Information in conflicts

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discussion papers

FS IV 01 – 11

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August 2001

ISSN Nr. 0722 - 6748

Forschungsschwerpunkt
Markt und politische Ökonomie

Research Area
Markets and Political Economy

Zitierweise/Citation:

Karl Wärneryd, **Information in Conflicts**, Discussion Paper
FS IV 01-11, Wissenschaftszentrum Berlin, 2001.

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ABSTRACT

Information in Conflicts

by Karl Wärneryd*

We consider a two-player contest for a prize of common but uncertain value. We show that less resources are spent in equilibrium if one party is privately informed about the value of a prize than if either both agents are informed or neither agent is informed. Furthermore, the uninformed agent is ex ante strictly more likely to win the prize than is the informed agent.

Keywords: Conflict, contest, asymmetric information, all-pay auction
JEL classification: C72, D44, D72, D82

ZUSAMMENFASSUNG

Information in Konflikten

Der Autor untersucht einen Wettbewerb mit zwei Spielern um einen Gewinn von allgemeinem, jedoch unbekanntem Wert. Er zeigt, daß im Vergleich zu einer Situation in der beide oder keiner von beiden Akteuren die Höhe des Gewinns kennen, weniger Ressourcen im Gleichgewicht verwendet werden, als in dem Fall in dem einer der beiden Spieler über die Höhe des Gewinns informiert ist. Des weiteren ist es ex ante streng genommen wahrscheinlicher, daß der uninformierte Spieler den Gewinn erhält und nicht der informierte.

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1 Introduction

Most of economic theory focuses on the analysis of voluntary market transactions, with well-defined property rights and costlessly enforceable contracts implicitly in the background. Many real-world transactions, in contrast, involve outright conflict. Such conflict takes many forms; theft, armed warfare, lobbying, and litigation are just a few examples. All have in common that they involve resources that could have been used productively being invested in effecting transfers of wealth. In this paper, we investigate the effects of asymmetric information in such conflicts.

Specifically, we model conflict as a *contest*. In a contest, the participants make efforts or expenditures in order to increase their probability of winning a prize or object. We consider the case of two agents competing for a prize that would be of the same value to both if both knew the value with certainty, but we assume this value is uncertain. This assumption of *ex post* common valuation seems natural in a number of potential applications, of which the following are just a few examples.

- Firms lobbying for monopoly privileges in a market, where one firm is the incumbent in the market.
- An investor trying to recover (e g, through the courts) funds from an entrepreneur.
- Two parties contesting each other's claim to a piece of property, where one party is the possessor of the property.

In all of these cases, one party has superior information about the *ex post* common value of the prize being contested.

In this paper, we exhibit a model in which this type of asymmetric information leads risk neutral agents to spend less in equilibrium than under symmetric information. This suggests, among other things, an explanation for the results of empirical studies on rent seeking (e g, Katz and Rosenberg [10] and Laband and Sophocleus [12]), which seem to indicate that expenditure on such activities is typically much lower than the expected value of the prize. While it is well known that, in a complete information setting, asymmetry of valuations tends to lower equilibrium aggregate expenditure (see, e g, Hillman and Riley [7]), in many cases asymmetric information seems the most natural source of such divergences in valuation.

Other contributions to the theory of informational asymmetry in contests include Harstad [6], who studies imperfectly, but symmetrically, privately informed agents in a contest where the highest bidder wins with certainty. Linster [13] and Hurley and Shogren [9] study models with independent valuations. The setup in the present paper is not a special case of either of these approaches. The model closest in spirit to the present one that we are aware of is found in Bernardo, Talley, and Welch [1], who study a variant of the two-type case in the larger context of a contracting problem with a court battle as the outside option, but focus on different issues and do not draw any general conclusions.

The contests studied in this paper are also special cases of imperfectly discriminatory *all-pay auctions*, i e, auctions where all participants pay their bids. (See, e g, Krishna and Morgan [11].) From the point of view of auction theory, perhaps the most interesting result in the present paper is the mildly paradoxical observation that the uninformed player in an asymmetric information contest wins the object with strictly higher probability than the informed player, something that cannot happen in a first-price auction.

The paper is organized as follows. Section 2 introduces the class of contests under study, and solves for equilibrium under the two possible symmetric information scenarios. This provides a benchmark for the analysis of asymmetric information in Section 3, where we prove, among other things, that aggregate expenditure in equilibrium is strictly less under asymmetric information, and that the uninformed player is more likely to win. Section 4 discusses some example distributions for the value of the prize. Section 5 suggests some extensions and concludes.

2 Symmetric Information

Two risk neutral agents, 1 and 2, compete for a prize of value y, where y is distributed according to the continuous cumulative distribution F, the support of which is contained in $[\underline{y}, \infty)$, where $\underline{y} > 0$. Let $\tilde{y}$ be the expected value of the prize. The agents make expenditures in order to increase their probability of winning. Specifically, if agent 1 expends x_1 and agent 2 expends x_2, we assume the probability of agent i winning the prize is[1]

$$p_i(x_1, x_2) := \begin{cases} x_i/(x_1 + x_2) & \text{if } x_1 + x_2 > 0 \\ 1/2 & \text{otherwise.} \end{cases}$$

This particular type of contest is sometimes referred to as a *lottery contest*, since it is equivalent to buying tickets at unit cost in a lottery where the winner is drawn from the set of tickets sold. We note, for future reference, that as long as we have $x_1+x_2 > 0$, p_i is convex in the *opponent's* expenditure.

The expected utility of agent i if neither agent is informed of the value y when making his expenditure decision is then

$$u_i^U(x_1, x_2) := \int_{\underline{y}}^{\infty} p_i(x_1, x_2) y \mathrm{d}F(y) - x_i.$$

It is easily verified that this objective function is strictly concave in own expenditure given the expenditure of the other party. Furthermore, it cannot be the case that nobody expends anything in equilibrium, since in case one agent's expenditure is zero the other can appropriate the prize with probability one for an arbitrarily small expenditure.

[1] This particular contest success function was introduced by Tullock [18, 19]. For further discussion, see, e g, Hirshleifer [8]. For an axiomatization, see Skaperdas [16]. Fullerton and McAfee [5], Esteban and Ray [4], and Nitzan [15] is a small sample of the large body of literature on applications of this class of contests. Results for more general success functions are hard to come by. By allowing for general distributions, however, we attain the same level of generality as auction theory in general.

Hence the best reply of agent i given the expenditure of the other agent is given by the first-order condition

$$\frac{\partial u_i^U(x_1, x_2)}{\partial x_i} = \frac{x_j}{(x_1 + x_2)^2}\tilde{y} - 1 = 0 \text{ for } j \neq i.$$

Solving the resultant set of simultaneous equations, we find that each agent expends $x^S := \tilde{y}/4$ in equilibrium.

Similarly, it is easily seen that if both agents are informed about y when making their expenditures, then for any y they will each expend $x_i := y/4$. Hence *ex ante* expected individual expenditure in this case is

$$\int_{\underline{y}}^{\infty} \frac{y}{4} \mathrm{d}F(y) = \frac{\tilde{y}}{4} = x^S.$$

We summarize these results as follows.

Proposition 1 *Under either symmetric information scenario, each player expends $x^S = \tilde{y}/4$ in expectation.*

That the two symmetric information scenarios are equivalent from an *ex ante* point of view is, of course, an artifact of risk neutrality.[2] But this setting makes the effect of informational asymmetry, which we study next, more interesting.

3 Asymmetric Information

Consider next what happens if one agent is informed of y but the other agent is uninformed. The uninformed agent is now potentially subject to an analogue of the winner's curse, and hence cannot rationally estimate the value of the prize at its expectation. We say an "analogue" of the winner's curse because the latter is the phenomenon where in a first-price auction

[2] For a discussion of the role of risk aversion in contests generally, see Skaperdas and Gan [17].

the uninformed agent only wins if he has overbid. In a lottery contest, an arbitrarily small expenditure always yields a positive probability of winning. Hence it may turn out *ex post* that the object has in fact been obtained rather inexpensively.

The informed agent's objective function, given the realization y, is now

$$u_I(y) := \frac{x_I(y)}{x_U + x_I(y)} y - x_I(y),$$

where x_U is the expenditure of the uninformed agent and $x_I(y)$ is the expenditure of the informed agent as a function of the value y. We shall sometimes refer to y as the *type* of the informed player. The first partial derivative of this objective function with respect to $x_I(y)$ is

$$\frac{\partial u_I(y)}{\partial x_I(y)} = \frac{x_U}{(x_U + x_I(y))^2} y - 1,$$

which is negative for all $x_I(y) \geq 0$ if we have $x_U > y$. The informed agent's best reply function, given y, is therefore

$$x_I(y) = \begin{cases} \sqrt{x_U y} - x_U & \text{if } x_U \leq y \\ 0 & \text{otherwise.} \end{cases}$$

Consider next the uninformed agent, whose objective function is now

$$u_U := \int_{\underline{y}}^{\infty} \frac{x_U}{x_U + x_I(y)} y \mathrm{d}F(y) - x_U.$$

It cannot be the case in equilibrium that $x_I(y) = 0$ for all y. This would imply $x_U \geq \bar{y}$. But given that no type of the informed player expends a positive amount, this cannot be a best reply on the part of the uninformed player, since he could lower his expenditure and still win with probability one in all states. (Of course, in the model as specified, the uninformed player does not even have a best reply when all types of the opponent expend zero.)

The relevant condition for an optimal expenditure level on the part of the uninformed agent, given $x_I(y)$, is therefore

$$\frac{\partial u_U}{\partial x_U} = \int_{\underline{y}}^{\infty} \frac{x_I(y)}{(x_U + x_I(y))^2} y \mathrm{d}F(y) - 1 = 0.$$

Note that the informed agent's first order condition for a positive expenditure may be written

$$(x_U + x_I(y))^2 = x_U y.$$

Substituting into the uninformed agent's first order condition, this implies that we have that

$$x_U = \int_{\underline{y}}^{\infty} x_I(y)\mathrm{d}F(y)$$

in equilibrium.[3] That is, we have proved the following.

Lemma 1 *In equilibrium, the uninformed and informed player expend the same amount in expectation.*

Expanding the uninformed player's first order condition further, using the informed agent's best reply function, we get the condition

$$\frac{1}{\sqrt{x_U}} \int_{x_U}^{\infty} \sqrt{y}\mathrm{d}F(y) - (1 - F(x_U)) - 1 = 0. \tag{1}$$

We shall also utilize that (1) may be rearranged to yield

$$x_U = \frac{1}{(2 - F(x_U))^2} \left(\int_{x_U}^{\infty} \sqrt{y}\mathrm{d}F(y) \right)^2. \tag{2}$$

Define $z := \int_{\underline{y}}^{\infty} \sqrt{y}\mathrm{d}F(y)$. Suppose we have $z^2/4 \leq \underline{y}$. Then setting $x_U = z^2/4$ implies $\int_{x_U}^{\infty} \sqrt{y}\mathrm{d}F(y) = z$, and hence fulfills (1). We shall call such an equilibrium, in which every type of the informed player expends a positive amount, an *interior* equilibrium.

More generally, we have the following, which is proved in the Appendix.

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[3] Consider also the more general class of contest success functions axiomatized by Skaperdas [16], where we have $p_i(x_1, x_2) = f(x_i)/(f(x_1) + f(x_2))$, with f an increasing function. Assuming $f(0) = 0$, the corresponding equilibrium property is $f(x_U)/f'(x_U) = \int (f(x_I(y)/f'(x_I(y)))\mathrm{d}F(y)$. One class of success functions for which equilibrium equality of expected expenditure can then immediately be seen to hold is the one where $f(x) = x^r$ for some positive constant r.

Proposition 2 *The asymmetric information contest has a unique equilibrium.*

We next consider aggregate expenditure in equilibrium. Recall that both parties expend the same amount in expectation. It is therefore sufficient to consider the amount x_U expended in equilibrium by the uninformed player. Although it holds generally that expenditure is strictly lower under asymmetric information than in either of the symmetric information scenarios, it is perhaps easiest to illustrate by considering the case when there is an interior equilibrium. In this case, we have $x_U = z^2/4$. Since $y \to \sqrt{y}$ is a strictly concave function, by Jensen's inequality we have $z < \sqrt{\tilde{y}}$, and therefore $z^2 < \tilde{y}$. Hence we must have $x_U < \tilde{y}/4$. Intuitively, in order to escape the winner's curse, the uninformed player behaves as if the prize was worth z^2 rather than $\tilde{y}$ in expectation. We prove the following more general result in the Appendix.

Proposition 3 *Aggregate expenditure is strictly lower under asymmetric information than under symmetric information.*

This result seems related to the observation (made by, e g, Hillman and Riley [7] and Che and Gale [2]) that, in a complete information environment, asymmetry of valuations tends to lower equilibrium expenditure.

Interior equilibria are of some special interest. We know from Proposition 3 that in any equilibrium, we have that

$$x_U < \frac{\tilde{y}}{4}.$$

A sufficient condition for the existence of a unique equilibrium with $x_U = z^2/4$ is therefore that we have $\tilde{y}/4 \leq \underline{y}$, i e, that $\tilde{y} \leq 4\underline{y}$. Hence we have the following.

Corollary 1 *Suppose we have $\tilde{y} \leq 4\underline{y}$. Then each type of the informed player expends a positive amount in equilibrium.*

A natural comparative statics experiment is to consider an increase in F in the sense of first order stochastic dominance. Unfortunately, this concept does not have much cutting power in the present model. To see why, let G strictly dominate F in the sense of first order stochastic dominance. Now consider the left-hand side of the equilibrium condition (1), and fix x_U. If F is replaced by G, we have $(1/\sqrt{x_U}) \int_{x_U}^{\infty} \sqrt{y} \mathrm{d}G(y) > (1/\sqrt{x_U}) \int_{x_U}^{\infty} \sqrt{y} \mathrm{d}F(y)$, but $-(1-G(x_U)) < -(1-F(x_U))$. Hence the net effect could go either way. It is therefore in principle possible to construct examples where first order stochastic dominance implies a lowering of equilibrium expenditure.

Let x_U^F and x_U^G be the respective equilibrium expenditures of the uninformed player associated with the two distributions. For the special case where equilibrium under both distributions is interior, we have that

$$x_U^F = \frac{1}{4} \int_{x_U^F}^{\infty} \sqrt{y} \mathrm{d}F(y) < \frac{1}{4} \int_{x_U^G}^{\infty} \sqrt{y} \mathrm{d}G(y) = x_U^G.$$

Similarly, we may consider the effects of an increase in riskiness, or second order stochastic dominance, under the same assumptions. Let G be a mean-preserving spread of F. It is then straightforward that $x_U^G < x_U^F$.

A sufficient condition for it to be the case that equilibrium under two different distributions involves positive expenditure by all types of the informed player in both cases is that the support of both distributions is small enough in a specific sense. Suppose both F and G are supported on $[\underline{y}, \overline{y}]$, and that we have $\overline{y} < 4\underline{y}$. Then the condition of Corollary 1 is satisfied. We have thus proved the following limited comparative statics result.

Proposition 4 *Suppose F and G are two distributions supported on $[\underline{y}, \overline{y}]$, where $\overline{y} < 4\underline{y}$. Then*

1. *if G strictly first order stochastically dominates F, we have $x_U^F < x_U^G$, and*

2. *if G is a mean-preserving spread of F, we have $x_U^G < x_U^F$.*

In equilibrium, the expected utilities of the respective players may be written as

$$\mathrm{E}u_U = \int_{\underline{y}}^{x_U} y\mathrm{d}F(y) + x_U(1 - F(x_U))$$

and

$$\mathrm{E}u_I = \int_{x_U}^{\infty} y\mathrm{d}F(y) - x_U(3 - F(x_U)).$$

Consider now a comparison with the scenario in which neither player knows y. In the special case of an interior equilibrium it is easily seen that the informed player is better off in expectation in the asymmetric information scenario, and the uninformed player worse off. In this case, we have $\mathrm{E}u_U = x_U = z^2/4$ and $\mathrm{E}u_I = \tilde{y} - 3x_U = \tilde{y} - 3z^2/4$. Since, again, $y \to \sqrt{y}$ is strictly concave, we have $z^2 < \tilde{y}$, and therefore $\mathrm{E}u_U < \tilde{y}/4$ and $\mathrm{E}u_I > \tilde{y}/4$. More generally, we have the following, which is proved in the Appendix.

Proposition 5 *In equilibrium of the asymmetric information contest, we have* $\mathrm{E}u_U < \tilde{y}/4$ *(the uninformed player is strictly worse off in expectation than if both were uninformed) and* $\mathrm{E}u_I > \tilde{y}/4$ *(the informed player is strictly better off in expectation than if both were uninformed).*

Since we have $\mathrm{E}u_U + \mathrm{E}u_I = \tilde{y} - 2x_U > \tilde{y}/2$, there is an efficiency gain from asymmetric information relative to the two symmetric information scenarios. The above result also tells us that this gain in its entirety accrues to the informed player.

This also allows us to say something about the incentives to acquire information. Suppose both players are initially uninformed, but before entering the contest have the option of independently becoming informed at a cost of $c > 0$. From an *ex ante* perspective, taking into account later equilibrium play of the contest, the information acquisition problem then looks like the game of Figure 1. By studying the equilibria of this game, we can find the subgame perfect equilibrium outcomes of the extensive form game as a whole.

		Player 2	
		Stay uninformed	Get informed
Player 1	Stay uninformed	$\tilde{y}/4, \tilde{y}/4$	$\mathrm{E}u_U < \tilde{y}/4, \mathrm{E}u_I - c$
	Get informed	$\mathrm{E}u_I - c, \mathrm{E}u_U < \tilde{y}/4$	$(\tilde{y}/4) - c, (\tilde{y}/4) - c$

Figure 1: The information acquisition game.

For sufficiently small c, i e, $c < \min\{\mathrm{E}u_I - (\tilde{y}/4), (\tilde{y}/4) - \mathrm{E}u_U\}$, getting informed will clearly be a dominant strategy for each player. Since the expected outcome of a contest where both parties are informed is the same as if neither was informed, under these circumstances the unique subgame perfect outcome of the game is inefficient. The game effectively has the structure of a Prisoners' Dilemma in which the contestants are led to inefficiently acquire information.

Finally, we consider each contestant's probability of winning the prize from an *ex ante* perspective. Define $\tilde{x}_I := \int_{\underline{y}}^{\infty} x_I(y)\mathrm{d}F(y)$, the expected equilibrium expenditure of the informed player. Now note that for any contest success function p that is strictly convex in the opponent's expenditure, as the one under consideration here is easily verified to be, by Jensen's inequality we have that

$$\int_{\underline{y}}^{\infty} p(x_U, x_I(y))\mathrm{d}F(y) > p(x_U, \tilde{x}_I).$$

In the present context, from Lemma 1 we have $p(x_U, \tilde{x}_I) = 1/2$. Hence we have proved the following somewhat surprising result.

Proposition 6 *In equilibrium, the uninformed agent's* ex ante *probability of winning is strictly greater than that of the informed agent.*

It is worthwhile to contrast this with equilibrium in a standard first-price auction under the same information assumptions. In the two-player case, it can be shown that there is a unique equilibrium where the uninformed player uses a mixed strategy, and the equilibrium distributions of bids are identical for the informed and uninformed player. Hence each player wins with probability one half. (See, e g, Milgrom and Weber [14].) But the two-player setting is a special case in the first-price framework. In case there are at least two informed players, the uninformed agent can only win when he has overbid. Hence for any positive bid, the expectation of the uninformed agent is negative. It follows that in equilibrium, uninformed players must bid zero and consequently have a zero probability of winning. The crucial difference between our contest and a first-price auction is that a player always has a positive probability of winning if he has expended a positive amount. That is, even very low bids leave a positive probability of winning a potentially valuable object.

Dixit [3], in a discussion of first-mover advantage in complete information contests, calls the player most likely to win in equilibrium the *favorite*. In the present incomplete information context, the *ex ante* favorite is thus somewhat paradoxically the uninformed player.

4 Examples

The Pareto Distribution

Suppose the value y is distributed on $[1, \infty)$ according to

$$F(y) = 1 - y^{1-\alpha},$$

where $\alpha > 2$. The density function associated with F is then

$$f(y) = (\alpha - 1)y^{-\alpha}$$

and the expected value of the prize is

$$\tilde{y} = \int_1^\infty (\alpha - 1)y^{1-\alpha}\mathrm{d}y = \frac{\alpha - 1}{\alpha - 2}.$$

We have that
$$z = \int_1^\infty \sqrt{y}\mathrm{d}F(y) = \frac{2\alpha-2}{2\alpha-3}.$$
Since we have $\alpha > 2$, we have $z < 2$. Hence we have $z^2/4 < 1 = \underline{y}$, so equilibrium is always interior.

The equilibrium expenditure of the uninformed player is therefore
$$x_U = \frac{z^2}{4} = \frac{1}{4}\left(\frac{2\alpha-2}{2\alpha-3}\right)^2.$$
Since we have that
$$\int_1^\infty \frac{1}{\sqrt{y}}\mathrm{d}F(y) = \frac{2\alpha-2}{2\alpha-1},$$
the expected win probability of the uninformed player is
$$\int_1^\infty \frac{x_U}{x_U + x_I(y)}\mathrm{d}F(y) = \int_1^\infty \frac{x_U}{\sqrt{x_U y}}\mathrm{d}F(y) =$$
$$\sqrt{x_U}\int_1^\infty \frac{1}{\sqrt{y}}\mathrm{d}F(y) = \frac{2(\alpha-1)^2}{3-8\alpha+4\alpha^2}.$$

Since this quantity is strictly decreasing in α and approaches $1/2$ as α approaches infinity, it is always strictly greater than $1/2$, as expected. For example, if we have $\alpha = 3$ the uninformed player wins with an *ex ante* probability of $8/15$.

The Two-Type Case

Suppose there are just two possible values of the prize, y_H and y_L, where we have $0 < y_L < y_H$. Let the probability of y_H be q. Although this discrete distribution does not strictly fall into the class considered previously, it usefully illustrates some central ideas. Since we can characterize this case completely, it is also of some independent interest.

We first note that there can be no equilibrium such that $x_I(y_H) = x_I(y_L) = 0$. As before, the informed agent's best reply expenditure will be positive if and only if we have $x_U < y$. Hence in the proposed equilibrium, we must have $x_U \geq y_H$. But this cannot be a best reply on the part of the uninformed agent, since he could lower his expenditure and still win with probability one in both states of the world. Therefore there is no equilibrium where the informed agent never expends a positive amount.

Two possibilities remain.

Case 1: Both informed types are active. Suppose we have $x_I(y_L) > 0$ and $x_I(y_H) > 0$. The uninformed agent's first order condition for a best reply expenditure is then

$$(1-q)\frac{x_I(y_L)}{(x_U + x_I(y_L))^2}y_L + q\frac{x_I(y_H)}{(x_U + x_I(y_H))^2}y_H - 1 = (1-q)\frac{1}{\sqrt{x_U}}\sqrt{y_L} + q\frac{1}{\sqrt{x_U}}\sqrt{y_H} - 2 = 0,$$

so equilibrium expenditure is

$$x_U = \frac{(q\sqrt{y_H} + (1-q)\sqrt{y_L})^2}{4}.$$

We must have $x_U < y_L$ for this to be an equilibrium, i e, that

$$q < \frac{\sqrt{y_L/y_H}}{1 - \sqrt{y_L/y_H}} =: \hat{q}.$$

Case 2: Only the highest informed type is active. Suppose we have $x_I(y_L) = 0$. The uninformed agent's first-order condition then reduces to

$$q\frac{x_I(y_H)}{(x_U + x_I(y_H))^2}y_H - 1 = q\frac{1}{\sqrt{x_U}}\sqrt{y_H} - 1 - q = 0,$$

so we have that

$$x_U = \left(\frac{q}{1+q}\right)^2 y_H.$$

In order for this to be consistent with the lowest type expending nothing, we must have $x_U \geq y_L$, i e, that

$$q \geq \hat{q}.$$

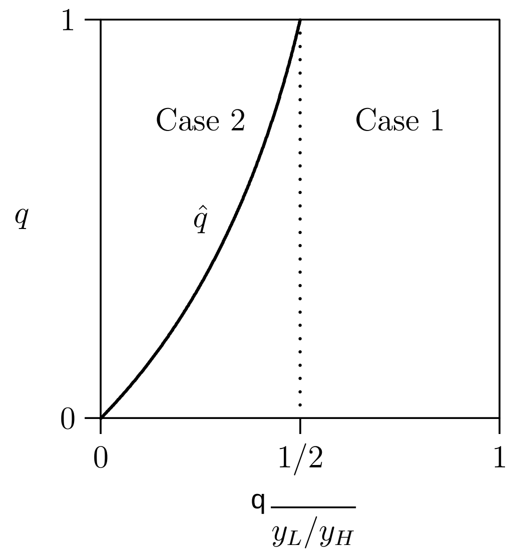

Figure 2: Equilibrium regions for the two-type case.

Figure 2 illustrates both types of equilibrium.

Note that in the Case 2 equilibrium, the uninformed player's probability of winning the object is

$$1 - q + q\frac{x_U}{x_U + x_I(y_H)} = 1 - q + \frac{q^2}{1+q} > 1/2.$$

Let $q \to 0$ and $y_L/y_H \to 0$, so that the condition for the existence of equilibrium is satisfied. Then the probability of the uninformed player winning approaches 1. That is, if the high value is much greater than the low value, and correspondingly unlikely to obtain, then the uninformed player is almost certain to win in equilibrium.

5 Concluding Remarks

We have studied a model of a common value contest under different assumptions about the information held by the players. We showed that when one player knows the value with certainty and the other player only knows its prior distribution, aggregate equilibrium expenditure is strictly less than in either the scenario where both players know the value with certainty or the scenario where neither player knows the value. In the terminology of rent-seeking theory, asymmetric information leads to less dissipation of the contested value.

This result opens the question of whether in a setting with more than two players there is some nontrivial allocation of information that minimizes aggregate expenditure. We suggest such multiplayer models as an interesting topic for further study.

We also found that in equilibrium under asymmetric information, the uninformed player wins with a strictly higher probability than does the informed player, in contrast with what would be the case in a first-price auction. It is clear that this result must also carry over to a larger class of models. It also has interesting implications for applications of the model. For instance, it suggests that, in the absence of any legal presumptions in favor of the possessor, in a legal battle over property rights in some object we would be most likely to see the object change hands.

Appendix

Proof of Proposition 2. We first prove existence. Define the function

$$g(x) := \frac{1}{\sqrt{x}} \int_x^\infty \sqrt{y}\,\mathrm{d}F(y) - (1 - F(x)) - 1.$$

A zero of g fulfills the first-order condition (1). We have that

$$g(\underline{y}) = \frac{z}{\sqrt{\underline{y}}} - 2.$$

Suppose we have $g(\underline{y}) \leq 0$. This implies $z^2/4 \leq \underline{y}$. Then setting $x_U = z^2/4$ is an equilibrium. Suppose instead we have $g(\underline{y}) > 0$. We also have that

$$\lim_{x\to\infty} g(x) = -1 < 0.$$

Hence if $x_U = z^2/4$ is not an equilibrium, then, since g is continuous, by the intermediate value theorem there is some finite $x > \underline{y}$ such that $g(x) = 0$.

We next prove uniqueness. We have that

$$\frac{\partial g(x)}{\partial x} = -\frac{\int_x^\infty \sqrt{y}\mathrm{d}F(y)}{2\left(\sqrt{x}\right)^3} < 0,$$

i e, g is strictly decreasing in x. Hence an x satisfying (1) must be unique. $\square$

Proof of Proposition 3. We know from Lemma 1 that both parties expend the same amount in expectation. Hence we need only consider x_U. Since the function $y \to \sqrt{y}$ is strictly concave, by Jensen's inequality we have that

$$\frac{1}{1-F(x_U)}\int_{x_U}^\infty \sqrt{y}\mathrm{d}F(y) < \left(\frac{1}{1-F(x_U)}\int_{x_U}^\infty y\mathrm{d}F(y)\right)^{1/2}.$$

Squaring both sides and multiplying by $(1-F(x_U))/4$, we get

$$\frac{1}{4(1-F(x_U))}\left(\int_{x_U}^\infty \sqrt{y}\mathrm{d}F(y)\right)^2 < \frac{1}{4}\int_{x_U}^\infty y\mathrm{d}F(y).$$

Since we have $1/(2-F(x_U))^2 \leq 1/(4(1-F(x_U)))$ for all $x_U < \infty$, we have that

$$x_U = \frac{1}{(2-F(x_U))^2}\left(\int_{x_U}^\infty \sqrt{y}\mathrm{d}F(y)\right)^2 < \frac{1}{4}\int_{x_U}^\infty y\mathrm{d}F(y) <$$
$$\frac{1}{4}\int_{\underline{y}}^{x_U} y\mathrm{d}F(y) + \frac{1}{4}\int_{x_U}^\infty y\mathrm{d}F(y) = \frac{\tilde{y}}{4} = x^S.$$

$\square$

Proof of Proposition 5. Define the function

$$h_U(x) := \int_{\underline{y}}^x y\mathrm{d}F(y) + x(1-F(x)).$$

We have that

$$\frac{\partial h_U(x)}{\partial x} = 1 - F(x) > 0.$$

Since from Proposition 3 we know that $x_U < \tilde{y}/4$, we then have that

$$\begin{aligned}\mathrm{E}u_U = h_U(x_U) < h_U(\tilde{y}/4) = \int_{\underline{y}}^{\tilde{y}/4} y \mathrm{d}F(y) + \frac{\tilde{y}}{4} - \frac{\tilde{y}}{4}F(\tilde{y}/4) \leq \\ \frac{\tilde{y}}{4}F(\tilde{y}/4) + \frac{\tilde{y}}{4} - \frac{\tilde{y}}{4}F(\tilde{y}/4) = \frac{\tilde{y}}{4}.\end{aligned}$$

Next define the function

$$h_I(x) = \int_x^{\infty} y \mathrm{d}F(y) - x(3 - F(x)).$$

We have that

$$\frac{\partial h_I(x)}{\partial x} = F(x) - 3 < 0.$$

Hence we have that

$$\begin{aligned}\mathrm{E}u_I = h_I(x_U) > h_I(\tilde{y}/4) = \int_{\tilde{y}/4}^{\infty} y \mathrm{d}F(y) - 3\frac{\tilde{y}}{4} + \frac{\tilde{y}}{4}F(\tilde{y}/4) \geq \\ \int_{\tilde{y}/4}^{\infty} y \mathrm{d}F(y) - 3\frac{\tilde{y}}{4} + \int_{\underline{y}}^{\tilde{y}/4} y \mathrm{d}F(y) = \tilde{y} - \frac{3}{4}\tilde{y} = \frac{\tilde{y}}{4}.\end{aligned}$$

$\square$

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