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ABSTRACT

Rent, Risk, and Replication – Preference Adaptation in Winner-Take-All Markets

by Karl Wärneryd*

We study the evolution of an economy where agents who are heterogeneous with respect to risk attitudes can either earn a certain income or enter a risky rent-seeking contest. We assume that agents behave rationally given their preferences, but that the population distribution of preferences evolves over time in response to material pay-offs. We show that, in particular, initial distributions with full support converge to stationary states where all types may still be present, risk lovers specialize in rent-seeking, and the available rents are perfectly dissipated.

Keywords: Preference evolution, risk attitudes, contests, winner-take-all markets
JEL classification: C72, D72, D80

ZUSAMMENFASSUNG

Rente, Risiko und Replikation – Präferenz-Anpassung in „Der-Sieger-bekommt-alles“ Märkten

Der Autor untersucht die Entwicklung einer Volkswirtschaft, in der sich die Akteure in ihrer Einstellung zu Risiken unterscheiden. Sie können entweder ein bestimmtes Einkommen erlangen oder sich in einen riskanten Rent-Seeking-Wettbewerb (Wettbewerb zum Erlangen einer Rente) begeben. Angenommen wird rationales Verhalten der Akteure bei gegebenen Präferenzen an, wobei sich die Verteilung der Präferenzen innerhalb der Bevölkerung als Antwort auf die materiellen Ergebnisse des Wettbewerbs entwickelt. Es wird gezeigt, daß im einzelnen, die ursprünglichen Verteilungen mit ganzer Unterstützung gegen stationäre Zustände konvergieren, in welchen noch immer alle Typen präsent sein können. Dabei spezialisieren sich risikofreudige Individuen auf Rent-seeking und die erzielbaren Renten sind perfekt gestreut.

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1 Introduction

In their popular book, Frank and Cook (1995) argue that Western economies increasingly have the characteristics of winner-take-all markets. Consider the market for opera singers. Before 20th century advances in recording technology there were local markets for opera singers, allowing also mediocre talents to earn a living. Today anybody anywhere can buy the performances of the world's foremost singers on CD. Hence only the best survive, and all the resources spent on, e.g., singing lessons, by hopefuls in contending for market leadership are wasted from a social point of view. This phenomenon of wasteful competition, more generally, is also known as **rent-seeking** (Tullock 1967; Krueger 1974), and also encompasses such things as patent races, lobbying to obtain a monopoly in a product market, and outright theft.

Frank and Cook seem to suggest that winner-take-all markets are a relatively new but increasingly important type of social interaction. It may be argued, however, that rent-seeking is as old as human social life itself. Consider the incentives for males to hunt in hunter-gatherer societies. Hawkes (1990, 1993) argues that the expected net nutritional benefit of hunting—a risky activity subject to the congestion externalities typical of rent-seeking—is largely negative. Hunting is primarily wasteful display behavior designed to attract female mates. In this paper, we adopt the view that contests of this nature have been a stable feature of human societies for a long time, during which preferences have been the subject of evolutionary selection, genetic and cultural. Since participation in a winner-take-all market is risky, individual attitudes toward risk are of central importance. Given that evolutionary pressures operate on the distribution of risk attitudes, we ask, among other things, whether evolution is likely to promote risk-taking and hence the social wastefulness pointed to by Frank and Cook.

Economic theories traditionally start with a set of agents with given preferences confronted by an interactive decision problem. When studying the evolution of an economy in the long run, however, it seems reasonable to think that preferences may change—for instance, because the set of agents participating in the economy changes. If the material reward structure defined by the particular decision problem changes at a slower rate than the distribution of preferences in the economy, it seems likely that the latter would adapt to the conditions defined by the former.

In this paper we shall assume that each individual acts rationally given his preferences and an interaction situation defined in terms of objective, material payoffs. But different individuals may have different preferences, and we shall assume that the population distribution of preference types evolves in the direction of locally higher material payoffs.

In the context of decision-making under uncertainty, naïve intuition perhaps suggests that preference evolution would weed out every type except the risk neutral one, since the risk neutral agent maximizes expected material payoffs and therefore evolutionary fitness. To see that this is not so, consider a situation where agents can choose between two activities, one safe and one risky. The risk neutral agent will choose the risky activity when its expected material payoff is higher. But then so will all risk lovers. They will receive the same objective payoff, and thus have the same evolutionary fitness, as the risk neutrals. Conversely, if a risk neutral agent chooses the safe activity, then so will all risk averse agents. If equilibrium payoffs are dependent on the relative frequencies of various types in the population the superiority of risk neutrality becomes even less obvious.

The main argument of this paper is that if the population distribution of risk attitudes evolves over time in the direction of locally higher material payoffs, then in the long run it will adjust to an equilibrium where rents are exactly dissipated. Thus any type of risk attitude may survive in the long run, but typically the stationary state reached will be associated with an in-period equilibrium that is in effect the same as if everybody were risk neutral.

The intuition behind the stationarity argument is the following. Consider a process of natural selection of preference types that makes types that are locally more successful in material terms grow in relative number. At a stationary state of such a system, that is, one where the relative numbers of different types do not change, the material payoffs from all activities undertaken by members of the population must be equal. Now say there are two available activities, one that yields a certain payoff and another that is risky. Suppose there is a stationary state such that both activities are undertaken. Then, since the equilibrium expected payoffs from the two activities are equal at the stationary state, it is necessarily the case that only risk neutrals and risk lovers would be engaged in the risky activity. In the rent-seeking economy, rents are perfectly dissipated since the payoffs from the two activities are equalized. It remains for us to show that the economy would in fact converge to such a stationary state.

The rest of the paper is organized as follows. Section 2 introduces a static model of an economy where agents, who may have different attitudes to risk, choose between, on the one hand, a certain income and, on the other, participation in a risky rent-seeking contest, where the expected value of participation is dependent on the number of others who also enter. The model is solved for a unique Nash equilibrium.

In Section 3 we study the dynamic case. The static equilibrium derived previously allows us to determine the material payoffs to every risk attitude type at any point in time. We assume that the replicator dynamics operates on the preference distribution based on these material rewards, and show that if the starting distribution is extreme in the sense that it lacks risk neutrals and either risk lovers or risk averse agents, in the long run the system will converge to a stationary state where only the type closest to risk neutrality in the starting distribution has survived. If the initial distribution is not extreme in this sense, the system converges to a stationary state where all types present at the beginning may have survived. In the static equilibrium associated with this stationary state rents are perfectly dissipated in the rent-seeking contest.

In dealing with preference evolution, this paper is related to contributions by Bester and Güth (1998), Cooper (1987), Dekel and Scotchmer (1999), Karni and Schmeidler (1986), Robson (1996a, 1996b), Rogers (1994), Rubin and Paul (1979), To (1999), and Waldman (1994). Some of these connections are explored in Section 4. Finally, Section 5 summarizes the argument and concludes the paper.

2 Static Equilibrium

We study an economy with two activities. The activity S(afe) yields the individual a certain payoff of $w > 0$ units of money, which may be thought of as the agent's initial wealth. The activity R(isky) yields a material payoff of $r > w$ with probability p and nothing with probability $1-p$. If an individual enters the risky activity he foregoes the certain payoff of w, which may therefore be viewed as his investment. Let $x_S \in [0, 1]$ be the population proportion of agents who choose the safe action, and $x_R := 1 - x_S$ the proportion who choose the risky investment. We assume the probability of getting a positive payoff from the risky activity declines in the total population proportion of agents who enter the activity according to the specification

$$p(x_R) := \begin{cases} 1 & \text{if } x_R \leq \rho \\ \rho / x_R & \text{otherwise,} \end{cases}$$

where $\rho \in (0, 1)$ is an exogenously given parameter. We interpret the risky activity as participation in a rent-seeking or winners-take-all contest with free entry, where ρ is the measure of contestants that can walk away with prizes. Think of ρ as the analogue in population measure terms of an integer number of prizes in a room. If fewer than ρ individuals enter the room, every person who enters gets a prize. If more than ρ people enter the room, some rationing of prizes must take place. If the individuals are identical with respect to arrival times, physical strength, etc, this discrimination must be essentially random, so that the probability of the single individual ending up with a prize is equal to the ratio of available prizes to the number of people entering the room.

The following assumption, which is maintained throughout, will turn out to guarantee the existence of an interior equilibrium, in the sense of an equilibrium where both activities are undertaken.

Assumption 1 We have $w/r > \sqrt{\rho}$.

Observe that this implies $w/r > \rho$.

Participation in the risky contest is socially wasteful in material terms if more is invested in aggregate than what the total of available prizes is worth. This simple model thus captures the essence of many more specific discussions of productive versus unproductive activities. In addition to the examples discussed previously, consider the potentially wasteful influence activities in organizations studied by, e.g., Milgrom (1988) and Milgrom and Roberts (1988, 1990). Within an organization, the individual may concentrate either on performing his currently allotted tasks, or spending time on political activities to get a promotion.

Let $x_R^\star$ be the equilibrium proportion of participants in the rent-seeking contest. If all agents were risk neutral, enough would enter to make the expected material payoffs from the two activities equal in an interior equilibrium. If we have $w/r > \rho$ there is a unique interior equilibrium such that

$$w = p(x_R^\star)r,$$

or, equivalently, $x_R^\star w = \rho r$. Since $x_R w$ is the per capita amount foregone by rent-seekers, that is, invested into the rent-seeking activity, and ρr is the per capita potential rent, this means the rents are perfectly dissipated in equilibrium. In general, let $\delta := x_R w/(\rho r)$ be the **dissipation rate**, with $\delta^\star = x_R^\star w/(\rho r)$ its equilibrium value. We say that rents are **underdissipated** if $\delta < 1$, **perfectly dissipated** if $\delta = 1$, and **superdissipated** if $\delta > 1$.

This definition of rent dissipation is the standard one in the literature. It is rather irrelevant from a welfare perspective, however, when individuals are other than risk neutral. A correct measure of rent dissipation in this case should take into account the costs and benefits of bearing risk. In general, the material dissipation rate defined above and a risk-adjusted one will diverge. We return to this issue below.

We turn now to the case where individuals are not necessarily risk neutral. We assume the economy has an uncountable infinity of individual agents, each of whom has measure zero, with a total mass of 1. We shall assume that all individuals have constant relative Arrow-Pratt risk aversion. This implies that an agent of type α may be taken to have the expected utility function

$$u_\alpha(m) := m^{1-\alpha}.$$

We have that $\alpha < 0$ corresponds to risk love, $\alpha = 0$ to risk neutrality, and $\alpha > 0$ to risk aversion.[1] For convenience, we restrict α to values in the interval $A := (-1, 1)$.

Suppose everybody had the same risk attitude value $\hat{\alpha}$. In equilibrium, enough would enter the risky activity to make the expected utilities from the two activities equal. That is, we would have that

$$w^{1-\hat{\alpha}} = p(x_R^\star) r^{1-\hat{\alpha}},$$

which implies that

$$\delta^\star = \left(\frac{w}{r}\right)^{\hat{\alpha}}.$$

We therefore have that

$$\delta^\star \lesseqgtr 1 \text{ as } \hat{\alpha} \gtreqless 0.$$

In particular, we note that the equilibrium rate of rent dissipation when everybody is of the same type is independent of ρ. Furthermore, this case

[1] Alternatively, we could eschew the use of explicit utility functions and order the individuals according to the p values that would make them indifferent between the two activities. This approach has the disadvantage of not allowing any definition of global risk neutrality.

illustrates the divergence between the rate of dissipation in material terms and a risk-adjusted one which evaluates the risky prospect at its certainty equivalent. Since in equilibrium each individual is indifferent between the two activities, the risk-adjusted rate of dissipation must always be equal to one if all individuals are of the same type.

In the most general case, different individuals may have different risk attitudes. We assume the risk attitude parameter is distributed in the population according to the right continuous, nondecreasing distribution function $F: \mathbb{R} \to [0, 1]$. The function value $F(\alpha)$ is then the population proportion of individuals whose types are α or less. More generally, if $E \subset \mathbb{R}$ is an arbitrary set, we write $F\{E\}$ for the mass on E.

We allow the population distribution of the risk attitude parameter to be discontinuous because under the evolutionary dynamic to be imposed later even a continuous starting distribution may eventually evolve into a discontinuous distribution. Since we shall track the evolution of static equilibria, we must be able to define equilibrium also given such discontinuous distributions.

Since F is monotonic, it has countably many mass points or **atoms**, corresponding to its discontinuities. We assume that F has no mass outside of the interval A, so that we have $F(\alpha) = 0$ for $\alpha \leq -1$ and $F(\alpha) = 1$ for $\alpha \geq 1$. We define $\underline{\alpha} := \sup\{\alpha: F(\alpha) = 0\}$ and $\overline{\alpha} := \min\{\alpha: F(\alpha) = 1\}$, the lower and upper bounds, respectively, of the interval on which F is concentrated. We shall also make use of the function $F(\alpha-) := \lim_{\alpha' \uparrow \alpha} F(\alpha')$, the left hand limit of F at α. If $F(\alpha-) < F(\alpha)$, then α is an atom of F. If for any open interval E containing a point α we have $F\{E\} > 0$, then α is said to be a **point of increase** of F. Denote by $S(F)$ the set of points of increase of F, called the **support** of F. The distribution F may or may not possess an associated density. (In other words, F behaves like a probability distribution. See, e.g., Feller 1971 for details.)

Consider now the decision problem of an individual of type α. Clearly, he will prefer to keep his certain income if

$$w^{1-\alpha} > p(x_R)r^{1-\alpha}.$$

Since there is no type that would prefer the safe activity if nobody entered the risky activity, this condition is equivalent to

$$\alpha > \frac{\log(w/(rp(x_R)))}{\log w/r}.$$

The right-hand side of this expression is a number independent of the agent's decision, since his decision has no measurable effect on x_R. It defines a threshold value for the risk aversion parameter such that all agents with α-values below the threshold would like to engage in the risky activity, and all agents with α-values above the threshold would like to keep their certain income. Individuals of exactly the threshold type are indifferent, and may therefore rationally undertake either activity. We stress that, in particular, it is not necessarily the case that all risk averse stay out of the contest and all risk lovers enter.

Suppose everybody enters the rent-seeking contest. In order to ensure that both activities are always undertaken in equilibrium, we shall assume that there is no type that would prefer the risky activity if everybody enters. A sufficient condition for this to hold is that

$$\frac{\log(w/(\rho r))}{\log w/r} < -1,$$

which is equivalent to Assumption 1.

An equilibrium is a situation such that all individuals are acting rationally. If F is continuous, the equilibrium measure of rent-seekers is the fixpoint $x_R^\star$ such that

$$x_R^\star = F\left(\frac{\log(w/(rp(x_R^\star)))}{\log w/r}\right).$$

We now generalize this idea to the case where F may be discontinuous. Let

$$\bar{F}(\alpha) := [F(\alpha-), F(\alpha)].$$

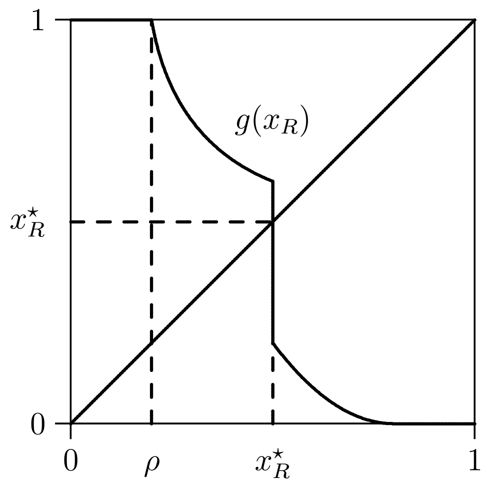

Figure 1: Rent-seeking equilibrium.

The correspondence $\bar{F}$ is singleton-valued everywhere except at the atoms of F, where it fills in the holes in the distribution. Now define

$$g(x_R) := \bar{F}\left(\frac{\log(w/(rp(x_R)))}{\log w/r}\right).$$

The equilibrium proportion of entrants into the rent-seeking contest is then a fixpoint of g, that is, we have that $x_R^\star$ is defined by the inclusion

$$x_R^\star \in g(x_R^\star).$$

Since g is a closed correspondence from the unit interval into itself, with nonempty convex and compact values, by Kakutani's fixpoint theorem it has a fixpoint. Furthermore, the fixpoint can easily be seen to be interior in the sense that $\rho < x_R^\star < 1$ (remember Assumption 1, which guarantees that $g(1) = \{0\}$). Thus both activities will always be undertaken in equilibrium, regardless of what the population distribution F looks like. Finally, the equilibrium must be unique since g is nonincreasing in the sense that $\max g(x'_R) \leq \min g(x_R)$ for $x'_R > x_R$. Figure 1 shows an example situation.

We summarize these observations in the following Proposition.

Proposition 1 Under Assumption 1 there is a unique equilibrium measure of rent-seekers $x_R^\star \in (\rho, 1)$.

The equilibrium proportion of rent-seekers uniquely determines the equilibrium threshold type

$$\alpha^\star = \frac{\log((w/(\rho r))x_R^\star)}{\log w/r}.$$

In case $\alpha^\star$ is an atom of F, some $\alpha^\star$-type individuals may have to do one thing and others the other. For completeness, we must specify exactly how they divide themselves in equilibrium. Let $k^\star$ be the proportion of $\alpha^\star$-types who enter the rent-seeking contest, with

$$k^\star = \begin{cases} \dfrac{x_R^\star - F(\alpha^\star -)}{F(\alpha^\star) - F(\alpha^\star -)} & \text{if } F(\alpha^\star) - F(\alpha^\star -) > 0 \\ 1 & \text{otherwise.} \end{cases}$$

That is, if F is continuous at $\alpha^\star$ we assume, with no loss of generality, that all $\alpha^\star$-types rent seek.

The degree of rent dissipation in equilibrium is determined by the proportion of risk lovers and risk neutrals in the population.

Proposition 2 In equilibrium,

1. if the proportion of risk lovers is greater than $\rho r/w$, then some risk lovers stay out of the contest and there is superdissipation of rents,

2. if the proportion of risk lovers is less than or equal to $\rho r/w$ and the proportion of risk lovers and risk neutrals is greater than or equal to $\rho r/w$, then all risk lovers enter, all risk averse stay out, and there is perfect dissipation, and

3. if the proportion of risk lovers and risk neutrals is less than $\rho r/w$, then some risk averse agents enter the contest and there is underdissipation.

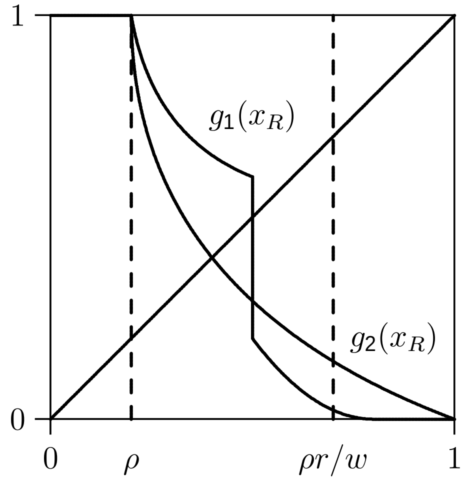

Figure 2: A counterexample.

Proof. First note that the proportion of risk lovers in the population is $F(0-)$ and the proportion of risk lovers and risk neutrals is $F(0)$. We also have $F(0-) = \min g(\rho r/w)$ and $F(0) = \max g(\rho r/w)$. Suppose we have $F(0-) > \rho r/w$. Since $\min g(x_R)$ is nonincreasing in x_R, we must have $\min g(x_R) > \rho r/w$ for all $x_R \leq \rho r/w$. It follows that $x_R^\star > \rho r/w$, $\delta^\star > 1$, and $\alpha^\star < 0$. Conversely if we have $F(0) < \rho r/w$. Finally, if we have $\rho r/w \in [F(0-), F(0)] = g(\rho r/w)$, we must have $x_R^\star = \rho r/w$, $\delta^\star = 1$, and $\alpha^\star = 0$. 2

It is **not** the case, however, that a distribution with more risk lovers necessarily has a higher proportion of rent-seekers in equilibrium than a distribution with fewer risk lovers. Figure 2 shows a counterexample. The distribution corresponding to g_2 has a higher proportion of risk lovers than the distribution corresponding to g_1 (i.e., we have $\min g_2(\rho r/w) > \min g_1(\rho r/w)$), but the fixpoint of g_2 is lower than the fixpoint of g_1.

The intuition for these results is the following. If there are "too few" risk lovers and risk neutrals, they can all enter without exhausting the rent and making the net expected value of rent-seeking negative. Therefore also some

w/r	ρ							
	.1	.2	.3	.4	.5	.6	.7	.8
.4	-.325963							
.5	-.452599	-.126616						
.6	-.556963	-.244605	$7.34845\,10^{-17}$					
.7	-.640905	-.352106	-.108907	.102019				
.8	-.707261	-.447494	-.213419	$9.0768\,10^{-17}$	.196401	.37851		
.9	-.75927	-.53003	-.311118	-.10155	.0995213	.292825	.478999	.6586

Table 1: Values of $\alpha^\star$ under the uniform distribution.

w/r	ρ							
	.1	.2	.3	.4	.5	.6	.7	.8
.4	1.34807							
.5	1.3685	1.09173						
.6	1.32911	1.13309	1.					
.7	1.25683	1.13381	1.03961	.964267				
.8	1.17096	1.10501	1.04878	1.	.957121	.919007		
.9	1.08328	1.05743	1.03332	1.01076	.989569	.969619	.950785	.932962

Table 2: Dissipation rates under the uniform distribution.

risk averse types will want to enter. How many of these latter will enter the rent-seeking contest is determined by the exact nature of the population distribution, however. For instance, if the risk averse individuals are predominantly `very` risk averse, then relatively few of them will enter. Conversely if there are "too many" risk lovers and risk neutrals. Therefore the equilibrium measure of rent-seekers is not a function simply of the proportion of risk lovers and risk neutrals.

Although we cannot solve for $\alpha^\star$ analytically, we may use numerical methods to compute it for selected parameter values and distributions. Table 1 shows fixpoint values when F is assumed to have the uniform density on A.[2]

Table 2 shows the corresponding dissipation rates. We note that while

[2] The `FindRoot` function of `Mathematica` by Wolfram Research, Inc, was used to compute this example. Any use of numerical methods necessarily involves approximation. One may readily verify that for $(w/r, \rho)$ equal to $(.6, .3)$ or $(.8, .4)$, $\alpha^?$ must actually be exactly equal to 0.

the fixpoint values range quite widely, the dissipation rates are always close to or equal to one.

As noted above, measuring the rate of dissipation in material terms is welfare-irrelevant when individuals have different attitudes toward risk. A better measure takes into account, e.g., that some individuals get utility from risk-taking in itself. To adjust the dissipation rate for risk, note that the certainty equivalent of the risky activity to an individual of type α is equal to $p(x_R)^{1/(1-\alpha)}r$. In equilibrium, we may thus measure the risk-adjusted rate of dissipation as the ratio of *per capita* investment to the *per capita* certainty equivalent of rent-seeking of the rent-seekers, that is, as

$$\hat{\delta}^\star := x_R^\star w \left(r \left(\frac{\rho}{x_R^\star} \right)^{1/(1-\alpha^?)} (x_R^\star - F(\alpha^\star -)) + \lim_{\alpha' \uparrow \alpha^?} \int_{-1}^{\alpha'} r \left(\frac{\rho}{x_R^\star} \right)^{1/(1-\alpha)} \mathrm{d}F(\alpha) \right)^{-1}. \tag{1}$$

We note that the risk-adjusted equilibrium rate of dissipation cannot be greater than what it would be if all rent-seekers were of type $\alpha^\star$, nor less than what it would be if all rent-seekers were of type $\underline{\alpha}$. That is, we have that

$$\frac{x_R^\star w}{x_R^\star r (\rho / x_R^\star)^{1/(1-\underline{\alpha})}} \le \hat{\delta}^\star \le \frac{x_R^\star w}{x_R^\star r (\rho / x_R^\star)^{1/(1-\alpha^?)}}. \tag{2}$$

Clearly, when all rent-seekers are risk lovers, the risk-adjusted rate of dissipation is bounded above by a number strictly less than the material rate of dissipation. We also know that if all individuals are risk averse, the risk-adjusted rate of dissipation is bounded below by a number strictly greater than the material rate of dissipation. The remaining cases are ambiguous. We thus have the following observation.

Proposition 3 Suppose we have $\alpha^\star < 0$. We then have $\hat{\delta}^\star < \delta^\star$; i.e., the material dissipation rate overestimates rent dissipation. On the other hand, suppose we have $\underline{\alpha} > 0$, i.e., that all individuals are risk averse. We then have $\hat{\delta}^\star > \delta^\star$; i.e., the material dissipation rate underestimates rent dissipation.

w/r	ρ							
	.1	.2	.3	.4	.5	.6	.7	.8
.4	.835865							
.5	.899058	.832272						
.6	.940053	.893058	.852768					
.7	.966282	.936303	.908557	.882136				
.8	.982912	.966293	.949862	.933367	.916547	.899087		
.9	.993403	.986584	.979476	.971996	.96403	.955413	.945886	.934979

Table 3: Risk-adjusted dissipation rates under the uniform distribution.

In case F has a density f, the risk-adjusted equilibrium dissipation rate reduces to

$$\hat{\delta}^{\star} = x_R^{\star} w \left(\int_{-1}^{\alpha^?} r \left(\frac{\rho}{x_R^{\star}} \right)^{1/(1-\alpha)} f(\alpha) d\alpha \right)^{-1} .$$

Table 3 shows the risk-adjusted rates of dissipation when F is the uniform distribution. We note that while they are also always close to one, they are always less than one, and furthermore less than the material dissipation rate except for one case. Since in each case a majority of the individuals who enter the risky activity are risk lovers, who derive utility from risk taking, the material dissipation measure typically overestimates the degree of dissipation.

3 Evolution

The literature on evolutionary game theory (see, e.g., Mailath 1998 for a survey) typically assumes that individuals are genetically or culturally programmed with certain behaviors or strategies. The population representation of a strategy then evolves in response to the payoffs it generates at a moment in time. A commonly used model of this process is the **replicator dynamics** (see, e.g., Maynard Smith 1982, Hofbauer and Sigmund 1988, or Weibull 1995), a model of asexual genetic reproduction.

In contrast, we shall assume that individual agents are carriers of preferences, rather than behaviors, and that they behave rationally given their preferences. Preferences, in turn, are assumed to evolve according to an adapted version of the replicator dynamics, with the material payoffs generated at a moment in time our measure of the evolutionary fitness of a preference type.[3]

It is implicit, of course, in this approach that we assume the rent-seeking game is the only stable situation relevant for the evolution of risk attitudes. That is, although individuals might face other games where risk attitude comes into play, we assume these are of negligible importance. Broadly, such situations fall into two categories: One where the probability distribution of payoffs is dependent on how many individuals pursue a given activity, and one where it is not. Our analysis is intended to cover the former case of frequency-dependent risk. The second case has no strategic aspects.

There are infinitely many agents of every measurable type, and we assume the stochastic trials are independent. We therefore invoke the law of large numbers and treat expected payoffs as actual average payoffs.[4] Let $\mu_t(\alpha)$ be the average payoff to individuals of type α at time t. Then

$$m_t\{E\} := \begin{cases} (1/F_t\{E\}) \int_E \mu_t(\xi) \mathrm{d}F_t(\xi) & \text{if } F_t\{E\} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

is the average fitness at time t of individuals whose types lie in the interval $E \subset A$ and $\bar{m}_t := m_t(1)$ average fitness in the population as a whole. In particular, we write

$$m_t(\alpha) := \begin{cases} (1/F_t(\alpha)) \int_{-1}^{\alpha} \mu_t(\xi) \mathrm{d}F_t(\xi) & \text{if } \alpha > \underline{\alpha} \\ 0 & \text{otherwise.} \end{cases}$$

[3] A referee points out that if material payoffs are thought of as, e.g., money, then the dependence of reproductive success on monetary income may be highly nonlinear. The main purpose here, however, is to distinguish between subjective utilities, which seem unlikely to be the direct basis for any form of evolutionary selection, and evolutionary fitness. Hence one can equivalently think of the objective payoffs as being specified directly in terms of offspring, biological or cultural.

[4] Although it is a common practice, it is well known that doing so with a continuum of individuals is not without problems. See, e.g., Judd (1985) or Boylan (1992).

for the average fitness at time t of individuals whose types are α or less.

The natural extension of the replicator dynamics to the case of a continuous trait is to require that every subinterval of types grow in mass proportionally to the relative average payoff of types in the interval, i.e., that we have that

$$F_{t+1}\{E\} = m_t\{E\}F_t\{E\}/\bar{m}_t \text{ for all measurable } E \subset A.$$

This is easily seen to be equivalent to the dynamics

$$F_{t+1}(\alpha) = \theta(F_t)(\alpha) := m_t(\alpha)F_t(\alpha)/\bar{m}_t \text{ for all } \alpha \in A.$$

This defines a discrete dynamical system on the set $\mathcal{F}$ of right-continuous population distributions on A.[5]

Now let

$$m_{St} := w$$

and

$$m_{Rt} := \rho r/x^\star_{Rt}.$$

We assume that in each period, the economy converges to the unique static equilibrium described in Section 2. This equilibrium determines the fitness of each type, which in turn determines the next-period distribution of types. Since in the static equilibrium we have that

$$\mu_t(\alpha) = \begin{cases} m_{Rt} & \text{if } \alpha < \alpha^\star_t \\ k^\star m_{Rt} + (1-k^\star)m_{St} & \text{if } \alpha = \alpha^\star_t \\ m_{St} & \text{if } \alpha > \alpha^\star_t, \end{cases}$$

the replicator dynamics may be written

$$F_{t+1}(\alpha) = \begin{cases} m_{Rt}F_t(\alpha)/\bar{m}_t & \text{if } \alpha < \alpha^\star_t \\ (m_{Rt}x^\star_{Rt} + m_{St}(F_t(\alpha) - x^\star_{Rt}))/\bar{m}_t & \text{otherwise,} \end{cases}$$

[5]For a discussion of the continuous-time replicator dynamics with a continuous trait, see Oechssler and Riedel (2001).

where $\bar{m}_t = x^\star_{Rt} m_{Rt} + (1 - x^\star_{Rt}) m_{St}$.

It is easy to see that F_t remains in $\mathcal{F}$ under the evolutionary dynamic θ. A stationary state is a distribution $F^\star$ such that $\theta(F^\star) = F^\star$. There are thus two types of stationary states. Either $F^\star$ has all its mass concentrated on a unique atom, or the fitnesses resulting from the two activities are equal.

Proposition 4 Let $F^\star$ be a stationary distribution under θ. Let $m^\star_R$ and $m^\star_S$ be the equilibrium fitnesses associated with $F^\star$. Then we either have $F^\star(\hat{\alpha}) = 1$ and $F^\star(\hat{\alpha}-) = 0$ for some $\hat{\alpha}$, or $m^\star_R = m^\star_S$.

(See the Appendix for a proof.)

Definition 1 We say a distribution F is *extreme* if either $\overline{\alpha}_0 < 0$ (in which case it is *left extreme*) or $\underline{\alpha}_0 > 0$ (in which case it is *right extreme*).

An extreme distribution is one where either only risk lovers or only risk averse have positive measure. Thus the admittedly immoderate distribution where only risk neutrals have positive mass is not considered extreme.

Proposition 5 Every left extreme (right extreme) initial distribution converges to a stationary distribution that has all its mass concentrated on the unique atom $\overline{\alpha}_0$ ($\underline{\alpha}_0$).

(See the Appendix for a proof.)

A different way of expressing the result is to say that an extreme starting distribution evolves to one where only the type closest to risk neutrality has survived. It follows that the long-run dissipation rate when the initial distribution is left extreme (right extreme) is equal to $(w/r)^{\overline{\alpha}_0} > 1$ ($(w/r)^{\underline{\alpha}_0} < 1$). The long-run risk adjusted dissipation rate in this case is equal to 1, since in the long run all individuals of the surviving type are indifferent between the two activities.

The replicator dynamics operating on an extreme starting distribution is unlikely, however, to be a good model of preference evolution, as it never allows the later introduction of types that were not present at the beginning. Both biological and cultural evolution are likely to be subject to shocks or mutations. In the case of cultural evolution, one source of mutation may be experimentation where individuals randomly assume attitudes that have not been tried before. A reasonable way to control for the effects of mutation without modelling it explicitly is to study what happens to initial distributions that have full support, or at least where not everybody is either risk averse or risk loving. This guarantees that both these broad categories of preferences are allowed to have an effect.

Proposition 6 Every nonextreme initial distribution converges to a stationary state where the population proportion of rent-seekers is $\rho r/w$, all risk lovers enter the rent-seeking contest, all risk averse stay out, and rents are perfectly dissipated.

(See the Appendix for a proof.)

From the inequality (2) we see that, in contrast with the case of an extreme starting distribution, the long-run risk adjusted dissipation rate is always less than or equal to 1. We also get a lower bound on the long-run risk adjusted dissipation rate, so we know that

$$\sqrt{w/r} \le \lim_{t\to\infty} \hat{\delta}_t^\star \le 1.$$

As an example, consider the case where the starting distribution consists of two atoms, $\underline{\alpha}_0 < 0$ and $\overline{\alpha}_0 > 0$. The long-run distribution will then also consist of these two atoms, with a mass of $\rho r/w$ on $\underline{\alpha}_0$. Therefore the long-run risk adjusted dissipation rate in this special case reduces to $\lim_{t\to\infty} \hat{\delta}_t^\star = (w/r)^{-\underline{\alpha}_0/(1-\underline{\alpha}_0)} < 1$.

Under certain circumstances we may directly find the long-run distribution of risk lovers, and thus an explicit expression for the long-run risk-adjusted dissipation rate.

Proposition 7 Let F_0 be nonextreme and continuous at zero, and suppose we have $\{\lambda\alpha_0^\star : \lambda \in [0,1]\} \cap S(F_0) \cap (-1,0] = \varnothing$. Then the long-run risk-adjusted rate of dissipation is given by

$$\lim_{t\to\infty} \hat{\delta}_t^\star = \left(\int_{-1}^{0} (w/r)^{\alpha/(1-\alpha)} \frac{1}{F_0(0)} \mathrm{d}F_0(\alpha) \right)^{-1}.$$

Proof. We have $F_{t+1}(\alpha) = m_{Rt}F_t(\alpha)/\bar{m}_t$ for all $\alpha \leq 0$ and all t, from which follows that $\lim_{t\to\infty} F_t(\alpha) = (\prod_{t=0}^{\infty} m_{Rt}/\bar{m}_t)F_0(\alpha)$ for $\alpha \leq 0$. Since F_0 is continuous at zero we know from Proposition 6 that $\lim_{t\to\infty} x_{Rt}^\star = \lim_{t\to\infty} F_t(0) = (\prod_{t=0}^{\infty} m_{Rt}/\bar{m}_t)F_0(0) = \rho r/w$, so we must have $\lim_{t\to\infty} F_t(\alpha) = (\rho r/w)(F_0(\alpha)/F_0(0))$ for $\alpha \leq 0$. The Proposition follows by insertion into (1). 2

We note, in particular, that the long-run risk-adjusted rate of dissipation in this case is independent of ρ. The condition on the support of F_0 guarantees that the trajectory of $\alpha_t^\star$ does not pass through regions where risk lovers have positive mass. The reason we require this is that we cannot (at least not easily) characterize the limiting distribution of risk lovers if the evolutionary dynamics requires certain types to both grow and diminish in representation at different times. A sufficient condition for the Proposition to hold is that we have $\alpha_0^\star > 0$.

Table 4 shows long-run risk-adjusted dissipation rates when the initial distribution is the uniform distribution on A. Values for the cases where $\rho r/w < .5$ cannot be computed using this formula, but the numbers in boldface give lower bounds according to (2).

That the set of stationary states is globally attracting does not necessarily mean that a particular stationary state is asymptotically stable. There is no guarantee that if we start at a stationary distribution and perturb it slightly, the system will return to that *particular* stationary state. Proposition 6 only guarantees stability in the face of a particular perturbation, namely, in the direction of the incoming trajectory. We may usefully distinguish, however, between the dynamic stability of a particular stationary distribution and that

w/r	ρ							
	.1	.2	.3	.4	.5	.6	.7	.8
.4	.632456							
.5	.707107	.707107						
.6	.774597	.774597	.852768					
.7	.83666	.83666	.83666	.895225				
.8	.894427	.894427	.894427	.933367	.933367	.933367		
.9	.948683	.948683	.948683	.948683	.968082	.968082	.968082	.968082

Table 4: Long-run risk-adjusted dissipation rates under the uniform initial distribution.

of certain of its aggregate properties. Thus we have trivially already proved that the long-run stationary population proportion of rent-seekers is stable in the following sense.

Corollary 1 Consider a stationary nonextreme distribution, i.e., one where the proportion of rent-seekers is $\rho r/w$. Perturb this distribution in such a fashion that the resulting perturbed distribution is also nonextreme. Then the system will in the long run return to a stationary state where the proportion of rent-seekers is $\rho r/w$.

We note that the permissible perturbations may be quite dramatic. In particular, if we limit attention to the set of nonextreme distributions, which includes the distributions that have full support, then the long-run proportion of rent-seekers is globally stable.

4 Related Literature

The idea that preferences may in the long run be shaped by the decision problem they are applied to is not new to this paper. Early discussions are found, e.g., in the work of Gary Becker. For instance, Becker and Michael (1973) say that

> [P]erhaps that common preference function has evolved over time by natural selection and rational choice as that preference

> function best adopted to human society. That is, in the short run the preference function is fixed and households attempt to maximize the objective function subject to their resource and technology constraints. But in the very long run, perhaps those preferences survive which are most suited to satisfaction given the broad technological constraints of human society (e.g., physical size, mental ability, et cetera).[6]

In a recent series of applications to specific problems (e.g., Güth and Yaari 1992, Bester and Güth 1998, and Güth and Nitzan 1997), Werner Güth terms the idea the **indirect evolutionary approach**. While the approach in these papers is explicitly game-theoretical, which translates into frequency-dependent fitness functions in the evolutionary context, the evolutionary literature on risk attitudes has hitherto taken a slightly different road.

Cooper (1987), Karni and Schmeidler (1986), and Rubin and Paul (1979) are early examples of attempts to derive expected utility maximization and metarational risk attitudes from evolutionary foundations. Karni and Schmeidler show that maximizing the probability of survival in a setting of sequential risky choices may imply expected utility maximization in the von Neumann-Morgenstern sense. Similarly, Cooper derives the Savage axioms of rational choice under uncertainty. Rubin and Paul are concerned with explaining risk-taking behavior among adolescent males. Such behavior may maximize fitness if there is an income threshold below which no females can be attracted. These contributions are all non-game-theoretical in the sense that the fitnesses of different behaviors are assumed independent of their relative representation in the population. It is therefore implicit that only one type of behavior may survive in the long run.

[6]The reference to a **common** preference function is because this discussion occurs in the context of a presentation of the so-called Z-good theory, which involves the idea that individuals have the same ultimate preferences but different technologies for satisfying them.

In a similar vein, Robson (1996a) shows that behavior violating expected utility maximization may win out in the long run if it distinguishes between gambles with idiosyncratic and aggregate risk. The idea here is that the law of large numbers will guarantee the survival of a type that favors a gamble with positive expected payoff and independent trials across individuals, whereas a gamble that has the same expectation but gives the same outcome to all individuals who take it may be associated with a positive probability of extinction of its adherents. Another paper, Robson (1996b), is game-theoretical in the sense that it extends the Rubin and Paul framework in such a way that relative income matters.

Dekel and Scotchmer (1999) continue the tradition of the papers mentioned earlier, but come closest to the present analysis in that they study a winner-take-all game. They study two different setups. In the first, individuals representing gambles are randomly matched in groups, with only the group winner receiving a positive fitness increment. In the second setup, all individuals in the population compete simultaneously. In the latter setting, which has some similarities with that of the present paper, the forces favoring risk-taking behavior are weaker.

All the papers mentioned have in common that they focus on conditions for a behavioral rule (which in these cases is identified with a probability measure on offspring) to be the unique ultimate survivor. In contrast, the present paper shows that different types of attitudes to risk, and the implied diffferent behaviors associated with them, may coexist in the long run.

5 Concluding Remarks

In this paper, we started by studying a simple rent-seeking economy with a population of agents of different risk attitude types. We solved this static model for a unique equilibrium and related the degree of rent dissipation in equilibrium to the population composition of risk attitude types. In doing so, we slightly generalized existing results on risk attitudes and rent dissipation. We also noted, however, that the orthodox notion of rent dissipation, which evaluates the degree of resource waste due to rent-seeking as if all agents were risk neutral, is misleading when different attitudes to risk are present. We therefore also provided a risk-adjusted measure of dissipation.

We then studied the implications of preference evolution based on material payoffs. In contrast with some other contributions to the theory of risk attitude evolution, based on non-game-theoretical approaches, we found that both risk lovers and risk averse types can plausibly coexist in the long run. These broad categories of risk attitude types will specialize, however, in such a fashion that only risk lovers undertake the risky rent-seeking activity in the long run. In general terms, this is an example of how preference evolution may lead to as-if risk neutrality in the sense of an aggregate result that mimics the outcome that results if all agents are risk neutral. From a rent-seeking perspective, it means that rents will be perfectly dissipated in material terms in the long run. We also showed, however, that if the different valuations of the risky prospect of different preference types is taken into account, then the risk-adjusted degree of rent dissipation is always less than perfect in the long run.

A natural question is to what extent the coexistence result is due to the restriction to two activities. Might it not be the case that if there was an even riskier, third alternative, then the most risk-loving types would gamble away their evolutionary prospects on this activity? We shall be satisfied here with the observation that if the levels of risk associated with different activities are endogenous, as in the present paper, then it may still be the case that there is an interior equilibrium that equalizes expected material payoffs. The population distributions associated with this equilibrium are then stationary. Thus coexistence is not simply an artifact of having just two activities. The full investigation of these possibilities must await future research.

Appendix

Proof of Proposition 4. Consider first the case where there is $\hat{\alpha}$ such that $F^\star(\hat{\alpha}) = 1$ and $F^\star(\hat{\alpha}-) = 0$. Then clearly we have $\mu^\star(\hat{\alpha}) = \bar{m}^\star$, so we have $\theta(F^\star)(\alpha) = 1$ for all $\alpha \geq \hat{\alpha}$ and $\theta(F^\star)(\alpha) = 0$ for all $\alpha < \hat{\alpha}$. Thus $F^\star$ is stationary. Consider next the case where $F^\star$ puts positive mass on more than one type, and suppose we have $m_R^\star \neq m_S^\star$. Then there is at least one type such that all agents of that type choose the same activity. Call this type $\hat{\alpha}$, and assume without loss of generality that the activity chosen is R. Then we have $\theta(F^\star)(\hat{\alpha}) = m_R^\star F^\star(\hat{\alpha})/\bar{m}^\star \neq F^\star(\hat{\alpha})$, so $F^\star$ is not stationary. 2.

Proof of Proposition 5. We prove this for the case of a left extreme initial distribution F_0. The right extreme case is, of course, entirely analogous.

We have $\alpha_0^\star \leq \overline{\alpha}_0 < 0$. Furthermore, it must hold that $\alpha_t^\star \leq \overline{\alpha}_t \leq \overline{\alpha}_0 < 0$ for all $t > 0$, so that $m_{Rt} < m_{St}$ for all $t > 0$. Consider now $F_{t+1}(\alpha)$ for any $\alpha < \overline{\alpha}_0$ and any $t > 0$. If we have $\alpha < \alpha_t^\star$, then we have either $F_{t+1}(\alpha) = F_t(\alpha) = 0$ or $F_{t+1}(\alpha) = m_{Rt}F_t(\alpha)/\bar{m}_t < F_t(\alpha)$. If we have $\overline{\alpha}_0 > \alpha \geq \alpha_t^\star$, then $F_{t+1}(\alpha) = (m_{Rt}x_{Rt}^\star + m_{St}(F_t(\alpha) - x_{Rt}^\star))/\bar{m}_t < F_t(\alpha)$. Therefore we must have $\lim_{t\to\infty} F_t(\alpha) = 0$ for all $\alpha < \overline{\alpha}_0$, and clearly $\lim_{t\to\infty} F_t(\overline{\alpha}_0) = 1$. 2

Proof of Proposition 6. The major part of this proof consists in showing that $\lim_{t\to\infty} \alpha_t^\star = 0$, or, equivalently, that $\lim_{t\to\infty} x_{Rt}^\star = \rho r/w$. If we have $\alpha_0^\star = 0$, we are done. Suppose we have $\alpha_0^\star < 0$, which implies $x_{R0}^\star > \rho r/w$ and $m_{S0} > m_{R0}$. We have $\max g_{t+1}(x_{Rt}^\star) = F_{t+1}(\alpha_t^\star) = (m_{Rt}x_{Rt}^\star + m_{St}(F_t(\alpha_t^\star) - x_{Rt}^\star))/\bar{m}_t < F_t(\alpha_t^\star)$ for all t such that $\alpha_t^\star < 0$. For any t with $\alpha_t^\star < 0$, there must then exist $n > 0$ such that $\max g_{t+n}(x_{Rt}^\star) < x_{Rt}^\star$, which implies $x_{Rt+n}^\star < x_{Rt}^\star$. That is, though $x_{Rt}^\star$ may stay the same for periods, it must eventually fall as long as we have $x_{Rt}^\star > \rho r/w$.

We next show that there cannot be equilibrium overshooting; that is, we cannot have $x_{Rt+1}^\star < \rho r/w$ and $x_{Rt}^\star > \rho r/w$ at any t. For suppose this was the case. Since we have $\alpha_t^\star < 0$, we have that

$$F_{t+1}(0) = \frac{m_{Rt} - m_{St}}{\bar{m}_t} x_{Rt}^\star + \frac{m_{St}}{\bar{m}_t} F_t(0) = 1 + \frac{m_{St}}{\bar{m}_t}(F_t(0) - 1).$$

We now note that $x^\star_{Rt+1} < \rho r/w$ implies $F_{t+1}(0) < \rho r/w$, which in turn implies that

$$\frac{m_{St}}{\bar{m}_t}(F_t(0) - 1) < \frac{\rho r}{w} - 1.$$

But since by assumption we have $x^\star_{Rt} > \rho r/w$, which implies $F_t(0) \geq F_t(0-) > \rho r/w$ and $m_{St} > m_{Rt}$, the left hand side of this expression is strictly greater than the right hand side. We thus have a contradiction.

Therefore we must have $\lim_{t\to\infty} x^\star_{Rt} = \rho r/w$. The case where $\alpha^\star_0 > 0$ is completely analogous. 2

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