1-\varepsilon)+\beta\dfrac{d}{(\theta+d)}}=\frac{\beta+(\delta-1)(1-\varepsilon)}{N}.
$$

The mark-up with respect to ε is

$$
\text{(B1)} \qquad M_\varepsilon=\frac{\beta(\delta-1)}{N^2}\frac{\theta}{(\theta+d)}>0.
$$

The mark-up with respect to effort *e* is $M_e=0$. Condition (13) is an implicit function of w. Thus the partial derivative with respect to, for example, a is

$$dw\left[1-\left(M_{\varepsilon}\varepsilon_{w}\right)\left(\frac{g(e^{*})+b-ta}{1-t}\right)-M\underbrace{\left(\frac{g'(e^{*})e_{w}}{1-t}\right)}_{+}\right].$$
$$=\left[\left(M_{\varepsilon}\varepsilon_{a}\right)\left(\frac{g(e^{*})+b-ta}{1-t}\right)+M\left(\frac{1+g'(e^{*})e_{b}}{1-t}\right)\right]da$$

First we have to sign the term in square brackets:

(B2) $$\Delta\equiv\left[1-M_{\varepsilon}\varepsilon_{w}\left(\frac{g(e^{*})+b-ta}{1-t}\right)-M\left(\frac{g'(e^{*})e_{w}}{1-t}\right)\right]$$

Adding the first and third term in the square brackets yields

$$1-M\left(\frac{g'(e^{*})e_{w}}{1-t}\right)=\frac{w(1-t)-Me^{\theta}\varepsilon}{w(1-t)}=M\frac{g(e^{*})+b-ta-g(e^{*})\theta\varepsilon}{w(1-t)}$$
$$=M\frac{\left(1-\frac{w(1-t)}{w^{n}-b}\right)g(e^{*})+b-ta}{w(1-t)}=M\frac{-\frac{(b-ta)}{w^{n}-b}g(e^{*})+b-ta}{w(1-t)}=M\frac{(b-ta)\left(w^{n}-b-g(e^{*})\right)}{w(1-t)(w^{n}-b)}>0.$$

Thus we can sign Δ:

(B3) $$\Delta=\left[\underbrace{M\frac{(b-ta)\left(w^{n}-b-g(e^{*})\right)}{w(1-t)(w^{n}-b)}}_{+}-\underbrace{\left(\underbrace{M_{\varepsilon}}_{+}\underbrace{\varepsilon_{w}}_{-}\right)}_{-}\left(\frac{g(e^{*})+b-ta}{1-t}\right)\right]>0$$

With $\Delta>0$, it is straightforward to sign the first term in equation (16) because $M_{\varepsilon}\varepsilon_{a}<0.$ The second term in equation (16) is also positive since

(B4) $$g'e_{a}-t=-t\frac{w^{n}-b-g(e)}{w^{n}-b}<0\,.$$

In equation (19), we have

(B5) $$g(e^{*})+b-a+(1-t)g'e_{t}=(b-a)\frac{w^{n}-b-g(e)}{w^{n}-b}\,,$$

which is positive if $b-a>0$. QED.

Appendix C: the sign of the numerator of (22)

Substituting the partial derivatives (19) and (20) into the numerator of (22) yields

$$
\begin{aligned}
& w_t + \frac{(w-a)}{t} w_a = \frac{M_\varepsilon}{\Delta}\left(\varepsilon_t + \frac{(w-a)}{t}\varepsilon_a\right) \\
& + \frac{M}{\Delta}\left[\frac{g(e)+b-a+(1-t)g'(e)e_t}{(1-t)^2} + (\frac{w-a}{t})\left(\frac{g'(e)e_a - t}{1-t}\right)\right] \\
& = -\frac{M_\varepsilon}{\Delta}\frac{d\theta w}{d+\theta}\left(\frac{(w^n-b-g(e))}{(w^n-b)^2}\right) + \frac{M}{\Delta}\frac{(w^n-b-g(e))}{(w^n-b)}\left[\frac{b-a}{(1-t)^2} - \left(\frac{w-a}{1-t}\right)\right] \\
& = -\frac{M_\varepsilon}{\Delta}\frac{d\theta w}{d+\theta}\left(\frac{(w^n-b-g(e))}{(w^n-b)^2}\right) - \frac{M}{\Delta}\frac{(w^n-b-g(e))}{(1-t)^2} < 0.
\end{aligned}
$$

Appendix D: the Slutzky-decomposition for the total effect of the wage tax on the negotiated wage

Differentiating the indirect Nash maximand $\Omega^* = U^\beta \pi^{*1-\beta} = \Omega^0$, where $U = sL^*$ and $\pi^* = f(e^* L^*) - w(1+s)L^*$, with respect to t and a gives

(D1) (i) $\Omega_t^* = \beta U^{\beta-1}\pi^{*1-\beta} U_t = -\beta U^{\beta-1}\pi^{*1-\beta} L^*(w-a) < 0$,

(ii) $\Omega_a^* = \beta U^{\beta-1}\pi^{*1-\beta} U_a = \beta U^{\beta-1}\pi^{*1-\beta} L^* t > 0$.

The wage tax has a negative effect and tax exemption has a positive effect on the Nash maximand. Using the comparative statics, the indirect Nash maximand can be inverted in terms of a for the function $a = h(t, \Omega^0)$. Substituting this for a in $\Omega^* = U^\beta \pi^{*1-\beta} = V^0$ gives the compensated indirect Nash maximand $\Omega^*(t, h(t, \Omega^0)) = \Omega^0$.[3] Differentiating this compensated indirect Nash maximand with respect to t gives $\Omega_t^* + h_t\, \Omega_a^* = 0$ so that $h_t = -\Omega_t^* / \Omega_a^* = (w-a)/t$. This describes the relationship of tax parameters to keep the Nash maximand constant.

[3] See, e.g., Diamond and Yaari (1972).

According to the duality theorem, the Nash maximand wage function w and the compensated wage function w^c at the same Nash maximand level are equal, so that we have $w(t, h(t, \Omega^0)) = w^c(t, \Omega^0)$. Differentiating this with respect to the wage tax gives $w_t + h_t w_a = w_t^c$ so that we obtain the Slutsky equation

(D2) $$w_t = w_t^c - \frac{(w-a)}{t} w_a \,,$$

where the total effect of the wage tax rate has been decomposed into the negative substitution effect ($w_t^c < 0$, see Appendix C) and the positive income effect $\left(-\frac{(w-a)}{t} w_a \right)$. QED.