

## Security needs and the performance of the defense industry

Blume, Andreas; Tishler, Asher

Veröffentlichungsversion / Published Version

Arbeitspapier / working paper

Zur Verfügung gestellt in Kooperation mit / provided in cooperation with:

SSG Sozialwissenschaften, USB Köln

### Empfohlene Zitierung / Suggested Citation:

Blume, A., & Tishler, A. (2000). *Security needs and the performance of the defense industry*. (Discussion Papers / Wissenschaftszentrum Berlin für Sozialforschung, Forschungsschwerpunkt Marktprozeß und Unternehmensentwicklung, Abteilung Wettbewerbsfähigkeit und industrieller Wandel, 00-04). Berlin: Wissenschaftszentrum Berlin für Sozialforschung gGmbH. <https://nbn-resolving.org/urn:nbn:de:0168-ssoar-115859>

### Nutzungsbedingungen:

Dieser Text wird unter einer Deposit-Lizenz (Keine Weiterverbreitung - keine Bearbeitung) zur Verfügung gestellt. Gewährt wird ein nicht exklusives, nicht übertragbares, persönliches und beschränktes Recht auf Nutzung dieses Dokuments. Dieses Dokument ist ausschließlich für den persönlichen, nicht-kommerziellen Gebrauch bestimmt. Auf sämtlichen Kopien dieses Dokuments müssen alle Urheberrechtshinweise und sonstigen Hinweise auf gesetzlichen Schutz beibehalten werden. Sie dürfen dieses Dokument nicht in irgendeiner Weise abändern, noch dürfen Sie dieses Dokument für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, aufführen, vertreiben oder anderweitig nutzen.

Mit der Verwendung dieses Dokuments erkennen Sie die Nutzungsbedingungen an.

### Terms of use:

This document is made available under Deposit Licence (No Redistribution - no modifications). We grant a non-exclusive, non-transferable, individual and limited right to using this document. This document is solely intended for your personal, non-commercial use. All of the copies of this documents must retain all copyright information and other information regarding legal protection. You are not allowed to alter this document in any way, to copy it for public or commercial purposes, to exhibit the document in public, to perform, distribute or otherwise use the document in public.

By using this particular document, you accept the above-stated conditions of use.

**discussion papers**

FS IV 00 – 04

**Security Needs and the Performance  
of the Defense Industry**

Andreas Blume  
Asher Tishler  
University of Iowa

March 2000

ISSN Nr. 0722 - 6748

**Forschungsschwerpunkt  
Marktprozeß und Unter-  
nehmensentwicklung**

**Research Area  
Market Processes and  
Corporate Development**

Zitierweise/Citation:

Andreas Blume, Asher Tishler, **Security Needs and the Performance of the Defense Industry**, Discussion Paper FS IV 00-04, Wissenschaftszentrum Berlin, 2000.

Wissenschaftszentrum Berlin für Sozialforschung gGmbH,  
Reichpietschufer 50, 10785 Berlin, Tel. (030) 2 54 91 - 0

## ABSTRACT

### **Security Needs and the Performance of the Defense Industry**

by Andreas Blume and Asher Tishler\*

Today, leading defense firms are concentrated into just two distinct blocs – those based in the US and those in Western Europe. All US defense firms and most European ones are private. Market structure may thus play an important role in determining procurement levels as well as defense policies in the US and Europe.

This paper focuses on the interactions between defense needs and market structure. It presents a model in which two producer blocs (representing the US and Europe) produce an identical homogeneous defense good. The “rest of the world” purchases the defense good from the two producer countries. The security level of each of the two producing countries depends on its purchase of the defense good relative to the amount of defense good purchased by the rest of the world. Each country measures its security level against a target that it sets for itself.

The main results of this paper are: (1) Generally, the total world quantity of the defense good is lower when the governments of the producers of the defense good pay the world price (rather than the marginal production cost plus a markup) to their defense industries. (2) The net defense cost (government expenditure on the defense good minus the profit of the defense industry) of each producing country is lower when producing-country governments pay the world price to their own defense industries. (3) Government expenditure on the defense good and the net defense cost for each producing-country are smaller when the number of defense firms in each country is relatively small. (4) Target security levels affect the optimal number of firms in each of the two producing countries. Higher target security levels result in a larger number of defense firms. (5) Multiple equilibria in the game where the developed countries independently choose their own procurement rules are possible.

*Keywords: defense industry, technology, security level, market structure*

---

\* We are grateful to M. Balch, A. Daughety, D. Dvir, G. Fethke, A. James, A.R. Nobay, J. F. Reinganum, W.P. Rogerson, A. Shenhar and D. Webb for valuable comments and suggestions.

## ZUSAMMENFASSUNG

### **Sicherheitsbedürfnisse und die Leistungsfähigkeit der Rüstungsindustrie**

Heute sind die führenden Rüstungsunternehmen in zwei unterschiedlichen Regionen konzentriert: in den USA und in Westeuropa. Alle US-Unternehmen und die meisten europäischen Unternehmen sind in Privatbesitz. Insofern kann die Marktstruktur als wichtige Einflußgröße des Beschaffungsniveaus und der Verteidigungspolitik in den USA und Europa angesehen werden.

Der Beitrag konzentriert sich auf die Interaktion zwischen Verteidigungsbedürfnissen und Marktstruktur. Es wird ein Modell vorgestellt, in dem zwei „Produzentenblöcke“ (die USA und Europa) ein identisches, homogenes Rüstungsgut produzieren. Der „Rest“ der Welt kauft die Rüstungsgüter von den beiden Herstellerländern. Das Sicherheitsniveau jedes der beiden Länder hängt von seiner Beschaffung der Rüstungsgüter im Vergleich zum Beschaffungsvolumen an Rüstungsgüter durch den Rest der Welt ab. Jedes Land bestimmt sein Sicherheitsniveau im Vergleich zu einem selbstgesteckten Ziel.

Das Hauptergebnis besagt: (1) Im allgemeinen ist das weltweite Gesamtvolumen an Rüstungsgütern niedriger, wenn die Regierung der Herstellerländer der Rüstungsgüter den Weltpreis an ihre Industrien zahlen (im Unterschied zu einer Preisbildung nach Grenzkosten der Produktion plus Zuschlag). (2) Die Nettoverteidigungsausgaben (staatliche Ausgaben für Rüstungsgüter minus der Gewinne der Rüstungsindustrie) eines jeden Herstellerlandes sind geringer, wenn die Herstellerländer den Weltpreis an ihre eigene Industrie zahlen. (3) Die Staatsausgaben für die Rüstungsgüter und die Nettoverteidigungsausgaben für jedes Herstellerland sind geringer, wenn die Anzahl an Rüstungsunternehmen in jedem Land verhältnismäßig gering ist. (4) Die Höhe des gewünschten Sicherheitsniveaus beeinflusst die optimale Anzahl von Unternehmen in jedem der beiden Herstellerländer. Höhere Sicherheitsniveaus führen zu einer höheren Anzahl an Rüstungsunternehmen. (5) Multiple Gleichgewichte des Spiels, in dem die entwickelten Länder unabhängig von ihrem eigenen Beschaffungsniveaus entscheiden, sind möglich.

## 1. Introduction

Since the end of the cold war, national defense budgets have shrunk drastically while production capacity has changed little. Consequently, export markets have become more competitive. Moreover, despite the increased competition and the use of cheaper, off-the-shelf commercial components instead of specially designed military ones, the prices of new weapons and defense systems seem to be rising inexorably. Economic necessity is wearing away the defense industry's segregation, forcing companies and governments to cooperate as well as to compete across borders. The outcome of this consolidation has been the emergence of a small group of defense giants in the US and Europe. Size, it seems, is a crucial factor in the defense industry (The Economist, 1997).

In this new environment, economic pressures are pushing defense industries up against restraints on the proliferation of advanced conventional military capabilities. This is because the best customers for sophisticated weapon systems are in regions where sales are likely to feed existing tensions and old disputes may erupt into war. Additionally, aggressive marketing by cash-strapped companies fighting to stay competitive may waken dormant rivalries.

Today, more than ever before, strong military powers are countries that have state-of-the-art technological know-how and the economic means to develop it into sophisticated weapon systems. Platforms such as the air-superiority fighter jet, the ballistic missile, precision guided weapon systems, integrated air defense systems, and integrated intelligence systems embody levels of sophistication and lethality that separate those who can produce and use them from all others (Ben Israel (1998), Rogerson (1994), The Economist (1998), James (1998a,b), Louscher, Cook and Barto (1998), Dvir and Tishler (2000)).

The US government has actively encouraged the consolidation process of its defense firms in order to reduce procurement costs and sustain a viable defense industry during a period of declining defense budgets. European companies, facing intense competition from the giant US firms, are also being forced to consider consolidation within Europe and, lately, with

American firms. Various political, economic and social developments have left the US and Western Europe as the only strong military powers with state-of-the-art technological know-how and the economic means to develop it into sophisticated weapon systems. Among these are the demise of the USSR, political constraints across Europe, the policy of European governments to purchase locally made – or at least European – defense systems (Cobble (1998), James (1998a), Lovering (1998), Serfati (1998)), and the Pentagon’s “buy American” policy (Flamm (1998), Markusen (1998)).

All the defense firms in the US and most of those in the European bloc are private. So, market structure may play an important role in determining procurement levels as well as defense policies in the US and Europe. (See Rogerson (1990, 1994), Kovacic and Smallwood (1994) and Flamm (1998) on the procurement process in the US).

This paper presents a model of the interactions between defense needs and market structure. The model represents two developed countries (corresponding to the US and Western Europe) that produce an identical homogeneous defense good. The “rest of the world” includes all the other countries in the world. There is no production of defense goods in the rest of the world. The two producer countries are allies (not enemies) but may have enemies in the rest of the world. The rest of the world features a downward-sloping demand function for the defense good and it purchases the defense good (produced by the two developed countries) at the equilibrium world price. The security level of each of the two developed countries depends on their purchase of the defense good relative to the amount of it that is purchased by the rest of the world. Target security levels are determined by the country’s culture, social fabric, political structure, religions, beliefs, etc.

The defense firms in the two producer countries play a Cournot game<sup>1</sup>. The optimal behavior of the defense industries in these two countries is analyzed under two types of pricing practices: (1) the government of each of the two producer countries purchases the defense good

---

<sup>1</sup> Kreps and Scheinkman (1983) describe a two-stage game in which firms choose their capacity in the first stage and engage in price competition in the second stage. They show that under some conditions the subgame perfect equilibrium outcome of this game is identical to the outcome of the Cournot game that is used here.

from its own defense industry at the world price and, (2) the price that the government of each of the two producer countries pays to its own defense industry equals the marginal production cost plus a markup.

Some of the results of this paper depend on strategic interaction between market structure and security, while other results may be more intuitive and straightforward. First, a lower price of the defense good shifts the world's demand function to the right. This is the main reason why, when the government of a developed country chooses to pay its own defense industry the world price, the defense industry prefers to have more than one firm in the market. Second, government expenditure on the defense good and the country's net defense cost (government expenditure on the defense good minus the profit of the defense industry) in the developed countries are smaller when the number of the defense firms in each country is small, which may explain the current consolidation process of the defense industries in these countries. Third, the developed countries jointly prefer to pay their defense industries the world price (rather than marginal cost plus a markup). However, multiple equilibria in the game where the developed countries independently choose their own procurement rules are possible.

The other main results of this paper are as follows. First, the world price of the defense good is higher when the governments of the developed countries pay the world price for their purchase of the defense good than when they pay the marginal cost plus a "reasonably small" markup. As a result, the purchases of the defense good by these countries, as well as the sales to the rest of the world, are lower in this case. Hence, there are less weapon systems in the world when governments choose to pay their defense industries the world price. The net defense cost of each producing country is, generally, lower in this case. Second, a larger number of defense firms in the developed countries makes the defense industries more competitive. This reduces the world price of the defense good, and thereby increases sales of the defense good to the developed countries and also to the rest of the world. Third, target security levels in the developed countries affect the optimal number of firms in these countries. A higher target security level results in a larger number of defense firms in each country. Fourth, if production of the defense good is significantly more efficient in one of the two countries than in the other (due to a larger



investment in R&D, say), that country may capture most or all of the exports of the defense good to the rest of the world. The larger the number of defense firms in each of the two developed countries, the more pronounced this phenomenon becomes. Finally, our analysis shows that judging the defense industry on economic performance alone is misleading because this does not take account of the threat from enemies (see Dvir and Tishler (2000) on this issue).

This paper is organized as follows. Section 2 provides a background and data on the defense industry. Section 3 presents the concept of security and describes the model. In Section 4 the model is solved. Section 5 analyses solutions of the model under different pricing practices. The profits of the defense industry and the optimal number of firms in that industry are derived in Section 6. Section 7 compares solutions of the model under the two types of pricing practice. Section 8 discusses different technologies, R&D, and the resulting exports of the defense industry. Section 9 discusses policy issues and concludes.

## **2. Background**

The reduction in defense spending, procurement, and exports around the globe is evident from Tables 1 and 2. World military expenditures declined from \$1203 billion in 1990 to \$840 billion in 1994. During the same period, world defense budgets declined by over 30% (Louscher et al. (1998)). These reductions are most notable in the developed countries (Table 1), in which the share of defense spending in GDP declined by about 50% from 1985 to 1998. On average, procurement budgets accounted for about 30% of total defense expenditure in the first half of the nineties. This figure declined somewhat in the second half of the decade (Flamm (1998)). Table 2 clearly shows that global arms exports were much smaller than global procurements. That is, the producer countries strictly enforced a “buy local” policy. Almost all of the procurements in the US and Western Europe were from local producers (Table 3 in Louscher et al. (1998) shows that most arms exports are to countries in the Third World).

**Table 1: Defense spending by major countries**

	Defense Spending % of GDP				Government Spending \$ billion
	1985	1990	1995	1998	1998
<b>US</b>	<b>6.5</b>	<b>5.7</b>	<b>3.7</b>	<b>3.1</b>	<b>264</b>
<b>UK</b>	<b>5.3</b>	<b>4.2</b>	<b>3.0</b>	<b>2.8</b>	<b>40</b>
<b>France</b>	<b>4.0</b>	<b>3.6</b>	<b>3.2</b>	<b>2.8</b>	<b>37</b>
<b>Germany</b>	<b>3.1</b>	<b>2.8</b>	<b>1.8</b>	<b>1.4</b>	<b>32</b>

Source: The Economist (1998).

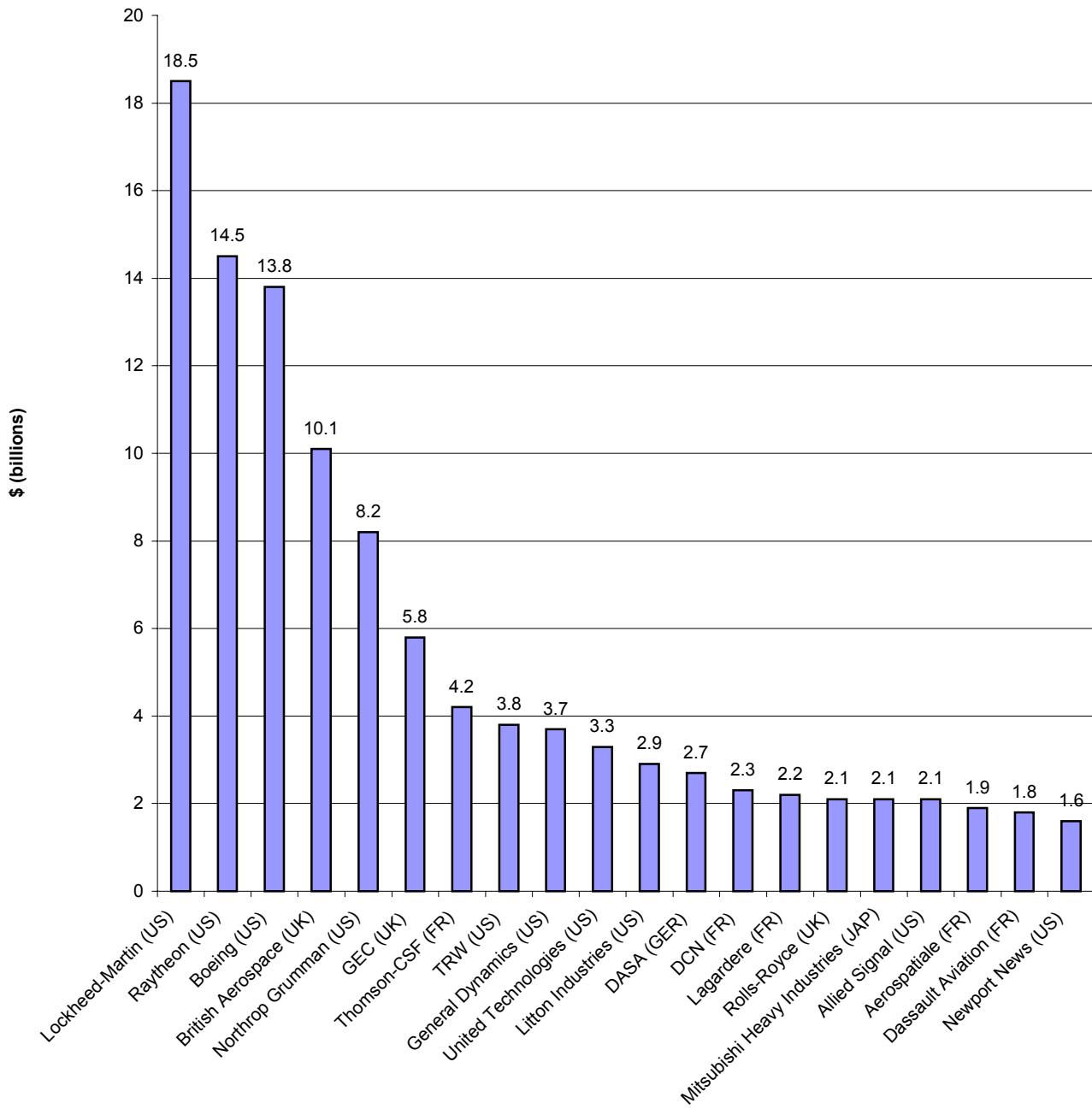
**Table 2: Global Military Expenditure, Procurement, and Arms Exports  
1990, 1994 (\$ billion, 1994 prices)**

	1990	1994
<b>Military Expenditure</b>	989	677
<b>Procurement Budget</b>	361	252
<b>Arms Exports</b>	57	32

Source: Louscher et al. (1998), The Economist (1997).

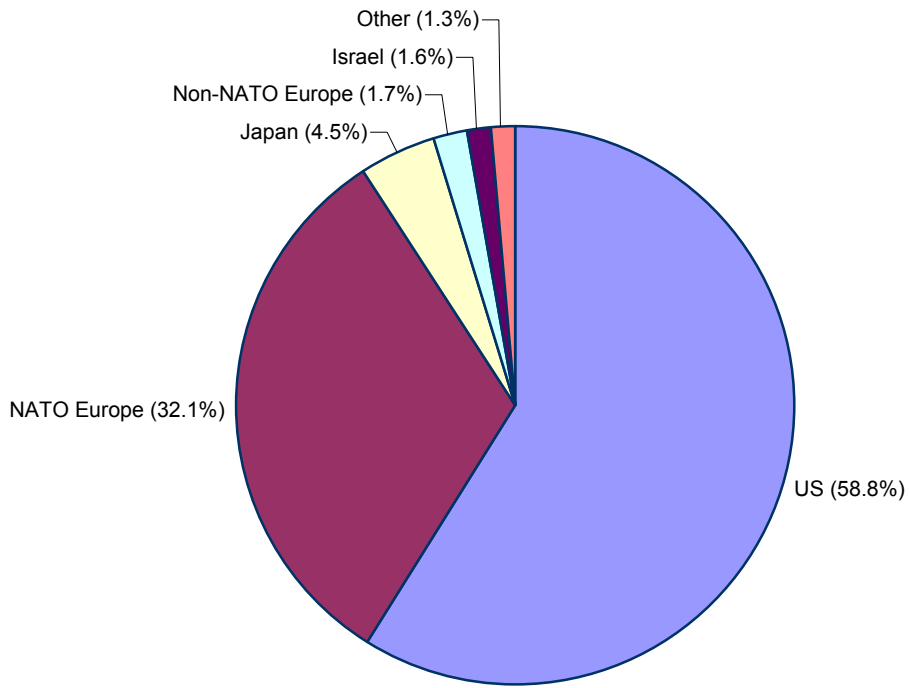
Figures 1 and 2 clearly show a concentration of leading defense firms into just two distinct blocs – the US and Western Europe. With one exception, the 20 largest defense firms in the world are all located in the US and Western Europe. In 1997, almost 60% of the defense sales of the largest 100 defense firms in the world originated in the US, 34% in Europe, 4.5% in Japan, 1.6% in Israel, and 1.3% in other countries. This is no surprise. These countries are among the few that have the technical capabilities to produce highly sophisticated weapon systems. They have also long recognized the importance of exports to the survival of their defense firms. In conclusion, we shall assume in this paper that producers of defense goods may be divided into two distinct blocs – the US and Western Europe. Note also that Western Europe and the US are allies.

Figure 1: Sales of Top 20 Defense Firms in 1997



Source: James (1998b)

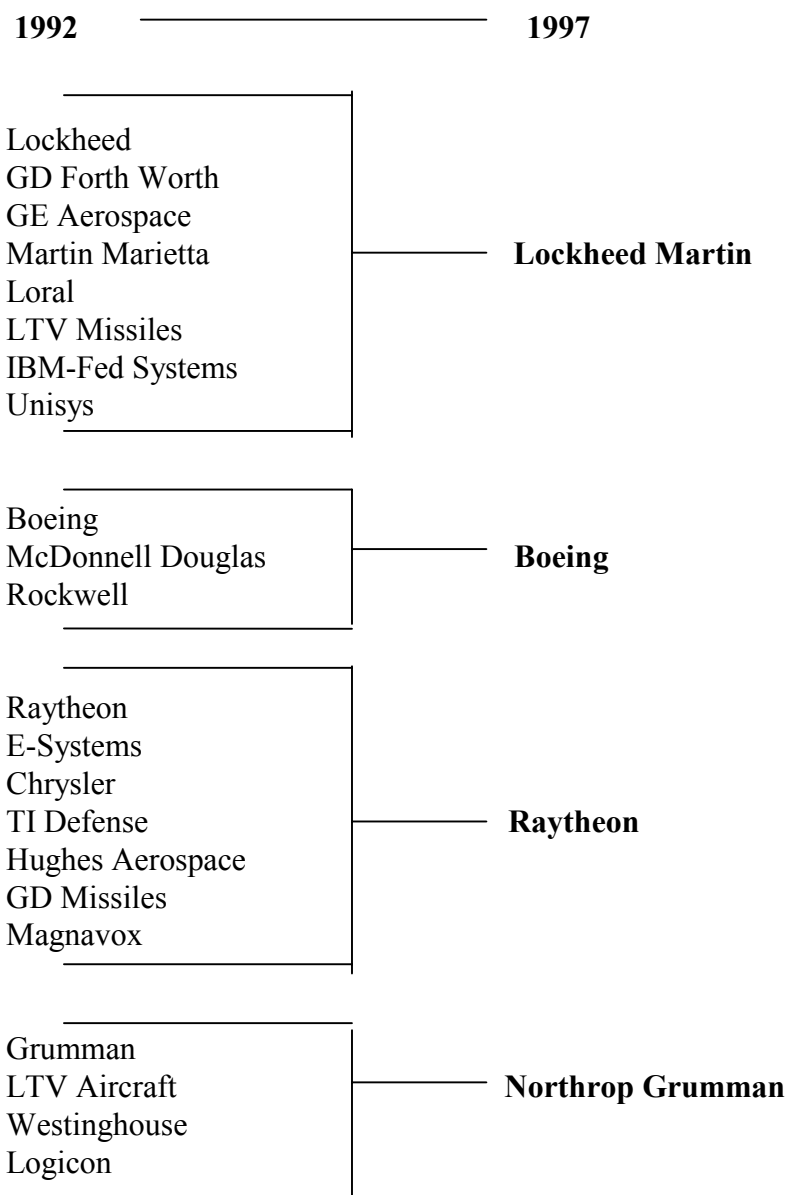
**Figure 2: Share of 1997 Defense Sales by Top 100 Defense Firms Worldwide By National Origin of Firm**



Source: Flamm (1998)

Finally, as defense budgets plummeted, the US altered its policy towards defense industry restructuring. The tradition of opposition to mergers among large defense firms during the cold war was reversed during the Clinton administration. During 1992-1997, four giant US defense firms emerged as a consequence of this policy change (see Figure 3).

**Figure 3: Mergers and Acquisitions in the US defense Industry during 1992-1997**



Source: The Economist (1997), James (1998b).

This consolidation process has had a profound impact on the industry. Table 3 shows the reduction of the number of producers supplying selected types of weapon systems. A similar, though much slower reduction has begun in Europe (Cobble (1998), the Economist (1997)) and in Israel (Dvir and Tishler (2000)). Note, however, that the reduction in the number of defense firms did not produce the savings anticipated, and surprisingly, few production lines have been closed (Markusen (1998)). Consolidation has reduced competition, and brought about a creation of a smaller number of defense manufacturers that are politically more powerful. This did not go unnoticed by the Pentagon, which successfully blocked the proposed merger of Lockheed Martin with Northrop in 1997, and the acquisition of Newport News by General Dynamics in 1999 (Wall Street Journal (1999)).

**Table 3: Prime Contractors in US Defense Industry**

	Number of Contractors	
	1990	1998
<b>Tactical Missiles</b>	13	4
<b>Strategic Missiles</b>	3	2
<b>Fixed Wing Aircraft</b>	8	3
<b>Rotary Wing Aircraft</b>	4	3
<b>Expendable Launch Vehicles</b>	6	2
<b>Satellites</b>	8	5
<b>Surface Ships</b>	8	5
<b>Torpedoes</b>	3	2
<b>Tactical Wheeled Vehicles</b>	6	4
<b>Tracked Combat Vehicles</b>	3	2

Source: Flamm (1998), James (1998b).

### 3. Security Level and the Defense Industry - the Basic Model

The security needs of a country are an expression of its desire and ability to deter war, or if war is unavoidable, to win it without sustaining excessive damage or loss of life. What amounts to “excessive” damage and loss of life is determined by the country’s culture, social fabric, religions, beliefs, and legal political structure. The amount of damage and loss of life that may result from a war depend on the country’s defense capabilities *relative* to those of its enemies. In reality, these defense capabilities are dependent on many factors – especially quantity and quality of personnel and weapon systems.

The concept of “winning” a war is not a simple one. Moreover, it changes over time because of changes in technology and other factors. There are many possible definitions and outcomes of a war. In technologically advanced countries, most of the defense budget is spent on the development, acquisition, and maintenance of major modern weapon systems designed for operation in large-scale wars. Hence this paper will be limited to the analysis of defense capabilities needed to deter or win a major (“all-out”) war. That is, we do not analyze defense systems and efforts that are designed specifically to deal with terror or minor local conflicts. Conventional weapon systems currently available to most technologically advanced countries are very potent. Therefore, unless the country wins the war quickly and convincingly it will suffer damage and loss of life that will be perceived within the country as devastating. Achieving a quick and convincing win requires a clear edge over the enemy (see Ben Israel (1998), Dvir and Tishler (2000)). A country may gain such an edge by building a much larger army (more weapon systems and personnel) than its enemy or, more importantly, weapon systems of distinctly higher quality.

This paper develops a model of two developed countries, country *A* (representing the US) and country *B* (representing Western Europe) which produce an identical homogeneous defense good<sup>2</sup>. The defense good is an aggregate of modern platforms (fighter planes, missiles, integrated weapon and intelligence systems, etc.) and their peripherals, as well as sophisticated munitions

---

<sup>2</sup> Extending the model to several countries would complicate the presentation but would not change the nature of the results.

and other high-tech equipment. The “rest of the world”, denoted  $W$ , includes all the other countries in the world. In this model there is no production of defense goods in the rest of the world. Countries  $A$  and  $B$  are allies – not enemies – but they may have enemies or potential enemies in the rest of the world<sup>3</sup>. The rest of the world features a downward-sloping demand function for the defense good (the quantity demanded is inversely related to the world price).

The defense industries in countries  $A$  and  $B$  consist of  $K$  and  $N$  profit-maximizing firms, respectively. Denote the output of the defense industry in country  $i$  by  $Y_i$ . The defense industries in countries  $A$  and  $B$  sell their output to their own governments and to the rest of the world. Country  $i$  buys defense goods only from its own defense industry (see Flamm (1998), James (1998a,b), Markusen (1998) and Serfati (1998) on this issue). The defense industry in country  $i$  must satisfy local demand by its government ( $Y_i^i$ ) before it is allowed to export the defense good to the rest of the world ( $Y_i^W$ ). Clearly,  $Y_i^i + Y_i^W = Y_i$ ,  $i=A,B$ .

Security in each country is a function of the size of the country’s stock of weapon systems relative to its enemies’. Thus, the security,  $S_i$ , of country  $i$  is dependent on two factors: the amount of the defense product  $Y_i^i$  in country  $i$ , and the amount of the defense good in the rest of the world (sold by the defense industries of countries  $A$  and  $B$  to the rest of the world)  $Y_A^W + Y_B^W$ . In this study we define the level of security of each country as follows:

$$S_A = Y_A^A / (Y_A^W + Y_B^W), \quad (1a)$$

and

$$S_B = Y_B^B / (Y_A^W + Y_B^W). \quad (1b)$$

---

<sup>3</sup> It is straightforward to show that the structure and nature of the solution of the model are unchanged when the producing countries are not allies (the security level of each country depends on its purchase of the defense good relative to the purchase of the defense good by all other countries in the world).



The rest of the world features the following demand function for the defense good:

$$P = a + bY^W, \quad (2)$$

where  $Y^W \equiv Y_A^W + Y_B^W$ . The terms,  $a > 0$ , and  $b < 0$  are known parameters. The rest of the world buys the defense good at the world price,  $P$ , which is determined by the model.

We assume that the *target* security level of each country,  $S_i^0$  (see expression (1)), is determined by military and political decision makers that assess the country's potential enemies. The target security level is exogenously given in this model. In addition, the number of defense firms in each country is known and exogenously given. The decisions in this model are taken in the following order:

**Stage 1** – conditional on  $S_i^0$ , the government of each country commits to the amount of the defense good that it will purchase from its defense industry. That is, each country seeks to make its actual security level,  $S_i$ , match its target security level  $S_i^0$  (which may be interpreted as a minimization of the quadratic loss function  $(S_i - S_i^0)^2$ ).

**Stage 2** – given the governments' commitments to purchase the defense good from their own defense industries, and the rest of the world's demand for the defense good, the defense firms in countries  $A$  and  $B$  decide how much to produce in order to maximize their profits. Thus, the  $N+K$  firms in countries  $A$  and  $B$  play a Cournot game to decide how much to produce and sell to the rest of the world.

The model is solved by first solving the Cournot game among the  $N+K$  defense firms. Then, given the reaction functions of the firms, the governments of  $A$  and  $B$  determine their optimal purchase of the defense good (their commitment levels) such that the initial security measures,  $S_i^0$ , are simultaneously attained in both countries.

To simplify the presentation and analysis, but without loss of generality, we assume that all the firms in country  $A$  are identical. Similarly, all the firms in country  $B$  are also identical. The firms in country  $A$  may be different from those in country  $B$ . We also assume that each

government divides its purchase of the defense good (its commitment) equally among the firms in its defense industry. The technology in country  $i$  is represented by the following quadratic cost function ( $Y_{ij}$  is the output of the  $j$ th firm in country  $i$ ,  $i=A,B$ ):

$$C(Y_{ij}) = \alpha_{0i} + \alpha_{1i}Y_{ij} + \frac{1}{2}\alpha_{2i}Y_{ij}^2, \quad (3)$$

where  $\alpha_{0i} > 0$ ,  $\alpha_{1i} > 0$ . We shall analyze cases in which  $\alpha_{2i}$  is positive (increasing marginal cost) or zero (constant marginal cost)<sup>4</sup>. Throughout the analysis we assume that  $a$ , the reservation price of the rest of the world (see (2)) is greater than the marginal cost of production (for example, if marginal costs are fixed,  $\alpha_{2A} = \alpha_{2B} = 0$ , then  $a > \alpha_{1A}$ , and  $a > \alpha_{1B}$ ).

#### 4. Solution of the Model

The model is solved in two steps. First we need to solve stage 2, the Cournot game among the  $N+K$  defense firms. Using the reaction functions derived for these firms, we then solve stage 1, in which the governments determine their optimal purchase of the defense good (their optimal commitment levels) such that the target security levels,  $S_i^0$ , are simultaneously attained in both countries. Clearly, the solution of the model depends on the price that each government pays for its purchase of the defense good. Because most of the cost of R&D of the defense industry in each country is borne by the government (Goolsbee (1998)), and because each government purchases its defense goods only from its own industry (see James (1998a), Markusen (1998) and Serfati (1998)), governments may request to pay less than the world price for their own purchase of the defense good. In the following exposition, we assume that government  $A$  and government  $B$  each independently choose one of two possible pricing structures: they may purchase the defense good from their defense industry either at the world price, or at a price which is equal to the marginal production cost of that country's defense industry plus a markup factor.

---

<sup>4</sup> The results of this paper hold also when  $[(K + N + 1)/(K + N + 2)]/b < \alpha_{2i} < 0$ .

**Stage 2**

The second stage solution is as follows. Suppose that government  $A$  purchases the defense good at the world price. The profit function of firm 1 in country  $A$  is given by:

$$\begin{aligned} \pi_{A1} = & [ a + b(Y_{A1}^W + \sum_{j=2}^K Y_{Aj}^W + \sum_{j=1}^N Y_{Bj}^W )](Y_{A1}^W + Y_{A1}^A) \\ & - [ \alpha_{0A} + \alpha_{1A}(Y_{A1}^W + Y_{A1}^A) + \frac{1}{2}\alpha_{2A}(Y_{A1}^W + Y_{A1}^A)^2 ]. \end{aligned} \quad (4)$$

If government  $A$  purchases the defense good at marginal cost plus a markup, the profit function of firm 1 in country  $A$  is given by:

$$\begin{aligned} \pi_{A1} = & [ a + b(Y_{A1}^W + \sum_{j=2}^K Y_{Aj}^W + \sum_{j=1}^N Y_{Bj}^W )] Y_{A1}^W + (1 + \mu)[\alpha_{1A} + \alpha_{2A}(Y_{A1}^A + Y_{A1}^W)] Y_{A1}^A \\ & - [ \alpha_{0A} + \alpha_{1A}(Y_{A1}^W + Y_{A1}^A) + \frac{1}{2}\alpha_{2A}(Y_{A1}^W + Y_{A1}^A)^2 ], \end{aligned} \quad (5)$$

where  $\mu \geq 0$  is the markup factor. Note that the price that government  $A$  pays for the defense good that it purchases from its defense industry equals  $(1 + \mu)[\alpha_{1A} + \alpha_{2A}(Y_{A1}^A + Y_{A1}^W)]$ .

Firm 1 in country  $A$  determines  $Y_{A1}^W$  (its exports to the rest of the world) such that its profits are maximized, conditional on exports of all the other firms in country  $A$  ( $Y_{Ak}^W$ ,  $k=2, \dots, K$ ) and country  $B$  ( $Y_{Bj}^W$ ,  $j=1, \dots, N$ ) and on the commitment levels of governments  $A$  and  $B$  ( $Y_{Ak}^A$ ,  $k=1, \dots, K$ , and  $Y_{Bj}^B$ ,  $j=1, \dots, N$ ). Similar expressions specify the profits of the other  $N+K-1$  firms in countries  $A$  and  $B$ .

The optimal solutions for  $Y_{Ak}^W$ ,  $k=1, \dots, K$  and  $Y_{Bj}^W$ ,  $j=1, \dots, N$  for the profit functions (4) and (5) are derived in the Appendix. The assumption that all firms in each country are identical ensures that the exports of all firms in each country are identical. Specifically, profit maximization by the defense firms in  $A$  and  $B$  yields the following solution:

$$Y_{A1}^W = \theta_{0A} + \theta_{1A} Y_{A1}^A + \theta_{2A} Y_{B1}^B, \quad (6a)$$

and

$$Y_{B1}^W = \theta_{0B} + \theta_{1B} Y_{A1}^A + \theta_{2B} Y_{B1}^B, \quad (6b)$$

where the parameters  $\theta_{0A} > 0$ ,  $\theta_{0B} > 0$ ,  $\theta_{1A}$ ,  $\theta_{2A}$ ,  $\theta_{1B}$ , and  $\theta_{2B}$  depend on the profit function ((4) or (5)), the parameters of the cost functions of  $A$  and  $B$  (expression (3)), the parameters of the demand function of the rest of the world (expression (2)), and the number of firms in each country ( $K, N$ ) (see Appendix).

The optimal solutions given by expressions (6a) and (6b) are not dependent on  $\alpha_{0A}$  and  $\alpha_{0B}$ . That is, fixed production cost does not affect the optimal solution in (6) (which equates marginal cost to marginal revenue). However, the profit-maximizing firm will operate only if its variable profit exceeds its fixed production cost. In the subsequent analysis, we assume that this condition holds. Clearly, the value of  $\alpha_{0A}$  and  $\alpha_{0B}$  may affect the number of firms in the defense industry (the high fixed cost of production of modern weapon systems is one of the major reasons for the consolidation of the industry, see Flamm (1998)). Fixed production costs may play an important role in policy decisions if the government can determine the optimal number of firms in the industry. That is, the optimal value of the objective function according to which the optimal number of firms is determined may have to be compared across several possible solutions.

Total sales of the defense good to the rest of the world (by all firms in  $A$  and  $B$ ) are:

$$Y^W \equiv Y_A^W + Y_B^W \equiv KY_{A1}^W + NY_{B1}^W = \gamma_0 + \gamma_1 KY_{A1}^A + \gamma_2 NY_{B1}^B, \quad (7)$$

where the parameters  $\gamma_0 > 0$ ,  $\gamma_1$  and  $\gamma_2$  depend on the type of profit function ((4) or (5)), on the number of firms in each country, and on the parameters of the cost functions (3) and the demand function (2).

**Stage 1**

Using the security measures (1) and the optimal solution in stage 2 (expressions (6) and (7)) we obtain the following equilibrium solution for total sales of the defense good to the rest of the world and the commitment levels of countries  $A$  and  $B$  (see Appendix):

$$Y^W = \gamma_0 / (1 - \gamma_1 S_A^0 - \gamma_2 S_B^0), \quad (8)$$

$$Y_A^A = \gamma_0 S_A^0 / (1 - \gamma_1 S_A^0 - \gamma_2 S_B^0), \quad (9a)$$

and

$$Y_B^B = \gamma_0 S_B^0 / (1 - \gamma_1 S_A^0 - \gamma_2 S_B^0). \quad (9b)$$

Clearly, in equilibrium, the commitment level of each country depends on its *target* security level and on the other country's *target* security level. This outcome is due to the effect that each country's level of security has on its own exports of the defense good to the rest of the world and, in turn, to the effect of the total quantity of the defense good that is acquired by the rest of the world on the commitment of each country (see (1)). Finally,  $Y^W$  is always positive (since  $\gamma_0 > 0$ ) and, generally,  $Y_A^A > 0$  if  $S_A^0 > 0$ , and  $Y_B^B > 0$  if  $S_B^0 > 0$ .

**5. Solution of the Model Under different Pricing Practices**

The solution of the model depends on the price that each government pays to its own defense industry for its own purchase of the defense good. We analyze the following three cases:

1. The governments of  $A$  and  $B$  pay the world price to their defense industries.
2. The governments of  $A$  and  $B$  pay the marginal cost plus a markup to their defense industries.
3. The government of  $A$  (say) pays its defense industry the marginal cost plus a markup and the government of  $B$  pays the world price.

To simplify the analysis we assume that all  $N+K$  firms in countries  $A$  and  $B$  use the same technology ( $\alpha_{jA} = \alpha_{jB}$ , for  $j=0,1,2$ ) and, if relevant, markup factor  $\mu$ .

**Case 1 (the governments of  $A$  and  $B$  pay the world price)**

It is straightforward to show that the government commitment level in country  $A$  is a substitute for the exports of country  $A$  ( $\theta_{1A} < 0$ , see (6a)). To simplify the explanation of this outcome, but without loss of generality, suppose that marginal cost is fixed. An increase in the commitment level of the government of country  $A$ ,  $Y_A^A$ , can be supplied only by firms in  $A$ . Thus, the firms in country  $A$  increase their sales to their own government. As a result, the marginal revenue of the firms in  $A$  is reduced. To increase their marginal revenue (in order to achieve  $MR=MC$ ) they reduce their exports to the rest of the world (by a smaller amount than the increase in  $Y_A^A$ ). Hence, the output of the defense industry in country  $A$  is higher than before the increase in  $Y_A^A$ . This reduction in  $A$ 's exports raises the world price for the defense good, which, in turn, causes the firms in country  $B$  to increase their sales to the rest of the world. That is, an increase in the commitment of government  $A$  increases global demand for the defense good (total demand of country  $A$  plus country  $B$  plus the rest of the world). Actually, all  $N+K$  firms increase their output by the same amount (the optimization of (4) yields  $Y_{A1} = Y_{B1}$ ). By the same argument, the commitment level of government  $A$  is a complement of the export of country  $B$  ( $\theta_{1B} > 0$ , see (6b)).

Global sales,  $Y^W$ , are always positive (since  $\gamma_0 > 0$ ,  $\gamma_1 < 0$  and  $\gamma_2 < 0$ ). Clearly,  $Y_A^A > 0$  if  $S_A^0 > 0$ . Moreover,  $Y_A^A$  increases when  $S_A^0$  increases or  $S_B^0$  declines. That is, when country  $A$  increases its target security level, its purchase of the defense good increases, and its firms decrease their sales to the rest of the world (see expressions (6) and (7)). This increases the world price of the defense good, causing the firms in  $B$  to raise their exports to the rest of the world. That is, an increase in  $S_A^0$  is achieved by increasing  $A$ 's commitment, and reducing  $A$ 's exports to the rest of the world by more than the increase in  $B$ 's exports (in response to the initial reduction of  $A$ 's exports). Likewise, if government  $B$  decides to increase its target security level, its commitment increases, the world price increases, and the quantity demanded by the rest of the world decreases, allowing country  $A$  to decrease its commitment in order to preserve its (unchanged) target security level.

It is straightforward to show that in equilibrium  $\partial Y_i^i / \partial K > 0$ , and  $\partial Y_i^i / \partial N > 0$  for  $i=A,B$ . Clearly, when the number of firms increases, the world price of the defense good,  $P$ , declines due to the increase in competition (decrease in monopoly power). The decline in  $P$  results in an increase in the quantity demanded by the rest of the world and, for given target security levels,  $S_A^0 > 0$  and  $S_B^0 > 0$ , an increase in the commitments of both  $A$  and  $B$ . That is, a more competitive defense market results in a world with larger amounts of weapon systems.

***Case 2 (the governments of A and B pay the marginal cost plus a markup)***

First, note that  $\theta_{1A} = \theta_{2A} = \theta_{1B} = \theta_{2B} = \gamma_1 = \gamma_2 = 0$  when the markup factor,  $\mu$ , equals zero, or when marginal production cost is constant ( $\alpha_{2A} = \alpha_{2B} = 0$ ) (see (6)-(9)). In this case, the commitment levels  $Y_A^A$  and  $Y_B^B$  are neither substitutes nor complements of the exports of the defense firms of  $A$  and  $B$ . Additionally, in this case the sales of the defense good to the rest of the world do not depend on the target security levels of countries  $A$  and  $B$  (that is,  $Y^W = \gamma_0$ ).

Suppose that the marginal production costs are increasing ( $\alpha_{2A} > 0$  and  $\alpha_{2B} > 0$ ). Here, in contrast to case 1, in which the governments of  $A$  and  $B$  pay the world price for their defense goods, the commitment level of government  $A$  is a complement of the exports of the defense firms of country  $A$  ( $\theta_{1A} > 0$ ) and a substitute for the exports of the defense firms of country  $B$  ( $\theta_{1B} < 0$ ). The explanation of this outcome is as follows. An increase in the quantity requested by government  $A$ ,  $Y_A^A$ , can be supplied only by firms in  $A$ . Thus, the firms of country  $A$  increase their production in response to an increase in the commitment level of their government and, hence, increase their marginal production cost of the last unit that they produce by  $\alpha_{2A}$ , which is less than  $(1 + \mu)\alpha_{2A}$ , the increase in their marginal revenue. Thus, the firms of country  $A$  increase their exports (an activity which lowers their marginal revenue) in order to equate their marginal revenue to their marginal cost. This change reduces the world price of the defense good, triggering a decline in  $B$ 's marginal revenue which, in turn, causes the firms in  $B$  to reduce their exports to the rest of the world.

Clearly,  $Y_i^i > 0$  if  $S_i^0 > 0$ . Here, in contrast to the case where the governments of  $A$  and  $B$  purchase their commitments at the equilibrium world price,  $Y_A^A$  increases when  $S_B^0$  increases. That is, if country  $B$  decides to increase its target security level, its government commitment increases, the world price declines and the quantity demanded by the rest of the world increases, forcing country  $A$  to increase its commitment in order to preserve its (unchanged) target security level. When country  $A$  increases its target security level, its purchase of the defense good increases, and its firms increase their sales to the rest of the world (see expressions (6)-(9)). That is, the increase in  $S_A^0$  is achieved by increasing  $A$ 's commitment and increasing  $A$ 's exports (by a smaller amount). At the same time  $B$ 's exports to the rest of the world are reduced (since the world price of the defense good is reduced).

It is straightforward to show that in equilibrium  $\partial Y_i^i / \partial K > 0$  and  $\partial Y_i^i / \partial N > 0$  for  $i=A,B$ . Clearly, when the number of firms increases, the world price of the defense good,  $P$ , declines due to the increase in competition (decrease in monopoly power). The decline in  $P$  results in an increase in the quantity demanded by the rest of the world and, for given target security levels,  $S_A^0 > 0$  and  $S_B^0 > 0$ , an increase in the commitments of both  $A$  and  $B$ . That is, a more competitive defense market results in a world with larger amounts of weapon systems.

***Case 3 (the government of  $A$  pays the marginal cost plus a markup, and the government of  $B$  the world price).***

This case is a hybrid of cases 1 and 2. That is, the commitment level of government  $A$  is a complement of the exports of the defense firms of country  $A$  ( $\theta_{1A} > 0$ ) and a substitute for the exports of the defense firms of country  $B$  ( $\theta_{1B} < 0$ ). At the same time, the commitment level of government  $B$  is a substitute for its own country's exports ( $\theta_{2B} < 0$ ) and a complement of the exports of country  $A$  ( $\theta_{2A} > 0$ ). In addition, when the markup factor,  $\mu$ , equals zero or when marginal production cost in  $A$  is constant ( $\alpha_{2A} = 0$ ), then the sales of the defense good to the rest of the world are not dependent on the target security level of country  $A$ .



## 6. Industry Profits, Defense Costs and the Number of Defense Firms

Determining the optimal number of firms in its defense industry is an extremely important government policy issue (Flamm (1998), James (1998b), Markusen (1998), Dvir and Tishler (2000)). Suppose that government  $B$  has set its number of defense firms to be  $N^0$ . (Large previous investments in reorganizing its defense industry or in renewing the industry's technology may be reasons for this commitment. For a discussion of this issue, see Besanko, Dranove and Shanley (1996).) The optimal choice of  $K$ , the number of defense firms in  $A$ , conditional on  $N^0$ , is dependent on government  $A$ 's objective. Three objective functions are analyzed here. That is, the government of  $A$  may set a value for  $K$  according to one of the following objectives:

1. To maximize the profits of its defense industry.
2. To minimize its cost of purchasing the defense good.
3. To minimize the net defense cost (government expenditure on the defense good minus the profit of the defense industry) of the country.

To simplify the analysis and presentation, we analyze the model under the assumptions that the same technology is used in countries  $A$  and  $B$  ( $\alpha_{jA} = \alpha_{jB} \equiv \alpha_j$  for  $j=0,1,2$ ) and that marginal production costs are fixed ( $\alpha_{2A} = \alpha_{2B} = 0$ ). Extensive simulated solutions of the model have shown that these assumptions do not change the nature of the results.

**Case 1** (countries  $A$  and  $B$  purchase the defense good at the world price).

The profit of each firm in country  $A$  and  $B$  is given by:

$$\pi_{A1} = \pi_{B1} = \frac{-(1+S^0)^2}{(N+K+1+S^0)^2} \frac{(a-\alpha_1)^2}{b}, \quad (10)$$

where  $S^0 \equiv S_A^0 + S_B^0$ . Profit is always positive because  $b < 0$ . Clearly, the profit of a single firm decreases when the number of firms in either country increases. This is the standard result in a Cournot game. That is, a larger number of firms implies a lower equilibrium price and lower

profits to each firm. Note also that the profits of any single firm are increased by a rise in the target security level in country  $A$  or country  $B$  ( $\partial\pi_{A1}/\partial S^0 > 0$ ). That is, a higher target security level is equivalent to an increase in the demand for the defense good, and thus profits increase.

Unlike the results of the standard Cournot model, the profits of the defense industry are not necessarily maximized when the number of firms in each country is one. Specifically, the profits of the defense industry are given by,

$$\pi = \pi_A + \pi_B \quad \text{where} \quad \pi_A = K\pi_{A1}, \quad \pi_B = N\pi_{B1}. \quad (11)$$

If the number of firms in each country is set to be the same,  $K=N$ , then maximal profits to the industry accrue when  $N=K=(1+S^0)/2$ . If, for example,  $S_A^0 = S_B^0 = 1.5$ , then maximal profits are obtained when  $K=N=2$ . The departure from the standard Cournot solution is due to the shift of the demand function to the right (left) when price declines (increases), and the requirement that government commitments are supplied first, prior to sales to the rest of the world<sup>5</sup>. That is, the demand for the defense good by the governments of  $A$  and  $B$  is not downward sloping. Total world demand (the demand of  $A$ ,  $B$  and the rest of the world) is obtained by shifting the demand function of the rest of the world to the right by the amount of the commitments of the governments of  $A$  and  $B$ . Note, however, that the maximal price of total world demand equals the reservation price of the rest of the world (that is,  $P=a$  for a quantity less than or equal to the commitments of  $A$  and  $B$ , and the slope of the demand function equals  $b$  thereafter).

When the number of firms is chosen such that industry's profits are maximized ( $N=K=(1+S^0)/2$ ) we obtain,

$$\max \pi \equiv \max(\pi_A + \pi_B) = (1+S^0) \frac{-(a-\alpha_1)^2}{2b}. \quad (12)$$

That is, the maximal profits in (12) equals that of the standard solution of a Cournot game

---

<sup>5</sup> See the discussion following expressions (6a) and (6b) regarding the effect of fixed production cost on the optimal number of firms in the defense industry.

without government commitments multiplied by  $(1+S^0)$ . (For a solution of the case where  $S^0=0$ , see Mas-Colell, Whinston and Green (1995).) Clearly, industry profits are higher in both countries when the target security level in either of the two countries is higher. In our case, industry profits may be somewhat lower than those in (12), because the number of firms must be an integer, and  $(1+S^0)/2$  is an integer only when  $S^0$  is an odd integer.

Consider the optimal choice of  $K$ , the number of defense firms in  $A$ , conditional on  $N^0$ . Our results show that for all three objective functions, the government of  $A$  is likely to set a value for  $K$  that is larger than the optimal  $K$  under the standard Cournot solution ( $K=N^0+1$ ) when  $S_A^0 = S_B^0 = 0$ . This outcome is due to the commitment of both  $A$  and  $B$  to meet target security levels. The number of firms that maximizes defense industry profits in country  $A$ , conditional on  $N^0$ , is given by:

$$K = N^0 + 1 + S^0. \quad (13a)$$

When  $S_A^0 = S_B^0 = 0$  one obtains  $K=N^0+1$ , which is the standard Cournot solution. In our model, the higher the target security levels in country  $A$  and/or  $B$ , the larger is the number of firms required to maximize the profits of country  $A$ 's defense industry. This is because the commitments of the governments of  $A$  and  $B$  do not depend directly on the world price.

Next, it is straightforward to show that the number of firms in  $A$  that minimizes this country's expenditure on the defense good is either  $K=1$  or  $K \rightarrow \infty$ .

The number of firms that minimizes net defense cost of  $A$  is:

$$K = \frac{(a - \alpha_1)(1 + S^0)^2 - a(1 + S^0)S_A^0}{(a - \alpha_1)(1 + S^0) + (2\alpha_1 - a)S_A^0} + \frac{(a - \alpha_1)(1 + S^0) - (2\alpha_1 - a)S_A^0}{(a - \alpha_1)(1 + S^0) + (2\alpha_1 - a)S_A^0} N^0. \quad (13b)$$

When  $S_A^0 = S_B^0 = 0$ , we get  $K=N^0+1$ , which is the standard Cournot solution. Requiring that  $a > (3/2)\alpha_1$  is sufficient (but not necessary) to ensure that  $\partial K / \partial N^0 > 0$ . This means that the

optimal  $K$  increases when  $N^0$  is larger. Note again that, in the solution,  $K$  and  $N$  must be integers. Finally, larger values of  $S_A^0$  and/or  $S_B^0$  yield a larger value of  $K$  in (13b). That is, an increase in the target security levels in  $A$  or  $B$ , will, conditional on  $N^0$ , increase the number of firms that minimizes the difference between  $A$ 's expenditure on the defense good and the profits of  $A$ 's defense industry.

**Case 2** (The governments of  $A$  and  $B$  pay their defense industries marginal cost plus a markup).

The defense industry profits of country  $A$  are:

$$\pi_A = \left\{ \frac{-K}{(N+K+1)^2} \frac{(a-\alpha_1)^2}{b} \right\} + \left\{ \frac{N+K}{N+K+1} \frac{(\alpha_1-a)}{b} \mu \alpha_1 S_A^0 \right\}. \quad (14)$$

The expression in the first set of curly brackets in (14) equals  $A$ 's profits from sales to the rest of the world (at the world price). The second part of (14) stands for the profits on sales to government  $A$ . In this case, for a given  $N^0$ , the value of  $K$  that maximizes  $\pi_A$ , the profits of the defense industry of country  $A$ , is given by the standard Cournot solution  $K = N^0 + 1$ .

The expenditure of government  $A$  on the defense good is:

$$GC_A = (1+\mu)\alpha_1 S_A^0 \frac{(N+K)}{(N+K+1)} \frac{(\alpha_1-a)}{b}. \quad (15)$$

For a given  $N^0$ ,  $\partial GC_A / \partial K > 0$ . Hence, government expenditures on the defense good are minimized when  $K=1$ . The reason for this result is as follows. Smaller  $K$  (and, equivalently, smaller  $N$ ) result in a higher world price for the defense good because smaller  $K$  and  $N$  imply greater monopoly power. Therefore, less of the defense good is purchased by the rest of the world. This, in turn, means that governments  $A$  and  $B$  can attain a given target security level with a lower purchasing commitment.

The net defense cost of A is not dependent on the markup factor since the profit of the defense industry from sales to its own government (see the second part of (14)) are part of the government expenditure on the defense good. That is,

$$GC_A - \pi_A = \frac{(a - \alpha_1)}{b} \left\{ \frac{(a - \alpha_1)K - \alpha_1(N + K)(N + K + 1)S_A^0}{(N + K + 1)^2} \right\}. \quad (16)$$

The value of  $K$  that minimizes  $GC_A - \pi_A$ , given  $N^0$ , is:

$$K = \frac{\alpha_1(1 + S_A^0) - a}{\alpha_1(1 - S_A^0) - a} + \frac{\alpha_1(1 + S_A^0) - a}{\alpha_1(1 - S_A^0) - a} N^0. \quad (17)$$

Clearly,  $S_A^0 = 0$  yields  $K = N^0 + 1$ . Here, the following optimal  $K$  emerges:

$$1 \leq K \leq N^0 + 1 \quad \text{when} \quad S_A^0 < \frac{a - \alpha_1}{\alpha_1}. \quad (18a)$$

and

$$K=1 \quad \text{when} \quad S_A^0 > \frac{a - \alpha_1}{\alpha_1}. \quad (18b)$$

That is, the optimal  $K$  is, generally, less than the standard result of a Cournot model. When  $A$ 's target security level is relatively large, the value of  $K$  that minimizes  $GC_A - \pi_A$ , given  $N^0$ , is one.

**Case 3** (government  $A$  pays its defense industry marginal cost plus a markup and government  $B$  pays its defense industry the world price).

When  $\alpha_{2A} = \alpha_{2B} = 0$ ,  $\alpha_{1A} = \alpha_{1B} = \alpha_1$ , we have:

$$\pi_{B1} = \frac{-1}{(N + K + 1 + S_B^0)} \frac{(a - \alpha_1)^2 (1 + S_B^0)^2}{b}, \quad (19a)$$

and

$$\pi_{A1} = \pi_{B1} + \frac{N+K}{N+K+1} \cdot \frac{\alpha_1 - a}{b} \cdot \frac{\mu\alpha_1 S_B^0}{K}. \quad (19b)$$

Clearly, the profits of the defense industry of country  $A$  are equal to or higher than those of country  $B$  (they are equal when  $\mu=0$ ). Suppose that  $N = N_0$  is given. What is the optimal  $K$ ?  $\partial\pi_A / \partial K = 0$  yields:

$$K = \frac{(a - \alpha_1) + \mu\alpha_1 S_A^0}{(a - \alpha_1) - \mu\alpha_1 S_A^0} (1 + S_B^0) + \frac{(a - \alpha_1) + \mu\alpha_1 S_A^0}{(a - \alpha_1) - \mu\alpha_1 S_A^0} N^0. \quad (20)$$

Clearly,  $\mu = 0$  yields  $K = N^0 + 1 + S_B^0$ . Setting  $\mu > 0$ , and  $a > \alpha_1(1 + \mu S_A^0)$  yields  $K > N^0 + 1 + S_B^0$ . In addition, it is straightforward to show that, for a given  $N = N_0$ ,  $GC_A$  is minimized at  $K=1$ . Finally,  $\partial(GC_A - \pi_A) / \partial K = 0$  yields,

$$K = \frac{[(a - \alpha_1)(1 + S_B^0)^2 - \alpha_1 S_A^0]}{(a - \alpha_1)(1 + S_B^0)^2 + \alpha_1 S_A^0} (1 + S_B^0) + \frac{(a - \alpha_1)(1 + S_B^0)^2 - \alpha_1 S_A^0}{(a - \alpha_1)(1 + S_B^0)^2 + \alpha_1 S_A^0} N^0. \quad (21)$$

$S_A^0 = S_B^0 = 0$  yields  $K = N^0 + 1$ .  $S_A^0 > 0$  and  $S_B^0 > 0$  yields  $K > 0$ , but  $K$  may be larger or smaller than  $N+1$  depending on the size of  $S_A^0$  and  $S_B^0$ .

Similarly, suppose that  $K = K^0$  is given, and country  $B$  can choose  $N$ , the number of its defense firms. The optimal value of  $N$  which maximizes profits, minimizes government expenditure, or minimizes  $GC_B - \pi_B$ , conditional on  $K = K^0$ , is the same as that in Case 2 when  $K$  and  $N$  are interchanged and when  $S_B^0$  replaces  $S^0$ .

## 7. Comparison of Solutions Across Pricing Practices

The optimal pricing practice of governments  $A$  and  $B$  for the purchase of the defense good from their own defense industries depends on the objectives of those governments. Here, we assume that a government chooses a particular pricing practice depending on the pricing practice of the other government and one of the following criteria: (1) maximization of the profits of its defense industry, (2) minimization of its cost of purchasing the defense good and, (3) minimization of its net defense cost. Next, we compare the solution of the model for the above three objectives across various pricing practices.

To make the comparison both simple and meaningful, we assume that  $A$  and  $B$  use the same technology and that marginal cost is fixed (that is,  $\alpha_{1A} = \alpha_{1B} \equiv \alpha_1$  and  $\alpha_{2A} = \alpha_{2B} = 0$ ). In addition, profits are dependent on the number of firms in each industry. Generally, if both countries use the same technology, higher profits will accrue to the industry with the larger number of firms. Thus, in the following analysis we assume that  $N=K$ .

Denote the profits of the defense industries of  $A$  and  $B$  by  $\pi_A^k$  and  $\pi_B^k$ , where  $k$  denotes the pricing practice ( $k=1$  when both governments pay the world price,  $k=2$  when both governments pay marginal cost plus a markup, and  $k=3$  when government  $A$  pays marginal cost plus a markup and government  $B$  pays the world price). Similarly, denote the government expenditures of  $A$  and  $B$  by  $GC_A^k$  and  $GC_B^k$ . We shall start by comparing the profits of the defense industry of  $A$  across different objectives of the government of  $A$ .

Clearly,  $\pi_A^2$  depends on the value of the markup factor  $\mu$ . It is easy to show that very large  $\mu$  may result in  $\pi_A^2 > \pi_A^1$ , in which case, government  $A$  will be paying more than the world price. In practice, the government's reason for setting the price of the defense good as equal to the marginal production cost plus a markup is to pay less than the world price. Hence, it is reasonable to assume that the firms' profit margin per unit of defense good from sales to their government is not larger than their profit margin from sales to the rest of the world. That is,

$\mu\alpha_1 \leq P - \alpha_1$ , where  $P$  is the world price. Then,

$$\pi_A^2 < \frac{-[K + (N + K)S_A^0](a - \alpha_1)^2}{(N + K + 1)^2 b}, \quad (22)$$

and (see (10)),

$$\pi_A^1 = \frac{-K}{(N + K + 1 + S_A^0 + S_B^0)^2} \frac{(a - \alpha_1)^2 (1 + S_A^0 + S_B^0)^2}{b}. \quad (23)$$

Expressions (22) and (23) imply

$$\xi \equiv \frac{\pi_A^2}{\pi_A^1} < \frac{(N + K + 1 + S_A^0 + S_B^0)^2}{(N + K + 1)^2} \frac{N + (1 + K)S_A^0}{K(1 + S_A^0 + S_B^0)^2}. \quad (24)$$

Then, since  $N=K$ , formula (24) implies

$$S_A^0 = S_B^0 < 1.5 \Rightarrow \xi < 1 \text{ for } N \geq 1,$$

$$S_A^0 = S_B^0 < 7.5 \Rightarrow \xi < 1 \text{ for } N \geq 2,$$

$$S_A^0 = S_B^0 < 17.5 \Rightarrow \xi < 1 \text{ for } N \geq 3.$$

Hence, in most situations we shall obtain  $\pi_A^2 < \pi_A^1$ . That is, when the number of firms is given, and the objective of both governments  $A$  and  $B$  is to maximize the profit of their defense industries, they will prefer to pay the world price for their purchases of the defense good.

Now consider government expenditure on the defense good. Again, it is a simple matter to show that a very large markup factor,  $\mu$ , yields  $GC_A^2 > GC_A^1$ . However, if the markup factor is “small”, we have  $GC_A^2 < GC_A^1$ . Formally, the conditions  $\mu=0$  and  $a/\alpha_1 > (1 + S_A^0 + S_B^0)(1 + \mu)$  are sufficient (but not necessary) to yield  $GC_A^2 < GC_A^1$ .



Clearly, if the pricing practices are compared on the basis of defense industry profits, or government expenditure on the defense good, the result will be dependent on the markup factor. The more natural objective is to minimize the net defense cost to the country. First, note that  $GC_A^2 - \pi_A^2$  is not dependent on the markup factor. Consider the case when  $S_A^0 = S_B^0$ . It is shown in the Appendix that the conditions,  $S_A^0 = S_B^0 \leq N^2 + 0.5$ , or  $S_A^0 = S_B^0 > N^2 + 0.5$  together with  $a/\alpha_1 \leq 4N^2 + 8N + 4$ , are sufficient, but not necessary, for

$$GC_A^1 - \pi_A^1 < GC_A^2 - \pi_A^2. \quad (25)$$

That is, when the governments' objective is to minimize net defense cost, they prefer to pay the world price for their own defense good rather than the marginal production cost plus a markup.

Consider case 3 (the government of  $A$  pays the marginal cost plus a markup and the government of  $B$  pays the world price). It is straightforward to show that at the optimum,  $Y_{A1}^W = Y_{B1}^W + Y_{B1}^B \equiv Y_{B1}$ . That is, the exports of each firm in country  $A$  equal production of each firm in country  $B$ . Also, it is obvious that government expenditure on the defense good in country  $A$  is lower than that in country  $B$  if the unit price that it pays to its defense firms is less than the world price. In this case, and for any markup, we have:

$$GC_A^3 - \pi_A^3 < GC_B^3 - \pi_B^3. \quad (26)$$

That is, the net defense cost of country  $A$  is smaller than that of country  $B$ .

Suppose that  $A$  is committed to its pricing practice. That is, government  $A$  pays on the basis of marginal cost plus a markup, regardless of the pricing practice chosen by government  $B$ . Which pricing practice is best for government  $B$  in this case? The following results hold:

1. For  $\mu=0$  (no markup) we have  $\pi_B^3 > \pi_B^2$ . However, for a sufficiently large  $\mu$ ,  $\pi_B^3 < \pi_B^2$ .
2. Generally, a sufficiently small markup factor,  $\mu$ , yields  $GC_B^3 < GC_B^2$ . However, for a sufficiently large  $\mu$  we have  $GC_B^3 > GC_B^2$ .

3. For some values of the model's parameters,  $B$ 's net defense cost is minimized when it pays its defense industry the marginal cost plus a markup. For other values of the model's parameters,  $B$ 's net defense cost is minimized when it pays its defense industry the world price.

Finally, if  $A$  is committed to paying the world price for the purchase of the defense good from its own defense industry,  $B$  is always better off if it also pays the world price. Thus, when governments independently choose procurement rules such that their net defense cost is minimized, the model may exhibit two different equilibrium points. If one country commits to paying the world price, so will the other country. If one government commits to paying marginal cost plus a markup, the other country may choose to pay its defense industry the marginal cost plus a markup or the world price, depending on the model's parameters.

## 8. Different Technologies, R&D, and Defense Industry Exports

Whether they are for civilian or defense use, technologies that are based on R&D are generally non-rival (their use by one firm does not limit their use by another). This means that the availability of these technologies will bring about strong spillovers across firms (see Romer (1990)). When defense R&D spillover across countries is not allowed (see Dvir and Tishler (2000) on this issue), countries with a large expenditure on R&D may gain a substantial technological advantage over countries that spend less on R&D (see Romer (1990), Leahy and Neary (1997), Goolsbee (1998) and Segerstrom (1998)).

In the current paper this technological advantage is reflected in the parameters of the cost function, formula (3) (for a general discussion, analysis, and examples of defense R&D, see Nelson (1993), Tishler et. al (1996), Serfati (1998), James (1998a), Ham and Mowery (1998) and Dvir and Tishler (2000)). Suppose that the government of country  $A$  spends more on defense R&D than the government of country  $B$ . There may be various reasons for such asymmetry. For example,  $A$ 's GDP may be larger than  $B$ 's, or country  $A$  may perceive a greater security threat than country  $B$  ( $S_A^0 > S_B^0$ ). In reality, both of these reasons drive the US to spend much more on defense R&D than Europe (see Flamm (1988), James (1998a), Markusen (1998) and Serfati

(1998)). Even if  $S_A^0 \leq S_B^0$ , country  $A$  may be technologically more advanced than country  $B$ . Clearly, government expenditure on defense R&D should be promoted because spending by private defense firms in countries  $A$  and  $B$  will be less than the country's optimal level (see Goolsbee (1998)). Thus, government expenditure on R&D is critical for advancing a country's defense technology know-how, particularly when defense R&D spillover across countries is not allowed. However, see Segerstrom (1998) for cases where the social rate of return on R&D spending does not exceed the private rate of return.

Suppose that countries  $A$  and  $B$  pay the world price to their own defense industries. For simplicity of exposition, but without loss of generality, set  $N=K$ ,  $S_A^0 = S_B^0$ , and consider the case of fixed marginal cost,  $\alpha_{2A} = \alpha_{2B} = 0$ . Suppose that country  $A$  spends more on R&D than country  $B$ , resulting in  $\alpha_{1A} < \alpha_{1B}$ . Using expressions (6)-(8), it can be shown that,

$$\frac{Y_A^W}{Y_A^W + Y_B^W} = \frac{1}{2N+1} \left\{ (2N+1 + 2S_A^0) \left( \frac{N(\alpha_{1A} - \alpha_{1B}) + (\alpha_{1A} - a)}{(\alpha_{1A} - a) + (\alpha_{1B} - a)} \right) - S_A^0 \right\}, \quad (27)$$

and

$$Y_{A1}^W - Y_{B1}^W = \frac{\alpha_{1A} - \alpha_{1B}}{b}. \quad (28)$$

Thus, country  $A$ 's exports of the defense good will exceed those of country  $B$ . Expression (27) shows that the lower is the marginal cost of production in country  $A$  relative to that in country  $B$ , the larger is country  $A$ 's share in the world market. That is, following common sense, more efficient (less costly) production confers greater market power. More specifically, if  $\alpha_{1B} - \alpha_{1A}$  is significant, even a small  $N$  may lead to  $Y_A^W \approx Y^W$ . The role of the cost differentials becomes more important as the number of defense firms increases since the excess exports of a single firm in country  $A$  over a single firm in country  $B$  are independent of the number of firms in each country. Hence, a larger number of firms in both producing countries increases country  $A$ 's share in the world market. This conclusion can be inferred directly from (28).

## 9. Summary and Conclusions

This paper presents a simple model that analyzes the interactions of two countries' defense needs with the market structure of their defense industries. We assume that the target security level in each country is (exogenously) determined by military and political decision makers who assess the countries' potential enemies. In addition, the number of defense firms in each country is known and given exogenously. The decisions in this model are taken in two stages. In stage 1, each of the two governments commits to the amount of the defense good that it will purchase from its defense industry. In stage 2, given the commitments of the governments to purchase the defense good from their own defense industries, and the rest of the world's demand for the defense good, the defense firms play a Cournot game to decide how much to produce and sell to the rest of the world in order to maximize their profits.

This paper shows that the concentration of all the major defense firms in the US and Western Europe, and the consolidation of the defense industries into a highly concentrated oligopoly play an important role in determining procurement levels, as well as defense policies, in the US and Europe. The main results of this arrangement are as follows.

1. Generally, the quantity of the defense good in the world is lower when the governments of producing countries pay the world price to their defense manufacturers than when they pay marginal cost plus a markup.
2. The net defense costs to a producing country (the government expenditure on the defense good minus the defense industry's profit) are lower when both governments pay the world price to their defense industries.
3. More competitive defense industries (a larger number of defense firms in the producing countries) reduce the world price of the defense good, and thus bring about more sales of the defense good to the producing countries as well as to the rest of the world.
4. Predetermined target levels of security affect the optimal number of firms in each of the producing countries. Generally, higher security levels result in a larger number of defense firms in each country.
5. If the production of the defense good in one of the producing countries is significantly more

efficient than that of the other country (due to a larger investment in R&D, say), the more efficient country may capture most or all of the exports to the rest of the world. The larger the number of defense firms in each of the two producing countries, the more pronounced this phenomenon becomes.

Finally, the results of this study may help to guide policy analysis. First, consolidation of defense firms is seen in the US and Western Europe as a way of reducing procurement costs and sustaining a viable defense industry during a period of declining defense budgets. This paper provides an additional explanation of the consolidation process. We show that government expenditure on the defense good and net defense cost are smaller when the number of defense firms in each country is relatively small. A smaller number of firms means a higher world price for the defense good which, in turn, results in lower sales of the defense good to the rest of the world. As a consequence, a lower commitment is required of the governments of the US and Western Europe to achieve predetermined target security levels. Thus, cooperation between the US and Western Europe will be beneficial to both. Allowing defense firms to consolidate across the Atlantic will reduce the total number of defense firms in the world, while increasing the size of the remaining firms. This will reduce the net defense costs of Western Europe and the US, at the same time raising the world price for the defense good, thus reducing the stock of weapon systems in the rest of the world. Second, the governments of the US and Western Europe should consider paying the world price (rather than marginal cost plus a markup) to their defense industries, thus reducing their own net defense costs.

## References

- Ben Israel, I., "Defense, Technology, and the Future Battlefield", Directorate of Defense Research and Development - "MAFAT" (in Hebrew), 1998.
- Besanko, D., D. Dranove and M. Shanley, The Economics of Strategy, Wiley & Sons, 1996.
- Cobble, E., "Ideas, Identity, and the Limits of European Defense Technology Cooperation". Presented at the Workshop on the Place of the Defense Industry in National Systems of Innovation, October 16-18, 1998, Cornell University.
- Dvir, D., S. Lipovetsky, A. Shenhar and A. Tishler, "In Search of Project Classification: A Non-Universal Approach to Project Success Factors", Research Policy, 27, 1998, 915-935.
- Dvir, D. and A. Tishler, "The Changing Role of the Defense Industry in Israel's Industrial and Technological Development". Forthcoming in Defense Analysis, 2000.
- Flamm, K. "The U.S. Defense Industry After the Cold War: Economic Pressures and Security Dilemmas". Presented at the Workshop on the Place of the Defense Industry in National Systems of Innovation, October 16-18, 1998, Cornell University.
- Ham, R.M. and D.C. Mowery, "Improving the Effectiveness of Public-Private R&D Collaboration: Case Studies at the US Weapons Laboratory", Research Policy, 26, 1998, 661-675.
- Goolsbee, A., "Does Government R&D Policy Mainly Benefit Scientists and Engineers?" American Economic Review, AEA Papers and Proceedings, 88, 2, 1998, 298-302.
- James, A. D., "The Place of the U.K. Industry in its National Innovation System: Co-evolution of National, Sectoral and Technological Systems". Presented at the Workshop on the Place of the Defense Industry in National Systems of Innovation, October 16-18, 1998a, Cornell University.
- James, A. D., "Post-Merger Strategies of the Leading US Defence Aerospace Companies", FOA Defence Research Establishment, Stockholm, FOA-R-98-00941-170-SE, 1998b.
- Kovacic, W.E. and D.E. Smallwood, "Competition Policy, Rivalries, and Defense Industry Consolidation", Journal of Economic Perspectives, 8, 4, 1994, 91-110.
- Kreps, D.M. and J. Scheinkman, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," Rand Journal of Economics, 14, 1983, 326-37.
- Leahy, D. and J.P. Neary, "Public Policy Towards R&D in Oligopolistic Industries", American Economic Review, 87, 4, 1997, 642-662.

- Louscher, J.D, A.H. Cook and V.D. Barto, "The Emerging Competitive Position of US Defense Firms in the International Markets", Defense Analysis, 14, 2, 1998, 115-134.
- Lovering, J., "The Defense Industry as a Paradigmatic Case of 'Actually Existing Globalization'". Presented at the Workshop on the Place of the Defense Industry in National Systems of Innovation, October 16-18, 1998, Cornell University.
- Markusen, A., "Should We Welcome a Transnational Defense Industry?" Presented at the Workshop on the Place of the Defense Industry in National Systems of Innovation, October 16-18, 1998, Cornell University.
- Mas-Colell, A., M.D. Whinston and J.R. Green, Microeconomic Theory, Oxford University Press, 1995.
- Nelson, R.R. (ed.), National Innovation Systems: A Comparative Analysis, Oxford University Press, 1993.
- Rogerson, W.P., "Quality vs. Quantity in Military Procurement", American Economic Review, 80, 1, 1990, 83-92.
- Rogerson, W.P., "Economic Incentives and Defense Procurement Process", Journal of Economic Perspectives, 8, 4, 1994, 65-90.
- Romer, P., "Endogenous Technical Change", Journal of Political Economy, 98, 5, 1990, S71-S102.
- Segerstrom, P.S., "Endogenous Growth Without Scale Effects", American Economic Review, 88, 5, 1998, 1291-1310.
- Serfati, C., "The Place of the French Arms Industry in Its National System of Innovation and in the Governmental Technology Policy". Presented at the Workshop on the Place of the Defense Industry in National Systems of Innovation, October 16-18, 1998, Cornell University.
- The Economist, "Linking Arms: A Survey of the Global Defense Industry", June 14, 1997.
- The Economist, "Platform Envy", December 12, 1998.
- Tishler, A., D. Dvir, A. Shenhar and S. Lipovetsky, "Identifying Critical Success Factors in Defense Development Projects: A Multivariate Analysis", Technological Forecasting and Social Change, 51, 1996, 151-172.
- Wall Street Journal, "General Dynamics to Press Ahead on Acquisition Trail", April 15, 1999.

## Appendix (SDI – 20)

**Case 1** (the governments of  $A$  and  $B$  pay their defense industries the world price).

### A. Stage 2

The profit function of the first firm in country  $A$  is given by:

$$\pi_{A1} = [a + b(Y_{A1}^W + \sum_{j=2}^K Y_{Aj}^W + \sum_{j=1}^N Y_{Bj}^W)](Y_{A1}^W + Y_{A1}^A) - [\alpha_{0A} + \alpha_{1A}(Y_{A1}^W + Y_{A1}^A) + \frac{1}{2}\alpha_{2A}(Y_{A1}^W + Y_{A1}^A)^2]. \quad (\text{A1})$$

$$\partial \pi_{A1} / \partial Y_{A1}^W = 0 \Rightarrow Y_{A1}^W = \frac{\alpha_{1A} - a}{(K+1)b - \alpha_{2A}} + \frac{\alpha_{2A} - b}{(K+1)b - \alpha_{2A}} Y_{A1}^A - \frac{Nb}{(K+1)b - \alpha_{2A}} Y_{B1}^W. \quad (\text{A2})$$

Similarly,

$$\pi_{B1} = [a + b(Y_{B1}^W + \sum_{j=2}^K Y_{Bj}^W + \sum_{j=1}^N Y_{Aj}^W)](Y_{B1}^W + Y_{B1}^B) - [\alpha_{0B} + \alpha_{1B}(Y_{B1}^W + Y_{B1}^B) + \frac{1}{2}\alpha_{2B}(Y_{B1}^W + Y_{B1}^B)^2]. \quad (\text{A3})$$

$$\partial \pi_{B1} / \partial Y_{B1}^W = 0 \Rightarrow Y_{B1}^W = \frac{\alpha_{1B} - a}{(N+1)b - \alpha_{2B}} + \frac{\alpha_{2B} - b}{(N+1)b - \alpha_{2B}} Y_{B1}^B - \frac{Kb}{(N+1)b - \alpha_{2B}} Y_{A1}^W. \quad (\text{A4})$$

Solving (A2) and (A4) for  $Y_{A1}^W$  and  $Y_{B1}^W$  yields:

$$Y_{A1}^W = \frac{[(N+1)b - \alpha_{2B}](\alpha_{1A} - a) - Nb(\alpha_{1B} - a)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2} + \frac{[(N+1)b - \alpha_{2B}](\alpha_{2A} - b)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2} Y_{A1}^A + \frac{Nb(b - \alpha_{2B})}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2} Y_{B1}^B, \quad (\text{A5})$$

$$Y_{B1}^W = \frac{[(K+1)b - \alpha_{2A}](\alpha_{1B} - a) - Kb(\alpha_{1A} - a)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2} + \frac{[(K+1)b - \alpha_{2A}](\alpha_{2B} - b)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2} Y_{B1}^B + \frac{Kb(b - \alpha_{2A})}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2} Y_{A1}^A. \quad (\text{A6})$$

Using (A5) and (A6) we obtain ( $Y_A^W \equiv KY_{A1}^W$  and  $Y_B^W \equiv NY_{B1}^W$ ):



$$Y^W \equiv Y_A^W + Y_B^W = KY_{A1}^W + NY_{B1}^W = \gamma_0 + \gamma_1 KY_{A1}^A + \gamma_2 NY_{B1}^B, \quad (\text{A7})$$

where:

$$\gamma_0 = \frac{K(\alpha_{1A} - a)(b - \alpha_{2B}) + N(\alpha_{1B} - a)(b - \alpha_{2B})}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2}, \quad (\text{A8a})$$

$$\gamma_1 = \frac{(b - \alpha_{2B})(\alpha_{2A} - b)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2}, \quad (\text{A8b})$$

$$\gamma_2 = \frac{(b - \alpha_{2A})(\alpha_{2B} - b)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2}. \quad (\text{A8c})$$

## B. Stage 1

Actual security levels are given by:

$$S_A = Y_A^A / (Y_A^W + Y_B^W) \quad \text{and} \quad S_B = Y_B^B / (Y_A^W + Y_B^W). \quad (\text{A9})$$

Target security levels in countries  $A$  and  $B$  are  $S_A^0$  and  $S_B^0$ .

Now, using the optimal level of  $Y^W \equiv Y_A^W + Y_B^W$ , given by (A7), in (A9), and denoting

$Y_A^A \equiv KY_{A1}^A$  and  $Y_B^B \equiv NY_{B1}^B$ , we have:

$$Y_A^A = S_A^0 (Y_A^W + Y_B^W) = S_A^0 (\gamma_0 + \gamma_1 Y_A^A + \gamma_2 Y_B^B), \quad (\text{A10a})$$

$$Y_B^B = S_B^0 (Y_A^W + Y_B^W) = S_B^0 (\gamma_0 + \gamma_1 Y_A^A + \gamma_2 Y_B^B). \quad (\text{A10b})$$

Solving (A10a) and (A10b) for  $Y_A^A$  and  $Y_B^B$  yields:

$$Y_A^A = \frac{\gamma_0 S_A^0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}, \quad \text{and} \quad Y_B^B = \frac{\gamma_0 S_B^0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}. \quad (\text{A11a})$$

By assumption, all firms in each country are identical, hence,

$$Y_{A1}^A = Y_A^A / K \quad \text{and} \quad Y_{B1}^B = Y_B^B / N. \quad (\text{A11b})$$

Using (A11) in (A7) implies:

$$Y^W = \frac{\gamma_0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}. \quad (\text{A12})$$

Substituting (A11) into (A5), we obtain total production of each firm in country  $A$ :

$$Y_{A1} = Y_{A1}^W + Y_{A1}^A = \frac{[(N+1)b - \alpha_{2B}](\alpha_{1A} - a) - Nb(\alpha_{1B} - a)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2} + \frac{b(b - \alpha_{2B})\gamma_0}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKb^2} \frac{S_A^0 + S_B^0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0} \quad (\text{A13})$$

**III.** Evaluation of  $\pi_{A1}$ ,  $\pi_A \equiv K\pi_{A1}$ ,  $GC_A$  and  $GC_A - \pi_A$

Assumptions: (1) Countries  $A$  and  $B$  use the same technology (this assumption is denoted ST).

(2) Marginal production costs are fixed (denoted FMC). That is,  $\alpha_{1A} = \alpha_{1B} \equiv \alpha_1$ , and

$$\alpha_{2A} = \alpha_{2B} = 0.$$

Using assumptions FMC and ST in (A8) implies:

$$\gamma_0 = \frac{N+K}{N+K+1} \frac{\alpha_1 - a}{b}, \quad \gamma_1 = \gamma_2 = \frac{-1}{N+K+1}. \quad (\text{A14})$$

Using FMC and ST in (A5) yields:

$$Y_{A1}^W = \frac{1}{N+K+1} \frac{\alpha_1 - a}{b} - \frac{N+1}{N+K+1} Y_{A1}^A + \frac{N}{N+K+1} Y_{B1}^B. \quad (\text{A15})$$

Using (A11) in (A15) and simplifying (or, using FMC and ST in (A13)) yields:

$$Y_{A1} = Y_{A1}^W + Y_{A1}^A = \frac{1}{N+K+1} \frac{\alpha_1 - a}{b} \left[ 1 + \frac{(N+K)S^0}{N+K+1+S^0} \right], \quad (\text{A16})$$

where

$$S^0 \equiv S_A^0 + S_B^0. \quad (\text{A17})$$

Using  $\gamma_i, i = 0, 1, 2$ , given in (A14), in (A12) yields:

$$Y^W = \frac{\gamma_0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0} = \frac{\alpha_1 - a}{b} \frac{N + K}{N + K + 1 + S^0}. \quad (\text{A18})$$

World price,  $P$ , is given by

$$P = a + bY^W = \frac{(N + K)\alpha_1 + a(1 + S^0)}{N + K + 1 + S^0}. \quad (\text{A19})$$

ST and FMT imply  $\pi_{A1} = (P - \alpha_1)Y_{A1}$ . Hence, Using (A18) and (A19) to evaluate  $\pi_{A1}$ , we have:

$$\pi_{A1} = \frac{-1}{(N + K + 1 + S^0)^2} \frac{(a - \alpha_1)^2 (1 + S^0)^2}{b}, \quad \text{and} \quad (\text{A20a})$$

$$\pi_A = K \cdot \pi_{A1}. \quad (\text{A20b})$$

Given  $N = N^0$ , the value of  $K$  that maximizes  $\pi_A$  in (A20) is obtained by solving for  $K$  such that  $\partial \pi_A / \partial K = 0$ . This yields:

$$K = N^0 + 1 + S^0. \quad (\text{A21})$$

It is straightforward to verify that  $\partial^2 \pi_A / \partial^2 K < 0$  at  $K = N^0 + 1 + S^0$ .

Government  $A$ 's expenditure on the defense good equals:  $GC_A = P \cdot Y_A^A$ . Using (A19) and (A11):

$$GC_A = \frac{(N + K)[(N + K)\alpha_1 + a(1 + S^0)]}{N + K + 1 + S^0} \frac{\alpha_1 - a}{b} S_A^0. \quad (\text{A22})$$

Solving for  $K$  such that  $\partial GC_A / \partial K = 0$  in (A22) yields (given  $N = N^0$ ):

$$K = \frac{a(1 + S^0)}{a - 2\alpha_1} - N^0. \quad (\text{A23})$$

$K$  in (A23) is the maximal value of  $GC_A$  ( $\partial^2 GC_A / \partial^2 K < 0$  at  $K$  defined by (A23)). The minimal value of  $GC_A$  in (A22) is obtained at one of the extremes:  $K=1$  or  $K \rightarrow \infty$  ( $K \rightarrow \infty$  implies

$$GC_A = \alpha_1 S_A^0 (\alpha_1 - a) / b).$$

Net defense cost,  $GC_A - \pi_A$  (where  $\pi_A$  is defined in (A20b) and  $GC_A$  is given in (A22)), equals:

$$GC_A - \pi_A = \frac{\alpha_1 - a(N+K)[(N+K)\alpha_1 + a(1+S^0)]S_A^0 - K(a-\alpha_1)(1+S^0)^2}{b(N+K+1+S^0)^2}. \quad (\text{A24})$$

$\partial(GC_A - \pi_A)/\partial K = 0$ , for  $N = N^0$ , yields,

$$K = \tau_1 + \tau_2 N^0, \text{ where} \quad (\text{A25a})$$

$$\tau_1 = \frac{(a-\alpha_1)(1+S^0)^2 - a(1+S^0)S_A^0}{(a-\alpha_1)(1+S^0) + (2\alpha_1 - a)S_A^0}, \text{ and } \tau_2 = \frac{(a-\alpha_1)(1+S^0) - (2\alpha_1 - a)S_A^0}{(a-\alpha_1)(1+S^0) + (2\alpha_1 - a)S_A^0}. \quad (\text{A25b})$$

Using (A25b) we observe that the condition  $a > (3/2)\alpha_1$  is sufficient (not necessary) for  $\tau_2 > 0$ , and  $a > 2\alpha_1$  is sufficient (not necessary) for  $\tau_2 > 1$ . Note that if  $\tau_2 > 1$ , an equilibrium exists only if  $\tau_1 < 0$ .

**Case 2** (the governments of  $A$  and  $B$  pay their defense industries marginal cost plus a mark-up)

### I. Stage 2

The profit function of the first firm in country  $A$  is given by:

$$\begin{aligned} \pi_{A1} = & [a + b(Y_{A1}^W + \sum_{j=2}^K Y_{Aj}^W + \sum_{j=1}^N Y_{Bj}^W)] Y_{A1}^W + (1+\mu)[\alpha_{1A} + \alpha_{2A}(Y_{A1}^A + Y_{A1}^W)] Y_{A1}^A \\ & - [\alpha_{0A} + \alpha_{1A}(Y_{A1}^A + Y_{A1}^W) + \frac{1}{2}\alpha_{2A}(Y_{A1}^A + Y_{A1}^W)^2]. \end{aligned} \quad (\text{A26})$$

$$\partial\pi_{A1}/\partial Y_{A1}^W = 0 \Rightarrow Y_{A1}^W = \frac{\alpha_{1A} - a}{(K+1)b - \alpha_{2A}} + \frac{-\mu\alpha_{2A}}{(K+1)b - \alpha_{2A}} Y_{A1}^A + \frac{-Nb}{(K+1)b - \alpha_{2A}} Y_{B1}^W. \quad (\text{A27})$$

Similarly,

$$\begin{aligned} \pi_{B1} = & [a + b(Y_{B1}^W + \sum_{j=2}^K Y_{Bj}^W + \sum_{j=1}^N Y_{Aj}^W)] Y_{B1}^W + (1+\mu)[\alpha_{1B} + \alpha_{2B}(Y_{B1}^B + Y_{B1}^W)] Y_{B1}^B \\ & - [\alpha_{0B} + \alpha_{1B}(Y_{B1}^B + Y_{B1}^W) + \frac{1}{2}\alpha_{2B}(Y_{B1}^B + Y_{B1}^W)^2] \end{aligned} \quad (\text{A28})$$

$$\partial\pi_{B1}/\partial Y_{B1}^W = 0 \Rightarrow Y_{B1}^W = \frac{\alpha_{1B} - a}{(N+1)b - \alpha_{2B}} + \frac{-\mu\alpha_{2B}}{(N+1)b - \alpha_{2B}} Y_{B1}^B + \frac{-Kb}{(N+1)b - \alpha_{2B}} Y_{A1}^W. \quad (\text{A29})$$

Solving (A27) and (A28) for  $Y_{A1}^W$  and  $Y_{B1}^W$  yields:

$$Y_{A1}^W = \frac{[(N+1)b - \alpha_{2B}](\alpha_{1A} - a) - Nb(\alpha_{1B} - a)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2} + \frac{-\mu\alpha_{2A}[(N+1)b - \alpha_{2B}]}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2} Y_{A1}^A$$

$$+ \frac{\mu\alpha_{2B}Nb}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2} Y_{B1}^B, \quad (\text{A30})$$

$$Y_{B1}^W = \frac{[(K+1)b - \alpha_{2A}](\alpha_{1B} - a) - Kb(\alpha_{1A} - a)}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2} + \frac{-\mu\alpha_{2B}[(K+1)b - \alpha_{2A}]}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2} Y_{B1}^B$$

$$+ \frac{\mu\alpha_{2A}Kb}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2} Y_{A1}^A. \quad (\text{A31})$$

Using (A30) and (A31) we obtain ( $Y_A^W \equiv KY_{A1}^W$  and  $Y_B^W \equiv NY_{B1}^W$ ):

$$Y^W \equiv Y_A^W + Y_B^W = KY_{A1}^W + NY_{B1}^W = \gamma_0 + \gamma_1 KY_{A1}^A + \gamma_2 NY_{B1}^B, \quad (\text{A32})$$

where:

$$\gamma_0 = \frac{K(\alpha_{1A} - a)(b - \alpha_{2B}) + N(\alpha_{1B} - a)(b - \alpha_{2A})}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2}, \quad (\text{A33a})$$

$$\gamma_1 = \frac{-\mu\alpha_{2A}(b - \alpha_{2B})}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2}, \quad (\text{A33b})$$

$$\gamma_2 = \frac{-\mu\alpha_{2B}(b - \alpha_{2A})}{[(K+1)b - \alpha_{2A}][(N+1)b - \alpha_{2B}] - NKB^2}. \quad (\text{A33c})$$

## II. Stage 1

Using the definitions of security in (A9), and the results in (A30) and (A31), yields:

$$Y_A^A = \frac{\gamma_0 S_A^0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}, \quad \text{and} \quad Y_B^B = \frac{\gamma_0 S_B^0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}. \quad (\text{A34a})$$

and (since all firms in each country are identical),

$$Y_{A1}^A = Y_A^A / K \quad \text{and} \quad Y_{B1}^B = Y_B^B / N. \quad (\text{A34b})$$

$$Y^W = \frac{\gamma_0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}. \quad (\text{A35})$$

### III. Evaluation of $\pi_{A1}$ , $\pi_A \equiv K\pi_{A1}$ , $GC_A$ and $GC_A - \pi_A$

Assumptions: (1) Countries  $A$  and  $B$  use the same technology (this assumption is denoted ST).

(2) Marginal production costs are fixed (denoted FMC). That is,  $\alpha_{1A} = \alpha_{1B} \equiv \alpha_1$ , and

$$\alpha_{2A} = \alpha_{2B} = 0.$$

Under these assumptions, (A30), (A34) and (A35) yield:

$$Y_{A1}^W = \frac{1}{N+K+1} \frac{\alpha_1 - a}{b}, \quad (\text{A36})$$

$$Y_A^A = \gamma_0 S_A^0 \quad \Rightarrow \quad Y_{A1}^A = Y_A^A / K = \gamma_0 S_A^0 / K, \quad (\text{A37})$$

$$Y^W = \gamma_0 = \frac{N+K}{N+K+1} \frac{\alpha_1 - a}{b}. \quad (\text{A38})$$

Hence, the world price of the defense good is:

$$P = a + bY^W = a + b \frac{N+K}{N+K+1} \frac{\alpha_1 - a}{b} = \frac{(N+K)\alpha_1 + a}{N+K+1}, \quad (\text{A39})$$

$$\text{and } P - \alpha_1 = \frac{a - \alpha_1}{N+K+1}. \quad (\text{A40})$$

The profit of the first firm in  $A$  is:

$$\pi_{A1} = PY_{A1}^W + (1 + \mu)\alpha_1 Y_{A1}^A - \alpha_1(Y_{A1}^A + Y_{A1}^W) = (P - \alpha_1)Y_{A1}^W + \mu\alpha_1 Y_{A1}^A. \quad (\text{A41})$$

Using (A36), (A37) and (A40) in (A41) we have

$$\pi_{A1} = \frac{a - \alpha_1}{N+K+1} \frac{1}{N+K+1} \frac{\alpha_1 - a}{b} + \mu\alpha_1 S_A^0 \frac{1}{K} \frac{N+K}{N+K+1} \frac{\alpha_1 - a}{b}, \quad (\text{A42})$$

and  $\pi_A = K\pi_{A1}$ , that is,

$$\pi_A = \frac{1}{(N+K+1)^2} \frac{\alpha_1 - a}{b} \{(a - \alpha_1)K + (N+K)(N+K+1)\mu\alpha_1 S_A^0\}. \quad (\text{A43})$$

Given  $N = N^0$ , the value of  $K$  that maximizes  $\pi_A$  in (A43) is obtained by solving for  $K$  such that  $\partial\pi_A / \partial K = 0$ . This yields:

$$K = \frac{(a - \alpha_1) + \mu\alpha_1 S_A^0}{(a - \alpha_1) - \mu\alpha_1 S_A^0} + \frac{(a - \alpha_1) + \mu\alpha_1 S_A^0}{(a - \alpha_1) - \mu\alpha_1 S_A^0} N^0. \quad (\text{A44})$$

It is straightforward to verify that  $\partial^2\pi_A / \partial^2 K < 0$  at this point (if  $K > 0$ , that is,  $a > \alpha_1(1 + \mu S_A^0)$ ).

Clearly,  $\mu = 0$  yields  $K = N^0 + 1$ .  $\mu > 0$  and  $a > \alpha_1(1 + \mu S_A^0)$  imply that  $K > N^0 + 1$ .

Government  $A$ 's expenditure on the defense good is  $GC_A = (1 + \mu)\alpha_1 Y_A^A$ . Using (A37) yields:

$$GC_A = (1 + \mu)\alpha_1 \frac{N+K}{N+K+1} \frac{\alpha_1 - a}{b} S_A^0. \quad (\text{A45})$$

To find  $K$  such that  $GC_A$  is minimized for a given  $N = N^0$ , we differentiate  $GC_A$  w.r.t.  $K$  to get

$$\partial GC_A / \partial K = (1 + \mu)\alpha_1 \frac{1}{(N^0 + K + 1)^2} \frac{\alpha_1 - a}{b} S_A^0 > 0. \quad (\text{A46})$$

Hence,  $GC_A$  is minimized at  $K=1$ .

Using (A43) and (A45) we obtain  $GC_A - \pi_A$  as follows:

$$GC_A - \pi_A = \frac{a - \alpha_1}{b} \left\{ \frac{(a - \alpha_1)K + (N+K)(N+K+1)\mu\alpha_1 S_A^0}{(N+K+1)^2} + (1 + \mu)\alpha_1 \frac{N+K}{N+K+1} S_A^0 \right\}. \quad (\text{A47})$$

To find the value of  $K$ , for a given  $N = N^0$ , that minimizes  $GC_A - \pi_A$  we set

$\partial(GC_A - \pi_A) / \partial K = 0$  to obtain

$$K = \frac{\alpha_1(1 + S_A^0) - a}{\alpha_1(1 - S_A^0) - a} + \frac{\alpha_1(1 + S_A^0) - a}{\alpha_1(1 - S_A^0) - a} N^0 \equiv \psi(1 + N^0), \quad (\text{A48})$$

where  $\psi < 1$ . Two possibilities arise:

$$0 < S_A^0 < a/\alpha_1 - 1 \Rightarrow 0 < \psi < 1 \Rightarrow 1 \leq K < N^0 + 1, \quad (\text{A49a})$$

$$S_A^0 > a/\alpha_1 - 1 \Rightarrow \psi < 0 \Rightarrow K = 1. \quad (\text{A49b})$$

**Case 3** (country  $A$  pays marginal cost plus a mark-up and  $B$  pays the world price)

**A. Stage 2**

The profit function of the first firm in country  $A$  is given by (A26) and the profit function of the first firm in country  $B$  is given by (A3). Profit maximization yields  $Y_{A1}^W$  as in (A30) and  $Y_{B1}^W$  as in (A6).

**B. Stage 1**

$$Y_A^A = \frac{\gamma_0 S_A^0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}, \quad Y_B^B = \frac{\gamma_0 S_B^0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}, \quad Y^W = \frac{\gamma_0}{1 - \gamma_1 S_A^0 - \gamma_2 S_B^0}, \quad (\text{A50})$$

where  $\gamma_0$  is given by (A8a),  $\gamma_1$  is given by (A33b) and  $\gamma_2$  is given by (A8c).

Assumptions: (1) Countries  $A$  and  $B$  use the same technology (this assumption is denoted ST).

(2) Marginal production costs are fixed (denoted FMC). That is,  $\alpha_{1A} = \alpha_{1B} \equiv \alpha_1$ , and

$$\alpha_{2A} = \alpha_{2B} = 0.$$

These assumptions imply that  $\gamma_0$  and  $\gamma_2$  are given by (A14), respectively, and  $\gamma_1 = 0$ .

Using (A50) we obtain:

$$P = a + bY^W = \frac{(N + K)\alpha_1 + a(1 + S_0^B)}{N + K + 1 + S_B^0}. \quad (\text{A51})$$

$$Y_{A1}^W = \frac{1}{N + K + 1} \frac{\alpha_1 - a}{b} \left[ 1 + \frac{N + K}{N + K + 1 + S_B^0} S_B^0 \right], \quad (\text{A52})$$

$$Y_{A1}^A = \frac{1}{K} \frac{N + K}{N + K + 1 + S_B^0} \frac{\alpha_1 - a}{b} S_A^0, \quad (\text{A53})$$

$$Y_{B1}^B = \frac{1}{N} \frac{N + K}{N + K + 1 + S_B^0} \frac{\alpha_1 - a}{b} S_B^0, \quad (\text{A54})$$

$$Y_{B1}^W = \frac{1}{N + K + 1} \frac{\alpha_1 - a}{b}, \quad (\text{A55})$$



Using (A51)-(A54) we obtain:

$$\begin{aligned}\pi_{A1} &= (P - \alpha_1)Y_{A1}^W + \mu\alpha_1 Y_{A1}^A \\ &= \frac{-1}{(N + K + 1 + S_B^0)^2} \frac{(a - \alpha_1)^2 (1 + S_B^0)^2}{b} + \frac{1}{K} \frac{N + K}{N + K + 1 + S_B^0} \frac{\alpha_1 - a}{b} \mu\alpha_1 S_A^0\end{aligned}\quad (\text{A56})$$

$$\pi_{B1} = (P - \alpha_1)(Y_{B1}^W + Y_{B1}^B) = \frac{-1}{(N + K + 1 + S_B^0)^2} \frac{(a - \alpha_1)^2 (1 + S_B^0)^2}{b}, \quad (\text{A57})$$

$$GC_A = (1 + \mu)\alpha_1 Y_{A1}^A = (1 + \mu)\alpha_1 \frac{N + K}{N + K + 1 + S_B^0} \frac{\alpha_1 - a}{b} S_A^0, \quad (\text{A58})$$

$$GC_B = PY_B^B = \frac{(N + K)\alpha_1 + a(1 + S_B^0)}{N + K + 1 + S_B^0} \frac{N + K}{N + K + 1 + S_B^0} \frac{\alpha_1 - a}{b} S_B^0, \quad (\text{A59})$$

Now, suppose that  $K$  is given. What is the optimal  $N$ ?

Comparing (A57) to (A20) and (A58) to (A22) shows that the two pairs of expressions are identical except that  $S^0$  in (A20) and (A22) is replaced by  $S_B^0$ . Hence, the optimal values of  $N$  for maximum profits, minimum government expenditure on the defense good, and minimum net defense cost are given by (A21), (A23) and (A25), respectively, when  $K$  and  $N$  are interchanged and when  $S_B^0$  replaces  $S^0$ .

Similarly, suppose that  $N = N^0$  is given. What is the optimal  $K$ ?

Using (A56),  $\partial\pi_A / \partial K = 0$  yields:

$$K = \frac{(a - \alpha_1) + \mu\alpha_1 S_A^0}{(a - \alpha_1) - \mu\alpha_1 S_A^0} (1 + S_B^0) + \frac{(a - \alpha_1) + \mu\alpha_1 S_A^0}{(a - \alpha_1) - \mu\alpha_1 S_A^0} N^0. \quad (\text{A60})$$

It is straightforward to verify that  $\partial^2\pi_A / \partial^2 K < 0$  at this point (if  $K > 0$ , that is,  $a > \alpha_1(1 + \mu S_A^0)$ ).

Clearly,  $\mu = 0$  yields  $K = N^0 + 1 + S_B^0$ .  $\mu > 0$  and  $a > \alpha_1(1 + \mu S_A^0)$  imply that  $K > N^0 + 1 + S_B^0$ .

To find  $K$  to minimize  $GC_A$  for a given  $N = N^0$ , we differentiate  $GC_A$  in (A58) w.r.t.  $K$  to get

$$\partial GC_A / \partial K = (1 + \mu)\alpha_1 \frac{1 + S_B^0}{(N + K + 1 + S_B^0)^2} \frac{\alpha_1 - a}{b} S_A^0 > 0. \quad (\text{A61})$$

and, clearly,  $\partial^2 GC / \partial^2 K < 0$ . Hence,  $GC_A$  is minimized at  $K=1$ .

Using (A56) and (A58) yields:

$$GC_A - \pi_A = \frac{(a - \alpha_1)}{b(N + K + 1 + S_B^0)^2} [(a - \alpha_1)(1 + S_B^0)^2 K - (N + K)(N + K + 1 + S_B^0)\alpha_1 S_A^0]. \quad (\text{A62})$$

$\partial(GC_A - \pi_A) / \partial K = 0$  yields

$$K = \frac{[(a - \alpha_1)(1 + S_B^0)^2 - \alpha_1 S_A^0]}{(a - \alpha_1)(1 + S_B^0)^2 + \alpha_1 S_A^0} (1 + S_B^0) + \frac{(a - \alpha_1)(1 + S_B^0)^2 - \alpha_1 S_A^0}{(a - \alpha_1)(1 + S_B^0)^2 + \alpha_1 S_A^0} N^0. \quad (\text{A63})$$

The condition  $S_A^0 = S_B^0 = 0$  yields  $K = N^0 + 1$ .  $S_A^0 > 0$  and  $S_B^0 > 0$  yield  $K > 0$ , but  $K$  may be larger or smaller than  $N^0 + 1$  depending on the size of  $S_A^0$  and  $S_B^0$ .

Assume that  $S_A^0 = S_B^0$  and  $N=K$ . Then,

$$(GC_A - \pi_A) - (GC_B - \pi_B) = \frac{(a - \alpha_1)^2 (1 + S_B^0)}{(2N + 1 + S_B^0)} \frac{2NS_B^0}{b} < 0. \quad (\text{A64})$$

That is, country  $A$  is better off.

### Comparison of profits, government expenditure and net profit among price practices

Denote the profit of the defense industry by  $\pi_A^k$ , where  $k$  denotes the pricing practice ( $k=1$  when both governments pay the world price,  $k=2$  when both governments pay marginal cost plus a markup,  $k=3$  when the government of country  $A$  pays marginal cost plus a markup and the government of country  $B$  pays its defense industry the world price, and  $k=4$  when the government of  $A$  pays its defense industry the world price and the government of  $B$  pays its defense industry marginal cost plus a markup). Similarly, denote the government expenditure of  $A$  by  $GC_A^k$ . We start by comparing profits across price practices 1 and 2.

Using (A43) and  $\mu\alpha_1 \leq (P - \alpha_1)$  (profit margins under price practice 2 are not larger than those under pricing practice 1), we have

$$\pi_A^2 < \frac{-1}{(N+K+1)^2} \frac{(\alpha_1 - a)^2}{b} \{K + (N+K)S_A^0\}. \quad (\text{A65})$$

Using (A20) and (A43) we have

$$\frac{\pi_A^2}{\pi_A^1} < \frac{(N+K+1+S^0)^2}{(N+K+1)^2} \frac{K(1+S_A^0) + NS_A^0}{K(1+S^0)^2}. \quad (\text{A66})$$

$\pi_A^2 / \pi_A^1$  may be larger or smaller than one. However, when  $N=K$  and  $S_A^0 = S_B^0$ ,  $\pi_A^2 / \pi_A^1$  tends to be less than 1, that is  $\pi_A^2 < \pi_A^1$ . Specifically, using (A66) one obtains  $\partial(\pi_A^2 / \pi_A^1) / \partial N < 0$  and,

$$S_A^0 = S_B^0 < 1.5 \quad \Rightarrow \quad \pi_A^2 < \pi_A^1 \quad \text{for } N \geq 1,$$

$$S_A^0 = S_B^0 < 7.5 \quad \Rightarrow \quad \pi_A^2 < \pi_A^1 \quad \text{for } N \geq 2,$$

$$S_A^0 = S_B^0 < 17.5 \quad \Rightarrow \quad \pi_A^2 < \pi_A^1 \quad \text{for } N \geq 3.$$

Using (A22) and (A45) it is easy to observe that for a sufficiently large mark-up,  $\mu$ ,  $GC_A^1 < GC_A^2$ .

When  $\mu=0$ ,  $GC_A^2 < GC_A^1$  if

$$\frac{(N+K)\alpha_1 + a(1+S^0)}{N+K+1+S^0} \frac{N+K+1}{N+K+1+S^0} > \alpha_1. \quad (\text{A67})$$

The relation between  $GC_A^1$  and  $GC_A^2$  when  $\mu=0$  is not simple because it depends on many parameters. However, when  $N=K$ , (A67) can be rewritten as follows:

$$K > \frac{(S^0)^2}{(a/\alpha_1 - 1) + (a/\alpha_1 - 2)S^0} - 1, \quad (\text{A68})$$

and  $a/\alpha_1 > 2 + S^0/3$  is sufficient for (A68) to hold. That is,  $N=K$  and  $a/\alpha_1 > 2 + S^0/3$  imply that  $GC_A^2 < GC_A^1$ .

Using (A24) and (A47) we have

$$(GC_A^2 - \pi_A^2) - (GC_A^1 - \pi_A^1) = \frac{\alpha_1 - a}{b} \psi, \quad \text{where} \quad (\text{A69})$$

$$\psi \equiv \left\{ \frac{N(a - \alpha_1)(1 + 2S^0) - 2NS^0[2N\alpha_1 + a(1 + 2S^0)]}{(2N + 1 + S^0)^2} - \frac{N(a - \alpha_1) - 2NS^0(2N + 1)\alpha_1}{(2N + 1)^2} \right\}$$

After some manipulations of (A69) we get

$$\begin{aligned} \psi \approx & 16N^3(S^0)^3\alpha_1 + 16N^2(S^0)^2\alpha_1 + 4N(S^0)^2\alpha_1 + 4NS^0\alpha_1 + 16N^2(S^0)^2\alpha_1 \\ & + 12N(S^0)^2\alpha_1 + 2NS^0\alpha_1 + 8N^3S^0a - 2S^0Na - 4(S^0)^2Na \end{aligned} \quad (\text{A70})$$

Using (A70) we observe that:

1. If  $S^0 > 1$ , then  $S^0 < N^2 + \frac{1}{2}$  is sufficient (but not necessary) for  $\psi > 0$ .
2. For any value of  $S^0$ ,  $a/\alpha_1 < 4N^2 + 8N + 4$  is sufficient (but not necessary) for  $\psi > 0$ .

That is, we expect that in most cases  $(GC_A^1 - \pi_A^1) < (GC_A^2 - \pi_A^2)$ .

We continue by comparing price practices for  $k=3$  and  $k=4$ . Suppose that country  $A$  is committed to pay its industry marginal cost plus mark-up. Then, if country  $B$  chooses to pay its industry marginal cost plus mark-up,  $\pi_B$  and  $GC_B$  are given by (see (A42), (A45)):

$$\pi_B^2 = \frac{a - \alpha_1}{N + K + 1} \frac{N}{N + K + 1} \frac{\alpha_1 - a}{b} + \frac{1}{K} \frac{N + K}{N + K + 1} \frac{\alpha_1 - a}{b} \mu \alpha_1 S_B^0, \quad (\text{A71})$$

$$GC_B^2 = (1 + \mu) \alpha_1 \frac{N + K}{N + K + 1} \frac{\alpha_1 - a}{b} S_B^0. \quad (\text{A72})$$

If  $B$  chooses to pay its industry the world price then  $\pi_B^3 = N\pi_{B1}^3$  where  $\pi_{B1}^3$  is given by (A57) and  $GC_B^3$  is given by (A59). Clearly, for large  $\mu$  we have,  $\pi_B^3 < \pi_B^2$ . For  $\mu=0$  we have  $\pi_B^3 > \pi_B^2$ .

Similar conclusion holds for  $GC_B$  (since  $a > \alpha_1$ ).

Define  $\xi \equiv (GC_B^2 - \pi_B^2) - (GC_B^3 - \pi_B^3)$ . It is straightforward to show that, depending on the model's parameters,  $\xi$  may be less than, equal to, or greater than zero.

Suppose that country  $B$  is committed to pay the world price to its industry. Then, if  $A$  chooses to pay the world price to its defense industry,  $\pi_A^1$  and  $GC_A^1$  are given by (see (A20), (A22)):

$$\pi_A^1 = \frac{-K}{(N + K + 1 + S^0)^2} \frac{(a - \alpha_1)^2 (1 + S^0)^2}{b}, \quad (\text{A73})$$

$$GC_A^1 = \frac{(N + K)[(N + K)\alpha_1 + a(1 + S^0)]}{N + K + 1 + S^0} \frac{\alpha_1 - a}{b} S_A^0. \quad (\text{A74})$$

For large  $\mu$  we have,  $\pi_A^1 < \pi_A^3$ . For  $\mu=0$  we have  $\pi_A^1 > \pi_A^3$ . For large  $\mu$  we have  $GC_A^1 < GC_A^3$ .

When  $\mu=0$ , the relation between  $GC_A^1$  and  $GC_A^3$  is dependent on the values of several parameters.

Again, suppose that  $A$  chooses to pay the world price to its defense industry. Using (A73) and (A74), net defense cost is given by (see (A24)),

$$GC_A^1 - \pi_A^1 = \frac{\alpha_1 - a}{b} \frac{(N+K)[(N+K)\alpha_1 + a(1+S^0)]S_A^0 - K(a-\alpha_1)(1+S^0)^2}{(N+K+1+S^0)^2} > \frac{\alpha_1 - a}{b} \frac{(N+K)(N+K+1+S^0)\alpha_1 S_A^0 - K(a-\alpha_1)(1+S^0)^2}{(N+K+1+S^0)^2}, \quad (A75)$$

Whereas when  $A$  chooses to pay marginal cost plus markup we have (see (A56) and (A58)):

$$GC_A^4 - \pi_A^4 = \frac{\alpha_1 - a}{b} \frac{(N+K)(N+K+1+S_B^0)\alpha_1 S_A^0 - K(a-\alpha_1)(1+S_B^0)^2}{(N+K+1+S_B^0)^2}. \quad (A76)$$

(A75) and (A76) have the same functional form with  $S_B^0$  in (A76) and  $S^0$  in (A75). Clearly,

$S^0 \equiv S_A^0 + S_B^0 \geq S_B^0$ . Denote

$$\Phi \equiv \partial \left\{ \frac{\alpha_1 - a}{b} \frac{(N+K)[(N+K)\alpha_1 + a(1+S^0)]S_A^0 - K(a-\alpha_1)(1+S^0)^2}{(N+K+1+S^0)^2} \right\} / \partial S_B^0 \propto \frac{\alpha_1 - a}{b} M, \quad (A77)$$

where,

$$M = -(N+K)[(N+K)\alpha_1 S_A^0 + (1+S_B^0)\alpha_1 S_A^0 + 2K(a-\alpha_1)(1+S_B^0)] < 0. \quad (A78)$$

Hence,  $\Phi < 0$ . Thus,  $GC_B^1 - \pi_B^1 < GC_B^4 - \pi_B^4$ .

That is,  $B$  is better off paying the world price to its defense industry.

## Bücher des Forschungsschwerpunkts Marktprozeß und Unternehmensentwicklung

### Books of the Research Area Market Processes and Corporate Development

(nur im Buchhandel erhältlich/available through bookstores)

Horst Albach, Ulrike Görtzen, Rita Zobel (Hg.)

#### **Information Processing as a Competitive Advantage of Japanese Firms**

1999, edition sigma

Dieter Köster

#### **Wettbewerb in Netzproduktmärkten**

1999, Deutscher Universitäts-Verlag/Gabler Verlag

Christian Wey

#### **Marktorganisation durch Standardisierung: Ein Beitrag zur Neuen Institutionenökonomik des Marktes**

1999, edition sigma

Horst Albach, Meinolf Dierkes, Ariane Berthoin Antal, Kristina Vaillant (Hg.)

#### **Organisationslernen – institutionelle und kulturelle Dimensionen**

WZB-Jahrbuch 1998

1998, edition sigma

Lars Bergman, Chris Doyle, Jordi Gual, Lars Hultkrantz, Damien Neven, Lars-Hendrik Röller, Leonard Waverman

#### **Europe's Network Industries: Conflicting Priorities - Telecommunications**

Monitoring European Deregulation 1

1998, Centre for Economic Policy Research

Manfred Fleischer

#### **The Inefficiency Trap**

Strategy Failure in the

German Machine Tool Industry

1997, edition sigma

Christian Göseke

#### **Information Gathering and Dissemination**

The Contribution of JETRO to

Japanese Competitiveness

1997, Deutscher Universitäts-Verlag

Andreas Schmidt

#### **Flugzeughersteller zwischen globalem Wettbewerb und internationaler Kooperation Der Einfluß von Organisationsstrukturen auf die Wettbewerbsfähigkeit von Hochtechnologie-Unternehmen**

1997, edition sigma

Horst Albach, Jim Y. Jin, Christoph Schenk (eds.)

#### **Collusion through Information Sharing?**

**New Trends in Competition Policy**

1996, edition sigma

Stefan O. Georg

#### **Die Leistungsfähigkeit japanischer Banken Eine Strukturanalyse des Bankensystems in Japan**

1996, edition sigma

Stephanie Rosenkranz

#### **Cooperation for Product Innovation**

1996, edition sigma

Horst Albach, Stephanie Rosenkranz (eds.)

#### **Intellectual Property Rights and Global Competition - Towards a New Synthesis**

1995, edition sigma.

David B. Audretsch

#### **Innovation and Industry Evolution**

1995, The MIT Press.

Julie Ann Elston

#### **US Tax Reform and Investment: Reality and Rhetoric in the 1980s**

1995, Avebury

Horst Albach

#### **The Transformation of Firms and Markets: A Network Approach to Economic Transformation Processes in East Germany**

Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum, Vol. 34

1994, Almqvist & Wiksell International (Stockholm).

Horst Albach

#### **"Culture and Technical Innovation: A Cross-Cultural Analysis and Policy Recommendations"**

Akademie der Wissenschaften zu Berlin (Hg.)

Forschungsbericht 9, S. 1-597

1994, Walter de Gruyter.

Horst Albach

#### **Zerissene Netze. Eine Netzwerkanalyse des ostdeutschen Transformationsprozesses**

1993, edition sigma.

Zoltan J. Acs/David B. Audretsch (eds)

#### **Small Firms and Entrepreneurship: An East-West Perspective**

1993, Cambridge University Press.

Anette Boom

#### **Nationale Regulierungen bei internationalen Pharma-Unternehmen: Eine theoretische Analyse der Marktwirkungen**

1993, Nomos Verlagsgesellschaft.

## DISCUSSION PAPERS 1998

Horst Albach	<b>Unternehmensgründungen in Deutschland Potentiale und Lücken</b>	FS IV 98 - 1
Dietmar Harhoff	<b>Vertical Organization, Technology Flows and R&amp;D Incentives - An Exploratory Analysis</b>	FS IV 98 - 2
Karel Cool Lars-Hendrik Röller Benoit Leleux	<b>Der Einfluß des tatsächlichen und des potentiellen Wettbewerbs auf die Rentabilität von Unternehmen der pharmazeutischen Industrie</b>	FS IV 98 - 3
Horst Albach	<b>Blühende Landschaften? Ein Beitrag zur Transformationsforschung</b>	FS IV 98 - 4
Shiho Futagami Tomoki Waragai Thomas Westphal	<b><i>Shukko</i> in Japanese Companies and its Economic and Managerial Effects</b>	FS IV 98 - 5
Dietmar Harhoff Timm Körting	<b>Lending Relationships in Germany: Empirical Results from Survey Data</b>	FS IV 98 - 6
Johan Lagerlöf	<b>Are We Better Off if Our Politicians Know How the Economy Works?</b>	FS IV 98 - 7
Justus Haucap Christian Wey Jens Barmbold	<b>Location Costs, Product Quality, and Implicit Franchise Contracts</b>	FS IV 98 - 8
Manfred Fleischer	<b>Patenting and Industrial Performance: The Case of the Machine Tool Industry</b>	FS IV 98 - 9
Dieter Köster	<b>Was sind Netzprodukte? - Eigenschaften, Definition und Systematisierung von Netzprodukten</b>	FS IV 98 - 10
Andreas Blume	<b>Coordination and Learning with a Partial Language</b>	FS IV 98 - 11
Andreas Blume Uri Gneezy	<b>An Experimental Investigation of Optimal Learning in Coordination Games</b>	FS IV 98 - 12
Andreas Blume Douglas V. DeJong George R. Neumann Nathan E. Savin	<b>Learning in Sender-Receiver Games</b>	FS IV 98 - 13
Hans Mewis	<b>The Stability of Information Cascades: How Herd Behavior Breaks Down</b>	FS IV 98 - 14
Lars-Hendrik Röller Mihkel M. Tombak Ralph Siebert	<b>The Incentives to Form Research Joint Ventures: Theory and Evidence</b>	FS IV 98 - 15
Christine Zulehner	<b>Econometric Analysis of Cattle Auctions</b>	FS IV 98 - 16
Catherine Matraves	<b>Market Structure, R&amp;D and Advertising in the Pharmaceutical Industry</b>	FS IV 98 - 17

## DISCUSSION PAPERS 1999

Suchan Chae Paul Heidhues	<b>Bargaining Power of a Coalition in Parallel Bargaining: Advantage of Multiple Cable System Operators</b>	FS IV 99 - 1
Christian Wey	<b>Compatibility Investments in Duopoly with Demand Side Spillovers under Different Degrees of Cooperation</b>	FS IV 99 - 2
Horst Albach	<b>Des paysages florissants? Une contribution à la recherche sur la transformation</b>	FS IV 99 - 3
Jeremy Lever	<b>The Development of British Competition Law: A Complete Overhaul and Harmonization</b>	FS IV 99 - 4
Damien J. Neven Lars-Hendrik Röller Zhentang Zhang	<b>Union Power and Product Market Competition: Evidence from the Airline Industry</b>	FS IV 99 - 5
Justus Haucap Uwe Pauly Christian Wey	<b>The Incentives of Employers' Associations to Raise Rivals' Costs in the Presence of Collective Bargaining</b>	FS IV 99 - 6
Jianbo Zhang Zhentang Zhang	<b>Asymptotic Efficiency in Stackelberg Markets with Incomplete Information</b>	FS IV 99 - 7
Justus Haucap Christian Wey	<b>Standortwahl als Franchisingproblem</b>	FS IV 99 - 8
Yasar Barut Dan Kovenock Charles Noussair	<b>A Comparison of Multiple-Unit All-Pay and Winner-Pay Auctions Under Incomplete Information</b>	FS IV 99 - 9
Jim Y. Jin	<b>Collusion with Private and Aggregate Information</b>	FS IV 99 - 10
Jos Jansen	<b>Strategic Information Revelation and Revenue Sharing in an R&amp;D Race with Learning Labs</b>	FS IV 99 - 11
Johan Lagerlöf	<b>Incomplete Information in the Samaritan's Dilemma: The Dilemma (Almost) Vanishes</b>	FS IV 99 - 12
Catherine Matraves	<b>Market Integration and Market Structure in the European Soft Drinks Industry: Always Coca-Cola?</b>	FS IV 99 - 13
Pinelopi Koujianou Goldberg Frank Verboven	<b>The Evolution of Price Discrimination in the European Car Market</b>	FS IV 99 - 14
Olivier Cadot Lars-Hendrik Röller Andreas Stephan	<b>A Political Economy Model of Infrastructure Allocation: An Empirical Assessment</b>	FS IV 99 - 15
Holger Derlien Tobias Faupel Christian Nieters	<b>Industriestandort mit Vorbildfunktion? Das ostdeutsche Chemiedreieck</b>	FS IV 99 - 16
Christine Zulehner	<b>Testing Dynamic Oligopolistic Interaction: Evidence from the Semiconductor Industry</b>	FS IV 99 - 17



Johan Lagerlöf	<b>Costly Information Acquisition and Delegation to a “Liberal” Central Banker</b>	FS IV 99 - 18
Ralph Siebert	<b>New Product Introduction by Incumbent Firms</b>	FS IV 99 - 19
Ralph Siebert	<b>Credible Vertical Preemption</b>	FS IV 99 - 20
Ralph Siebert	<b>Multiproduct Competition, Learning by Doing and Price-Cost Margins over the Product Life Cycle: Evidence from the DRAM Industry</b>	FS IV 99 - 21
Michael Tröge	<b>Asymmetric Information Acquisition in Credit Auction</b>	FS IV 99 - 22
Michael Tröge	<b>The Structure of the Banking Sector, Credit Screening and Firm Risk</b>	FS IV 99 - 23
Michael Tröge	<b>Monitored Finance, Usury and Credit Rationing</b>	FS IV 99 - 24
Silke Neubauer	<b>Multimarket Contact, Collusion and the International Structure of Firms</b>	FS IV 99 - 25

## DISCUSSION PAPERS 2000

Justus Haucap Uwe Pauly Christian Wey	<b>Collective Wage Setting When Wages Are Generally Binding: An Antitrust Perspective</b>	FS IV 00 - 01
Stephanie Aubert Andreas Stephan	<b>Regionale Infrastrukturpolitik und ihre Auswirkung auf die Produktivität: Ein Vergleich von Deutschland und Frankreich</b>	FS IV 00 - 02
Achim Kemmerling Andreas Stephan	<b>Political Economy of Infrastructure Investment Allocation: Evidence from a Panel of Large German Cities</b>	FS IV 00 - 03



Absender/From:

Versandstelle - WZB  
Reichpietschufer 50  
D-10785 Berlin

**BESTELLSCHEIN / ORDERFORM**

Bitte schicken Sie mir aus der Liste der  
Institutsveröffentlichungen folgende Papiere zu:

Bitte schicken Sie bei Ihren Bestellungen von WZB-Papers  
**unbedingt** eine **1-DM-Briefmarke pro paper** und einen an  
Sie adressierten **Aufkleber** mit. Danke.

For each paper you order please send a **"Coupon-  
Réponse International"** (international money order)  
plus a **self-addressed adhesive label**. Thank You.

Please send me the following papers from your Publication List:

---

Paper Nr./No.                      Autor/Author + Kurztitel/Short Title

---

