Mathematical Model of Societal Productive Forces Structural Evolution

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Abstract

Mathematical model of societal productive forces structural evolution is discussed, particularly with regard to evolution of social production of life. There are analyzed some methodological grounds and issues of mathematical modeling, like correlation between accuracy and complexity/simplicity of models. There are presented both conceptual model of social production of life (by V.Khmelko) and mathematical model developed according to the conceptual one.

"There are more than a dozen of global models created under the influence of Roman Club initiatives to facilitate research and evaluation of ways the world and particular regions would develop. But there's a social unit missing in most of them", — says a modern culture sociology researcher [1]. She explains this omission by the fact of ultimate complexity of social processes. Nevertheless, Forrester's [2] "world dynamics" methodology, that was in fact used by Roman Club, allows to make a simplified model of even a very complicated system. Though this simpli-

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fication caused some fierce purist criticisms of world dynamics theory, we are inclined to join D.Gwishiani's view of that this simplification allowed to "successfully construct mathematical models … and to use them in order to improve the understanding of complex systems' qualitative behavior" [2, p. 8].

This article is not aimed at eliminating of the above-mentioned omission, but at giving a general reader at least one example of mathematical modeling methodology application to dynamics of social evolution process.

1. Conceptual model of social production of life (according to V.Khmelko) and some methodology fundamentals of mathematical modeling

Ukrainian sociologist V.Khmelko developed his concept of social production of life as a process of society reproduction in early 70ies of the last century and presented it in a number of works [see 3–8 and others]. The author views social production of life as a complex phenomenon that is an integral process only by a final result which is the human society. At the same time this integrity, connected with heterogeneity of the socium itself as a system, is a system of interrelated and interdependent processes of production of: 1) social life bearers — the people, 2) material prerequisites to their existence — the means to life and 3) social means of their existence — the social relations [6, p. 125].

Due to lack of space in this article we can not consider all the elements of this concept and the author's argumentation in it's favor. We pursue a more modest aim: to build a mathematical model of just the "productive" subsystem of those complex processes investigated by V.Khmelko, namely the processes of production of society elements — people themselves and their means of life.

Such a choice results from two thoughts: first, in our opinion, the subsystem is a core element of the overall author's conception and determines the nature of other subsystems considered; second, it has an evident structure, brightly expressed dynamic character and due to this it is the best subject for mathematical modeling.

V.Khmelko views social production of society elements as a system of processes that emerged from the division of labor. The author structures this system by the decomposition of entire process of people production by two (and the means to life — by three) spheres according to specific qualitative features of their end products. According to this methodology, social reproduction decomposes into the next five spheres: 1) the reproduction of people as living beings, which initially was the consumption of natural products activity (by hunting-and-gathering), that enables just the biological reproduction; 2) the production of food — farming, when a human not only does simple appropriation of nature-created consumption products but assists in the process of creation; 3) the production of material means of life (an industrial society); 4) the production of symbolic (informational) means of life and 5) the production and reproduction of people as social beings, as personalities [6, p. 136–148].

Then the author uses historical and archival sources, population census data and such to evaluate the share (percentage) of every productive sphere in total social labor costs and the change of this ratio throughout history. The results of this evaluation give us a very interesting picture of structural evolution of societal productive forces that takes a pronounced wave shape (see Figure 1) [4; 5; 9]. This picture was published two years before E.Toffler's work "The Third Wave". Though, it obviously corresponds with this American researcher's theory of three waves of human civilization — the agricultural, industrial and the post-industrial one.

At the same time we think the Figure 1 provides a more integrate picture, with graphics 1 and 5 on the figure having the same wavelike shape as others (numbers 2, 3 and 4). Some differences that strike one's eye are that the first wave is falling now and the fifth is rising.

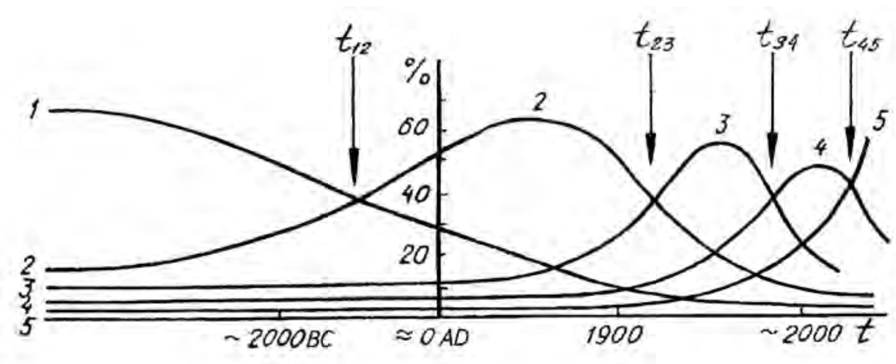

Figure 1. The empirical picture of structural evolution of productive forces (nonlinear time scale): 1 — primitive appropriation activity; 2 — agricultural production; 3 — industrial production; 4 — information production; 5 — production of the entire human as a creative person; t , t , t , t — change of dominance moments (parity points).

Analysis of the historical process of productive forces structural evolution enabled the author to discover and formulate quite important patterns that will be described in this article later. Every stage of historical development of social production has its dominating sphere — the category of products that consumes most of social labor and determines the place and value of other production spheres. While diverse elements of productive forces are developing, the rate of the overall social labor allocation between different spheres changes, that requires the flow of labor force from one productive sphere to other.

It was like that when on the beginning of the "industrial revolution" (see Figure 1) the number of people employed in the 2nd (agricultural) sphere decreased, while in the 3rd (industrial) the number was growing. But reduction of social labor in the 2nd sphere in any case does not mean reduction of agricultural production. Just the opposite, the need of agricultural products continues to increase (at least due to population growth) and actual amount of production grows. It means that at the moment the graph 2 reaches its apogee the process of extensive development of agricultural production ends and the process of intensification begins, that means growth of labor productivity. It obviously results from new implements and technologies provided by the 3rd (industrial) and the 4th (informational) productive spheres. In return, this process would be impossible without extensive development of the 3rd and the 4th spheres that is actually taking place at the time (see Figure 1).

Thus, during every stage of historical development the fastest extensive growth is observed in productive sphere which provides products that have the greatest significance to intensification of production processes of the dominating sphere. This causes flow of labor force from the dominating sphere to the sphere that is extensively growing and, finally, causes change of domination [6, p. 165–173].

This conclusion not only explains the wavelike character of productive forces structural evolution, but uncovers its deep objective causal essence. This makes for great prognostic scope of the Khmelko's conceptual model.

In this connection we cannot refrain from one important methodological observation. The above-mentioned purists may rebuke Khmelko for the fact that his conceptual model is too simplistic and does not cover all forms and kinds of human activity, especially at present time of highly sophisticated division of labor. We cannot deny these arguments because they are true to some extent. For instance, while we can include to the model numerous public administration elements, it is hard to find any place for the activity of army, police or criminals just as we cannot do so with most activities that aim at protection or destruction (as to particular society).

So, we agree that the model is in fact simplistic. But does it form a deficiency or, just the contrary, an advantage? That is the question we are going to answer.

The most eminent example in history is Newton's laws of mechanics. Nowadays after Einstein's achievements we know that Newton's laws are not exact laws but simplistic models. Nevertheless, these laws made grounds for celestial mechanics, allowed high-precision evaluations of planetary motion, prediction of solar and lunar eclipses and so forth. Refinements introduced by Einstein were applied to nuclear physics, cosmogonical studies etc.

Having this example, the answer to the methodological question seems to be clear. Science in general does not provide us with absolutely precise models but with simplistic ones.

Nonsimplified, or "absolutely precise" model does not make sense at all, because it should have been identical with the object of research itself. So, if we imagine a researcher who may have created such a model, he would have encountered a problem. Confronted once again with the object of research and its infinite complexity, he would have had to start his research from the very beginning. Certainly this is an extreme situation for, if "absolutely precise" models ever exist, they are no more than Plato's philosophical "eidoses" that have no practical importance.

In practice when any model (either conceptual or mathematical) of a real environmental phenomenon is being constructed, we always have to find a compromise between complexity (precision) and simplicity (approximation) of the model. It is evident that every specific answer to the problem depends mostly on the aim of modeling, on what we are going to investigate using this model, on the questions we want to answer. Just as like as most other compromise application tasks, this one has no exact answer, though we can say that the general rule is the following: "specification of laws leads to deterioration of their predictive ability ... Just the contrary, decreasing of completeness of description brings an increase in predictive ability, though ... precision weakens" [10]. Profound thoughts on this matter can be found in the works of prominent mathematician and mechanician H.Poincare [11].

However, let us get back to the main theme of the article and once more to Figure 1. The thing that strikes the eye is that the graph is drawn using nonlinear time scale, and the closer we approach present time along the time axis t, the more stretched becomes the scale. The author uses such nonlinearity to make graphs 1–5 more demonstrative and harmonic. To make a comparison, same graphs are drawn on the Figure 2 using linear time scale. It allows to see acceleration of evolution processes as they approach the present time. For instance, during the past two centuries the structure of societal productive forces suffered far more changes than during previous thousands of years of human history, when the changes are hardly evident.

We think that this acceleration is coherent with the exponential population growth noticed by Thomas Malthus and generally approved by modern demographers. Causal relationship between the population growth and the acceleration of evolution processes is quite clear: the density of population grows, trade and intellectual communication intensifies, new ideas, inventions and goods spread faster. All this raises the efficiency of humankind "collective intellect" and facilitates acceleration of the evolution.

Existence of this acceleration was noted by social processes researchers (economists, sociologists, philosophers etc.) a long time ago and introduced into science with series of "revolutionary" terms: neolithic (agricultural) revolution, industrial revolution, postindustrial (informational) revolution. What next? Such terminology can be based to some extent on not just significant acceleration of social processes, but on that their speed sometimes exceeds adaptive abilities of either particular person or entire (not high-dynamic) modern societies.

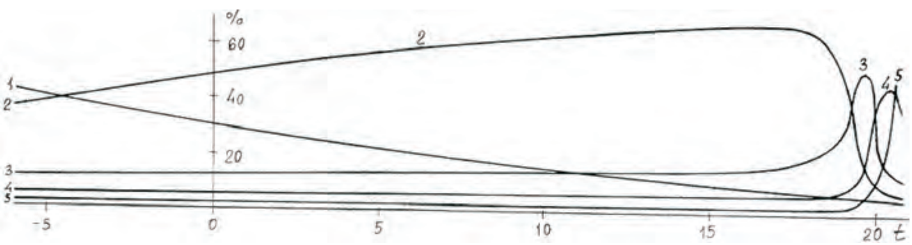

Figure 2. Empiric picture of the productive forces structural evolution (linear time scale).

This circumstance brought about both a number of social crises and huge amount of publications of apocalyptic predictions and thoughts on crisis of modern civilization. The meaning of this situation is that overall acceleration of social processes and particularly — of the structural evolution processes in our presence subdues some borders and becomes a global problem that requires every possible attention of scientific community. Exactly in this context we are going to make a modest investigation.

As mentioned earlier, the “revolutionary” terminology can be grounded only “to some extent”. In fact, we are generally against such terminology. According to one of the dictionaries, a revolution is “a radical qualitative change, drastic leap-like transition from one qualitative state to another…” [12].

But the empirical graphs on Figures 1 or 2 shows only change of speed of evolution processes but no sign of leap, or according to mathematical terminology, no discontinuous change. Aside from the empirical knowledge itself we can explain logically. Technological ideas that are one of the main motive powers of productive forces evolution can in fact occur in one’s mind in a moment (“by leap”), but their practical realization and spreading (only this can finally cause structural changes of productive forces) is a process that develops during the time and is not by any mean a “leap”. For instance, it took nineteen centuries to put in practice the principle of steam engine invented by Heron from Alexandria.

We realize that any terminological dispute is in most cases losing undertaking. It is why we stress in this problem not the terminology but the methodology. We have set the task to create a mathematical model of quite complex process that requires to maintain some correspondence between the terminology of social sciences and mathematics.

In mathematics there is a notion of smoothness of function, the smoothness even can be evaluated according to a special scale. Say, there are so-called discontinuous functions with limited discontinuity situated on the lower pole of this scale. Such function has not a derivative in the breaking point (the speed of the function growth in this point is «infinite») and passes by the leap from one value to another. This functions are to be used to model “leap-like transitions” or revolutions. The opposite pole is occupied by the most smooth functions — so-called analytic functions that not only have no breaches or ruptures, but have an infinite quantity of continued derivatives. Graphs of such functions are smooth, just like that shown on Figures 1 and 2.

The aforesaid explains why the question of “leaps” and “revolutions” has not just terminological, but either principal importance. Guided by Figure 1 we should look for the mathematical model of that kind which generates analytic functions. Besides, the first attempt of mathematical modeling of the process in question imitated the "revolutionary" scheme of development that we consider as inappropriate and that is why we would not discuss it in details [9, p. 173–180][1].

2. The mathematical model

Figures 1 and 2 demonstrate that while historical time passes, manpower resources flow from any productive sphere *i* to the (*i*+1), (*i*+2), that is towards the increase of *i* index, and there is no backward flow. Naturally, in fact the labor force migration is to some extent chaotic and random but the aforesaid is evident when we look at the resultant of the process. But having such an obvious unidirectionality of the structural evolution of societal productive forces, there has to be some *fundamental cause* of this effect. There has to be some *permanent factor* that causes such an unidirectionality. Further we are going to investigate the nature of this factor and to give it a quantitative assessment, even if it would be simplified and generalized.

Before we start to make the evaluation, let us answer one general visionary question. How it come to happen that during thousands of years of human history such a harmonious succession of changes of productive spheres' dominance brought about, as the empirical data shows? V.Khmelko's historical analysis, briefly described in the previous chapter, clearly explains immediate causes of its emergence, but if viewed as a

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1 The model proposed by I.Chernenko is a dynamic system with *n* stationary states (*n* is the number of chosen social production spheres), among which during the particular part of time one is stable and others unstable. The model also includes one variable C_0 (aggregate amount of labor), that continuosly grows and in the process of growth creates bifurcational («revolutionary») situations — primarily stable 1st stationary state transmits the stability property to the 2nd, the 2nd to the 3rd and so forth. Every phase when the productive dominance approaches its apogee the author views as the whole system approaching to its stable stationary state. The change of dominances is the system's stationary states exchange of the stability property.

We think that the main defect of this model is the structural evolution process' principal dependence on the continuous growth of the C_0 parameter that has no proper conceptual grounds. We think the parameter directly influences the tempo of evolutional processes, but not the essence of these processes. Furthermore, the growth of the C_0 parameter obviously will not be everlasting. The growth will stop some time, just at the moment when population growth will come to the end. Let us suppose that this end came when the agricultural production dominated. In this case according to I.Chernenko's model, the agricultural society would stay forever and there would not be any industrial level of culture. Besides, the author have chosen as a prototype the P.Allen's model of cities growth and decay dynamics that has very little in common with the process in question.

whole the picture makes an impression of so to say "rational purposefulness" that might have some mystery beyond it.

Still, it is obvious that if we do not to take into account the dispensation, the rationalistic teleology of evolution appears no more than a figment of our imagination. In general, it applies not only to the process in question but to any evolutional process. For instance, when we look at rationally expedient and balanced interaction of plant and animal populations within one ecosystem or at incredible adaptation ability of particular organism, it is quite hard to resist the feeling of that this rational expedience "is programmed by someone". This feeling arises despite the Darwin's explanation that this phenomenon is just the result of natural selection, the process of survival of every "rational" and "expedient" being while any "irrational" and "inexpedient" (or just not expedient enough) dies out. Modern terminology calls such processes the *self-organization* processes, or the processes of "order emerging from chaos" [14].

Just in the same way the process of the societal productive forces structural evolution occurs. We realize that social processes differ from the biological ones at least due to the fact that the subjects of elementary events that take place on the lowest level of these processes are the people, the intellectual beings. The history presents plenty of attempts to direct social processes of particular society according to some predetermined plan. The distressing results of these attempts are well-known, but their main feature was the fact that they just brought about slight fluctuation to the historical evolution process. When we look at the evolution in macro, they appear to be negligible. So it is obvious that the biological and social evolution are more similar than different.

Having the assumption of the self-organizational nature of social evolution and of the mentioned similarity we conclude that the fundamental factor that influences the kind of societal productive forces structure is the society's, the humankind natural aspiration for the survival and prosperity, with the general efficiency serving as the criterion for "natural" selection of the changes in the social production structure. This means that those changes of the societal productive forces allocation structure that facilitate the growth of effectiveness of social production are supported by society while changes aiming at the opposite direction are suppressed and die out. With all this going on, the society's (or its particular members') awareness of its aspiration for increase in productive efficiency is of little significance, because the final result lightly depends on this fact.

Let us now try to evaluate the social productivity of labor of every productive sphere singled earlier in the text. We can logically suppose that such valuation be the ratio of the amount of objects of consumption (i.e. material and cultural benefits) that a worker gets for his labor time:

$$e_i = \frac{M(t)}{\tau_i} = \frac{\sum_{j=1}^{n} \alpha_j(t) \cdot m_j(t)}{\tau_i}. \qquad (1)$$

The $M(t)$ is the average socially accepted annual rate of consumption that allows normal (as to particular society in the particular moment t) life of both the worker and other family members supported by him; $m_j(t)$ — the production of sphere j that is produced by one average worker per year; $\alpha_j(t)$ — weight coefficients that determine the share of production j in the average rate of consumption ($0 < \alpha_j < 1$); n is the number of productive spheres defined (in our case $n = 5$); τ_i is the total labor time spent by the worker annually in the productive sphere i; e_i is the value of the labor productivity of the sphere i.

It is obvious that the quantities m_i and α_i are time functions and change greatly during the time flow. Say, in the time just before the sociogenesis when social division of labor is almost absent, $\alpha_1 \approx 1$, $m_1 \approx M$, and with any $i > 1$ the quantities α_i and m_i are not much more than zero. But in the course of time they grow that is evidently proved by the pattern of consumption of modern developed societies.

Since α_i the coefficient is nondimensional by nature, the formula (1) implies the existence of some universal unit for the production m_i of all productive spheres. Nowadays such a unit is money being the universal equivalent of any product. But the money as well as the commodity production in the modern sense emerged relatively not long ago and had not existed before that time. However, it does not change the structure of the formula (1) because it has sense only at the time when at least primary division of labor emerges that is necessary accompanied by exchange of its products. The exchange in its turn requires an introduction of some equivalent that may change in time (furs, cattle, some measure of capacity etc.) but should exist in any case. Since the nature of this equivalent does not influence the structure of formula (1), we are no more interested in it.

The formula (1) may take another appearance if the numerator and the denominator are multiplied by the average duration of human ability to labor L:

$$e_i = \frac{M(t)L}{\tau_i L} = \frac{M(t)L}{T_i}. \tag{2}$$

ML is the total amount of consumer objects earned by the worker through his life, and $T_i = \tau_i L$ is lifelong spending of labor time.

The weak point of formulas (1) and (2) is that their numerators have the function $M(t)$ that grows greatly due to impetuous growth of human needs. Thus it is better to connect the valuation of social labor efficiency of the productive sphere i with the quotient of the rate of consumption $M(t)$ to socially accepted average annual needs $P(t)$. Then we divide both left and right parts of formula (2) by $P(t)L$ (with PL being the lifelong needs of worker) and instead of labor productivity e_i we receive a more convenient efficiency valuation:

$$c_i = \frac{e_i}{P(t)L} = \frac{M(t)L}{P(t)L} \cdot \frac{1}{T_i} = \frac{M(t)}{P(t)} \cdot \frac{1}{T_i}, \tag{3}$$

where the efficient M/P can be viewed as the rate of satisfaction of the actual needs of worker. Apparently, that $M/P = 1$ if these needs are fully satisfied; $M/P > 1$ if they are satisfied excessively and $M/P < 1$ if the needs are partly satisfied.

It is also evident that the most probable value of the M/P quotient cannot be far from one. Any deviation of this function from either way inevitably causes the emergence of positive or negative stimulus to change the volume of production and thus the system automatically stabilizes in the equilibrium position around $M/P = 1$. This means that the first efficient of the right part of the formula (3) can be discarded as negligible. Then

$$c_i = \frac{1}{T_i}, \tag{4}$$

that means we can consider that the labor efficiency valuation of the sphere i is the value that is inverse to lifelong time spending T_i of an average worker needed to earn the average socially accepted level of consumption $M(t)L$.

The value $T_i = \tau_i L$ surely changes in time too. At that, τ_i at the course of time decreases, while L increases. Thus at first crude approximation we will consider their product constant ($T_i \approx$ const), that can be seen as average (as to the whole time) value of the $T_i(t)$ function. Owing to this assumption the evaluation of the labor efficiency c_i, that we call *the development potential of the productive sphere i* is simplified as much as possible and becomes constant, thus giving an opportunity to make quite simple mathematical model.

To build such a model, on the assumption of above-mentioned analogy of biological and social evolutional processes we use the well-known methodology of mathematical modeling of so-called “living” systems. These systems have some specific features. First, it is the striving of the “living” subject for survival and self-development, that mathematically is formulized as positive (biological) feedback to the subject itself; second, extensive interaction with the environment that quite often counteracts and limits one’s self-development. Due to this, the known mathematical model of “living” systems has the following general appearance:

$$x = x\varphi(x). \qquad (5)$$

The $x = x(t)$ is the level of the process investigated, say, population: $x = dx/dt$ — the speed of process (first derivative); $\varphi(x)$ — the limiting function that is the model of environment influence [14–16].

The same structure have the well-known models of “living” systems: 1) the model of self-restoration of population with limited living resources, or the so-called logistic equation [14, p. 253–257, see also 15, p. 465; 17, p. 184–187]; 2) the model of interaction of “predator-prey” populations, or Lotka-Volterra equation [15, p. 172; 16, p. 67; 17, p. 135–139]; 3) Fisher-Aigen system [18]; 4) May’s ecological models [19] etc.

Two efficients in the right part of differential equation (5) represent two specific features of “living” systems: the first represents positive feedback, recognized by modern science (e.g. molecular biology) as a ground of the life itself [14, p. 20], and the second — influence of the environment, or the competition for access to limited means of subsistence (resources).

In general, when system of n competing subjects is in question, the equation (5) transforms into n-dimensional system of differential equations:

$$x_i / x_i = \varphi(x_i, \ldots, x_n), \quad i = 1, \ldots, n. \qquad (6)$$

This means that the evolution speed of the process i (the speed of the process divided on its level) is determined by the influence of its environment. Positive environment facilitates growth of the process speed ($x_i > 0$), the negative causes a decline ($x_i < 0$).

Thus, building of mathematical model of evolution of any “living” system comes to construction of limiting function(s), in other words, the modeling of the competitive environment. In our case, when evolution of productive sphere i is investigated, competitors (for manpower resources) are all other $n - 1$ spheres. To consider their competitive influence, we use a notion of *average evolutional development potential* $c(t)$, that is the result of averaging of potentials (4) of all social production spheres, taking into account their respective shares:

$$c(t) = \sum_{i=1}^{n} c_i x_i(t). \qquad (7)$$

The $x_i(t)$ is a relative (in shares of one) share of total society manpower of the productive sphere i. (Actually, total manpower is the ordinate of graphs on Figures 1, 2).

Now we can suppose that the difference of potentials

$$\varphi_i(t) = c_i - c(t), \quad i = 1,\dots,n, \qquad (8)$$

reflects the influence of environment on the evolution of sphere i. Then from (6)–(8) we get a system of nonlinear differential equations [20; 21]

$$x_i / x_i = c_i - c(t) = \sum_{j=1}^{n} (c_i - c_j) x_j, \ i = 1,\dots,n, \qquad (9)$$

that is supposed to model the structural evolution of productive forces.

The system (9) has the first integral like

$$\sum_{i=1}^{n} x_i(t) = 1. \qquad (10)$$

This total can be interpreted as “law of conservation” in “living” systems, and according to evolutional point of view, it is the competitive condition, or (according to M.Aigen) “general organization constant” [22].

It is not often possible to find a general analytic solution for a nonlinear differential equation, but in case of system (9) it is possible, and further we will show the way it can be accomplished.

It follows from (9) that the difference between the evolution speed of the spheres i and k is equal to the difference between their development potentials

$$x_i / x_i - x_k / x_k = c_i - c_k. \qquad (11)$$

If we use a symbolic notation

$$y_{ik} = x_i / x_k, \qquad (12)$$

then from (11), (12) we get the linear differential equation

$$y_{ik} = y_{ik}(c_i - c_k), \qquad (13)$$

which general solution we can find easily [23]. This solution and the condition (10) allows to arrive to a general solution of the system (9) in the following analytic form:

$$x_i(t) = \frac{x_i(t_0)}{\sum_{j=1}^{n} x_j(t_0) \exp[(c_j - c_i)(t - t_0)]}, \ i = 1,\dots,n. \qquad (14)$$

The t_0 is any fixed point of time on the axis t, and $x_i(t_0)$ — so-called initial conditions, that is the value of ordinate of process i in the moment $t = t_0$.

The solution (14) not just only completes the model making of the process in question, but also provides an essential simplification of its use in practice by excluding the necessity of numeric integration of the non-linear differential equations' system (9).

3. Some results of primary investigation of mathematical model (9)

3.1. Mathematical model parameter identification. The mathematical model (9) and formula (14) contain n a priori unknown parameters of c_i. Thus, in order to perform numerical calculations (model experiments) these parameters have to be given particular numerical values. This may be accomplished at least in two ways: 1) using the meaning of c_i parameters that can be calculated on a basis of statistical data; 2) try to find unknown parameters by identifying empirically known history of evolution process with the resolution (14) of the differential equation system (9).

The second way belongs to a group of so-called inverse problems of the differential equation theory. In mathematical modeling it is called model parameter identification [24]. It is used when unknown parameters do not have precise meaning interpretation and are of phenomenological nature. We will make use of this second way because we realize that the meaning interpretation that we gave to c_i parameters in the previous chapter of this article is just hypothetical.

Let us look at the intersection points of the graphs of the i and k processes (Figure 1) which we call parity points. At this points,

$$x_i(t_{ik}) = x_k(t_{ik}), \tag{15}$$

the t_{ik} is the point of time when graphs of the i and k processes intersect (Figure 1), or the abscissa of parity points.

From (14), (15) we get the formula

$$c_i - c_k = \left[\ln \frac{x_k(t_0)}{x_i(t_0)}\right] \Big/ (t_{ik} - t_0). \tag{16}$$

By the way, other means can be used to evaluate c_i parameters and it is not necessarily to use the data of parity points. Say, if we knew the value of any two processes x_i and x_k at two (arbitrary) points of time t_0 and t_1, we could use the formula

$$c_i - c_k = \left[\ln \frac{x_k(t_0)x_i(t_1)}{x_i(t_0)x_k(t_1)} \right] \Big/ (t_1 - t_0). \qquad (17)$$

As it is seen from equations (9) and its resolution (14), levels of processes $x_i(t)$ are defined by the difference of development potentials c_i, not by their absolute values. This is why the identification formulas (16) and (17) have similar specific look. From the point of "pure" mathematical view this means we can arbitrary assign the value of one of n unknown c_i parameter. Hence, the investigator has some freedom in interpretation of the meaning of model parameters; here we propose one of possible interpretations. But as soon as one chooses any particular interpretation and on this basis suggests a numerical value of one of the model's parameters, he looses the freedom since numerical values of other unknown parameters unambiguously are calculated according to formulas (16) and (17). The result of comparing of this calculations to numbers that correspond to the meaning interpretation we can use to verify the chosen interpretation hypothesis.

3.2. Model experiment. To perform evolution processes $x_i(t)$ calculations according to formula (14), we have to set "initial conditions" for an arbitrary moment t_0 of time and to determine the difference of development potentials $c_i - c_j$ according to formulas (16) and (17). Aforesaid putting forward a hypothesis and as a basis for definition of one c_i parameter is, properly speaking, unnecessary.

The differences between the result of this evaluations (Figure 3) and known empirical data (Figure 2) occurred to be insignificant, taking into consideration great simplification and incompleteness of the identification procedure used and moreover, the simplicity of the mathematical model itself.

3.3. Verification of the interpretation hypothesis, that is not necessary for the performance of model experiment, allows to make some additional interesting conclusions. With the use of particular values of differences in development potentials we find following simultaneous inequalities [21, p. 66]:

$$c_1 < c_2 < c_3 < c_4 < c_5; \quad T_1 > T_2 > T_3 > T_4 > T_5. \quad (18)$$

This shows some time directivity of the productive forces structural evolution process towards increasing of the development potential, or decrease of average lifelong spending of working time T_i, that are required for life reproduction. In the extreme case when $T_1 = T_2 = T_3 = T_4 = T_5$, time changes within the system stop and stagnation begins: $x_i = \text{const}$. Let us make use of our right to assign one parameter of the model and value T_3 the (industrial sphere) at 20 years. Hence we get a succession

$T_1 = 32$, $T_2 = 31$, $T_3 = 20$, $T_4 = 15$ и $T_5 = 12$ years, that corresponds to parity points coordinates.

The numerical values (whatever approximated) found we consider to be very reliable. They show that the T_i value does not exceeds a human life duration, although at “the beginning of evolution” with a quite short life-span at the time these values were almost equal. The insignificance of the difference between T_1 and T_2 (or c_1 and c_2) probably indicates very low agriculture level in the beginning of the agricultural era and explains very long period of active competition between spheres 1 and 2 (Figures 1, 2). With the beginning of industrial era the situation swiftly improves, and the value falls far behind the average life-span, that can be viewed as the most important indicator of the mankind progress. But it worth mentioning that during several thousand years the T_i value only halved.

To the point, the notion of the progress itself is currently rising a lot of scepsis due to its unavoidable concomitant losses. Thus we are pleased to rehabilitate, at least in part this notion on the basis of the suggested model and its examination. And more to the point, a famous biologist M.Tymofeiev-Resovskyi has proposed to scientific community to discuss the introduction of the third general biological historical principle of *progressive evolution* into the discourse of biology theory (along with such fundamentals of biology as the principles of natural selection and of convariant reduplication) [25].

3.4. The analysis of the model’s steady states stability. The mathematical model (9) belongs to a group of autonomous dynamic systems [26]. Farther we will try to evaluate general trends of its evolution on the qualitative level by examining of steady states and their stability.

Steady state of a dynamic system is the state in which the system can persist for indefinite time, given that no external factor will interfere. The steady state is called stable if slight deviations caused by single external excitation do not grow in time or stay limited. In other case the state is called unstable. Finally, if the initial deviations decrease in time, and if $t \to \infty$, then disappear completely, the state is called asymptotically stable.

To find all possible steady states of the system (9), we have to equate its right parts with zero and to find the resolution of the nonlinear equations system. Omitting this simple but toilsome procedure, we would like to mention that the system (9) has n steady states of “monopolistic” type, when the share of one productive sphere $x_i = 1$, and other’s shares are equal to zero. With the use of Liapunov’s first stability theorem [27] we conclude that processes in the system (9) have aperiodic nonoscillatory nature, that fully corresponds with their real history that is empirically known, and all its steady states excluding the "final" (x_n = 1, x_1 ...= x_{n-1} = 0) are unstable. The "final" state is, on the contrary, asymptotically stable, that means the system is tending to it with any initial conditions.

The "any" conditions mentioned require some comments. At first it seems that there can not be any uncertainty as to initial conditions of the evolutional processes in question since they are preset historically: the structural evolution of the productive forces started somewhere about 1st ("starting") unstable stationary point (x_1 = 1 while $t \to -\infty$) and since that ancient time has been going for many centuries (Figure 1, 2). This is a historical fact that is not subjected to any change. But in general, speaking of "initial conditions" we associate them not with the "beginning" of evolution, but with the state of system in any moment of time t_0. Change of initial conditions, or change of the state of system in the moment t_0 may occur under the influence of any external excitation that abruptly disturb the natural course of evolutional processes. These may be world wars, global ecological or natural catastrophes etc. That is why stability testing of the model (9) is so interesting — it allows to make the following conclusion: any external excitations, except for if they destroy all the system (in other words, if the mankind manages to avoid self-destruction from the atomic war or similar trouble), can not radically change the main trend of the productive forces structural evolution. It will anyway move towards its "final" steady state (x_n = 1) since there's no other stable steady points in the system in question.

Finally we will try to explain the phenomenon of unstable steady states. Let's imagine that on the "beginning" of evolution the system is in the stationary point (x_1 = 1). It can stay there all the time under the condition that there would be no external disturbance, that is surely impossible in nature. Such a "first" disturbance could be a discovery by the first primitive that a grain once thrown into earth comes up, or a domestication of an animal. Such a disturbance pushes the system off the unstable stationary point and the process of evolution begins in compliance with its (dynamical) laws. According to them, the system with started evolutional process will never come to any other unstable stationary point omitting them and move to an asymptotically stable one.

Thus, the investigation of steady states and the stability of the dynamic system allows to define main trends of the system behavior on the qualitative level without any calculation of the resolution of the differential equations system. This is why the study of stability is one of the fundamental stages of mathematical modeling of any dynamic (evolutionally changeable) system.

4. Some final statements

4.1. Is it possible to combine the simplicity of a model with its precision? We have already mentioned in the beginning of this article that the simplicity and the prognostic ability of the model is contradictory with its precision. But can we somehow omit this contradiction and increase the precision of the prognosis without loosing the simplicity of model? It is possible to some extent and we will describe it in short.

If we identify the parameters of model in a chosen moment of time t_0, at least in this point the model completely corresponds to empirical reality. If the model is made successfully and describes at least main trends of the real process evolution, the correspondence allows us to suppose that the mistakes of the model around this point are not significant. Hence, the general rule is as follows: the more we move away from the point of identification along the time axis, the more evident become model's mistakes. We can overcome this quite easily by performing the identification procedure at different points of time. This also helps to compensate the negative influence on the model by our crude assumption that the development potentials c_i are constant. Just like that with the use of the formula (17) and the value of t_0=19 (the beginning of 20th century) the Figure 3 is drawn.

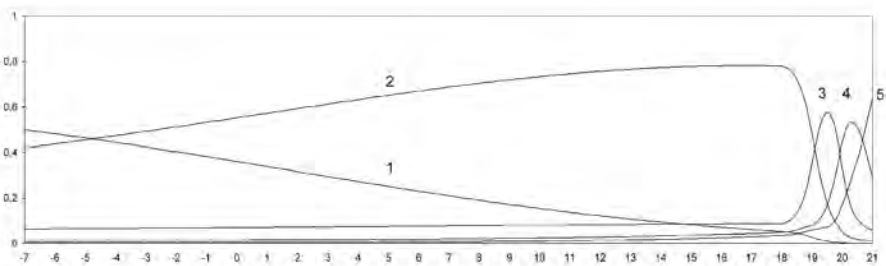

Figure 3. The result of model-based calculation of the productive forces structural evolution (century scale).

4.2. Do the "laws of history" exist? K.Popper, a famous 20th century social philosopher, said that "the future depends on us while we do not depend on any historical necessity" [28]. On this basis he hardly criticized the so-called "historicism", that tried to "understand the laws of historical development" [28, p. 21]. He opposed the "historicism" with the "social engineering of peace-meal decisions" that he thought of as an actual social development strategy. Properly speaking, it means he was an adherent of social self-organization, though the term was not used widely in that time. By the way, Popper's follower F.Hayek had been using the term with a reference to I.Prigogin, one of the greatest founders of self-organization theory [29].

By criticizing "historicism" Popper actually disaffirms any possibility of existence of a scientifically substantiated strict succession of historical events. But Khmelko's principal model demonstrates such succession, with great part of this succession based on the historical experience, on the established facts. As long as Khmelko is a self-organizationist too, do we have an insoluble antinomy?

We do not think so. Popper was writing his main work about the open society during the World War II when there was no self-organization theory. And the fact explains his rigidity in this question. Now thanks to I.Prigogin and his school works, we know that self-organization process, if developed without any obstacles, always comes to formation of a time-space structure (according to Prigogin, a "disipatic structure"). Just such a time structure (space disappeared by integration on the whole space) is shown on the Figures 1–3 (more about modern interpretation of Popper's views see in the above-cited work [10]).

4.3. "The end of history?" This provoking question was asked by F.Fukuyama and is very topical in the context of this article, though the author has almost renounced his hasty and insufficiently grounded conclusions. Properly speaking, the article is mostly dedicated not to history as a whole, but to its "productive" part, that means the history (evolution) of the societal productive forces structure. The model constructed seemingly prognoses the "end" of the history, since it is moving towards the asymptotic stable state with the dominance of 5th productive sphere.

And now the question arises: what then? Is it true, that the evolution has an "end", after which there will be no changes? The model proposed in this article does not allow to give an answer, since we can trust its prognosis only around the established historical facts. According to our vision with a consideration of the ancient wisdom of "all flows, all changes", we would like to answer the question by the question of Fukuyama: "Maybe, the perspective of centuries of boredom will make the history to take one more new start?" [30].

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