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Stochastic Demographic Dynamics and Economic Growth: An Application and Insights from the World Data

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Abstract: *»Stochastische Bevölkerungsdynamik und Wirtschaftswachstum: Anwendungen und Einsichten auf der Basis von Weltdaten«*. This research has two broad objectives: First, to model population growth in a stochastic framework such that the effects of possible non-mean convergent shocks could be studied theoretically on long-run economic growth and planning. Second, an empirical strategy for modelling stochastic population growth over time is provided. Forecasting exercise has been rigorously carried for population growth and income by embedding the stochastic growth feature of population. For modelling purpose, a long-memory mechanism for population growth is suggested so that the classical economic growth assumption of constant and/or non-stochastic population growth in economic growth models appear as a limiting case. The analytical results show that embedding the stochastic features of population growth helps in explaining the economic growth volatility. In particular, it is found to be a formidable cause of the presence of long-memory in output. The empirical analysis shows that unless the stochastic feature of population growth is taken into empirical growth models, we will not be able map out the significant effects of demographic variables consistently over time. It is also shown that how corroborating the information of stochastic shocks of population alters our forecast vision by impacting significantly on the precision of the estimates.

Keywords: Stochastic population growth, long-memory, convergence patterns approach, population and income forecasting.

1. Introduction: Cross Country Growth Variations – Tracing Causes, Sources and Consequences

The study of demographic-economic growth relation is quite old dating back to the days as early as Malthus (1798). Scores of theoretical papers since then have attempted to unravel the dynamics of population growth and economic development, most of them pointing to the fact that 'excess population' would retard economic growth via excess resource consumption. Serious empirical dissections also flourished in the past decades, however, a major part of the

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findings were mired by confusion about whether population has any perceptible effect on economic development. If so, in which direction? Concrete conclusions about the effect started to occur in the 1990s, thanks to the advent of new growth theory that initiated research in this direction. Banking upon the latter's influence, cross-country empirical analysis showed that 'concrete and meaningful' result would occur when we disentangle the aggregate population into its components, viz., age-specific population (Kelley and Schmidt, 1995, 2001). However, despite the steady progress of theoretical and empirical research in this line, the consequences of stochastic nature of population, particularly the possibility of a persistent shock, has remained largely unaddressed by mainstream economic theory. This dissertation aims at studying the effect of such possible stochastic demographic shock on economic growth and development.

A common assumption in growth models is that population growth is exogenous. Ramsey and the typical Solow-Swan models are the two standard examples. Population has also been considered endogenous in some studies where the level of economic development determines the growth of population and vice versa. In this vein, some recent research (viz., Boucekkine et al., 2002) has stressed the effect of vintage age structure on economic growth. Whether 'population' is exogenously or endogenously treated, as economic applied growth theorists, we advertently assume that population is 'stationary' in nature, moves slowly without continuous drift, and in effect remains more or less 'stable' over time.

The curse of time is that nothing in the universe escapes the effect of shocks, big or small, physical or non-physical. Shocks do not remain constant and travel in emptiness. They accumulate as time advances and moves across space. Shocks, with even a smaller magnitude at some point in time, might destabilize the system after long time of accumulation. Therefore, the progress of the shock over the historical trajectory of a variable is of utmost relevance while monitoring the evolution of the system. In the demographic and economic growth context, extant empirical and theoretical research outrightly assume 'stationarity' of aggregate and age-structured population growth, and therefore they relegate any possibility of the influence of stochastic demographic shocks in economic growth. It seems to us that the 'stationary' assumption is far too simplistic in the analysis of complex economic system. Indeed, the lack of substantive application of stochastic behavior of population from temporal dimension may be attributed partly due to the ease of economic modeling and partly due to the unavailability of sophisticated econometric tool. Modeling population in this setting is alarmingly sparse but for some modest contributions of Diebolt and Guirard (2000) and Gil-Alana (2003).

Taking lead from these, in this book we aim to provide a comprehensive analysis of stochastic behavior of demographic shocks studied from temporal perspective and discuss how these shocks interact with the economic system of developed and developing countries. In particular, utilizing the stochastic demographic properties in the temporal domain, the book aims to offer explanations of cross-country growth variations, trace the source and extent of fluctuations, and study their consequences for long-term economic planning. Although appreciable attempts have been made in the past to explain cross-country growth variations, the extant literature have primarily focused on non-persistent demographic system with stationary shocks, and have attributed the growth variations to the variations in technological innovations, human capital progress and proliferation of knowledge. While the contributions of these factors cannot be denied, the underlying mechanism to explain economic process over time has not been justified. Consequently, there is no single explanation to cross-country growth variations. In light of this, the book integrates the complex contribution of innovation, knowledge creation and human capital development in economic growth by building a unified framework, viz., demographic-economic growth system in the temporal domain. We posit that demographic pressure leads to higher innovation due to excess demand push, knowledge creation occurs with higher trained population which at the same time increases the volume of human capital.

Therefore, characterization of demographic and economic growth system together can explain much of the underlying dynamics of growth variations. More accurate explanations will emerge when their time properties are exploited. This is precisely the main objective of this book. To this end, we study the stochastic nature of demography-economic growth system. The specific thematic threads on which this research has been carried are outlined below.

1.1 Cliometric investigation on demographic components and economic growth

As a prelude to examining the stochastic demographic characteristics, which forms the core of Chapter 3, in Chapter 2 we investigate how the effects of demographic components viz., age specific population have changed over the decades. Following the standard practice of assuming 'stationary' features of population growth, we first evaluate and extend the popular empirical economic growth models. Specifically, we extend Kelley and Schmidt (1995, 2001) by adding additional growth regressors and increasing the sample span. In this exercise, we note certain inconsistencies and irregularities of growth variations. Further we find that decadal changes have brought forth variations in economic growth of developed and developing economies. Therefore, we tend to argue that accounting for temporal features of the demographic and economic growth system would provide clear insights into persistent growth fluctuations.

Although the dynamic response of output to the variations in age-specific population is studied in a panel setting, the assumption of stationarity limits us to further explore the artefact of stochastic shocks within large cross-section of countries and with large time dimension. We hail this problem as one possible source of explanation to why in the 1980s some empirical results showed positive effect of population growth on economic development and some found even no measurable effect. The dynamical changes that occur over time, specifically the recognition of stochastic shocks in population, is important in its own right and therefore must be incorporated in the study of demographic-economic growth models.

1.2 Long-memory demography and economic growth

Reflecting on the limitations of Chapter 2, in Chapter 3 we develop a new mechanism to characterize stochastic nature of demographic shocks in which population series with large temporal dimension is assumed to be governed by certain amount of stochastic shocks. This allows us to characterize aggregate and age-structured population growth by a long-memory (or fractional memory) data generating process. By doing so, the conventional 'stationary' assumption underlying the current theoretical and empirical exploration is relaxed and more dynamic information about the persistence of shocks is accommodated in the economic growth models. In the framework of endogenous economic growth with endogenous population change, this chapter sets out to build a long-memory model for population and its components (viz., age structure) to delineate the effects of demographic changes on developed and developing country economies. The chapter mainly focuses on the validity and plausibility of stationary assumption of population growth, studies their effects on standard growth model and empirically illustrate the effects of such shocks on the development objectives. To this end, we first provide a theoretical framework to show that long-memory shocks in demographic age structure or population might induce long-memory in economic growth. An empirical illustration of both developed and developing countries is carried out to demonstrate that population age structure in these countries are characterized by long-memory. In general, we find non-mean-convergent demographic shocks for some countries, while for others, stationary long-memory guides their growth processes.

1.3 Population forecasting and stochastic long-memory

Following the theoretical development and empirical illustration in Chapter 3, in Chapter 4 we propose to employ fractionally integrated ARMA (in short, ARFIMA) model for forecasting total population and demographic age-structure. The conventional methods of population forecasting is discussed in this chapter evaluating the advantages and potential weaknesses of these methods. Our approach to population forecasting while accounting for stochastic shocks is a major shift from the conventional 'low, medium, and high' variant of the population projection. Moreover, our approach is a departure from the stochastic population forecast based on Leslie matrix. We also examine in this chapter why forecasting techniques in demography have not been so advanced though the methods have not remained too traditional either.

The ARFIMA methodology suggested in Chapter 4 is an extension of the ARMA methodology used by Pflaummer (1992) and Lee and Tuljapurkar (1994). By employing the ARFIMA methodology we allow both short-run and long-run dynamics of the demographic system. We also evaluate how endogenous demographic shifts contributes to the dynamics of the demographic processes in terms of non-linearity and how the use of such features impact upon the forecasts. Accordingly, long-memory population forecasting in the presence of endogenous phase switching is performed. Pflaumer (1992) used Box-Jenkins ARMA methodology to forecast US total population. ARMA method suffers from the typical drawback that it relies heavily on the unit root assumption. We relax this integer restriction and employ a fractionally integrated ARMA method to forecast total population for a sample of developed and developing countries. Another important distinction of our model is that we incorporate stochastic regime switching features in the ARFIMA estimation assuming that demographic variables are prone to endogenous phase switchings. Forecasting properties in the presence of regime switching are discussed. Using the data of a set of developed and developing countries total and age-structured population, we forecast them by utilizing their properties of stochastic shocks characterized by ARFIMA processes. We also show in what way the forecasts are different from the earlier research. Specifically we compare the results with UN forecast and depict that while our demographic forecasts are different from the earlier approaches, our method can also act as a complementary tool to gain accurate information on the future projections.

1.4 Income-forecasting with long-memory demographic dynamics

Chapter 5 is a cogent extension of Chapter 4. In this chapter we incorporate the memory properties of demographic age-distribution to forecast Gross Domestic Product (or National income) of some developed and developing economies. Based on a panel data framework, Malmberg and Lindh (2005) proposed a demography-based global income forecasting. An apparent assumption in their model is that total as well as age-structured population growths are stationary and therefore stochastic demographic shocks would contribute little to demographic variations. We relax this assumption in the forecasting model by noting that the growth of total age-structured population need not be stationary and that any degree of stochastic shocks in these series can affect forecast performance. Given that a long-memory panel method is yet to be comprehensively built for forecasting, we perform forecast of demography-based income in the univariate context assuming a stochastic long-memory process for age-structured population growth. Finally, Chapter 6 summarises the main findings of this research and outlines possible directions for further research.

2. Cliometrics of The Abiding Nexus Between Demographic Components and Economic Development

2.1 Introduction

The mystery of economic growth has not been cracked in economics. The movement of the production potential of the industrialised nations over long periods of time is still in the centre of the very latest economic (Aghion and Howitt, 1998; Temple, 1999) and cliometric (Abramovitz, 1986; Crafts, 1987; Darné and Diebolt, 2004; Goldin, 1995; McCloskey, 1987; North, 1994; Wright, 1971) debates. This preoccupation is far from new. The classical economists were already concerned about how to increase welfare by increasing growth (Smith, 1776). The subject remained controversial after World War II, with the theoretical debate on the long-term stability of market economies. However, through Solow's (1956) economic growth model neoclassical thinking gradually exerted its power. Its reasoning is clear and it also explains numerous aspects related to economic growth which are summarised perfectly in Kaldor's (1963) six 'stylised facts'. At the same time-perhaps paradoxically-scientific interest in work on growth and economic cycles disappeared. There were two main reasons for this. Firstly, the shortsightedness of economists whose attention was centered almost exclusively on the study of short movements and secondly, the comparative weakness of theoretical models incapable of solving the aspects that remain unexplained by the different theories of growth. This partially explains why the post-war neo-classical models are unsatisfactory.

Indeed, in the long run, they only account for economic growth by involving exogenous factors (except for Ramsey's (1928) model that was rediscovered very recently) and in this case the technical progress achieved without cost outside the economic system. In addition, Solow's reference model does not provide any way of explaining the divergence in growth rates at the international level, as with the idea of long-run equilibrium, all countries should progress at identical, exogenous rates of technical progress. Similarly, it should be noted that the hypothesis of the systematic existence of a negative correlation between income level and economic growth rate is not based on any satisfactory empirical verification. Finally, nothing really corroborates the convergence hypothesis, that is to say the transfer of capital from the richest to the poorest countries (Barro and Sala-I-Martin, 1992, 1995). However, the work of Lucas (1988) and Romer (1986, 1990) attracted attention, and the 1980s marked a renaissance of the neo-classical theory of growth. The prime objective was to go beyond the weakness of the old theoretical models. The aim was also to answer new questions: what are the determinants of sustainable economic growth? Can technical progress alone increase social welfare or can capital accumulation also lead to a permanent increase in per capita income? What are the factors of production that engender sustainable economic growth: physical capital, environmental capital, human capital, social capital or technological knowledge? What are the mechanisms that guarantee growth over a long period for a market economy? Finally, what is/are the market structure/s within which economic growth can be achieved? Strengthened through these questions, the debate on the determinants of the economic growth process has attracted considerable attention, both in the importance of its implications in terms of economic policy and in the number of theoretical and empirical analyses that it engendered. Curiously, population dynamics is often absent from the theoretical developments and empirical verifications (Fogel, 1994; Jones, 1998) or appears implicitly under the heading 'human capital'. As a possible response to this, our focus in this chapter is to identify, in econometric history terms, the role of the components of demographic change in economic development, here with an in-depth analysis of the demography-economic growth nexus in the last four decades (1960-2000).

Lately, demographic variables have been found to play central role in economic growth fluctuations[1] have been found to play central role in economic growth fluctuations in many developed and developing countries. However, not very long ago the contributory role of demography to the process of economic growth was considered redundant and was mostly mired by confusions about the sign and magnitude of population growth in the economy. It is only very recently that certain convergence of views have started to occur indicating that population age structure, and not the aggregate population matters for per capita income growth. The conservationist ideas used to dominate economic thinking in 1970s, for instance, the mercantalists viewed that a large population would stimulate economic growth, while Malthusian arguments (due to Malthus, 1798) unequivocally persuaded most economists to the point that due to decreasing returns, population growth would lead to lower per capita income. A neutralist view also emerged during the 1970s and 1980s concluding that population growth rates are not influential behind variations in per capita income. In effect, demographers and socio-economic policy makers found themselves in delirium as to 'which theory to believe' until Kelley and Schmidt's (1995) (hereafter, KS) seminal work threw a definite answer to the persistent mess of confusions. Building on the nuances of new growth theory, KS (1995) and latter Crenshaw et al. (1997) demonstrated that segregating population size

[1] For example, birth and death rates, life expectancy at birth, etc.

into different components[2] could offer the much needed solution to understand the exact consequences of population change in developed and developing economies.

Based on the sample spanning over three decades (from 1960-1990), KS (1995) found that components of population, viz., births and deaths had notable but offsetting impacts in the earlier periods (in the 1960s and 1970s). In contrast, as KS note, while the short-run costs of births increased significantly in the 1980s, especially in the developing countries, the short-run benefits of mortality reduction decreased during the same time. Moreover, KS also found the significant growth-enhancing effect of population density over 1960-1990. In what ensued the growing empirical developments in the last decade, Boucekkine et al. (2002) offered theoretical justifications to the short-run and long-run effects of the vintage nature of population on economic growth. In a more recent contribution, Boucekkine et al. (2005) illustrated that 'population pressure' (due to higher population density) could play significant role in productivity inducement, thereby contributing to the sustainable economic growth.

Though the theoretical developments in demographic-economic growth relation provide, *inter alia,* the basis for sound empirical modeling the attempt for the latter has so far been slowly-paced and less well-documented. Interestingly, due to the spurt of demographic fluctuations occurring in each decade, one cannot be sure if the conclusions of KS (1995) would still remain significant for the current decade. KS (1995) model is based on purely demographic factors, viz., birth and death rates, and the effect of labor force (captured by lagged birth rate by 15 years). However, the impacts of these variables on the economy may vary over time and even to the extent of inclusion of non-demographic variables such as inflation, number of schools per capita, etc. Thus, in view of the built-in complex demographic-economic relationship and the recent findings that population age structure exerts significant impact on a wide array of macroeconomic activities (Lindh and Malmberg, 1999) (hereafter ML), this chapter extends KS (1995) in two directions.

First, to assess the pattern of demographic change in the current decade we extend KS sample by 10 years (from 1990-2000). Two, in order to study the robustness of KS (1995) results we also add other demographic and non-demographic variables, viz., life-expectancy at birth and inflation to the KS demographic model. In fact, life-expectancy at birth is held as one of the most important factors in income variation in developed and developing countries and has been employed in the demography-based income forecasting by ML (2005). Moreover, in the standard macroeconomic models, inflation pressure is depicted by many authors to covary with the age distribution unless accommodated by monetary policy. For instance, Lindh and Malmberg (2000) estimate

[2] These are crude birth and death rates and lagged birth rate to account for labor force influence on economic growth.

the relation between inflation and age structure on annual OECD data 1960-1994 for 20 countries and found that an age pattern of inflation effects is consistent with the hypothesis that increases in the population of net savers dampen inflation, whereas especially the younger retirees fan inflation as they start consuming out of accumulated pension claims. The inclusion of these two variables in the demographic regression is purported to provide an idea of how the demographic-economic growth relationship can be sensitive to the inclusion of other relevant variables in the model.

Though KS (2001) use life expectancy and inflation and compares different demographic models propounded in the literature including their own, their sample is limited to the period 1960-1995. Interestingly, KS (1995) stress that adding non-demographic variables, such as inflation to their model, did not change the overall conclusion of the results, therefore they were induced to keep the pure demographic model with only birth, death rates and lagged birth rate (which takes account of labour force). The sample span for KS (1995) was from 1960-1990. In KS (2001), the authors add five years more to their sample and also add other non-demographic and demographic variables to the model. Though the variables appear to be significant and have expected signs, it is not clear whether addition of 5 years changed their results, which were not significant in their earlier study.

There is yet another important consideration in KS modeling. KS (2001) approximate the effect of the last decade by a five year average (1990-95). KS (2001) also use life expectancy at birth and inflation in their demographic model and find that the results are robust to the inclusion of other variables in the regression. However, KS data is beset with one notable problem; the authors they do not treat 'influential observations' in the sample. For instance, population density for Hong Kong and Singapore is exceptionally higher than other countries in the sample and this might influence the implications of the results. Moreover, KS (2001) could have used a five-year average throughout (from 1960-95) to lend consistency to the data segmentation. While the use of five year average (as claimed by KS, 2001) to approximate the 'changes occurring in the last decade' is not expected to bring alterations in the implications of results, it is important to know that this strategy provides at best an approximate and not the real descriptions of demographic effect on economic growth. Undeniably, given rather fast demographic changes in the last decade, inclusion of decadal variations in the regression as we have done in this chapter could provide a more realistic picture of demographic effects on economic growth, more so, the sensitivity of the last decade's impact in the regression.

Broadly speaking, the purpose of this chapter is to contribute o the interface literature of demography and economic growth by depicting the changing weight of demographic components effects on economic development. Specifically, we show that the consequences of rise (fall) in CBR and CDR has changed over the last four decades. Using an up-to-date sample period (till

2000) and GDP per capita income compatible with purchasing power parity (in 1995 international prices), we illustrate how the weights of these effect have changed over time. In contrast to KS (1995), we find that there is little gain to expect from further reductions in mortality in developing countries. Interesting implications follow as the effect of CBR is observed to become positive for developed countries. In the wake of the current demographic transition and the recently propounded 'zero population growth' as optimum for higher economic prosperity in the developed countries[3], the finding of positive effect CBR in these countries calls for a rethinking on the population policy. Moreover, we also find that the growth-enhancing effect of population density is only limited to 1960s instead of a perceived significant positive effect for all the decades. This chapter also argues that influential observations in the sample, like the presence of Hong Kong and Singapore in the population density due to their very high density figures in comparison to others in the sample, need to be treated so that demographic regression as in KS (1995) can provide meaningful intuition to demo-economic fluctuations. Given that the standard econometric literature explains how the presence of 'influential observations' in the sample can cause biased inference, our extended model (to be discussed shortly) finds the significant growth-enhancing effect of population density in for all the decades after dropping Hong Kong and Singapore from the sample.

The plan of this chapter is as follows. A synoptic view on the current state of the population debate is provided in Section 2.2. The idea of this section is to track down the exact direction of the trend, that is, till recently what has been held and concurrently proved about the effect of demographic components on economic growth. The evaluation will be based on three popular approaches, viz., correlation approach, production function approach, and convergence patterns model. A critical review of these models is provided in this section. In section 2.3, we discuss the importance of the components of demographic change on economic development with a specific attention on their short-run and long-term consequences. Section 2.4 outlines the model to be estimated and discusses the econometric methodology to be used. Features of the data and design of the variables are also noted in this section. Empirical results are presented in section 2.5. Section 2.6 concludes with the major findings of the chapter and discusses their implications in the current development context.

2.2 Stylised facts

In this section we provide a synoptic review of the empirical economic growth literature emphasizing on the discussion of different channels through which demographic dynamics can potentially affect economic growth. The theoretical armor and empirical comprehensiveness of recent demographic-economic

[3] See Boucekkine et al. 2002 for an analysis in this respect.

growth model was in its infancy in the 1970s and early 1980s. The role of the demographic process was underemphasized and consequently the effect of past and future demographic trends on growth used to remain largely unexplored (Boucekkine et al., 2002). Instead, technological change[4] tended to be used as the guiding force in models of economic growth. The recent theoretical advances (e.g., Boucekkine et al., 2002) thanks to the advent of new growth theory (viz., Lucas, 1988; Romer, 1986, 1990) and commendable progress in empirical macroeconomic literature, noted an exception to this trend by initiating a comprehensive research in demographic-economic growth relation. However this development was not easy, rather it took a baffling thirty years to establish a clear delineation between demography and economic growth. To address this we explicate first the development of theoretical and empirical construction of demography-economic growth relationship and then critically examine the empirical findings based on them.

2.2.1 The Construct

In the literature, three basic theoretical formulations, viz., correlation approach, production function approach, and convergence-pattern approach are found which describe the economy-demography nexus. Traditionally, **correlation approach** used to be extensively applied in early empirical growth models to explore demographic-economic interactions:

$$(Y/Ngr) = \Gamma_1(X_D) \tag{2.1}$$

$(\mathrm{Y/N}gr)$ is per capita output growth, X_D indicates demographic variable(s) which may include *ngr* , the contemporaneous growth of population (N), age structure of the population, crude birth rates (CBR) and/ or crude death rates (CDR), N and/ or population density, life expectancy, and migration. Performing investigation for various countries[5] over periods of time, several models during and before 1980s drew on unconditional correlations between per capita output, $(\mathrm{Y/N})$, and population growth, *ngr* . Empirical results widely vary: some providing evidence of 'no measured impact' of *ngr* on $(\mathrm{Y/N})$, many studies showing negative impact, and even some providing evidence of positive association between the two.

The results from this approach are not easy to explain as it delivers at best the first hand information on the effects of demography.[6] Moreover, *ngr* being an aggregate phenomenon does not provide any indication about the specific

[4] Some other variables were also used for explaining aggregate output growth, viz., human capital, free market institutions, and budgetary disciplines.
[5] Mostly developing countries considering the spur of rapid population growth.
[6] KS (2001) describe this as 'first pass assessments.'

'channels' population affects output growth. Given that an individual acts upon the economy's resource differently over his life cycle, it is instructive to segregate *ngr*,[7] into various components viz., young generation, working age, and retired cohorts. Correlation approach does not lend to segregation of demographic variables into various components. This feature is aptly carried by convergence-patterns approach which we will discuss shortly.

Production function framework provides an alternative mechanism to explore economic-demographic relationship. This approach as described in Eq.2.2,

$$Y = \Gamma_2(K, L, H, R, A) \tag{2.2}$$

involves lot of growth variables like physical capital (K), human capital (H), technology (A), and natural resources (R) to explain variations in output (Y). Data on these variables are not easily available and therefore, the variables are often transformed into growth terms for empirical execution. Eventually, demographic processes are linked to the growth of the factor inputs in the production function. Due to its limited empirical rendering an alternative formulation is suggested which is free from the drawbacks of these two approaches and enables itself to exhaustive econometric analysis. **Convergence-patterns model** provides an answer to this problem.

This model builds on the production function framework of Solow-Swan type. Following this construct the economic growth of a country is allowed to vary with the levels of economic development. Nonetheless, initial endowments of the economy play important role along with the demographic factors. The underlying idea of this model is to study the pace at which countries move from their current level of labour productivity to their long-run, or steady-state level. The rate of labour productivity is assumed to be proportional to the gap between the logs of the long-run steady state and current level of labour productivity.[8] Formally,

$$(Y/Ngr) = \delta[log((Y/N)^* - log((Y/N)] \tag{2.3}$$

Here $(Y/N)^*$ is the steady state (Y/N). The greater this gap, the greater are gaps of physical capital, human capital, and technical efficiency from their

7 *ngr* equals fertility rate minus the death rate. The net migration rate may be added, but this has been suppressed in the empirical model of this chapter.

8 We are using here the per capita output and per-laborer output interchangeably. For the latter, it would have been written as (Y/L), where L denotes labour. $L = \alpha N$, where α is usually unity.

long-run levels. Depending on country specific characteristics, the long-run (Y/Ngr) differs across countries.

Dasgupta (1995) observed that due to the poor resources base and lower level of initial development, developing countries with persistent poverty confront the problem of catching up with developed nations. Convergence-pattern model underlines this idea. Generally, the relationship between per capita income growth, $(Y/Ngr)_{t,t+n}$ and the initial level of per capita income following this model, is negative (See Solow, 1956). In line with the construct of the convergence-patterns approach, KS (1995) provided the following econometric model which has tended to be a bench mark for future research. Allowing that the demographic processes vary by stages of economic development, KS (1995) describe the convergence-patterns model[9] as:

$$(Y/Ngr)_{t,t+n} = \Gamma_3[(Y/N)_t, I_t, S_t, \{(X_D)_{t,t+n}, (X_D)_{t,t+n} * (Y/N)_t\}] \quad (2.4)$$

$\Gamma 3(.)$ is assumed to be a linear function of the variables. $(Y/Ngr)_{t,t+n}$, represents per capita output growth, $(Y/N)_t$ is the initial level of per capita income, I variables supplies information on the 'initial' state of the economy, for example, population density and educational attainment. Following eq.2.1, the demographic variables (X_D) include the contemporaneous $CBR_{t,t+n}$, $CDR_{t,t+n}$, CBR lagged by 15 years (CBR_{-15}), and life expectancy at birth, etc. S_t variables represent factors influencing economic development as well as changes in the stocks, viz., savings, investment returns, state of democracy, inflation, etc.

Recall that the inverse relation between $(Y/Ngr)_{t,t+n}$ and $(Y/N)_t$ forms the basis of convergence-pattern model. The interaction term, $(X_D)_{t,t+n} * (Y/N)_t$ implies that the effect of population growth and its components are allowed to vary by levels of economic development (Y/N). The competing hypotheses associated with this are consistent with a declining (increasing) negative (positive) effect of population growth as the country develops. Before providing a detailed analysis (in the ensuing section) of the direction and implications of (X_D) and I variables influences on per capita income growth, a summary view of their effects are in order. A rise in CBR is harmful for an economy as higher births prompt higher dependence on resources, but a rise CDR enhances economic growth as resource dependence is reduced. This observation is subject to subtle evaluation as higher deaths of workers impede economic growth while higher death of dependents stimulate it. Moreover, population density induces growth through 'demographic-pressure'. So far we described

[9] Note that, while in theory all variables are measured in exact instant t, in implementation, the measurement (of say, $(Y/N)_{t,t+n}$) is over the period $(t, t+n)$. Studies employ five-, ten-, 25-year, or even longer periods (KS, 2001).

the theoretical underpinning of the recent empirical model exploring demographic-economic growth association. In the next section, we summarise the findings (specifically the sign and direction of impacts) based on these empirical models (viz., correlation and convergence patterns).

2.2.2 Revisiting Empirical Literature

Analysis from correlation approach

Based on the correlation approach, Simon Kuznet found that there is in *general* a lack of correlation between population and economic growth in the early 1970s. However, such an astounding finding drew heavy criticisms as people were persuaded to believe that rapid population growth deters the pace of economic progress. This is because, increasing population has been customarily regarded as mere consumers of resources and that its faster growth is associated with diminishing returns to capital. Based on this strong prior belief, using international cross-country data for the 1980s some research (e.g., United Nations, 1973), as reviewed by KS (1994), found a negative association between population and economic growth. KS (1994) posit that a high rate of growth can not be supported by a corresponding increase in investment, thus lowering growth of per capita output.

Simon (1981) was probably the first to challenge this pessimistic view in his *The Ultimate Resource* depicting that population growth was likely to exert a *positive* net impact on economic development in many Third World countries 'in the intermediate-run'. In fact, this 'revisionist' approach of Simon (1981) changed much of the dogmatic thinking of population growth's consequence on economic development in subsequent years. He illustrated that the outcome of the population growth on the economy are likely to depend both on the *time dimension* of the assessments, and whether *feedbacks* are included in the analysis. Feedback effects arise, in the model, due to the population pressure that would ultimately cause natural resource exhaustion. Going by his illustration it means, over longer periods most natural resource prices actually declined. This happened despite the existence of rising demand from increasing population.[10] Hence the 'time dimension' is important for these 'adjustments' or 'feedbacks' to be assessed, which arises due to population pressure.

Based on both the time series and cross-sectional data, the investigators of National Research Council (NRC, 1986)[11] put a rather balanced and 'non-alarmist' assessment: 'on *balance,* we reach the qualitative conclusion that

[10] This is due to price-induced substitutions in production and consumption of natural resources, and an increase of supply due to technical advancement and innovation.

[11] The investigation purported for the economic consequences of population growth in poor countries, recognized that instead of regarding population growth as exogenously given, it should be treated as a causal factor.

slower population growth would be beneficial to economic development of most developing countries... (but) there is no cause of alarm over the high rates being experienced there'.The NRC's findings suggest that population growth has both positive and negative effects, and "given the current evidence, though the actual size of the impact can not be determined, the direction of the impact...can be detected". Since the sample considered by the council involved developing countries, a cautionary note was always due as 'persistent' high births[12] in those countries negate the positive feedback from the labor force. But since 'persistency' of higher births is likely to be attenuated in the longer run, youth's dependency on resources would go down giving rise to substantial contribution from the labor force.

Some other studies in the late 1980's (Srinivasan, 1988; Kelley, 1988) found that while slower population growth would indeed advance the economic well-being of most developing countries, the size of the net impact would not likely be especially remarkable by comparison with many other determinants of economic growth. Precisely, Kelley (1988) states that "*economic growth ...would have been more rapid in an environment of slower population growth, although in a number of countries the impact was probably negligible and in some it may have been positive.*"

Kapuria-Foreman (1995) in the more recent literature, found a non-significant correlation in cross country studies (in general) and a slightly positive causality from population to growth while considering time series analysis. In a sense, though it can be said that, research started in the 1980s 'revisioned' a comprehensive development in analyzing the impact of population growth mainly due to the consideration of 'time' dimension (i.e., modifying shorter-run direct effects of demography with feedbacks occurring in the longer run) and impact of separate components of demographic change (births, deaths, age, size, and density), however, until the 1990s, uncertainties remained concerning quantitative assessments of the impact.

Analysis from convergence-patterns approach

Extending Brander and Dowrick (1994) and Barlow (1994) framework,[13] KS (1995) used the convergence-patterns approach (eq.2.4) to explore the relation between economic growth and demographic factors. KS (1995) explored if direction of the impact of population growth has changed over three decades (1960-1990). Building on Barro's (1991) core variables,[14] KS (1995) modeled

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[12] We explain this feature and its consequence more clearly in the next section.

[13] These models showed that past births contribute to current labor force and hence promote economic growth, whereas current births impede economic growth due to adverse effects on resource base and thus on investment.

[14] The variables are age structure, life-expectancy, level of per capita income, level of education, crude birth and death rates, etc.

aggregate population growth taking into account its different demographic components and provided intuition about the direction of impact by untangling the short- and long-run effects of the components of demographic change. They distinguished between several alternative demographic influences on the economy's potential output in the long-run (e.g., the impact of population size and density), and timing of demographic impacts (e.g., the timing of birth rate and death rate reductions) which affect both the short and long-run.

The central results in KS (1995) summarized in Section 2.1 may be recalled: (i) 'A decrease in the crude death rate induces economic growth in developing countries', (ii) Although death rate reductions contributed positively in each decade, this contribution declined monotonically over time, (iii) Concerning the impact of birth rates, the short-term costs of high birth rates increased in the developing countries in the 1980s. For developed countries a different picture emerge: negative effects of birth rate are found, the magnitude of which is large in the 1960s and 1970s, but fairly small in 1980s. Nonetheless, the effect of past births is depicted to be small. Interestingly, the population pressure depicted by 'density' variable is found to significantly affect output growth for all the decades (1960-1990). The growth-enhancing effect of density in all these periods means that increasing population prompt higher innovation and hence higher economic growth. The result is found to be consistent for all the countries over three decades (1960-1990).

KS (2001) extends their earlier model by enhancing the sample size till 1995 and introducing some other variables like total population size, life expectancy at birth, and inflation to their original demographic model (as in eq.2.4). In KS (2001), the authors compare their extended model with different demographic-economic models propounded by other researchers, e.g., Bloom and Williamson (1997) and find in general that given the demographic trends (mainly declining mortality and fertility) over the period 1960-95, economic growth is favorably impacted by demography. Following KS (2001) estimates, for instance, fertility and mortality changes have each contributed around 22 percent to changes in output growth. One important result of KS (2001) is that emerging economies which are now experiencing early stages of demographic transition can expect an increase in working-age population growth for some time in future.

As different (non-)demographic components change exert varying effects on growth of income, we outline in the next section their short-run and long-turn consequences on economic growth.

2.3 Direction and implications of demographic effects

Taking the recent flavor of demographic explanation of cross-country variation in income, socio-economic policy makers and demographers are now flocking to the point to understand the short- and long-run consequences of changes in demographic variables on a country's income and welfare. Specifically, how these 'time profile' of effects lead to the choice of various long-term development objectives for the respective economies. Both theoretical and empirical economic growth literature is abound with numerous findings in this regard. For instance, Boucekkine et al. (2002) argue that the effect of population growth on per capita output growth should be interpreted in light of the vintage structure of the aggregate human capital. Based on demographic shifts, the authors provide explanation to the transition from a stagnant to a modern-growth economy by noting that an exogenous increase in longevity leads to higher schooling time and can induce an economy from a no-growth path to a balanced growth path. Essentially this means that the short-run and long-run consequences of population growth depends on the growth of its vintage components. Thus, contrary to the orthodox perception of 'people being only the *consumers* of resources' (Crenshaw et al., 1997), the rise of the population number should not, *per se,* be thought of retarding economic growth. The number may in fact spur economic growth depending on which 'segment' of the population is on the rise.

The dynamic impact of population growth on the growth of per capita output depends on the *vintage structure* of population viz., the young generation, the labor force, and the retired cohorts. The magnitude of short-run and long-run effects depends on the magnitude and pace of growth of these generations. To start with, the growth in the number of children may impede economic growth as scarce economic resources are invested in goods and services that yield 'few immediate economic multipliers' (Crenshaw et al., 1997). But growth in the economically active population, i.e., labor force, is rather beneficial as it can propel economic growth due to their resource creating abilities. Thus population policy aiming at birth rate and mortality reductions will generate short-run and long-run consequences in the economy.

• *Effects of CBR and lagged CBR*

For a clear elucidation, consider first the direction and sign of impacts of CBR on income growth. Higher birth rates generally add to the population mass of a country and that the short-run effect of a birth is likely to be negative[15], which may incline a national government to adopt birth rate reductions policies. Evidently, this bears an immediate positive short-run impact on growth due to its emphasis on the economization of child-rearing expenses. The role gets reversed though in about say fifteen years, as 'there will be fewer persons entering their productive work force years' (KS, 2001). However, the dynamics due to birth rate reductions can be explained in terms of the 'autocorrelated' nature of past and current births.

[15] Because the children or young generation are net 'resource users'.

A strong economic logic and empirical evidence follows this fact. Take for instance, the case of developing countries. Current high population growth of these countries is autocorrelated, implying that they experienced hight past population growth rates. This observation has two-way effects: On the one hand, the stock of accumulated 'resource users' shoots up over time exhibiting negative impact on the economy, while on the other hand, as the new births in the past turn out to be 'resource creators' in the life cycle, accumulation of them in the economy infuses positive externalities. In terms of time impact, population growth can have short-run negative effect on economic growth due to youth dependency, and long-term positive impact resulting from labor force growth and a subsequent boost in aggregate demand (Bloom and Freeman, 1998; Barlow, 1994).

Following neoclassical economic thinking, where a labor force growth is assumed to be key for economic growth, the positive correlation of labor force growth and economic growth can be explained by taking into account the scale and demand effects. Following Crenshaw et al. (1997) a growing labor force encourages scale effects in terms of larger domestic markets, more complex division of labor, greater volume of diffused technology, and lower per capita costs of public infrastructure (e.g., roads and transports). Similarly, the demand effects can be explained using 'Kuznets cycles' of U.S. economic history; increasing population has been associated with increasing production – the possible reason being increased demand for consumer goods in the wake of family formation (Easterlin, 1968).

• *Effects of CDR*

Mortality reductions, especially infant/child mortality, can have similar time impacts as birth rate reduction. If mortality decline is concentrated among infants and young children, this may create a baby boom in the initial years. Subsequently, due to increased use of contraception, the consequent declining fertility generates a large cohort of young people. When this cohort enters the labor force, it produces a period of 40 to 50 years in which the existence of relatively high worker-dependent ratio creates a potential boost to per capita income (Bloom and Canning, 2001). Eventually, as the cohort ages, the effect disappears, though it can still have notable significance while it lasts.

Population growth has also been explained in some recent studies (e.g., Nielsen and Alderson, 1995) as generating income inequality in the long run. According to them, rapid labor force growth deteriorates wage rates and generates inequalities. As Crenshaw et al. (1997) argued, to justify the logic of positive association between labor force growth and economic development, it is necessary to have individual and family adversity, sharp levels of income inequality, and declining wages due to stiff labor competition. Following Lewis' (1954, 1958) two-sector model, the authors explain that during early and intermediate stages of development, rapid labor force growth boosts the profit margins of the capitalists in the modern sector by reducing the average wage levels. This profit, in turn, gets invested in the modern sector, and productivity, and hence the job opportunities improve in the long run.

• *Effect of Density*

Excessive population growth with respect to higher population density and size also have a long term impact on economic growth. In economic geography literature, explanations have been put forward how density of population positively affects production and consequently, growth. In the extant literature on empirical growth, models which explored density and size in addition to population growth posited that, population density and all components of demographic change exerted significant effect on output growth. They found that population density had a significant positive effect in 1960s and 1970s and the net effect of all three components of demographic change is negative, for all countries on the average.

Boucekkine et al. (2005) in a more recent contribution theoretically show how the transition from economic stagnation to sustained growth is often modelled due to "population-induced" productivity improvements. They derive the effect of population on productivity from from optimal behavior. As per their finding, both the number and location of education facilities are important as individuals determine their education investment depending on the distance to the nearest school, and also on technical progress and longevity. In this setting, higher population density enables the set-up costs of additional schools to be covered, opening the possibility to reach higher educational levels. Using counterfactual experiments the authors find that 'one third of the rise in literacy can be directly attributed to the effect of density, while one sixth is linked to higher longevity and one half to technical progress' (Boucekkine et al., 2005). To sum up, population density will have positive relation with income growth. However, the sign may appear negative or may be positive and insignificant if density variable is not properly defined, for instance the locational effects due to education and proximity to productive resources.

• *Effect of Inflation and Life Expectancy*

Moreover, in the standard macroeconomic models, inflation pressure is depicted by many authors to covary with the age distribution unless accommodated by monetary policy. For instance, Lindh and Malmberg (2000) estimate the relation between inflation and age structure on annual OECD data 1960-1994 for 20 countries and found that an age pattern of inflation effects is consistent with the hypothesis that increases in the population of net savers dampen inflation, whereas especially the younger retirees fan inflation as they start consuming out of accumulated pension claims. On the whole, inflation and per capita income growth are expected to be negatively correlated. In the similar vain, higher life expectancy at birth can induce higher income growth in a country via increased productivity due to enhanced working years and higher savings rate at every stage of life-cycle, even when retirement is endogenous. Therefore, life-expectancy and per capita income growth are expected to have positive sign in the regression.

Nonetheless, a complete explanation of statistically robust correlations between population growth and per capita output growth, summarised above, needed a complex and extensive statistical modeling.[16] From theoretical perspective, the formulation of population in terms of vintage structure and studying its impact on the economy in an endogenous growth framework (Boucekkine et al. 2002) is a major leap. The remarkable contribution followed from KS (1995) in the empirical front in line with Brander and Dowrick's (1994) tradition, where they decomposed the aggregate population into components of change. This laid the foundation for untangling the long-run and short-run effects of population growth on economic development using an exhaustive econometric methodology. In this chapter, we study how the weight of each demographic effect has changed over time. The following section describes data and methodological framework used in this chapter.

2.4 Empirical Framework

2.4.1 Methodology

The methodological framework described in this chapter heavily draws on KS (1995). The empirical specifications as well as the econometric methodologies used in KS (1995) have been retained in our investigation so that appropriate comparisons[17] can be made. KS (1995) specified the following equation:

$$\begin{aligned}(Y/Ngr)_{i(t,t+n)} &= \alpha_i + \eta_t + \beta \ln (Y/N)_{it} + \theta I_{it} + \zeta S_{it} + \delta (X_D)_{it} \\ &\quad + \varsigma (X_D * Y/N)_{it} + \varepsilon_{it}\end{aligned} \tag{2.5}$$

where $\varepsilon_{it} \sim \mathrm{IN}\left(0,\sigma^2\right)$.

Three empirical specifications (of equation 2.4 and so equation 2.5), are used, each one succeeding the other inductively following the addition of different demographic variables in the model (see equations 2.6-2.8). In equations

16 Because of the anomaly in the 1980s due to world recessions, war, and droughts and because of a possible association of negative consequences of population growth with diminishing returns to capital and the environment (KS, 1995).

17 The methodologies used in this chapter can be substantially changed incorporating more time series features in the framework and studying the memory property of the demographic components. However, this exercise is reserved for future investigation. Nonetheless, the intuition here also provides motivation for Chapter 3.

2.7 and 2.8 we have added life-expectancy at birth and inflation so to give a more general specification of the demographic model. First, we estimate KS (1995) specification with modified data set and next we also estimate the general model. Note that KS (1995) models exclude life-expectancy and inflation from equations 2.7 and 2.8, though these variables are included in KS (2001).

We start with the simple model where per capita output growth[18] is explained by the log of initial level of per capita income, the aggregate population growth, population density, and the interaction term, i.e., $\mathrm{N}gr * (\mathrm{Y}/\mathrm{N}gr)$. It may be noted that the variable, 'density', has been entered in each (of the three) equation as it represents the information about the initial state of the economy and most importantly it captures the concept of technological change which is induced by faster population growth. In the next model, a decomposition of $\mathrm{N}gr$, viz., CBR, and CDR, and their interaction terms are considered. The third model is the most general one, where another demographic component, CBR_{-15} is added to model 2 to take account of the net effect of past births on the growth of per capita output. The empirical models derived from eq.2.5 have the following specifications:

$$\begin{aligned} Model1: (Y/Ngr)_{i(t,t+n)} &= \alpha_i + \eta_t + \beta \ln{(Y/N)_{it}} + \theta(Density)_{it} \\ &+ \delta_1 Ngr_{it} + \delta_2(Ngr * Y/N)_{it} + \varepsilon_{it} \end{aligned} \tag{2.6}$$

$$\begin{aligned} Model2: (Y/Ngr)_{i(t,t+n)} &= \alpha_i + \eta_t + \beta \ln{(Y/N)_{it}} + \theta(Density)_{it} \\ &+ \zeta(Infl) + \delta_1(CBR)_{it} + \varsigma_1(CBR * Y/N)_{it} + \delta_2(CDR)_{it} \\ &+ \varsigma_2(CDR * Y/N)_{it} + \delta_3 ln(e_0) + \varepsilon_{it} \end{aligned} \tag{2.7}$$

$$\begin{aligned} Model2: (Y/Ngr)_{i(t,t+n)} &= \alpha_i + \eta_t + \beta \ln{(Y/N)_{it}} + \theta(Density)_{it} \\ &+ \zeta(Infl) + \delta_1(CBR)_{it} + \varsigma_1(CBR * Y/N)_{it} + \delta_2(CDR)_{it} \\ &+ \varsigma_2(CDR * Y/N)_{it} + \delta_3 ln(e_0) + \delta_4(CBR_{-15})_{it} \\ &+ \varsigma_4(CBR_{-15} * Y/N)_{it} + \varepsilon_{it} \end{aligned} \tag{2.8}$$

An important question arises about the functional form of Y/N. Basically three types functional forms, viz., linear, log, and cubic forms have been used by different authors in various studies. Among them the log form of Y/N is chosen following KS (1995), as adjusted R^2 for log form was among the highest with no notable differences in demographic effects. Moreover, one may be inclined to think about the status of equations 2.4 and 2.5 as a data generating process (DGP). Note that the per capita income growth is allowed to depend on the initial income per capita, say at the level 1960 for each country. Equations 2.4 and 2.5 suggest that Y/N has a long-run equilibrium level which depends on productivity (viz., technology, capital stock, etc.). These effects are in fact captured by S_{it} variable in eq. 5 (S_t in eq.2.4). The growth also depends on a

[18] Note that we do not refer here to the instantaneous growth, rather growth over period.

host of other demographic factors. So this serves as a DGP. The purpose of introducing $\mathrm{Y/N}$ initial is to test for convergence pattern model. One can of course go without this variable. The methodological framework of the chapter depends on the convergence-type model, hence is the inclusion of $\mathrm{Y/N}$ initial. S variables are assumed to hold constant in the model. Technological changes are assumed to be induced by demographic pressure. Time trend is irrelevant in the model, as we are interested in decennial growth changes. Time-effects are captured in η_t.

The method of estimation[19] of the convergence-patterns model (equations 2.6-2.8) is in line with KS (1995). In these equation, α_i and η_t denote country specific and time specific intercepts. Depending on the assumptions made about α_i and η_t different kinds of models could be generated viz., Pooled estimation, Fixed Effect Model (FEM) and Random Effect Model (REM). In the empirical growth theory, there are numerous applications and discussions about the relative significance of these models. Specific discussions in this regard centers on whether α_i is properly viewed as a random variable (known as random effect model) or as parameter to be estimated (known as fixed effect model). Nevertheless, the application of these methods (viz., FEM and REM) is sometimes limited by the choice of the economic model and intuition guiding the theory of specific interest to the researchers.

REM assumes α_i and η_t as random which are separate components of the error terms. Though theoretically REM edges over FEM on efficiency ground, its estimates may be biased if α_i and η_t are correlated with independent variables, i.e., X_D type of variables, and $\ln(\mathrm{Y/N})$ in our case. For instance, if natural resources are a part of the individual intercept α_i, and are correlated with $\ln(\mathrm{Y/N})$, then use of REM will result in biased coefficient estimates. Moreover, FEM is appropriate if individuals in the sample, i can not be viewed as random draw of some underlying distribution. In our case i denotes countries, and therefore it is not a random draw Hence we use FEM. KS (1995) note that FEM 'edges out REM in situations in which the sample represents a sizable proportion of the population', which is a feature of our specific case. Using Hausman specification test, the authors showed that FEM consistently dominates REM and Pooled estimation methods employed in his study.

Recall that the most important assumption on which panel estimation methods (FEM or REM) are based is the cross-section independence of observations. There is a growing literature in this field recently which considers 'de-

[19] We will discuss shortly that to avoid persistency problem in Panel data, data can be aggregated to decennial periods. Then the standard fixed effects and / random effects methods are applied for estimation of the parameters. However, the time series characteristics have not been incorporated in the panel data which can be done using for instance, panel unit root, or even more flexible formulation using panel long memory. The latter has not been studied yet though effort is being made in this direction.

pendence' structure and not the standard 'structure of independence' of observations in the model. Empirical applications of this new consideration are rather sparse because it involves a lot of methodological complexities. Nonetheless, the 'independence' assumption is very standard in panel data literature[20] and for the purpose of this chapter we have followed the same.

Using FEM means inclusion of country intercepts and the determined time periods intercepts in the estimation. The country dummies control for the influences of per capita income growth, viz., cultural attitude or natural resource base, which vary across countries but reasonably remain constant over time within countries (KS, 1995). The time dummies control for period-specific and/ or global influences.[21] Under the assumption of single intercept, we perform a 'pooled regression'. At the same time, to capture the separate effect of each decade, cross section regressions for four decennial periods have been carried out in this chapter.

2.4.2 Data

We have extended KS (1995) sample till 2000. The empirical estimation is based on the regression of 86 countries (63: developing and 23: developed countries). Two separate regressions were performed. First with Hong Kong and Singapore (86 countries) and second, without them (84 countries). These two countries have very high density in comparison to other countries in the sample and therefore appear to be influential observations in the sample. The consequences of the latter's presence in the sample is well studied in the econometric literature, one of them being the biased parameter estimates. As we will notice shortly, dropping the two countries from our sample substantially improves the parameter estimates of the density variable in the panel regression. A note on KS(1995) data is in order. By carefully studying KS data we find that for some countries including Hong Kong, the values are incorrectly reported, the most common of them is anomaly in terms of interchanged data for some countries. For instance, the values for density in case of Hong Kong is incorrectly specified as 37, 48, and 63, which are averages for each decade 1960-70, 1970-80, and 1980-90 respectively. For Honduras the specified values are 3080, 3880, and 5040 respectively for each decade. Similar misspecifications are also found for other countries. In the current data set, which are from the World Bank Development Indicators, we have corrected these mis-specifications. In effect, some changes in the results for the effect of density variable in the panel regression are expected, and might not match with that of KS (1995).

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[20] Recently Pesaran (2003) and Bai and Ng (2002) model dependence structure in the panel data.

[21] For instance, the oil price shock in the 1970s and recessionary periods in the 1980s.

The dependent variable is per capita growth rate of GDP[22] (at constant purchasing power parity (PPP) at 1995 base). The source of the data is from Penn World Table 5.1. This GDP adjusts for the actual buying power of national currencies, and excludes the factor income from abroad (See, Summers and Heston,1994, and Penn World Table 5.1). Data on population growth, the crude birth rate (CBR) and the crude death rate (CDR) have been collected from the US Census Bureau, while density, and other relevant data have been collected from the World Bank Development Indicators. CBR, and CDR are measured per 100 population, and density is measured per 1000 population. The available length of data is for 40 years (from 1960 till 2000). Data on these variables have been aggregated over decennial periods keeping in mind the possibility of persistence and simultaneity between the dependent and explanatory variables. Aggregating over longer growth periods (say 10 year aggregation in our case), the differential $\mathrm{Y/N}\,gr$ growth rates can alter $\mathrm{Y/N}$ enough to influence substantially the pace of demographic change. KS (1995) reiterate that decennial periods embody more 'real' demographic information because lower-degree aggregation, say 5 years, rely on the assumptions inherent in extrapolations between decennial censuses. Thus in the estimation we have four decennial periods: 1960-70, 1970-80, 1980-90, and 1990-2000.

The per capita output growth (the dependent variable of our model) is not an 'instantaneous' growth rate. In the empirical literature, growth models often use 'growth over periods' and not 'instantaneous growth'. Hence confusion might arise about the calculation of the per capita output growth rate in our model, which in our case is: $\sqrt[n]{(\mathrm{Y/N}gr)_{t+n}/(\mathrm{Y/N}gr)_t}-1$. n, in our case, is 10 years. To take into account the dynamic effects of the birth rate, we have lagged crude birth rate by 15 years, as netting out the effect of lagged birth rate, denoted as (CBR_{-15}) establishes the significance of labor force in the estimation. Consequently, it can reduce the magnitude of 'negative effect' of CBR in the estimation. Since demographic data were unavailable before 1950, lagged values for CBR_{-15} for 1960s apply to 195055, for 1970s, it applies to 1955-1965. Again for 1980s and 1990s we have used the periods 1965-1975 and 1975-1985, respectively.

2.5 Results

The results discussed in this section are based on the methodological framework outlined in the preceding section. The results are analyzed from two perspectives. First, based on fixed effect model, we summarize the effect of each demographic component over four decades. A pooled regression under common country intercepts for all countries (both developed and developing) has been carried out for model 3 (eq.2.8). Second, to perceive the separate

[22] Real GDP per capita is assumed to the best indicator of a nation's affluence.

effect of these components in each decade cross-section regressions of per capita income growth on demographic components have been performed using model 3. Based on the estimates from these regressions, partial derivatives of the per capita income with respect to each demographic components have been calculated so that their exact effect on per capita income growth can be assessed. A confidence band $(\pm 2\sigma)$ has been constructed for the partial effect of demographic components so that significance of each effect can be judged statistically.

The general characteristics of the data may be noted from Tables 2.1 and 2.2, which present the standard deviation and median values of the demographic variables under investigation. Notable differences in the variation in CBR and CDR can be observed between developed and developing countries during 1990-2000. While σ_{CBR} for developed countries stands at 0.357, an expectedly high variation is observed for developing countries. Similarly, death rate variations is also very high for developing $(\sigma_{CDR} = 0.590)$ in comparison to the developed counterpart $(\sigma_{CDR} = 0.143)$. Indeed, the nature of variability of these two demographic components indicate about the relative stability of their demographic system. Developing countries have typically experienced high birth rates and death rates. However, the rate of births and deaths have varied from country to country, the foremost cause being the inability of various developing countries to implement the population control policy and generate resources to contain them.

On the other hand, developed countries have maintained more or less a consistent and low variability due to their degree of development. From Table 2.2, observe the growing difference of initial level of per capita income, $(\mathrm{Y/N})$ for the two sets of countries since 1960s till 2000. Even though the median $(\mathrm{Y/N})$ has doubled for both, the wide difference remain between them; in 1960s developed countries median $(\mathrm{Y/N})$ was about 5 times higher than developing countries which in forty years have proliferated to approximately 7 times differences. The median CBR and CDR in developing countries have mitigated from 1960s while the median labor force (i.e., CBR_{-15}) has increased over the years. This is an expected outcome due to the heavy investment in education and human capital in those countries and implementation of birth control and death reduction policies.

Before elaborating on the empirical results, some notes on the general features of the tables would help in understanding the intuition of reported results and regressions which are different from KS. Basically, our results can be divided into two separate theme: First, with KS (1995) basic variables and second, an extended model with some other demographic and non-demographic indicators. The purpose of the latter is to examine the robustness of KS conclusion under extended model specification. Moreover, as we noted earlier, influential observations in the sample may bias results, we have run separate panel and cross-section regressions with and without influential observations (viz., Hong kong and Singapore) from the sample. The effect of the latter is clearly visible in terms of improved significance level of density variable in the regression. We elucidate the point shortly.

Tables 2.3 and 2.4 present our estimation of KS three-period regression (from 1960-1990) with original KS (1995) variables but with new data collected from the World Bank and Penn World Table. In Table 2.3 we report how population growth would impact per capita income growth when the basic state variable, here density, is not considered. Table 2.5 represents our regression results of the extended sample (but not with addition of other variables) with Hong-Kong and Singapore in the sample. Table 2.6 reports the cross-section regression results which is derived from Model 3 and has the same features as Table 2.5. Table 2.7 is based on cross-section regression results and calculates partial effect of the variables for each decade using the median values 2.2. In Tables 2.9, 2.10, and 2.11, we present panel and cross-section regression and the corresponding partial effects based on 84 countries after dropping Hong-Kong and Singapore from the sample. Finally, Tables 2.12 and 2.13 present results of extended model which include other demographic and non-demographic variables. Note that we have estimated models with and without dropping the influential data in the sample. The results are reported in Tables 2.5, 2.9, 2.12, and 2.13. Tables 2.5-2.7 report regression results based on 86 countries which include Hong Kong and Singapore. In Tables 2.8-2.13, estimation based on 84 countries (after deleting Hong Kong and Singapore) are reported.

(i) Effects of $\ln(\mathrm{Y/N})$ *and Density*

To begin with, we first examine the convergence-patterns hypothesis from our estimation. The empirical relation between $\mathrm{Y/N}\,gr$ and $\mathrm{Y/N}$ was described in section 2.2. It was noted that $\mathrm{Y/N}\,gr$ varies inversely with $\mathrm{Y/N}$, the initial level of per capita income. The negative sign of the coefficient of $\mathrm{Y/N}$ explains the the logic of the convergence patterns model. Three models (see Table 2.5), viz., from the most restrictive to the most general (Model 3) outline the coveted demographic-economic growth relation. In the restrictive (Model 1) the demographic components have been excluded in order to perceive the effect of only aggregate population growth and the state variable, the density of population in the model. In Model 2 and Model 3, demographic variables have been inductively included (See Table 2.5). Observe that adding more demographic variables (in Model 2 first and then Model 3 for the most general one) increased the explanatory power of the model. As expected, R^2 is the highest with the most general model (Model 3 in Table 2.5). The estimates of the panel estimation using FEM (Table 2.5), exhibit the expected sign for the convergence pattern model, i.e., significant negative estimates of $\ln(\mathrm{Y/N})$ are observed in all the three models. Precisely, this vindicates the invariance of 'convergence' hypothesis to the decomposition of aggregate population into various components, viz., CBR, CDR, CBR_{-15}.

We now investigate how population density impacts economic growth in our estimation. In the literature, it has been argued that population density would propel economic growth via technological progress. High population density would in fact exert pressure on the economy to vie for more innovation so that innovation-led technological progress promotes economic growth. Therefore, empirically we would expect a statistically positive coefficient of density in the growth regression. Table 2.5, which reproduces KS (1995) regression with our new and modified data set shows that though the coefficient of population density is found to be positive across model specifications, they are not significant. This is not surprising given our discovery that KS data contained some error. Another feature of the KS data is that they do not eliminate or treat influential observations (e.g., high population density for Hong Kong and Singapore) from the sample. Once these observations are dropped from the sample, the density variables appear to be highly significant.

Specifically, KS (1995, 2001) found that population density has had growth-enhancing effect on economic growth. Both panel and cross-sectional regressions supported their findings. However, in view of our discovery of some anomaly in the KS data, for instance, the interchanged values of density and per capita income for many countries, it is hard to rely on the significance of their results. We have estimated KS basic model with three decennial periods as in KS (from 1960-1990) after correcting for these anomalies. The results are reported in Table 2.4. Note that density is not found to be significant across model variations even though they are still found to be positive. This is evidently in contrast with KS (1995, 2001). Moreover, as we extend the sample till 2000, we find similar pattern (see Table 2.5). With the exception of Model 2, the density variable is insignificant in other models as well as in all the cross-section regressions except for 1960-70 (see Table 2.6).

An additional problem might occur if there are influential observations in the sample, which are likely to bias the estimation of the regression. Therefore, it is necessary to either drop them or treat them in the regression. To keep analysis simple, we have dropped these two countries from the regression[23]. After correcting for the anomaly in the data and dropping these two countries we find the density variable to be positive and significant both in the panel and cross-sectional regressions as depicted by Tables 2.9 and 2.10.

[23] Note also that KS did a similar trick by dropping China from the sample by arguing that inclusion of this country would bias regression (because of very high population figure, etc.) and therefore a separate analysis may be required for this country.

(ii) Effects of Demographic Components

We now investigate how different components of population impact on economic growth. For the purpose, we begin by studying the significance of aggregate population on economic growth during the estimation period. Next we segregate population into different components, viz., CBR, CDR and lagged CBR (CBR_{15}) and map out their separate impacts on economic growth. To incorporate the effect of the interaction terms, notice that $\mathrm{Y/N}$ has been multiplied with each demographic component (see Model 2 and Model 3) so that the magnitude of the partial effect of each demographic term on per capita output growth, $\mathrm{Y/N}\,gr$ can be assessed. Our three decennial regression (in Table 2.4) depicts that population growth is not significant, even their interaction term is found to be insignificant. Moreover, population growth and its interaction term are depicted to be jointly insignificant which is in contrast to original KS (1995) results. To assess the robustness of their findings, in Table 2.3 we have estimated models without density variable in the regression. The results are reported for original KS data and our data. It is evident from Table 2.3 that population growth and interaction terms are neither individually nor jointly significant. Then one may question about the influence of population density for the joint significance of population growth on economic growth as depicted in KS (1995). From KS regression, we find the high influence of density variable in all the models and cross-sectional periods. It is not clear if demographic variables will still be significant in the absence of population density in the regression.

However, in the extended sample, we find joint negative significance of growth rate of population on per capita output growth over 40 years (1960-2000) (see Table 2.5). Conclusion about negative effect of population on $\mathrm{Y/N}\,gr$, following Model 1, at best provides preliminary gross assessments, as nothing is revealed about its exact profile of impact on $\mathrm{Y/N}\,gr$. For the purpose, ngr has been segregated into different components according to their resource using and creating abilities. In Models 2 and 3 (Table 2.5), the results from this regression are reported.

First, consider the effect of CBR on $\mathrm{Y/N}\,gr$. The coefficients of CBR and its interaction term, $CBR * \mathrm{Y/N}$, in Model 2 (Table 2.5), are insignificant. In KS (1995, Pp 551), the coefficient of CBR has always been significant which is a contrast to our results. However, we find CBR and its interaction term are jointly significant and negative which is the same as KS (1995). Intuitively, this means higher births retard economic growth possibly though higher dependence of younger population on economic resources which ultimately reduces savings and hence economic growth via negative multiplier effect. To understand how demographic components have changed over time, a cross-sectional regression for different time periods have been performed, viz., separately for each decade from 1960-70 to 1990-2000 (in Table 2.6). Significance of these variables are evaluated based on t-statistics. Joint significance levels are reported for each component with their interaction terms. Though joint significance is not found for each set of variables, additional testing reveals statistically significant coefficient changes over time for each of the combined pairs of CBR, CDR, and CBR_{-15}. Based on this table along with the calculated variable medians (Table 2.2), we have also estimated the partial effect of CBR, CDR, and CBR_{-15} on per capita income growth for both developed and developing countries. Table 2.7 summarises the results.

The partial effects of CBR, CDR, and CBR_{-15} are evaluated at $(\mathrm{Y/N})$ medians of developed and developing countries (see Table 2.2), which are plotted in Figures 2.1 and 2.2. These figures can be compared with KS (1995) which are plotted for the periods 1960-1990 (see Figures 2.3 and 2.4). Evidently, the comparison is made till 1990 since KS (1995) sample ends in 1990. In Figures 2.3 and 2.4, CBR-own indicates our estimation and CBR-KS indicates KS (1995) result. Looking at Figure 2.3, it is observed that the partial effect of CBR in developing countries, in contrast to KS (1995) is not monotonically declining over time. Our estimates depict a step-like pattern. Monotonic decline of the partial effect of CBR for developing countries in KS (1995) means that higher births in succeeding decades in these countries consistently and continuously impinge more harm on economic growth. Steplike pattern of the partial effect, as found in our case instead implies this is not the case. There could be the effect of some population policy which can cause the effect of CBR to be felt severely in some decades, but not consistently for all the decades together.

In our case, the negative effect of birth rise reached the lowest in the 1980s. An upward trend (though negative) of the effect in 1990-2000 implies that in future the effect of CBR can have positive effect on per capita income growth for developing countries. At least a forecast based on our estimates would imply so. The reason for the step-like pattern could be laid as follows. Due to high investment in education and human capital in the developing countries the recent years, the negative effects of CBR may recede over time. Moreover, the adoption of population control policy in developing countries that began as early as 1960 and 1970s, did not register immediate impact on economic growth. The possibility of revival in the 1990s contemplates the fact that population control policy adopted earlier would bear some positive impact on economic growth which might take another decade or so to completely settle in.

This empirical fact is much closer to the logic that depending on the nature of the structural parameters of the economy (e.g., in our case it could be the social environment and educational attainment of the population), a shock (in terms of policy adoption for population control) intended to put the economy in a different path – might take a long time for the effect to be felt. Indeed, this is the case with population control policy: the economy might have to wait a long time for the full effect to be internalised (see Dasgupta, 1995 for a discussion on these issues).

Within developed countries (see figure 2.4), effect of CBR gives rise to some interesting features. Beginning from high negative partial effect of higher births in the 1970s, the current decade (1990-2000) shows positive impact of CBR on developed countries income growth. In view of KS (1995) finding, the negative impacts of higher births seemed to recede over time, being 'fairly small' in 1980-90, though it still remained negative during the same decade. From our cross-section estimation, we find that the effect of birth rates have become positive for the developed countries during the last decade (1990-2000). In view of the recent demographic changes in the developed world, this contributory effect of CBR is very important.

A possible reason for this trend is as follows. Developed countries experienced a huge drop in mortality and consequently followed a continuous and steady decline in fertility. In most part of the European continent, as Boucekkine et al. (2002) note, "fertility has now reached or even fallen below the replacement level". Therefore, though "the future scenario of zero population growth is considered seriously", the finding of positive effect of higher births for developed countries in our study, calls for a re-examination of the hypothesis. It is nevertheless true that, due to drastic fall in fertility level in the developed countries, along with an increase in the schooling time, the 'replacement level' of population must be substituted for higher births. There is large literature that confounds this logic. At the simplest level, it can be said that higher CBR in the countries which experience higher fertility is as dangerous as lower births in low fertility countries. The consequence being the same, only the time profile of effects differs.

Compensating for the negative overall effect of higher births, a decrease in CDR is expected to accelerate economic growth. Coefficient of CDR in Model 2 (Table 2.5) is not significant though the interaction term, $CDR*(\mathrm{Y/N})$, as well as their joint significance, summarises the partial negative effect of CDR on economic growth. The conclusion is strengthened when we consider the general model (model 3, where the effect of lagged birth rate (CBR_{-15}) is incorporated). As can be seen, both CDR and $CDR*(\mathrm{Y/N})$ exert significant impact on $\mathrm{Y/N}\,gr$. The finding of negative effect of CDR implies that reduction in CDR will enhance economic growth. But which country block (i.e., developing or developed) stand to gain more from a further reduction of CDR? A close look at Table 2.5 and Figures 2.1 and 2.2 provide some insights. Column 3 of Table 2.7 presents the partial effect of CDR on per capita income growth for both developed and developing countries. Figures 2.1 and 2.2 (part b) plot the effects of CDR in both sets of countries. Additionally, to provide a comparison with KS (1995), Figures 2.3 and 2.4 represent the comparison for developing and developed countries respectively.

To perceive the implications of these findings, first begin with the case of developing countries. A drop in the median value of CDR from 0.985 (in 1980-90) to 0.765 (in 1990-2000) (see Table 2.2) in these countries was expected to increase output growth per capita during the decades. As mentioned, this can be known from the partial effect of CDR reduction on per capita output. Considering the estimates from Tables 2.5 and 2.7, the partial effect of reducing CDR for developing countries during 1980-2000, in fact improved per capita output. Even if we compare our estimates for the period 1980-90 with that of KS (1995), a clear distinction emerges: the size of the effect of CDR on output growth in case of KS (1995) is smaller than our estimates (Table 2.5 and Figure 2.1, part b for developing countries). It is true that death reductions in developing countries is mostly concentrated in younger and working age people who by and large contribute to the output growth process.

Going by KS (1995) estimates, a still positive partial effect can be expected from death reductions in developing countries. Considering our estimates, this optimism somehow fades in. As can be seen from Figures 2.3, part b, CDR reduction diminish over time following KS (1995), even becoming negative during 1980-1990. The effect of CDR reduction for 1990-2000 can be observed from our estimation (Table 2.7, col.3). We find even large negative effect of CDR during this decade. This result gives another intuition to our earlier explanation why CBR has contributed positively to the growth in the developed countries.

So far we discussed the effect of CBR and CDR reductions on the growth of output taking the case of both developed and developing nations. Another demographic factor which is also important is the lagged effect of CBR, namely CBR_{-15} in our model. Generally, CBR_{-15} is likely to scale down the negative partial effect of CBR in the model (see Model 3) so that the net effect of CBR can be correctly specified. Going by our results, we find significant negative effect of CBR_{-15} on output growth per capita (Model 3, Table 2.5). In fact, our pooled estimation also confirms this finding (Table 2.5). In KS (1995), initially starting from a negative effect in the 1960s, it latter became large and positive for the developing countries in the ensuing decades. Surprisingly, large negative effect of CBR_{-15} is found in our study for developing countries for all the decades, whereas for developed countries, the lagged birth rate contributed to per capita output growth in the 1980s, finally setting down to large negative value in 1990 (see Figures 2.3 and 2.4, part c). It can be said that the expected (positive) effect of CBR_{-15} as per our estimation does not show much promise as a catalyst for growth since its growth-enhancing effect could have very much been confused by the 'persistent high births' in most of the developing countries. The caveat here is that as the number of births keep on rising every year, this 'persistent effect' becomes so large that it outweighs the effect of lagged birth rate $\left(CBR_{-15}\right)$ for those countries. Therefore, the growth-enhancing effect of CBR_{-15} may not be so prominent in developing countries in comparison to the developed counterpart.

(iii) Effects of demographic variables after dropping Hong Kong and Singapore from the sample

In the previous section we discussed how the components of population and density have had varying influences on economic growth of developing and developed countries. Following our earlier discussion that influential observations need to be dropped from the sample, in this section we elaborate on the results of such regression which are depicted in Tables 2.8, 2.9, 2.10, and 2.11. In general we find that density variable has now become highly significant, as opposed to the earlier regression with 86 countries which included Hong Kong and Singapore. Moreover, cross-section results (in Table 2.10) also support that population density has growth-enhancing effect in all decades. Concerning the separate effects of CBR, CDR and CBR_{-15}, from Table 2.11, we note that CBR has become positive for both developing and developed countries. In case of the former, it implies, further reductions in birth rate in the developing countries will not enhance economic growth. For developed countries, higher birth will also do good as number of births in these countries have drastically fell over the years. Consequently, in the coming decades bulk of the population in developed countries will comprise of retired cohorts. Therefore, governments in these countries need to adopt policies to encourage higher births. These results are similar to our earlier cross-section regression with Hong Kong and Singapore (Table 2.6).

Among developing countries, three trends are apparent. First, the effect of CBR has become positive for 1990-2000. We may speculate that due to the restrictive population policies adopted by some developing countries, for instance India, despite growth of population in these countries, higher births are not expected to witheld economic progress. This might be attributed to the increasing number of work force to the total population and high level of investment to improve the quality of human capital in those countries. In effect, positive births are expected to add to the growing labor force which is good for economic growth. Second, death rate reductions appeared to contribute positively in the decades 1960-70 and 1970-80. However, the positive effects of CDR reduction in fact becomes negative in 1980-90 and 1990-2000. Third, in contrast to KS (1995), the impact of lagged births (presumably the labor force) appears to be positive initially, and then becomes negative in the last two decades (1980-90 and 1990-2000). A somewhat different picture emerge for developed countries. Negative effects of birth are found only in 1970-80. In the ensuing decades, the effects become positive and large. The positive effects of CDR reduction appeared to recede over 1960-1990, however, for the recent decade the benefits of CDR reduction appears to pick up (see Table 2.11). Finally, the effect of past birth are found to be fairly large (as opposed to KS (1995) over the decades. These results support the strategy of assessing demographic impacts by stage of development. For instance, as KS note, reductions in CDR may be concentrated in the older cohorts in the developed countries, and in younger cohorts in the developing countries. Figures 2.5 and 2.6 plot these partial effects for 84 countries (after excluding Hong Kong and Singapore).

(iv) Effects of additional variables

To assess the robustness of KS results extended to the current decade, we have incorporated two other variables, viz., life-expectancy at births and a non-demographic variable, viz., inflation. Their precise impacts on economic growth are explained in the previous section. To compare if the implications of demographic impacts change due to the addition of extra regressors in the model can be studied from Tables 2.12 and 2.13. Note that these results correspond to the truncated sample, i.e., N =84 (after dropping Hong Kong and Singapore from the regression). In Table 2.12 we notice that the implications of demographic impacts remain more or less the same, additionally life expectancy and inflation also appear to be significant with expected sign in Model 2. In Table 2.13 we have included only inflation and not the life expectancy at birth based on the logic that if the inclusion of non-demographic factor change the results. Evidently, we find inflation to be negative and significant in Models 2 and 3, thus vindicating the fact that addition of non-demographic factors in the regression will enlarge the analysis.

2.6 Discussion and Conclusion

The question of the sources of growth has been the subject of renewed interest since the early 1980s. The so-called endogenous growth theories have been used to extend and go beyond the traditional growth model. The main factors of endogenous growth, that may or may not generate externalities, are the accumulation of knowledge (Romer), public infrastructure (Barro), human capital (Lucas) and expenditure on research. Population is often absent from theoretical and empirical observations, but the fundamental character of the demographic variable for economic growth is far from new. In the sixteenth century, Jean Bodin marked the interest shown in the notion of population and more generally in subjects related to demography, as he affirmed that 'There is no wealth but in people'. The relations between demographic growth, technological changes and the standard of living have therefore been the subject of numerous analyses. The most famous-that of Malthus-holds that the population will regulate itself and above all stagnate. Although this is pertinent for a large part of our history, the changes observed since 1750 call the idea into question. Many currents have emerged in the analysis of population and there are two opposing views of the subject.

The Malthusian line of thinking considers that populations grow geometrically while resources grow arithmetically. So either the population voluntarily agrees to limit its growth (with 'moral restraint' or abstaining from marriage) or it will be destroyed by war, famine and plague. The creative pressure approach developed by Boserup (1981) puts forward the hypotheses that demographic pressure causes the reorganisation of agricultural production. The size of the population and hence the level of resources needed leads to changes in farming methods. Boserup thus answers the Malthusian trap (insufficient food production) with the low population density trap (poor technical progress). Encouraged by Kelley and Schmidt (1995, 2001) and Crenshaw et al. (1997) empirical assertions that demographic components play pivotal role in explaining economic growth of developed and developing countries, in this chapter we studied how the weight of each component effect has changed over time.

Extending Kelley and Schmidt (1995) data till 2000, (more precisely, including another decennial period in the model), we showed that the weight of the effect of demographic components have varied over the last four decades. For the purpose of exposition and drawing comparisons we have used the convergence-pattern model as in KS (1995). Our results can be viewed from two perspectives. First, results from the complete sample as in KS (1995) without dropping Hong Kong and Singapore. And second, results of the regression after dropping them. Significant changes in the results occur, important of them is the significance of population density in the regression. Moreover, some distinctions can be observed for the effect of CBR, CDR and lagged CBR for two sets of regressions. Conclusions are drawn based on the common findings of the two regressions and obvious distinctions emerging from the two. For methodological reasons (concerning the influential observations as discussed in the previous section) we base our analysis on the truncated regression (i.e., without Hong Kong and Singapore in the regression).

Most important conclusions emerging from our analysis are: *(i)* very little gain can be expected from further reductions in mortality in the developing countries. Mortality reductions in these countries are heavily concentrated among children, which is costly in terms of economies output. With higher and persistent birth rates the effort to materialise the positive effect of mortality reductions in the developing countries can do no more good. In effect, the national governments in these countries should control for the momentum of persistent high birth rate effects.

(ii) Despite the fact that higher birth rates retard economic progress in developing countries, interestingly the same may not be true for developed nations. We found that the effect of CBR has become positive in the developed countries in the recent decade. This finding can be put into perspective given that the future of zero population growth as optimum for higher economic growth is considered recently by some researchers (For instance, Boucekkine et al. 2002). Given the recent trend of demographic transitions and declining fertility level in these countries, economic growth may in fact get slowly paced. A positive effect of CBR as found in our chapter provides an interesting and intuitively a healthy sign for economic growth. *(iii)* The effect of CDR in the

developed countries is very large in all the decades – larger during the current decade (1990-2000). As we know, death rate reductions contribute positively to economic growth in each decade. During the last decade, the effect seems to be quite large. Important to note that unlike developing countries, death rate reductions in the developed countries are concentrated not only among younger generations but most importantly among the working age people. Hence, as the greater the number of working age people, the faster is the economic progress.

(iv) Growth-enhancing effect of population density is observed for all decades which is similar to KS findings. Population pressure due to high density would lead to higher innovation and consequently higher economic progress. Looking at the magnitudes of density for all decades from Table 2.10, it is evident that population density has contributed relatively highly among all decades, to the economic growth of developed and developing countries.

(v) Additional non-demographic variables would enlarge the analysis and implications of demographic variables on economic growth. KS (1995) conclusion that only demographic variables are robust in explaining economic growth may no longer be robust to the varying demographic and economic growth relation.

The finding of large negative partial effect of *CBR* in developing country for the past decade can be put both in historical and theoretical perspectives: given poor resource base, higher birth rate (accumulated over time from successive higher birth rates in the past) will put the developing countries economic prosperity into dismay. The effect of population policies aimed at controlling birth rate reductions in developing nations, will take time to make the positive effects being felt. Since these countries historically suffer from past high population growth, the rate of accumulation of lagged birth rate (CBR-15) might have been slower in the 1970s and 1980s and the net effect of CBR could have less than an offsetting amount. The period 1990s experienced a slackening effect of lagged birth rate both in case of developing and developed countries especially in the last decade.

This is in contrast to KS (1995) finding: in case of developing countries, the favorable effect of past births starting from 1970s continued to be positive till 1990s, while the effect turned out to be negative during 1970-90s for developed nations. Overall it seems that the impact of lagged birth rate in the last decade is highly increased both in developed and developing country economies. Our estimates show that the favorable effect of lagged birth rate is felt only in 1980s in case of developed countries, while the effect tended to be negative for developing nations over time. The short-term costs of high birth rates has been increasingly felt by developing countries over past four decades. Mortality reductions in those countries (concentrated mainly on infants) showed a sign of improvement though still remained negative till date.

Finally a note on the model and assumption is in order. Recall that throughout the chapter we have assumed stationarity of the demographic variables.

Consequently, a stationary panel method was used for estimation. However, recent research (Gil-Alana, 2003) shows that population growth can possess a kind of memory property or long-range dependence. With stationary assumption all inherent dynamics of the process is assumed out. However, allowing for memory structure to prevail in population variable shows high degree of memory which of course affects other variables like per capita output growth. Hence the time series effect of demographic variables needs to be taken into account in Panel data which is disregarded in stationary panels. In this light, we think that even if one includes many non-demographic and demographic variables in the regression, it would certainly improve the robustness of the model but will reveal little about the stochastic behavior of demographic components and their consequences on economic growth. One therefore needs to go beyond the conventional stationary assumption of population and study the properties of evolution of the series. This concern forms the core of the next chapter (Chapter 3).

2.7 Appendix

2.7.1 Calculation of Partial Derivatives

The partial derivatives reported in Table 4 are calculated in the following way. Recall Model 3 of Section 2.4:

$$\begin{aligned}&Model2: (Y/Ngr)_{i(t,t+n)} = \alpha_i + \eta_t + \beta \ln{(Y/N)_{it}} + \theta(Density)_{it} + \\ &\zeta(Infl) \quad + \delta_1(CBR)_{it} + \varsigma_1(CBR * Y/N)_{it} + \delta_2(CDR)_{it} + \\ &\varsigma_2(CDR * Y/N)_{it} \quad + \delta_3 ln(e_0) + \delta_4(CBR_{-15})_{it} + \\ &\varsigma_4(CBR_{-15} * Y/N)_{it} + \varepsilon_{it}\end{aligned} \tag{2.9}$$

The cross-section estimates of this regression (for four decennial periods) are used to calculate partial effect of each demographic variable. Given those parameter estimates, our purpose is to find the partial derivates of $\left(\mathrm{Y/N}gr\right)_t$ with respect to the variable vector $\underline{x} = \left(CBR, CDR, CBRL\right)$. For instance, in case of CBR, the partial derivative is simply calculated as:

$$\frac{\partial(Y/Ngr)_{i(t,t+n)}}{\partial CBR_{it}} = \hat{\delta_1} + \hat{\delta_2} * Median(Y/N)_{it} \tag{2.10}$$

In the same way, the partial derivatives for CDR and CBRL can be calculated from the cross section regression.sing the format above. Since we are interested in comparing the partial derivatives of developing and developed countries, we have used the median of $\mathrm{Y/N}$ separately for those two sets of countries.

2.7.2 Confidence band for estimates of partial derivatives

Denote the estimate of the partial derivative of say, CBR (at period t and for the set of countries belonging to developing nations) as P_{st}, where s = (1, 2) and t= (1960-70, 1970-80, 1980-90, 1990-2000). Confidence band of P_{st} at 95 percent significance level (given its mean, $\bar{P_{st}}$ and standard deviation, σ_P) is

$$CI = \bar{P_{st}} \pm 1.96 * \frac{\sigma_P}{\sqrt{N}} \tag{2.11}$$

N = 23 for developed and 63 for developing countries. $\bar{P_{st}}$ is assumed to be the same as the estimated $\hat{P_{st}}$ as

$$\bar{P_{st}} \equiv E[\hat{\delta_1} + \hat{\delta_2} * Median(Y/N)] = \hat{P_{st}} \tag{2.12}$$

Similarly,

$$\sqrt{Var(P_{st})} = \sqrt{Var(\hat{\delta_1}) + Var(\hat{\delta_2}) * (Median(Y/N))^2 + 2Median(Y/N)Cov(\hat{\delta_1}, \hat{\delta_2})}$$

Table 2.1: Descriptive Statistics: Standard Deviation; N = 86, sample: 1960-2000

Developing countries	Y/Ngr	Y/N	Dns	Ngr	GBR	GDR	$CBR_{\_15}$
1960-70	1.931	1.448	0.585	0.587	0.681	0.625	0.449
1970-80	2.495	1.993	0.700	0.606	0.885	0.607	0.434
1980-90	2.264	2.668	0.870	0.651	0.973	0.557	0.601
1990-2000	2.572	3.643	1.036	0.617	1.016	0.590	0.938
Developed countries							
1960-70	1.605	3.045	0.104	0.766	0.515	0.2 IS	0.573
1970-80	0.949	3.368	0.112	0.665	0.586	0.190	0.501
1980-90	0.739	3.603	0.118	0.604	0.523	0.164	0.472
1990-2000	1.097	4.494	0.125	0.670	0.357	0.143	0.571

Table 2.2: Variable Medians (N = 86 countries; sample: 1960-2000)

Years	*Y/Ngr*	*Y/N*	*Density*	*Ngr*	*CBR*	*CDR*	$CBR_{\_15}$
Developing Countries							
1960-70	2.056	1.628	0.023	2.654	4.559	1.880	3.852
1970-80	1.491	2.018	0.031	2.635	4.345	1.479	3.869
1980-90	-0.167	2.634	0.049	2.571	4.032	0.985	3.746
1990-2000	0.872	2.851	0.050	2.301	3.285	0.765	4.090
Developed Countries							
1960-70	3.455	7.801	0.088	0.811	1.829	0.960	1.881
1970-80	2.532	12.085	0.091	0.847	1.523	0.976	1.774
1980-90	1.864	15.782	0.092	0.444	1.268	0.948	1.659
1990-2000	1.524	19.813	0.096	0.430	1.269	0.932	1.415

Table 2.3: Effect of Density in Basic Model: Dependent Variable, $(\mathrm{Y/N}gr)$

Variables	KS-Original Data Model 1: No density	Modified KS data Model 1: No density
ln (Y/N)	-3.722(-3.95)	-3.512(-4.37)
Ngr	-0.744(-1.61)	-0.635(-1.42)
Ngr*(Y/N)	-0.020(-0.21)	-0.002(-0.05)
Constant	11.241(6.10)	9.311(5.67)
R^2	0.716	0.683
sigma	2.271	1.567
No. Obs.	267	257
chi-square(2):	4.50(p=0.11))	3.388(p=0.18)

Note: Bracketed values are t-statistics.

Table 2.4: KS Basic Model (own estimation): Dependent Variable, (Y/Ngr)

	Panel Estimation: Fixed effect model			Cross-Sectional Estimation:
	Model 1	Model 2	Model 3	Pooled
Ln(Y/N)	-3.53(-4.43)	-5.501(-3.98)	-5.75(-3.80)	0.013(0.03)
Ngr	-0.587(-1.30)			
Ngr*(Y/N)	-0.008(-0.182)			
CBR		[−1.704(−2.84)	[−1.427(−2.20)	[−0.626(−1.38)
CBR*(Y/N)		−0.021(−0.263)]	−0.005(−0.066)]	−0.088(1.22)]
CBR-15			-0.686(-1.33)	-0.236(-0.53)
CBR-15*(Y/N)			-0.002(-0.03)	-0.017(-0.34)
CDR		[−0.627(−0.637)	[−0.798(−0.771)	[−0.768(−1.66)
CDR*(Y/N)		0.298(2.83)]	0.312(3.05)]	−0.052(−0.770)]
Density	1.005(1.19)	1.350(1.47)	1.534(1.13)	0.380(1.16)
Constant	9.336(5.73)	16.64(6.04)	18.74(5.97)	7.17(5.28)
R-square	0.68	0.70	0.71	0.20
No.Obs	257	257	257	257

Note: Bracketed values are t-statistics;
square brackets indicate joint significance at 5 percent level.

Table 2.5: KS Extended Model (N = 86 countries; sample: 1960-2000): Dependent Variable, $(\mathrm{Y/N}gr)$

	Panel Estimation			
	Model 1	Model 2	Model 3	Pooled
Ln(Y/N)	-2.389(-3.66)	-2.567(-2.15)	-3.55(-2.58)	-0.115(-0.184)
Ngr	⌈ −0.409(−1.00) ⌉			
Ngr*(Y/N)	⌊−0.025(−0.728)⌋			
CBR		⌈ −0.544(−0.853) ⌉	⌈ −0.334(−0.504) ⌉	-0.272(-0.785)
CBR*(Y/N)		⌊−0.100(−1.44)⌋	⌊−0.120(−1.75)⌋	-0.042(-0.732)
CBR_{-15}			⌈ −1.05(−3.08) ⌉	⌈ −0.901(−3.24) ⌉
$CBR_{-15} * (Y/N)$			⌊0.049(0.817)⌋	⌊−0.019(−0.502)⌋
CDR		⌈ −0.884(−1.38) ⌉	⌈ −1.22(−2.12) ⌉	⌈ −0.571(−1.14) ⌉
CDR*(Y/N)		⌊0.186(2.50)⌋	⌊0.163(2.16)⌋	⌊−0.094(−1.52)⌋
Density	0.719(1.36)	0.740(1.69)	0.281(0.567)	0.614(8.23)
Constant	8.61(6.63)	11.06(4.16)	14.63(4.44)	7.47(5.49)
R^2	0.60	0.62	0.63	0.20
Std. Error (σ)	2.057	2.052	2.01	2.05
No. of Observations	343	343	343	343

Note: (i) t-statistics are in parentheses (at 0.05 level), (ii) This table presents KS estimation with extension, (ii) Square brackets over two variables indicate joint significance at 0.05 level.

Table 2.6: Cross-Section Estimation (Model 3)(N = 86 countries; sample: 1960-2000): Dependent Variable, (Y/Ngr)

	1960-70	1970-80	1980-90	1990-2000
Ln(Y/N)	-0.133(-0.140)	1.219(1.064)	-2.653(-2.90)	-1.615(-1.313)
CBR	-0.346(-0.393)	0.902(1.182)	⌈ −1.549(−2.623) ⌉	-1.658(-1.712)
CBR*(Y/N)	0.031(0.108)	-0.279(-1.75)	⌊−0.025(−0.229)⌋	0.142(0.923)
CBR_{-15}	-0.468(-0.679)	-0.525(-0.792)	-0.545(-0.997)	-0.840(-1.031)
$CBR_{-15} * (Y/N)$	-0.040(-0.197)	0.033(0.291)	0.052(0.574)	-0.031(-0.231)
CDR	⌈ −1.066(−1.272) ⌉	⌈ −2.864(−3.13) ⌉	-1.259(-1.548)	⌈ 0.305(0.315) ⌉
CDR*(Y/N)	⌊−0.176(−0.704)⌋	⌊0.043(0.267)⌋	0.065(0.607)	⌊−0.200(−1.69)⌋
Density	1.01(2.37)	0.634(1.474)	0.527(1.522)	-0.027(-0.100)
Constant	8.033(4.217)	5.492(2.26)	11.340(5.405)	10.395(3.643)
R^2	0.37	0.38	0.48	0.27
Adj.R^2	0.31	0.31	0.43	0.19
Std. Error (σ)	1.65	1.82	1.57	2.05
No. of Observations	86	86	86	86

Note: (i) Bracketed values are t-statistics at 5 per cent level.
(ii) Square brackets over two variables indicate joint significance at 0.05 level.

Table 2.7: Partial Derivatives Evaluated at (Y/N) Medians (N = 86 countries; sample: 1960-2000)

Years	*CBR*	*CDR*	*CBR*$_{-15}$
Developing Countries			
1960-70	-0.295	-1.352	-0.533
1970-80	0.338	-2.78	-0.458
1980-90	-1.614	-1.09	-0.408
1990-2000	-1.253	-0.265	-0.928
Developed Countries			
1960-70	-0.104	-2.439	-0.780
1970-80	-2.470	-2.344	-0.126
1980-90	-1.944	-0.233	0.276
1990-2000	1.155	-3.658	-1.454

Table 2.8: Variable Medians for 84 Countries: Sample 1960-2000

	Y/ngr	*Y/N*	*Dns*	*Ngr*	*CBR*	*CDR*	*CBR*$_{-15}$
Developing countries							
1960-70	2.056	1.628	0.023	2.654	4.500	1.800	3.852
1970-80	1.491	2.018	0.031	2.635	4.400	1.400	3.869
1980-90	-0.167	2.634	0.039	2.571	3.800	1.100	3.746
1990-00	0.872	2.851	0.050	2.301	3.300	0.900	4.090
Developed countries							
1960-70	3.455	7.80	0.08766	0.811	1.80	1.00	1.881
1970-80	2.532	12.09	0.09101	0.847	1.50	1.00	1.774
1980-90	1.864	15.78	0.09152	0.444	1.30	0.90	1.659
1990-00	1.524	19.81	0.0955	0.43	1.30	0.90	1.415

Table 2.9: KS Extended Model Without Singapore and Hong Kong (N = 84; sample: 1960-2000): Dependent Variable, (Y/Ngr)

	Panel Estimation			
	Model 1	Model 2	Model 3	Pooled
Ln(Y/N)	-2.312(-3.46)	-2.025(-1.81)	-3.007(-2.35)	-0.075(-0.117)
Ngr	-0.284(-0.736)			
Ngr*(Y/N)	-0.010(-0.377)			
CBR		⌈ −0.091(−0.151) ⌉	⌈ 0.200(0.325) ⌉	-0.411(-1.03)
CBR*(Y/N)		⌊−0.121(−1.78)⌋	⌊−0.151(−2.21)⌋	-0.042(-0.440)
CBR_{-15}			⌈ −1.108(−3.12) ⌉	⌈ −0.806(−2.40) ⌉
$CBR_{-15} * (Y/N)$			⌊0.047(0.762)⌋	⌊−0.015(−0.290)⌋
CDR		⌈ −1.010(−1.54) ⌉	⌈ −1.370(−2.36) ⌉	⌈ −0.316(−0.544) ⌉
CDR*(Y/N)		⌊0.182(2.48)⌋	⌊0.172(2.58)⌋	⌊−0.094(−0.117)⌋
Density	10.457(3.27)	11.101(3.19)	12.846(3.59)	0.434(0.463)
Constant	6.845(4.95)	9.274(3.85)	13.083(4.06)	7.217(4.70)
R^2	0.60	0.61	0.63	0.15
Std. Error (σ)	1.637	2.003	1.595	2.07
No. of Observations	335	335	335	335

Note: (i) t-statistics are in parentheses (at 0.05 level), (ii) This table presents KS estimation with extension, (iii) Square brackets over two variables indicate joint significance at 0.05 level.

Table 2.10: Cross-section regression for KS Extended Model Without Singapore and Hong Kong (N=84; sample: 1960-2000): Dependent Variable, (Y/Ngr)

	1960	1970	1980	1990
Ln(Y/N)	0.414	1.724	-0.905	-0.489
CBR	-1.345*	0.581	-0.650	0.210
CBR*(Y/N)	0.195	-0.134	0.120	0.066
CBR_{15}	0.287	-0.023	-1.099*	-1.952*
CBR_{15} * (Y/N)	-0.140	-0.088	-0.043	0.026
CDR	0.557	-1.933*	0.452	0.764
CDR* (Y/N)	-0.373	0.044	-0.095	-0.270**
Density	2.040**	1.951**	3.994**	3.041*
Constant	6.335**	2.805	6.115**	7.184**
R^2	31	26	26	26
Std. Error (σ)	1.63	1.84	1.60	2.05
No. of Observations	83	84	84	84

Note: *: significance at 10 percent. **: significance at 5 percent level;
(ii) Square brackets over two variables indicate joint significance at 0.05 level;
(ii) For 1960, data on Uganda is missing. Therefore, N — 83 instead of 84.

Table 2.11: Partial Effects: KS Extended Model (N = 84; sample 1960-2000)
(Without Singapore and Hong Kong)

Developing			
Years	CBR	CDR	CBR_{15}
1960	-1.027	-0.05	0.06
1970	0.310	-1.84	0.00
1980	-0.335	0.20	-0.08
1990	0.397	0.00	-0.11
Developed			
Years	CBR	CDR	CBR_{15}
1960	0.177	-2.35	-0.81
1970	-1.043	-1.40	-1.41
1980	1.237	-1.06	-1.93
1990	1.513	-4.58	-2.49

Table 2.12: KS Extended Model With Other Variables (N = 84; sample: 1960-2000) (Without Singapore and Hong Kong): Dependent Variable, $(\mathrm{Y/N}gr)$

	Panel Estimation			
	Model 1	Model 2	Model 3	Pooled
Ln(Y/N)	-2.31(-3.46)	-2.35(-2.18)	-3.311(-2.77)	-0.139(-0.221)
Ngr	-0.284(-0.736)			
Ngr*(Y/N)	-0.010(-0.377)			
CBR		$\lceil -0.172(-0.337) \rceil$	$\lceil -0.037(-0.071) \rceil$	-0.530(-1.34)
CBR*(Y/N)		$\lfloor -0.128(-1.96) \rfloor$	$\lfloor -0.141(-2.19) \rfloor$	-0.021(-0.337)
CBR_{-15}			$\lceil -1.172(-3.06) \rceil$	$\lceil -0.853(-2.43) \rceil$
$CBR_{-15} * (Y/N)$			$\lfloor 0.028(0.446) \rfloor$	$\lfloor -0.023(-0.473) \rfloor$
CDR		$\lceil -0.235(-0.294) \rceil$	$\lceil -1.310(-1.69) \rceil$	$\lceil -0.878(-1.49) \rceil$
CDR*(Y/N)		$\lfloor 0.185(2.50) \rfloor$	$\lfloor 0.185(2.69) \rfloor$	$\lfloor -0.108(-1.94) \rfloor$
Density	10.45(3.27)	10.49(2.02)	10.49(2.02)	0.515(0.556)
log(LifeExpect)		6.83(2.11)	2.326(0.715)	-2.22(-1.37)
Inflation		-0.020(-3.65)	-0.002(-3.79)	-0.03(-8.52)
Constant	6.84(4.95)	-18.79(-1.36)	5.47(0.38)	17.66(2.41)
R^2	0.61	0.67	0.70	0.24
Std. Error (σ)	1.527	1.960	1.905	1.97
No. of Observations	335	307	307	307

Note: (i) t-statistics are in parentheses (at 0.05 level), (ii) This table presents KS estimation with extension. Square brackets over two variables indicate joint significance at 0.05 level.

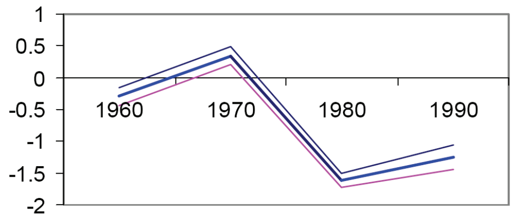
Table 2.13: KS Extended Model without Life Expectancy N = 84; sample 1960-2000)) (Without Singapore and Hong Kong): Dependent Variable, $(\mathrm{Y/N}gr)$

	Panel Estimation			
	Model 1	Model 2	Model 3	Pooled
Ln(Y/N)	-2.31(-3.46)	-1.89(-1.70)	-3.08(-2.42)	0.066(0.650)
Ngr	-0.284(-0.736)			
Ngr*(Y/N)	-0.010(-0.377)			
CBR		⌈ 0.164(0.281) ⌉	⌈ 0.338(0.538) ⌉	-0.462(-1.19)
CBR*(Y/N)		⌊−0.150(−2.18)⌋	⌊−0.165(−2.39)⌋	-0.020(-0.318)
CBR_{-15}			⌈ −1.20(−3.29) ⌉	⌈ −0.731(−2.06) ⌉
$CBR_{-15} * (Y/N)$			⌊0.039(0.615)⌋	⌊−0.035(−0.690)⌋
CDR		⌈ −1.51(−2.52) ⌉	⌈ −1.797(−3.20) ⌉	⌈ −0.462(−0.840) ⌉
CDR*(Y/N)		⌊0.175(2.32)⌋	⌊0.167(2.38)⌋	⌊−0.130(−2.22)⌋
Density	10.45(3.27)	9.46(1.83)	11.76(2.43)	0.50(0.510)
Inflation		-0.018(-4.06)	-0.020(-4.05)	-0.03(-8.33)
Constant	6.84(4.95)	8.78(3.56)	14.12(4.45)	7.44(4.90)
R^2	0.61	0.65	0.67	0.23
Std. Error (σ)	1.527	1.574	1.525	1.99
No. of Observations	335	309	309	309

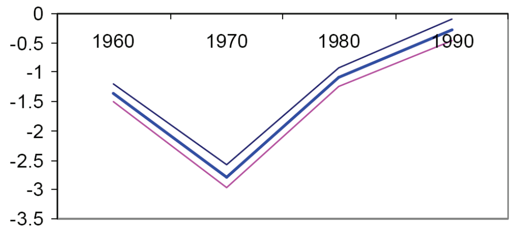
Note: (i) t-statistics are in parentheses (at 0.05 level), (ii) This table presents KS estimation with extension. Square brackets over two variables indicate joint significance at 0.05 level.

Figure 2.1: Partial Effects of CBR, CDR, and CBR-15 for Developing Countries (Total Countries = 86)

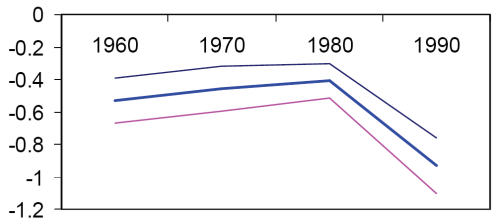
(a) Partial Effect of CBR for LDCs



(b) Partial Effect of CDR for LDCs



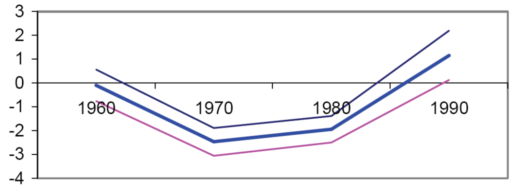
(c) Partial Effect of CBR-15 for LDCs



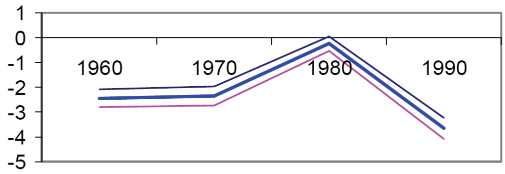
The solid lines represent the empirical estimates of the partial effects. Lines above and below the ‘solid line’ (which is in the middle) represent upper and lower confidence band.

Figure 2.2: Partial Effects of CBR, CDR, and CBR-15 for Developed Countries
(Total Countries = 86)

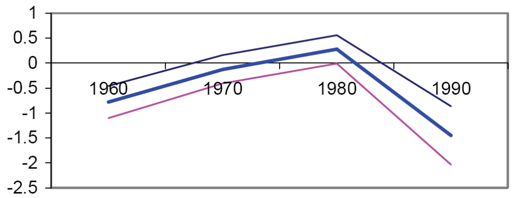
(a) Partial Effect of CBR for Developed Countries



(b) Partial Effect of CDR for Developed Countries



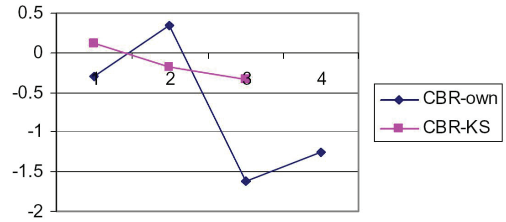
(c) Partial Effect of CBR-15 for Developed Countries



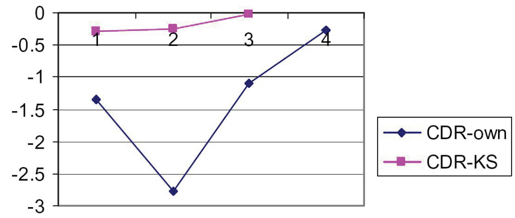
The solid lines represent the empirical estimates of the partial effects. Lines above and below the 'solid line' (which is in the middle) represent upper and lower confidence band.

Figure 2.3: Comparison of Partial Effects of CBR, CDR, and CBR-15 with KS (1995): Developing Countries (N=86)

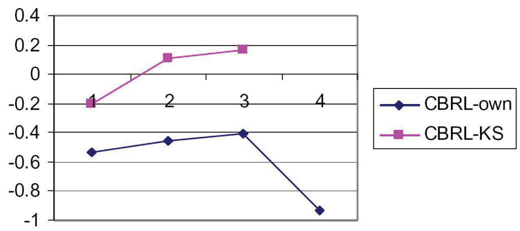
(a) Comparison of CBR



(b) Comparison of CDR



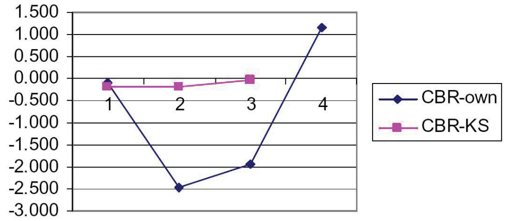
(c) Comparison of CBR(-15)



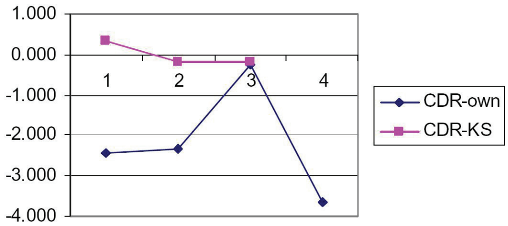
Note: Numbers 1,2,3,4 in the X-axis in the above figures represent the decades 1960-70, 1970-80, 1980-90, 1990-2000 respectively.

Figure 2.4: Comparison of Partial Effects of CBR, CDR, and CBR-15 with KS (1995): Developed Countries (N=86)

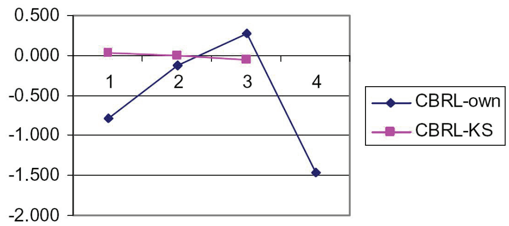
(a) Comparison of CBR



(b) Comparison of CDR

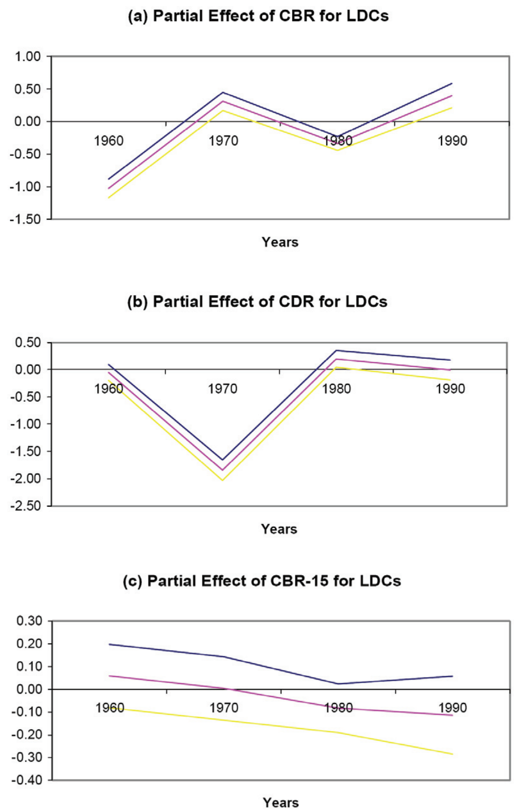


(c) Comparison of CBR(-15)



Note: Numbers 1,2,3,4 in the X-axis in the above figures represent the decades 1960-70, 1970-80, 1980-90, 1990-2000 respectively.

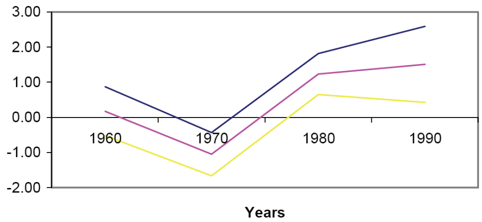
Figure 2.5: Partial Effects of CBR, CDR, and CBR-15 for Developing Countries (Total Countries = 84)



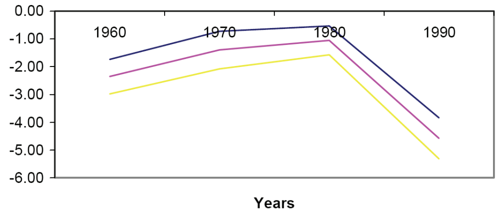
The solid lines represent the empirical estimates of the partial effects. Lines above and below the ‘solid line’ (which is in the middle) represent upper and lower confidence band.

Figure 2.6: Partial Effects of CBR, CDR, and CBR-15 for Developed Countries
(Total Countries = 84)

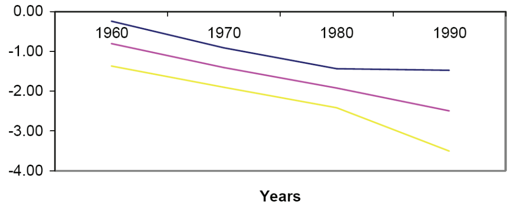
(a) Partial Effect of CBR for Developed Countries



(b) Partial Effect of CDR for Developed Countries



(c) Partial Effect of CBR-15 for Developed Countries



The solid lines represent the empirical estimates of the partial effects. Lines above and below the 'solid line' (which is in the middle) represent upper and lower confidence band.

3. Stochastic Demographic Dynamics, Economic Growth and Long-memory

3.1 Introduction: Tracing the source of fluctuations and the need for a new approach

After a long hiatus of sustained population debates,[24] a surge of recent theoretical and empirical research demonstrates how (age-specific) population growth acts upon economic growth of developed and developing countries. By segregating total population into different components (viz., age structure, crude birth and death rates, etc.), Kelley and Schmidt (KS: 1995, 2001), for instance empirically showed that growth of these components explain growth variations in these countries. In particular, KS outlined the long-term and short-term effects of age-distribution on the future growth trajectory. Using cross-country data, Malmberg and Lindh (2005) in an intriguing research showed that 'around half of the variation in growth since the last war – especially trend variation – is explained by age distribution'. Incorporating age-structure information in their forecasting model, the authors were also able to perform a stable and better forecast of global income than the commonly used technology based forecasting approach. It is also held that the age variations of greater magnitude, as observed in most of the industrialized nations, would have a dramatic impact on macroeconomic consequences in addition to the fact that the pace of growth of these components would also determine the speed and pattern of convergence of developing economies economic growth. Thus, recent empirical investigation has clearly established the critical role of population, especially the role of age structure distribution in economic growth fluctuations.

From theoretical perspective, though the delineation between population and economic growth is quite old, the literature exploring the dynamic relationship between age-structure and economic growth is rather sparse. Employing an overlapping generations model (OLG) in endogenous economic growth framework, Boucekkine et al. (2002) studied the economic growth consequences of age-structured population growth variations. The authors found that growth of working age population and investment in them, have various short-term, intermediate-term, and long-term consequences on economic growth. Moreover, the 'transition from a stagnant economy to a modern-growth economy,' as evinced in their research, could be made solely on the basis of 'demographic shifts'. Therefore, demographic factors, especially the age-structure distribution

[24] See Kelley and Schmidt (2001) and Birdsal et al. (2001) for a survey of the debate that concerns about the exact effect of population growth and its components in economic growth and development.

would play critical role in the current and future economic policy decision making.

The core assumption underlying the extant theoretical and empirical growth models is that the growth of aggregate and age-structured population is stationary. They are assumed to remain stable over time so that possible stochastic dynamics affecting in each generation of population is ruled out. Following this assumption, population and age-structure variables would have only short-run growth consequences, as the shocks are likely to completely disappear in the long-run. Indeed, the OLG model as employed by Boucekkine et al. (2002) would remain difficult to use unless a stable age structure is assumed. Nonetheless, the generational accounting models are also highly sensitive to the stylised assumption of stability.

Under the conventional assumption of stationarity, stability, and exogeneity, standard dynamic economic growth analyses are easily carried out because it helps avoid methodological and technical complexities. And most possibly, with such assumption plausible empirical growth models could be built which to some satisfactory extent could reproduce real life demographic and economic growth variations. In fact, exogeneity assumption of population acts as a ready proxy for 'stationary' population growth because if the latter is assumed to affect the economy from outside the system, idiosyncratic population shocks would have no measurable impact on the economy. However, real life economic situations prove otherwise. Population growth is more likely to be characterized as endogenous due to the interaction with the economy and due to its own course of evolution. Therefore, instead of the assumed linear effect on the economy, it would have a non-linear and persistent effect. A shock to the population growth, broadly to the demographic system thus, is most likely to have a long-run impact on the economic system.

Indeed, in the wake of recent demographic changes (e.g., shifts in age structure distributions, and the demographic age structure's own inherent dynamics), the stationary assumption appears to us to be too restrictive because it inadvertently downplays the role of demographic shocks and their magnitude of persistence in economic growth fluctuations. It has been demonstrated by some authors (e.g., Prskawetz and Feichtinger, 1995) that due to its endogenous[25] nature, population growth may imply chaos.[26] Endogenous phase switch in the form of demographic variations like high or low fertility, higher work force growth etc., observed over a long period of time can induce non-linearity in the series, which can result in chaotic population dynamics. Day (1993) explains the high non-linear and chaotic nature of population due to multiple

[25] In the sense that past population growth affects the economy so that it is endogenously determined as part of an interacting system.

[26] This refers to the extreme sensitivity of the future growth path of these variables to their initial distributions.

phase switch in the series. Thus, in view of these theoretical and empirical development, it appears to us that the conventional 'stationary' assumption of population and its components is far too narrow and the assumption needs to be relaxed to accommodate broader dynamics of demographic changes, which in turn could have more than mere short-run effects on the economy.

Empirical research which studies the effects of stochastic demographic shocks on economic growth is rather sparse except the modest contributions of Diebolt and Guiraud (2000) and Gil-Alana (2003). By depicting the fractional nature of population in OECD countries, Gil-Alana showed that the order of integration of the population series substantially varies across countries and depends on how we specify the $I(0)$ disturbances. Among OECD countries, while some countries (like Germany and Portugal) present the smallest degrees of integration, population in Japan appears as the most non-stationary series. Examining the case of France, Diebolt and Guiraud (2000) also showed that the fractional nature of population series will exert significant impact on future consumption and more so on the socio-economic relation. The research carried out by these authors provide preliminary first hand information about the nature of persistence of population series, though very little intuition concerning the degree of persistence on economic growth and development could be drawn from their research. The underlying theoretical mechanism delineating the long memory population and economic growth relationship is apparently missing in these papers.

Moreover, long-memory characterization in total population provides only an unclear picture of the true nature of demographic dynamics, their degree of persistence and the effect of persistence on economic growth. This chapter makes a modest attempt to fill the void in the literature. It is apparent that modeling age-structure distribution in the long-memory setting provides answer to our worries. Nevertheless, in view of the myriad implications of age-structure distribution on (macro)-economic growth of developed and developing economies, a long memory characterization will be very helpful in mapping the degree of persistence of demographic shocks leading to a clear understanding of their impact on future economic growth and policy. We suggest a long-memory model for population growth and age-structure distribution in an endogenous economic growth setting to study the persistence of shocks of population and age structure on economic growth.

The contribution of this chapter to the literature is two-way. Assuming that population growth is endogenous, *first,* we provide a theoretical construct to show that long-memory characteristics of population and age structure might induce long-memory in output growth. *Second,* we empirically illustrate the long-memory effects of population and age-structure growth on economic development of both developed and developing countries. By allowing long-memory data generating process (DGP) to population growth and age-structure, we naturally allow the possibilities of both stationary and non-stationary dynamics in the demographic system. Moreover, we assume that the 'strength and length of memory' of demographic variables governs their future growth path and shapes the pattern of interaction with the economic system. This observation forms the core motivation of the chapter.

Clearly, this chapter builds on the assumption laid out above – that the DGP for demographic components and economic growth are long-memory. Long memory DGP of demographic and output variables provides us the much needed platform to check for the magnitude of persistence so that a distinct conclusion about the long-term and short-term effect of shocks can be laid. Moreover, as we know higher is the persistence, lesser is the possibility of shocks converging to the steady state values. Therefore, counter-cyclical policies are often recommended in the literature. From our 'memory estimates' we would know the exact magnitude of persistence and its property of convergence to the mean value in the long run so that appropriate policies can be recommended keeping in mind the development objectives of developed and developing countries. To this end, our empirical examination comprises of 152 countries (31 developed and 121 developing countries), thus provides an exhaustive exploration of long-memory demographic dynamics of a large set of countries.

Drawing on the intuition of the theoretical construct we show that (non)-stationary long memory in population growth and age distribution may induce long-memory in economic growth. Michelacci and Zaffaroni (2000) tried to estimate the long-memory behavior of output growth of developed countries. The authors did not provide any indication of the source of long memory in output growth. In this chapter, we show that long memory in output growth might have resulted from long-memory in demographic variables, like population and age structure. Therefore, besides, uncertain technological changes, fluctuations in demographic age distribution is considered in our chapter as the main contributor to long-memory output dynamics. Our empirical estimation shows that significant long-memory can be found in most of the developed and developing countries. Some of them experiencing non-stationary long-memory but assured of long-run convergence. While for others the memory estimates exhibit destabilising forces inducing non-convergence of the series to the steady state level. Keeping in mind the development objectives of those countries, these memory estimates provide essential information about their long-term relationship with income growth as well as the feature of long-run convergence.

The next section (Section 3.2) summarizes the concept of long memory and implications for shock persistence. We also outline a theoretical link between the long memory features of demographic variables and economic growth in this section. Section 3.3 summarizes the estimation techniques of the memory parameter. Data and empirical results of the chapter are presented in Section

3.4. Section 3.5 concludes with the major findings by critically examining them in the light of development objectives of developing and developed countries.

3.2 Theoretical construct

3.2.1 The Concept of long memory and analysis of shocks in demographic components

For a time series P_t, long-memory (and short-memory) depicts the strength of 'dependence' between its current and remote past values: between P_t and P_{t-k}, where k is the lag length. The nature of 'correlatedness' between them gives more intuition about the dynamics of the system where P_t evolves and at the same time affects other associated variables in the system (see for instance Baillie and Bollerslev, 1994 for review). Stronger (weaker) correlation between P_t and P_{t-k} indicates higher (lower) persistence of shocks. The more persistent a shock is, the more vulnerable is the system. In the time domain, the process P_t can exhibit long-memory property if its autocorrelations $p(k)$ exhibit slow decay and persistence. In the frequency domain, long memory is defined when we evaluate spectral density at frequencies close to zero. The memory parameter, which defines the nature of shock persistence, is assumed to be 'fractional' rather than an integer in the typical autoregressive integrated moving average *(ARIMA)* model so that we can define a long memory DGP for P_t. The fractional ARMA (in short, *ARFIMA)* model is described as

$$\Phi(L)(1-L)^d P_t = \Theta(L)u_t. \tag{3.1}$$

or $(1-L)^d P_t = \Phi^{-1}(L)\Theta(L)u_t$, where, L is the lag operator, $\Phi(L)$ and $\Theta(L)$ are *AR* and *MA* polynomials of order p and q respectively. *d* is the integration or *memory* parameter which can be defined on the real line for *ARFIMA* model. Restricting d to the integer values of 0 and 1 gives rise to standard *ARMA* model. $u_t(t \geq 0)$ is assumed to be *iid* with zero mean and continuous spectrum $f_u(\lambda)$ (see for instance, Granger and Joyeux, 1980 and Baillie and Bollerslev, 1994 for details).

The assumption of real d values, combined with the filter $(1-L)$, displays various memory characteristics of P_t. Usually, this can be known by looking at the following binomial expansion of $(1-L)^d$:

$$(1-L)^d = \sum_0^{\infty} h_j L^j = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 - \ldots \tag{3.2}$$

$h_0 \equiv 1$, L^j is backward operator j times, and $h_j \equiv (1/j!)$ $(d+j-1)$ $(d+j-2)$ $(d+j-3)\ldots$ $(d+1)$ (d). It may be noted from the above that the coefficient of lagged P_t provides the rate of declining weights. However, based on the noninteger values and sign of d, the following memory properties are observed.

With $d = 0$ in Eq.3.1, the process exhibits 'short memory' as the autocorrelations in this case is summable and decay fairly rapidly so that a shock has only a temporary effect completely disappearing in the long run. Long memory and persistence is observed for $d > 0$. In this case, the shock affects the historical trajectory of the series. However, greater is the magnitude of d, stronger is the memory and shock persistence. For $d \in (0,0.5)$, the series is covariance stationary and the autocorrelations take much longer time to die out. When $d \in [0.5,1)$, the series is a mean reverting long-memory and non-stationary process. This implies even though remote shocks affect the present value of the series, this will tend to the value of its mean in the long run. For $-1/2 < d < 0$ the process is known to be fractionally over-differenced. In this case, there is still short memory with summable autocovariances, but the autocovariance sequence sums to 0 over $(-\infty,+\infty)$.

For $d < -1/2$ the series is covariance stationary but not invertible. And finally, when $d \geq 1$ the series is nonstationary and exhibits 'perfect memory' or 'infinite memory'. There is no unconditional mean defined for the series in this case. The process defined by this value of d is non-stationary and non-mean reverting. In this case, the mean of the series has no measured impact on the future values of the process. Important to note that for $0.5 \leq d < 1$, there is no variance, so the existence of the mean would need to be established in each case. There is a median, however. So this case may be described by 'median reversion'. The results are summarised in Table 3.1.

Table 3.1: Fractional components and their interpretation

d	Interpretation
0	: Short-memory population growth, log population is $I(1)$
1	: Non-stationary population growth, log population is $I(2)$
$< 0, 0.5 >$	: Long-memory population growth, log population is $I(\mathrm{d}+1)$

3.2.2 The long memory demography and economic growth linkage

In this section we outline a theoretical mechanism exploring the long-memory demography and economic growth linkage. Our objective is to show that long-memory in demographic variables can give rise to long-memory in output growth. We approach the problem in two ways. First, the econometric formulation is given which explains how the presence of long-memory in demographic components may give rise to long-memory in output growth. Second, a stochastic Solow-Swan type of economic growth model is constructed where the source of stochasticity comes from the long-memory structure of population growth rate, which while embedded in the growth-theoretic set-up can cause substantial variations in output, consumption, and saving behaviour of the economy. Our idea is to allow population growth in Solow-Swan model as a long-memory data generation process (DGP) and study the effect of stochastic memory on output. For the purpose, we define below the data generating process of population and economic growth.

Recall that population growth is defined as fertility rate minus death rate plus net migrations. Denoting F_t as the fertility rate, D_t as death rate, and Nm_t as the net migration rate, population growth at time t is given as

$$n_t = (F_t - D_t) + Nm_t \tag{3.3}$$

Assume for simplicity that Nm_t is zero in the model, so that population growth can be accounted strictly by demographic characteristics, viz., F_t and D_t. Following Dasgupta (1995) who illustrates that high births at the current period might have resulted from the high births in the past, we assume in this chapter that current high n_t at t is a result of high n_t in the previous periods, say at $t-1$. This allows n_t to be modelled as an autoregressive (AR) process. This assumption carries significance for the development objectives of developing countries. For instance, high persistence in n_t in developing countries, among many reasons, might indicate the ineffectiveness of population control policy. We elaborate on this point later in this chapter.

Statistically, the long-memory DGP for n_t can be defined as follows

$$(1-L)^d \Phi(L) n_t = \theta(L) u_t \tag{3.4}$$

where $u_t \sim iid\left(0, \sigma_u^2\right)$, $\Phi(L)$ and $\theta(L)$ are autoregressive and moving average polynomials, and $(1-L)^d$ is a filter giving a picture of rate of decline of memory, d, as lag length, L increases. Setting $\Phi(L)/\Theta(L) \equiv \Psi(L) = 1$ for simplicity, the above equation is written as

$$(1-L)^d n_t = (1-L)^d (F_t - D_t) = u_t \tag{3.5}$$

That is n_t is governed by the strength of the memory parameter, d. Intuitively this means the shock in the population growth at the current period is regulated by shocks in the past periods with certain memory structure. Higher memory estimates implies higher persistence of shocks and the converse.

Recently Michelacci and Zaffaroni (2000) investigated the long memory characteristics of output (Gross Domestic Products, GDP) of developed countries and found substantial evidence of long memory. The long memory in the growth of output can be written in the similar way as long memory in population growth. Denoting y_t as the growth of output, then

$$(1-L)^d y_t = u_t \tag{3.6}$$

represents long-memory in y_t with the usual restrict ions of d on the real line. An obvious question that may arise in this context is the source of long memory in output. Economic growth models in the last two decades have suggested many different models explaining fluctuations in growth or persistence of shocks, the most important variable being the technological progress. However, as we explained before, demographic variables have recently occupied central place in the explanations of economic growth fluctuations, therefore, any shock persisting in the growth of output could be interpreted originating from the growth of demographic variables along with technological progress. A natural way to present whether, say y_t is a short-memory or a long memory process, is to know the shape of the spectral density of y_t.

If y_t is described by $y_t = \bar{y} + u_t$, i.e., the process is independently distributed around the mean, then the spectral density of y_t is

$$f_y(\lambda) = \frac{\sigma_u^2}{2\pi}.$$

If shocks persist in y_t and is characterized in long memory setting, then y_t follows $(1-L)^d y_t = u_t$ with the spectral density

$$f_y(\lambda) = \left|1 - e^{i\lambda}\right|^{-2d} f_u(\lambda) = \left|2\sin\left(\frac{\lambda}{2}\right)\right|^{-2d} f_u(\lambda),$$

where $f_u(\lambda)$ is the spectral density of the error term. Now assuming the demographic-economic relation as above, the long memory in output growth, y_t can be represented by the long memory in the growth of demographic components, V_{it}. V_{it} denotes population of different age structure, viz., $V_{it} = ((Age0-14)_t, (Age15-64)_t, (Age65+)_t)$, where i refers to each age group that varies over time t. Malmberg and Lindh (2005) provide the following explicit formulation of demography-economic relationship in their forecasting model, which is our typical interest to show how the memory characteristics in different age-structure population directly contribute to the long-memory in output. The model is

$$y_t = \beta_0 + \beta_1 Age(0-14)_t + \beta_2 Age(14-65)_t + \beta_3 Age(65+)_t + \epsilon_t \tag{3.7}$$

A compact expression of the above equation is

$$y_t = \beta_0 + \sum_i \beta_i V_{it} + \epsilon_t \tag{3.8}$$

where $i = [1,2,3]$ refers to population of age group $0-14, 15-64, 65+$ respectively $\varepsilon_t \sim I(0)$.

To demonstrate how demographic shocks represented by long-memory population growth and age structure gives rise to long-memory in output growth, it is necessary to refer to the shock expansion mechanism (Eq. 3.2). If demographic shocks exist and have persistence, then the memory structure is given by

$$V_{it} = \overline{V_i} + 1 + dV_{i,t-1} - \frac{d(d-1)}{2!}V_{i,t-2} + \ldots - (-1)^j \frac{d(d-1)\ldots(d-j+1)}{j!}V_{i,t-j} + \ldots, \tag{3.9}$$

with $\overline{V_i} = 0$ and $d \geq 0$. Note that shocks driving a fractional process must have mean zero, otherwise V_{it} will exhibit a time trend of $O(t^d)$. In Eq. 3.2, we showed that $(1-L)^d$ could be expressed by h_j where the expression for h_j indicates declining weights, we can denote this as impulse-response coefficient of L^j. Hamilton (1994) showed that for large $j, h_j \sim (j+1)^{d-1}$ with $d < 1$. Given the demographic-economic relation, the growth of output, y_t is represented as a function of the impulse-response coefficient h_j. The magnitude of shocks originating from the demographic variables growth over time will affect long-run behavior of y_t. The higher the estimate of d, the more intensively y_t will respond to the shock, and hence there would be fluctuations.

Important considerations emerge concerning the 'order of long-memory in economic growth as a result of the linear combination of different orders of memory in demographic components'. Putting differently, what would be the order of integration of aggregate population if its components display various order of integration? Statistically, the question is: what likely impacts the linear combination of different orders of $I(d)$ processes of age shares will have on the order of y_t? In this case, one would be interested in analyzing the long-run equilibrium relationship between y_t and V_{it} given different orders of d. A study of this consideration is beyond the scope of this chapter and therefore is reserved for future research.

A Stochastic Solow Model

In this section we build a theoretical model for interlinking the long-memory characteristics of demography and economic growth. Stochastic version of Solow-Swan model is used where population growth in the model, instead of being constant, is assumed to have stochastic shocks so that dynamics of population growth can determine the dynamics of output in the economy. Drawing on the intuition and construct of long-memory population growth described in the preceding section, we allow population in Solow-Swan model to follow a long-memory DGP. The economy is assumed to be closed. The production function of the representative agent is given a Cobb-Douglas type:

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha} \tag{3.10}$$

where $0 < \alpha < 1$, Y_t is output at time t, K_t is capital input at t. Labor input, N_t governed by the growth of population, n_t so that

$$N_t = (1 + n_t) N_{t-1} \tag{3.11}$$

where population growth, n_t, in our system is assumed to follow a long-memory data generating process which evolves as

$$(1 - L)^d \Phi(L) n_t = \Theta(L) \epsilon_t \tag{3.12}$$

L is the lag operator as defined before and

$$(1 - L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j - d)}{\Gamma(j + 1)\Gamma(-d)} L^j \tag{3.13}$$

$\Phi(L) = (1 + \phi_1 L + \ldots + \phi_p L^p)$ and $\Theta(L) = (1 - \theta_1 L - \ldots - \theta_q L^q)$ are *AR* and *MA* polynomials respectively. Moreover, the investment, I_t and capital stock equations are described as

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{3.14}$$

In the above equation, capital stock is assumed to decline at a constant rate of $\delta(0 < \delta < 1)$ per period. Given that s is the faction of Y to be invested, then

$$I_t = sY_t \tag{3.15}$$

Consumption is defined according to

$$C_t = (1 - s) Y_t \tag{3.16}$$

The immediate effect of long-memory population growth on economy's long-term output, consumption and investment growth can be observed by plugging the long-memory DGP of n_t in the production, capital, and consumption equations. Assuming[27] that

$$\psi(L) = \frac{(1-\theta_1 L-\ldots-\theta_q L^q)}{(1+\phi_1 L+\ldots+\phi_p L^p)} \equiv 1$$

in Eq. 3.12 and substituting it in Eq. 3.11 and then in Eq. 3.10, we obtain

$$Y_t = AK_t^{\alpha}[(1+(1-L)^{-d}\psi(L)\epsilon_t)N_{t-1})]^{1-\alpha} \tag{3.17}$$

The output per capita, $y_t = (Y_t / N_t)$ in this case is a function of sequence of shocks, thus regulating the 'efficiency unit of output' by the stability of shocks. Moreover, since $(1-L)^d$ can be represented by impulse-response mechanism, viz.,

$$\sum_{j=0}^{\infty}(j+1)^{d-1}$$

, inducting this in Eq. 3.17 then depicts

$$Y_t = AK_t^{\alpha}[(1+\sum_{j=0}^{\infty}(j+1)^{1-d}\psi(L)\epsilon_t)N_{t-1})]^{1-\alpha} \tag{3.18}$$

Assuming the effect of technology, A, to be constant on Y_t, or by assuming that growth in A is caused by population pressure, a unit shock in n_t in Eq. 3.18 can exhibit how Y_t responds to it. Nevertheless, it is clear that depending on the magnitude of d, the behaviour of N_t can determine the nature of output growth in the economy. Now, since consumption and investment are a function of output, the persistence of shocks in output, consumption and investment growth in the economy. Denoting, aggregate output, aggregate consumption, and aggregate capital stock at T, as Q_T, C_T, and K_T, it can be perceived that

$$\sum_{t=1}^{T} Y_t = f(K, n(d)),\ \sum_{t=1}^{T} C_t = g(s, Y(d)), \text{ and } \sum_{t=1}^{T} K_t = v(\delta, I(d)),$$

where $n(d)$ denotes long-memory population growth, $Y(d)$ as long-memory output, and $I(d)$, long-memory investment. The steady-state growth of output and investment can be derived from the above characterizations of stochastic output, consumption and investment equations. The effect of long-memory population shock on output is demonstrated in Figure 3.1. It may be observed from Figure 3.1 that as we increase the value of d from 0.1 till 0.8, i.e., from stationarity to high non-stationarity, the response of output to such variation

27 This assumption is not binding but assumed for simplicity.

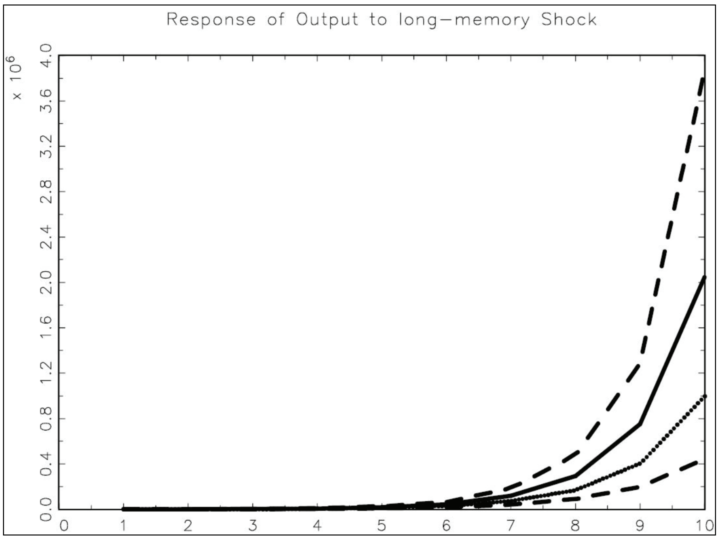
also increases over time, viz., from a slow response to a very steep response as the economy progresses. This depiction does not fully capture the exhaustive dynamics that can arise in stochastic Solow-Swan model due to its response to long-memory population shocks. However, it provides a preliminary idea of the consequences of stochastic demographic system in the economic growth processes.

3.3 Estimation of d

An important parameter in the equations described above is the memory parameter, d, which needs to be estimated to give a clear picture of the magnitude of shock persistence. This section deals with the estimation method of d used in the chapter. Elaboration of other methods can be found in Robinson (1995), and Kim and Phillips (1999, 2000). Among several approaches of estimating d, notable among them are the parametric approach by Sowell (1992) (exact maximum likelihood estimator) and the approximate Whittle estimator due to Whittle (1951), and Fox and Taqqu (1986). In semiparametric class, Geweke and Porter-Hudak estimator (GPH, 1983), and Gaussian semiparametric estimator due to Robinson (1995) are extensively used in the literature. For the empirical investigation of the chapter, we use the modified log periodogram regression (LPR) method developed by Phillips (1999a, 1999b). Agiaklglou et al. (1993), Cheung (1993) and Hurvich et al. (1998) argue that GPH has severe small-sample bias and very inefficient if the results are likely to be contaminated by possible short-memory parameters (i.e., *AR* and *MA* parameters). They also argue that the estimator is not invariant to first differencing so that there might be an over-differencing issue.

Moreover, distinguishing unit-root behavior from fractional integration may be problematic, because the GPH estimator is inconsistent against $d > 1$ alternatives. This weakness of the GPH estimator has been addressed by Kim and Phillips' (2000) Modified Log Periodogram Regression method (MLPR), in which the dependent variable is modified to reflect the distribution of d under the null hypothesis that $d = 1$. This is in fact the modified version of GPH (1983). The estimator gives rise to a test statistic for $d = 1$, which is a standard normal variate under the null.

Figure 3.1: Long-memory effect on Output



GPH is a standard estimation technique for *d*, however, the modifications suggested to GPH method needs to be explained. Specifically, we briefly explain the difference between the two by first defining the discrete Fourier transform (DFT) of the time series, and pointing out how the dependent variable, viz., the periodogram, is modified in case of MLPR. To elucidate the problem, recall Eq.l, and simplify the DGP of the process P_t as $(1-L)^d P_t = u_t$ for convenience. Note that the stationary component of u_t (in Eq. 3.1) is a linear process of the form:

$$u_t = C(L)\varepsilon_t = \sum_{j=0}^{\infty} c_j \varepsilon_{t-j}, \sum_{j=0}^{\infty} j|c_j| < \infty, C(1) \neq 0$$

for all t and with $\varepsilon_t = iid(0, \sigma^2)$ with finite fourth moments. Under this assumption, the spectrum of u_t is

$$f_u(\lambda) \;=\; \tfrac{\sigma^2}{2\pi} | \textstyle\sum_{j=0}^{\infty} c_j e^{ij\lambda} |^2 .$$

Then, the spectrum of P_t can be defined as

$$f_P(\lambda) = |1 - e^{i\lambda}|^{-2d} f_u(\lambda) \tag{3.19}$$

where P_t is stationary, i.e., $|d| < 1/2$. This is also the analogue of the spectrum in the nonstationary case when $d \geq 1/2$. Taking logs of Eq. 3.19 produces:

$$lnf_P(\lambda) = -2dln(|1 - e^{i\lambda}|) + lnf_u(\lambda). \tag{3.20}$$

GPH (1983) propose that d can be estimated from the above by a linear log periodogram regression, where $f_P(\lambda)$ is replaced by the periodogram ordinates, $I_P(\lambda)$ evaluated at the fundamental frequencies

$$\lambda_\zeta = \frac{2\pi\zeta}{n}, n = 0,1,...,n-1.$$

Here $I_P(\lambda_\zeta) = F_P(\lambda_\zeta)F_P(\lambda_\zeta)^*$, $F_P(\lambda_\zeta)$ is the dft

$$F_P(\lambda_\zeta) = \frac{1}{\sqrt{2\pi n}}\sum_{t=1}^{n} P_t e^{it\lambda_\zeta}$$

of P_t and F^* is the complex conjugate of F. Substituting these informations in Eq. 3.20 now becomes

$$ln[I_P(\lambda_\zeta)] = -2dln|1 - e^{i\lambda_\zeta}| + ln(f_u(\lambda_\zeta)) + \eta_j \tag{3.21}$$

where $\eta_j = ln[I_P(\lambda_\zeta)/f_P(\lambda_\zeta)]$. Note that f_u is continuous, and therefore $f_u(\lambda_\zeta)$ is effectively constant for frequencies in a shrinking band around the origin (Phillips, 1999a), thus enabling a linear least square regression of $ln[I_P(\lambda_\zeta)]$ on $ln|1 - e^{i\lambda_\zeta}|$ over frequencies $\zeta = 1,...,\upsilon$. The regression gives rise to the GPH estimation of d, where asymptotically $\hat{d}_{GPH} \sim N(d, \frac{\pi^2}{6})$. This method has been extensively used in practice due to ease of handling. However, Phillips (1999a) note that Eq. 3.20 is a moment condition and not a data generating mechanism, and the analysis of this regression estimator is complicated while characterising the asymptotic behavior of the dft $F_P(\lambda_\zeta)$ which is central to determining the properties of the regression residual η_j in Eq. 3.21.

Phillips (1999a) suggested modification in the GPH method by properly demonstrating the asymptotic properties of dft. Moreover, the modified GPH, knowns as MLPR provides a wider range for both the stationary and nonstationary areas such as $d \geq 0$ and for *AR* and *MA* errors and test for nonstationarity. The author argues that since we usually have no prior information about the order of integration, d, hence it is instructive to cover a wide range plausible parameter values of d. The biggest concern in GPH technique is that little is known about the short memory component of u_t in our DGP and that its spectrum $f_u(\lambda)$ is treated nonparametrically. In log periodogram regression, viz., in GPH, this is accomplished by working with the dft $F_P(\lambda_\zeta)$ of the data P_t over a set of υ frequencies

$$\lambda_\zeta = \frac{2\pi\zeta}{n} : \zeta = 1,...,\upsilon$$

that shrink slowly to origin as the sample size $n \to \infty$ by virtue of a condition on υ of the type

$$\frac{\upsilon}{n} \to 0 .$$

As $d \to 1$, the dft $F_P(\lambda_\zeta)$ behaves differently due to the effects of leakage in semiparametric estimation of d. Therefore it needs modification. For $d \in (1/2,1)$, Phillips (1999a) derives t he dft of $F_P(\lambda_\zeta)$ as

$$F_P(\lambda_\zeta) = (1-e^{i\lambda_\zeta})^{-d}F_u(\lambda_\zeta) - \frac{e^{i\lambda_\zeta}}{1-e^{i\lambda_\zeta}}\frac{P_n}{\sqrt{2\pi n}} + o_p\left(\frac{(1-e^{i\lambda_\zeta})^{1-d}}{\zeta^{1-d}}\right) \quad (3.22)$$

where

$$\frac{\zeta}{n} + \frac{n^\alpha}{\zeta} \to 0$$

as $n \to \infty$, for some $\alpha \in (1/2,1)$. The asymptotic behavior of $F_P(\lambda_\zeta)$ is dominated by the first two terms in Eq. 3.22, however as $d \to 1$ the importance of the second term in Eq. 3.22 is reflected. Apparently, it is necessary to correct the dft $F_P(\lambda_\zeta)$ by adding the correction term represented by the known form of expression in Eq. 3.22. For log periodogram regression this amounts to using the quantity

$$V_P(\lambda_\zeta) = F_P(\lambda_\zeta) + \frac{e^{i\lambda_\zeta}}{1-e^{i\lambda_\zeta}}\frac{P_n}{\sqrt{2\pi n}} \quad (3.23)$$

in place of $F_P(\lambda_\zeta)$ in the regression. Thus in stead of the usual least square regression (over $\zeta = 1,\ldots,\upsilon$)

$$ln(I_P(\lambda_\zeta)) = \hat{\alpha} - \hat{d}ln|1 - e^{i\lambda_\zeta}|^2 + error \quad (3.24)$$

which is motivated by the form of the moment equation in the frequency domain, the argument above suggests the linear least square regression

$$ln(I_V(\lambda_\zeta)) = \tilde{\alpha} - \tilde{d}ln|1 - e^{i\lambda_\zeta}|^2 + error \quad (3.25)$$

in which the periodogram ordinates,

$$\ln(I_P(\lambda_\zeta))$$

are replaced by $\ln\left(I_V\left(\lambda_\zeta\right)\right) = V_P\left(\lambda_\zeta\right)V_P\left(\lambda_\zeta\right)^*$. This is known as modified log periodogram regression *a la* Phillips. Thus in place of the 'regression model'

$$ln(I_P(\lambda_\zeta)) = \alpha - dln|1 - e^{i\lambda_\zeta}|^2 + u(\lambda_\zeta), \tag{3.26}$$

with $\alpha = \ln\left(f_u(0)\right)$ and

$$u(\lambda_\zeta) = ln[I_P(\lambda_\zeta)/f_P(\lambda_\zeta)] + ln(f_u(\lambda_\zeta)/f_u(0))$$

as in Eq.3.21, we now have from Eq. 3.22

$$I_V(\lambda_\zeta) = |1 - e^{i\lambda_\zeta}|^{-2d} I_u(\lambda_\zeta) \left| \left[1 + o_p\left(\frac{1}{\nu^{1-d}}\right) \right] \right|^2 \tag{3.27}$$

The new regression (Eq. 3.25) seems likely to be most useful in cases where nonstationarity is suspected, especially when $d > 1$. As in the case of GPH, the distribution of

$$\hat{d}_{MLPR} \sim N(d, \tfrac{\pi^2}{24\nu}).$$

Among several advantages in the modified LPR method, Phillips notes that it modifies the periodogram ordinates to find the correct form of the data generating process for the discrete fourier transforms (DFT) which is simple and involves no unknown parameters. Moreover, consistency can be obtained under weaker conditions without assuming distributional forms – which is a big advantage in comparison to GPH.

A practical problem is the choice of υ, the number of periodogram ordinates to be used in the regression. GPH (1983) suggests that the optimal $\upsilon = T^\alpha$ where $\alpha = 1/2$ and T is the sample size. The choice involves a tradeoff that may be described as follows. The smaller the bandwidth, the less likely the estimate of d is contaminated by higher frequency dynamics, i.e., the short-memory. However, at the same time smaller bandwidth leads to smaller sample size and less reliable estimates. As in the case of GPH method, the smaller value of α: (as in $\upsilon = T^\alpha$) implies the smaller number of harmonic ordinates (i.e., the smaller bandwidth) will be used for the estimation of d. Generally, in empirical analysis, preference is given to increasing the value of α to check for the consistency of the estimate of d although simulation experiments can confirm the validity of the selection. For our purpose, we have used $\alpha = 0.60$ through $\alpha = 0.80$ to estimate d. We choose[28] $\alpha = 0.7$ based

[28] The estimates of *d* for other bandwidth are available with the authors though we have not reported in the main text due to space limitation.

on a Monte Carlo simulation experiment (see table below) where we have minimum bias for that bandwidth. Davidson's (2005) TSM software is used to carry out the simulation experiment which is built for the GPH model (assuming that the simulation results will not drastically change if we had used MLPR).

Table 3.2: Monte Carlo Simulation for Choice of Bandwidth

Bandwidth	Estimated bias	Significance	RMSE bias
α=0.60	0.018	3.03	0.0189
α=0.65	0.021	2.86	0.023
α=0.70	0.014	2.20	0.015
α=0.75	0.015	2.47	0.016
α=0.8	0.017	2.83	0.018

3.4 Data and Empirical Evidence

3.4.1 Data and estimation issues

Data

We estimate d for four demographic variables, viz., aggregate population, nd population of different age shares, i.e., 0-14, 15-64, and 65 and above. A total of 152 countries have been considered of which 31 are developed and the 121 are developing countries. The definition of 'development' follows the guidelines of World Bank which is based on the per capita income level of various countries. In the World Bank Development Indicators distinction of low income and high income countries are made. For the high income countries there are high-income OECD and high-income non-OECD countries. However, to avoid complexity, we have categorised all the high-income (both OECD and non-OECD) countries as developed and the rest as developing countries.

Aggregate population data has been collected from both World Bank and Maddison. Maddison does not report the data on other demographic components, hence the World Bank data source has been used for the purpose. The sample span of the aggregate population is from 1950-2003 (which is from Maddison) and for different age shares the sample span is from 1960-2003. Though Maddison data comprise sample long time back in the past, e.g., from 1800, data for all the countries are not available at the same time. Therefore, 1950 has been selected as the starting date for aggregate population. It may be noted that 50 years of demographic data is not a very small sample for long-memory estimation given the slow paced demographic variation. Since our purpose is to capture the demographic dynamics over time, 50 years data works well for our purpose. To know the effect of long memory parameter on per capita output growth, we have regressed the estimated memory parameter on per capita output growth of both developed and developing countries over five decades.

Estimation Issues

An important problem in the long-memory literature is whether to detrend he series before estimating the memory parameter. Since the presence of a trend can dominate the dynamics of the series, it is suggested by many researchers (e.g., Michelacci and Zaffaroni (MZ, 2000)) to extract the trend in order that the true dynamics of the series can be known. Elucidating on beta convergence of output growth, MZ (2000) suggested that a linear trend be extracted from the per capita income series and then a truncated filter $(1-L)^{1/2}$ would be applied to the residuals. Long memory parameter then, can be estimated for the transformed series using Robinson's semi-parametric estimation procedure. Commenting on MZ's methodology, Silverberg and Verspagen (2001) found that Robinson's methodology suffers from serious small-sample bias and that the use of filter $(1-L)^{1/2}$ after linear trend extraction is seriously flawed. Instead they suggested the use of first-difference filter, $(1-L)$, to remove the trend. In the more recent literature Dolado et al. (2003) investigate this issue using their fractional Dickey-Fuller (FDF) method and conclude that the presence of trend may indeed affect the behavior of the series and generally supported MZ (2000) methodology of extracting linear trend from the data. Moreover, there are some literature (for instance, Silverberg and Verspagen, 2001) which shows that 'fractionally differencing the series by $1/2$ entails an approximation and loss of data and the series must be intialized' with the loss of some observations. So it is better to simply *first difference* the series (in logs) to remove the trend.

Although controversies still remain as to whether detrend the series before estimation of d , in this chapter we follow the logic that the logarithmic difference of population and its components, i.e., their growth rates, would give rise to the same kind of effect as trend extraction from the raw series. Since we are interested in investigating the memory structure of the growth of demographic variables, the logarithmic first differences have been used in this chapter. We have applied Kim and Phillips (1999) modified log periodogram regression to estimate d for the first difference of the log of the population and other demographic variables, viz., population of different ages. Thus we use growth of the variables not the level (which is a stock concept) to look into the long memory dynamics. Presence of long memory in the growth variables have interesting economic implications, which is generally related to the endogenous growth model. Though the relation between long memory and endogenous growth has not been studied, implications of unit root in the endogenous growth set up has been recently investigated (see Lau, 1997, 1999). In this chapter we do not provide an exhaustive theoretical link between long memory in demographic variables and economic growth, we have provided a preliminary background on this relation in Section 3.2.

Another important consideration in the long-memory estimation of the aggregate series is to consider 'aggregation' problem for exhibiting long memory in the series and a possible structural break. This consideration needs mentioning as one of our demographic variables, the population growth, is an aggregate variable. A typical question asked by Granger (1980) was that whether aggregation of individual components of an aggregate series gives rise to long memory in the latter. Indeed, Granger (1980) proved this possibility by showing that long-memory can arise due to aggregation of cross-section (individual components) units of economic time series. Using a beta distribution for individual components, Granger showed the aggregate series can exhibit long-memory. Lippi and Zaffaroni (1999), for example, generalize Granger's result by replacing the assumed beta distribution with weaker semiparametric assumptions, and Chambers (1998) considers temporal aggregation in addition to cross-sectional aggregation, in both discrete and continuous time. Diebold and Inoue (2001) show that a mixture model with a particular form of mixture weight linked to sample size gives rise to an $I(d)$ behavior of a time series. Parke (1999) proposes a duration model for the explanation of long memory in employment series.

An interesting question arises concerning whether individual components display he same characteristics as the aggregate series? Sonnenschein (1972) and Debreu (1974) proved that *'aggregate outcomes may not reflect individual behaviors'* because the structure and the time profile of interactions between agents, vary over time and across cross-section units. This is indeed the case, as the time profile of interaction between the population of ages 0-14 and 15-64, for instance, vary across different countries. Therefore, finding of long-memory say in the aggregate population in the developed or developing countries may not be reflected in the individual countries. Though aggregation problem needs to be considered seriously for future research, for the purpose of the chapter, we will not delve too much into the theme. Our purpose in this chapter is not to study the source of long-memory in the aggregate population series. Rather, we investigate if at all aggregate population and its different components can be characterized by long-memory behavior.

Some authors (see for instance, Diebold and Inoue, 2001) also argue that long memory can often be confused with structural break. Structural break arises most often due to policy changes in the economy and sometime due to the introduction of exogenous shocks (say oil price shock for instance). A considerable amount of work exist for unit root and structural break though some recent research has started to recognize the effect of structural break on the memory characteristics in a time series. Diebold and Inoue (2001) provides Monte Carlo evidence to support the claim that long memory and structural change are easily confused, all in the context of simple and intuitive econometric models. Majority of the financial and economic time series exhibit a kind of structural break (some have endogenous and some exogenous). However, aggregate population and its components do not often show such kind of break although it is quite possible to have one. For demographic variables, there are very little or rare evidence where a major break changed the demographic pattern consequently affecting growth of the economy. Nevertheless, it is true that demographic changes are occurring frequently due to economic policy changes in different countries and these changes work in such a way that it affects long-term demographic-economic relation. In the strict sense of the term, therefore, we do not consider the possibility of structural break in our model.

3.4.2 Empirical results

(a) First-pass assessment

For a first pass assessment of the existence of long-memory in the demographic components, we have estimated Lo's (1991) R/S long-range dependent statistics (Table 3.3). The statistics provides us with a summary view of the strength of dependence between remote past and current values of the demographic variables. The (modified) R/S statistic is the range of partial P_t from its mean, rescaled by its standard deviation which is given by:

$$Q_n = \frac{1}{\sigma_n(l)}\left[Max_{l\leq k\leq n}\sum_{j=1}^{k}(P_j-\overline{P_n}) - Min_{l\leq k\leq n}\sum_{j=1}^{k}(P_j-\overline{P_n})\right] \quad (3.28)$$

Where σ_n^2
$= \frac{1}{n}\sum_{j=1}^{n}(P_j-\overline{P_n})^2 + \frac{2}{n}\sum_{j=1}^{l}\omega_j(l)\left(\sum_{i=j+1}^{n}\omega_j(l)(P_i-\overline{P_n})(P_{t-j}-\overline{P_n})\right)$
is the sample variance estimation of P computed over n samples and

$$\omega_j(l) = 1 - \frac{j}{l+1}, l < n$$

given the sample variance σ_n of P_t and the sample mean, $\overline{P_n}$. The first erm of the bracket is the maximum of the partial sums of the first k; deviations of P_j from the sample mean, $\overline{P_n}$ which is non-negative. The second term corresponds to the minimum of the partial sums, which is non-positive. Therefore the difference of these two quantiles, called 'range' is always non-negative, so that the rescaled range, $Q_n \geq 0$. Equation 3.28 computes the range of partial sums of deviation from the time series P_t from its mean $\overline{X}_n$ rescaled by $\sigma_n(l)$.

The null hypothesis we tested here is that there is no long-range dependence n the population series. This test is performed by calculating the confidence intervals with respect to some significance level, and then checking whether the rescaled range statistic, Q_n, lies in or outside the desired level. We have estimated this statistic for aggregate countries as per the classifications of the World Bank. For instance, we have high-income, low-income, least developed, and others. For each aggregate group we calculated Lo's statistic for aggregate as well as other demographic variables, i.e., the population of different ages (See Table 3.3). Observe that null hypothesis of long-range dependence is rejected in most of the cases at 10 percent significance level.

(b) Estimates of d: Modified Log Periodogram Regression

The support for long-range dependence can be made by evaluating the Modified PR (MLPR) estimates of d, reported in Tables 3.4, 3.5 and 3.6. Table 3.4 presents the MLPR estimates of d for the aggregate countries. Table 3.5 provides d estimates of total population growth for a sample of developed countries which corresponds to a longer data (since 1870-2003) collected from Maddison's World Table. Tables 3.4 and Table 3.6 present the memory estimates for the exhaustive list of developed and developing countries including the aggregate countries, viz., high income and low income, European Union, and Sub-Saharan Africa. The data span here is from 1960-2003 assembled from the World Bank. Due to the unavailability of data on age-specific population before 1960s, d has been estimated for the available sample span. The effect of large sample on long-memory estimates is compared using Tables 3.5 and 3.6.

From Tables 3.4 and 3.6, it is evident that aggregate population and population of different ages depict significant memory characteristics. The World aggregate population have lower d estimates than World population of different ages. The higher the estimates of d, the higher is the persistence of shocks. According to our estimates, for population of 65+ age group, i.e., the retired cohort, we have very high persistence than other categories for World. Specifically, while the world's aggregate population, younger generation (age 0-14) and the working people age (15-64) growth possesses a memory which is in the range $1/2 < d < 1$ implying long-run mean convergence of the the aggregate population shock, however the same is not true for the retired cohort. This series is highly nonstationary implying that a certain non-invertible intrinsic shock is affecting the working age population, one of them being the problem of faster aging. Very high persistence is observed for younger cohorts and working age population in high-income OECD countries, negative memory is found for high-income non-OECD countries. Not surprisingly the degree of persistence for low income countries is far more than high income countries for all demographic components. This provides credence to the fact that low income countries are yet to contain on the pace of population growth and more importantly to restrain the shock in the productive population sector, viz., the working age cohorts.

In Table 3.6, MLPR estimates for all individual countries are reported. For ease of understanding, we have provided (Kernel) density plots of these estimates for all countries (see Figure 3.2) as well as for developed (Figure 3.3) and developing country (Figure 3.4) blocks. Note that $Totpop_{MLPR}$ denotes estimates of MLPR for total population, similar denotation holds for different age groups. Long memory is observed for both developed and developing country blocks (with anti-persistence property for a handful few).[29] The density plots depict precisely this result. Non-stationary long-memory is evident for both the country blocks, since the mean of the density is concentrated around 1. However, a distinction can be made between the degree of persistent shocks between developed and developing countries. Comparing Figures 3.4, it is clear that higher persistence is observed in case of developing country aggregate population growth. For younger age population, the mean of the estimates of d is higher for developed countries than that of the developing countries. The observation may be put into perspective due to the lower growth of younger age population, specifically the lower new births in developed countries. The working age population (Population of age 14-65) have lower persistence of shocks in developing countries (Figure 3.4 than developed counterparts since the average memory estimates for these countries are much above 1.

Taking the effect of all the countries together (World), aggregate population in this category is characterised by a non-stationary and non-mean-reverting process. It can also be noticed that the working age population (14-65) of high income countries are less affected by the stochastic shock than the low income countries. Considering the modified LPR estimates, a long-memory characteristic for aggregate population as well as 0-14 age groups can be concluded. The long memory regression on the per capita output growth depict long memory parameter does not have significant effect on per capita growth for both developed and developing countries. However, the shocks do exert significant positive effect (though very negligible) on the working age population and the retired cohorts for developing countries, while it affects the young age population in the developed countries.

The extant research holds that the 'length' of the time series affects the magnitude of memory estimates, viz., shorter time series might show higher fluctuations than longer time series because fluctuations somehow smooth out in case of the latter. Moreover, as the magnitude of the memory parameter may vary, it might force substantial variation in the conclusion about the nature of shock persistence, except some special cases. To take into account this possibility, we have estimated MLPR for aggregate population for 20 developed

[29] Only for Kuwait and Angola.

countries from Maddison data with the sample span 1870-2003 and compare the results with our earlier estimates (with the sample span 1950-2003). The results are reported in Table 3.5. Comparing Table 3.5 and Table 3.6, we observe that the magnitude of d for aggregate population growth with the extended sample span (i.e., from 1870-2003) is less than the estimates using the period 1950-2003. However, there is no substantial change in the conclusion about long-memory characteristics for half of the 20 countries, the exception being Australia, Denmark, Germany, Greece, UK, Switzerland, Spain, Norway, Ireland, and New Zealand. In fact, we find anti-persistence in case of Ireland using the extended sample which is contrary to the high persistence as per the old sample (1950-2003). It is obvious that longer time series contain smoothed out shocks where the shocks are prominently visible in the shorter sample span. In any case, except for Ireland, Greece, Germany, Spain, and UK, non-stationary long-memory is observed for the rest of the 20 developed countries in the extended sample.[30]

(c) Cross-section growth regression with long-memory population: Tracing cross-country growth persistence and variations

From the discussion above, it is apparent that stochastic demographic shocks have significant effect on economic growth in a standard Solow-Swan model where the aggregate output, capital and investment will be guided by some function of stochastic population shocks. Drawing on this analytical results, we intend to empirically demonstrate how the stochastic shocks exert impact on growth variations (and the converse) of developed and developing countries. The basic framework of the model is as follows:

$$d_{i,(t,T)} = \Gamma_1(log(GDP_{i,(t,T)}), Z_{i,(t,T)}) + \epsilon_{it} \tag{3.29}$$

or

$$log(GDP_{i,(t,T)}) = \Gamma_2(d_{i,(t,T)}, Z_{i,(t,T)}) + \psi_{it} \tag{3.30}$$

where $t = (1,...,T)$, $d_{i,(t,T)}$ is the estimate of long-memory population growth during the period $(t,...,T)$ for i^{th} country, $Z_{i,(t,T)}$ represent a vector of other demographic variables, such as average schooling rate, life expectancy at birth and population density, etc. ε_{it} and $\psi_{it} \sim (0,\sigma^2)$. GDP growth is calculated by taking the log differences between period t and T. The two equations de-

[30] Maddison data does not contain information on different population age shares. For this reason, we have used World Bank data for which the sample span is shorter than Maddison, i.e., from 1960 onwards.

scribe a system where first long-memory shock appears as dependent variable and in the second model it is endogenized in the economic growth regression.

Tables 3.7 and 3.8 present results from cross-section regression of memory parameter of population on economic growth and the converse for a set of both developed and developing countries. Consider first the developed country regression results (Table 3.7). The first part of this table present results when dependent variable of the regression is growth of GDP per capita over 40 years. The explanatory variables we have chosen are growth of stochastic memory estimates of total population, life expectancy at birth, population density, and average years of schooling over the four decades. We expect that if stochastic population shock has had any significant effect on long-run output growth, then the coefficient of this variable *(d-totpop)* should be statistically significant (positive or negative indicating the direction of impact of stochastic shocks). Similarly, life expectancy at birth, increase in the average years of schooling and population density are expected to exert positive influence on growth. For instance, enhanced life expectancy encourages people to stay more productive over years thus contributing to growth, while density propel economic growth via population-induced technological change. Additionally, average schooling years is a proxy for human capital, which is significant to the extent that educated people are more productive to the economy than the uneducated the 'buffer' of human capital via educated mass pushes the economy forward.

As expected, we find that per capita GDP growth in the last four decades has been significantly affected by stochastic population shocks during this period. Though population is traditionally believed to negatively affect output growth, our result supports the recent finding that growth of total population could have positive effect on economic growth depending on which segment of the population is growing faster. Fast growth of younger age population will impede growth via excessive resource dependence, while rapid growth of working age population propels economic growth via resource creation. It all depends on the net effect accounting for which segment of the population is growing faster than the rest. In light of this view, our finding suggests that total population growth shocks had growth-enhancing effect on output over the last four decades indicating the significance of the interplay of stochastic population shocks with long term memory and economic growth. We also find that life-expectancy at birth $\ln(e0)$, *Density,* and average years of schooling, *AvgSchooling* have significant and expected effect on output growth. The significant F value shows overall significance of all the explanatory variables.

On the other hand, when stochastic memory estimate of population is used as the dependent variable in the regression, some interesting results emerge. For instance, even though the coefficient of per capita GDP growth is found to be positive, it is not found to be significant in the regression suggesting that the stochasticity in the population or long-memory in the growth of total population in the developed countries are not found to be significantly influenced by the growth of per capita output over four decades. Rather, we find that life expectancy at birth and population density have significant negative effect on long-memory estimates of population growth. The reasons may be explained as follows. The theoretical explanation of long-memory parameter suggests that higher memory can inflict harm on economic growth as the economy will not return or will take very long time to return to its steady state situation. Alternatively, higher memory estimate would indicate the tendency of the economy towards a chaotic situation. From Table 3.8, we found a significant negative effect of population density, which indicates as density of population increases, it will 'negate' the effect of higher memory on economic growth via 'population induced technical change'. Similar effect can be seen for life expectancy. As people live longer, their increasing contribution to the economy in fact reduces the magnitude of stochastic population shock over time.

Now, when we analyse the pattern for developing countries, the following results emerge from our regression. We find that stochastic population shock and per capita GDP growth in the past forty years are negatively correlated. Using GDP growth as dependent variable, it is evident from Table 3.8 that stochastic population shock negatively affects GDP growth at about 10 percent significance level. Similarly, with population shock as dependent variable we find per capita GDP growth being negatively and significantly influenced during the last four decades. Density is found to have negative effect on output growth for developing countries. The result seems to be akin to the recent finding that density should not just mean the scatterness of population over all arable and non-arable land, but it should mean the scatterness of people over productive land. In that sense, the finding of negative effect of density for developing countries imply the lack of growth-motivating effect of population pressure via technical change.

3.5 Conclusion

This chapter provided a theoretical basis and empirical formulation for understanding the effects of stochastic population shocks for economic growth fluctuations. We stressed on modeling population in a long-memory framework so that demographic variables dynamic relations with economic growth can be better understood. Conventionally demographic fluctuations were considered to have little impact on long-run economic growth due to the assumption of 'constancy' of population in the model and often a stationary behavior. Drawing on realistic situations and some empirical evidence, we argued that such assumptions are too stringent which evidently downplays the role of demographic shocks in economic growth. We analytically showed that long memory in demographic components may possibly give rise to long memory in economic growth. Using the framework of stochastic Solow-Swan model we depicted that the length of memory in population growth would affect aggregate output and hence consumption, and investment behavior of the economy. Population growth with 'short-memory' would induce less fluctuations in the economy's output which could have important implications for investment decisions for the economy to generate further growth.

An exhaustive empirical illustration has been carried out by estimating the memory parameter for both developed and developing countries. We have also estimated the persistent effect of shocks for various country aggregates, viz., high-income, low-income, etc., to make appropriate comparisons. However, caution should be made while interpreting these results. We know that aggregation always smooths out individual irregularities in the data and gives rise to a nice pattern. The differences in the country blocks are due to World Bank classification which is based on per capita income level of these countries. It is therefore, advisable to take note of the memory estimates and study the nature of shock persistence for individual countries.

The estimates of persistence evince that very high degree of shock persistence is observed in both developed and developing countries. A comparison can be made between say low income and high income countries following World Bank classification. We observe that low income countries possess higher degree of memory than high income countries implying persistence of population growth shocks is higher in low income countries than the high income countries. High degree of persistence in the aggregate population growth calls for policy check keeping in mind the development objectives of high income and low income countries. Stationary memory and non-stationary memory of different growth implications; while stationary memory feature possessed by a series would settle for long run convergence to the steady state values, non-stationary memory have generally non-convergent characteristics. Generally, non-convergent shock characteristics are possessed by 'supply or productivity side factors in the economy as argued by several authors (like Durlauf, 1989) while convergent memory characteristics of a series exhibit 'short-run' randomness which is a characteristics of 'demand side factors.

Our results hint at the existence of nonstationary memory for younger generation in the developed countries. This might suggest that developed countries population growth policy should be put into perspectives taking into consideration of 'positive or zero population growth'. Growth shocks in younger generation can not be sustained rather needs to be retarded keeping in mind the fact that younger generation will contribute to the work force of the economy in succeeding years. We observe convergent memory features for some developing countries and non-convergent for others. This outcome is very natural given that the population control policy has not born equal success or even have not been implemented with equal pace in all the countries, hence depending on the structure of the economy, different magnitude of growth shocks have been observed within developing countries. For these countries there is a need to sustain economic development by further investment in education. Our estimates also show that non-mean reversion with non-stationarity is a feature of for these countries other age-structure variables. Moreover, from our regression of memory parameter on the growth of per capita income for both country blocks show that the effect of long-memory is significant for per capita income growth of developing countries for aggregate population as well as the growth of different components of population. High investment in education and other necessary measures for accumulating human capital will reduce the effect of this memory on growth of income. For developed countries, we find significant effect of long memory in retired cohorts on per capita income growth. Though these regression results are preliminary, it gives at least the first hand information of the effect of memory of demographic components on per capita income growth.

Finally, the memory estimates are merely indicative of the stochastic demographic structure which needs careful attention while designing macroeconomic or public policy for future, for instance the distribution and intergenerational transfer and management of resources. High degree of persistence in population does no good for economic growth. One needs to check the implications of demography-economic growth theorisations in light of our findings. Moreover, an important point in our results concerns the possibility of stacking countries with 'common memory' features so that for each block with different persistence profile, the dynamics of demography and economic growth relation can be studied.

Table 3.3: Lo's long-range dependence test: Sample 1960-2003

	Tot Pop	Age 0-14	Age 15-64	Age 65+
	Lo's RS test	Lo's RS test	Lo's RS test	Lo's RS test
World	1.593 (< 0.2)	1.772(< 0.05)	1.714(< 0.1)	1.187(< 0.6)
High Income:				
OECD	1.833(< 0.05)	1.590(< 0.2)	1.993(< 0.025)	1.408(< 0.3)
Non-OECD	1.780(< 0.05)	1.057(< 0.8)	1.032(< 0.8)	1.036(< 0.8)
Low Income	1.632(< 0.1)	1.677(< 0.1)	1.334(< 0.4)	1.363(< 0.4)
Sub-Saharan Africa	1.381(< 0.3)	1.735(< 0.1)	1.516(< 0.2)	1.735(< 0.1)
Least Developed Countries	1.289(< 0.5)	1.782(< 0.05)	1.551(< 0.2)	1.447(< 0.3)
European Monetary Union	1.635(< 0.1)	1.581(< 0.2)	1.589(< 0.2)	1.263(< 0.5)

Note: Probability values in brackets.

Table 3.4: Modified LPR estimates of d for aggregate countries (Sample 1960-2003): Variables are in first difference in logs

Country	Tot Pop		Age 0-14		Age 15-64		Age 65+	
	d (0.7)	SE	d (0.7)	SE	d (0.7)	SE	d (0.7)	SE
Europe&Central Asia	0.556	0.112	0.678	0.141	1.096	0.233	1.042	0.266
European Monetary Union	0.912	0.145	1.181	0.234	1.037	0.214	1.044	0.189
High Income	1.034	0.178	0.239	0.087	0.014	0.101	0.417	0.111
High Income: Non-OECD	0.81	0.187	-0.064	0.127	-0.1	0.109	-0.058	0.124
High Income: OECD	1.068	0.266	1.209	0.254	1.175	0.198	0.764	0.145
Least Devel. Countries	0.995	0.213	1.257	0.249	0.819	0.167	0.894	0.165
Low&Middle Income	1.189	0.277	0.936	0.137	0.968	0.199	0.612	0.113
Low Income	1.116	0.264	1.164	0.244	0.915	0.178	0.849	0.146
Sub-Saharan Africa	0.994	0.187	1.294	0.234	0.936	0.188	0.572	0.121
World	0.596	0.112	0.845	0.163	0.999	0.213	1.052	0.256

Note: SE denotes standard errors.

Table 3.5: Long-memory estimates of Aggregate Population Growth: Developed Countries (Maddison data): Sample: 1870-2003

MLPR estimates of d		
Countries	d(0.7)[31]	Standard Error
Australia	0.581	0.14
Austria	0.453	0.101
Belgium	0.721	0.156
Canada	0.791	0.153
Denmark	1.036	0.11
Finland	0.632	0.118
France	0.605	0.152
Germany	0.299	0.124
Greece	0.099	0.113
Ireland	-0.03	0.051
Italy	0.508	0.087
Netherlands	0.585	0.139
NewZealand	0.735	0.175
Norway	0.85	0.134
Portugal	0.539	0.081
Spain	0.246	0.112
Sweden	0.621	0.158
Switzerland	0.646	0.175
UK	0.262	0.123
USA	1.046	0.116

[31] $d(0{\cdot}7)$ is the bandwidth of d.

Table 3.6: Estimates of memory parameter: Modified LPR method. For Aggregate Population: Sample is from 1950-2003. For Age-structured population the sample is from 1960-2003

Country	Code	Total Pop	Total Pop	Age 0-14	Age 0-14	Age 15-64	Age 15-64	Age 65+	Age 65 +
		d(0.7)[32]	SE	d(0.7)	SE	d(0.7)	SE	d(0.7)	SE
Australia	D	1.142	0.243	1.089	0.322	1.021	0.497	1.157	0.170
Austria	D	0.741	0.197	1.210	0.337	1.047	0.163	1.096	0.186
Bahrain	D	0.801	0.084	0.557	0.27	1.049	0.226	-0.312	0.157
Belgium	D	0.999	0.172	1.164	0.324	1.422	0.341	1.066	0.238
Canada	D	1.303	0.198	1.041	0.249	1.318	0.314	1.270	0.156
Denmark	D	1.121	0.177	1.018	0.204	0.842	0.124	1.191	0.193
Finland	D	0.852	0.126	1.097	0.403	0.888	0.193	1.469	0.295
France	D	0.757	0.236	1.014	0.173	1.176	0.199	0.938	0.226
Germany	D	0.962	0.133	1.277	0.418	1.034	0.17	1.043	0.125
Greece	D	0.875	0.208	1.059	0.187	0.981	0.153	1.093	0.251
Hong Kong, China	D	0.334	0.098	0.746	0.176	0.933	0.213	0.932	0.149
Ireland	D	0.919	0.143	1.266	0.319	0.924	0.252	0.873	0.199
Israel	D	0.279	0.111	1.147	0.25	1.383	0.425	1.085	0.150
Italy	D	0.705	0.240	1.090	0.143	0.885	0.164	0.852	0.271
Japan	D	0.953	0.101	1.345	0.266	0.822	0.084	0.988	0.098
Korea, Rep.	D	1.605	0.235	1.025	0.133	1.199	0.217	0.617	0.205
Kuwait	D	-0.130	0.127	1.216	0.183	1.020	0.361	0.000	0.263
Netherlands	D	0.988	0.252	1.043	0.317	1.141	0.131	1.385	0.203
New Zealand	D	1.119	0.227	1.179	0.414	1.168	0.303	1.301	0.262
Norway	D	1.126	0.154	1.384	0.377	1.120	0.107	1.243	0.214
Portugal	D	0.479	0.156	1.300	0.381	1.115	0.204	1.059	0.425
Puerto Rico	D	0.401	0.313	1.083	0.212	1.140	0.297	1.103	0.233
Qatar	D	1.007	0.193	0.927	0.184	1.189	0.267	-0.345	0.210
Singapore	D	0.942	0.278	1.015	0.271	1.096	0.14	0.946	0.194
Slovenia	D	0.372	0.273	0.949	0.244	1.284	0.209	0.987	0.232
Spain	D	1.209	0.128	1.164	0.274	1.035	0.178	0.766	0.217
Sweden	D	0.974	0.246	1.062	0.218	1.103	0.196	1.107	0.202
Switzerland	D	1.062	0.335	1.260	0.307	1.179	0.316	1.159	0.258
United Arab Emirates	D	1.465	0.221	1.084	0.237	1.186	0.156	0.497	0.401
United Kingdom	D	0.895	0.315	1.080	0.434	0.992	0.229	1.034	0.337
United States	D	1.192	0.116	1.275	0.237	1.413	0.354	0.760	0.194
Afghanistan	LDC	0.614	0.188	0.864	0.146	0.927	0.255	0.876	0.151
Albania	LDC	0.937	0.232	0.724	0.129	1.305	0.206	0.842	0.268
Algeria	LCD	0.702	0.202	1.001	0.118	0.714	0.185	0.983	0.249
Angola	LCD	-0.043	0.171	0.692	0.127	0.350	0.232	0.598	0.164
Argentina	LCD	1.152	0.103	0.934	0.224	1.075	0.113	1.173	0.120
Armenia	LCD	0.524	0.148	0.277	0.154	1.188	0.21	0.760	0.228
Azerbaijan	LCD	0.889	0.136	0.749	0.171	1.163	0.209	0.885	0.180
Continued....									

[32] $d(0{\cdot}7)$ is the bandwidth of d.

Country	Code	Total Pop	SE	Age 0-14	SE	Age 15-64	SE	Age 65+	SE
Continued ...									
Bangladesh	LCD	0.854	0.148	0.849	0.07	0.710	0.213	1.066	0.175
Belarus	LCD	0.759	0.154	0.602	0.132	1.196	0.328	0.883	0.242
Benin	LCD	0.717	0.042	0.993	0.188	0.667	0.204	1.057	0.261
Bolivia	LCD	0.877	0.199	1.229	0.227	0.856	0.064	0.590	0.173
Bosnia and Herzegovina	LCD	0.882	0.177	0.766	0.392	1.133	0.434	0.834	0.244
Botswana	LCD	1.513	0.150	0.886	0.117	1.156	0.163	0.583	0.178
Brazil	LCD	1.226	0.060	1.019	0.111	1.201	0.101	1.269	0.195
Bulgaria	LCD	1.105	0.138	0.638	0.162	0.947	0.205	1.035	0.198
Burkina Faso	LCD	0.502	0.100	1.099	0.11	0.469	0.223	1.060	0.156
Burundi	LCD	0.556	0.209	1.173	0.185	0.776	0.181	0.833	0.295
Cambodia	LCD	1.067	0.197	0.943	0.151	1.005	0.194	0.892	0.258
Cameroon	LCD	0.888	0.146	1.183	0.165	0.727	0.15	0.990	0.122
Cape Verde	LCD	1.246	0.139	0.838	0.229	0.721	0.18	0.325	0.215
Central African Republic	LCD	0.856	0.262	1.304	0.209	0.786	0.183	0.759	0.144
Chad	LCD	0.091	0.160	0.532	0.219	0.507	0.109	0.648	0.172
Chile	LCD	1.084	0.084	0.886	0.156	1.163	0.124	1.110	0.139
China	LCD	0.256	0.349	0.905	0.355	1.007	0.161	0.971	0.279
Colombia	LCD	1.242	0.063	1.026	0.129	1.169	0.092	1.181	0.138
Congo, Dem. Rep.	LCD	1.299	0.378	0.877	0.234	0.930	0.202	1.060	0.144
Costa Rica	LCD	1.080	0.144	0.500	0.233	1.188	0.117	0.601	0.190
Cote d'Ivoire	LCD	1.106	0.190	1.186	0.117	1.075	0.073	1.026	0.166
Croatia	LCD	0.453	0.201	1.051	0.192	1.181	0.322	1.129	0.338
Cuba	LCD	1.050	0.188	1.109	0.359	1.252	0.169	1.029	0.194
Czech Republic	LCD	1.028	0.273	0.977	0.149	0.937	0.376	1.067	0.244
Djibouti	LCD	0.622	0.192	1.362	0.383	1.191	0.48	-0.089	0.226
Dominican Republic	LCD	1.287	0.043	0.964	0.076	1.230	0.199	0.702	0.187
Ecuador	LCD	1.159	0.057	1.076	0.062	1.021	0.123	0.856	0.293
Egypt, Arab Rep.	LCD	0.952	0.149	0.925	0.111	0.826	0.166	1.177	0.145
El Salvador	LCD	0.879	0.187	1.364	0.234	1.079	0.185	0.677	0.190
Equatorial Guinea	LCD	0.708	0.265	1.027	0.287	0.915	0.291	0.380	0.182
Eritrea	LCD	0.623	0.259	0.991	0.307	0.984	0.06	0.233	0.258
Estonia	LCD	0.962	0.255	0.632	0.22	1.139	0.184	1.159	0.221
Gabon	LCD	0.707	0.192	0.938	0.196	0.886	0.144	0.497	0.167
Gambia, The	LCD	0.912	0.058	1.323	0.168	0.808	0.172	0.203	0.219
Georgia	LCD	1.030	0.244	0.281	0.122	0.905	0.188	0.450	0.273
Ghana	LCD	0.493	0.252	0.461	0.162	0.840	0.139	0.780	0.198
Guatemala	LCD	0.988	0.086	1.248	0.146	0.891	0.071	0.575	0.229
Guinea	LCD	0.430	0.206	0.682	0.286	0.526	0.286	0.494	0.275
Guinea-Bissau	LCD	0.361	0.172	0.674	0.254	0.894	0.265	0.490	0.257
Haiti	LCD	1.089	0.188	1.120	0.202	0.645	0.107	0.692	0.238
Honduras	LCD	1.192	0.190	1.247	0.133	0.957	0.095	0.572	0.171
Hungary	LCD	0.599	0.171	0.963	0.175	1.087	0.323	1.121	0.198
India	LCD	1.048	0.116	1.076	0.099	1.065	0.075	1.215	0.253
Indonesia	LCD	1.106	0.119	1.154	0.094	1.062	0.116	0.631	0.296
Continued…									

Country	Code	Total Pop	SE	Age 0-14	SE	Age 15-64	SE	Age 65+	SE
Continued ...									
Iran, Islamic Rep.	LCD	1.075	0.218	0.901	0.227	0.927	0.18	0.871	0.228
Iraq	LCD	0.338	0.173	1.099	0.171	0.939	0.104	0.878	0.222
Jamaica	LCD	0.938	0.181	1.020	0.374	0.855	0.361	1.059	0.252
Jordan	LCD	0.547	0.170	1.318	0.227	0.818	0.276	0.699	0.211
Kazakhstan	LCD	0.709	0.216	0.411	0.173	1.263	0.237	0.915	0.202
Kenya	LCD	1.276	0.127	1.223	0.136	0.959	0.065	1.042	0.235
Korea, Dem. Rep.	LCD	1.305	0.220	1.255	0.246	1.112	0.186	0.426	0.215
Kyrgyz Republic	LCD	1.376	0.199	0.513	0.144	0.856	0.29	0.930	0.275
Lao PDR	LCD	0.828	0.091	0.920	0.249	0.742	0.26	1.205	0.513
Latvia	LCD	1.138	0.183	0.599	0.144	1.175	0.202	1.067	0.181
Lebanon	LCD	1.173	0.163	0.902	0.133	1.003	0.185	0.432	0.276
Lesotho	LCD	1.370	0.064	0.747	0.201	1.136	0.157	0.418	0.269
Liberia	LCD	0.196	0.165	1.154	0.204	1.087	0.206	0.900	0.334
Libya	LCD	0.838	0.197	1.308	0.263	1.127	0.117	0.449	0.186
Lithuania	LCD	0.766	0.175	0.366	0.126	1.162	0.228	1.161	0.234
Madagascar	LCD	0.603	0.050	1.143	0.294	0.709	0.138	0.751	0.129
Malawi	LCD	0.840	0.207	1.098	0.218	0.944	0.205	0.843	0.170
Malaysia	LCD	1.158	0.058	0.871	0.235	0.984	0.158	0.807	0.175
Mali	LCD	0.701	0.168	0.925	0.075	0.707	0.169	1.015	0.236
Mauritania	LCD	0.267	0.238	0.960	0.193	0.715	0.09	-0.316	0.283
Mauritius	LCD	1.131	0.118	1.148	0.173	1.410	0.202	0.378	0.275
Mexico	LCD	1.201	0.052	0.854	0.115	1.047	0.182	1.137	0.319
Moldova	LCD	0.770	0.213	0.521	0.126	1.202	0.375	0.307	0.239
Mongolia	LCD	1.157	0.242	1.090	0.204	0.878	0.192	0.820	0.270
Morocco	LCD	1.067	0.088	0.997	0.15	0.824	0.208	1.247	0.349
Mozambique	LCD	0.624	0.180	0.923	0.263	0.736	0.331	0.772	0.183
Myanmar	LCD	1.082	0.154	0.924	0.111	1.078	0.044	0.942	0.084
Namibia	LCD	0.536	0.143	1.054	0.157	0.822	0.198	-0.121	0.307
Nepal	LCD	0.583	0.078	1.201	0.179	0.681	0.062	0.552	0.144
Nicaragua	LCD	0.989	0.280	1.119	0.162	0.872	0.089	0.424	0.208
Niger	LCD	0.893	0.063	1.053	0.104	0.786	0.081	0.604	0.383
Nigeria	LCD	0.877	0.150	1.229	0.161	1.024	0.154	0.869	0.131
Oman	LCD	0.943	0.136	0.873	0.205	0.712	0.251	0.031	0.289
Pakistan	LCD	0.956	0.185	1.209	0.205	0.839	0.167	0.672	0.169
Panama	LCD	1.128	0.126	0.996	0.099	1.123	0.086	0.745	0.202
Paraguay	LCD	1.004	0.050	1.237	0.236	0.826	0.304	1.335	0.387
Peru	LCD	1.189	0.048	0.989	0.031	1.107	0.073	1.255	0.304
Philippines	LCD	1.166	0.060	1.006	0.08	1.039	0.031	1.041	0.260
Poland	LCD	1.1 IS	0.095	0.891	0.153	1.254	0.231	1.397	0.282
Romania	LCD	0.942	0.146	0.674	0.381	1.106	0.209	1.205	0.238
Russian Federation -	LCD	0.961	0.106	0.729	0.133	0.818	0.147	1.133	0.295
Rwanda	LCD	-0.132	0.089	0.762	0.251	0.952	0.187	0.514	0.262
Senegal	LCD	0.939	0.140	1.170	0.165	0.869	0.117	0.866	0.316
Serbia and Montenegro	LCD	0.378	0.444	0.955	0.137	1.169	0.326	0.964	0.233
Sierra Leone	LCD	0.137	0.181	0.813	0.224	0.551	0.195	0.759	0.124
Continued…									

Country	Code	Total Pop	SE	Age 0-14	SE	Age 15-64	SE	Age 65+	SE
Continued ...									
Slovak Republic	LCD	0.443	0.171	0.796	0.176	1.183	0.205	1.059	0.309
Somalia	LCD	0.550	0.111	1.010	0.241	0.966	0.227	1.017	0.217
South Africa	LCD	1.102	0.073	1.066	0.061	0.981	0.261	0.832	0.138
Sri Lanka	LCD	1.223	0.114	1.107	0.163	1.428	0.228	0.768	0.301
Sudan	LCD	0.771	0.171	1.091	0.192	0.908	0.114	0.586	0.201
Swaziland	LCD	0.916	0.238	0.671	0.13	1.174	0.227	0.436	0.181
Syrian Arab Republic	LCD	0.997	0.088	0.948	0.109	0.718	0.388	1.002	0.258
Tajikistan	LCD	0.986	0.174	0.446	0.149	0.783	0.282	1.023	0.288
Tanzania	LCD	0.970	0.206	1.298	0.189	1.001	0.103	0.872	0.183
Thailand	LCD	1.257	0.046	1.123	0.21	1.349	0.206	0.848	0.256
Togo	LCD	0.228	0.159	0.896	0.21	0.749	0.193	0.898	0.227
Trinidad and Tobago	LCD	0.867	0.245	0.654	0.226	1.117	0.197	0.581	0.160
Tunisia	LCD	0.687	0.202	0.778	0.117	0.844	0.193	1.061	0.322
Turkey	LCD	1.242	0.088	1.413	0.347	1.028	0.137	1.238	0.259
Turkmenistan	LCD	1.344	0.222	0.590	0.154	0.932	0.276	0.667	0.263
Uganda	LCD	1.039	0.182	1.127	0.111	1.124	0.133	1.142	0.250
Ukraine	LCD	0.865	0.149	0.586	0.152	0.728	0.103	1.185	0.298
Uruguay	LCD	1.153	0.176	0.910	0.15	0.900	0.224	0.951	0.195
Uzbekistan	LCD	1.092	0.184	0.452	0.139	0.806	0.139	1.039	0.174
Venezuela, RB	LCD	1.267	0.083	1.018	0.093	1.184	0.139	1.096	0.110
Vietnam	LCD	1.366	0.221	0.579	0.115	0.760	0.231	0.832	0.169
Yemen, Rep.	LCD	0.577	0.113	1.152	0.246	0.890	0.354	1.114	0.397
Zambia	LCD	1.155	0.159	1.214	0.153	1.067	0.124	0.922	0.083
Zimbabwe	LCD	1.006	0.201	0.708	0.087	1.273	0.167	0.738	0.153

Note: SE denotes standard errors.

Table 3.7: Cross-section regression of long-memory demography effect on economic growth (Sample 1960-2003): Developed Countries

Depdent Var:	grgdp40	
Variables	Coef.	t-stat
d-totpop	3.425	2.190
ln(eO)	0.568	4.850
Density	6.226	1.650
AvgSchooling	1.847	1.690
Constant	-0.435	-0.140
N = 23	$F(3, 19) = 16.11$**	$R^2 = 0.50$
Dependent Var:	d-totpop	
Variables	Coef.	t-stat
grgdp40	0.048	1.530
ln(eO)	-0.045	-2.200
Density	-1.453	-2.770
AvgSchooling	0.053	0.290
Constant	1.080	4.660
N = 23	$F(4, 18) = 2.71$**	$R^2 = 0.30$

Note: (i) Regression with robust standard errors, (ii) **: Significance at 5 percent level.

Table 3.8: Cross-section regression of long-memory demography effect on economic growth (Sample 1960-2003): Developing Countries

Depdent Var:	grgdp40	
Variables	Coef.	t-stat
d-totpop	-1.007	-1.65
ln(eO)	0.003	0.18
AvgSchooling	-0.035	-1.05
Density	-1.211	-1.86
Constant	4.425	2.82
N = 44	$F(4, 39) = 1.59$	$R^2 = 0.18$
Dependent Var:	d-totpop	
Variables	Coef.	t-stat
grgdp40	-0.078	-2.08
ln(eO)	0.004	0.48
AvgSchooling	-0.006	-0.28
Density	-0.076	-0.47
Constant	0.950	2.99
N = 44	$F(4, 39) = 1.19$	$R^2 = 0.09$

Note: (i) Regression with robust standard errors, (ii) **: Significance at 5 percent level.

Figure 3.2: Kernel density plots of long-memory estimates of aggregate population and age shares (All countries)

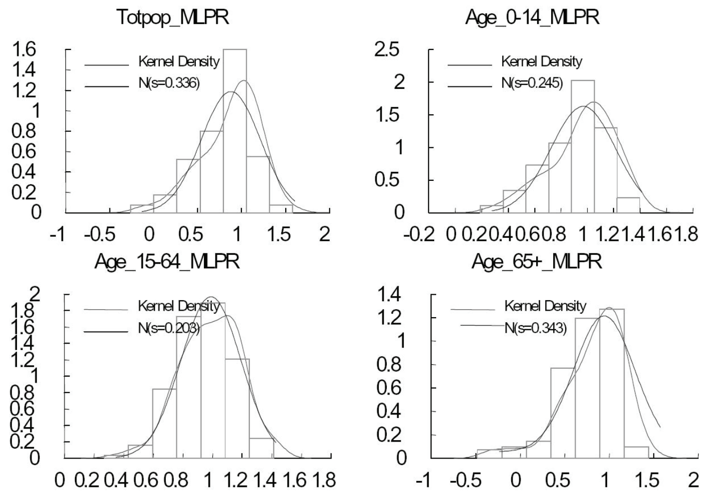


Figure 3.3: Kernel density plots of long-memory estimates of aggregate population and age shares (Developed countries)

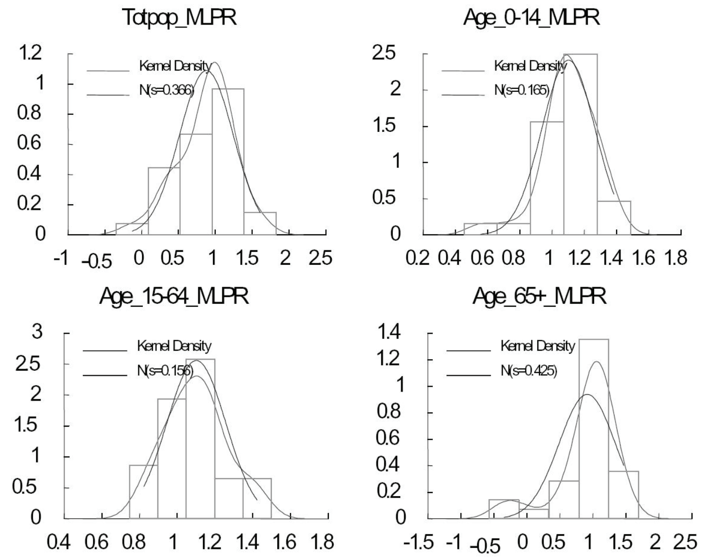
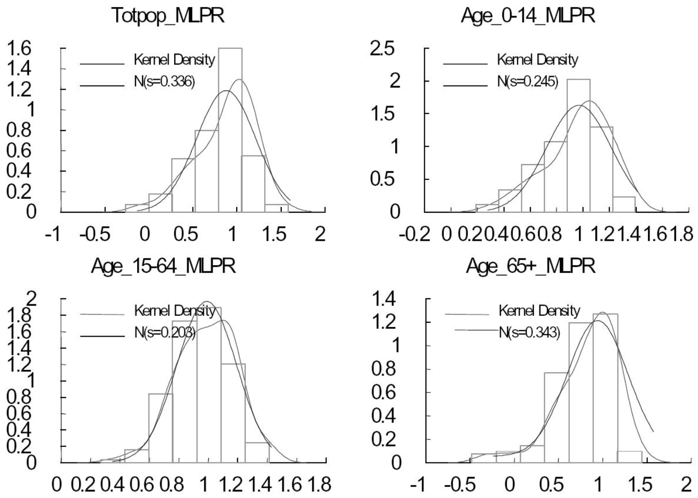


Figure 3.4: Kernel density plots of long-memory estimates of aggregate population and age shares (Developing countries)



4. Population Forecasting with Stochastic Long Memory Framework

4.1 Introduction: Problems in demographic forecasting

For about two centuries, population forecasting has been a national agenda of many countries who follow this historical practice to account for present and future economic contingencies. The preoccupation has recently got momentum as both the methodological tools and the perceived relation between demography and economic growth have undergone paradigmatic changes. On the methodological side of demographic forecasting, notable progress has been made which are mostly 'probabilistic' (e.g., using expert opinions and using random scenarios) in nature. Temporal approaches were infrequently suggested although demographic variables, like total population can be apparently described by a data generation process which owns temporal features. The lack of rigorous enforcement of time series methods in population forecasting is understandable as the development of these methods even in the mainstream economics experienced major changes only after 1980s[33].

[33] Particularly after Nelson and Plosser' (1982) finding that most of the macroeconomic variables contain a stochastic trend, implying the interplay of stochastic exogenous shocks in the system which must be treated before any meaningful economic conclusions can be drawn.

Two possible causes could explain the long-sought seriousness in demographic forecasting. *First,* there is a recent surge of interest in the study of economic-demographic relation in endogenous economic growth setting.[34] The rise or fall in the 'number' of population can have direct impact on economic growth as the change in the population number either accentuates growth via resource creation or retards it via resource destruction. For designing long-term socio-economic policies related to distribution and allocation of resources it is vital to know the 'number of people (young, working age population, or old) in future' so that precautionary measures can be undertaken to preserve sufficient resources for future younger cohorts and provide necessary social security benefits to the older ones. After all, at least a 'best guess' of the future total population or particularly forecasts of population of different age-structures are mandatory to plan for future economic growth and at the same time to keep up with the momentum of current economic growth with proper intergenerational transfer of resources. Though this explanation appears illustrative of the coveted demographic-economic growth relation, it casts light on a neglected dimension, viz., the stationary assumption of population growth. Recent literature – which are quite diversified in nature – have attempted (as will be explained shortly) to model uncertainties or stochasticities in the growth of population and various age-specific population growth rates.

Second, population forecasting methods have undergone rapid changes in the past few years, mainly due to the initiatives of some experts who challenged the traditional approach (for instance, high, low, and medium variant of forecast, or even probabilistic projections). It is no wonder that for many years, population forecasters have worked largely in isolation from other forecasters. Ahlburg and Land (1992) argue that this separation is due in part, to the uniqueness of the demographic subject matter, and, in part, due to the typical approach of the population forecasters (e.g., 'cohort component approach'). The latter is ideally suited for dealing with the unique properties but not for most other forecasting tasks. Consequently, for the last six decades, population forecasters have attempted to put their efforts in a different domain than that of other forecasters. Yet, a significant change has been brought in the population forecasting methodologies in the last decade mainly due to the *adoption* and fast *adaptation* of new methodological techniques, viz., stochastic process models, structural equation models, and due to the recent most-widely used time series techniques. We call the first source of motivation as **growth-induced-motivation,** and the latter as **methodology-induced-motivation** for population forecasting.

Recently, Pflaumer (1992) and Lee and Tuljapurkar (1994) employed time series based method, viz., autoregressive moving average (in short, ARMA)

[34] See for instance, Boucekkine et al. 2002 for an excellent theorization of the relation.

technique to forecast population series and mortality for United States. The early attempts by these authors for applying time series methods to demographic forecasting certainly draws appreciation. However, mostly convulsed with the apprehension that time series models might not be useful for demographic forecasting and believing that demographic processes are different from say, macroeconomic processes, the application and wide spread use of these methods for demographic variables so far have not been astounding. The possible reason could be that although temporal methods have been extensively used in the context of macroeconomic and financial econometric domain, their properties and implications for demographic processes have not been fully investigated so far, except some inquisitive researchers like Pflaumer (1992) and Lee and Tuljapurkar (1994). The authors utilize Box-Jenkins' autoregressive integrated moving average method (ARMA) to show how the exploitation of time series properties of the demographic processes can eke out the intrinsic characteristics of stochasticitic shock of the demographic system. The ARMA approach, while proved to be a good alternative to probabilistic forecasting method in demography, its inherent weakness against distinguishing between unit root nonstationarity and 'gradual' nonstationarity (i.e., which is between stationarity and unit root) is widely studied and criticized in the literature. A more flexible formulation is therefore suggested where the degree of integration, unlike ARMA process, can assume the whole range of values on the real line that evidently includes zero and one, the typical stationary and non-stationary ARMA processes. The flexible formulation is called fractionally integrated ARMA, or ARFIMA process.

We employ this method for forecasting total and age-structured population and extend the earlier research in two directions. From methodological perspective the study of stochastic shocks in a historical time series is better understood in an ARFIMA framework because in this case we are able to characterize the evolution of shocks better with different magnitudes of persistence in contradistinction to the knife-edge assumption of low or high persistence as in ARMA. In practice, it is more possible that demographic shocks can be persistent but over time it would recede to come back to the equilibrium path than to assume strictly that it is either stationary with no persistence or non-stationary with persistence where the series will drift forever and would not be back to equilibrium. Many 'gradual nonstationary' features might characterize the demographic process as well. However, the use of Box-Jenkins ARMA method would little reflect on the detailed dynamics in the sense stated above.

Indeed, even a smaller amount of stochastic shock can make the long-term demographic projections volatile, enforcing the process to be contingent upon some degrees of stochasticities of the system itself along with the fact that the evolution of other components of the system also counts. For instance, the forecast of total population will certainly depend upon the growth of its components, and intrinsic behavior of birth and death processes at a point of time.

Using ARFIMA method, we model total population which can be governed by the growth of its components and the stochastic shocks which can be generated as an outcome of the interaction mechanism with the economy (endogenous) or as a result of the influence of external environment (exogenous). Using thus ARFIMA framework we forecast total population for a set of developed and developing countries (Pflaumer, 1992 and Lee and Tuljapurkar, 1994 experimented with United States only using ARMA method).

These motivations clearly explain the seriousness of the population forecasters to aptly combine mainstream forecasting techniques with the typical idiosyncrasies of the population series.[35] Conventional population projections are based on a set of assumptions that are only occasionally stated explicitly. Projections assume there will be *(i)* no deep structural changes like catastrophic events, *(ii)* no feedback effect in the sense that vital rates vary independently of the distribution of the population across the categories to which they apply, and *(iii)* neutrality of public policy response and environmental pressure.[36] However, these assumptions appear to be too stringent because for instance in practice structural shifts might occur to the population series and that feedback effect is a natural consequence of the interaction mechanism of population and economy. In light of these considerations, this chapter attempts to go beyond the conventional methodologies by suggesting a time series based forecasting method, viz., ARFIMA, which encompasses a broad range of memory structure with interesting short-run and long-run dynamics. Feedback mechanism works in our model as it is in-built and given by the data generating process. In section 4.2 we summarise the main contribution and flaws of conventional population forecasting methods and discuss advantages of ARFIMA model. Section 4.3 discusses the ARF1MA model with and without regime switching. Section 4.4 summarizes the data characteristics and thoroughly discusses the empirical findings. Conclusion and implications of the results are presented in Section 4.5.

4.2 Revisiting literature

Two main approaches have been put forth in the population forecasting literature. (1) The conventional high, low, medium variant and probabilistic projections approach, (2) and the very recent time series based methods. The spe-

[35] The data generating mechanism of population series differs widely from many economic variables due to the fact that the dynamics involved in the movement in this series is more complex than many economic variables, e.g., inflation, industrial production, output, etc.

[36] Though some studies state that environment will bite back due to unsustainable high world population or due to continuous growth of fertility, there will be a public policy-response in the form of powerful pronatalist policies, econometric and demographic studies suggest, however, that the ability of governments to affect fertility in industrial nations is quite weak.

cial issues of *International Statistical Review, 2004* discusses the usefulness and drawbacks of probabilistic projection methods and the use of expert-opinions in population forecasting at length. The basic idea of these approaches is to model uncertain future where the degree of uncertainty critically depends on the functioning of the demo-economic systems. The population forecasting techniques should therefore correctly deal with minimizing the amount of uncertainty. The conventional way to deal with uncertainty in demographic forecasts is to employ *high, medium and low scenarios.* This approach suffers from the flaw that it is based on very strong and implausible assumptions about the correlation of forecast errors over time, and between fertility and mortality (Lee and Tuljapurkar (LT), 1994). The *random scenario* method is an improvement, but it retains some of the same flaws.

The probability-based forecasting method, considered by many as an improvement over population scenarios, do not take uncertainty into account. Probabilistic population forecasts differ from deterministic forecasts in that they quantify the uncertainty of the course of future rates and therefore must specify future total fertility rates, life expectancies, and net migration rates as *distributions* and not as *points.* Probability analysis itself is uncertain to some extent. Some critics are of the opinion that no mathematical model can accurately measure the socioeconomic factors that affect population growth. Fertility decline, for example, which is the major factor in a slowdown of population growth, can happen in a variety of settings for unpredictable reasons. For instance, Eastern Europe always had a higher fertility rate than Western Europe, but when the former Soviet Union broke up, fertility rates in those former Soviet countries plummeted unexpectedly. The actual outcome of world population growth will depend on how people's social behavior changes. This is really very difficult to predict using any kind of mathematical method. Drawing on the recent importance of the effect of age structure on economic growth, Prskawetz et al. (2005), for instance derive the uncertainty of predicted economic growth rates using probabilistic demographic forecasts.

A complementary method recently proposed in population forecasting is to employ time series based technique, viz., autoregressive integrated moving average (ARIMA). The idea of the time series based approach is to incorporate the dynamic information of demographic variables accumulated over time and forecast growth and/or level of population based on stability or stationarity of the demographic system. It may be noted that time series based forecasting methods take into account the endogeneity of the demographic and economic system, incorporate the dynamics that arise due to the interaction of the population and the aggregate economy. Following this framework, importance is laid on the magnitude and pace of any endogenous and /or exogenous shocks in the system. Lee (1992) provides particular emphasis on developing time series models for the vital rates which are suited for long-run forecasting, with horizons of 75 years or so. LT (1994) further developed methods based on stochastic Leslie matrix in which the vital rates are modelled as stochastic time series, viz., ARMA. The use of ARMA method by LT shows high degree of consistency with the latest Census Bureau forecasts and bounds.

Cohen (1986) and Pflaumer (1992) use ARMA method to forecast total population. In particular, employing Box-Jenkins ARMA method Pflaumer (1992) forecast United States total population till 2080 and showed that Box-Jenkins approach is equivalent to a simple trend model when making long-range predictions for the United States. Further investigation of the forecasting accuracy by the author reveals that the Box-Jenkins method produces population forecasts that are at least as reliable as those done with more traditional demographic methods. Moreover, while ARMA approach has been found as efficient as other complex demographic models, the former is simple to use and can incorporate short-run and long-run demographic dynamics.

The underlying idea in the ARMA approach is that the data generating process (DGP) of population growth is characterized by both endogenous nature (i.e., past population growth affecting the current population – the autoregressive or AR structure) and some unforeseen shocks (i.e., the moving average or MA structure). If population growth is non-stationary, the series is first differenced following conventional wisdom. In this case the order of integration is 1. A zero integration order would indicate that the population series is stationary. Alternatively speaking, it is said to possess short-memory shocks. The nonstationary case is also referred to as long-memory, where a unit shock in the series leaves a permanent effect on the historical growth path. However, the in-between case, the fractional integration method exists, which relaxes the integer order of restriction to non-integer or fractional values so that the data generating process of the level population or its growth can be characterized by a fractional ARMA process. By doing so, we are able to investigate different short-run and long-run effects of shocks in the population series. In Chapter 3, we have elaborated on the concept and implications of long-memory population growth.

To provide a quick note, we explicated in Chapter 3 that the fractional structure of population growth gives rise to an important class of model, viz., long-memory. This implies that a shock to the population series continues to remain and that it is characterized by very high persistence where the series may not return to the mean level in the long-run. This is called non-stationary long-memory. Another possibility is that the shock will continue to affect the series for some time in future, will have slower decay but ultimately converges to the mean level in the long-run. This property of the population series can be characterized by stationary long-memory. Depending on the length and magnitude of memory, policy measures are adopted to stabilize the series. Notably, this dynamic characteristic of a time series provides rich stock of information about the future evolution of the series, help in identifying clear dynamics and provide a better forecast accuracy.

As will be explained in the following section, the conventional ARIMA model is a special case of the more general fractional ARMA class. Forecasting models which possesses the ability to incorporate rich dynamic information on the growth of the variables, for instance, short-memory and longmemory behavior, better approximate real demographic situations and are therefore able to deliver better forecast. In this chapter, we have employed fractional ARMA model for forecasting with and without consideration of stochastic regime switch. In the presence of stochastic regime switch, our objective is to study the possible impact of endogenous demographic shifts on forecasting performance.

4.3 The Model

The fact that population forecasting is different from other kinds of forecasting, that it should warrant its own special methods, and its own special discussion, is true, in particular, when we try to do long term demographic forecasts many decades into the future, which is typically the case of this chapter. The uniqueness of the population forecasting method lies in the usefulness of the stock of information that demographic system readily have. For instance, (1) The initial age distribution of the population provides early information about future population size, age distribution, and growth rates, e.g., since their birth, we have known exactly when the baby boom generations would swell the numbers of elderly. (2) The relative slowness, smoothness and regularity of change in fertility and mortality facilitate long term forecasts. Compared to real productivity growth or to real interest rates, for example, the vital rates are less volatile. (3) Fertility, mortality and nuptiality have highly distinctive age patterns which have persisted over the several centuries for which they have been observed. These regular and distinctive age patterns reinforce the preceding two points, by making the consequences of initial age distributional irregularities more predictable. Demographers have developed methods and models for exploiting these features of population evolution in their projections. This does not mean, of course, that demographers have built a sterling record of success in long term forecasting. Their record, nonetheless, has been a mixture of success and failure (Lee and Tuljapurkar, 1998).

This section presents a model that ideally takes into account the turning points, regular changes in the series and more so endogenous shifts which might make the population series unstable or chaotic. The chaotic nature of population has been thoroughly investigated in the literature, see for instance Prskawetz and Feichtinger (1995) and Day (1993). Following these authors, contrary to the standard assumption of stationarity of population growth, the series can in fact display chaotic pattern because of the presence of either high non-linearity between population and the economy and/or due to the presence of multiple endogenous shifts in the demographic system which is often overlooked in the empirical demographic literature. Therefore population series needs to be modelled in an econometric framework which accommodates these intuitions and based on them projections need be made.

As said before, we would employ ARFIMA framework for long-term population projections as this method can accommodate many complex features of stochastic shock behavior, viz., endogenous nature (by AR process), exogenous shocks (by MA process) and stochastic memory (the integration parameter) – all of them can form a non-linear structure as well. A description of the long-memory process and the properties of shock persistence have been discussed in Chapter 3. For motivating our analysis here we would briefly touch upon the concept and explain the forecasting properties of the ARFIMA process.

Consider the following model for y_t :

$$\Phi(L)y_t = \Theta(L)\epsilon_t; \epsilon_t \sim (0, \sigma^2); t = 1, ..., T. \tag{4.1}$$

$\Phi(L)$ and $\Theta(L)$ are autoregressive (AR) and moving average (MA) polynomials of order p and q respectively. $\Phi(z) = (1 - \phi_1 z - ... - \phi_p z)$ and $\Theta(z) = (1 + \theta_1 z + ... + \theta_q z)$ are viewed as functions of a complex number z and have no zero in the unit circle, $|z| \leq 1$. This is a stationary ARMA model. If y_t is non-stationary and $I(1)$, following *ARM A* framework, it needs to be (first) differenced to make it stationary so that invertibility of the model is assured. In this case,

$$\Phi(L)\Delta y_t = \Theta(L)\epsilon_t; \epsilon_t \sim (0, \sigma^2) \tag{4.2}$$

where $\Delta = (1 - L)$, L is the lag operator such that $(1 - L)y_t = y_t - y_{t-1}$. Note that in the typical *ARIMA* model Δ has the power $d = 1$, which is the standard unit root assumption and for *ARMA*, $d = 0$. This is the typical Box-Jenkins *ARMA(p, q)* model, which has been adopted by Lee and Tuljapurkar (1994) and Pflaumer (1992) for forecasting vital rates and total population for United States. Theoretical and empirical literature have extensively discussed about the limited dynamics of the integer order of restriction of d. Instead, a more powerful and flexible method in the form of fractional ARMA method has been proposed in the literature (e.g., Granger and Joyeux, 1980; Hosking, 1981). Beran (1994) discusses the main asymptotic results for regression models with long memory errors and Baillie and Bollerslev (1994) provide details on the models. Relaxing the integer order of integration for d (i.e., $d = 0$ or $d = 1$), if we allow it to lie on the real line, i.e., assume fractional values, then *ARIMA* model can be described as ARFIMA(p,d,q).

y_t can be described as *ARFIMA* process if

$$(1 - L)^d \Phi(L)(y_t - \mu_{01} - \mu_1 t) = \mu_{02} + \Theta(L)u_t. \tag{4.3}$$

where μ_{01} and μ_{02} are type 1 and type 2 intercepts. Note that if type 1 intercept is considered, then we are actually treating the series, y_t after eliminating the effect of an autonomous or constant factor from the model. The type 2 intercept enters as an explanatory variable in the model. At most one of the intercepts can be non-zero. Assuming that $\mu_1 t = 0$, i.e., there is no trend in the model[37], then with type-1 intercept y_t can be expressed as

$$y_t = \mu_{01} + (1-L)^{-d}\Phi(L)^{-1}\Theta(L)u_t. \tag{4.4}$$

and with type 2 intercept

$$y_t = (1-L)^{-d}\Phi(L)^{-1}\mu_{02} + (1-L)^{-d}\Phi(L)^{-1}\Theta(L)u_t. \tag{4.5}$$

Considering the expressions in 4.4 and 4.5, we observe that the main distinction between the two intercepts is that type 2 intercept will induce a trend of $O(t^d)$ because it is discounted by the memory parameter d as well as by the autoregressive polynomial $\Phi(L) = (1-\phi_1 L - \phi_2 L - ... - \phi_p L)$, whereas the type 1 intercept is simply a location shift of the fractionally integrated process.

Additionally, $u_t(t \geq 0)$ is assumed to be stationary with zero mean and continuous spectrum $f_u(\lambda)$. When y_t is first differenced μ_{01} disappears and the model is governed by trend term and type 2 intercept, μ_{02}. The latter is sometimes incorporated in the empirical analysis if y_t is believed to be influenced by some stochastic exogenous factors which exert constant effects on the historical trajectory of y_t. Demographic system possesses this peculiarity in that the dynamics of the demographic system is often led by both its endogenous nature and some external constraints. If the model is suspected to be governed by a trend term, then the trend must be extracted from y_t before estimation. However, if y_t is believed to be a non-stationary process, then it should be differenced till stationarity is achieved. A leading special case of the ARFIMA model above is without any intercept or trend term, so that eq.4.3 can be written as

$$(1-L)^d\Phi(L)y_t = \Theta(L)u_t. \tag{4.6}$$

Imposition of unit root in this model, that is the first difference of y_t would mean

[37] If the trend term is considered to be non-zero in the model, then this would of course superimpose a drift of $O(t)$. If there is a unit root, in effect the trend term replaces the type 1 intercept to locate the differenced sequence, whereas the type 2 intercept is asymptotically unidentified. In other words, $(1-L)^{1-d}\mu_{02} = 0$ when $(1-d) > 0$. In finite samples it would represent an initial negative trend diminishing to 0.

$$(1-L)^{d-1}\Phi(L)\Delta y_t = \Theta(L)u_t. \tag{4.7}$$

where $\Delta = (1-L)$, so that the estimated differencing parameter now becomes $d-1$. An attraction of fractional processes is that they allow more flexibility in the dynamic responses of economic variables to shocks than is permitted under the unit root model. Correspondingly, the slow decay of the effect of shocks allows for slow adjustments to equilibrium in models of cointegration and is a potential advantage of long memory error processes in the econometric modelling of long-run economic equilibrium. These favorable properties of flexibility and slow convergence to equilibrium levels make fractional integration an attractive model of time series behavior, that accommodates both unit root type persistence as well as long range dependence and mean reversion. In short, these processes give us a more general mechanism for studying the phenomena of co-movement among economic time series.

The prediction from ARFIMA processes is usually carried out by using an infinite *A II* representation. Writing $z_t = (y_t - \mu_{01})$ in 4.3 where $\mu_1 t = 0$ is assumed to be zero, then AR(∞) representation of z_t is defined as

$$z_t = \sum_{j=1}^{\infty} \pi_j z_{t-j} + u_t \tag{4.8}$$

In obvious notation:

$\Pi(L) = \sum_{j=0}^{\infty} \pi_j L_j = \Delta(L)\Phi(L)(1-L)^{d-1}$ and $\pi_0 = 1$.

Note that there is an *AR* unit root for $d > 0$. When pre-sample values, i.e., z_j for $j < 0$, are set to zero in forecasting, the corresponding predictions are called 'naïve' forecasts. These predictions are optimal if the observations are known into the infinite past. And the corresponding one-step-ahead forecast errors are labelled naive residuals, denoted by $\widetilde{e}_t$. Similarly the *MA* representation of z_t is:

$$z_t = \sum_{j=1}^{\infty} \psi_j u_t = \Psi(L)u_t = \Phi(L)^{-1}\Delta(L)^{-1}(1-L)^{1-d}u_t \tag{4.9}$$

has an MA-unit root when $-1 < d \le -0.5$. Note that $\psi_j \to 0$, when $j \to \infty$ for $d < 1$. The process is therefore mean-reverting in this case, and innovations u_t only have a transitory effect on the time- series process. Given these formalizations, the likelihood of the model is derived, which can be of three types, viz., exact maximum likelihood (EML), modified profile likelihood (MPL), and approximate likelihood based on nonlinear least squares (NLS). Doornik and Ooms (2004) provides a detailed discussion of the properties of these likelihood functions. EML is calculated from 4.3 based on the normality assumption and with a procedure to compute the autocovariances in the $T \times T$ covariance matrix

$$\sum = \sigma^2 I \text{ of a } T \times 1 \text{ vector of observations } y.$$

The aim of the MPL is to develop more accurate inference on parameters of interest in the presence of a large number of nuisance parameters. Both EML and MPL require $\sum$ and its inverse to exist and therefore require stationary *ARFIMA*-errors. For NLS, the approximate ML estimator is based on minimizing ARFIMA-processes with non-stationary ARFIMA errors. In general, the criterion function of the above model when u_t is standard Gaussian is the standard ML function.[38]

The best linear prediction of z_{T+H} given the information in z and knowing the parameters of the *ARFIMA* process, is given by

$$\hat{z}_{T+H|T} = (\gamma_{T-1+H} \dots \gamma_H)(\Sigma_T)^{-1} z = \acute{f}_H z \tag{4.10}$$

where γ is the autocovariance function and H is the step of forecast. Eq. 4.10 can be viewed as a regression of Z_{T+H} on z. Now denoting

$$F_{H|T} z$$

as the optimal forecast for

$$\acute{z}_{H|T} = (\acute{z}_{T+1|T}, \dots, \acute{z}_{T+H|T}),$$

$$\text{the } var(\acute{z}_{H|T} - z_H) = \Sigma_H - F_{H|T} \Sigma_T \acute{F}_{H|T}.$$

It is often of interest to forecast partial sums of z_t, e.g., when the log-population predictions are constructed as partial sums of population growth forecasts. A recursive prediction

$$z_{T+1}, \tilde{z}_{T+2}, \dots$$

using AR-representation up to order $(T, T+1, \dots)$ can be calculated. In that case, pre-sample values are set to zero and the predictions are optimal since the observations are known into the infinite past. The corresponding variances of $\tilde{z}_{T+H}$ are computed using the *MA* coefficients of Eq. 4.9:

$$var(\tilde{z}_{T+H}) = \sigma^2 \left(1 + \Sigma_{j=1}^{H-1} \psi_j^2\right) \tag{4.11}$$

[38] This is given by:
$L_T = -\frac{T}{2} log 2\pi - \frac{1}{2} \sum_{t=1}^{T} (u_t^2)$.

The results of empirical and Monte Carlo investigation carried by some authors (e.g., Bharadwaj and Swanson, 2006) establish the usefulness of *ARFIMA* models in practical prediction[39] based applications. Bharadwaj and Swanson (2006) present ex ante forecasting evidence based on an updated version of the absolute returns series as in Ding et al.(1993) who suggested ARFIMA models to be estimated using a variety of standard estimation procedures. The latter are shown to yield 'approximations' to the true unknown underlying DGPs that sometimes provide significantly better out-of-sample predictions than the wide class of non-ARFIMA models including AR, ARMA, ARIMA, random walk, Generalized Autoregressive Conditional Heteroscedasticity (GARCH), simple regime switching, and related models. They showed that very few models were better than ARFIMA models, based on the analysis of point mean square forecast errors (MSFEs) and the predictive accuracy tests of Diebold and Mariano (1995) and Clark and McCracken (2001). Bharadwaj and Swanson (2006) find strongest evidence in favor of ARFIMA models and show that these models frequently outperform linear alternatives around one third of the time. Moreover, the authors depict via discussion of a series of Monte Carlo experiments that ARFIMA models perform better for greater forecast horizons, while this is clearly not the case for non-ARFIMA models.

4.3.1 Markov Switching ARFIMA (MS-ARFIMA) model

Recently econometricians (e.g., Granger and Terasvirta, 1999; Diebold and Inoue, 2001; Gourieroux and Jasiak, 2001) have begun to consider the relationship between structural changes in time series and long-memory showing analytically and via Monte Carlo that models with regime changes may exhibit long-memory properties. One may question then the implications of these results for forecasting, for example posing: despite spurious long-memory effects due to regime shifts, will an ARFIMA specification still be an effective tool of forecasting? In this regard, Diebold and Inoue (2001) suggest that 'even if the *truth* is structural change, long-memory may be a very convenient shorthand description, which may remain very useful for tasks such as prediction.'

Gabriel and Martins (2004) investigate whether a long-memory approach will be robust to structural breaks in a time series, in terms of providing good forecasts for financial and macroeconomic data. The authors posit that considering the way predictions are constructed for long-memory models (i.e., taking into account the information of distant lags), one may anticipate that

[39] As is often the case, when the ultimate goal of an empirical investigation is the specification of predictive models, then a natural tool for testing for the presence of long memory-is the predictive accuracy test. In this case, if an ARFIMA model can be shown to yield predictions that are superior to those from a variety of alternative linear (and nonlinear) models, then one has direct evidence of long memory, at least in the sense that the long memory model is the best available 'approximation' to the true underlying DGP.

ARFIMA models would experience difficulties in forecasting future immediate regime changes, unless of course the switching is transitory. Moreover, they found that although long-memory models may capture some in-sample features of the data, when shifts occur in the series, their forecast performance is relatively poor when compared to MS models. However, this result is contingent upon some specific data considered and may change depending upon varying forecast settings, e.g., as in multivariate forecasting.

In our case, the population series of a number of countries are believed to be heavily affected by demographic shifts which are likely to possess both endogenous and exogenous features. Natural calamities, like the recent Tsunami can cause heavy demographic instability in the form of demographic shifts. This is exogenous in nature. And also sometimes, due to mounting population pressure, public policies respond in stringent way to get the population equilibrium down to stable growth. This is endogenous in nature. To address this possibility in our forecasting exercise, we estimate an MS-ARFIMA model which is typically represented by

$$(1-L)^d\Phi(L)(P_t-\mu_{01}(s_t))=\Theta(L)u_{s_t,t}; \quad u_t \sim NID(0,\sigma^2_{s_t}) \qquad (4.12)$$

where $\mu_{01}(s_t)$ is the switching unconditional mean. s_t is a binary random variable on $S = 1,2$, indicating the unobserved regime or state driving the process at date t. As it will be explained shortly s_t is a stationary first-order Markov chain in S. If the switch is expected to occur for the 'intercept' then it would mean that some unobserved structural changes are guiding the process P_t. If the switch occurs for d, then we would expect that some stochastic (persistent or non-persistent) shocks are accountable for the behavior of P_t. For instance, it might be possible that government intervention in population policy may induce changes in the growth pattern of population. One can identify this with a structural shift of the parameter in question. Structural shift can also occur if some natural calamities tell upon the growth of population. All these can be reflected in the persistent property of shocks, i.e., shifts in the value of d. To ensure stability in the system, one would assume that the switch from one state to the other is transitory in order to get a good forecast (See, Gabriel and Martins, 2004).

Generally the switching unconditional mean is first estimated and then y_t is demeaned and is made available for estimation. Switching may as well occur in the memory parameter, d, so that

$$(1-L)^{d(s_t)}\Phi(L)(P_t-\mu_{01})=\Theta(L)u_{s_t,t}; \quad u_{s_t,t} \sim NID(0,\sigma^2_{s_t}) \qquad (4.13)$$

The regime switch in fact can occur in both intercept and memory parameter, so that

$$(1-L)^{d(s_t)}\Phi(L)(P_t-\mu_{01}(s_t))=\Theta(L)u_{s_t,t};u_{s_t,t}\sim NID(0,\sigma^2_{s_t}) \quad (4.14)$$

We elucidate below the basic idea of Markov Switching model. Generally the strategy is to decompose a series in a finite sequence of distinct stochastic processes, or regimes. The current process in each regime is linear, but the combination of processes produces a nonlinear regime. Specifically, Markov-switching models allow for two (or more) processes to exist with a series of shifts between the states occurring in a probabilistic fashion, so that shifts occur endogenously rather than being imposed by the researcher. The modelling strategy thus imposes a simpler-than-conventional structure on the demographic process within any given regime, but gains power to fit the historical data by allowing regimes to change. The idea is to describe the stochastic process that determines the switch from one regime to another by means of a Markov Chain. Markov Chain is used to model the behavior of a state variable (or of a combination of variables) that determines which regime is current, as this variable cannot be directly observed.

A Markov chain can be represented as follows. Suppose that the $I(d)$ series y_t is subject to regime shift, s_t being the state of the variable y_t being in regime $1,...,M$ such that $s_t \in \{1,...,M\}$. The regime s_t can be modelled according to a discrete-state homogeneous Markov-chain generating mechanism:

$$Pr(s_t|\{s_{t-j}\}_{j=1}^{\infty})=Pr(s_t|s_{t-1};\rho) \quad (4.15)$$

where ρ is the vector of parameters of the regime generating process. Suppose that the probability of y_t with state s_t assuming some particular value j and depending only on the previous value s_{t-1}, is given by the following equation

$$P\{s_t=j|s_{t-1}=i,...\}=P\{s_t=j|s_{t-1}=i\}=P_{ij} \quad (4.16)$$

This process is described as a Markov chain with m-states, whose probability P_{ij} indicates the probability of state i being followed by state j with the property that

$$P_{i1}+P_{i2}+...+P_{im}=1 \quad (4.17)$$

Given eq.4.17 we can build the transition matrix, where line i, column j, give the probability of state i being followed by state j.

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix} \tag{4.18}$$

The main characteristic of this Markov transition matrix of first order is that the probability of transition to the next regime relies only on the current regime, which simplifies the modeling and, especially, the estimation methods. The switching parameters in the *ARFIMA* forecasting in the presence of Markov switching can occur due to significant shifts in the memory parameter, d in two regimes, or significant shifts in *ARMA* parameters and intercept of the model as well as the variance in different regimes.

The switching is under the control of a Markov-chain updating mechanism with fixed transition probabilities. A compact description is given in Davidson (2005). Let $f\left(y_t \middle| s_t = j, \Omega_{t-1}\right)$ denote the probability density of the dependent variable, y_t at time t when regime j is operating. Ω_{t-1} represents the history of the process to date $t-1$, and let the probability of falling in regime j at time t evolve according to

$$P(s_t = j|\Omega_t) = \frac{f(y_t|s_t = j, \Omega_{t-1})P(s_t = j|\Omega_{t-1})}{\sum_{i=1}^{M} f(y_t|s_t = i, \Omega_{t-1})P(s_t = i|\Omega_{t-1})} \tag{4.19}$$

where $P(s_t = j|\Omega_{t-1}) = \sum_{i=1}^{M} p_{ji}[p(s_{t-1} = i|\Omega_{t-1})]$.
The transition probabilities, p_{ji} are fixed parameters that need to be estimated. The likelihood function to be estimated is described as

$$L = \sum_{t=1}^{T} log \sum_{i=1}^{M} f(y_t|s_t = j, \Omega_{t-1})p(s_t = j|\Omega_{t-1}) \tag{4.20}$$

The series $p\left(s_t = j \middle| \Omega_{t-1}\right)$, for $j = 1,...,M-1$, the 'filter probabilities', are a by-product of the estimation (Davidson, 2005). The MS-ARFIMA forecast formulae are described in Davidson (2005). The formula takes account of the uncertainty about which regime an observation represents, but ignores parameter uncertainty (Davidson, 2004; 2005). The empirical results are discussed in the following section.

4.4 Empirical Analysis

4.4.1 Data and Estimation Issues

Before we elaborate the empirical results and their implications, some notes on the data characteristics and estimation issues are in order. We use annual data series on aggregate population series (from 1870-2002) for a sample of developed and developing countries.[40] The data have been gathered from Maddison (2002, 2004). In accordance with a recommendation by Kashyap and Rao (1976) and later by Pflaumer (1992), we estimate *ARFIMA* for the logarithm of the total population series in the first differences. To shed light on the memory structure we have estimated d using both log-periodogram regression and parametric regression with Sowell (1992) maximum likelihood method. For the former, Phillips (1999a,b) modified log-periodogram regression (MLPR) has been employed. For a detailed discussion of the method see Phillips (1999a,b) and Kim and Phillips (1999). This method estimates only the long memory parameter, d and does not depend on the effect of short-memory structure, like *AR* and *MA* parameters. In fact, The MLPR computes a modified form of the Geweke/Porter-Hudak (GPH, 1983) estimate of the long memory parameter, d , of y_t when distinguishing unit-root behavior from fractional integration becomes problematic, given that the GPH estimator is inconsistent against $d > 1$. This weakness of the GPH estimator is solved by ML PR. in which the dependent variable is modified to reflect the distribution of d under the null hypothesis that $d = 1$. The estimator gives rise to a test statistic for $d = 1$, which is a standard normal variate under the null.

We have estimated *ARFIMA (p, d, q)* model for the first difference of the logarithm of total population series, which is the growth rate of population. All estimations have been carried out using Time Series Modelling package (Davidson, 2005), except the modified log periodogram estimation of d for which we have used STATA. Trend term is not included in the model as it is expected that first difference of the population series will automatically eliminate the trend effect. The *ARFIMA* estimation has been performed with and without type 2 intercept to study whether the presence/absence of this intercept present substantial variations in the forecast. Note that the first difference of the model naturally eliminates type-1 intercept. In Doornik and Ooms (2004), their model does not consider the presence of type-2 intercept term. Their model roughly looks like a version of Eq. 4.3 with trend term and type-2 intercept being set to zero in the model. Here we follow the generalization as in David-

[40] The developed countries are: Austria, Australia, Belgium, Germany, France, Spain, Sweden, United States, and United Kingdom. For developing countries we have considered: Brazil, China, India.

son (2005) and consider type-2 intercept in the model to allow for the presence of a possible unforeseen stochastic shocks in the model.

Going by theoretical wisdom, we expect higher forecast estimates in case of intercept model since a part of the variation in the series is attributed to the variation in a trend-like term, independent of the variable's endogenous changes. It may be mentioned that the ARFIMA estimation will be carried for $\ln \Delta P_t$, therefore type-1 intercept automatically vanishes after differentiation of P_t. What remains now is whether to use type-2 intercept in the estimation. There is of course no *a priori* reason to choose one over another although it is often guided by the economic/demographic assumption and reasoning. For instance, in our case, we would favor without type-2 intercept model because while first differencing P_t we have eliminated the effect of a trend term to some extent and therefore accounting for a further inclusion of intercept in the model might bias the result (see for instance, Silverberg and Verspagen, 2001).

However, a comparison of the forecast errors from the two models and comparison with UN and other forecasts along with the in-sample forecasts would lend a clue to our choice. Figures 4.4 through 4.14 present forecast plots of the estimated models with and without type-2 intercept. For each country, the Kernel density plot is also provided (lower panel of each forecast) for 2050. We estimate a parametric ARFIMA model for the first difference of the logarithmic of the total population and then perform dynamic forecasts till 2050. Estimation in first difference is guided by the rule-of-thumb that Robinson's semiparametric d value is more than 0.4, which gives evidence of long-memory process (See Davidson, 2005 for the selection procedure). Moreover, modified log-periodogram estimates presented in Table 4.1 (first column) also provide similar conclusion. Based on these information, we have first-differenced the population series before estimation.

Different orders of ARFIMA, viz., ARFIMA(3,d,3) model were tried with maximum *AR* order $p = 3$ and maximum *MA* order $q = 3$. In contrast to the standard log-periodogram or semiparametric estimation procedure for d, for instance, the Whittle ML method, where d is estimated without imposing any *a priori* restriction on the *AR* and *MA* parameters, in the present context we estimate d directly from the model. Clearly, the magnitude of d depends on the magnitudes of *AR* and *MA* parameters when estimated without Markov switching model and at other time in the presence of different regimes while imposing regime switching in the ARFIMA estimation. Selection of the best ARFIMA model for forecasting has been based on Schwarz Information Criteria and comparing the highest likelihood estimates of various ARFIMA models.

Finally, a note on the confidence band of the forecast values. We have used Bootstrap standard errors to build confidence band for the estimated forecast values. Ex-ante multi-step forecast using Monte Carlo method has been performed. In contrast to the analytic method, in the monte carlo method the dynamic model is stochastically simulated *F*-steps forward using three shock generating mechanisms, viz., Gaussian, Likelihood matching, and bootstrap. In this case, the median of the simulations provides the point forecast, and 2.5% and 97.5% quantiles are also reported in order to provide a 95% confidence band around the forecast. Moreover, Kernel density plots for forecast point have also been provided against the normal density to have additional information on forecast values (see the lower panel of each forecast plot in Figures 4.4 through 4.14).

4.4.2 Empirical Results: Forecast comparisons and accuracy measures

Analysis of Population Size

This section demonstrates the application of the ARFIMA technique using the logarithm of total population figures from 1870-2002 for some selected countries. Eight developed countries and three developing countries have been chosen for empirical demonstration though the number can be increased to accommodate wide range of countries. The selected developed countries, viz., Austria, Belgium, France, Germany, Sweden, United Kingdom, Australia and United States appear to capture a common variability in the demographic structure. While among developing countries China and India have much in common as they possess the highest population share in the world. Moreover, Brazil's economic and demographic system also resembles to that of India. Note that the choice of the countries is rather arbitrary and to some extent is governed by common demo-economic structural affinities as mentioned about. To have insight into the forecasting results, it is useful to study their time series plots.

First, we provide a graphical presentation of the logarithm of total population and their first differences for the selected developed and developing countries. Second, we analyze the *memory* structure of total population growth (i.e., logarithmic first difference of the total population) with respect to semi-parametric log periodogram regression and parametric maximum likelihood method where we estimate a full ARFIMA model.

In case of the log-periodogram method, long memory $(1-L)^d$ is estimated without taking account of short-memory structure. For the latter short-memory structure is directly investigated by allowing *AR* and *MA* parameters in the model. In the sequel, long memory parameter, d is also estimated in the presence of *AR* and *MA* parameters to check if long-memory parameter is contaminated by short-memory behavior. The logarithm of the plots of population series and their first differences (to be interpreted as growth rates) are described in Figures 4.1 through 4.3. In Figures 4.1-4.2, developed countries logarithmic population plots and the first differences (in logarithmic scale) are presented. Figure 4.3 depicts the same for developing countries.

It can be observed that except for United States, Australia, and Sweden, other developed countries like Austria, Belgium, Germany, France, and United Kingdom show fluctuations. Similarity of structure is observed in most of the European countries, like Austria, Belgium and Germany. Among developing countries (shown in Figure 4.3), Brazil and China exhibit a similar trend with steady rise in the growth.[41] For India, a structural break can be observed which occurred in 1947 during the partition of the undivided India into Pakistan. Overall, all developing countries experience rapid total population growth. The first difference of $\log P_t$ (i.e., $\Delta \log P_t$) for developed and developing countries are described in Figure 4.2 and Figure 4.3 (lower panel only) respectively. From the plots we observe that the first difference of the population series are stationary as the values fluctuate around zero. However, slightly positive values for some countries, like US for the first difference plots may indicate that the series needs to be differenced further. Pflaumer (1992) showed that US population is second difference stationary from 1900-1988. However, first or second differencing is a rather extreme transformation of the series while following fractional integration setup, a fractional transformation is often recommended because this is expected to give an accurate measure of integration parameter d.

Table 4.1 reports the estimates of d from semiparametric MLPR and parametric ARFIMA models. In the parametric case, results are reported for ARFIMA with and without Type 2 intercept (second and third column of Table 4.1). The modified log-periodogram (MLPR) estimate clearly indicates existence of stationary long-memory in population growth rates as in all cases $0 < d < 1$ except for United States for which $d \geq 1$. This is the case with non-stationary long memory. The level population in this case is $I(d+1) = I(2.046)$. The evidence of long-memory in growth rates is indicative of the nature of persistent shocks, which must be taken into account for longrange forecasting. The reason is that estimates of d indicates the convergence pattern of shocks in the long-run – a feature which will be of enormous use for a stable and accurate long-range forecast of level population. The ARFIMA estimation of d evince less persistence compared with MLPR estimates as the former accommodates short-memory structure in the form of estimated AR and MA parameters. This is not surprising as the MLPR method provides estimates of d using a semiparametric approach. The data generation process is designed in MLPR such that AR and MA terms effects are not directly accounted

[41] A structural break seems to have occurred for China around 1959 and for Brazil in 1949. However, a slight change in the slope of the curve may not always be considered as structural changes, because such changes do often occur in the economy. By structural changes here we mean a clear and measurable shift in the slope of the curve which is expected to occur due to some drastic or massive changes in economic or socio-economic or demographic regime.

for rather the maximum likelihood designed for this method in the frequency domain is expected to account for some of their effects. However, since the short-run and long-run effects are not directly taken into account as in parametric method the estimates would differ (see for instance, Tolvi, 2003). The superiority of one method over other cannot be gauged as they just provide two different mechanisms to approximate short-memory and long-memory features of a given model. The MLPR estimates like other semiparametric methods (such as Robinson, 1995) test for the presence of a short and/or long-memory, while in parametric case, the long-memory parameter is directly estimated in the presence of some other parameters.

Clearly, the estimates of d from ARFIMA models seems to have been contaminated by short-memory behavior.[42] Table 4.3 precisely illustrates this possibility, where ARFIMA models have been estimated[43] and selected according to *SIC* criterion. As expected AR parameters are found to significantly influence the DGP of population growth rates. For UK, MA parameters are very significant and negatively affect the growth path of population. Mixed results are found for India. The interpretation of ARFIMA components with estimated d is reported in Table 4.2. To summarise, the estimated parameters of the full ARFIMA model indicate about the significance of AR and/or MA shocks in the population series; while significant AR parameters hint at the relevance of past shocks from historical point of view (this is also endogenous to the system), the significant MA parameters tell us about the influence of stochastic external shocks to the demographic system. The interpretation of the memory features may hold true even with log periodogram estimates of d (see Chapter 3). The chosen ARFIMA models are reported in Table 4.3. From the estimates of d of the differenced population series, it is apparent that the actual population series is $(1+d)$ (estimated by the model), which depict non-stationary behaviour.

[42] Note that *d* is estimated from MLPR method as a result of log-periodogram regression without accounting for AR or MA parameters. While in ARFIMA model, *d* is estimated along with other parameters, like AR and MA orders. This is a fully parametric estimation of ARFIMA. The difference between MLPR and ARFIMA estimates of *d* would remain as long as the data generation mechanism in the two cases are different. Both the models have their respective weaknesses though and it is difficult to choose one over the other. We may note that we estimated *d* in Chapter 3 using MLPR method to know about the influence of stochastic memory on economy growth. That is, independent of any other endogenous or exogenous effects how stochastic memory could affect growth. In forecasting we would like to use the fully parametric ARFIMA model, first to lend comparison with ARMA method used by Pflaumer for US and second, one would know how future population projection will be substantially led by AR, MA and *d* altogether.

[43] We have not reported the estimated results of ARFIMA model without type-2 intercept due to our a priori choice of with type-2 intercept model for forecasting.

Analysis of Forecasts

In this section we discuss the forecasting results of ARFIMA and Markov Switching ARFIMA (MS-ARFIMA) estimation and compare them with United Nations projections till 2050. The forecasting performance of the total population is evaluated based on the computation of dynamic forecasts. The models have been estimated recursively and dynamic forecasts generated starting from 2003 till 2050 except for United States for which the forecasting horizon extends upto 2080 in order to compare with Pflaumer's (1992) and Census Bureau's estimates. For United States the sequence leads up to 78-step ahead dynamic forecasts though we have calculated forecasts till 2050 for comparison with United Nations projections. Finally we try to provide some preliminary estimation of the forecasts when the DGP of the total population follows long memory with stochastic regime switching, i.e., the Markov switching model in our case. The idea here would be to hint at the changes in forecast values when a possible regime switch is suspected in the *ARFIMA* model. Our purpose here is not to compare the forecast accuracy of the regime switch and simple ARFIMA process as this has been carried in some studies, e.g., Gabriel and Martins (2004), etc. While finding rather a trivial answer to the performance pattern of regime switch and ARFIMA processes, the authors reiterate that in case of *longer sample,* markov switching fares better than simple ARFIMA. However, the validity of their results might also be contingent upon the specific data that they used. It may also depend on the chosen length of a sample. However, keeping in mind the possible drawbacks of each method, we present here the forecast values of each method and compare them to give a preliminary idea on the performance of these methods in our data. Table 4.4 presents a comparison of our ARFIMA forecasts with UN projections and Table 4.6 reports the 95 percent lower and upper limit confidence interval for the forecast points. Figures 4.4 through 4.14 depict forecast till 2050.

To provide credence to our forecast, we have performed in-sample forecast for the selected countries and have generated forecasts for the years 2000-2005. In Table 4.7, we present forecast figures for 2005 which are compared with actual population figures for 2005 (obtained from the World Fact Book[44]) and UN projection values for this year. The 95% confidence band for our actual forecast (based on ARFIMA projection) is also reported (the last two columns of the Table 4.7). It may be observed that out of 11 countries considered here, for about 8 countries our forecasts are closer to actual figures (from the World Fact Book) for 2005.[45] Except for Austria, Australia and Sweden, for other

[44] http://www.cia.gov/cia/publications/factbook/rankorder/2119rank.html. Figures for 2000-2004 are not reported in the Table to save space.

[45] It may be noted though the 'closeness' of our estimates to that real figures is merely an indication of the performance of the method we have used for forecasting. However, to de-

countries the forecast values are more or less the same as the actual figures in 2005. The UN estimates are also closer to the actual figures but in some cases, for example India and China, the projections are higher than the actuals. On the whole, ARFIMA and UN projections show comparable estimates though in most cases ARFIMA projections seem to approximate actual figures more closely than UN projections.[46]

Our estimation is based on time series methods while UN and others projections are based on high, low, medium variant and probabilistic methods. Unless we compare the existing methods with some criteria, we cannot say here whether the probabilistic or time series methods are better. Pflaumer (1992) admits that time-series based method, such as ARMA can be a complementary tool along side probabilistic or other methods to give credence to the forecasts. The competitive features of time series and other methods are not yet known and investigation of this sort is also beyond the scope of the present chapter. For our purpose we can conjecture that time series methods as a demographic forecasting tool can be as useful as other methods. However, in the temporal domain, ARFIMA models are said to be better than ARIMA due to the inherent characteristic of the former to account for stochastic shocks accurately. It is also true that if one relaxes the probabilistic assumption on demographic variables in favor of a more accurate characterization of stochastic shocks which have evolved over time, then the revelation of the in-sample characteristics in terms of memory properties of shocks can sometimes better indicate how demographic structure has evolved over the years.

The superiority of this method is yet to be proven systematically although in terms of the 'assumptions' about demographic processes and some features of the model behavior the usefulness of time series methods can be gauged. For instance, Tuljapurkar et al. (2004) compare the forecast of vital rates (e.g., fertility rates) from random scenario and time series methods and find several key differences between these methods. The authors found that serial correlations in the forecast were much smaller in time-series methods and trajectories in these methods were much more irregular than in random scenario. Based on these theoretical investigation therefore advantages of time-series based methods can be held over other competing methods.[47]

liver a rigorous analysis on the 'closeness' some additional tests and information maybe required, viz., the width of the confidence band.

46 It needs to be stressed that it is not possible at this point to prove the superiority of our forecast method over UN or others. All these models have different assumption about the underlying demographic process and they model uncertainty differently. Hence, a more rigorous comparative analysis is required, which is beyond the scope of the present research.

47 Tuljapurkar et al. (2004) investigate the validity of their findings for US data on mortality and fertility for which a robust amount of literature on different forecasting methods and their applications exist. In this chapter we have extended the analysis to many countries and to different age-specific population. We do not have available information to compare our

As invoked earlier, although population forecasting employing time series methods is getting increasing attention recently, so far the application of these techniques, such as ARM A model has been employed for United States population, life-expectancy and/or other demographic variables as could be found in Pflaumer (1992) and Lee and Tuljapurkar (1994). Therefore to render appropriate comparison of our ARLIMA forecast with ARMA and other stochastic forecasting methods, we elaborate in this section the forecast of total population for United States.

From Table 4.3, it is evident that US logarithmic total population is an *ARFIMA* $(1,1+d,0)$ process, where the estimated d is zero, implying no significant memory structure. (Note that considering MLPR estimates, we found evidence of non-stationary long-memory characteristics of this series). Following ARFIMA representation, the estimated model is

$$\Delta \ln(y_t) = 0.000 + 0.984\Delta \ln(y_{t-1}) \tag{4.21}$$

Since $\Delta \ln(y_t) = \ln(y_t) - \ln(y_{t-1})$, Eq. 4.21 can be written as $\Delta \ln(y_t) - 1.984 \ln(y_{t-1}) + 0.984(y_{t-2}) = 0.000$ The characteristic equation is

$$\lambda^2 - 1.984\lambda + 0.984 = 0 \tag{4.22}$$

Upon finding the solution of the equation (where $\lambda_1 = 1$ and $\lambda_2 = 0.984$), the general solution of the difference equation is written as

$$\ln y_t = M_1 1^t + M_2(0.984)^t + zt \tag{4.23}$$

Taking the initial conditions y_0 we can obtain a definite solution of the difference equation from which long-term forecast of population can be calculated with constant annual population growth rate.[48] Similarly, the characteristic equations for other countries with different orders of ARFIMA can be written

forecast with other methods with criterion like checking the strength of serial correlation and comparing the variance of these models. Nevertheless we do not also presume that this is an ideal way to compare as the data generation mechanism for random scenario and time-series methods are different.

[48] Note that our forecasts are basically point forecasts sequentially performed over long period of time. Although these forecasts do not reveal much about parameter uncertainty in comparison to interval forecasts, a study of the estimated confidence interval for the point forecast provides some idea about the range of values the forecast would fall. Moreover, all the forecast plots accompany density forecast figures to help explain the amount of uncertainty. Standard practice in time series based forecasts is to consider point forecasts, at the least while estimating an ARFIMA type of model.

which are then solved to achieve a definite solution of the corresponding difference equation. Finally, this is used for long-term forecast.

Two different projections can be observed, viz., with and without type-2 intercept. The motivation of the presentation of forecasts with these two models is to understand how the inclusion of an additional intercept (concerning the exogenous autonomous factor) change forecast values. Since our estimated model is the first difference of P_t, the trend and type-1 intercept have automatically vanished, therefore, the imposition of another intercept which corresponds to MA term in the model would provide higher forecast than without type-2 intercept. Doornik and Ooms (2004) prefer without type-2 intercept model as most of the economic time series, after first differencing do not necessarily need to have another independent term to govern the overall dynamics of the ARFIMA model.[49] Unless of course there is a strong reason to believe that this intercept should be incorporated in the demographic model, it is better to follow the standard practice, i.e., to compare our forecast from without type-2 intercept model with other competing methods of population forecasting.

To elucidate the point, note that our forecast for US total population (in Table 4.4) shows that in 2050 the total population will reach around 392.3 million without type-2 intercept and 525.9 millions with type-2 intercept, which is lower than Pflaumer (1992) projections based on the sample period 1900-1988. Evidently forecast with type-2 intercept is higher than without it simply because we have assumed that unforeseen stochastic shocks might play a role in the succeeding years and would act upon forecast values. Note that the longer sample span (as in our case from 1870-2002) incorporates more dynamic information about the prevalent shock's evolution for which the effect of shocks is reflected on the forecasts.

Based on our sample, the ARFIMA forecast for US population for 2080 is 783.0 million in contrast with Pflaumer (1992) which is 830.7 million. The lower limit of the 95 percent confidence interval for our ARFIMA projections are at 563.5 million and upper limit at 1125.7 million (See Table 4.5). The ARFIMA forecast without Type 2 intercept presents rather lower forecast value for US, which is about 392.3 million. The United Nations provides estimates of 459.8 million for high variant, 337.5 for low and 394.4 for medium variant for the year 2050. Ahlburg and Vaupel's (AV, 1990) alternative projections of US population with high variant is about 553 million (See AV, 1990 for detailed analysis on the projection). This figure takes into account 2% mortality progress large fertility cycles and 1-2 million immigrants. The middle variant estimate of AV is 402 million population in 2050 which takes into account 1% mortality progress and 1-2 million immigrants.

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[49] See for example the ARFIMA 1.21 estimation package of Doornik and Ooms.

Notice that the high variant projection of AV corresponds closely with our ARFIMA $(1,1+d,0)$ forecast with Type-2 intercept for US. The estimated population in our case is 525.9 million, whereas for AV it is 553 million. Similarly, for 2080, AV's hight variant projections stand at 811 million, Pflaumer's (1992) ARIMA (1,1,0) projections at 830.7 million. The ARFIMA $(1,1+d,0)$ forecast for US shows lower estimates (783.0 million) which is less than both AV and Pflaumer's projections. It may be mentioned at this point that AV (1990) criticize US Census Bureau (the estimates are not presented here) for imposing too conservative assumptions in the middle and high projections. They conclude that the Census Bureau high projection might be treated as a reasonable middle forecast. The same argument seems to hold true for UN projections too. We have shown here that for US, the estimates are closer to AV (1990) high variant projections. Figure 4.11 demonstrates the projection from ARFIMA $(1,1+d,0)$ model with 95% confidence band.

• *Analysis for other countries*

A similar analysis can be carried for other countries as well. Take for instance the case of Austria as in Table 4.4. The total population size for this country in 2050 is expected to be 8.49 million (without type-2 intercept) and 9.42 million when type-2 intercept is introduced in the forecasting model. The UN high variant projection for 2050 is 9.27 million and medium variant is 8.07 million. For Sweden, the revised UN high projection is estimated at 11.58 million total population and 10.05 million for medium variant projection, whereas our ARFIMA $(1,1+d,0)$ forecast estimates (without the intercept) is 8.75 million and 11.48 million with intercept. In 2050, France will have a total population of 72.9 million with ARFIMA $(1,1+d,0)$ in the presence of type 2 intercept and 60.35 million without intercept model. Thus it is evident that the presence of type-2 intercept consistently gives higher forecast than without it. This is natural given the explanation as above that the presence of an additional regressor, specifically a constant term will include some amount of additional variation of the forecast.

Notable differences between UN and ARFIMA projections occur for developing countries. Consider for instance, China for which the medium variant UN forecast is 1392 million (approximately 1.4 billion) in 2050. Our ARFIMA $(1,1+d,0)$ model with Type 2 intercept leads to significantly higher forecast, which is 2063 million (approximately 2 billion) in 2050. However, ARFIMA $(1,1+d,0)$ model without Type-2 intercept provides lower forecast, which is approximately 1.5 billion in 2050. For India, the ARFIMA forecast significantly differ from UN projections. For instance, the high variant UN forecast for India is 1889 million (which is 1.88 billion), where the ARFIMA $(1,1+d,2)$ forecast in the presence of Type-2 intercept leads to significantly high forecast of 2380 million (that is 2.38 billion) in 2050. The ARFIMA model without intercept estimates the forecast at 2182 million or 2.18 billion during the same time. Comparing the forecasts of population size for China and India, it is observed that India is expected to surpass China as the most populous country in the world. The same observation also follows from UN forecasts. To have an idea about how our ARFIMA forecasts is proximus to the actual figures in 2005, we have compared India's current population figure[50] with our forecast for 2005. We find that our projections are quite close to the actual figure in 2005, providing credence to the accuracy of the forecasts.[51] However, the forecast values of India and China should be explained with caution as the confidence band for these countries are very wide.[52]

• *MS-ARFIMA forecast analysis*

So far we have discussed results based on simple ARFIMA models. However, in the preceding section we have also noted that demographic system can experience possible structural shifts. Endogenous changes might also occur in the system. To capture their effects, MS-ARFIMA model is employed. In the estimation we have allowed switching to occur in d as well as in the intercept. The explanation of switching parameter has been stated in the preceding section. Recall that we have assumed two regimes in the model. So the estimated transition probabilities in this case would indicate how the population series has experienced a stochastic shift from one regime to the other. In terms of regime shift in d, the first regime could be no stochastic shocks and regime 2 could be with stochastic shocks. Since we have assumed a stationary Markov process, the transition probabilities should indicate stationarity so that based on this a stable forecast can be drawn.

Note that we have not considered all countries for investigation in MS-ARFIMA model. We have selected a few where we felt, the switch is more distinct than others. Among the countries we have performed forecast, India, Belgium, UK, and Austria have been selected. For instance, in case of India demographic shifts occurred in 1947 during the independence. A similar pattern can be observed for UK, where a break in the series is visible around 1920s. Important to remember that while a one-off shock like the partition of India in 1947 or the 1920-21 shock in the UK cannot be described in a Markov-switching framework, there can be many small but significant demographic

[50] Source for World Factbook: http://www.cia.gov/cia/publications/factbook.

[51] Instead of the confidence band, the relevant standard errors could be reported. However, our preference for the confidence band is based on the standard reporting in the forecasting literature. Added motivation is that the confidence band gives us an idea about the tightness or wideness of the actual forecast.

[52] Despite trying with many alternatives and re-running the estimation we did not find substantial changes in the results. This might indicate the population series of these countries need additional treatment to counter high-uncertainty as represented by the wide confidence interval.

shifts in the series, which might give rise to a long-memory pattern as the 'shifts can cause non-linearity' in the model. Switching can occur in the model which may be known to us a-priori. Often some switchings can occur endogenously and may not be super-imposed while performing estimation. Therefore, the model is allowed to be estimated assuming that the switching has occurred in intercept and/or in the memory parameter (in the current estimation, for example). Following Davidson (2005) TSM package where it allows the switching to be selected in ARMA parameters, d and/or intercept, we have estimated the Markov Model where switching has been assumed to occur both in intercept and in long-memory parameter. However, to take account of these shifts MS-ARFIMA model is employed and total population forecast till 2050 is performed. Two regime-shifts have been permitted in the model. The MS-ARFIMA forecasting results and transition probabilities for the regime switch are reported in Tables 4.8 and 4.9.

To start with, the MS-ARFIMA $(1,1+d,0)$ forecast for France puts a lower estimate. In the presence of Markov Switching the ARFIMA estimates put the total population for France at 59.1 million in 2050, which is 4.8 million higher than UN low variant projection (54.3 million). The Markov transition probabilities are, $p_{11} = 0.80$, $p_{12} = 0.19$, $p_{21} = 0.84$, and $p_{22} = 0.15$. There is 84 percent probability that changes in the long-memory parameter and intercept from Regime 1 to Regime 2 is significant which governs the ARFIMA process.[53] Similarly, comparing the ARFIMA and MS-ARFIMA forecasts for India, we observe that ARFIMA forecasts (both with and without intercepts) have higher estimates than MS-ARFIMA. Notably, when endogenous shifts are considered in the model, the Markov Switching process leads to lower estimates. The MS-ARFIMA forecast for India stands at 1684 million which is lower than ARFIMA forecasts (2182 million without intercept). For Austria, similarly the MS forecast leads to 7987 million which is lower than simple ARFIMA forecasts. The transition probabilities for Austria are $p_{11} = 0.98$, $p_{12} = 0.02$, $p_{21} = 0.69$, and $p_{22} = 0.31$. This implies there is 98% probability that regimes changes in memory parameter and intercept significantly affect the DGP of total population series.[54] Note that since the estimated transition probabilities are quite large, this indicates that the regimes are persistent. Therefore, it is not surprising to find long-memory in the series.[55]

[53] Following Davidson (2005) regime 1 denotes no structural parameter and non-stochastic shocks persisted and regime 2 would mean structural changes in the economy and persistence of stochastic shocks played crucial role in the series.

[54] We have assumed that the regime change occurs both in the intercept and parameter, i.e., the memory parameter here.

[55] Given the nature of stochasticity of population series and the evident persistence of regimes, the forecast from this method may not be reliable. Therefore, we do not extend a rigorous analysis of this method in this chapter which is preserved for future research. A

The intriguing question is: which forecast is to be taken into confidence? Going by Pflaumer's (1992) logic it can be said that time series based models do not perform worse than more complex demographic models in projecting future population size. Many demographic methods involve complex computational mechanism, for instance computation of stochastic Leslie matrix, or modelling stochasticity in life-expectancy and incorporating for population projection. These involve great risk of lending to complex modelling, and depends on the pattern of future population growth. In terms of methodological simplicity and better embedding of dynamic information in the demographic system, time series methods seem to be simpler and more informative. The suggested ARFIMA model for total population forecast is a significant step in this direction which can aptly combine the dynamic demographic intuitions in the econometric model by permitting the 'memory' of shocks to assume fractional value. This makes the model more general and flexible giving the forecaster an opportunity to take account of a shock, of whatever magnitude it is, which can influence the future trajectory of population growth. A shock of small magnitude today can accumulate over time and may cause instability in the series based on which no definitive conclusion about its interaction with the rest of the economy can be made.

Theoretically, *ARFIMA* models have been shown to perform better than *ARIMA* method to identify and model stochastic shock behavior in a series. The *ARIMA* method as employed by Pflaumer (1992) can be suitably extended to *ARFIMA* paradigm which is more flexible domain and where the characteristics of shocks can be better understood than in the *ARIMA* framework. Moreover, using conventional practice of forecasting as in Doornik and Ooms (2004), where type-2 intercept is omitted from the model, and since there is little a priori ground to induct such term in the model to capture unseen future stochasticities (which is already captured by the behavior of error term), we stick to the forecast values of *ARFIMA* model without type-2 intercept. Additionally, the choice of the without type 2 model for forecast is also motivated by comparing the likelihood and Schwartz criterion of the two models.[56]

possible direction would be to treat the persistent properties of the transition matrix and use them for forecasting population series.

[56] A possible method to lend a choice over the presence of such intercept is to perform a simulation experiment for each country. But we do not deem it fit for our case due to the reasons explained before that we do not have a priori justification of incorporating such a term in the model. Of course, whence the estimation is performed on the raw series instead of the differenced series, a choice of one of these intercepts would have led us to fix a criteria for selection. Since in our case we have differenced the series for estimation, one of the intercepts has already been accounted for in the model.

4.5 Conclusion

Two leading concerns underlined the analysis of this chapter. First, most of the notable research in population forecasting were concentrated for US data and rarely for some other countries. Specifically, the application of time-series methods for demographic forecasting has not found much importance beyond US demography analysis. Our primary purpose was to use this method for other countries, viz., for a set of developed and developing countries whose demographic patterns have been the cause of concerns over the years. While developed countries are experiencing a fall in population, developing countries are experiencing an upward trend. Using time-series method as one of the stochastic methods of forecasting, we attempted in this chapter to shed some light on the future pattern of population in these countries. Another aim of this chapter was to introduce ARFIMA framework as a more flexible method to forecasting population. With regard to this, we intended to extend Pflaumer (1992) ARIMA method to a general ARFIMA framework. By extending the model, we intended to take into consideration the varied nature and effect of demographic shocks in the model which was absent in the ARIMA case. The consideration of ARFIMA method for forecasting has been motivated by the results of Chapter 3 where we found significant evidence of long-memory of total population and age-structured population using historical time series.

Indeed, the advantages of ARFIMA method is established in the literature, however, our consideration of a fractional class instead of integer order of integration of the memory parameter has allowed us to incorporate different short-run and long-run features of the population series for long-range forecasting. Some important points emerge from the forecasting exercise. First, the comparison of in-sample forecast values show that our *ARFIMA* projections are indeed closer to the actual figures and are comparable to UN projections. As invoked earlier, the models are not comparable due to different assumptions about the data generation process of the population and most importantly due to the difference in method of treatment of uncertainty in these methods. While pure demography based methods, like stochastic Leslie projection matrix, etc, involve complex assumptions about the evolution of the demographic process, the time series characterization of demographic variables actually simplify this complexity to a great extent by directly incorporating endogenous and exogenous shocks in the demographic variables which originate due to evolution of the demographic system. As Alho and Spencer (2005) note, time series based methods are not strict alternative for demographic forecasting, but the former can complement the latter, and even at times, if combined efficiently with complex demographic processes, can prove to be very powerful forecasting tool. Like Pflaumer (1992), we may conjecture that ARFIMA forecasts can complement though not compete with forecasts from other methods.

Nevertheless, within time series methods the ARFIMA method have more advantages than standard ARIMA and most other demographic forecasting methods like high, low, and medium variant projection techniques used by US Census Bureau and United Nations, for instance. For any forecasting exercise, it is always handy to incorporate as much dynamic information in the model as possible to have a reliable and economically meaningful forecasts. Population, by nature acts endogenously and interacts with the economic system in such a way that the feedback of shocks that influence the series can be handled by stochastic modelling, more accurately by time series methods.

Based on the standard empirical convention and relying on the comparison with actual in-sample forecast and tight confidence band we choose without intercept model for our forecast. Our forecast shows that European countries total population will steadily grow while a faster growth is deemed for developing countries like India and China. Importantly, by 2050 India is expected to replace China as the most populous nation on the earth. The two largest countries follow quite different population policies. China's policies are in fact binding and follows 'controlling' population as opposed to the liberal attitude in India which has a normative approach of 'stabilizing population'. However, it is expected that the relatively faster growth of population in India will also add to the stock of human capital and in fact the high cost of births can be attenuated by higher contribution from the accumulated human capital.

Table 4.1: Estimation of d for $\Delta \ln P_t$

Countries	d-Modified (log-periodogram)	d-ARFIMA (with intercept)	d-ARFIMA (no intercept)
Austria	0.453 (0.101)	0.005 (0.002)	0.011 (0.001)
Australia	0.581 (0.140)	0.007 (0.004)	0.091 (0.031)
Belgium	0.721 (0.156)	0.000 (0.100)	0.024 (0.012)
France	0.632 (0.152)	-0.003 (0.132)	-0.002 (0.101)
Germany	0.3 (0.124)	0.007 (0.003)	0.012 (0.001)
Sweden	0.621 (0.158)	-0.001 (0.104)	-0.001 (0.108)
UK	0.262 (0.123)	0.012 (0.001)	0.813 (0.152)
USA	1.046 (0.116))	0.000 (0.002)	0.000 (0.002)
Brazil	0.776 (0.231)	0.000 (0.001)	0.000 (0.003)
China	0.502 (0.121)	0.000 (0.121)	0.000 (0.003)
India	0.113 (0.034)	0.241 (0.114)	1.083 (0.342)

Note: Bracketed values are standard errors. The ARFIMA is estimated for the first difference, so d for actual population will be 1+estimated d.

Table 4.2: ARFIMA (p,d,q) components and their interpretation

d	Φ	Θ	Interpretation: Population growth
0	$< 0,1 >$	$< -1,0 >$	: Short-memory, log population is $I(1)$
1	$< 0,1 >$	$< -1,0 >$	: Non-stationary, log population is $I(2)$
$< 0, 0.5 >$	0	0	: Long-memory, log population is $I(d+1)$

Table 4.3: ARFIMA (p,d,q) model estimation for $\Delta \ln P_t$ (No intercept)

Country	*ARFIMA (p,d,q)* Model	*d*	*AR(1)*	*AR(2)*	*MA(1)*	*MA (2)*	*MA(3)*
Austria	ARFIMA(l,l+d,0)	0.011	0.427 (0.121)				
Australia	ARFIMA(2,l+d,0)	0.091	1.007 (0.269)	-0.181			
Belgium	ARFIMA(2,l+d,0)	0.024	0.948 (0.421)	-0.150 (0.040)			
France	ARFIMA(l,l+d,0)	-0.002	0.778 (0.274)				
Ger-many	ARFIMA(l,l+d,0)	0.012	0.457 (0.173)				
Sweden	ARFIMA(l,l+d,0)	-0.001	0.918 (0.413)				
UK	ARFIMA(0,l+d,3)	0.813			0.757 (0.328)	-0.612	-0.169 (0.086)
USA	ARFIMA(l,l+d,0)	0.000	0.984 (0.423)				
Brazil	ARFIMA(2,l+d,0)	0.000	0.534 (0.215)	0.458 (0.252)			
China	ARFIMA(l,l+d,0)	0.000	0.953 (0.487)				
India	ARFIMA(2,l+d,0)	1.083	-0.587 (0.329)	-0.308 (0.195)			

Note: Bracketed values are standard errors.

Table 4.4: Comparison of Total Population Forecasts with UN Projections (in thousands)

	UN Projections till 2050			Our Forecasts till 2050	
Countries	High	Medium	Low	ARFIMA forecasts	ARFIMA forecasts
				With 2 intercept	Without 2 intercept
Austria	9277	8073	7022	9423.86	8493.021
Australia	32050	27940	24300	41856.4	26212.718
Belgium	11347	10302	9331	12911.84	10989.549
France	72785	63116	54342	72911.38	60355.056
Germany	90909	78765	68086	101518.87	86855.405
Sweden	11587	10054	8710	11486.18	8757.800
UK	77910	67143	57711	69494.3	81226.758
USA	459862	394976	337519	525970.286	392385.479
Brazil	301352	253105	210188	491884.9	266465.484
China	1647189	1392307	1171259	2063677	1480662.232
India	1889631	1592704	1332527	2380926	2182540.231

Table 4.5: Comparison of our forecasts for USA with Pflaumer's (1992) and AV (1990) (Figures in millions). For 2005 forecast for India, figures are in thousands

	Own (for 2080)	Pflaumer (for 2080)	AV (2050)	AV (2080)
	783	830.7	553	811
95% upper CI	1125.7	1045.5		
95% lower CI	563.5	660		
	Our estimate	World Factbook		
India's Population in 2005	1074106-8	1080264.3		

Note: Our forecast for 2005 is compared with actual population figure from the World fact Book.

Table 4.6: 95% Confidence Interval for ARFIMA forecast for 2050 (in thousands)

Countries	With Intercept Model		Without Intercept Model	
	95% lower limit	95% upper limit	95% lower limit	95% upper limit
Austria	7849.46	10766.56	7217.04	9823.18
Australia	33523.43	52470.16	17490.27	39379.47
Belgium	10588.25	16004.10	9091.54	13160.83
France	57296.80	90309.69	45615.37	80660.16
Germany	76191.10	122149.38	64472.88	105767.65
Sweden	10371.32	12924.76	6491.83	12814.08
UK	67575.45	71610.72	35560.83	170757.55
USA	419995.80	693842.31	231653.96	734540.20
Brazil	425066.11	614767.85	95320.48	948844.30
China	1476226.90	3032809.57	684196.20	3534208.75
India	2014738.56	2833434.11	1497039.43	3246214.16

Table 4.7: Comparison of Actual Population Figures with Forecast Values for 2005 (in numbers): Without intercept model

Countries	Actual Fig.	UN projection	Own estimation	Lower 95% CI	Upper 95% CI
Austria	8184691	8189000	8214039	7925178	8485380
Australia	20090437	20155000	20211360	19539630	20922910
Belgium	10364388	10419000	10334055	10107166	10538600
France	60656178	60496000	60475887	57699281	62818193
Germany	82431390	82689000	82867647	77419967	87116362
Sweden	9001774	9041000	8883936	8719350	9001081
UK	60441457	59668000	60839833	58806049	62755406
USA	295737134	298213000	298045071	293607758	303458457
Brazil	186112794	186405000	184609858	180412288	186465218
China	1306313812	1315844000	1304069086	1273144013	1357289155
India	1080264388	1103371000	1074106810	1066614317	1082734128

Note: Actual population figures for 2005 are obtained from World Fact Book.

Table 4.8: MS-ARFIMA Forecast: Total Population (in '000)

Countries	MS-ARFIMA Model	Median Forecast
AUSTRIA	(0,l+d,0)	7987.2
INDIA	(2,l+d,0)	1684534.7
UK	(l,l+d,0)	73791.5
Belgium	(l,l+d,0)	11950.1
France	(l,l+d,0)	59144.1

Table 4.9: Transition Probabilities

Countries	p_{11}	p_{12}	p_{21}	p_{22}
AUSTRIA	0.98	0.02	0.69	0.31
INDIA	0.99	0.01	0.51	0.49
UK	0.99	0.01	1.00	0.00
Belgium	0.933	0.067	0.05	0.95
France	0.81	0.19	0.84	0.15

Figure 4.1: Logarithm of Total Population plots for developed countries

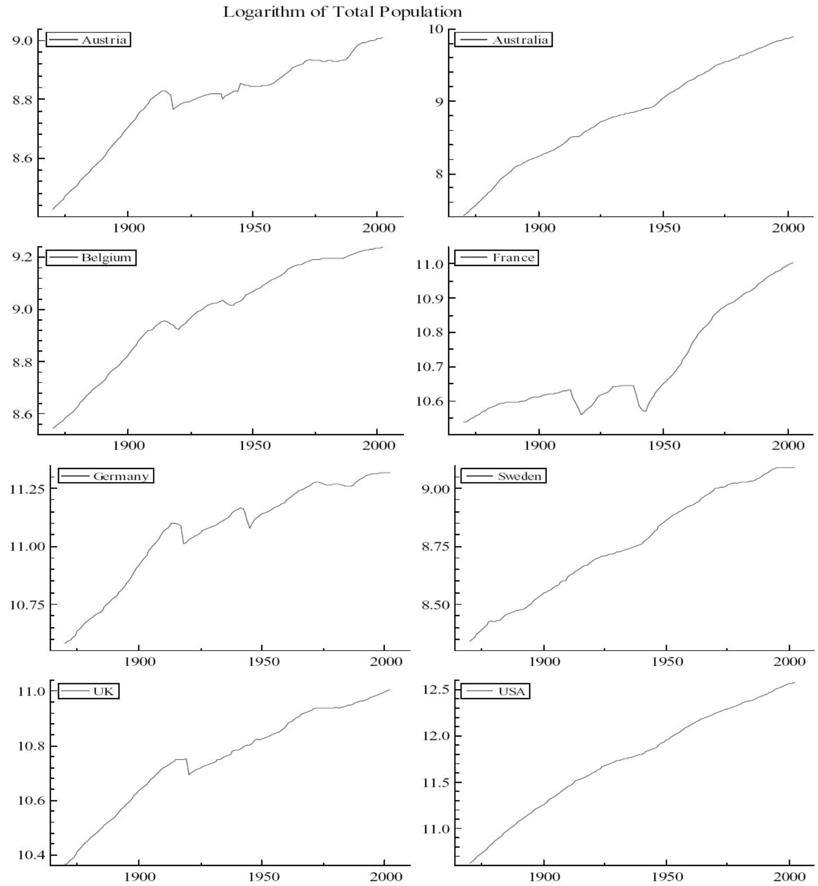


Figure 4.2: First Difference of Logarithm of Total Population plots for developed countries

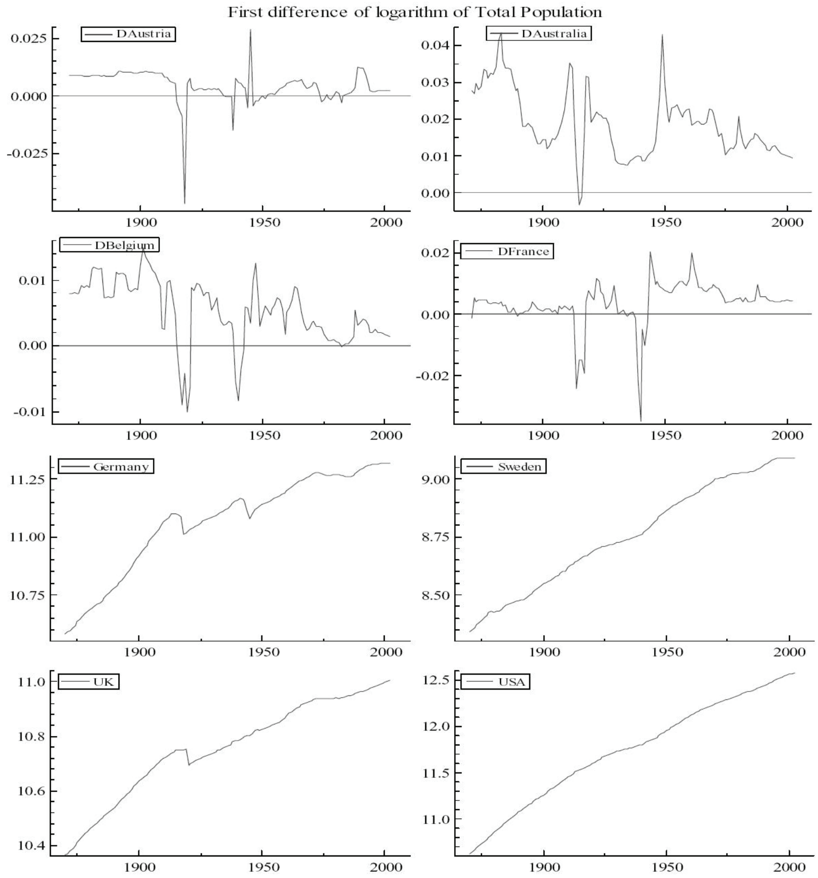


Figure 4.3: Logarithm of Total Population and first difference plots for developing countries

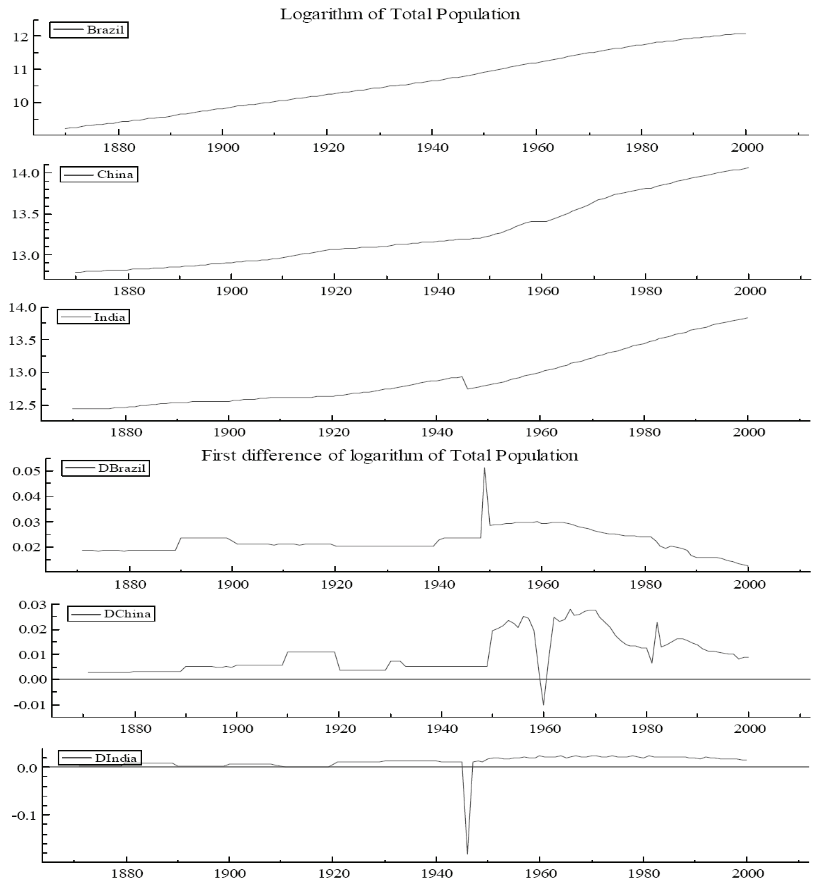


Figure 4.4: ARFIMA total population forecast for Austria

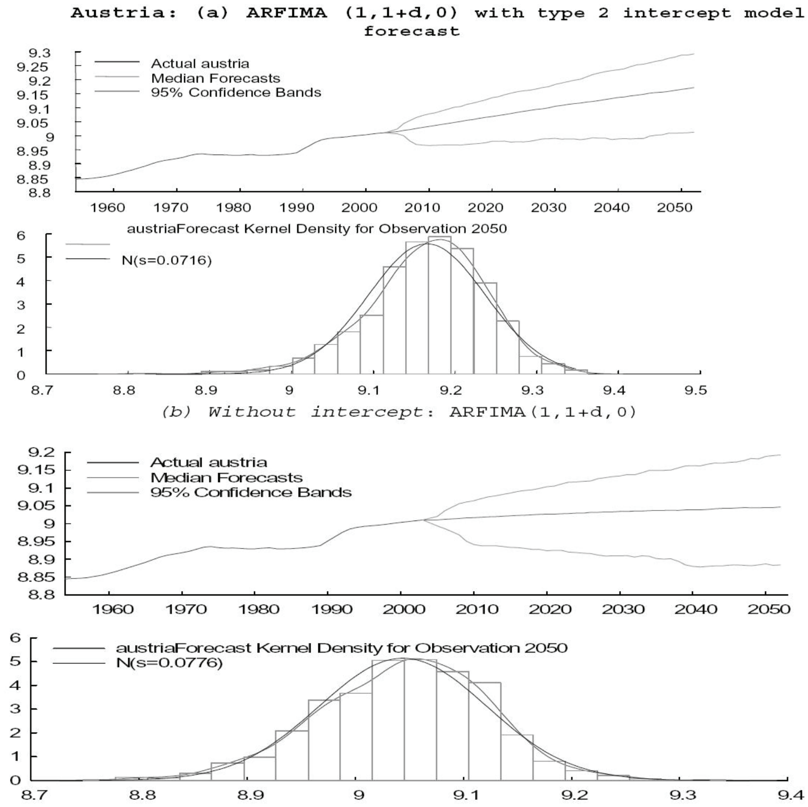
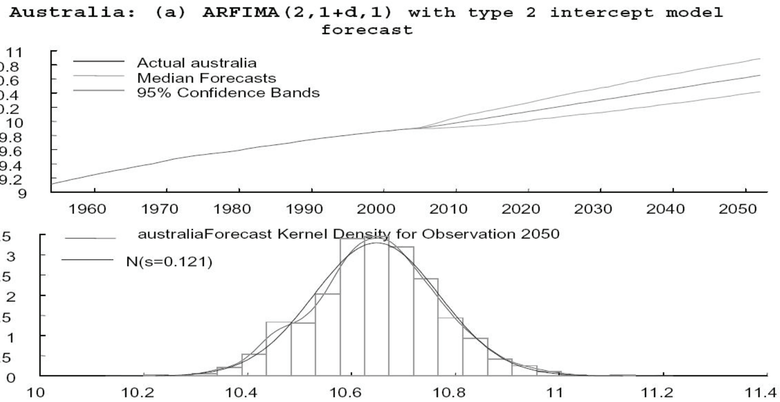


Figure 4.5: ARFIMA total population forecast for Australia



(b) Without intercept: ARFIMA(2,1+d,0)

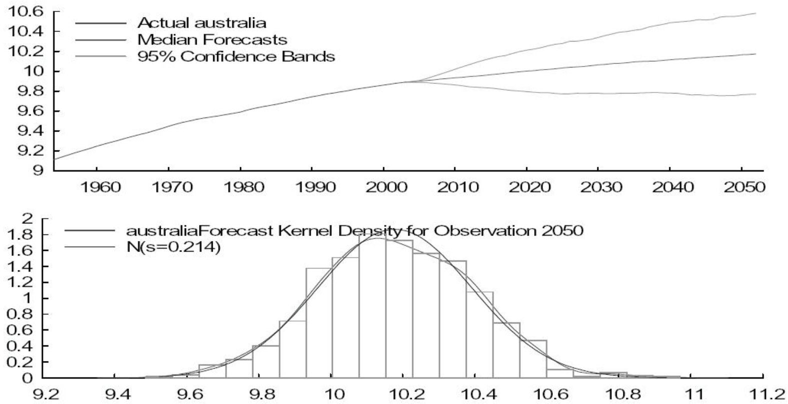


Figure 4.6: ARFIMA total population forecast for Belgium

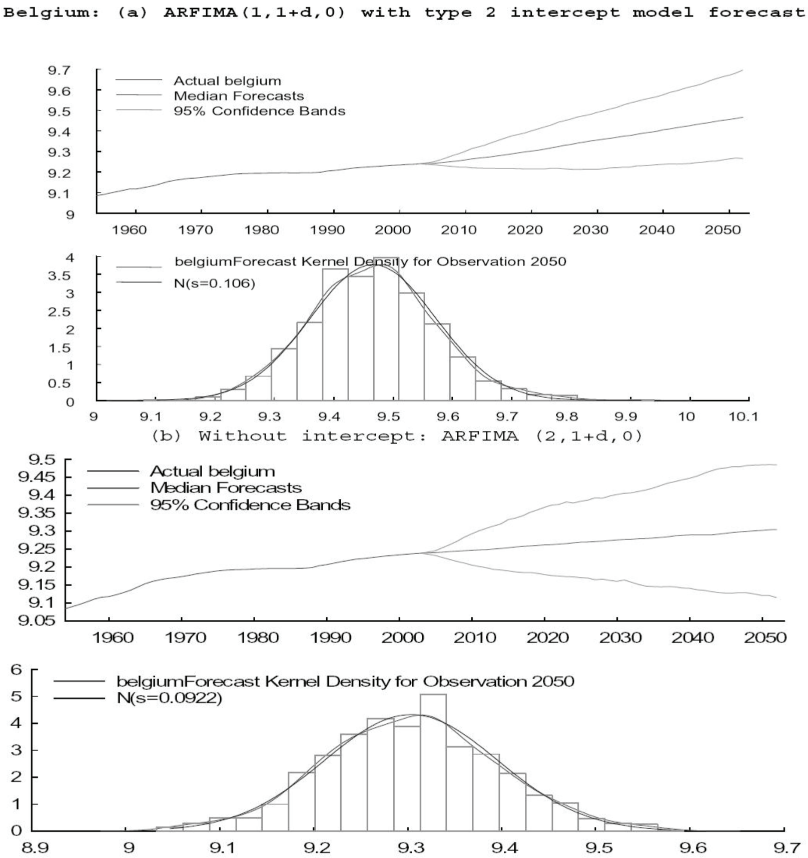


Figure 4.7: ARFIMA total population forecast for France

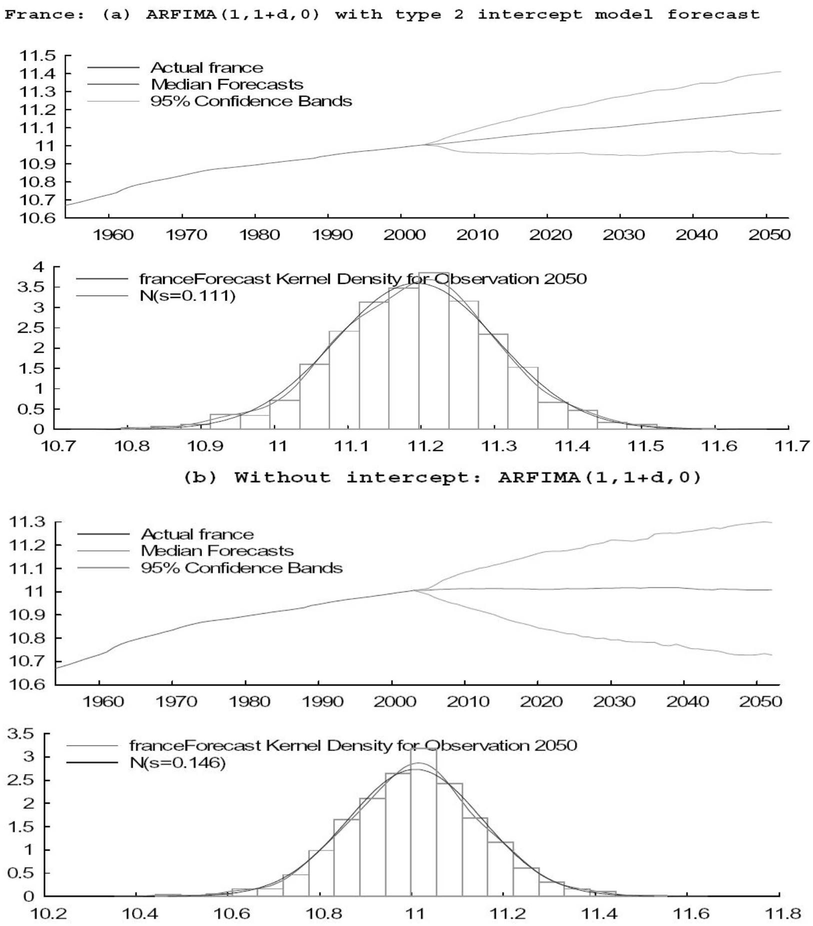


Figure 4.8: ARFIMA total population forecast for Germany

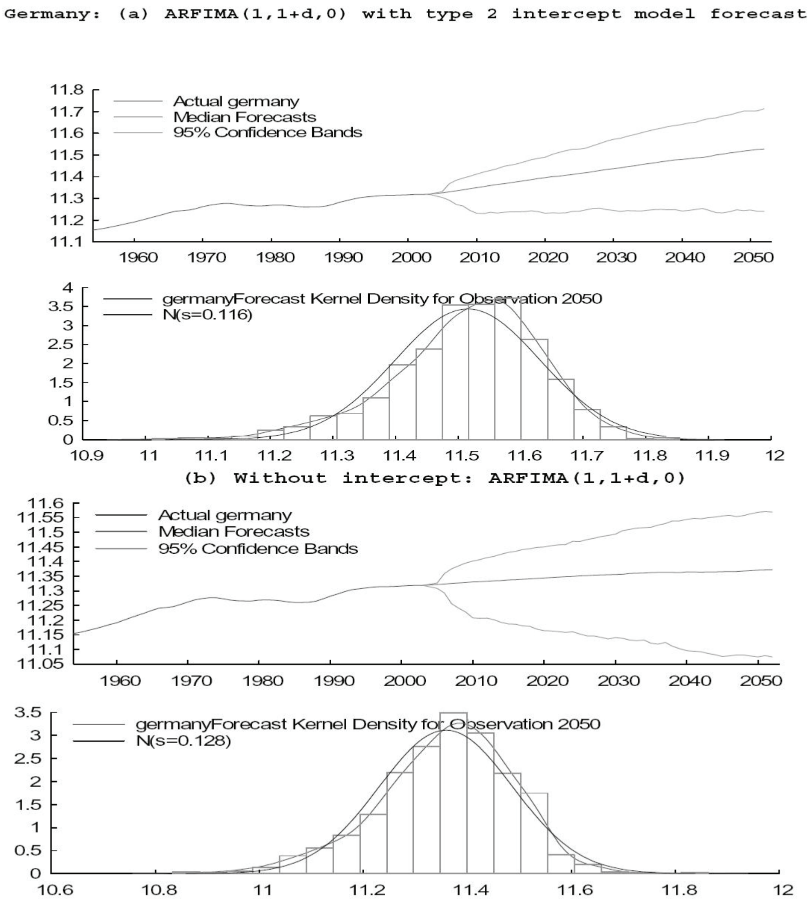
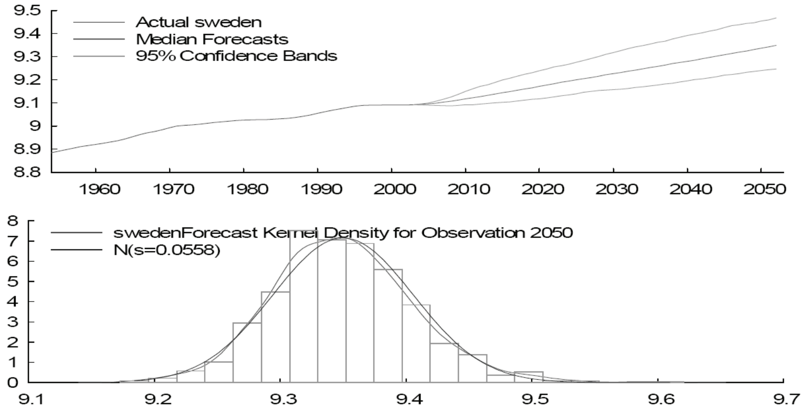


Figure 4.9: ARFIMA total population forecast for Sweden



(b) Without intercept: ARFIMA(1,1+d,0)

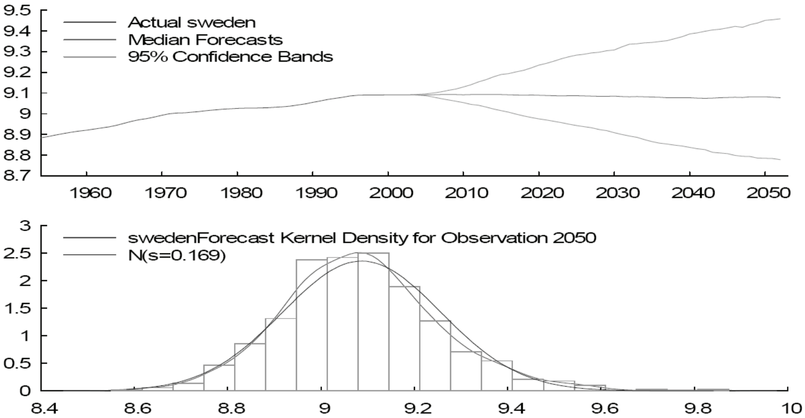


Figure 4.10: ARFIMA total population forecast for United Kingdom

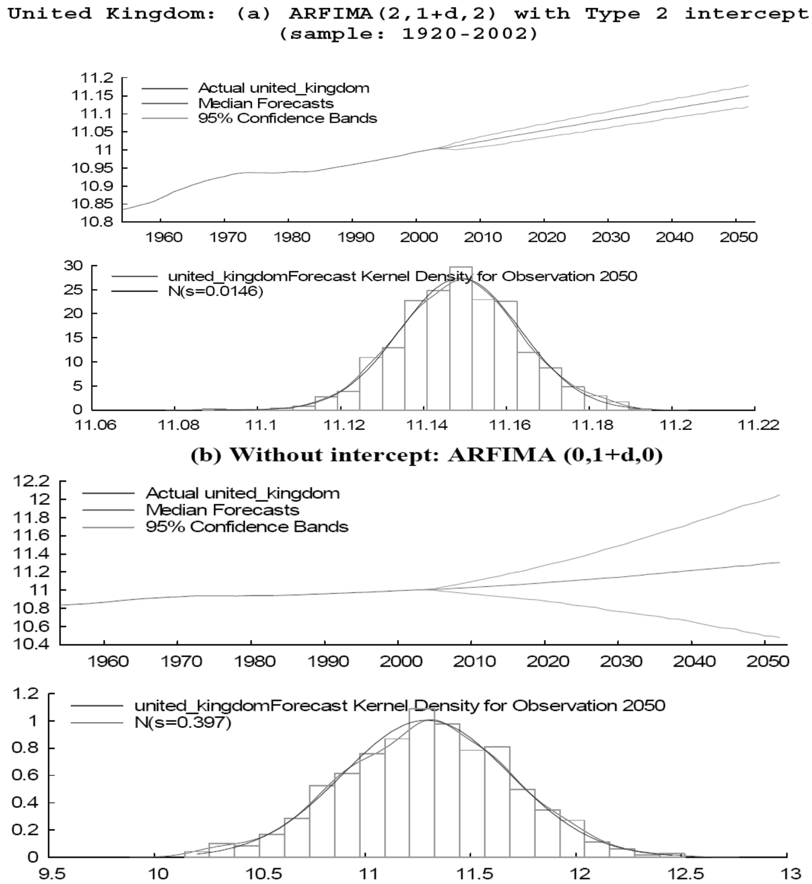
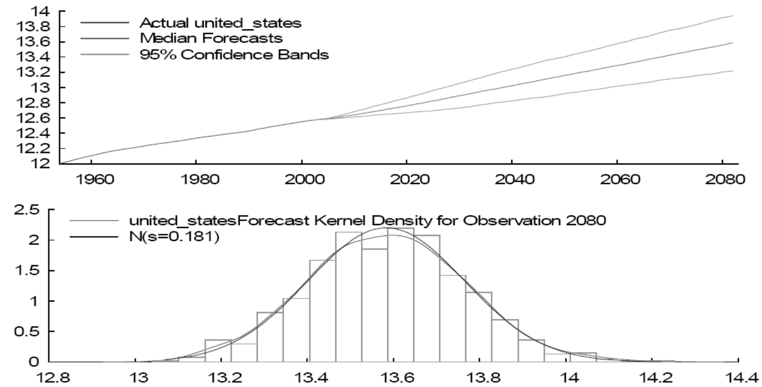


Figure 4.11: ARFIMA total population forecast for United States

United States of America: (a) ARFIMA(1, 1+d, 0) with type 2 intercept



(b) Without intercept: ARFIMA(1, 1+d, 0)

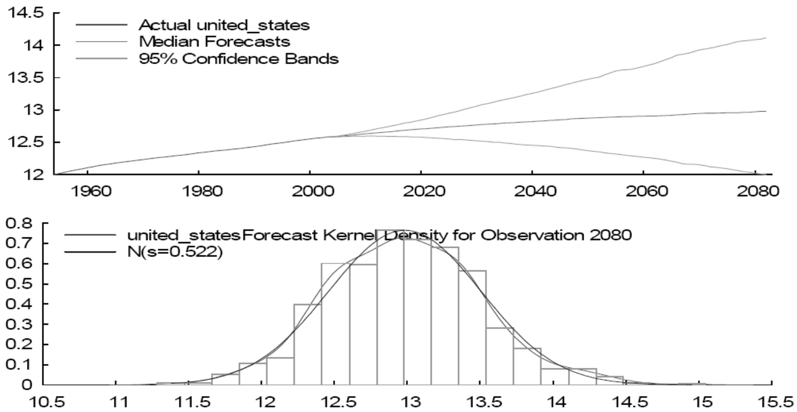
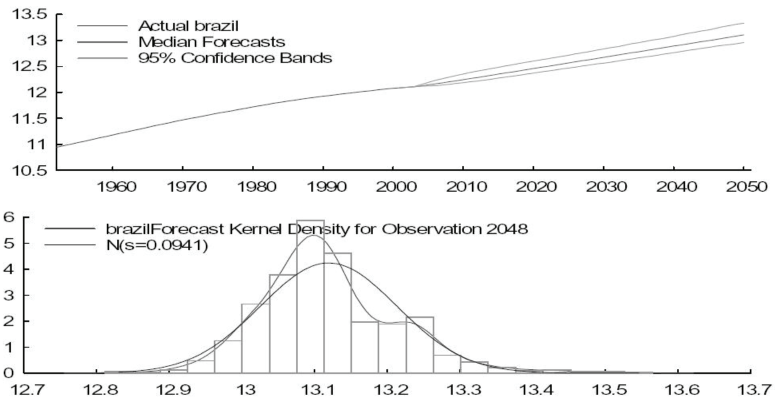


Figure 4.12: ARFIMA total population forecast for Brazil

Brazil: (a) ARFIMA(1, 1+d, 0) with type 2 intercept



(b) Without intercept: ARFIMA(2, 1+d, 0)

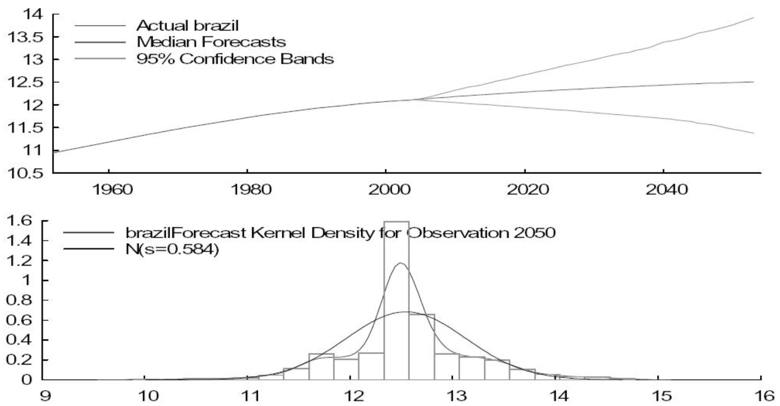


Figure 4.13: ARFIMA total population forecast for China

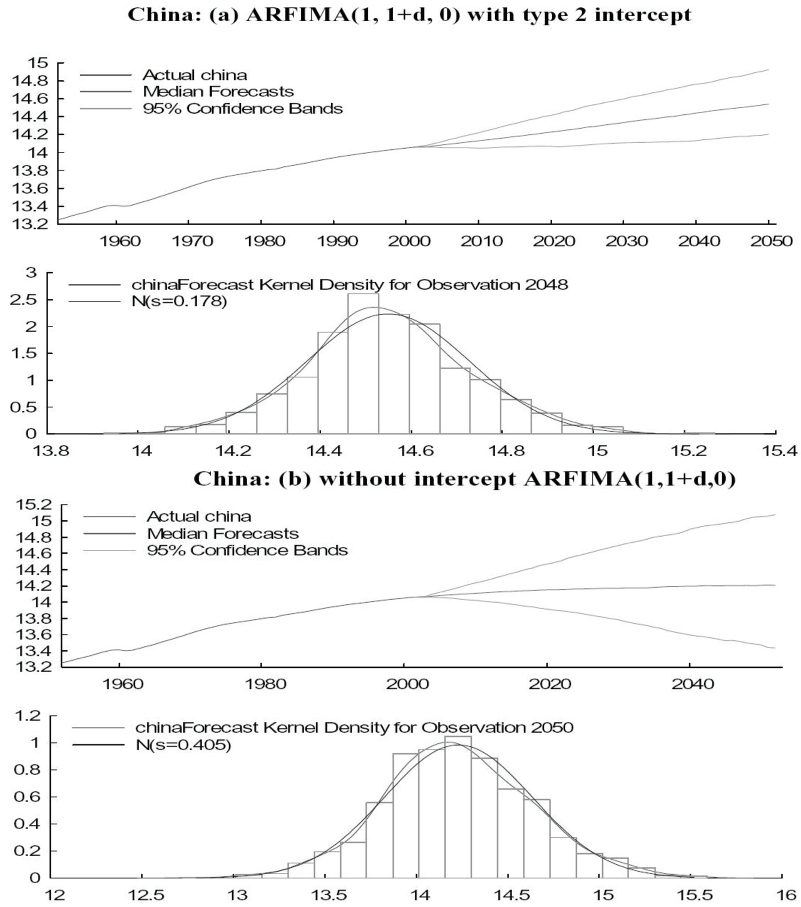
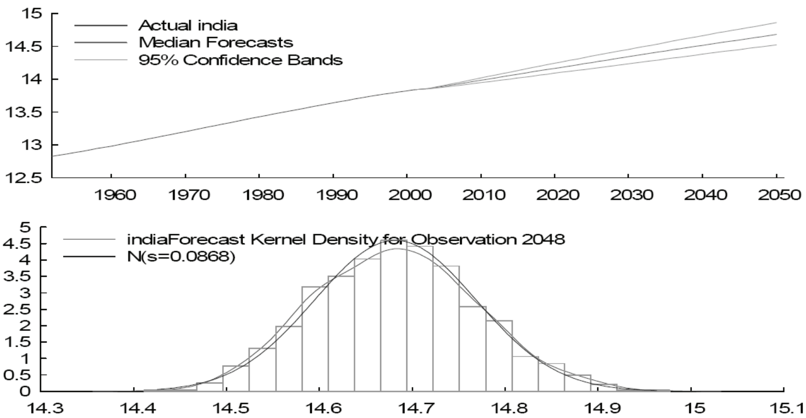
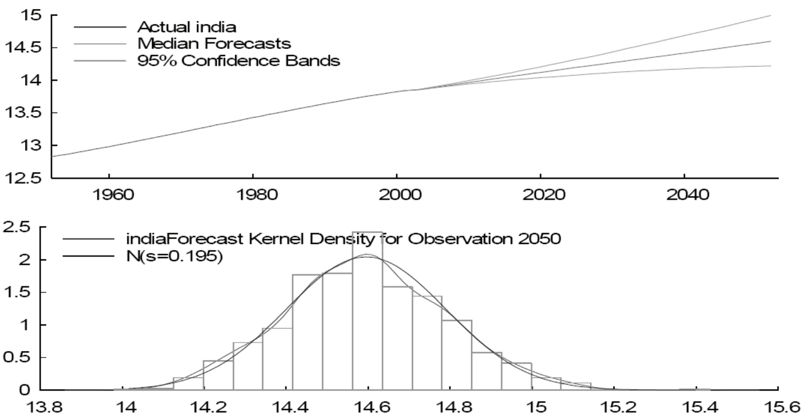


Figure 4.14: ARFIMA total population forecast for India

India: (a) ARFIMA(1, 1+d, 2) with type 2 intercept



(b) Without intercept: ARFIMA(2,1+d,0)



5. A Further Look into the Demography-based Income Forecasting Method

5.1 Introduction: The role of stochastic demographic process in income forecasting

An overwhelming spurt of research in the last two decades both in theory (e.g., Boucekkine et al. 2002) and empiric (e.g., Kelley and Schmidt, 1995; Malmberg and Lindh, 2005) emphasize that population growth, specifically the changes in the demographic components (viz., age structure, life expectancy rate, fertility and mortality rates, etc.,) exert substantial influence on economic growth and development. As Malmberg and Lindh (hereafter, ML, 2005) state, three arguments underline the importance of age structure for per capita income. First, the savings argument, which states that countries with high child-dependency rates will be low and this may lead to low productivity (Coale and Hoover, 1958). Second, a high dependency rate implies a low worker per capita ratio which directly leads to low per capita income due to pure accounting effect (Krueger, 1968; Janowitz, 1973). Third, as demonstrated by Lindh and Malmberg (1999), age structure within the working-age population is also of enormous importance. These arguments have important implications for long-term per capita income forecasting. Historically, such forecasts have been based primarily on assumptions about the rate of technological change.

In the empirical growth literature it has been suggested that a stable statistical relation exist between age structure and per capita income. Therefore, conventional population projections can be used to forecast future trends in income growth. An apparent outcome of this perceived advantage is a paradigmatic shift in economic growth forecasts – from the conventional *technology-based* forecasting to the recent *demography-based* forecasting. Malmberg and Lindh (2005) in an important research have provided the underpinning and usefulness of demography-based income forecasting method. In this chapter, we evaluate the key assumptions of the demography-based income forecasts and suggest modifications in the forecasting model by accounting for the 'dynamics' of demographic changes and possible presence and persistence of shocks while forecasting per capita income. We are motivated by the fact that embedding historical information in a model actually enriched its explanatory power of a future event. Demographic shocks of any magnitude – smaller or bigger – while being embedded in the income forecasting model is expected to take into account the (hidden) demographic shocks exogenous or endogenous.

The conventional methodological underpinning of the demography-based income forecasting method rests on the critical assumption that (components) of population remain 'stationary' (or stable over time) implying that a shock to the population growth series would not bring about remarkable changes in the future growth trajectory. In other words, demographic components are assumed to possess 'short memory' ability to remember past shocks. This typical feature of demographic variables provided the forecasters the necessary platform to increasingly employ them in long-run economic forecasting. Interestingly, a recent theoretical development – which demonstrates that due to its endogenous nature[57] population growth and its components may imply unstable (or chaotic) pattern – seems to have been overlooked in the forecasting literature. Following this theorization, a shock to the population series in the remote past can significantly affect its future growth trajectory and in turn, economic growth. Therefore, the future growth path of these variables can become very sensitive to their initial distributions (Prskawetz and Feichtinger 1995). Moreover, the population series may even experience many shifts due to endogenous cycles (or phase switch) caused by frequent demographic changes and changes in the demo-economic policy (Day, 1993). Thus, endogenous nature of population growth combined with endogenous phase switching can give rise to chaotic or unstable pattern in demographic variables. Recent empirical findings (e.g., Gil-Alana, 2003) also provide credence to this claim.

In view of these developments, it appears to us that the stationary assumption underlying the growth of population and its components is far too narrow as it downplays the role of possible shocks in the series which could have more than mere short-run impacts on their long-term projections as well as that of national income and other macroeconomic aggregates. In fact, the 'strength and length of memory' of demographic variables to remember past shocks governs their future growth path and shapes the pattern of interaction with the economic system. Taking this as the starting point, this chapter aims to provide a new dimension to the demography-based GDP forecasting methods by extending the domain of demographic variables from stationarity (i.e., no possibility of stochastic shocks) to nonstationarity (i.e., possibility of stochastic shocks which are characterized by long memory).

This point was taken in chapter 3, where components of population change are shown to be characterized by non-stationary processes. Building on this research, in this chapter we employ long memory data characteristics of population growth and its components to forecast per capita income of selected developed and developing countries. By doing so we embed historical information about the stochastic behavior of demographic variables in the forecasting model – a fact which was so far sidelined in the empirical demographic research. Though time series methods are receiving immense popularity in demographic forecasting processes (for instance Lee and Tuljapurkar, 1994), the propounded methods still lack flexibility and does not appropriately ac-

[57] In the sense that past population growth affects the economy so that it is endogenously determined as part of an interacting system.

count for the different demographic dynamics, precisely, the length of demographic shocks and its corresponding impact on long-run growth of the economy. The central aim of this chapter is to propose modifications in the conventional demographic forecasting methods by suggesting a long-memory process for the evolution of the demographic variables and consequently incorporate these dynamics in the forecast of per capita income of some developed and developing countries. To gain quick insights into income forecasting problems we summarize the main concerns and approaches in section 5.2. In section 5.3, we discuss the properties of our model and compare with ML (2005). Data and estimation issues are presented in section 5.4. Section 5.5 summarizes the main findings of the chapter.

5.2 Problems in Income Forecasting

Income forecasting is a challenging task. The conventional technology-based forecast suggests that future trajectory of income growth principally depends on technological change. However, empirical specification of the parameter of technology change is endowed with many problems, important of them is the extent of uncertainty inherent in technological change and the specification of other growth related variables such as inflation, money supply growth etc. Interestingly, recent research seems to have overcome some of the inherent difficulties in the technology-based forecasting method by suggesting an alternative, viz., demography-based forecasting of economic growth. Lindh and Malmberg (1999) show that variations in age-specific population growth account for a significant variations in some of the key macroeconomic fundamentals, like inflation, savings, etc. Therefore, demographic variables, like age shares are highly recommended as instrument for forecasting income growth. Nevertheless, this apparent appeal of demographic variables as instruments for economic growth forecasts may not be taken too easily as the implementation of the approach still remains a wide academic debate.

A recommended way to perform demography-based income forecast is to regress GDP on demographic variables and make forecast for some future date. It may be noted that, demographic projections are uncertain in nature. To a first order approximation this is a question of the assumptions made on fertility, migration and mortality in demographic projections (ML, 2005). Moreover, probabilistic demographic forecasts can also be included into the model to deal with this issue in an explicit way (e.g., Prskawetz et al. 2004). Drawing on the effect of age structure on economic growth, the authors derive the uncertainty of predicted economic growth rates using probabilistic demographic forecasts in case of India, where they combine the effect of social infrastructure alongside demographic variable (i.e., age structure) for forecasting economic growth. Though probabilistic methods provide certain range of values with confidence interval for forecast, its biggest limitation as being probabilistic has called for alternative methods. The regression approach *a la* ML (2005) assumes significance in this context.

Following Lindh and Malmberg (1999), the age-structure information can be used in the growth regression in panel data and forecast economic growth based on the demographic information. However, a common worry in panel data is the problem of heterogeneity both across countries and over time. Thus the question is whether it is legitimate to assume a homogeneous model for such a variety of countries, different in size, location, history, institutions and natural resources. In fact, in some sense, every country is a unique economic system related to its neighbors by a multitude of different relations (ML, 2005). The authors posit that:

> Using a panel estimation approach confers substantial advantages. Not only does the number of observations increase substantially, but it also allows us to control for unobservable that are constant over the estimation period as well as common time-specific effects. The price to be paid for this is that we need to assume that a more or less general model applies to all countries in the sample. It is, however, neither inconceivable nor impossible to account for some country differences within the model.

Among many possible problems in panel regression analysis (for instance, the presence of structural break), ML (2005) note the importance of regression of non-stationary time series which result in spurious outcomes. Phillips and Moon (1999) demonstrate that this problem can be substantially ameliorated in a panel context by the cross-section information. However, the extent of non-stationarity can still induce problem in panel regression. Bai and Ng (2003) and others suggest the use of stochastic common factor model to take account of the possible non-stationary feature of regressors in the panel data observing that in recent years panel data for many demographic and macroeconomic variables are available for large time and cross-section dimension. If non-stationarity is is a serious concern in individual time series, it can infest the same problem when considered in a panel data. Therefore, substantial amelioration of spurious outcome may not eliminate the problem completely. Care needs to be taken to treat the non-stationary nature of variables in the panel in order that one attempts to achieve a good forecast. A panel forecast method of GDP in line with ML but taking non-stationary demography features needs further theoretical and empirical development, which is not the focus of this chapter although we have preserved it for future research. In this chapter, we take note of the non-stationary problem of demographic variables and induct their characteristics in the GDP forecast for each individual country.

Finally, there is a problem of assuming a common data generating process (DGP) for all countries in the panel and perform income forecast for a set of countries in the global level. The standard way is to assume that cross-country observations are drawn from a DGP that is at least partly common to all countries, viz., the demographic transition and the concurrent industrialization and aging of the population. ML (2005) reiterate that while their observations from more developed countries provide some information to forecast the evolution of the less developed countries, their sample contains little information regarding the aging society and how it will adapt to a rising dependency burden. Recent work on non-stationary panel seems to offer a solution to this problem by identifying the set of countries in terms of the common stochastic shocks they share and then it is possible to forecast for those blocks of countries. While this could be an interesting direction of research, complying with the scope of this chapter we would only focus on univariate income forecast by incorporating stochastic demographic information in the model.

5.3 Model

This section outlines the usefulness of long-memory methodology for demography-based income forecasting. ML's (2005) forecasting technique is described first before we elaborate on the long-memory framework in the income-forecasting framework. ML start with a model for a panel regression in levels of the logarithm of per capita GDP, *y,* on the logarithm of age shares, x , and a trend function $V(t), t$ being the time period:

$$\log(y_{it}) = V(t) + \sum_{k=0-14}^{65+} \gamma_k X_{kit} + \zeta_i + \epsilon_{it} \tag{5.1}$$

ζ_i is the country-specific intercept, $k = 0-14, 15-29, 30-49, 50-64, 65+$. In this equation, GDP per capita is assumed to be described by Cobb-Douglas index of age-shares, and $V(t)$ is intended to capture technological change. This is a standard production function specification with the exception that population age shares have been substituted for production factor intensities. To incorporate the effects of life expectancy and heterogeneity, ML proposed the following model:

$$\log(y_{it}) = \alpha \log(e_{0it}) + \sum_{k=0-14}^{65+} (\beta_k + \theta_k \log(e_{0it}))\gamma_k X_{kit} + \zeta_i + \nu_t + \epsilon_{it}, \tag{5.2}$$

where $\log(e_{0it})$ is the log of life expectancy at birth. Theoretically, the model allows for changing age share coefficients contingent on how far the demographic transition has progressed. To account for time-specific effects, v_t has been added to the equation. ML (2005) thus describes *simple model* (Eq. 5.1) and *interaction model* (Eq. 5.2) to analyse the effect of demographic variables on per capita income. Based on their previous work (Lindh and Malmberg, 1999), ML suggests that an aggregation of the age groups (viz., children 0-14, young adults 15-29, mature adults 30-49, middle aged 50-64, and old age 65+)

works well in growth equations without running into the collinearity problems. The limits for these functional groups are not exact. However they vary both with time and culture, as well as the institutions that transmit and govern the economic effects of the age group. ML assumes that this specification is a pragmatic approximation for estimating growth effects from the continuous age distribution. The age distribution in turn proxies for the actual functional changes over the life cycle which are the real causes for the income effects.

Equations 5.1 and 5.2 assume that demographic variables, viz., age-specific population and life expectancy, are stationary. Non-stationary nature of these variables in the panel data would cause spurious regression as mentioned above. However, as we have demonstrated in chapter 3 that age-specific population may contain long-memory component and therefore shocks are persistent in these series. This observation might create additional problem in the panel regression. Bai and Ng (2002, 2003), and Im et al. (2003), among others provide formulation to deal with nonstationarity in panel data that enables testing of unit root in a panel framework. A natural question that may arise is: what if one or all of the demographic variables (in Eq. 5.1 or 5.2) are non-stationary? How do we forecast if demographic variables are characterized by a long-memory process? It is closer to reality to assume that demographic variables might be affected by some shocks, endogenous or exogenous, which can stay with the series for some period of time in the future. Apparently, ML's specification rules out the possibility of such shocks and if at all some exist, the authors argue, are ameliorated by panel structure.

Granger and Joyeux (1980) showed that long-memory in a time series can arise due to aggregation of individual series. Even though the extent of memory of shocks is ameliorated in the panel, the effects are not completely neutralized. Moreover, individual countries demographic dynamics can substantially affect forecasting performance. To address these concerns let's consider the DGP of y_{it} in a long-memory framework. The income growth equations are described with and without demographic variables, similarly as ML's (2005) *simple* and *interaction* model. We re-write ML's equations in a long memory framework with one notable exception. The formalization of our model concerns with univariate long-memory framework, as there is virtually little literature on the study of long-memory characteristics in a panel data set up. Two variants of the model are considered, viz., model with and without demographic structure. In case of the former, we first introduce population growth, and then induct age-shares information. Aggregate population growth is assumed to suppress dynamic information in the model, as aggregation greys out dynamic behavior of individual components of population. This problem can be ameliorated by introducing age-shares, which exhibit wide variability and are dynamically linked to GDP fluctuations. For a discussion on this refer to Lindh and Malmberg (1999). The following equations describe our model.

$$Model1: (1-L)^d\Phi(L)(\log(y_{it})-\mu_{it}^1) = \mu_{it}^2 + \Theta(L)\epsilon_{it} \quad (5.3)$$

$$Model2: (1-L)^d\Phi(L)(\log(y_{it})-\mu_{it}^1) = \mu_{it}^2 + \gamma n_{it} + \Theta(L)\epsilon_{it} \quad (5.4)$$

$$Model3: (1-L)^d\Phi(L)(\log(y_{it})-\mu_{it}^1) = \mu_{it}^2 + \sum_{k=0-14}^{65+}\beta_k x_{kit} + \Theta(L)\epsilon_{it} \quad (5.5)$$

L is the lag operator, $\Phi(L)$ and $\theta(L)$ are autoregressive (AR) and moving average (MA) polynomials. $i=(1,...,N)$ refers to countries, and $t=(1,...,T)$ denotes time. ε_{it} is assumed to be normally distributed. n_{it} is the aggregate population growth rate. μ^1 and μ^2 are (Type 1 and Type 2) intercepts. Type 1 intercept accounts for structure and changes in the dependent variable, i.e., whether an independent and/autonomous factor govern the growth of the dependent variable. Type 2 intercept enters as an explanatory variable, which in the absence of other regressors, account for some independent exogenous changes occurring in the system. x_{kit} are population age shares. $(1-L)^d$ describes the fractional differencing operator which is given by

$$(1-L)^d = \sum_{j=0}^{\infty} h_j L^j \quad (5.6)$$

where $h_0 = 1$ and

$$h_j = \frac{-d\Gamma(j-d)}{\Gamma(1-d)\Gamma(j+1)} = \frac{j-d-1}{j}h_{j-1}, j \geq 1. \quad (5.7)$$

Note that Type 2 intercept is induced in the model as an exogenous drift. The contribution to the process takes the form $(1-L)^{-d}\mu_{it}^2$, which since the pre-sample terms are truncated, gives a sequence of the form

$$\mu_{it}^2 \sum_{s=1}^{t} h_s = O(t^d), \quad (5.8)$$

when $h_t = O(t^{d-1})$, by a standard result on summation series. This implies that the process is non-stationary, with infinite mean and variance in the limit, for $d > 0$. For fractional process without drift, the model is stationary for $0 < d < 0.5$.

Equation 5.5 is the univariate long memory representation of ML's (2005) *simple model* which incorporates only population age shares. *Interactive* model as in ML (2005) with life-expectancy at birth can be introduced; however we given the objective of this chapter, it is not required at this stage. Since the thrust of the chapter lies in introducing fractional feature of demographic variables in GDP forecasting, long-memory dynamics in age-specific population can to some extent account for inclusion of life-expectancy in the forecast, although with no absolute certainty. Our idea is to keep the model simple and study the effects of long-memory on GDP forecast. Therefore, the interaction variables as in ML can be introduced in future research.

The DGP described by Eq. 5.3 states that y_{it} is governed by the structure of memory, the autoregressive and by moving average polynomial representation of *iid* shocks. Eq. 5.4 has broader encompassing as it accounts for the effect of aggregate population growth. Eq. 5.5 is still broader as it segregates the total population into age shares and plugs them into the model. The peculiarity of these equations is that we allow for the possibility of demographic dynamics in the growth equation, where shocks can have more than mere short-run impacts on the historical trajectory of y_{it}. In fact, depending on the non-integer values and sign of d, short, long, or intermediate memory properties can arise. For instance when $d < 1/2$, the series has finite variance, but for $d = 1/2$, the series has infinite variance. The y_t is stationary and invertible when $-1/2 < d < 1/2$. For $d = 1/2$, standard Box-Jenkins techniques will indicate that differencing is required and provided that $d < 1$, differencing will produce a series whose spectrum is zero at zero frequency. This heavily-used model is a special case of an autoregressive fractionally integrated moving average *(ARFIMA(p, d, q))* process.

A detailed description of the properties can be found in the survey of Baillie and Bollerslev (1994). In the demographic context see Chapter 3 for a comprehensive analysis. Ding, Granger and Engle (1993) suggests that ARFIMA models estimated using a variety of standard estimation procedures yield 'approximations' to the true unknown underlying DGPs that sometimes provide significantly better out-of-sample predictions than AR, MA, ARMA, GARCH, simple regime switching, and related models, with very few models being "better" than ARFIMA models, based on analysis of point mean square forecast errors (MSFEs).

• *Estimation strategy*

ML (2005) employ a panel data framework to forecast global income. In this chapter, we resort to univariate forecast of world income as well as the income of a sample of developed and developing countries with and without consideration of demographic age structure. Our strategy is as follows. First, employing ARFIMA methodology we forecast total and age-specific population of different countries till 2050. The population age-structure in our case comprise of three categories, viz., young 0-14, working age 15-64, and retired cohorts 65+. Thus, we perform long-memory forecast of total population and each age group and based on the forecasts, we calculate population growth rate,

$n_t = [ln(TotalPop_t) - ln(TotalPop_{t-1})]$
for each country.

The age shares are calculated as:

$$x_{(0-14)it} = \left(\frac{PopulationAge(0-14)}{TotalPopulation}\right)_{it},$$

$$x_{(15-64)it} = \left(\frac{(PopulationAge(15-64)}{TotalPopulation}\right)_{it},$$

$$\& \ x_{(65+)it} = \left(\frac{(PopulationAge(65+)}{TotalPopulation}\right)_{it}$$

for $t = (1960,...,2050)$.

Second, we perform ARFIMA regression of y_{it} using the regressors as in Models 2 and 3 taking into account the demographic information and availability of GDP data till current period (in our case it is 2000), and then use the parameter estimates to forecast GDP till 2050 given our forecast population growth and age shares till 2050. ML (2005) use medium variant population projection till 2050 to forecast GDP. Contrarily, we have used our time series forecast of age-specific population which takes into account the demographic variations and persistence of possible shocks in the economic and demographic system. For notational convenience, we will refer to Eq.5.3 as Raw model, and Eq. 5.4 and 5.5 as Demographic models.

5.4 Data and Empirical Results

5.4.1 Data

In this section we discuss the forecasting results based on raw and demographic models of GDP. The results are compared with ML (2005) and implications are drawn from the analysis. We have used data for real GDP per capita (collected from Penn World Table version 6.1 by Heston et al. 2002) at purchasing power parity in 1996 US dollar. Information on age-specific population has been collected from the World Bank Development Indicators. For real GDP, the sample is from 1960-2000 and for age-specific population the sample extends till 2003. We have selected a set of developed and developing countries to compare our results with ML (2005). The selected countries are Belgium, Sweden, USA and Japan among developed countries and India and China, among developing countries. We have also performed forecasting for the World real GDP data to study the pattern of global variation of income till 2050. The results are discussed in two steps. First, we analyse the pattern of age-specific population till 2050 for different countries. Specifically, we will concentrate on providing intuition on how population of different vintages would act upon economy's resources. Second, based on the calculation of age-shares and population growth rate, the forecasting results are discussed. In course of comparison, reference is made to the forecast from the raw model as it would provide an idea about how the allowance of demographic information in the model changes forecast pattern.

5.4.2 Empirical results

Variations in age shares

In this sub-section, we analyze the variations in age shares till 2050 for the selected developed and developing countries. The analysis is purported to give an idea of the effect of different age shares on economy's resources. Rising younger age population (i.e., 0-14) mounts pressure on the economy by consuming resources that could have otherwise been used for capital formation. Working age population (i.e., 15-64) contributes to economy's growth by creating resources. The retired age population (i.e., 65+) also exerts pressure on the economy, because like younger cohorts they also force government to plan a chunk of economy's resource for consumption, pension, and retirement benefits. An economy therefore, needs to plan beforehand for the inter-generational distribution of resources considering how different age-shares would look like in future, say five decades from now. A meticulous economic planning is therefore proves handy for efficient management and mobilization of resources. More so, the dynamics of age-share movement is important for explaining long-term growth of income. To have an idea about how various age shares in some developing and developed countries behave, refer to Figure 5.1.

Population age shares till 2050 have been calculated based on the time series projections of age-specific and total population of different countries till 2050. Unlike ML (2005), who relied on the medium variant UN projections, we have performed an ARFIMA forecast for total and age-specific population. Figures for total population forecasts have been adopted from our earlier estimates (from Chapter 4).

A striking 'common feature' among all the developed and developing countries (in Figure 5.1) is that young age population share (0-14) will continue to fall in the coming decades, at a faster rate for developed countries (viz., Belgium, Japan, Sweden, and USA) and slower for the world and the developing ones (viz., China and India). This is not surprising given the recent trend of population growth in developing and developed economies. For the former, population growth of young cohorts will decline slowly as the current high rate will guide its future trajectory. Similar logic applies for developed countries where the current lower rate of young population growth would further lower the rate in the coming decades. The pattern is a clear indication of an autoregressive structure, where past high (low) growth of population results in current high (low) growth.[58]

Some striking features emerge from the plot of age-specific population age-shares (see Figure 5.1). First, the number of worker (i.e., population 1564) seems to experience a steady global rise. Similar trend is observed among developing countries, viz., India and China, which will continue to dominate the economic power in the coming decades. Though the share of younger cohorts will continue to fall for these two most populous countries, China is likely to experience a rise in retired cohorts, which is more than India's in 2050. Due to the smaller and declining share of retired age population, India is likely to be in a better position in terms of economic growth, as she would divert lesser resources for consumption end.

A typical situation is observed for the European countries, e.g., Belgium and Sweden, where till the recent period, young and retired people age share are almost at par, however the share of the latter is deemed to gradually exceed the young age share till 2050. Similar structure is also observed for working age share, therefore these countries will experience a similar trend in GDP growth in the next decades. Among developed countries, USA's working age people share will remain constant throughout the coming decades though a steady decline will be observed for young and retired age shares. Given these dynamic demographic information, in the next subsection we examine the pattern of income forecast for these countries and compare our results with ML (2050).

Real GDP per capita and age-specific population forecast

Table 5.1 summarizes the ARFIMA forecasting models, which are selected on the basis of Schwarz criteria and highest likelihood of the estimated models. The GDP forecast plots[59] for each country and that of the World are presented in Figures 5.2 through 5.8. We have estimated *ARFIMA(p,d,q)* model with a maximum order of p and q set equal 2. The chosen model for each country has been used for forecasting. The regression results are reported in Table 5.2. Forecasting results with and without demographic information are presented in Table 5.3. All estimations have been performed in *Time Series Modelling (TSM)* package of James Davidson (2005).

[58] Note that, demographic process evolves in a slower pace than other economic processes, as multitude of factors act and interact with demographic process to ensure faster and slower evolution.

[59] The age-specific forecast plots have not been reported in the chapter to limit space.

Figure 5.1: Plot of Age-specific population age shares: 1960-2050

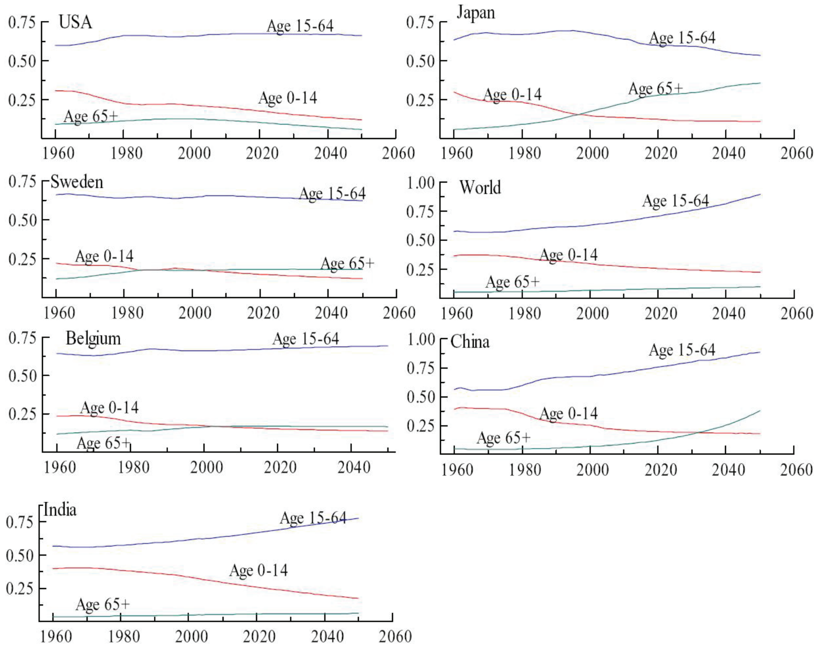


Table 5.1: Selected ARFIMA(p,d,q) Models for Forecasting

Countries	Age 0-14	Age 15-64	Age 65 +	Real GDP	Real GDP
				(Pop Growth)	(Age Shares)
Belgium	(l,l+d,2)	(2,l+d,2)	(l,l+d,0)	(2,d,0)	(l,d,0)
Sweden	(l,l+d,0)	(l,l+d,0)	(l,l+d,0)	(l,d,0)	(l,d,2)
Japan	(l,l+d,0)	(l,l+d,l)	(l,l+d,0)	(l,d,l)	(l,d,0)
USA	(l,l+d,0)	(l,l+d,0)	(l,l+d,0)	(2,d,l)	(l,d,2)
China	(l,l+d,0)	(l,l+d,2)	(l,l+d,0)	(0,d,0)	(0,d,2)
India	(l,l+d,0)	(2,l+d,0)	(l,l+d,0)	(2,d,0)	(l,d,0)
World	(l,l+d,0)	(l,l+d,0)	(l,l+d,0)	(l,d,0)	(0,d,0)

Note: Model selection based on Schwarz criteria.

Table 5.2 presents the parameter estimates[60] of ARFIMA regression of real GDP per capita (at 1996 price PPP) and share of age-structured population for a set of developed and developing countries. In general age-specific population are observed to exert expected impacts on the countries income per capita, viz., theoretical caveat is that age 0-14 have negative, age 1564 exert positive and age 65+ have negative effect on the resources of an economy. While theoretical prediction about the sign of effects stand true under the most general circumstance, say under linearity assumption of the model, it verily depends upon the economy's strength in the form of 'how quickly the feedback-effect' takes place from the accumulation of these groups of population. From Table 5.2 we observe that all the countries exhibit expected effects for the specific age-groups.[61] We also find evidence of significant stochastic shocks in the models. Therefore, non-inclusion of such shocks in the forecasting model (as assumed in ML for instance) may not reveal much about the trajectory and impact of demographic shocks in delivering a better forecast. Positive and larger d in the model indicates long-memory population shocks, which in our estimates are mostly mean-convergent; larger demographic shocks can induce high non-linear interaction between the demography and economic growth system.

From Table 5.3 it can be observed that younger age population is likely to fall in all developed and developing countries. For instance, among the European countries, Belgium will experience a fall from about 1690.4 thousand in 2010 to 1485.5 thousands in 2050 (See Fig. 5.3 for the corresponding estimates). Whereas for Sweden it is 1499.7 thousands in 2010 and 1383.1 thousands in 2050. Given the current trend this would mean, Sweden will experience about 10.3 percent decline and Belgium with a 15.5 percent decline in the younger population. At the same time, these countries would see an increase in the number of working age people, viz., Sweden about 18 percent and Belgium about 8.37 percent. Given the number of retired age people in 2005 (viz., 1754.6 and 1587.6 thousands for Belgium and Sweden respectively), there won't be substantial change in these age groups in 2050. The effect of these age-structure changes can be calculated for GDP in 2050.

[60] For Belgium, China, India, and the World the GDP series have been detrended before estimation such that age share variations do not get affected by a deterministic trend in the ARFIMA regression. The purpose of de-trending these series is to remove the possible cyclical components from the series such that age-share variations can adequately capture variations in GDP over the period of regression.

[61] While the parametric specification of the model often delivers expected sign of impacts of the respective age shares (sometimes eliminating the stochastic trend term from the data and taking care of the possible heteroscedasticity), Azomahou and Mishra (2006), using a non-parametric panel method showed that this may not necessarily be the case. For instance, retired age group may not necessarily exert negative impact on growth as the working age people once retired, do not become instantaneously unproductive. However this statement has to be qualified in light of the development in developing countries where majority of the population are unemployed.

Examining the case of Sweden in Table 5.3 (and the corresponding forecast plot, Fig. 5.5, we find that the per capita GDP would stand at about 45752.4 dollars in 2050 without demographic variations in the forecasting model (Model 1). However, once stochastic aggregate population dynamics is embedded in the model a significant rise in the per capita GDP forecast is observed (which is 53156.7 dollars in 2050). Further improvement in the forecast is warranted once stochasticities in the population components (Model 3) are taken into account. The results are indicative of the fact that (1) a demography-based GDP forecast puts an optimistic figure for future, (2) Comparing the estimate (which is 59754.5 dollars) with ML (2005) estimate for Sweden for the year 2050 (54000 dollars in the interaction model), we find a slightly higher forecast for GDP per capita, possibly due to our consideration of long-memory features of demographic shocks. Notice that although inducting demographic information in the forecasting model delivers higher prediction than the raw model (where no demographic information is included), it is difficult to judge whether consideration of demographic dynamics alone can exude better and higher forecast. We have not so far considered any competing model (like a forecast model with other non-demographic variables) to lend a comparison. However, our assessment is based on a priori finding of some researchers like ML (2005) that demography-based income forecasting models are as good as other competing framework. In general our results comply with ML's conclusion about the relevance of demographic dynamics in income forecasting.

In ML(2005) a hump around 2010 for Sweden is observed, where after reaching an estimated income level of about 48000 dollar, the amount declines to about 36000 dollar in four decades. Our forecast does not predict such humps,rather it shows a steady increase over time. The possible reason may be in the assumption of the data generating process (DGP). Long-memory DGP assumes certain degree of smoothness, where the forecast is made simultaneously considering the effect of shocks (the memory parameter), the endogenous system (the autoregressive parameter) and some possible external shocks (the moving average parameter), besides the built in demographic information for the forecast. ML's DGP follows a panel structure, and it is possible that due to the differences in DGPs, the smoothness of the forecast may follow in one and disappear in the other.

ML (2005) showed that due to recent baby boom around 1990, Sweden would have very fast growth over the next two decades while the US would stagnate earlier and Japan already stagnated. The interaction model which loads increased longevity has a much more positive path but still stagnating in the long run. Similarly, considering some examples of less developed economies, the authors showed that the 'difference between forecasts between India and China have a similar pattern as for the USA although at lower levels and the simple model stagnates later in China and later still in India'.

Table 5.2: Parameter estimates of ARFIMA regression between GDP and age-structured Population relation. Sample (1960-2000)

Country	Belgium	Sweden	Japan	USA
Intercept	11.280 (0.511)	9.335 (0.04)	6.626 (2.310)	11.553 (0.34)
Age 0-14	-7.404 (4.283)	-1.364 (1.816)	-1.052 (0.858)	-0.795 (1.291)
Age 15-64	1.918 (3.250)	1.503 (2.236)	1.571 (1.019)	8.939 (2.460)
Age 65+	-5.937 (3.787)	-0.308 (1.231)	-1.752 (1.081)	-0.231 (3.578)
d	-0.113 (0.030)	0.653 (0.221)	0.029 (0.004)	0.241 (0.220)
AR1	0.587 (0.060)	-0.153 (0.181)	0.492 (0.050)	
AR2				
MAI		-0.987 (0.170)		-0.128 (0.125)
MA2		-0.289 (0.130)		-0.015 (0.022)
R^2	0.99	0.99		0.99
	China	India	World	
Intercept	8.936 (0.915)	6.444 (0.163)	9.056 (3.336)	
Age 0-14	-8.132 (0.970)	-1.766 (0.536)	-3.316 (0.510)	
Age 15-64	1.389 (1.309)	2.041 (0.833)	0.482 (0.292)	
Age 65+	-0.835 (6.094)	-9.847 (8.843)	-0.410 (0.506)	
d	-0.022 (0.008)	0.544 (0.140)	0.017 (0.002)	
AR1	0.438 (0.265)	0.196 (0.124)		
AR2	-0.089 (0.140)			
MAI	-1.064 (0.072)			
MA2	-1.040 (0.072)			
R^2	0.98	0.99	0.97	

Note: Standard errors are in parentheses.

Comparing our long memory GDP forecasts[62] with ML's it can be observed that India's annual per capita GDP will grow to about 11000 dollar in 2050 using Model 3, i.e., with age shares. In case of China, the same model forecasts 17094 dollars in 2050. These estimates are higher than ML's interaction model (Table 5.4). Note that the raw model does not predict substantial difference in the forecast between India and China. However, as we induct demographic information in the model, viz., model 2 and 3, the differences become prominent. For instance, accounting for population growth in the model, China would have per capita GDP of 14401 dollars in 2050, while India will have 10532 dollars during the same time. For India the increase is very little, which is about

[62] Note that our forecasts are basically point forecasts sequentially performed over long period of time. Although these forecasts do not reveal much about parameter uncertainty in comparison to interval forecasts, a study of the estimated confidence interval for the point forecast provides some idea about the range of values the forecast would fall. Moreover, all the forecast plots accompany density forecast figures to help explain the amount of uncertainty. Standard practice in time series based forecasts is to take account of point forecasts, at the least while estimating an ARFIMA type of model.

200 dollars more than the raw model. For China, Model 2 improves forecast about 4000 dollar more than the raw model (model 1 without demographic information). The forecast further widens when we accommodate age-shares in the model. For India, there is a significant change of forecast values from 10532.3 dollars with Model 2 to 11204.5 dollars in 2050. For China, although the figure is much higher than India's, looking at the growth (from 2010 till 2050), it can be easily seen that, India's income growth is faster than that of China. The possible reason could be due to the specific pattern of age-share variation (as explained in the preceding subsection). The 95% confidence band for these estimates are provided in Table 5.5.

Table 5.3: ARFIMA(p,d,q) Forecast of Global Income and Population Age

	Years	Age 0-14	Age 15-64	Age 65+	RGDP	RGDP(Pop)	RGDP(Age)
	(in ’000)	(in ’000)	(in ’000)	(US dollar)	(US dollar)	(US dollar)	(US dollar)
Belgium	2010	1690.4	6996.0	1790.1	25616.7	26849.4	26574.0
	2020	1621.3	7176.7	1812.7	27173.6	30424.4	30481.3
	2040	1528.1	7392.3	1802.4	29941.5	36461.1	40598.0
	2050	1485.5	7461.4	1792.9	31101.1	38330.5	48565.1
Sweden	2010	1499.7	5984.9	1641.7	27419.2	27364.4	27419.2
	2020	1453.9	6242.3	1749.2	31319.6	32663.1	33189.9
	2040	1404.7	6794.0	1954.5	40578.8	45297.2	49069.8
	2050	1383.1	7089.7	2077.0	45752.4	53156.7	59754.5
Japan	2010	16903.9	83033.5	29173.0	28652.6	32209.0	33657.8
	2020	15610.5	76496.5	40741.4	32273.4	37835.4	41647.6
	2040	13492.6	57988.5	81145.6	39458.3	48242.7	62630.0
	2050	12499.0	47382.1	114119.3	44002.4	53050.5	75886.9
USA	2010	61944.9	210238.9	36351.8	34787.0	34821.8	44311.5
	2020	62317.7	237755.9	36098.3	36827.5	39104.8	54176.3
	2040	61389.9	304979.5	32048.3	39695.8	45889.9	63196.2
	2050	61759.3	343863.5	29378.0	41233.2	49217.3	66303.6

table continued…

China	2010	279847.5	989544.5	119252.7	4226.0	5312.9	7050.1
	2020	252710.5	1143952.6	166208.8	5406.7	7244.5	12400.1
	2040	207939.0	1510573.6	325787.3	8326.5	11864.4	17213.2
	2050	187587.4	1734101.5	456343.0	10024.6	14401.7	17044.1
India	2010	349759.1	766814.3	65447.3	3320.9	3449.6	3501.2
	2020	347666.9	938464.2	87640.6	4427.5	4571.5	4456.1
	2040	317426.0	1405635.4	157156.9	7704.0	7797.0	8558.7
	2050	294195.6	1715131.0	210449.3	10328.9	10532.3	11204.5
World	2010	1856122.1	4497354.9	508387.8	6517.2	6478.2	6447.3
	2020	1920315.3	5261885.6	606221.0	6840.4	6985.6	7302.1
	2040	2028891.2	7060313.4	805324.0	7416.0	7728.0	8037.4
	2050	2078173.6	8244031.1	914378.6	7615.2	7956.1	8351.8

Note: RGDP: Real GDP forecast without demographic information, RGDP (POP) is with population growth and RGDP(Age) is with age share information.

Table 5.4: Malmberg and Lindh GDP Forecasts (in dollars)

Countries	Simple model	Interaction model
China	9000	13500
India	7200	10200
Japan	40000	75000
USA	44000	62000
Sweden	42000	54000

Note: Figures are calculated from Malmberg and Lindh (2005) for the year 2050. Actual figures are not available, hence these are closer approximations.

Among developed countries, Japan's per capita GDP is estimated to be higher than other countries (both developed and developing). The raw model forecasts 44002.4 dollars in 2050, which increases to 53050.5 dollars when we introduce population growth in the model. However, a hopping 75886.9 dollars is reached when we incorporate age-share dynamics in the forecasting model.[63] ML's interaction model forecasts about 75000 dollars, although the simple model (without life-expectancy rate) projects GDP about 40000 dollars for Japan in 2050. For USA, the demographic model with population growth projects GDP per capita at 49217.3 dollars with the lower limit of the 95% confidence band calculated at 45615.4 dollars and upper limit at 51948.1 dollars (Table 5.4). While for age-share model, the forecast is still higher (65186.0 dollars) which is also more than ML's estimates from simple and interaction models. The lower and upper 95% confidence band for our age-share model are 59994.0 dollars and 70122.6 dollars respectively. Generally tighter confidence bands are indication of lower amount of uncertainty where the forecast value would range between 5 percent confidence interval. Looking at the forecast values and their confidence band we observe narrow confidence band for our forecasts which is a rough measure of predictive uncertainty. Predictions from a model with lower uncertainty are the ones which are more reliable. However, to examine if our forecasts are 'accurate', we need to find an alternative measure.

Note that accuracy of forecast is related to reliability in the following way: Accuracy = precision + reliability. For our purpose it is necessary to comment

[63] Even though Japan's population shows in general a declining trend except an expected continuous rise of retired population till 2050, a declining population does not necessarily entail negative economic growth. Productivity growth can still boost GDP per capita, and if large enough, even overcome the effect of population decline. The coefficients of stochastic shocks, *d* is higher for USA and lower for Japan. Greater magnitude of long-memory demographic shocks would reduce predicted values while interacting with different population components. Although the concomitant rise in retired population and a fall in the workforce, Japan is accompanied by systemic social changes including the employment system, the social security system and the financial system where this high-per capita income appears plausible and sustainable.

upon the precision of our forecasts. Standard convention to check for forecast accuracy is to examine either the *ex post* error terms or to simulate the *ex ante* errors. Concerning the first possibility, a rough measure of forecast accuracy is therefore to compare the mean-square error of the models although a study of the simulated ex-ante error terms can throw light on the predictive accuracy. To compare between 'raw and demographic' models we may take note of the AIC (Akaike Information Criterion) values; a model with higher AIC is generally a better model and more informative. Comparison of the mean square error across models would reveal which of them have better predictive accuracy. Table 5.6 reports results of the mean square errors from the estimation of forecast models with and without demographic information (Model 3 here). It is evident that the mean square error is smaller for the demographic model for all countries and therefore our stochastic demography-based income forecast model can be said to provide better prediction than the raw model.

For USA the younger population will remain more or less constant over the decades, while work force would increase and the number of retired people will also experience concurrent decrease. This seems to have an income effect which would mean that the less number of retired people would continue to contribute to the income growth along with the then current work force. Given the constant growth of younger cohorts, USA is likely to be in the advantage and might experience accelerating growth in income in the coming decades. However, Japan's income growth will far exceed USA in 2050 and would be the richest nation on the earth. Also it may be noted that World income will continue to grow along with each age-specific population group. In 2050, the per capita GDP for the world will be 8453.2 (with Model 2) which is a growth of about 36 percent in 5 decades. Inclusion of demographic variations increase forecast from 7615.2 dollars to 7956.1 dollars (using population growth) and 8453.2 dollars (with age shares). A general trend thus may be noted from 5.3 – that inclusion of demographic information improves forecast. Raw model does not incorporate demographic variations, and therefore, GDP forecasts can be assumed to be governed mainly by exogenous shocks in the form of moving average parameters, or some endogenous shocks (reflected in the form of autoregressive structure). However, corroboration of demographic information enriches the forecasting model so that variations in income can be accounted for by demographic variations.

Table 5.5: Confidence Band for Real GDP per capita Forecast (in US dollars)

Country/Variables	Lower 95% CI	Upper 95% CI
BELGIUM		
1. Pop Growth	32016.3	45752.4
2. Age Shares	45379.4	51042.1
CHINA		
1. Pop Growth	2070.4	74906.8
2. Age Shares	6730.5	22814.4
INDIA		
1. Pop Growth	5171.4	20054.0
2. Age Shares	7492.1	18774.4
JAPAN		
1. Pop Growth	49365.2	57411.5
2. Age Shares	64796.1	87203.5
SWEDEN		
1. Pop Growth	17038.0	161943.0
2. Age Shares	56670.0	62818.2
USA		
1. Pop Growth	45615.4	51948.1
2. Age Shares	59994.0	70122.6
WORLD		
1. Pop Growth	6646.9	9269.6
2. Age Shares	8073.2	8556.6

Table 5.6: Comparison of Models: Mean Square Error

Country	No Demography	Demography
Belgium	0.056	0.029
Sweden	0.035	0.023
Japan	0.054	0.034
USA	0.069	0.039
China	0.201	0.110
India	0.100	0.072
World	0.018	0.013

5.5 Discussion and Conclusion

Using long memory (age-structured) population projections, this chapter provided income forecast of the world economy along side a selected developed and developed countries. ML (2005) research has been extended in the long-memory framework (in a univariate setting). Literature is replete with the evidence that long-memory DGP of a time series permeate more dynamics of the observed system and has the ability to model future with rich information about stochasticity of a variable. Indeed, the use of ARFIMA framework for modeling age-specific population and concomitantly employing fractional framework for GDP forecasting offers advantage in that we are able to incorporate more dynamic information of the demographic and economic system in the forecasting model. Endogenous nature of population growth is assumed (this is a model characteristics due to the occurrence of possible feedback effect from demography to the economy and vice versa) which contributes to the economic growth and affect long-term variation in growth. The assumption of autoregressive population structure accommodates endogenous nature of population and when it interacts with economic output, an endogenous economic system is generated. In this sense the inclusion of long-memory demographic information in the GDP forecasting model describes economic system quite distinctly.

The thrust of the chapter lies in the recognition that demographic variables, like other macroeconomic variables, may be subject to shocks, and that the shocks may have more than mere short-run impacts on the demographic system. Hence there is a need to model demographic variables in a flexible framework that incorporates both short- and long-run dynamics. Outright assumption of stationarity of these variables straightaway eliminate the possibility of shocks having long-run impact. Therefore, the assumption of long-memory data generating process for demographic variables allows us to understand its interaction with the rest of the economy. Important points emerge from the comparison of forecasts between our long-memory demographic model (Model 2 and 3) and ML's simple and interaction models.

Note that ML's simple model (which incorporates only age-shares) is in fact the long-memory demographic model of this chapter. We have not added life-expectancy in the equation, and hence there is no interaction. Effect of life-expectancy is expected to be captured by the long-memory dynamics of the demographic system. The main idea of the inclusion of life expectancy in the demographic model is that the relationship between income and demographic variables is likely to shift over time and stage of development. Interactions occur in the system between the expected rate of return from education and life expectancy, which ultimately govern the growth of income. However, this interaction – which appears to be complex in nature, needs an exhaustive modelling. Once again there could arise the questions of stochastic or non-stochastic nature of the variable and implications of its interactions given this backdrop.

In ARFIMA, the memory parameter is expected to capture the nature of persistent shock in the economy. Autoregressive structure would capture endogeneity in the system, specifically the way the current state of the economy reacts to or depends on the past. Independent or autonomous changes are captured by intercepts. It is not surprising to see that the forecasts based on long-memory demographic model (with age share) is similar to the forecasts from ML's interaction model. Though more investigation is required to substantiate the argument that forecasts from interaction model (with life expectancy) is comparable to ARFIMA forecast with demographic model (without life expectancy), it provides a first-hand information about the simplicity of ARFIMA model and the rich stock of information it carries with to explain the demographic system. Some distinct differences in the forecast emerge as summarised below.

For the convenience of comparison we have estimated a raw model (without demographic information) and compared the projections from this model with that of demographic models. We find that inclusion of demographic variations predicts higher forecasts and given the smallest mean square error this is reliable. The relevance of demography in our forecast model however qualifies ML's argument that demography-based income forecast is more informative as economy responds to demographic changes more acutely (Lindh and Malmberg, 1999). In general we find that long-memory forecasts with demographic variations have a little higher projection than ML's interaction model, though the difference is not substantial. The forecast accuracy has been checked looking at the 95% confidence interval.[64] Narrow confidence band is indicative of better accuracy of forecast.

Our age-specific population forecasts show that young age population (0-14) will experience decline both in developed and developing countries, a bit faster for the latter, for instance India and quite steadily for countries like USA and the European countries, viz., Belgium and Sweden. Working age population will substantially fall for Japan but at the same time there will be an alarming rise of the retired people. Belgium and Sweden are likely to experience steady increase both working age and retired people, almost by an offsetting amount. Generally, a visible difference in the population number of three age groups exert various income effects on the economy and thereby affect the intergenerational transfer and management of resources. European countries will mostly experience a steady rise in the growth rate. The currently aging developed countries will experience a stagnating or even negative growth trend in GDP. Most developing countries will, however, experience accelerating growth and converge to although not reach the income levels of the developed world. The main exceptions to this are to be found in sub-Saharan Africa where the impact of AIDS on the age distribution postpone any growth take-off. However, even in these countries the UN assumptions that the AIDS epidemic will be brought to an end results in increasing growth rates toward the end of the period.

Fractional framework (be it in a univariate or panel setting) is a very useful tool to accommodate movement of shocks and model their interaction with the economy. This recognition is getting popularity in the demographic analysis

[64] Instead of the confidence band, the relevant standard errors could be reported. However, our preference for the confidence band is based on the standard reporting in the forecasting literature. Added motivation is that the confidence band gives us an idea about the tightness or wideness of the actual forecast.

recently, though the literature is very sparse. Our strategy of modelling demographic variables in a long memory framework and use the dynamic information for long-run forecasting of income would provide a new direction of research in the demographic context – a departure from conventional wisdom. Though we have extended ML (2005) framework in a long-memory set up in the univariate context, an extension to panel framework will be interesting. The efficacies of long-memory in panel data are yet to be theoretically established, which we preserve for an extension of the current research.

Figure 5.2: Global real GDP per capita forecast

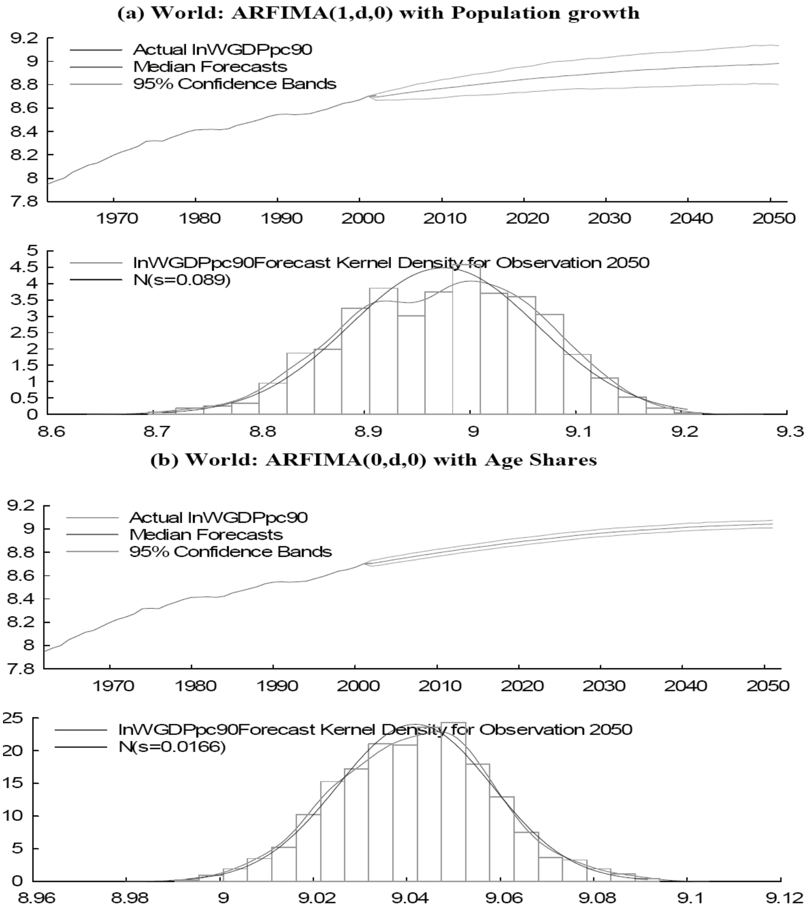


Figure 5.3: Belgium real GDP per capita forecast

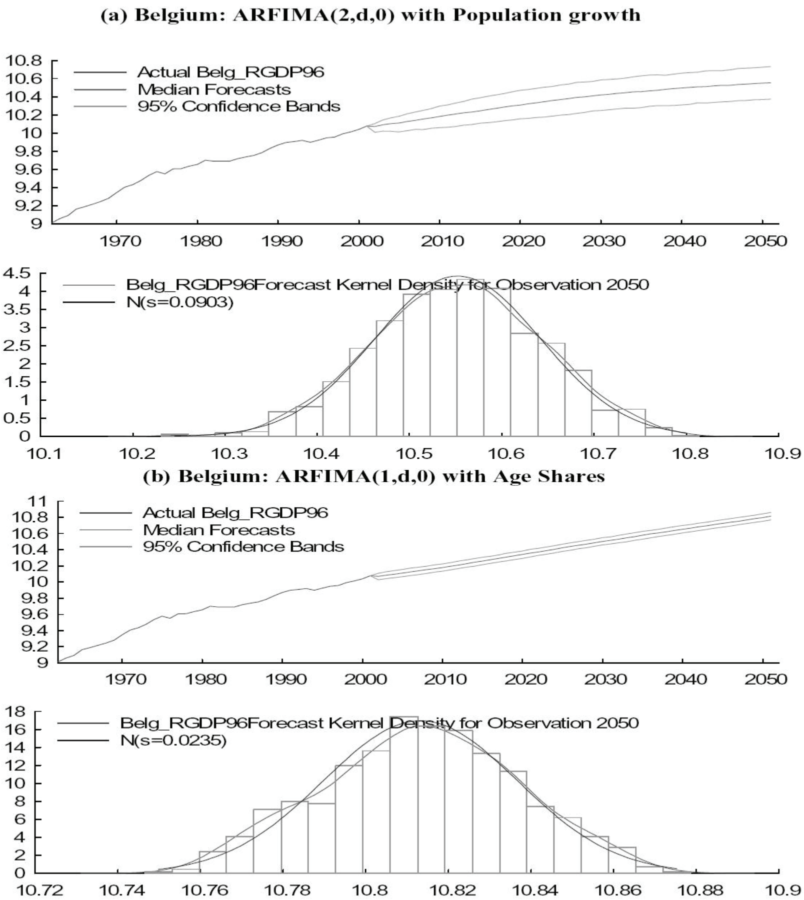


Figure 5.4: USA real GDP per capita forecast

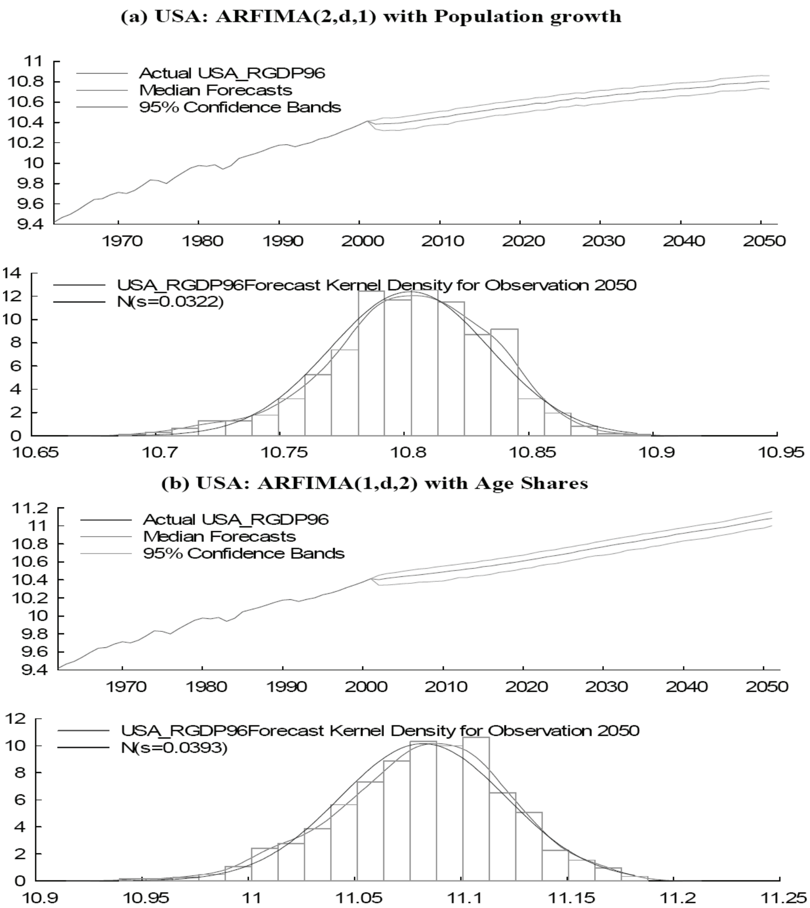


Figure 5.5: Sweden real GDP per capita forecast

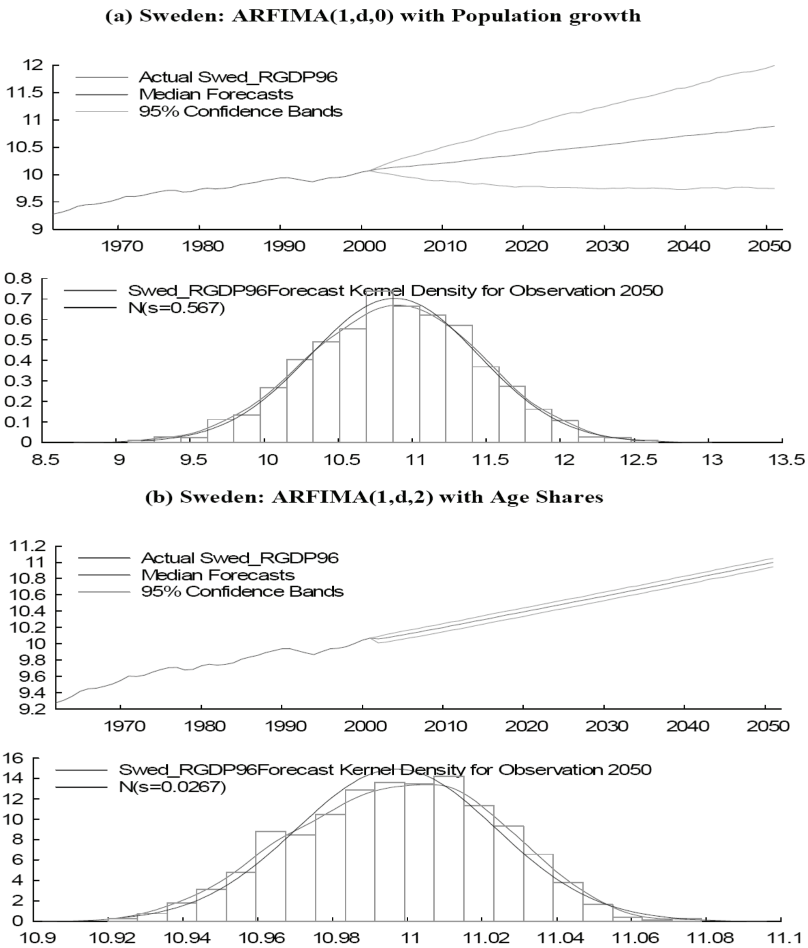


Figure 5.6: Japan real GDP per capita forecast

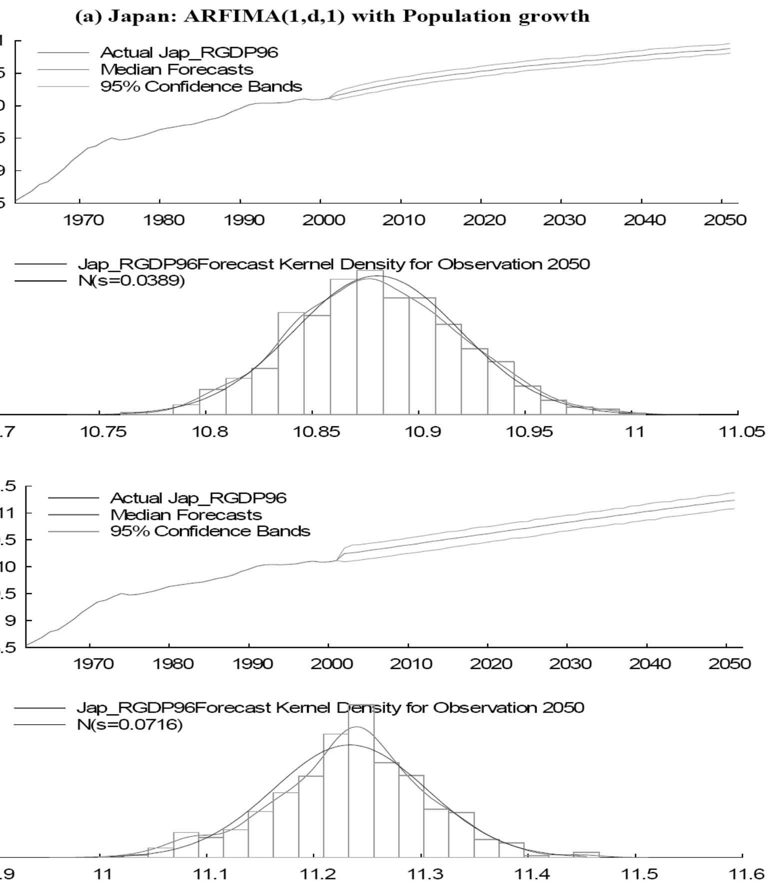
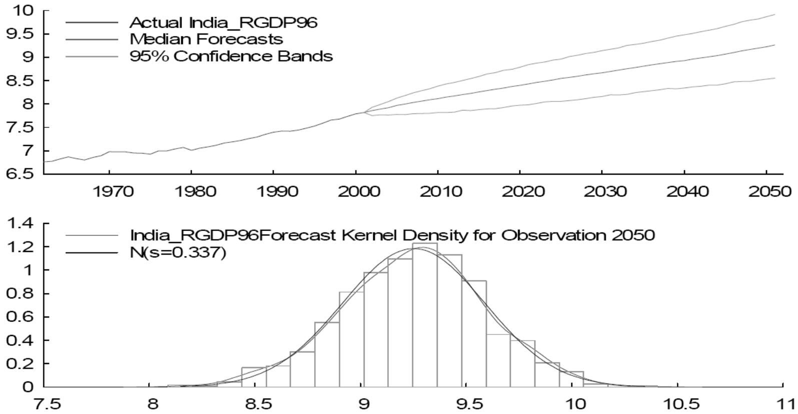


Figure 5.7: India real GDP per capita forecast

(a) India: ARFIMA(2,d,0) with Population growth



(b) India: ARFIMA(1,d,0) with Age shares

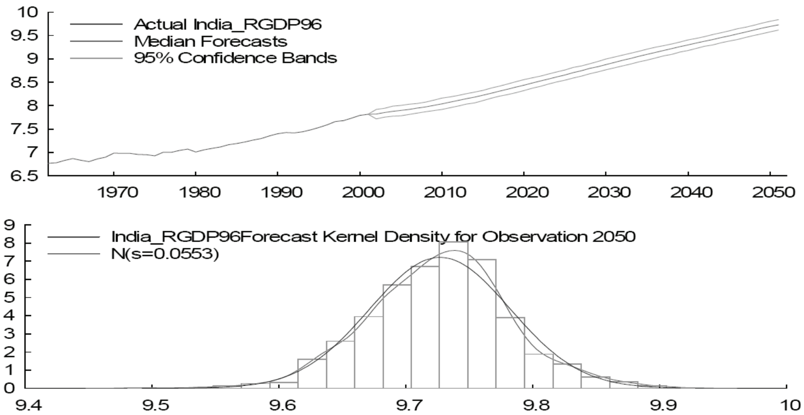
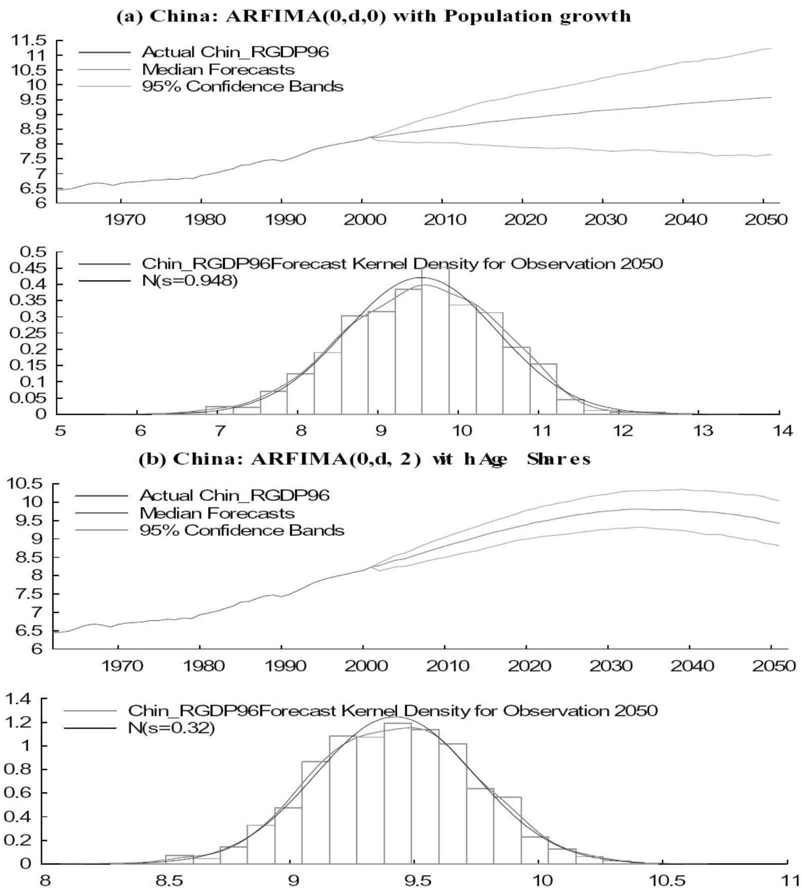


Figure 5.8: China real GDP per capita forecast



6. Conclusions: Policy Analysis and Development Objectives

The central objective of this book has been to study the consequences of stochastic demographic system in economic growth processes. It was assumed that stochasticities in the demographic system could be an outcome of endogenous variation (due to its interaction with the economic system and own evolution) and exogenous changes (forced upon by the environment). Based on this assumption, the book introduced stochastic long-memory features of the demographic system and studied how it would contribute to economic growth fluctuations. From a broader perspective, the book made an exception to the conventional assumption in empirical and theoretical growth literature that 'population growth is stationary'. It was observed that this simplistic assumption was largely forced by methodological complexity in the demographic processes and to some extent by the unavailability of the state-of-the-art econometric technique two decades ago. However, drawing upon evidences from real life economic situations and demographic variations, we found that demographic system need not be characterized as a stationary system. We made an attempt to tread beyond this conventional assumption and allowed the interplay of non-stationary demographic shocks with the economic system. From broader perspective, the book endeavored to provide a new analysis of demography-economic growth relation based on stochastic characterization of the demographic and so demography-economic system.

By explicitly emphasizing the relevance of temporal variation in the evolution of demographic system, we brought out the underlying dynamics of demography-economic growth relation. We stressed that despite their inherent complex mechanism demographic variables cannot escape the impact of shocks accumulated over time. Indeed, any process, physical or non-physical, does evolve over time. Therefore, time must play important role in deciphering their influences on other variables in the system. The traditional assumption that demographic processes remain stable or stationary over time, is to some extent misleading, though for the sake of a broader theoretical analysis, it seems very useful. For meaningful and accurate empirical measurement, it is all the more necessary to recognize the influence of shocks, which may be transitory in nature, but over time the aggregation of which can cast spell on the entire economic system. From a fairly broader perspective, the core of the book lied in dealing with such issues and introducing a mechanism through which demographic and economic growth relation can be better studied. For long term policy and development objectives of different countries it is pertinent that the government measure the magnitude of demographic shocks and based on the nature of cross-persistence of shocks of stationary or non-stationary demographic system, appropriate policies be undertaken. For instance, in Chapter 3 we found evidence of long-memory component of demographic system which had more than mere short-run effects on the aggregate economy. This implies that the demographic system needs to be stabilized first because it forms the core of the process in economic growth. For developing countries perspective it would mean, for example, that the spurt of young age population must be contained and more resources must be devoted to education to increase the buffer of human capital such that the growth of the economy can be accelerated to the optimum as economy progresses over time.

The four principal chapters (Chapter 2 through Chapter 4) of this research addressed the above issues in parts. First, in Chapter 2, by first sticking to the conventional stationary assumption and extending Kelley-Schmidt's analysis to the current decade we observed some variations in the results which are different from Kelley and Schmidt. In particular, we observed that addition of a decade did change the effect of birth and death rates on economic growth of developed and developing countries. Though different explanation might accrue to such decadal variations, the most important one appears to us is the lack of treatment of time dimension in the panel regression. With standard panel and cross-section regressions, the major conclusions that emerge from this chapter are: (1) further birth rate reductions in developing countries would do no more good for the economic progress of these countries. (2) Effect of growth-enhancing effect of population density is found for all decades.

An important conclusion from this chapter is the finding of positive effect of birth for developed countries. In view of this fact, the recently proposed 'zero-population growth' policy for the bulk of developed countries might need a rethinking. It also emerged from our analysis that the standard methodology to translate demographic and economic growth variables to five, ten or more years of averaging in fact average out the hidden dynamics present in the system which in our opinion could be delineated via accounting for time variation by each year. In the extant empirical growth model, we stick to such averaging as this avoids the problem of persistency in the data. However, in view of the development of modern methodological tools, the analysis of demography-economic growth nexus can be further broadened. It was also brought out in this chapter that unless one explicitly accounts for temporal dynamics in the demography-economic growth models (such as the convergence-patterns), the consistency of the effect of demographic variables on economic growth cannot be fully ascertained. Long-years of aggregation and averaging as often used in empirical economic growth literature only discounts the persistent shocks in the system if there were any. For the sake of approximating realistic economic situations, it is necessary to study the persistent behavior of growth variables while making long-term economic decision and projections.

Due to the apparent need of accounting for detailed temporal dynamics in the empirical economic growth regressions, in Chapter 3 we investigated if demographic variable, like aggregate population and age-specific population display some stochastic memory features. And if so, how does one incorporate them in the analysis of economic growth model where population is assumed to be endogenous to the system via an interaction and feedback mechanism. We addressed these questions by examining the memory properties of aggregate and age-specific population and their impact on economic growth. Our analysis for about 200 countries covering both developed and developing countries evinced that population series contain substantial stochastic memory and thus warrant implications for the development objectives of developing and developed countries. On the basis of their memory features, we were able to classify the countries sharing 'common stochastic memories,' such that appropriate policies could be designed looking at whether the population series of these countries suffer from non-mean convergent or stationary shocks. We also provided a theoretical framework to show that variations in the degree of stochastic shocks of population would in fact welcome variations in the response of output in the long-run. Specifically, steeper response of output could be observed as population growth becomes gradually non-stationary. Our research also brought out the importance of stochastic demographic shocks in explaining and being explained by the economic growth variations of developed and developing countries.

Having demonstrated that aggregate and age-specific population series exhibit significant memory features (in Chapter 3), in the next chapter (Chapter 4) we provided population forecast based on their long-memory features. Population projection has held nerve for the modern forecasting theory, however, application of time series methods have not been so popular except some notable but sparse contributions from demographers. We forecasted population and age-specific population for a set of developed and developing countries using the popular ARFIMA methodology while also allowing for endogenous demographic shift in the series. We found that ARFIMA models produced forecasts which were at least as reliable as more traditional demographic forecasts. Moreover, ARFIMA method have more advantages than the standard ARIMA and most other demographic forecasting methods viz., high, low, and medium variant projection techniques used by US Census Bureau and United Nations. Thus it seems apparent that for any forecasting exercise it is always handy to incorporate as much dynamic information in the model as possible to deliver reliable and economically meaningful forecasts.

The forecast values of UN high variants appeared to be more or less compatible with our ARFIMA forecasts. We demonstrated that European countries' total population would steadily rise while a faster growth was deemed for developing countries like India and China. Interestingly, going by our projections, India is expected to replace China as the most populous nation on the earth by 2050. This might result from China's population control policy which significantly controls its future growth trajectory in contrast to India's so-called normative approach of 'stabilizing population' where, *not* 'controlling population' might cost the country with heavy demographic pressure. For India, the task is rendered even more difficult by a relatively rapid rate of population growth that results in the addition of nearly 20 million every year. However, the conventional family planning approach with its emphasis on contraceptive targets (i.e. the number of couples using contraception) has shown to be ineffective over the years, which gives credence to the fact that the population size will soar in five decades.

By employing a flexible data generation process in forecasting, like ARFIMA, we depicted that the desirable demographic dynamics could be displayed and the complex core of the system in terms of 'response to shocks' can be better characterized. Therefore, the ARFIMA forecast for total and age-structured population, as resorted to in the book, could possibly serve as providing complementary information, if not completely an alternative to the oft-practiced methods such as probabilistic projection or high, medium, low variant projections.

Utilizing the stochastic behavior of the demographic system, we also performed demography-based income projections for some developed and developing countries in the book (Chapter 5). The major conclusions are as follows. We noted that long-memory forecasts with demographic variations exhibit higher projection values than earlier forecasts, such as ML although the difference was not substantial. Our forecasting model was presumed to be more informative due to the fact that we could account for the possible subtle variations of shocks while performing projections. We found that our forecasts were higher than ML who assumed stationary behavior for age-structured and total population. Demography-based income projections were shown to fare better than non-demographic models. Moreover, under further demographic decomposition and consideration of stochastic dynamics, the forecasts were found to improve which indicates that the more dynamic information one would embed in the forecasting model, the better would be the forecasts.

To conclude, the book made a modest attempt in trying to introduce the role of stochastic features of the demographic system in economic growth. While this research has answered some basic questions concerning the long-run economic growth consequences due to stochastic demographic variations, it can be further extended in several other directions. There are several possible aspects of the research on which further work could be initiated. First, although in Chapter 3 we provided a long-memory demography and economic growth linkage, a complete formulation of the problem in an endogenous growth theoretic setting can give further insights about their long-run behaviour. Different welfare implications can be derived and intergenerational transfer of resources can be studied from such formulation. Second, forecasting of demography-based GDP in the book has been carried out in the univariate setting. However, a long-memory panel framework could be built and forecasting of individual as well as 'blocks' of countries could be performed based on common stochastic memory and their interactions with the economic system. Forecasting accuracy of ARFIMA models for demography and demographic-economic system could also be checked given the various non-linear models, such as AR, ARMA, Markov Switching, etc.

Another possible extension of the book would be to examine the long-run equilibrium relationship between demographic variation and economic growth assuming that long-memory shocks characterize both the demography and economic system. A fractional cointegration analysis can provide insights into their co-movement and most importantly to answer some of the important questions in economics growth: Does stochasticity in the demographic system cause persistent growth variations in the world economy and/or in different countries? To what extent economic growth variations could be contributed to the demographic dynamics? On a different angle, the causal link between demographic pressure induced innovation and economic growth could also be studied in a stochastic growth setting.

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