

## Good jobs, bad jobs and redistribution

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**Good Jobs, Bad Jobs and Redistribution**

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## ABSTRACT

### **Good Jobs, Bad Jobs and Redistribution**

by Kjell Erik Lommerud, Bjørn Sandvik and Odd Rune Straume\*

We analyse the question of optimal taxation in a dual economy, when the government is concerned about the distribution of labour income. Income inequality is caused by the presence of sunk capital investments, which creates a ‘good jobs’ sector due to the capture of quasi-rents by trade unions. We find that whether the government should subsidise or tax investments is crucially dependent on union bargaining strength. If unions are weak, the optimal tax policy implies a combination of investment taxes and progressive income taxation. On the other hand, if unions are strong, we find that the best option for the government is to use investment subsidies in combination with either progressive or proportional taxation, the latter being the optimal policy if the government is not too concerned about inequality and if the cost of income taxation is sufficiently high.

*Keywords:* Rent sharing, segmented labour markets, optimal taxation, redistribution  
*JEL classification:* H2, J42, J51

## ZUSAMMENFASSUNG

### **Gute Arbeitsplätze, schlechte Arbeitsplätze und Umverteilung**

In dem Beitrag wird die Frage der optimalen Besteuerung in einer Wirtschaft mit zwei Sektoren untersucht, in der die Regierung an der Verteilung des Arbeitseinkommens interessiert ist. Einkommensungleichheit wird dann durch versunkene Kapitalinvestitionen verursacht, die einen Sektor mit „guten“ Arbeitsplätzen schaffen, der durch die Anreignung von Quasi-Renten durch Gewerkschaften entsteht. Ob die Regierung Investitionen subventionieren oder besteuern soll, hängt entscheidend von der Verhandlungsstärke der Gewerkschaften ab. Wenn die Gewerkschaften schwach sind, dann sieht die optimale Steuerpolitik eine Kombination aus Investitionssteuern und progressiver Einkommensbesteuerung vor. Im Falle von starken Gewerkschaften zeigt sich, daß die beste Handlungsalternative für die Regierung aus dem Einsatz von Subventionen in Kombination mit entweder progressiver oder proportionaler Besteuerung besteht, wobei die letztere dann optimal ist, wenn die Regierung nicht zu sehr an Ungleichheit interessiert ist und wenn die Kosten der Einkommensbesteuerung hinreichend hoch sind.

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# 1 Introduction

This paper concerns itself with redistribution policy specifically and optimal taxation more generally in a dual labour market setting. We portray the economy as divided in a primary 'good jobs' sector and a secondary sector with 'bad jobs'. The good jobs sector is characterised by sunk capital investments and by the fact that labour, by forming a trade union or otherwise, manages to capture parts of the quasi-rents that are generated. The good jobs sector is 'good' because wages are higher there, and all workers would prefer a good job if they could only get one. In turn, the fact that labour captures quasi-rents will typically lead to underinvestment and a too small primary sector, as in Grout (1984) and Manning (1987).

There are several options available for a government that cares about distribution. Should the government try to tax away some of the income of the primary sector workers and redistribute towards secondary sector ones? Should the government instead seek to tax away the quasi-rent from the primary sector directly through an investment tax? Or by using a profit tax? Or should the government try to expand the primary sector by subsidising investment, so that more workers can enjoy high primary sector wages? The attempt to disentangle questions as these is the core contents of the current work.

The literature on optimal redistributive taxation (e.g. Dixit and Sandmo (1977), Sandmo (1983, 1998) and Parker (1999)) discusses redistribution in a setting with *competitive* labour markets. Here a high income is typically the result of high ability, the relevant trade-off is between more redistribution and distorted labour supply incentives. As Sandmo (1998) stresses, without distributional concerns it is difficult even to give a welfare-theoretic justification for the use of distortive income taxation, as uniform lump-sum taxes then could be used.

There is also a large literature on trade unions, wage bargaining and taxation.<sup>1</sup> A key focus in many of these papers is on the effect of taxation on wage determination and employment in various model formats. Both theoretically and empirically, results appear ambiguous. Dual labour markets and redistributive considerations are not treated. We know of only a few papers that study unions and tax policy in two-

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<sup>1</sup>Some early papers are Oswald (1982), Layard (1982), Hersoug (1984), Malcolmson and Sartor (1987), Lockwood (1990) and Lockwood and Manning (1993). More recent work include Altenburg et al. (2001), Aronsson et al. (2002), Brunello and Sonedda (2002), Fuest and Huber (2000), Kolm (2000), Koskela and Vilmunen (1996) and Sørensen (1999). Recent empirical evidence can be found in Brunello et al. (2002), Hansen et al. (2000), Holmlund and Kolm (1995) and Lockwood et al. (2000). For a survey, see Røed and Strøm (2002).

sector models. Holmlund and Lundborg (1990) study the incidence of different ways of financing unemployment benefits in a Harris-Todaro framework.<sup>2</sup> Kleven and Sørensen (1999) study taxation in dual labour markets with efficiency wages in the primary sector. This holds also for Wauthy and Zenou (2002), who introduce educational subsidies as a policy element. We use a different type of dual labour market model, with locked-in investment, and the interplay between tax progressivity and investment subsidies/taxes will be important. Finally, we mention Agell and Lommerud (1997), where the dual labour market framework is close to the one used here, but where a minimum wage is the only policy instrument.

Of related interest is the empirical literature on labour rent sharing more specifically and firm and industry wage differentials more generally. A recent study is Margolis and Salvanes (2001), that also contains many further references. Several of these studies support the notion that labour shares in rents even in countries as the US where unions tend to be weak, but contrasting views are also aired.

We will now sketch the main findings of the paper. If trade unions are strong and a planner's preference for equality is high, it turns out to be the best policy to combine progressive income taxation and investment subsidies. In the choice between trying to transfer money from primary to secondary workers and seeking to expand the number of primary sector jobs, one chooses both. If trade unions still are strong, but the preference for redistribution is weaker, one will choose only to try to expand the good-jobs sector. We find that if income taxation becomes more costly, this will expand the parameter space when the only policy used is to get more people into primary sector jobs while the parameter space where this policy is used in conjunction with progressive taxation contracts.

Moreover, if trade unions are not very strong, the investment subsidy policy will be abandoned to the benefit of an investment tax. Expanding the primary sector now mostly benefits capitalists (that by assumption do not count in the welfare function), which cools the desire for investment subsidies. Instead, the government seeks to capture those rents that the union could not by taxing investment. The investment tax is supplemented by progressive income taxation. This result, that as mentioned apply for low values of the union's ability to capture rent, is only to a minor degree influenced by the strength of the planner's preference for redistribution from primary to secondary workers.

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<sup>2</sup>Also the framework we use has links to Harris-Todaro's (1970) model of two-sector developing economies, although our focus is not on wait unemployment. Calvo (1978) introduced primary sector trade unions into the Harris-Todaro model.

Finally, we assume that a tax on pure profits is possible up to an exogenously given capped level. If the possibility of profit taxation goes up, this contracts the parameter space with investment taxation, basically because another instrument to redistribute from capitalists to workers now becomes available.

## 2 Model

There are two sectors in the economy; a capital-intensive 'primary sector' and a labour intensive 'secondary sector'. As a simplification we assume that primary sector production necessitate the use of both labour and capital, whereas self-employed labour is the only factor of production in the secondary sector.

By forming a trade union, workers in the primary sector are able to capture a share of the rents generated by sunk capital investments. This is the key assumption of the model.

There are two periods in our model. In the first period primary sector investments,  $K$ , are sunk. For simplicity we assume a fixed relationship between employment and capital in the primary sector. Thus, employment is indirectly determined in the first period by

$$L = \frac{1}{\gamma}K \quad (1)$$

where  $\gamma$  is a measure of the degree of capital intensity in production.

Production takes place in the second period. Workers are initially identical, and sorting to the sectors is assumed to be arbitrary. The wage rate in the primary sector,  $w_1$ , is determined through ex-post bargaining between the union and the firm(s). In the secondary sector, labour is used in a constant-returns-to-scale process and the workers in this sector earn  $w_2$ . All workers, in both sectors, supply one unit of labour inelastically.

### 2.1 Wage bargaining

Since employment is determined prior to bargaining, primary sector workers are only concerned about maximising wages. We assume that the number of working hours per worker in the primary sector is fixed by the employers before bargaining.<sup>3</sup>

Due to some entry costs, there is a fixed number of firms in the primary sector. Assuming a linear revenue function, with the marginal

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<sup>3</sup>If employers and unions had bargained over an hourly wage and then let workers choose their desired work effort, we would have oversupply of labour effort. Working more would for each individual worker be a way of securing more rents for himself.

revenue of capital given by  $\phi$ , the second period profit of a representative employer is

$$\pi = \phi K - w_1 L \quad (2)$$

We assume Nash bargaining and let the relative bargaining strength of the trade union be denoted by  $\alpha \in [0, 1]$ . Primary sector workers can always obtain  $w_2$ , so this is taken to be the threat-point of the union. The threat-point of the firm is zero. Let  $t \in [0, 1]$  be the marginal tax rate on labour income. The primary sector wage rate is given by the solution to the following problem:

$$\max_{w_1} \alpha \ln \{(w_1 - w_2) (1 - t)\} + (1 - \alpha) \ln \{\phi K - w_1 L\}$$

which yields

$$w_1 = w_2 + \alpha(\phi\gamma - w_2) \quad (3)$$

and

$$\pi = (1 - \alpha) (\phi K - w_2 L) \quad (4)$$

The equilibrium wage is given by the fall-back wage and a share in the higher productivity of labour in the primary sector,  $\phi\gamma - w_2$ .

Obviously, we have to assume that  $\phi\gamma > w_2$ . Note that pre-tax wages does not depend on tax rates. This result depends on the linearity assumptions of the model and means that we bypass a discussion of questions of incidence.

Normalising the size of the total labour force to 1, the average pre-tax labour income in the economy is given by

$$\bar{w} = Lw_1 + (1 - L) w_2 \quad (5)$$

## 2.2 Capital investments

Abstracting from discounting, the first-period problem facing the firm is to choose the level of investments that maximises present-value profits, given by

$$\Pi = \pi - C(K, s) \quad (6)$$

The cost of investment depends on the amount of capital acquired and the investment subsidy,  $s$ . We propose a very simple convex cost function, given by

$$C(K, s) = \frac{1}{2} K^2 - sK \quad (7)$$

Using (1), (3) and (4) we find that the optimal investment level is given by

$$K = (1 - \alpha) \left( \phi - \frac{w_2}{\gamma} \right) + s \quad (8)$$

Using (1), (4), (6) and (7) we also find that

$$\Pi = \pi - C(K, s) = \frac{1}{2}K^2 \quad (9)$$

Without unionisation, the firm would be able to hire workers at the wage  $w_2$ . In this case it is easily shown that the optimal investment level is given by

$$K^* = \phi - \frac{w_2}{\gamma} \quad (10)$$

Thus, we see from (8) that in the absence of investment subsidies, the presence of unions always leads to under-investment, as long as the union has some bargaining power.

## 2.3 Government

In designing an optimal tax scheme a policy maker, sometimes dubbed as "the government", is assumed to have three different policy instruments at its disposal; a proportional income tax rate ( $t$ ), a uniform lump-sum transfer to all workers ( $b$ ) and a capital investment subsidy ( $s$ ). Whereas  $b$  is assumed to be non-negative,  $s$  is allowed also to take on negative values, making it then a tax on investments. Note that  $t > 0$  in conjunction with  $b > 0$  implies progressive income taxation.

As we will return to later, we also assume that there is profit taxation in the economy, but that this rate is capped at an exogenous level,  $\tau$ .

### 2.3.1 Welfare

The purpose of this paper is to characterise optimal policy not for *any* social planner, but for a policy maker that cares for equality. This policy maker concerns himself not only with total labour income, but cares also for the *distribution* of income across the working population. Capital income typically accrues disproportionately to the richest in society, so it seems reasonable that such a policy maker would place less weight on capital income than on labour income. To keep the analysis simple, we make the exaggerated assumption that profits do not enter the policy maker's objective function at all. As a by-product of this assumption, it becomes absolutely clear that when the policy maker in some circumstances chooses to subsidise investments, this is only because not only capitalists but also workers benefit. A relatively simple welfare function that has the required properties is<sup>4</sup>

$$W = \tilde{w} (1 - \beta G) \quad (11)$$

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<sup>4</sup>The welfare function is non-welfarist in the sense that it does not build on individual utility functions and arbitrary assumptions about cardinal properties of these functions. The present function has also tractability advantages in the current context. A social welfare function of this kind is suggested by Lambert (1993). Using

where

$$\tilde{w} = \bar{w}(1 - t) + b$$

is average *post-tax* labour income.  $G \in [0, 1]$  is the Gini coefficient, a standard measure of income inequality. A more equal distribution of income would imply a lower value of  $G$ . The parameter  $\beta \in [0, 1)$  is thus a measure of the policy maker's aversion towards income inequality.<sup>5</sup>

In our model, the Gini coefficient is given by<sup>6</sup>

$$G = \frac{L(1 - L)(w_1 - w_2)(1 - t)}{\tilde{w}} \quad (12)$$

If the government wants to decrease income inequality, there are several options available. From (12) we find that

$$\frac{\partial G}{\partial t} = -\frac{L(1 - L)(w_1 - w_2)b}{\tilde{w}^2} < 0$$

and

$$\frac{\partial G}{\partial b} = -\frac{L(1 - L)(w_1 - w_2)(1 - t)}{\tilde{w}^2} < 0$$

Thus, an increase in  $t$  or an increase in  $b$  will both contribute towards a more equal distribution of labour income.

Another, and less obvious, option for the government is to alter the relative share of 'good jobs' in the economy, for instance by subsidising capital investments. Let  $\hat{w}_i$  be post-tax labour income per worker in sector  $i$ . An expansion of the primary sector can then be found to have the following impact on income distribution:

$$\frac{\partial G}{\partial L} = -\frac{(\hat{w}_1 - \hat{w}_2)[\hat{w}_1 L^2 - \hat{w}_2(1 - L)^2]}{\tilde{w}^2}$$

We see that a marginal expansion of the primary sector has an ambiguous effect on the Gini coefficient. More precisely, we see that

$$\frac{\partial G}{\partial L} < 0 \quad \text{if} \quad \frac{\hat{w}_1}{\hat{w}_2} > \frac{(1 - L)^2}{L^2}$$

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survey data, Amiel et al (1999) find that this functional form generally gives a better fit to individuals' attitudes towards inequality than more traditional forms based on constant relative or constant absolute inequality aversion.

<sup>5</sup>The assumption that  $\beta < 1$  not only ensures that welfare always is positive, but makes further sure that the welfare function is Paretian in that it obeys the Pareto criterion. We show below that moving a worker from the bad-jobs to the good-jobs sector always improves welfare, and it can also be shown that increasing the primary sector wage, *ceteris paribus*, increases welfare.

<sup>6</sup>See Appendix A for an explicit derivation of the Gini coefficient.

This is always true if  $L > \frac{1}{2}$ . Thus, expanding the primary sector contributes to a more equal income distribution if more than half of the working population is already situated in this sector, or if the difference in post-tax income between the sectors is sufficiently high. Expanding the good-jobs sector can be good or bad for income distribution as measured by the Gini coefficient: For illustration assume that all workers work in bad jobs, giving a good job to only one worker then makes the income distribution more uneven. If all but one worker have good jobs, moving this last worker to the primary sector obviously evens out the income distribution.

Inserting the expression for  $G$  into the welfare function, we obtain

$$W = (1 - t) [\bar{w} - \beta L (1 - L) (w_1 - w_2)] + b \quad (13)$$

We can see how a marginal change in the relative sizes of the two sectors affects welfare by considering the following partial derivatives:

$$\frac{\partial W}{\partial L} = (\hat{w}_1 - \hat{w}_2) [(1 - \beta) + \beta 2L] > 0$$

$$\frac{\partial^2 W}{\partial L \partial \beta} = (\hat{w}_1 - \hat{w}_2) (2L - 1) > (<) 0 \quad \text{if } L > (<) \frac{1}{2}$$

A marginal expansion of the primary sector may not necessarily make income distribution more even, but it will always increase welfare. Even if the distribution should become more skewed, the effect of the increased average will dominate. Had this not been the case, our welfare function would have been non-Paretian. Furthermore, if the primary sector is larger than the secondary sector initially, the welfare gain of a further expansion will be larger the more weight the government puts on income equality. The converse will be the case if  $L < \frac{1}{2}$  initially.

To sum up this discussion, expanding the primary sector is 'redistribution policy' at least in the imprecise sense that some previously disadvantaged workers benefit from the policy. If 'redistribution policy' is given the sharper meaning to decrease a spread measure of incomes as the Gini coefficient, we see that primary sector expansion is a rather blunt instrument to achieve this goal. Such expansion does only contribute to redistribution in this more narrow sense when the primary sector is 'large' to begin with.

### 2.3.2 Budget

In order to avoid trivial solutions we must introduce impediments to the use of tax instruments. We make the standard assumption that income taxation is costly, and that the marginal cost of taxation is increasing. Technically, this cost is incorporated in a very simple way by assuming

that for every extra tax dollar levied on the working population, the government receives  $1 - \lambda t$  in extra tax revenues, where  $\lambda \in [0, 1]$ .<sup>7</sup>

In addition to labour income taxes, there is also a proportional tax on the present value of profits. However, we assume that firms can evade this tax whenever the profit tax rate exceeds some exogenously given level. This feature is captured in the simplest possible way by assuming an exogenous profit tax rate  $\tau \in [0, 1]$ . The two parameters  $\lambda$  and  $\tau$  represent the limits to taxation: Without them, tax authorities could simply collect all income in society and redistribute after their own liking.

With these assumptions, net government revenues are given by

$$B = \tau\Pi + \left(1 - \frac{\lambda}{2}t\right)t\bar{w} - b - sK \quad (14)$$

### 3 Optimal taxation

The optimal tax package is a vector  $(t^*, b^*, s^*)$  that solves the following problem

$$\max_{t,b,s} W(t, b, s; \beta, \gamma, \alpha, \phi, w_2) \quad s.t. \quad B(t, b, s; \tau, \lambda, \gamma, \alpha, \phi, w_2) \geq 0$$

Before turning to a partly graphical discussion of optimal taxation, some insights into the mechanisms of the model can be obtained by inspection of the first-order conditions of the problem. Assuming that the public budget constraint holds with equality, we can solve it for  $b$  and insert into the maximand, thereby eliminating one choice variable.

Using (5), (13) and (14), the first-order conditions for an interior solution can be expressed as

$$(t) : \quad \frac{\partial b}{\partial t} + \beta L(1 - L)(w_1 - w_2) = \bar{w} \quad (15)$$

$$(s) : \quad (w_1 - w_2)(1 - t)(1 - \beta(1 - 2L)) \frac{\partial L}{\partial s} = -\frac{\partial b}{\partial s} \quad (16)$$

where

$$\frac{\partial b}{\partial t} = (1 - \lambda t)\bar{w} \quad (17)$$

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<sup>7</sup>As labour supply is exogenous in the model, the cost of taxation must be thought of as administrative costs of tax collection. In the primary sector, there is a rent element in the wage, so workers might want to work as much as they are allowed to, but at least in the secondary sector, income taxation will in practice influence labour supply. Endogenous labour supply would complicate the model quite dramatically, and we think that the present, simpler formulation with an increasing marginal cost of taxation at least in a crude way captures the essential point that there is an efficiency cost of driving progressive labour income taxation too far.

and

$$\frac{\partial b}{\partial s} = \tau \frac{\partial \Pi}{\partial s} + \left(1 - \frac{\lambda}{2}t\right) t (w_1 - w_2) \frac{\partial L}{\partial s} - \left(K + s \frac{\partial K}{\partial s}\right) \quad (18)$$

The left hand side of (15) can be interpreted as the marginal social benefit of income taxation. The first term is the marginal increase in average post-tax income due to a tax-financed increase in lump-sum transfers, whereas the second term is the benefit from a more equal distribution of income. We see that the magnitude of this second term is dependent on the degree of inequality aversion, the wage gap between the sectors, and the relative size of the primary sector. The more equal the sectors are in terms of employment, the higher is the effect of increased income taxation on reducing income inequality, as measured by the Gini coefficient.

The right hand side of (15) is the direct income loss for workers due to a marginal increase in income taxation, which is the average pre-tax wage. In this context, we can interpret  $\bar{w}$  as the marginal social cost of income taxation. An important observation is that if  $\beta > 0$ , the marginal benefit of income taxation always exceeds the social costs at  $t = 0$ . There is no efficiency loss associated by collecting the first income tax dollar, and by handing out this dollar in equal portions to the entire working population, social welfare is increased. Since complete equalisation of income is only obtained at  $t = 1$ , this must be true irrespective of the sizes of  $s$  and  $b$ . Thus, corner solutions with  $t^* = 0$  can be ruled out.

Now turning to the first-order condition for the investment subsidy (tax). If  $s > 0$ , the left hand side of (16) can be interpreted as the marginal social benefit of investment subsidies. The primary social benefit of increasing the investment subsidy is a higher average wage, due to the fact that a larger share of the working population is allowed into the 'good' sector. The magnitude of this benefit is determined by the initial wage gap between the sectors. As already noted and which also as can be seen from (16), if  $L > \frac{1}{2}$  initially, an influx of workers into the primary sector contributes towards decreasing income inequality.

The right hand side of (16) can be interpreted as the marginal social costs of investment subsidies. Looking at (18), the last term in the square brackets are the direct costs of a marginal increase in  $s$ . These costs are mitigated, however, by an increase in tax revenue when the primary sector is expanded. Of course if  $s < 0$ , the interpretation of (16) is reversed.

We can use the above analysis to identify the different tax regimes in the model. We know that  $t^* = 0$  cannot be a solution as long as  $\beta > 0$ . Furthermore, from the budget constraint we know that  $b^* = 0$  must imply  $s^* > 0$ . This leaves us with three possible regimes:

Regime I: Progressive income taxation and investment subsidies ( $b^* > 0, s^* > 0$ )

Regime II: Proportional income taxation and investment subsidies ( $b^* = 0, s^* > 0$ )

Regime III: Progressive income taxation and an investment tax ( $b^* > 0, s^* \leq 0$ )

Optimal taxation in the present economy concerns basically two issues: The planner has a taste for redistribution and there is also an underinvestment problem due to the fact that labour captures parts of quasi-rents. Regime I then seems to be a natural choice, one uses progressive taxation for redistribution purposes and investment subsidies to expand the primary sector (which also can have beneficial distributional effects). However, it is possible (regime II) that all proceeds from the income tax is used on investment subsidies (and not on a lump-sum grant to all workers). It can also be (regime III) that one uses the capital investment in the primary sector as a tax base, knowing that this aggravates the underinvestment problem, in order to pay out higher lump-sum grants to the working population. The central question in the remainder of the paper is when which strategy is chosen. The central parameters of the model are labour's rent share ( $\alpha$ ), the planner's taste for redistribution ( $\beta$ ), and the two measures of impediments to taxation, the income tax collection cost parameter ( $\lambda$ ) and the cap on profit taxation ( $\tau$ ). The full analytical solution turns out to be rather messy. Instead of performing standard comparative statics experiments, our main strategy will be to fix two of these variables (the tax impediment variables) at various given levels, and then illustrate graphically how combinations of  $\alpha$  and  $\beta$  lead optimal policy to lie in one of the three possible regimes. Before we do this, however, we will study two special cases that we think shed some light on the workings of the model, namely the cases where income taxation is costless,  $\lambda = 0$  and when the planner has no redistribution preference,  $\beta = 0$ .

At this point, we make some rather innocent simplifications, by setting  $\phi = 1$ ,  $\gamma = 1$  and  $w_2 = 0$ . These simplifications preserve all the important mechanisms of the model. Furthermore, in order to secure an optimal solution for the entire set of parameter values we will assume that the exogenous profit tax rate,  $\tau$ , is not too high.<sup>8</sup>

With these simplifications, the optimal taxation problem is reduced

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<sup>8</sup>It can be shown that when substituting for  $b$  in the welfare function, concavity of the maximand requires that  $\tau \leq 2(1 - \alpha\beta(1 - t)) - \alpha\lambda(1 - \alpha + s)$ .

to

$$\max_{s,t,b} W = (1-t)\bar{w}[1-\beta(1-L)] + b \quad (19)$$

subject to

$$B = \tau\Pi + \left(1 - \frac{\lambda}{2}t\right)t\bar{w} - b - sL \geq 0 \quad (20)$$

As forewarned, we start out by considering two 'polar' cases, one in which income redistribution is costless and one in which the policy maker has no preference for such redistribution.

### 3.1 Costless income taxation ( $\lambda = 0$ )

If income taxation is costless, then income redistribution is also costless. We assume that the government is concerned about income distribution at least to some small degree ( $\beta > 0$ ). It can costlessly achieve complete equality by setting  $t = 1$ . Setting  $\lambda = 0$  and  $t = 1$ , the problem is reduced to

$$\max_s W = b$$

where

$$b = \frac{1}{2}(1-\alpha+s)(2(\alpha-s) + \tau(1-\alpha+s))$$

The first thing to note is that  $\beta$  is eliminated from the problem. This is very intuitive. The trade-off between income redistribution and income maximisation for the whole working population is only present when redistribution is costly. Otherwise, the optimal policy for the government is to collect all labour income, and hand it out as lump-sum transfers. Thus, the optimal value of  $s$  is the one that maximises government revenue. The optimal solution is given by

$$t^* |_{\lambda=0} = 1 \quad (21)$$

$$b^* |_{\lambda=0} = \frac{1}{2(2-\tau)} \quad (22)$$

$$s^* |_{\lambda=0} = \frac{2\alpha + \tau(1-\alpha) - 1}{2-\tau} \quad (23)$$

We see that  $b^* |_{\lambda=0} > 0$ , so the optimal tax regime is either I or III, depending on the parameter values.

From (23) we see that

$$\frac{\partial s^*}{\partial \tau} |_{\lambda=0} = \frac{1}{(2-\tau)^2} > 0$$

and

$$\frac{\partial s^*}{\partial \alpha} \Big|_{\lambda=0} = 1 > 0$$

A higher profit tax rate, or stronger unions, increases the optimal value of  $s^*$ . There are three potential sources of tax revenue from the primary sector: Income taxation, investment taxation and profit taxation. Increasing the investment tax (or reducing the subsidies) yields direct revenues for the government, but the resulting contraction of the primary sector means that revenues from income and profit taxation are reduced. If unions are strong, and thus primary sector wages are high, an expansion of the primary sector would yield a large increase in tax revenues. The same is true if the profit tax rate is high. Consequently, it is optimal for the government to subsidise capital investments if the relative bargaining strength of the unions, or the profit tax rate, is sufficiently high.

From (23) we also find that

$$s^* \Big|_{\lambda=0} > (<) 0 \quad \text{if} \quad \alpha > (<) \frac{1-\tau}{2-\tau}$$

At one extreme, if  $\tau = 0$ , then  $s^* \Big|_{\lambda=0} = \alpha - \frac{1}{2}$ . Thus, without profit taxation, the optimal tax policy implies investment subsidies (taxes) if  $\alpha > (<) \frac{1}{2}$ . At the other extreme, if  $\tau = 1$ , then  $s^* \Big|_{\lambda=0} = \alpha$ , which is the first-best level of subsidies, from an efficiency point of view.<sup>9</sup> Under the assumption that profit to capital owners has no value for the social planner, any profit tax rate below the confiscatory level means that some part of capital revenues are wasted, from the planner's point of view, and thus the optimal subsidy rate implies investment below the first-best level.

### 3.2 No inequality aversion ( $\beta = 0$ )

If the government is not concerned about income distribution among workers, the tax parameters are optimally set so that average labour income is maximised. Setting  $\beta = 0$  we find the optimal solution to be given by

$$b^* \Big|_{\beta=0} = \begin{cases} \frac{1-2\alpha}{2(2-\tau)} & \text{if } \alpha \leq \frac{1}{2} \\ 0 & \text{if } \alpha > \frac{1}{2} \end{cases} \quad (24)$$

$$t^* \Big|_{\beta=0} = \begin{cases} 0 & \text{if } \alpha \leq \frac{1}{2} \\ \frac{2\alpha(2+\lambda)-2\sqrt{\alpha\psi}}{6\alpha\lambda} & \text{if } \alpha > \frac{1}{2} \end{cases} \quad (25)$$

---

<sup>9</sup>For this special case we do not need any assumption that caps the profit tax at a level strictly below one.

$$s^* |_{\beta=0} = \begin{cases} \frac{2\alpha - \tau\alpha - 1 + \tau}{2 - \tau} & \text{if } \alpha \leq \frac{1}{2} \\ \frac{2\alpha(2+5\lambda) - 2\sqrt{\alpha}\sqrt{\psi}(1-\lambda) + 9\tau\lambda(1-\alpha) - 2\lambda(3+\alpha\lambda)}{9\lambda(2-\tau)} & \text{if } \alpha > \frac{1}{2} \end{cases} \quad (26)$$

where

$$\psi = 4\alpha - 8\alpha\lambda + \alpha\lambda^2 + 6\lambda$$

When income inequality is not an issue for the government, the income tax parameter  $t$  is only used when it is necessary to finance investment subsidies, so  $t^* |_{\beta=0} > 0$  only if  $\alpha > \frac{1}{2}$ . Like in the case with  $\lambda = 0$ , there is no trade-off between income redistribution and income maximisation, so the government's problem is reduced to setting the tax parameters in a way that maximises average after-tax income.

From (26) we find that  $s^* |_{\beta=0} < 0$  if  $\alpha < \frac{1-\tau}{2-\tau}$ . Consequently, there are three possible tax regimes:

If  $\alpha < \frac{1-\tau}{2-\tau}$ , there is a tax on investments, and the revenues from this tax, and the profit tax, are distributed to workers as lump-sum transfers. If  $\alpha \in (\frac{1-\tau}{2-\tau}, \frac{1}{2})$ , there are investment subsidies, and these are fully financed by revenues from the profit tax, which also cover the lump-sum transfer. If  $\alpha > \frac{1}{2}$ , the optimal policy also imply investment subsidies, but these are now jointly financed by profit and income taxation, and there are no lump-sum transfers.

### 3.3 Inequality aversion and costly income taxation ( $\lambda > 0$ and $\beta > 0$ )

Now to the more general case when the government cares about inequality, and income taxation is costly. The optimal solution is determined by four parameters:  $\alpha$ ,  $\beta$ ,  $\lambda$  and  $\tau$ . The explicit solutions, which are rather messy, are reported in Appendix B.

We know from the previous analysis that there are three possible tax regimes, where Regime II is a corner solution with  $b^* = 0$ . Furthermore, Regime I also contains a possible corner solution with  $t = 1$ . If the cost of income taxation is sufficiently low, combined with a certain degree of inequality aversion, we know that the government will set the income tax at the confiscatory level.

The three possible tax regimes are most easily characterised graphically. In Figures 1-3 we have plotted  $b^* = 0$  and  $s^* = 0$  for fixed values of the limits-to-taxation variables  $\tau$  and  $\lambda$ , to illustrate how different combinations of  $\alpha$  and  $\beta$  determine the optimal tax regime. Using Figure 1 as a benchmark, Figure 2 shows the effect of more costly income taxation, whereas Figure 3 shows the effect of a higher profit tax rate.

We see that whether the government should subsidise or tax capital investments (the choice between regimes I or II on the one side and regime III on the other) is primarily dependent on the relative bargaining

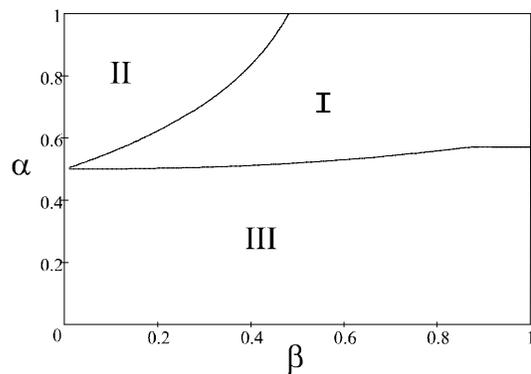


Figure 1: Optimal tax regimes when  $\tau = 0$  and  $\lambda = \frac{1}{2}$ .

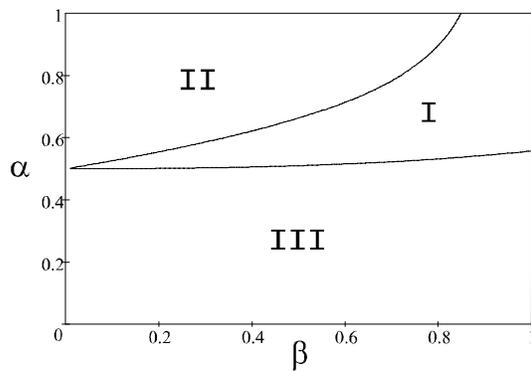


Figure 2: Optimal tax regimes when  $\tau = 0$  and  $\lambda = 1$ .

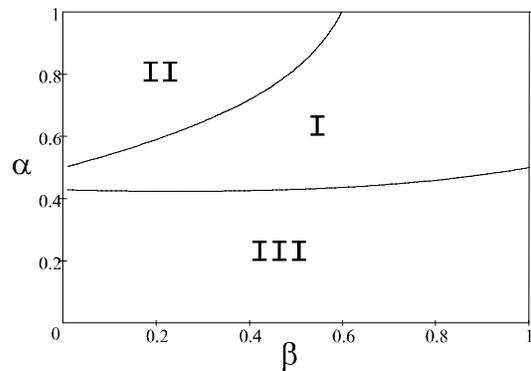


Figure 3: Optimal tax regimes when  $\tau = \frac{1}{4}$  and  $\lambda = \frac{1}{2}$ .

strength of the trade unions. If unions are strong, then the wage gap between the sectors is large, and an expansion of the primary sector will have a pronounced effect on both the average wage and income tax revenues. Consequently, the optimal policy implies investment subsidies. Conversely, if the unions are weak, the major share of the revenues created by investment subsidies will go to the capital owners, so the government would do better by taxing investments.

An increase in the degree of inequality aversion,  $\beta$ , implies a slight expansion of Regime III. If the government has very strong preferences for income redistribution, it is important to raise sufficient revenue for this purpose. A tax on investment is thus a somewhat more likely outcome. The main effect of a reduction in  $\beta$  is that it makes it less likely that progressive taxation is used in conjunction with investment subsidies. The fact that  $\beta$  does not play an important role for whether or not capital investments should be subsidised suggests that industrial policy of the kind we are considering is a relatively poor instrument for achieving income redistribution, compared to using income tax parameters.

The optimal tax policy implies proportional income taxation (Regime II) if the unions are relatively strong, and the degree of inequality aversion is relatively low. This is also quite intuitive. We know that  $b^* = 0$  only when investments are subsidised, and this will be the case when unions are sufficiently strong. The primary benefit of using tax revenues to subsidise investments is an increase in the average wage. On the other hand, the purpose of progressive taxation is income redistribution, and the benefit of redistribution is determined by  $\beta$ . When  $\beta$  is relatively low, it is more beneficial for the government to use all tax revenues to increase the average wage, by subsidising primary sector investments. Consequently, the optimal policy implies  $b^* = 0$  if  $\alpha$  is sufficiently high and  $\beta$  is sufficiently low.

An increase in the profit tax rate,  $\tau$ , implies a contraction of Regime III. This is very intuitive. The cost of investment taxation is the (indirect) loss of tax revenue when the primary sector contracts. Consequently, a higher profit tax rate means that this cost increases. The flip side of the coin of this argument is that when investment is subsidised, a high profit tax means that the subsidy is partly retrieved through profit taxation, which obviously makes subsidies more attractive.

We also see that an increase in  $\lambda$  implies an expansion of Regime II. When the cost of income taxation increases, the optimal income tax rate goes down. Thus, less tax revenue is generated, and, since the marginal benefit of investment subsidies is decreasing, a larger share of the tax revenues is spent on subsidising the primary sector. Consequently,  $b^* = 0$  for a larger set of parameter values.

What about within-regime effects of stronger concerns for income equality? Consider Regime I, with progressive taxation and investment subsidies. A priori, it is not obvious how a higher degree of aversion towards income inequality might affect the optimal policy in this regime, in which tax revenues are used for both lump-sum transfers and investment subsidies. Should the government spend relatively more of its tax revenues on one or the other alternative, or should total tax revenues be increased in order to facilitate an increase in both lump-sum transfers and investment subsidies? In the case where profit taxation is not feasible (i.e.  $\tau = 0$ ) it can be shown (see appendix B) that  $\frac{\partial s^*}{\partial \beta} < 0$  and  $\frac{\partial L^*}{\partial \beta} > 0$  for the entire parameter space in Regime I. When  $\beta$  is not too low, numerical simulations also suggest similar results for the case of  $\tau > 0$ . Thus, increased concerns for income equality mean that the government should increase income taxation, but at the same time *reduce* investment subsidies, implying that a larger share of total tax revenues is used for lump-sum transfers. Once again, this suggests that income tax progressivity and investment subsidies are not really complementary policy instruments with respect to achieving a more even income distribution. In our model, the main effect of investment subsidies is to increase the average post-tax income among workers, whereas income redistribution is better achieved through progressive income taxation.

## 4 Concluding Remarks

The purpose of this paper has been to examine optimal taxation issues in a dual economy, where labour shares in rents in a primary sector but not in the other, secondary sector of the economy. Labour rent sharing at the same time introduces an underinvestment problem in the primary sector and distributional issues within the working population. How should a benevolent planner tackle this?

Assume that we are in a situation with strong unions and a high planner preference for income equality. Optimal policy is then to use progressive taxation and investment subsidies in conjunction. A pure profit tax will be used to the extent this is possible, and a high profit tax points at more investment subsidies, as the subsidy cost then to a high degree is retrieved by the profit tax. This policy package is not dissimilar to actual policies used for example in Scandinavia in the first decades following World War II, and these countries were arguably marked by strong unions and a preference for equality. 'Progressive taxation' here means to tax away money from relatively high earning unionised workers to the less fortunate, so the welfare state system should indeed be seen as part of this 'progressivity'. Both redistributive taxation and active industrial policy have often been criticised as costly deviations from

an efficient economy, but here we have argued that a package of these policies can be the optimal response to labour rent sharing, a package that partly is meant to restore efficiency.

The above package of policies is not optimal in all settings, however. Assume for example that unions are weakened. Rent-sharing is less important within the rent-generating sector. We have then shown that the optimal policy package is still to use progressive taxation, but now to tax rather than subsidise investments. Profit taxation is still used to the extent that this is possible. We have assumed that the planner only care for the income of the working population, not for that of capital owners. With little rent-sharing, the motivation to restore efficiency in the primary sector disappears: Unionised workers will nevertheless be held down to something close to the competitive wage. A planner will rather think of the primary sector as a tax base: He will try to capture parts of the quasi-rents himself, on behalf of the working population, now that the union is less able to do so. Unionised workers still earn more than the workers in the 'bad jobs' of the economy, so progressive taxation is still a relevant tool for redistribution.

Another thing that might change from the starting point with strong unions and a high preference for equality, is that the planner's own preference against inequality within the working population might change. The planner is of course merely an abstraction, so if the attitudes towards equality change in the society, this should be reflected in the inequality aversion parameter that is used for policy analysis. A change in attitudes on equality turns out mainly to be important when unions are strong enough to make investment subsidies part of the optimal policy package. Less weight on equality will make it more likely that we combine investment subsidies with proportional rather than progressive taxation. Tax revenue from the income tax has two competing uses: They can be redistributed to the working population in a lump-sum manner or used to subsidise investment. The less weight on equality considerations, the more likely is the corner solution where all tax revenue is used to increase the size of the primary sector rather than for lump-sum transfers.

Lastly we want to highlight what happens if the cost of collecting income taxes changes. We have represented tax collection costs in a rather simple manner, but we could argue outside the model that they are connected to the labour supply responses of people not working in the rent-sharing sector. An increasing cost of tax collection could therefore mean that rent-sharing is less important, not because unions *within* the rent-sharing sector are weaker, but because the rent-sharing sector has contracted in size relative to another sector where workers earn much simply because they are highly productive. Then labour supply issues

will probably mean that the distortions created by income taxation become more important. Crudely put, the economy might have developed from a setting with rent-sharing unionised workers in heavy industries to a setting with highly productive high-tech workers that are paid the value of their marginal product. The cost of taxation might also have increased due to globalization, with more mobile workers.

While weaker unions implied moving from investment subsidies to investment taxes, a higher collection cost implies that the corner solution when investment subsidies are used in combination with proportional taxation becomes more probable. With more expensive tax collection, there is less tax revenue available. This should point to less redistribution and less investment subsidies. But why should this mean that it is more likely that we are in a corner solution where only investment subsidies and not lump-sum redistribution is chosen? The basic reason is that the marginal benefit of investment is decreasing, so with contracting tax revenues a larger share of these revenues is used to subsidise investments. A fuller analysis should take into account that if the reason that the collection cost has gone up is contraction of the rent-sharing sector, then there is also probably less need for investment subsidies. In turn, this might imply that progressive taxation might survive as part of the optimal policy package even in a 'modernised' economy.

By way of conclusion, it is interesting to discuss if the proposed policies can be characterised as 'union busting through the tax system'? Take as an example the situation with strong unions and strong preferences for equality. The joint policy of investment subsidies and progressive taxation does of course go some way towards undoing the actions of the unions. The primary sector wage premium that the unions have secured is partly taken away and given to less fortunate workers. The efficiency loss from unionisation is also partly restored through the investment subsidy. In the long run, the incentives for workers to form unions are weakened. This might very well be referred to as union busting, if one chooses. The planner is assumed to care for the whole working population, though, so an alternative angle is to say that policy here adjusts for the insider-outsider problem inherent in primary sector unionisation. The interests of *all* workers of course cannot be worse represented when union power is supplemented with the power to tax.

## A Derivation of the Gini coefficient

Let  $x \in [0, 1]$  denote the proportion of the working population when workers are ordered according to income, and let the function  $L(x)$  denote the share of total income accruing to the share of the population given by  $x$ . So, for instance,  $L(0.3)$  is the share of total income accruing

to the 30 per cent poorest of the population.  $L(x)$  is commonly known as the Lorenz curve. In our model, with only two types of workers, the Lorenz curve is a piece-wise linear function with a kink at  $x = 1 - L$ . The functional expression is found to be

$$L(x) = \begin{cases} \frac{\hat{w}_2}{L(\hat{w}_1 - \hat{w}_2) + \hat{w}_2} x & \text{if } 0 \leq x \leq 1 - L \\ -\frac{(1-L)(\hat{w}_1 - \hat{w}_2)}{L(\hat{w}_1 - \hat{w}_2) + \hat{w}_2} + \frac{\hat{w}_1}{L(\hat{w}_1 - \hat{w}_2) + \hat{w}_2} x & \text{if } 1 - L < x \leq 1 \end{cases} \quad (\text{A.1})$$

where  $\hat{w}_i$  is the post-tax income per worker in sector  $i$ . From the Lorenz curve we can derive the Gini coefficient by the formula

$$G = 1 - 2 \int_0^1 L(x) dx \quad (\text{A.2})$$

Inserting the expression for  $L(x)$  from (A.1) into (A.2) and integrating, yields

$$G = \frac{L(1-L)(\hat{w}_1 - \hat{w}_2)}{(L\hat{w}_1 + (1-L)\hat{w}_2)} \quad (\text{A.3})$$

Finally, substituting for  $\hat{w}_i = w_i(1-t) + b$  yields

$$G = \frac{L(1-L)(w_1 - w_2)(1-t)}{\tilde{w}} \quad (\text{A.4})$$

where  $\tilde{w}$  is the average post-tax income.

## B Explicit expressions for the optimal solution

Solving the budget constraint, (20), for  $b$  yields

$$b = \frac{1}{2}(1 - \alpha + s)[t\alpha(2 - t\lambda) + \tau(1 - \alpha) - s(2 - \tau)]$$

Inserting this expression into the welfare function and maximising, yields the following interior solution (i.e. Regime I and III) for the optimal tax parameters:

$$t^* = \frac{\tau\lambda + 2\beta\alpha\lambda + \beta^2\alpha - 2\lambda + \sqrt{\eta}}{3\beta\alpha\lambda} \quad (\text{B.1})$$

$$s^* = \frac{-\tau\lambda - 2\beta\alpha\lambda + 3\beta^2\alpha^2 - \beta^2\alpha + 2\lambda - \sqrt{\eta}}{3\beta^2\alpha} \quad (\text{B.2})$$

$$b^* = \frac{(\rho(\beta - \lambda) + \lambda\sigma - \sqrt{\eta})(\sqrt{\eta}(\rho(3 - \beta - 2\lambda) + 5\sigma\lambda) - \kappa)}{54(\beta^4\alpha^2\lambda)} \quad (\text{B.3})$$

where

$$\eta = 4\tau\lambda^2\beta\alpha - 4\beta^2\alpha\tau\lambda + 4\beta^2\alpha^2\lambda^2 - 4\tau\lambda^2 + 4\lambda^2 + 2\beta^2\alpha\lambda - 2\beta^3\alpha^2\lambda$$

$$\begin{aligned}\kappa &= 5\tau^2\lambda^2 - 20\tau\lambda^2 - 6\beta\alpha\tau\lambda + 14\tau\lambda^2\beta\alpha - 8\beta^2\alpha\tau\lambda + 2\beta^4\alpha^2 + 6\beta^2\alpha^2\lambda \\ &\quad - 6\beta^3\alpha^2 + 2\beta^3\alpha^2\lambda + 8\beta^2\alpha^2\lambda^2 + 12\beta\alpha\lambda + 20\lambda^2 - 8\beta^2\alpha\lambda - 28\beta\alpha\lambda^2 \\ \rho &= 2\beta\alpha, \quad \sigma = (2 - \tau)\end{aligned}$$

There are two possible corner solutions. If the preference for income redistribution is sufficiently high, and if the cost of income taxation is sufficiently low, the optimal solution implies  $t^* = 1$ . In this case we find that the optimal values of  $s$  and  $b$  are given by

$$\begin{aligned}s^* &= \frac{2 - 2\tau - 4\alpha + \alpha\lambda + 2\tau\alpha}{2(\tau - 2)} \leq 0 \\ b^* &= \frac{(2 - \alpha\lambda)^2}{8(2 - \tau)} > 0\end{aligned}$$

On the other hand, if the inequality aversion is sufficiently low and the unions are sufficiently strong, the optimal solution implies  $b^* = 0$ . Furthermore, from the budget constraint we know that  $b^* = 0$  implies  $s^* > 0$ , so this particular corner solution uniquely determines Regime II.

## B.1 Comparative statics in Regime I

Assume  $\tau = 0$ . From (B.1) and (B.2) we derive

$$\frac{\partial s^*}{\partial \beta} = -\frac{1}{3}\lambda(2 - \beta\alpha) \frac{\Omega}{\beta^3\alpha\sqrt{k}} \quad (\text{B.4})$$

$$\frac{\partial t^*}{\partial \beta} = \frac{1}{3} \frac{\Psi}{\lambda\beta^2\alpha\sqrt{k}} \quad (\text{B.5})$$

where

$$\begin{aligned}\Omega &= 2\sqrt{k} + \beta\alpha(4\lambda - \beta) - 4\lambda \\ \Psi &= (2\lambda + \beta^2\alpha)\sqrt{k} + \beta^3\alpha^2(\beta - \lambda) - 4\lambda^2(1 - \beta\alpha) \\ k &= \beta^4\alpha^2 + 2\lambda(1 - \beta\alpha)(\beta^2\alpha + 2\lambda(1 - \beta\alpha)) > 0\end{aligned}$$

From (B.4) and (B.5) we see that

$$\text{sign}\left(\frac{\partial s^*}{\partial \beta}\right) = \text{sign}(-\Omega)$$

and

$$\text{sign}\left(\frac{\partial t^*}{\partial \beta}\right) = \text{sign}(\Psi)$$

It is not straightforward to determine the sign of  $\Omega$  and  $\Psi$ . Our approach will be the following: First we define the set

$$D = \{(\alpha, \beta, \lambda) : \alpha \in (0, 1), \beta \in (0, 1), \lambda \in (0, 1)\}$$

We will then solve  $\Omega = 0$  and  $\Psi = 0$  for one of the parameters. The solutions then define two sets

$$D_{\Omega} = \{(\alpha, \beta, \lambda) : \Omega = 0\}$$

and

$$D_{\Psi} = \{(\alpha, \beta, \lambda) : \Psi = 0\}$$

If  $D_{\Omega} \cap D = \emptyset$ , we know that  $\Omega$  is either negative or positive for the entire set of permissible parameter values, and we can then simply check the sign by inserting numerical values for the parameters. Of course, the same logic applies for  $\Psi$ .

Solving  $\Omega = 0$  for  $\alpha$  yields a unique solution,  $\alpha = 0$ . Thus,  $D_{\Omega} \cap D = \emptyset$ . By inserting numerical values, it is straightforward to check that  $\Omega > 0$  for all  $(\alpha, \beta, \lambda) \in D$ .

Solving  $\Psi = 0$  for  $\lambda$  yields three roots:

$$\lambda_1 = 0$$

$$\lambda_2 = \frac{\left(-8 - \beta^2\alpha^2 + 8\beta\alpha + \sqrt{(16\beta^2\alpha^2 + \beta^4\alpha^4 - 16\beta^3\alpha^3)}\right) \alpha\beta^2}{16(1 - \beta\alpha)^2}$$

and

$$\lambda_3 = \frac{\left(-8 - \beta^2\alpha^2 + 8\beta\alpha - \sqrt{(16\beta^2\alpha^2 + \beta^4\alpha^4 - 16\beta^3\alpha^3)}\right) \alpha\beta^2}{16(1 - \beta\alpha)^2}$$

It is easily checked that  $\lambda_2 < 0$  and  $\lambda_3 < 0$  for all combinations of  $\alpha \in (0, 1)$  and  $\beta \in (0, 1)$ . Thus,  $D_{\Psi} \cap D = \emptyset$ . By numerical insertion it is confirmed that  $\Psi > 0$  for all  $(\alpha, \beta, \lambda) \in D$ . Thus, we conclude that  $\frac{\partial s^*}{\partial \beta} < 0$  and  $\frac{\partial t^*}{\partial \beta} > 0$  in Regime I.

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