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# Scotland the Grey: A Linked Demographic-CGE Analysis of the Impact of Population Ageing and Decline

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Abstract: This paper links a multi-period economic Computable General Equilibrium (CGE) modelling framework with a demographic model to analyse the economic impact on Scotland of its projected ageing and declining population. The model quantifies the effect on aggregate economic variables, such as GDP, employment and competitiveness, and also on individual sectors. With the principal demographic projections, the fall in population, and particularly working-age population, has a depressing impact on economic activity. By changing the demographic parameters, we track the impact of increasing net migration, a policy actively being pursued by the Scottish Executive. However, the required size of the annual net-migration needed to neutralise the adverse natural demographic changes is far higher than the current trends.

JEL classification: R12, R13, R15; J11

Key words: Scotland, regional CGE modelling, ageing population, migration

#### 1. Introduction

There is currently apprehension amongst policy makers concerning Scottish demographic trends. The principal UK Government Actuaries Department's (GAD) projections for the Scottish population to 2040 are shown in Figure 1.1. These projections indicate that under the principal assumptions concerning demographic parameters, by 2040 Scottish total and working age populations are 2.6% and 14.4% below the base year (2000) values respectively.

Demographic change has many economic implications. The novelty of the present analysis is that population projections, based upon assumptions concerning key demographic parameters, are used to generate exogenous shocks to a Computable General Equilibrium (CGE) model. That is to say, a demographic model (POPGROUP) is linked to an economic model of Scotland (AMOS) in order to isolate and identify the system-wide economic impacts of changes in population size and composition.

The strength of the linked modelling strategy is that changes in the size and composition of the local population will have both direct demand- and supply-side effects, particularly in a regional economy facing nationally determined and financed fiscal, social security and pension arrangements. These direct impacts generate endogenous economic responses on variables such as the level of investment, wage rates, labour market participation and price competitiveness. The interaction of these variables produces the changes in aggregate indicators, such as total employment, GDP and consumption. These linked model simulations can be performed using official demographic projections. However, scenario analysis, where the demographic parameters, such as fertility rates, longevity or the extent of net migration, deviate from their projected values is also available.

In the present paper we concentrate primarily, though not exclusively, on the effect on the level of economic activity in Scotland, and we focus especially on the labour market impacts. We are particularly interested in the extent to which migration counteracts the negative effects of a declining and ageing population. This is of particular policy relevance as the Scottish Executive, through its Fresh Talent

Initiative, Working in Scotland programme is attempting to retain foreign students studying in Scottish higher education establishments, essentially attempting to increase immigration. Whilst we identify the impacts in individual sectors, these impacts are not driven here by changes in the composition of public and private sector demand to meet the requirements of an ageing population. Rather the sectoral impacts occur through the less frequently studied supply-side effects linked to a tighter local labour market.

The paper is organised in the following way. Section 2 outlines the theoretical analysis of the economic effects of population change on the labour market. Sections 3 and 4 detail the POPGROUP demographic model and the AMOS computable general equilibrium (CGE) economic model (Simpson, 2005;.Harrigan *et al*, 1991). Section 5 gives the economic impacts that would be associated with the principal UK Government Actuaries Department's (GAD) Scottish population projections. Section 6 identifies the extent to which increased net migration to Scotland would reverse the negative changes in activity associated with population ageing and decline. Section 7 is a short conclusion.

# 2. Theoretical Analysis of the Impact of Population Change on the Aggregate Scottish Labour Market.

In this section we provide the conceptual underpinning for the simulation results that are reported in Sections 5 and 6. We focus on the labour market, which is analysed using an aggregate labour supply and demand framework, an appropriate method given the economic issues raised by an ageing population. The analysis in this section is implicitly long run so that the identified equilibrium wage and employment levels are those towards which the economy is being attracted over time.

Figure 2.1 represents the interaction of the labour supply and general equilibrium labour demand curves in the unified Scottish labour market. The wage is the real consumption wage. We assume that there are no geographical or skill submarkets within the Scottish economy and that labour can move freely between sectors. The analysis is comparative static in that it identifies the impact on the

equilibrium real consumption wage and employment of an exogenous demographic disturbance.

The upward sloping labour supply curve is straightforward. It indicates that with a fixed working-age population, an increase in the real wage will increase the quantity of labour supplied. The labour demand curve is a little more complex. It represents the relationship between the quantity demanded of labour and the real consumption wage in long-run equilibrium. This is where incomes and all other relevant prices are endogenous, and the capital stock is fully adjusted to the new commodity outputs and input prices.

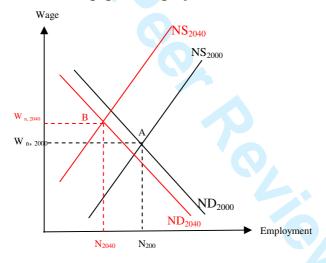
The general-equilibrium labour demand curve is drawn as downward sloping and this reflects the case in the AMOS model with default Scottish parameter values. Essentially if the real consumption wage is increased exogenously, the price of all domestically produced goods will rise relative to imports. As a result competitiveness falls, exports fall and the marginal propensity to import rises. The endogenous household income and consumption will fall and capital stock adjustments will reduce the labour intensity of production so that domestic output and quantity of labour demanded falls.

However, the general equilibrium nature of the function means that other relationships are possible. This can be illustrated for extreme assumptions. For example, if the economy were closed to trade, or if the law of one price held in all the region's export markets, then the long-run general equilibrium labour demand curve would be horizontal. In these circumstances, fixing the rate of interest, as we do in the AMOS model, fixes the real wage (Sraffa, 1960). Under these extreme assumptions demand plays no part in determining employment.

<sup>&</sup>lt;sup>1</sup> As explained in Section 4 we actually use a wage curve specification for the regional labour market. (Blanchflower and Oswald, 1994; Layard *et a1*. 1991). However, the wage curve approach gives results that are observationally equivalent to a labour supply curve although the interpretation is somewhat different.

On the other hand, for combinations of extreme low export demand and factor substitution elasticities, the general equilibrium labour demand curve can be vertical or even perversely upward sloping (McGregor *et al*, 1995). With a vertical labour demand curve, of course, the labour supply would now have no role to play in determining aggregate employment. However, to reiterate, we would expect, and with the AMOS model find, a downward sloping general equilibrium labour demand curve, so that changes in employment and real wages can result from either the demand or supply side of the labour market.

Figure 2.1: The Labour Market in 2000 and in 2040 under the GAD population projections.



In Figure 2.1 we compare the long-run labour market equilibrium in 2040 with that in 2000, under the assumption that the Government Actuaries Department (GAD) population projections apply and that real *per capita* Scottish government expenditure remains constant. The initial equilibrium is represented by point A, where the base-period (2000) labour demand and supply curves  $ND_{2000}$  and  $NS_{2000}$  intersect. This generates the initial equilibrium employment and real consumption wage levels  $w_{n,2000}$ ,  $N_{2000}$ .

According to the GAD projection, over the period to 2040, the population in Scotland will decline and age. These exogenous changes in population size and age composition shift both the labour demand and supply schedules. First, the fall in the

working age population reduces labour supply at each real consumption wage level generating an inward shift of the labour supply curve. This is illustrated in the Figure 2.1 by the new labour supply curve  $NS_{2040}$ , which lies to the left of the original labour supply curve  $NS_{2000}$ .

However, the change in population also affects labour demand. In particular, we assume that real government expenditure per head remains constant, so that Scottish real government expenditure varies as the total population varies. This is a realistic assumption given the framework of our analysis and the way in which the funding for Scottish public sector budget is administered. Funding is determined through the Barnett formula, which has two key features. First, public expenditure in Scotland is mainly financed through the Westminster Parliament. Second, Barnett is a population-based formula that has operated in practice to maintain the Scottish *per capita* public expenditure relative to the English level (Bailey and Fingland, 2004; Christie and Swales, 2005).<sup>2</sup>

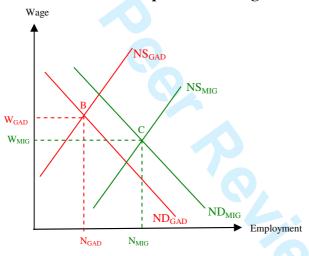
In the AMOS model, any exogenous change in product demand shifts the labour demand curve in the same direction. AMOS operates with linear homogeneous production and consumption functions and an exogenous interest rate. For a given real consumption wage rate, AMOS responds to a demand injection in the long run as an extended (investment and benefits endogenous) Input-Output (IO) system (McGregor *et al*, 1996). This means that the demand curve shifts by the extent of the exogenous employment change plus the appropriate Type II IO employment multiplier. Because total population has fallen, exogenous government expenditure is lower, so that labour demand also declines at each real consumption wage rate. This shifts the labour demand curve to the left, to ND<sub>2040</sub>.

The new equilibrium is at point B, the intersection of the general equilibrium labour demand and supply curves  $ND_{2040}$  and  $NS_{2040}$ . Both the labour demand and supply shifts lead to lower employment, but the impact on the wage depends on the

<sup>&</sup>lt;sup>2</sup> It is important to note that this is not the way the formula should work if it is applied precisely (Cuthbert, 1998; McGregor and Swales, 2005; McLean and McMillan, 2003 )

relative size of the shifts. Our prior expectation is that the reduction in labour supply will be much greater than the reduction in labour demand. There are two reasons for this. First, for Scotland the projected proportionate fall in the working age population is much greater than the fall in the population as a whole. Second, government expenditure is only one of the elements of final demand. Under these conditions, the larger proportionate reduction in labour supply will stimulate wage increases as the labour market tightens.

Figure 2.2: The Labour Market in 2040 comparing GAD's projections and positive net migration every year.



In Figure 2.2 we extend the analyses to show the impact on the labour market in 2040 of increased inward migration. Figure 2.2 compares the labour market in 2040 under two different scenarios. One scenario simply takes the GAD population projections. The second represents a situation in which there is positive net migration, perhaps as a result of Scottish Executive policies, such as the Fresh Talent Initiative (www.scotlandistheplace.com).

The situation under the GAD projection is given by point B, which is the intersection of the labour demand and supply functions  $NS_{2040}$ ,  $ND_{2040}$ . This was the final position illustrated in Figure 2.1. Now impose net in-migration at a rate higher than that assumed under GAD. The Scottish working age population, and therefore the Scottish labour force, is going to be higher than under the GAD prediction for

2040. This is illustrated in Figure 2.2 by an outward shift of the labour supply curve to  $NS_{MIG}$ . Labour demand will also shift outwards to  $ND_{MIG}$  as a result of higher population-linked government expenditure. The new equilibrium is at point C, the intersection of the new labour supply and demand curves. In this case the shifts in labour demand and supply both work to increase equilibrium employment (compared to the equilibrium at B). Further, because we expect that the increase in labour supply will dominate the increase in labour demand, pressure in the labour market should ease and the wage fall.

Figures 2.1 and 2.2 show the qualitative impact of population ageing and inmigration on the labour market. These labour market changes will have major impacts on the economy overall. In particular, with an ageing population, we expect a negative effect upon competitiveness due to the increased wage, which is a key production cost. This reduced competitiveness has a negative impact on GDP. On the other hand, increased net migration eases the labour market, improves competitiveness and increases GDP. However, the interactions within the economy are naturally more complex and detailed than this and are explored in the simulations reported in Sections 5 and 6. However, note that the figures that are reported in Sections 5 and 6 are for time periods over which such long-run adjustment is not yet complete. Similarly, where we report period-by-period simulations, we are tracing out the adjustment path to a long-run equilibrium that is perpetually changing.

### 3. POPGROUP: The Demographic Model.

Demographic projections were produced using the POPGROUP software developed by the Cathie Marsh Centre for Census and Survey Research (CCSR) at the University of Manchester (Simpson, 2005). POPGROUP uses the conventional cohort component method for the demographic projections. This method tracks each cohort and its mortality, fertility, and migration over time. Starting with a base population, year-by-year deaths are subtracted, and births and net migration are added to the population. This program allows us to project future population by sex and age

structure, based on the current age-sex structure and additional assumptions about the main demographic variables: mortality, fertility and migration.

A simple algebraic account of the model follows. The population of sex, s, aged x + 1 on his or her last birthday in year t + 1,  $P_s(x+1, t+1)$ , is determined as the sum of two elements. The first is the population of sex s whose age at his or her last birthday in year t was x,  $P_s(x, t)$ , who survive to the next year. The probability of survival is age, sex and time specific, and is given as  $S_s(x+1, t+1)$ . The second element is the level of net migration, which is again sex, age and time specific and given as  $NM_s(x+1,t+1)$ . Therefore:

$$P_{s}(x+1,t+1) = P_{s}(x,t)S_{s}(x+1,t+1) + NM_{s}(x,t) \qquad x \ge 0$$

The population of sex s who are aged 0 in year t+1 is determined by the number of births in time t,  $B_s(t)$ , times the probability of surviving to period t+1 plus the number of migrants in this category, so that:

$$P_s(0,t+1) = B_s(t)S_s(0,t+1) + NM_s(0,t)$$

The birth rate is given by the population of females, f, broken down by age, multiplied by the age specific fertility rates, ASFR:

ertility rates, ASFR: 
$$B(t) = \sum_{x} 0.5 \Big[ P_f(x,t) + P_f(x,t+1) \Big] ASFR(x,t)$$
 between males and females according to the sex ratio of

with births divided between males and females according to the sex ratio of 106 males to 100 females.

Adding up the components of change across ages and sexes, we obtain the total population in year t+1 as:

$$P(t+1) = P(t) + B(t) - D(t) + NM(t)$$

where D(t) is the number of deaths and is given as:

$$D(t) = \sum_{s} \sum_{x} \left\{ P_{s}(x,t) \left[ 1 - S_{s}(x+1,t+1) \right] \right\} + \sum_{s} B_{s}(t) \left[ 1 - S_{s}(0,t+1) \right]$$

In all the simulations the actual population structure estimated by the Government Actuaries Department (GAD) are used for the period from 2000 to 2004. The population projections therefore begin from 2004. Our projections are based on fertility and mortality assumptions employed by the GAD in their 2004-based principal Scottish population projection, which was published on October 20, 2005. Using this as a base we have produced several scenarios of population projections depending on the assumed level of migration. For the age distribution of migrants, we generally assume that 1/3 are in the age group 0 - 19 and 2/3 in the age group 20 - 39. Within these age groups, migrants are distributed equally by age and sex. This age-sex structure can be summarized as "young couples with one child".

#### 4. AMOS: The Economic Model.

AMOS is an acronym for *A Macro-Micro Model Of Scotland*. It is best regarded as a Computable General Equilibrium modelling framework "... because it encompasses a range of behavioural assumptions, reflected in equations which can be activated and configured in many different ways" (Harrigan *et al*, 1991, p. 424). This allows the user to choose from a variety of model closures and key parameter values, as appropriate for particular applications. A good general description of CGE modelling is given in Greenaway *et al* (1992, Ch. 2) and an extensive review of regional CGE models can be found in Partridge and Rickman (1998).

<sup>&</sup>lt;sup>3</sup> Available from the GAD website <u>www.gad.gov.uk</u>, population projections section

We have calibrated the AMOS modelling framework to data on the Scottish economy given in the form of a Social Accounting Matrix (SAM) for the year 2000. The model has 3 transactor groups - households, firms and government - and 2 exogenous external transactors – the rest of the UK (RUK) and the rest of the world (ROW). The model has 25 activities/commodities and these are listed in Table A1.1 in Appendix 1. A condensed account of the model structure is given in Table A2.1.

In the version of AMOS used here, production takes place in perfectly competitive industries using multi-level production functions. This means that in every time period all commodity markets are in equilibrium, with price equal to the marginal cost of production. Value-added is produced using capital and labour via standard production function formulations so that, in general, factor substitution occurs in response to relative factor-price changes. Typically, constant elasticity of substitution (CES) technology is adopted, which is the case in simulations reported here.<sup>4</sup> In each industry intermediate purchases are modelled as the demand for a composite commodity with fixed (Leontief) coefficients. These are substitutable for imported commodities via an Armington link. The composite input then combines with value-added (capital and labour) in the production of each sector's gross output. Cost minimisation drives the industry cost functions (equation 1 in Table A2.1) and the factor demand functions (equations 7 and 8 in Table A2.1).

Whilst the AMOS framework offers a wide choice of labour market closures, in the simulations reported here real consumption wage determination is characterised by a regional bargaining function (also expressed as a wage curve, represented as equation 5 in Table A2.1). This establishes a negative relationship between the real wage and the unemployment rate (Minford *et al*, 1994). Empirical support for this wage curve specification is now widespread, even in a regional context (Blanchflower and Oswald 1994). The bargaining function is parameterised using the regional econometric work reported in Layard *et al.* (1991):

$$\ln rw_{n,t} = a - 0.113 \ln u_t + 0.40 \ln rw_{n,t-1}$$

<sup>&</sup>lt;sup>4</sup> Leontief and Cobb-Douglas options are available as special cases.

where rw is the Scottish real wage, u is the Scottish unemployment rate, t is the time subscript and a is a calibrated parameter.<sup>5</sup> To transform the real wage to the nominal wage, we multiply by the consumer price index (equation 2 in Table A2.1).

Perfect labour mobility is assumed between sectors, generating a unified labour market. Therefore, although wage rates vary between sectors in the base-year data set, in the simulations wages in all sectors change by the same proportionate amount in response to exogenous shocks. The real consumption wage in each time period is then derived through the interaction of the resulting wage curve and the general equilibrium labour demand curve (equation 9 in Table A2.1). In the derivation of the general equilibrium labour demand curve, it is important to note that all prices and incomes are taken to be endogenous.

The four main components of commodity final demand (represented by equation 12 in Table A2.1) are consumption, investment, government expenditure and exports. Household consumption is a linear homogenous function of real disposable income and relative prices (equations 2, 11 and 13 in Table A2.1). Real government expenditure per head is assumed to be constant (equation 17, Table A2.1) and in these simulations the population is typically determined exogenously using the demographic model. Exports are determined by exogenous external demand via an Armington link, making exports relative price sensitive (equation 18, Table A2.1).

The modelling of investment demand is a little more complex. In the multiperiod variant of the model, capital stock adjustment at the sectoral level, which ultimately determines aggregate investment demand, is dealt with in the following way. Within each time period, both the total capital stock and its sectoral composition are fixed. The interaction between this fixed capital supply and capital demand at the sectoral level determines each sector's capital rental rate (equation 10, Table A2.1).

The capital stock in each sector is then updated between periods via a simple capital stock adjustment procedure, according to which investment equals

depreciation plus some fraction of the gap between the desired and actual level of the capital stock (equations 6, 14 and 15 in Table A2.1). Desired capital stocks are determined on cost-minimisation criteria, using the user cost of capital as the relevant price of capital (equations 3 and 4 in Table A2.1). In the base period the economy is assumed to be in long-run equilibrium, where desired and actual capital stocks are equal, with investment simply equal to depreciation. Investment as a source of product demand is then determined by running the demand for increased capital stock by sector through the capital matrix (equation 16, Table A2.1).

In the model, we do not impose macro-economic constraints, such as balance of payments or budget deficit limits or targets. There is no reason for these to bind at the regional level. However, we do track these surpluses. We also interpret the conceptual time periods of the model as years: annual data are used for the calibration and, where applicable, the estimation of parameter values.

As stated earlier in this section, the structural characteristics of the AMOS model are parameterised on a Social Accounting Matrix (SAM) for Scotland for 2000. In all sectors, the elasticity of substitution between capital and labour in the production of value added is 0.3. However, we adopt fixed coefficients in the generation of the intermediate composite. This is required because of the large number of zero entries. The Armington trade elasticities for imports and exports are 2.0. The speed of adjustment parameter for the adjustment of actual to desired capital stock is 0.5.

Before discussing the simulation results it is important to clarify a key characteristic of the AMOS model. AMOS is not a forecasting model. When it is parameterised on the base year data set, it is assumed that the economy is in long-run equilibrium. If there are no changes to the exogenous variables and the model is run in period-by-period mode, then the model will simply report an unchanging economy. This assumption of initial long-run equilibrium is clearly a convenient fiction. We

<sup>&</sup>lt;sup>5</sup> The calibration is made so that the model, together with the set of exogenous variables, will recreate the base year data set. This calibrated parameter does not influence simulation outputs, but the assumption of initial equilibrium is, of course, important.

<sup>&</sup>lt;sup>6</sup> This process of capital accumulation is compatible with a simple theory of optimal firm behaviour given the assumption of quadratic adjustment costs. The whole process is analogous to Tobin's q.

make this assumption because it greatly increases the transparency of the model and eases initial parameterisation.

In the simulation results that we report in Sections 5 and 6, the only exogenous changes that are introduced into the model are the direct government demand and labour force changes associated with the population projections. Therefore the results should be interpreted as deviations from what would have occurred with a population that is constant both in size and age composition.

# 5. The Consequences for the Scottish Economy of the Government Actuaries Department (GAD) Population Projections.

The assumptions underlying the principal GAD projection for Scotland are a fertility rate of 1.6, life expectancies of 79.2 years for men and 83.7 for women and net immigration at a constant level of 4000 *per annum*. In Figure 1.1 we tracked the percentage changes from the base year in total population and total working-age population generated by these assumptions. According to this scenario, the working age population is expected to fall below its base year (2000) value in 2012 and decline by 14.37% by 2040. Total population falls below the base-year value in 2032 and is 2.62% below it by 2040.

The demographic data represented in Figure 1.1 are used to generate exogenous disturbances in the AMOS model as discussed in Section 4. This gives the simulation results summarised in Table 5.1.

## Table 5.1 here

As we explain in Section 2, the population change produces two simultaneous exogenous impacts. One operates on the demand side and the other on the supply side. Changes in total population affect the demand side, generating changes in government demand as we hold real *per capita* government expenditure constant. Changes in working age population are modelled as supply-side changes, operating through adjustments in the labour force, which affect the tightness of the labour market.

One important technical point needs to be made here. We can enter the period-by-period government demand changes so that they precisely match the period-by-period changes in population. However, as the model is presently configured, we can only impose changes in the labour force by means of a linear trend. Therefore, for example, with the GAD projection shown in Figure 5.1 above, the 14% fall in the working age population over the 40 years between 2000 and 2040 is modelled as a linear reduction in the labour force equal to an annual rate of 0.5% of the original 2000 value. This means that whilst the end-point simulation results are correct, there is some loss of accuracy in the adjustment path.<sup>7</sup>

Also recall that the results in Table 5.1 should be interpreted as variations away from what would have occurred with population held constant in both size and age composition. Begin by looking at the Scottish employment and GDP figures. As we expect from the theoretical discussion in Section 2, employment falls, in this case by 9.0% in 2040, with a corresponding decline in GDP of a little less at 8.2%. Note that because of the linearisation of the change in the working age population in the model, the results shown in Figure 5.2 will overestimate the reduction in the initial years, where an increase in output and employment is expected, but underestimate the rate (but not the level) of decline in the latter period of the simulation where working age population is falling.

Table 5.1 reveals two important points. The first is that the fall in employment is much lower than the fall in working age population. Working age population declines by 14.4% % whilst employment declines by only 9.0%, implying an increase in the participation rate and a fall in the unemployment rate to partially offset the negative supply side impacts. This tightening of the Scottish labour market is apparent in the real wage results where the real Scottish take-home wage of increases by 7.7% by the year 2040.8

The second key point is that the fall in GDP closely follows the decline in employment. In considering the fall in GDP, these are driven primarily by the

 $<sup>^{7}</sup>$  In fact, the imposed adjustment path will affect the endpoint results, but this influence should be small.

exogenous supply side changes introduced by the fall in the labour force, given that the actual population reduction is relatively small. There is a reduction in Scottish exports generated by the decline in Scottish competitiveness that accompanies the tighter labour market. Table 5.1 shows the Scottish consumer and export price (competitiveness) indices. Note that consumer price index increases by 2.6% to 2040, but the increase in export price index is even higher, at 3.4%. As a consequence, the demand for exported goods falls by 6.4%. The capital stock will adjust to changes in output demand but more slowly than the changes in employment in particular sectors so that the change in GDP will slightly lag the changes in employment. There will also be a tendency for production to become more capital intensive as the nominal wage increases, so that there is some substitution of capital for labour.

In measuring the aggregate impact on the Scottish economy of the projected demographic changes, up to now we have focussed on GDP and employment. This is primarily because total employment and GDP are figures that attract concern from both the Scottish Executive and the UK central government as measures of economic performance. However, from a welfare perspective, the change in real consumption is probably of greater concern for the Scottish people. In this respect, public consumption is exogenous in the simulations reported in this paper and is taken to change in line with population. As discussed in Section 2, this replicates the present actual operation of the funding formula for the UK devolved regions. However private consumption is endogenous. To 2040, the fall in private consumption, by 5.2%, is less than the fall in GDP and employment. This reflects the increase in real wage that accompanies the decline in working age population. However, the fall in household consumption is greater than the fall in total population, so that there is a fall in overall Scottish *per capita* consumption to 2040.

### Figure 5.1 here

Figure 5.1 shows the expected changes by 2040 in sectoral output and employment generated by the GAD demographic projections for Scotland. Note first

<sup>&</sup>lt;sup>8</sup> There is not full pass through of the increased wages to prices, that is prices increase by less than wages, primarily because of the presence of imports from outwith Scotland as elements of the consumption basket and as intermediate inputs in production.

that these simulations do not take into account any variation in the composition of government and household consumption demand that will be driven by population ageing. For example, there is much discussion of the implications for health care and education provision resulting from longer life expectancy (Economic Policy Committee and European Commission, 2006). Such compositional demand changes are not considered here.

These disaggregated results therefore partly reflect more general demand-side factors, such as general movements between private and public consumption as the population structure changes. However, the disaggregation primarily picks up the often-neglected supply-side factors that are operating through the tightening of the labour market and the impact that these have on the competitiveness of individual sectors.

Figure 5.1 indicates that by 2040 the outputs of all sectors are affected negatively. However, there is a wide variation in the impacts, ranging from an output reduction of 2.8% for Public Administration to 11.8% for Construction. In general those sectors selling most of their output to government demand (Public Administration, Social Work, Health, Education) are affected least because government expenditure, which in these simulations is linked to total population, remains relatively constant over the whole simulation period, falling by only 2.6% by 2040. In addition these sectors are sheltered in the sense that they are subject to limited international competition.

The extent of the negative effect upon other sectors (which are mostly hit much harder) is determined by two factors. First, labour intensive sectors are worst affected because of the increased cost of labour. Second, the sectors that are more exposed to international trade feel the negative competitiveness effect more strongly. For example, sectors such as Agriculture clearly suffer these negative competitiveness effects. However, Other Manufacturing, which is the most export intensive sector, is not so severely hit because it is not labour intensive. In all sectors, employment falls

<sup>&</sup>lt;sup>9</sup> We measure the price of the exported good as the weighted average of export prices using base year weights.

by more than output because, as the price of labour rises, firms substitute caapital for labour. Also, it takes more time to optimally adjust the capital stock.

Before discussing the effect of changing the demographic assumptions that underpin the principal GAD projections, it is instructive to test how sensitive the measured economic impacts are to changing the values of key economic parameters in the AMOS model. We focus on two parameters here. These are the Armington trade elasticities and the elasticity of the wage bargaining function. For both parameters we consider the variation in key endogenous variables in the final period, 2040. The results are given in Tables 5.2 and 5.3.

Table 5.2 shows the results of varying the trade price elasticities. Our default value for the Armington trade elasticities is 2, and this is the value used to generate the results shown in Table 5.1. Where the trade elasticities are increased this implies that the markets for both exports and imports are more price elastic. This is often taken to represent a more open economy where the relevant product markets are more competitive. In terms of the analysis in Section 2 we expect an increase in trade elasticities to produce an increase in the elasticity of the general equilibrium labour demand curve, so that the quantity of labour demanded becomes more sensitive to changes in the real wage. Where the trade elasticities are reduced, the opposite is the case. The results shown in Table 5.2 support this interpretation.

#### Table 5.2 here

Focussing first on the labour market variables, where the trade elasticity is increased to 4, the 2040 fall in employment is greater (10.1%) and the increase in the real wage is lower (5.6%), than under the default simulation. Increasing the Armington elasticities reduces the movement in the terms of trade that accompany a tightening of the labour market and increases the output adjustments that are required to bring the economy back into equilibrium. There is a larger fall in GDP and consumption than under the default elasticity and the cpi and export price (competitiveness) adjustments are lower. Where the Armington elasticities are reduced to unity, the adjustments are in the opposite direction. In this case the real wage, cpi and export prices increase by more and employment, GDP and consumption

fall by less than for the default simulations. Whilst with the low export demand elasticity *per capita* GDP still falls, *per capita* consumption shows a slight increase.

In order to test how sensitive the results are to variation in labour market assumptions, we conducted simulations using the GAD population projection under two limiting cases. The first imposes within-period fixed labour supply. This means that labour supply is a given proportion of the labour force, where the labour force is adjusted period-by-period through demographic changes. There is therefore assumed to be no unemployment or participation rate adjustment as the labour market tightens. This implies a wage curve that is infinitely elastic. <sup>10</sup> In the analytical framework outlined in Section 2, in any individual time period, this situation is represented by a vertical labour supply curve.

The second, alternative, assumption is that there is so much slack in the local labour market that any change in labour demand is met by a corresponding change in employment but no adjustment to the real wage. In this case, in each time period the labour supply curve would be horizontal and employment adjusts to changes in labour demand through changes in the unemployment and participation rates. This corresponds to a wage curve where the elasticity is zero.

In the fixed labour supply case we expect the employment reduction and the increase in the real wage to be larger than under the default simulations. In the fixed real wage scenario, the opposite results hold: we expect the employment to fall and the wage increase to be less than in the conventional wage curve account. However, when we attempt to run the model with a fixed real wage, the model fails to solve. The problem is that in this case the model produces a negative unemployment rate. The key practical point is simply this: the population constraints implied by the GAD demographic projections, combined with a fixed government expenditure per head, must put upward pressure on real wages. The fixed real wage scenario is unfeasible.

On the other hand, where labour supply is completely inelastic, the whole adjustment to the labour force contraction must come through higher wages. As we

have argued already, we expect employment and output to fall by more under this labour market closure, as participation and unemployment rates are not allowed to adjust. This will be combined with an even grater pressure upon wages. In Table 5.3 we present the percentage changes in the main aggregate indicators for 2040 under the fixed labour supply and wage curve labour market closures.

#### Table 5.3 here

Indeed our results are consistent with what economic theory suggests. Employment with fixed labour supply is expected to fall by 14.4%, exactly in line with the reduction in the labour force (and working age population), whilst under the bargaining closure the employment reduction is only 9.0%. The corresponding figures for GDP and the wage rate show a reduction of 13.3% in GDP and an increase in the real wage of 13.7% with the fixed labour supply, as compared to a GDP reduction of 8.2% and a real wage increase of 7.7% with the wage curve closure. By definition, under fixed labour supply, the unemployment rate remains unaltered, whilst under bargaining the unemployment rate is almost halved.

#### 6. A Wider Range of Migration Scenarios.

Elsewhere we have tested the sensitive of these economic results to changes in a range of demographic parameters. (Lisenkova *et al*, 2006). These demographic parameters are the birth rate, the male and female life expectancy, and the migration rate. We used the GAD high, principal and low estimates for these parameters as guidelines. The demographic parameter with the biggest economic impact is the rate of net migration, and it is this variable that has attracted the most policy interest. For this reason we have undertaken extensive simulations where we vary the migration parameter. However, the model is entirely flexible concerning demographic effects and could be equally used to simulate the impact of an increasing birth rate, further increases in life expectancy or demographic disturbances, such as those associated with wars or mass migration.

<sup>&</sup>lt;sup>10</sup> Remember that the wage curve represents the wage as a function of the unemployment rate. AA perfectly elastic wage curve is therefore observationally equivalent to a perfectly inelastic labour supply curve.

Figure 6.1 shows the GDP changes over time under the high, principal and low values for Scottish net migration parameter given by GAD. In the principal GAD Scottish projections, the net migration is set at 4,000 *per annum*. GAD suggests 12,000 in-migrants and 4,500 out-migrants as appropriate high and low annual values for migration.

# Figure 6.1 here

We here extend this analysis of the impact of migration by performing a range of simulations where we vary the migration rate, whilst holding the other demographic parameters constant at their GAD principal values. Specifically, we report simulations where the annual rate of in-migration is: -10,000, zero, 4,000 (the current GAD principal projection), 5,000, 10,000, 20,000, 30,000, 40,000, 50,000, and 60,000.

Before considering the simulation results it is instructive to see how these projected rates compare to the actual Scottish net migration figures over the recent past. Figure 6.2 shows these values since 1950. Up to 1990 Scotland experienced substantial net out migration. However, since 1990 net migration has fluctuated around zero. An important point in assessing these simulations is that the maximum net migration figure for any one year is 27,200, though this did occurred in 2003. Values of annual net migration of 30,000 and above have simply not been observed in the recent past for Scotland.

### Figure 6.2 here

Summaries of the main aggregate economic and demographic indicators for these alternative migration simulations are given in Table 6.1. For the simulations with migration rates that differ from those for which we have GAD projections, the demographic projections are made using the POPGROUP model.<sup>11</sup> The subsequent time sequences for total and working age population are then used as exogenous shocks in the AMOS model.

#### Table 6.1 here

We focus attention here on a subset of five indicative migration scenarios, together with the GAD principal projection. These are for Scottish in-migration of – 10,000, zero, 10,000, 20,000 and 40,000 per annum. The first key point is that only with a net migration above 10,000 per annum is total population projected to be greater than the base year (2000) value in 2040, and only with a net migration of above 20,000 *per annum* is working age population higher.

In Figures 6.3 to 6.6 we present the changes in GDP, employment, real wage and consumption for these indicative net migration scenarios. The economic results are consistent with the analysis in Section 2. We begin with Figure 6.3 and 6.4, which give the percentage changes in GDP and employment. Note that only for net migration rates of 20,000 *per annum* and over do GDP and employment levels rise as a result of population change. With in-migration values below this, both GDP and employment fall.

# Figure 6.3 here Figure 6.4 here

The mechanisms driving these results stem primarily from the labour market and the subsequent impact on competitiveness. Figure 6.4 shows the real wage change associated with the different migration scenarios. In this case, the 20,000 in-migration case generates a small increase in the real wage. We know that in this simulation, the working age population is rising, but more slowly than the total population. In terms of the analysis in Section 2, the outward shift of the labour demand curve, as government expenditure rises in line with total population, is greater than the outward shift in the labour supply curve as the working age population increases. There is

<sup>&</sup>lt;sup>11</sup> Our population projections are not exactly comparable to the GAD projections, so that some inconsistencies occur. Specifically, we assume an age distribution for migrants that is slightly different

therefore an increase in employment but also a small increase in the wage. However, for the examples where net migration is less than 20,000, the wage increase is much greater and is not accompanied by any expansion in government expenditure. The tightening of the labour market is reflected in reduced competitiveness, with the price of exported goods falling only for those simulations where in-migration is greater than 20,000 *per annum*.

### Figure 6.5 here

Finally, it is useful to look at the impact of alternative rates of net migration on consumption. This is shown in Figure 6.6. As with GDP, the change in total consumption in 2040 becomes positive only where net migration is 20,000 per annum. However, the variation in consumption is less than the variation in GDP. Further, changes in migration do not improve *per capita* consumption, but actual reduce it.

# Figure 6.6 here

These results suggest that immigration has a positive impact on economic activity but that net migration of around 20,000 *per annum* is required in order to fully offset the negative effects of a "naturally" declining and ageing population. This level of in-migration will prevent working age population from shrinking up to 2040.

#### 7. Conclusions

In this paper we explore the macroeconomic consequences of the principal GAD projected declining and ageing population in Scotland using methods which combine both the POPGROUP demographic model and the AMOS economic model for Scotland. Our main conclusion is that the resulting tightening of the labour market will have adverse consequences for the Scottish employment level, growth and competitiveness. For the principal GAD demographic projections by 2040 Scottish total and working age populations are 2.6% and 14.4% below the base year (2000) values. This generates a fall in Scottish employment and GDP of 9.0% and 8.2%

respectively below the levels that would have occurred had the population size and composition remain constant.

The impact that demographic change might have on economic activity has been a concern for Scottish policy makers. The demographic variable with the biggest economic impact is the rate of net migration and it is this variable that has attracted the most policy interest. The advantage of increased net migration is that immigrants are mainly of working age and are accompanied by children. However, if current demographic trends continue, policy initiatives, such as the Scottish Executive's Fresh Talent Initiative need to produce a positive annual net migration of 20,000 in order to prevent the labour force shrinking by 2040.

There are, of course, issues that merit further investigation and may play a crucial role in forming future macroeconomic trends in the light of changing demographics. Two seem particularly significant. First there will be effects upon the composition of both private and public sector consumption demand associated with population ageing. This concerns not only the differences in the tastes and needs as an individual ages but also issues such as the political power of older voters. Secondly, if young and old workers are qualitatively different, with different skills and other work characteristics, the ageing of the workforce will generate qualitative labour market effects not captured in the AMOS model at present. This will affect not only the overall productivity of the workforce but also potentially the distribution of wage income between young and old workers.

Figure 1.1- Percentage changes from the base year (2000) values for working age and total population using the principal GAD projection.

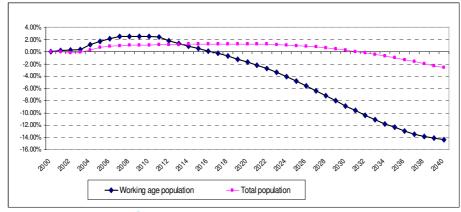


Table 5.1- Percentage change of aggregate economic and demographic variables under the principal GAD projection

principal GMD projection						
GAD	2000	2005	2010	2020	2030	2040
GDP	0.00	-0.42	-1.12	-3.03	-5.41	-8.15
Real wage	0.00	1.12	2.15	4.12	5.98	7.65
Consumption	0.00	-0.20	-0.60	-1.75	-3.28	-5.22
Working Age Population	0.00	1.71	2.50	-1.69	-8.87	-14.37
Total population	0.00	0.67	1.09	1.28	0.25	-2.62
Total employment	0.00	-0.60	-1.43	-3.54	-6.07	-8.97
Competitiveness index	0.00	0.26	0.62	1.51	2.48	3.39
CPI	0.00	0.20	0.46	1.14	1.88	2.55

Figure 5.1- Impact on sectoral output and employment. (Percentage changes by the year 2040.

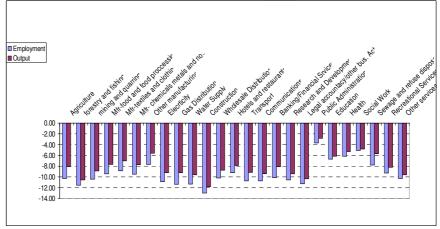


Table 5.2: Percentage change of aggregate economic and demographic variables for 2040 with the GAD net migration scenario, but varying the Armington trade elasticities

	Trade Elasticities			
	1	2	4	
GDP	-6.32	-8.15	-9.25	
Real wage	11.74	7.65	5.61	
Consumption	-2.01	-5.22	-6.95	
Working Age Population	-14.37	-14.37	-14.37	
Total population	-2.62	-2.62	-2.62	
Total employment	-7.29	-8.97	-10.07	
Competitiveness index	6.30	3.39	1.95	
CPI	4.94	2.55	1.36	

Table 5.3: Percentage change of aggregate economic and demographic variables for 2040 with the GAD net migration scenario, but varying the labour market closure assumptions

	Bargaining	
	(LNJ estimates)	Fixed Labour Supply
GDP	-8.15	-13.26
Real wage	7.65	13.65
Consumption	-5.22	-6.48
Working Age Population	-14.37	-14.37
Total population	-2.62	-2.62
Total employment	-8.97	-14.37
Competitiveness index	3.39	6.42
CPI	2.55	4.79

Figure 6. 1 Trends of GDP for alternative migration assumptions

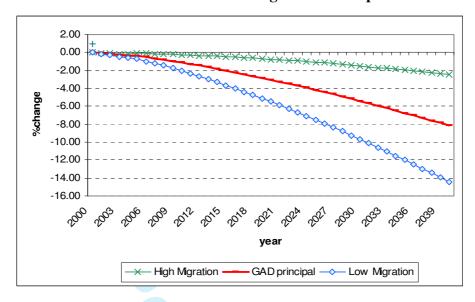


Figure 6.2- Net migration Scotland 1950-2005. Source General Register Office for Scotland.

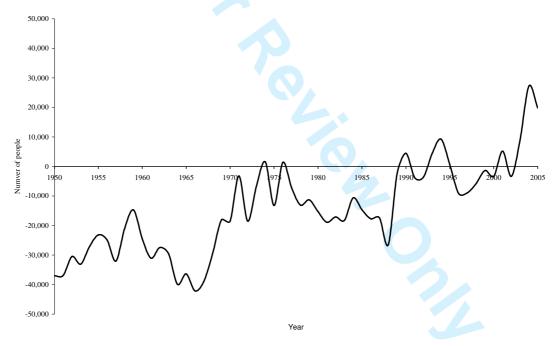


Table 6. 1 Main aggregate economic indicators for different net migration scenarios

2040 -20.37 19.09 -13.29 -29.73 -16.69 -22.39 7.94 5.66 2040 -13.19 12.29 -8.49 -19.08 -7.53 -14.53 5.29 3.90 2040
19.09 -13.29 -29.73 -16.69 -22.39 7.94 5.66 2040 -13.19 12.29 -8.49 -19.08 -7.53 -14.53 5.29 3.90
-13.29 -29.73 -16.69 -22.39 7.94 5.66 2040 -13.19 12.29 -8.49 -19.08 -7.53 -14.53 5.29 3.90
-29.73 -16.69 -22.39 7.94 5.66 <b>2040</b> -13.19 12.29 -8.49 -19.08 -7.53 -14.53 5.29 3.90
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5.66 2040 -13.19 12.29 -8.49 -19.08 -7.53 -14.53 5.29 3.90
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-13.19 12.29 -8.49 -19.08 -7.53 -14.53 5.29 3.90
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-19.08 -7.53 -14.53 5.29 3.90
-7.53 -14.53 5.29 3.90
-14.53 5.29 3.90
5.29 3.90
3.90
2040
-8.15
7.65
-5.22
-14.37
-2.62
-8.97
3.39
2.55
2040
-7.78
7.13
-5.06
-13.75
-2.92
-8.57
3.16
2.38
2040
-4.14
4.50
-2.44
-8.41
1.66
-4.54
2.08
1.60

Table 6.1 (continued)

Table 6.1 (continued)					1	
20000	2000	2005	2010	2020	2030	2040
GDP	0.00	0.16	0.50	1.28	1.98	2.48
Real wage	0.00	-0.06	0.04	0.32	0.51	0.49
Consumption	0.00	0.18	0.57	1.44	2.17	2.63
Working Age Population	0.00	1.66	4.04	4.58	2.78	2.24
Total population	0.00	0.65	2.43	6.30	9.34	10.84
Total employment	0.00	0.21	0.61	1.49	2.26	2.80
Competitiveness index	0.00	0.01	0.10	0.29	0.42	0.43
СРІ	0.00	0.04	0.14	0.32	0.41	0.39
30000	2000	2005	2010	2020	2030	2040
GDP	0.00	0.53	1.51	3.82	6.19	8.41
Real wage	0.00	-0.73	-1.08	-1.52	-1.91	-2.40
Consumption	0.00	0.46	1.38	3.50	5.61	7.51
Working Age Population	0.00	1.84	5.50	8.79	10.29	12.88
Total population	0.00	0.85	3.67	9.95	15.66	20.01
Total employment	0.00	0.72	1.87	4.45	7.03	9.40
Competitiveness index	0.00	-0.12	-0.15	-0.27	-0.47	-0.77
CPI	0.00	-0.03	0.00	-0.05	-0.22	-0.49
40000	2000	2005	2010	2020	2030	2040
GDP	0.00	0.89	2.47	6.19	10.09	13.84
Real wage	0.00	-1.37	-2.09	-3.07	-3.83	-4.57
Consumption	0.00	0.74	2.19	5.53	8.98	12.25
Working Age Population	0.00	2.08	6.96	13.03	17.76	23.56
Total population	0.00	1.05	4.94	13.61	21.98	29.19
Total employment	0.00	1.22	3.08	7.22	11.45	15.47
Competitiveness index	0.00	-0.23	-0.38	-0.74	-1.17	-1.66
CPI	0.00	-0.10	-0.12	-0.35	-0.71	-1.15
50000	2000	2005	2010	2020	2030	2040
GDP	0.00	1.24	3.37	8.41	13.71	18.87
Real wage	0.00	-1.96	-3.01	-4.38	-5.38	-6.25
Consumption	0.00	1.02	2.99	7.53	12.27	16.87
Working Age Population	0.00	2.33	8.40	17.24	25.23	34.20
Total population	0.00	1.24	6.18	17.26	28.30	38.36
Total employment	0.00	1.70	4.22	9.82	15.57	21.11
Competitiveness index	0.00	-0.35	-0.58	-1.12	-1.72	-2.34
CPI	0.00	-0.16	-0.22	-0.58	-1.08	-1.64
60000	2000	2005	2010	2020	2030	2040
GDP	0.00	1.57	4.23	10.51	17.13	23.62
Real wage	0.00	-2.52	-3.84	-5.51	-6.65	-7.59
Consumption	0.00	1.30	3.78	9.50	15.49	21.39
Working Age Population	0.00	2.57	9.83	21.45	32.71	44.84
Total population	0.00	1.44	7.43	20.92	34.62	47.54
Total employment	0.00	2.16	5.31	12.28	19.45	26.43
Competitiveness index	0.00	-0.45	-0.75	-1.44	-2.16	-2.86
CPI	0.00	-0.21	-0.30	-0.76	-1.37	-2.01
<u>.</u>	0.00	0.21	0.00	0.70	1.07	2.01

Figure 6.3 – Trends of GDP for alternative migration scenarios

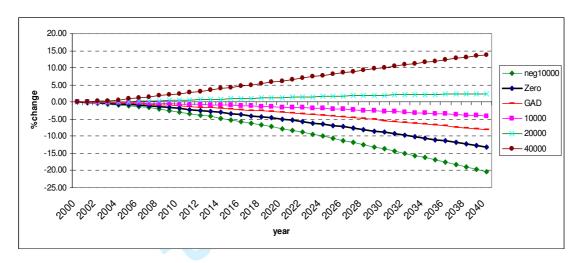


Figure 6.4- Trends of employment for alternative migration scenarios

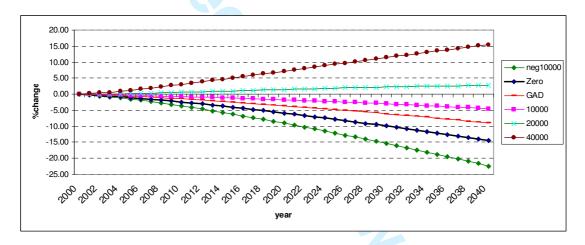


Figure 6.5 Trends of real wage for alternative migration scenarios

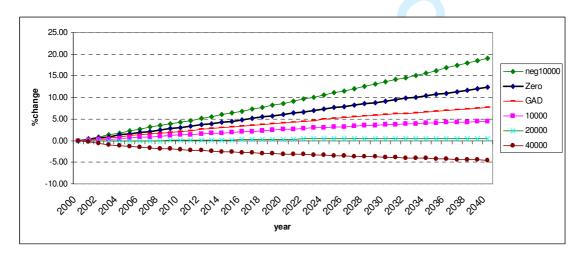
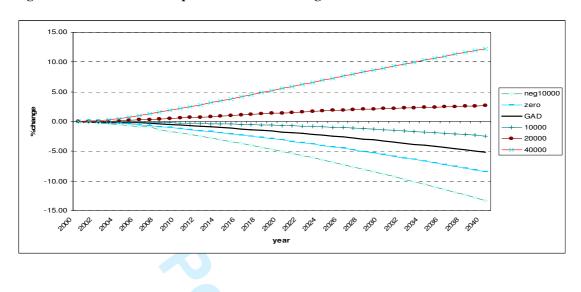


Figure 6.6 Trends of consumption for alternative migration scenarios



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# **Appendix 1: The Sectoral Disaggregation in AMOS**

Table A1.1 Sectors (Activities/Commodities) Identified in AMOS

	Sector name	IOC codes	SIC
1	Agriculture	1	1
2	Forestry and fishing	2 & 3	2, 5.01 & 5.02
3	Mining and quarrying	4-7	10-14
4	Mfr – food and food processing	8-20	15 & 16
5	Mfr - textiles and clothing	21-30	17-19
6	Mfr - chemicals, metals and non-metals	36-61	24-28
7	Mfr - other manufacturing	31-35, 62-84	20-23, 29-37
8	Electricity	85	40.1
9	Gas	86	40.2 & 40.3
10	Water	87	41
11	Construction	88	45
12	Wholesale and retail distribution	89-91	50-52
13	Hotels and restaurants	92	55
14	Transport	93-97	60.1-63
15	Communications	98-99	64
16	Banking and other financial services	100-107	65-72
17	Research and development	108	73
18	Legal, accountancy and other business services	109-114	74
19	Public administration	115	75
20	Education	116	80
21	Health	117	85.1 & 85.2
22	Social work	118	85.3
23	Sewage and refuse disposal	119	90
24	Recreational services	121	92
25	Other services	120, 122 & 123	91, 93 & 95

# **Appendix 2: The AMOS Model**

### Table A2.1: A Condensed Version of the AMOS CGE Model

1. Commodity Price $p_{i} = p_{i}(w_{n}, w_{ki})$ $cpi = \sum_{i} \theta_{i} p_{i} + \sum_{i} \theta_{i}^{UK} \overline{p}_{i}^{UK} + \sum_{i} \theta_{i}^{ROW} \overline{p}_{i}^{ROW}$ 3. Capital Price Index $kpi = \sum_{i} \gamma_{i} p_{i} + \sum_{i} \gamma_{i}^{UK} \overline{p}_{i}^{UK} + \sum_{i} \gamma_{i}^{ROW} \overline{p}_{i}^{ROW}$ 4. User Cost of Capital $uck = uck (kpi)$ 5. Wage Equation $w_{n} = w_{n}(N, \overline{L}, cpi)$ 6. Capital Sock $K_{i,t}^{S} = (1 - d_{i})K_{i,t-1} + \Delta K_{i,t-1}$ 7. Labour Demand $N_{i}^{D} = N_{i}^{D}(Q_{i}, w_{n}, w_{k,i})$ 8. Capital Demand $K_{i}^{D} = K_{i}^{D}(Q_{i}, w_{n}, w_{k,i})$ 9. Labour Market Clearing $\sum_{i} N_{i}^{D} = N$ 10. Capital Market Clearing $K_{i}^{S} = K_{i}^{D}$ 11. Household Income $Y = \Psi_{n}Nw_{n} + \Psi_{k} \sum_{i} K_{i}w_{k,i}$ 12. Commodity Demand $Q_{i} = C_{i} + I_{i} + G_{i} + X_{i}$ 13. Consumption Demand $C_{i} = C_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_{i}^{*} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ 15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i}(K_{i}^{*} - K_{i})$ 16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \overline{D}^{UK}, \overline{D}^{ROW})$ 17. Government Demand $G_{i} = \varphi_{i}p_{i}\overline{P}$ 18. Export Demand $X_{i} = X_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \overline{D}^{UK}, \overline{D}^{ROW})$	1.0	
$cpi = \sum_{i} \theta_{i} p_{i} + \sum_{i} \theta_{i}^{NN} \overline{p}_{i}^{NN} + \sum_{i} \theta_{i}^{NN} \overline{p}_{i}^{NN} $ $3. \text{ Capital Price Index} \qquad kpi = \sum_{i} \gamma_{i} p_{i} + \sum_{i} \gamma_{i}^{UK} \overline{p}_{i}^{UK} + \sum_{i} \gamma_{i}^{NOW} \overline{p}_{i}^{ROW} $ $4. \text{ User Cost of Capital} \qquad uck = uck(kpi)$ $5. \text{ Wage Equation} \qquad w_{n} = w_{n}(N, \overline{L}, cpi)$ $6. \text{ Capital Sock} \qquad K_{i,t}^{S} = (1 - d_{i}) K_{i,t-1} + \Delta K_{i,t-1} $ $7. \text{ Labour Demand} \qquad N_{i}^{D} = N_{i}^{D}(Q_{i}, w_{n}, w_{k,t})$ $8. \text{ Capital Demand} \qquad K_{i}^{D} = K_{i}^{D}(Q_{i}, w_{n}, w_{k,t})$ $9. \text{ Labour Market Clearing} \qquad \sum_{i} N_{i}^{D} = N$ $10. \text{ Capital Market Clearing} \qquad K_{i}^{S} = K_{i}^{D}$ $11. \text{ Household Income} \qquad Y = \Psi_{n} N w_{n} + \Psi_{k} \sum_{i} K_{i} w_{k,t} $ $12. \text{ Commodity Demand} \qquad Q_{i} = C_{i} + I_{i} + G_{i} + X_{i}$ $13. \text{ Consumption Demand} \qquad C_{i} = C_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ $14. \text{ Desired Capital Stock} \qquad K_{i}^{*} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ $15. \text{ Capital Stock Adjustment} \qquad \Delta K_{i} = \lambda_{i}(K_{i}^{*} - K_{i})$ $1_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ $17. \text{ Government Demand} \qquad G_{i} = \varphi_{i}p_{i}\overline{p}$	1. Commodity Price	$p_i = p_i(w_n, w_{ki})$
$kpi = \sum_{i} \gamma_{i} p_{i} + \sum_{i} \gamma_{i}^{ROW} \overline{p}_{i}^{ROW} + \sum_{i} \gamma_{i}^{ROW} \overline{p}_{i}^{ROW}$ 4. User Cost of Capital $uck = uck(kpi)$ 5. Wage Equation $w_{n} = w_{n}(N, \overline{L}, cpi)$ 6. Capital Sock $K_{i,j}^{S} = (1 - d_{i})K_{i,j-1} + \Delta K_{i,j-1}$ 7. Labour Demand $N_{i}^{D} = N_{i}^{D}(Q_{i}, w_{n}, w_{k,j})$ 8. Capital Demand $K_{i}^{D} = K_{i}^{D}(Q_{i}, w_{n}, w_{k,j})$ 9. Labour Market Clearing $\sum_{i} N_{i}^{D} = N$ 10. Capital Market Clearing $K_{i}^{S} = K_{i}^{D}$ 11. Household Income $Y = \Psi_{n}Nw_{n} + \Psi_{k}\sum_{i} K_{i}w_{k,i}$ 12. Commodity Demand $Q_{i} = C_{i} + I_{i} + G_{i} + X_{i}$ 13. Consumption Demand $C_{i} = C_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_{i}^{S} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ 15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i}(K_{i}^{S} - K_{i})$ 16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i}p_{i}\overline{P}$	2. Consumer Price Index	$cpi = \sum_{i} \theta_{i} p_{i} + \sum_{i} \theta_{i}^{UK} \overline{p}_{i}^{UK} + \sum_{i} \theta_{i}^{ROW} \overline{p}_{i}^{ROW}$
5. Wage Equation $ w_n = w_n(N, \overline{L}, cpi) $ 6. Capital Sock $ K_{i,j}^S = (1 - d_i) K_{i,t-1} + \Delta K_{i,j-1} $ 7. Labour Demand $ N_i^D = N_i^D(Q_i, w_n, w_{k,i}) $ 8. Capital Demand $ K_i^D = K_i^D(Q_i, w_n, w_{k,i}) $ 9. Labour Market Clearing $ \sum_i N_i^D = N $ 10. Capital Market Clearing $ K_i^S = K_i^D $ 11. Household Income $ Y = \Psi_n N w_n + \Psi_k \sum_i K_i w_{k,i} $ 12. Commodity Demand $ Q_i = C_i + I_i + G_i + X_i $ 13. Consumption Demand $ C_i = C_i(p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, Y, cpi) $ 14. Desired Capital Stock $ K_i^* = K_i^D(Q_i, w_n, uck) $ 15. Capital Stock Adjustment $ \Delta K_i = \lambda_i (K_i^* - K_i) $ 16. Investment Demand $ I_i = I_i(p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, \sum_i b_{i,j} \Delta K_j) $ 17. Government Demand $ G_i = \varphi_i p_i \overline{P} $	3. Capital Price Index	$kpi = \sum_{i} \gamma_{i} p_{i} + \sum_{i} \gamma_{i}^{UK} \overline{p}_{i}^{UK} + \sum_{i} \gamma_{i}^{ROW} \overline{p}_{i}^{ROW}$
6. Capital Sock $K_{i,i}^{S} = (1 - d_i)K_{i,i-1} + \Delta K_{i,i-1}$ 7. Labour Demand $N_i^D = N_i^D(Q_i, w_n, w_{k,i})$ 8. Capital Demand $K_i^D = K_i^D(Q_i, w_n, w_{k,i})$ 9. Labour Market Clearing $\sum_i N_i^D = N$ 10. Capital Market Clearing $K_i^S = K_i^D$ 11. Household Income $Y = \Psi_n N w_n + \Psi_k \sum_i K_i w_{k,i}$ 12. Commodity Demand $Q_i = C_i + I_i + G_i + X_i$ 13. Consumption Demand $C_i = C_i(p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_i^* = K_i^D(Q_i, w_n, uck)$ 15. Capital Stock Adjustment $\Delta K_i = \lambda_i (K_i^* - K_i)$ 16. Investment Demand $I_i = I_i(p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, \sum_i b_{i,j} \Delta K_j)$ 17. Government Demand $G_i = \varphi_i p_i \overline{P}$	4. User Cost of Capital	uck = uck(kpi)
7. Labour Demand $N_{i}^{D} = N_{i}^{D}(Q_{i}, w_{n}, w_{k,i})$ 8. Capital Demand $K_{i}^{D} = K_{i}^{D}(Q_{i}, w_{n}, w_{k,i})$ 9. Labour Market Clearing $\sum_{i} N_{i}^{D} = N$ 10. Capital Market Clearing $K_{i}^{S} = K_{i}^{D}$ 11. Household Income $Y = \Psi_{n}Nw_{n} + \Psi_{k}\sum_{i} K_{i}w_{k,i}$ 12. Commodity Demand $Q_{i} = C_{i} + I_{i} + G_{i} + X_{i}$ 13. Consumption Demand $C_{i} = C_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_{i}^{*} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ 15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i}(K_{i}^{*} - K_{i})$ 16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i}p_{i}\overline{P}$	5. Wage Equation	$w_n = w_n(N, \overline{L}, cpi)$
8. Capital Demand $K_{i}^{D} = K_{i}^{D}(Q_{i}, w_{n}, w_{k,i})$ 9. Labour Market Clearing $\sum_{i} N_{i}^{D} = N$ 10. Capital Market Clearing $K_{i}^{S} = K_{i}^{D}$ 11. Household Income $Y = \Psi_{n}Nw_{n} + \Psi_{k}\sum_{i} K_{i}w_{k,i}$ 12. Commodity Demand $Q_{i} = C_{i} + I_{i} + G_{i} + X_{i}$ 13. Consumption Demand $C_{i} = C_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_{i}^{*} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ 15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i}(K_{i}^{*} - K_{i})$ 16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i}p_{i}\overline{P}$	6. Capital Sock	$K_{i,t}^{S} = (1 - d_i)K_{i,t-1} + \Delta K_{i,t-1}$
9. Labour Market Clearing $\sum_{i} N_{i}^{D} = N$ 10. Capital Market Clearing $K_{i}^{S} = K_{i}^{D}$ 11. Household Income $Y = \Psi_{n}Nw_{n} + \Psi_{k}\sum_{i}K_{i}w_{k,i}$ 12. Commodity Demand $Q_{i} = C_{i} + I_{i} + G_{i} + X_{i}$ 13. Consumption Demand $C_{i} = C_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_{i}^{*} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ 15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i}(K_{i}^{*} - K_{i})$ 16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i}p_{i}\overline{P}$	7. Labour Demand	$N_i^D = N_i^D(Q_i, w_n, w_{k,i})$
$\sum_{i} N_{i}^{D} = N$ $10. \text{ Capital Market Clearing} \qquad K_{i}^{S} = K_{i}^{D}$ $11. \text{ Household Income} \qquad Y = \Psi_{n} N w_{n} + \Psi_{k} \sum_{i} K_{i} w_{k,i}$ $12. \text{ Commodity Demand} \qquad Q_{i} = C_{i} + I_{i} + G_{i} + X_{i}$ $13. \text{ Consumption Demand} \qquad C_{i} = C_{i} (p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ $14. \text{ Desired Capital Stock} \qquad K_{i}^{*} = K_{i}^{D} (Q_{i}, w_{n}, uck)$ $15. \text{ Capital Stock Adjustment} \qquad \Delta K_{i} = \lambda_{i} (K_{i}^{*} - K_{i})$ $16. \text{ Investment Demand} \qquad I_{i} = I_{i} (p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ $17. \text{ Government Demand} \qquad G_{i} = \varphi_{i} p_{i} \overline{P}$	8. Capital Demand	$K_i^D = K_i^D(Q_i, w_n, w_{k,i})$
11. Household Income $Y = \Psi_n N w_n + \Psi_k \sum_i K_i w_{k,i}$ 12. Commodity Demand $Q_i = C_i + I_i + G_i + X_i$ 13. Consumption Demand $C_i = C_i (p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_i^* = K_i^D (Q_i, w_n, uck)$ 15. Capital Stock Adjustment $\Delta K_i = \lambda_i (K_i^* - K_i)$ 16. Investment Demand $I_i = I_i (p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, \sum_i b_{i,j} \Delta K_j)$ 17. Government Demand $G_i = \varphi_i p_i \overline{P}$	9. Labour Market Clearing	$\sum_{i} N_{i}^{D} = N$
$Y = \Psi_{n}Nw_{n} + \Psi_{k} \sum_{i} K_{i}w_{k,i}$ 12. Commodity Demand $Q_{i} = C_{i} + I_{i} + G_{i} + X_{i}$ 13. Consumption Demand $C_{i} = C_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_{i}^{*} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ 15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i}(K_{i}^{*} - K_{i})$ 16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i}p_{i}\overline{P}$	10. Capital Market Clearing	$K_i^S = K_i^D$
13. Consumption Demand $C_{i} = C_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, Y, cpi)$ 14. Desired Capital Stock $K_{i}^{*} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ 15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i}(K_{i}^{*} - K_{i})$ 16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i} p_{i} \overline{P}$	11. Household Income	$Y = \Psi_n N w_n + \Psi_k \sum_i K_i w_{k,i}$
14. Desired Capital Stock $K_{i}^{*} = K_{i}^{D}(Q_{i}, w_{n}, uck)$ 15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i}(K_{i}^{*} - K_{i})$ 16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i} p_{i} \overline{P}$	12. Commodity Demand	$Q_i = C_i + I_i + G_i + X_i$
15. Capital Stock Adjustment $\Delta K_{i} = \lambda_{i} (K_{i}^{*} - K_{i})$ 16. Investment Demand $I_{i} = I_{i} (p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i} p_{i} \overline{P}$	13. Consumption Demand	$C_i = C_i(p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, Y, cpi)$
16. Investment Demand $I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i} p_{i} \overline{P}$	14. Desired Capital Stock	$K_i^* = K_i^D(Q_i, w_n, uck)$
$I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{VK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$ 17. Government Demand $G_{i} = \varphi_{i} p_{i} \overline{P}$	15. Capital Stock Adjustment	$\Delta K_i = \lambda_i (K_i^* - K_i)$
$\bigcup_i \varphi_i P_i$	16. Investment Demand	$I_{i} = I_{i}(p_{i}, \overline{p}_{i}^{UK}, \overline{p}_{i}^{ROW}, \sum_{i} b_{i,j} \Delta K_{j})$
18. Export Demand $X_i = X_i(p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, \overline{D}^{UK}, \overline{D}^{ROW})$	17. Government Demand	$G_i = \varphi_i p_i \overline{P}$
-	18. Export Demand	$X_i = X_i(p_i, \overline{p}_i^{UK}, \overline{p}_i^{ROW}, \overline{D}^{UK}, \overline{D}^{ROW})$

#### **NOTATION**

#### **Activity-Commodities**

i, j are activity/commodity subscripts.

#### **Transactors**

UK = United Kingdom, ROW = Rest of World

#### **Functions**

p (.) cost function

 $w_n(.)$  wage equation

uck(.) user cost of capital formulation

K<sup>D</sup>(.), N<sup>D</sup>(.) factor demand functions

C(.), I(.), X(.) Armington consumption, investment and export demand functions,

homogenous of degree zero in prices and one in quantities

#### **Variables**

C consumption

D exogenous export demand

G government demand for local goods

I investment demand for local goods

 $\Delta K$  investment demand by activity

K<sup>D</sup>, K<sup>S</sup>, K<sup>\*</sup>, K capital demand, capital supply, desired and actual capital stock

L labour force

N<sup>D</sup>, N labour demand and total employment

P population

Q commodity/activity output

X exports

Y household nominal income

b elements of capital matrix

cpi, kpi consumer and capital price indices

d physical depreciation

p price of commodity/activity output

t time subscript

uck user cost of capital

$\mathbf{w}_{\mathrm{n}},\mathbf{w}_{\mathrm{k}}$	wage, capital rental
Ψ	share of factor income retained in region
θ	cpi weights
γ	kpi weights
φ	government expenditure coefficient
λ	capital stock adjustment parameter

#### Notes:

Variables with a bar are exogenous.

A number of simplifications are made in this condensed presentation of AMOS

- 1. Intermediate demand is suppressed throughout e.g. only primary factor demands are noted in price determination in equation (1) and final demands in the determination of commodity demand in equation (12).
- 2. Income transfers are generally suppressed.
- 3. Taxes are ignored.
- 4. There are implicit time subscripts on all variables. These are only stated explicitly in the capital updating equation (6).