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A LOGICAL RECONSTRUCTION OF PURE EXCHANGE ECONOMICS

INTRODUCTION

Most economists think that a logical reconstruction of economic theories is trivial, some even would say that it is nonsense to force their theories into the Procrustean bed of logics. That such work, in fact, is not trivial can be seen from existing textbooks. There are practically no textbook presentations for which the following quotation from [7] would be wrong[1]:

> From the standpoint of logical rigour and precision, however, none of the existing treatments... seems to be entirely free from serious defects. None of them comes even close to satisfying the standards set, let us say, by Hilbert in his axiomatization of Euclidean geometry.

Precision, perhaps, is not necessary in order to be able to *use* a theory. It *is* necessary, however, if we want to say something non-trivial *about* such a theory. Formal reconstructions of existing theories are not intended primarily to induce progress in the field reconstructed. They are intended to provide a basis for reflecting on theories in general, i.e. they provide a basis for the philosophy of science.

My aim in this paper is to present a logical reconstruction of pure exchange economics (PEE). PEE contains the very core of micro-economic theories which use utility functions. It is called 'pure' because money and production are not treated explicitly. So PEE yields a rather simple logical structure. This allows to depict methodological issues in a clear and simple way without drowning in a flood of technicalities. On the other hand, the principle issues of PEE are essentially the same as in more complicated, 'realistic' and 'modern' micro-economic theories – just because the latter with practically no exception logically contain the principles of PEE. This indicates two more special aims. First, PEE's reconstruction is intended to yield a basis for further investigation of different micro economic theories and their intertheoretic relations, and second, it is intended to elucidate the status of the utility function which is the central and crucial concept of micro-economic theories.

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My reconstruction is based on the standard texts [1], [4], [6] and [8], and will follow very closely the exposition in [6], p. 126 ff. As a descriptive tool I will use set theoretic predicates, and I also will adopt some ideas of Sneed's concerning the logical structure of empirical theories (compare [3], [9], [10]). The reader is assumed to be familiar with set theoretic notation.

I. BASIC CONCEPTS

We will need seven basic concepts in order to describe PEE's models. First, we need 'economic agents' or simply persons. There are finitely many of them involved in an economic system and the corresponding formal notion is a set $\mathscr{F} = \{i_1, \ldots, i_n\}$, where $i_1, \ldots, i_n$ represent those persons. Second we need kinds of commodies. It is assumed that there are finitely many of them, e.g. lettuce, milk, corn, VWs, Boeing 727's etc. The corresponding notion is a set $G = \{1, \ldots, m\}$ of natural numbers. Each natural number in G represents a distinct kind of commodity. This identification has the advantage that the kinds of commodities are linearly ordered in a natural way. Third, we need quantities of commodities. All information about quantities of commodities as distributed among the individuals of the economic system will be lumped together into one single concept: a function $q^0 : \mathscr{F} \times G \to \mathbb{R}^+_0$. '$q^0(i, g) = \alpha$' means 'person i owns or consumes quantity α of commodity number g'. $\mathbb{N}$, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{R}^+_0$ denote the natural, real, real positive and real non-negative numbers, respectively. Fourth, we have a function $\tilde{q}: G \to \mathbb{R}^+$ which is interpreted as follows. To each kind of commodity $g \in G$ $\tilde{q}$ assigns the total amount $\tilde{q}(g)$ of that kind of commodity which 'exists' in the economic system. If g, for instance, denotes 'milk' and the economic system consists of a distinct village then $\tilde{q}(g)$ is the total quantity of milk which at the moment of consideration is present.

The fifth notion is that of price. We use a price function $p: G \to \mathbb{R}^+$ assigning to each kind of commodity a number $p(g)$ which denotes the price of one unit of this commodity. Prices are assumed to be positive for zero prices indicate that arbitrary amounts of this 'commodity' are available. Such a 'commodity' is no commodity is the sense of economics. The most important concept is, sixth, the concept of utility. Each person derives a certain utility from given amounts of each kind of commodity he ownes or consumes. Formally, we introduce a function[2] $U: \mathscr{F} \times \mathbb{R}^m \to \mathbb{R}$ '$U(i, \alpha_1, \ldots, \alpha_m) = \beta$' means 'person i's utility derived from quantities α_1,

..., α_m of commodities 1, ..., m respectively is β'. That is, if i owns α_1 units of commodity 1 and ... and α_m units of commodity m then his utility derived from the commodities is β. If we replace α_1, ..., α_m by what i actually possesses, namely $q^0(i, 1)$, ..., $q^0(i, m)$ we obtain an expression $U(i, q^0(i, 1), \ldots, q^0(i, m))$ for i's actual utility. Finally, we need a set E of equilibrium distributions. By a distribution we mean any function of the type of q^0. Any such function describes some *possible* way of distributing commodities among the individuals. One way of stating the aim of PEE is to say that it aims at characterizing equilibrium distributions. PEE states the conditions under which a given distribution yields economical equilibrium.

Integrating these notions into a structure we obtain

*D*1 x is a potential model of PEE (in symbols: $x \in M_p$) iff there exist $\mathscr{F}$, G, $\tilde{q}$, q^0, p, U, E such that

(1) $x = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E \rangle$

(2) $\mathscr{F}$ is a finite, non-empty set and $G = \{1, \ldots, m\} \subseteq \mathbb{N}$

(3) $\tilde{q}: G \rightarrow \mathbb{R}^+$

(4) $q^0: \mathscr{F} \times G \rightarrow \mathbb{R}_0^+$

(5) $p: G \rightarrow \mathbb{R}^+$

(6) $U: \mathscr{F} \times \mathbb{R}^m \rightarrow \mathbb{R}$ is smooth

(7) $E \subseteq \{q / q: \mathscr{F} \times G \rightarrow \mathbb{R}_0^+\}$

We do not allow for negative values for quantities owned by persons – (D1–4) Intuitively, this would amount to allowing for debts. (D1–3) guarantees that all kinds of commodities considered are available to a certain extent. Commodities with zero total amounts are not relevant and can be excluded. The values of U (D1–6) are not required to be non-negative. But the additive structure of $\mathbb{R}$ is not really relevant here. No intuitive distinction is attached with the distinction between positive and negative real numbers. Only the ordering matters. It may be noted that prices are independent from individuals – i.e. they are the same for all persons – and that the individual utilities depend only on the amounts of commodities owned by the respective individuals. i's utility does not depend on what other persons possess which excludes external effects.

In order to state the axioms we need two auxiliary concepts.

D2 Let $x = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E\rangle \in M_p$.

(a) $Z_x := \{q/q:\mathscr{F} \times G \to \mathbb{R}_0^+ \text{ and } \forall g \in G(\sum_{i\in\mathscr{F}} q(i, g) \leq \tilde{q}(g))\}$ is called the *consumption set (of* x).

(b) $B_x := \{q/q \in Z_x \text{ and } \forall i \in \mathscr{F} (\sum_{g\in G} p(g)(q(i, g) - q^0(i, g)) = 0)\}$ is called the *budget set* of x).

Z_x is a special kind of choice set, where a choice set is a set of possible alternatives from which one alternative can be chosen. In the present case the individuals have to choose among different possible distributions. They will try to choose that distribution for which their utility is highest. The choice set in question then is the set of all possible distributions. Since its elements are distributions of commodities to be consumed this set in the present context is called consumption set. Z_x contains all distributions which are possible under the constraint of the given total amounts of commodities ($\tilde{q}$). For each possible distribution q in Z_x and for each $g \in G$ the sum of the quantities owned by all individuals may not exceed the total existing amount $\tilde{q}(g)$. The budget set B_x contains only those distributions which are compatible with the given budgets of the individuals. Individual i's budget is just the value of the bundle of commodities he owns: $\sum_{g\in G} p(g) q^0(i, g)$. A distribution $q \in B_x$ must leave all these values unchanged: $\sum_{g\in G} p(g)\, q(i, g) = \sum_{g\in G} p(g)\, q^0(i, g)$ for all $i \in \mathscr{F}$. Intuitively, B_x contains those redistributions of q^0 under which the individuals do not gain or lose something. The transition from q^0 to a $q \in B_x$ represents an economical change in the course of which nobody has made profits, losses or debts.

II. THE AXIOMS

The models of PEE now can be described as follows.

D3 x is a model of PEE ($x \in M$) iff there exist $\mathscr{F}$, G, $\tilde{q}$, q^0, p, U and E such that

(1) $x = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E\rangle$

(2) $x \in M_p$

(3) $q^0 \in Z_x$

(4) $E \in B_x$

(5) $\forall q(q \in E \rightarrow \forall i \in \mathscr{F}\ \forall q' \in B_x(U(i, q'(i, 1), \ldots, q'(i, m)) \leq U(i, q(i, 1), \ldots, q(i, m))))$

(6) $E \neq \emptyset$

A model contains exactly the concepts described in (D1), i.e. it is a potential model. In order to understand the axioms (D3–3)–(6) it is helpful to imagine an economic system as evolving in three steps. Initially, a distribution of commodities q^0 is given. In a second step exchanges are performed until nobody wants to exchange any more or cannot exchange any more. The latter may occur if some person has come to a point where he needs all his commodities for his own provision. In a final stage, after exchanges have been performed, we have a new distribution q of commodities. It must be stressed that this is only an auxiliary picture. No time is involved in (D3): PEE is a static theory.

(D3–3) says that the initial distribution (or the actual distribution) q^0 is possible relative to the total amounts of commodities given by $\tilde{q}$. (D3–4) requires the equilibrium distributions to satisfy the budget constrains expressed in B_x. Distribution q originating from q^0 by exchange is an equilibrium distribution only if in the course of exchange nobody has made economical gains or losses. (D3–5) is the central axiom expressing 'maximization of utilities'. Intuitively, it says that q is an equilibrium distribution only if all individual utilities derived from q are maximal with respect to B_x. That is, there is no distribution q' in B_x which yields greater utilities. Roughly, this amounts to saying that all individuals try to maximize their utility. But in doing so they are restricted to exchange within the scope of their budgets. They maximize their utilities under the constraints imposed on them by owning only finite values under the initial distribution q^0. Such a 'maximal' distribution is rightly called an equilibrium distribution for if people have maximized their utilities under the budget constraints there is no reason for further exchanges. Further exchanges could not increase their utilities. So they will stay in such a state – in equilibrium – until 'external' influences (e.g. production of new commodities) change the situation. (D3–6) require that there exists at least one equilibrium distribution.

In order to get an idea of what such a model is like imagine a small village in the early middle-ages. Men working on the fields, women making clothes, perhaps some people with special abilities being able to make e.g. a finer brand of shoes or saddles or earthenware. Twice a year a tradesman passes with luxuries from the great world. On certain occasions exchange will take place. Peasant A having three cows and meadows for at most four cows will try to get rid of one of the two calves just born by two of his cows. Perhaps he makes the change to complete the dowry of his daughter by some nice pieces made by Mrs. B. Peasant C whose single cow presently gives no milk, fetches some milk for his children from his neighbour, perhaps he takes some salads with him in order to compensate. Money is not needed, the whole system economically is (nearly) closed. (This naive description must not bring up the impression that PEE has a very limited scope. In principle western economical societies (countries) can be described alike; only they are exposed to various other – partly non-economical – constraints.)

From such a piece of reality one could find out the total amounts of commodities and their initial distribution among the individuals, i.e. one could determine $\tilde{q}$ and q^0. The real situation is a model of PEE if, roughly, people can exchange their commodities such that everybody maximizes his utilities relative to the constraints of not changing the value of his budget. If such an exchange is possible there is some plausibility for its being performed. And after exchange has taken place there will be no need – at least for a short time – to exchange further. In this period the real system will be an immediate model of the axioms: the axioms will be satisfied for the actual distribution.

It may be noted that we have not required all markets to be cleared. This axiom is a special axiom which can be added to those of (D3) but, we think, it does not belong to the core of those basic axioms to be satisfied in all micro-economic situations. We will introduce this axiom in Sec. VII by means of a specialization. Note also that E may contain several members. In models as described by (D3) the mathematical form of U is not specified so that models with different equilibrium distributions are possible. There is no reason to require exactly one such distribution from the beginning.

III. SOME THEOREMS

We will summarize here some results about the formal theory just described. The first three theorems are well known in economics. Our main concern with these is to formulate them precisely and to adjust them to our terminology. A further theorem will clarify the logical relations of the basic concepts.

Let us denote by U_i the function $U_i:\mathbb{R}^m \to \mathbb{R}$, defined by $U_i(\alpha_1, \ldots, \alpha_m) := U(i, \alpha_1, \ldots, \alpha_m)$. Let us further denote by D_jU_i the j-th partial derivation of U_i and by $\frac{\partial g}{\partial \kappa}(\tau, \kappa)$ the functional matrix $\left(\frac{\partial g_i}{\partial \alpha_j}(\tau, \kappa)\right)_{i,j \leq m}$ at $\langle \tau, \kappa \rangle$ for $g:\mathbb{R}^{2m} \to \mathbb{R}^m$, $\tau = \langle \alpha_1, \ldots, \alpha_m \rangle$ and $\kappa = \langle \beta_1, \ldots, \beta_m \rangle$.

T1 If $x = \langle \mathscr{F}, G, \tilde{q}\ q^0, p, U, E \rangle$ is a model of PEE and $q \in E$ is such that for all $i \in \mathscr{F}$ and $g \in G$: $q(i, g) > 0$ then there exist real numbers λ_i such that for all $g \in G$: $D_gU_i(q(i, 1), \ldots, q(i, m)) = \lambda_i p(g)$

Proof: (1) $U_i:\mathbb{R}^m \to \mathbb{R}$ is continuously differentiable. Define $g:\mathbb{R}^m \to \mathbb{R}$ by $g(\alpha_1, \ldots, \alpha_m) = \sum_{j \leq m} p(j)\ (\alpha_j - q^0(i, j))$. Then (2) g is continuously differentiable, too. Let $\tau := \langle q(i, 1), \ldots, q(i, m) \rangle$. We have $\langle D_1g(\tau), \ldots, D_mg(\tau) \rangle = \langle p(1), \ldots, p(m) \rangle$, so (3) $\langle D_1g, \ldots, D_mg \rangle$ has maximal range at τ. By (D2–b) and (D3–4): (4) $g(\tau) = 0$. Now let $\langle \alpha_1, \ldots, \alpha_m \rangle$ be such that $g(\alpha_1, \ldots, \alpha_m) = 0$ and $\alpha_j > 0$ for $j = 1, \ldots, m$. There exists a $q' \in B_x$ such that $q'(i, j) = \alpha_j$ for $j = 1, \ldots, m$. By (D3–5) $U_i(\alpha_1, \ldots, \alpha_m) \leq U_i(q(i, 1), \ldots, q(i, m))$. So (5) U_i, restricted to the set $\{\kappa / g(\kappa) = 0 \wedge \kappa > 0\}$ has a maximum at point τ. (T1) now follows from (1)–(5) and a well known theorem on extrema under subsidiary conditions (see e.g. (12), p. 350).

The expression

$$RS(i, j, k, q) := \frac{D_jU_i(q(i, 1), \ldots, q(i, m))}{D_kU_i(q(i, 1), \ldots, q(i, m))}$$

is called the rate of substitution of individual i for commodities j and k. (T1) implies the standard result that in equilibrium the rates of substitution are the same for all individuals and are equal to the price ratios of the respective commodities.

T2 If $\langle \mathscr{F}, G, \tilde{q}\ q^0, p, U, E \rangle$ is a model of PEE and $q \in E$, $q > 0$ then for all $i, i' \in \mathscr{F}$ and $j, k \in G$:
$RS(i, j, k, q) = RS(i', j, k, q) = p(j)/p(k)$

Proof: Obvious from (T1).

T3 Let $\langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E\rangle$ be a model of PEE, $i \in \mathscr{F}$, $q \in E$, $q > 0$ and let U be such that for $\lambda \in \mathbb{R}$ the function g_λ: $\mathbb{R}^{2m} \to \mathbb{R}^m$, defined by $g_\lambda(\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_m) := \langle D_1 U_i(\beta_1, \ldots, \beta_m) - \lambda\alpha_1, \ldots, D_m U_i(\beta_1, \ldots, \beta_m) - \lambda\alpha_m\rangle$ satisfies the following condition:

$$\det\left(\frac{\partial g_\lambda}{\partial \kappa}(p(1), \ldots, p(m), q(i, 1), \ldots, q(i, m))\right) \neq 0.^5$$

Then there exists a function f defined on a neighbourhood of $\langle p(1), \ldots, p(m)\rangle$ such that $f(p(1), \ldots, p(m)) = \langle q(i, 1), \ldots, q(i, m)\rangle$

Proof: By (T1) there is $\lambda \in \mathbb{R}$ such that $\forall g \in G(D_g U_i(q(i, 1), \ldots, q(i, m)) = \lambda p(g))$. Choose such a λ and consider $g := g_\lambda$ as defined above. Then (1) $g(p(1), \ldots, p(m), q(i, 1), \ldots, q(i, m)) = 0$. (D1–6) implies that g is continuously differentiable at point $\langle p(1), \ldots, p(m), q(i, 1), \ldots q(i, m)\rangle$. There are neighbourhoods V of $\langle p(1), \ldots, p(m)\rangle$ and W of $\langle q(i, 1), \ldots, q(i, m)\rangle$ such that (2) g is continuously differentiable on $V \times W$ and (3): $\det\left(\frac{\partial g}{\partial \kappa}(\tau, \kappa)\right) \neq 0$ for all $\langle \tau, \kappa\rangle \in V \times W$. From (1)–(3) it follows by the theorem on implicit functions (e.g. (11), p. 277) that there are neighbourhoods $V_0 \subseteq V$ of $\langle p(1), \ldots, p(m)\rangle$ and $W_0 \subseteq W$ of $\langle q(i, 1), \ldots, q(i, m)\rangle$ and a unique function $f: V_0 \to W_0$ such that for all $\tau \in V_0$: $g(\tau, f(\tau)) = 0$. Especially, this implies $g(p(1), \ldots, p(m), f(p(1), \ldots, p(m))) = 0$ and, since f is unique, $f(p(1), \ldots, p(m)) = \langle q(i, 1), \ldots, q(i, m)\rangle$.

T4 In PEE the terms $\tilde{q}$, q^0, p, U and E are mutually independent from each other.

Proof: Let $x = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E\rangle$ be defined as follows: $\mathscr{F} := \{i, i'\}$, $G := \{1, 2\}$, $\tilde{q}(1) = \tilde{q}(2) = 1$, $q^0(j, k) = 1/2$ for $j \in \mathscr{F}$ and $k \in G$, $p(1) = p(2) = 1$. $E = \{q^0\}$ and for $i \in \mathscr{F}$ let U_i:$\mathbb{R}^2 \to \mathbb{R}$ be defined by $U_i(\alpha, \beta) = \alpha + \beta$. Clearly, x is a model of PEE. Now we have to show that for $\gamma \in \{\tilde{q}, q^0, p, U, E\}$ there exists a $\gamma' \neq \gamma$ and an y such that (1) y is a model of PEE, (2) γ' is a component of y and γ' occurs in y at the same place at which γ

occurred in x, and (3) all components $\in \{\tilde{q}, q^0, p, U, E\}$ different from γ are the same in x and in y.

(1) $\gamma = \tilde{q}$. Define $\gamma' := \tilde{q}'$ by $\tilde{q}'(k) = \tilde{q}(k) + 1$ for $k \in G$. Then $y = \langle \mathscr{F}, G, \tilde{q}', q^0, p, U, E \rangle$ is a model of PEE.

(2) $\gamma = q^0$. Let $q^{0\prime}(i, 1) = q^{0\prime}(i', 2) = 1/4$ and $q^{0\prime}(i, 2) = q^{0\prime}(i', 1) = 3/4$. Then $y = \langle \mathscr{F}, G, \tilde{q}, q^{0\prime}, p, U, E \rangle$ is a model of PEE.

(3) $\gamma = p$. Let $p'(1) = p'(2) = 2$. Then $y = \langle \mathscr{F}, G, \tilde{q}\ q^0, p', U, E \rangle$ is a model of PEE.

(4) $\gamma = U$. For $j \in \mathscr{F}$ define $U_j': \mathbb{R}^2 \rightarrow \mathbb{R}$ by $U_j'(\alpha, \beta) = \alpha\beta$. It is easily checked that $y = \langle \mathscr{F}, G, \tilde{q}\ q^0, p, U', E \rangle$ is a model of PEE.

(5) $\gamma = E$. Let $E' = \{q^0, q^{0\prime}\}$ where $q^{0\prime}$ is defined as in clause (2). Then $y = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E' \rangle$ is a model of PEE.

Note that $\mathscr{F}$ and G are not independent from the other concepts just because they are contained, for instance, in q^0 as components of q^0's domain.

IV. THEORETICAL CONCEPTS

We now turn to the question which of the basic concepts of PEE are theoretical with respect to PEE and which are not. According to Sneed component t of theory T is theoretical with respect to T iff every method of determination of t already presupposes that T is true (for details see (2)). If t is a function (or a predicate) the method of determination consists of finding out the function value $t(x_1, \ldots, x_n)$ (the truth value $t(x_1, \ldots, x_n) \in \{W, F\}$) for given arguments $x_1, \ldots, x_n$. And that such a method presupposes T to be true means that the axioms describing a measuring apparatus of the relevant form logically imply the basic axioms of T. This criterion of theoreticity has been successfully applied to several theories and need not be defended here. What are the results of applying it in the present case?

Clearly, $\mathscr{F}$ and G are PEE-non-theoretical. In order to determine whether something is a person or a kind of commodity we neither make use of nor presuppose the principle of maximization of utility[3]. It is also clear that quantities of commodities belonging to distinct persons can be determined without presupposing economical axioms. So $\tilde{q}$ and q^0 are PEE-non-theoretical, too.

In case of p we have to ask: How can we determine $p(g)$ for a given kind of commodity g? The first answer which refers to the realistic situation of a market is that we just have to ask the merchants, i.e. those individuals which offer commodities. But this method might yield different prices for g due to different answers of different merchants. This conflicts with our treatment of p as independent from the individuals. In order to obtain *one* price to which all individuals would subscribe it seems that we have to presuppose a state of economical equilibrium for only in equilibrium different individuals will agree on one price (compare T2) of Sec. III). This would suggest p being PEE-theoretical. There is, however, a PEE-independent method of determination for p. One has to observe the quantities people exchange. For two commodities g and g' and two persons i and i' we can calculate the ratio of the quantities of g and g' actually exchanged by i and i'. This ratio might be called 'individual price ratio of g and g' for i and i'. From individual price ratios we can come to absolute individual prices by convention, i.e. by declaring the price of a unit of some fixed commodity to be one. Now we take the arithmetical means of all the individual prices of any commodity to be the price of that commodity. This method certainly leads to prices even in situations where the principle of maximization of utility is not realized. So we have a method of determination not presupposing PEE and therefore p is PEE-non-theoretical[4].

The determination of utility is still more involved. How can we determine $U(i, \alpha, \ldots, \alpha_m)$? This question leads to a far reaching methodological issue which in this paper can only be outlined. A detailed treatment must be postponed to another paper for reasons of space.

Our claim is that U is PEE-theoretical. In order to defend this claim we have to make plausible that *all* methods of determination for U presuppose PEE. This, of course, cannot be logically proved for we have no axiomatic description of the class of all methods of determination for U. But still there are good arguments in favour of this claim. We distinguish two classes of methods of determination for U which seem to be complete insofar as all known methods belong to one of them.

Methods of determination of the first class work as follows. One presupposes U's mathematical form. The determination of $U(i, \alpha_1, \ldots, \alpha_m)$ then amounts to specify the arguments $i, \alpha_1, \ldots, \alpha_m$ and to *calculate* the function value $U(i, \alpha_1, \ldots, \alpha_m)$. If, for instance, U has the form

$$U(i, \alpha_1, \ldots, \alpha_m) = \sum_{k=1}^{m} \beta_k \log(\alpha_k - \delta_k)$$

and the real numbers $\beta_1, \ldots, \beta_m, \delta_1, \ldots, \delta_m$ are given then, in fact, $U(i, \alpha_1, \ldots, \alpha_m)$ can be calculated by purely mathematical means. Such methods of determination presuppose PEE in the following sense. In order to be accepted as a reasonable method of determination for *utilities* (and not just for abstract real numbers of the form $U(i, \alpha_1, \ldots, \alpha_m)$) the mathematical form of U has to be 'economically reasonable'. But for such economically reasonable U's one can prove the existence of equilibrium distributions[6]. That is, 'reasonable economical' form of U implies PEE's axioms. So all methods of the first class presuppose PEE and therefore suggest U being PEE-theoretical. Intuitively, these methods presupposing U to be known in advance were felt to be unsatisfactory because it is unclear how U's special form is found.

Methods of determination from the second class were developed in order to solve this problem. One was looking for methods of determining U without presupposing PEE. The basic idea is to start with a notion of preference and to formulate axioms sufficiently strong to allow a *definition* of U in terms of preferences. Thus the problem of determination of U is reduced to determining truth values of sentences of the form 'Person i prefers quantity α of g to quantity α' of g''. Intuitively, it seems that there are methods of determination of the latter kind which do not pressupose PEE, for to prefer one thing to another seems to be something different from maximizing one's utility. Thus carrying out this idea – which meanwhile has been done successfully – seems to yield a proof of U's non-theoreticity in PEE. There seem to be methods of determination for U where we have to check only preferences and where U's values can be derived from the results obtained about preferences. This observation has led to a decrease of interest in utility functions among economists. The prevailing tendency seems to be that utility can be dispensed with, and that it makes more sense to 'stay closer to the facts' by investigating preferences or developing econometrical theories.

But the conclusion from the existence of theories of preference allowing definitions of utility to U's PEE-non-theoreticity turns out to be too rash. For by looking closer at the relation between theories of preference and PEE it turns out that theories of preference can be reduced to PEE in the precise sense of 'strong reduction' of [3]. But such a reduction yields a strong logical connection between both theories which gives rise to a weaker version of 'presuppose' as used in Sneed's criterion of theoreticity.

Term t in this weaker sense is T-theoretical if every method of determination for t either presupposes T in the old sense or yields a model of some theory T' which can be reduced to T. This weaker version has been introduced and discussed in [2] where its plausibility was demonstrated by two examples from classical mechanics. Now according to this weaker criterion U remains PEE-theoretical despite the fact that U can be defined via 'underlying' theories of preference. For all reasonable methods of determination of preference can be shown to imply the basic axioms of suitable theories of preference. And since the latter can be reduced to PEE the weak form of the criterion is satisfied. Intuitively, this is so because in order to determine whether person i prefers x to y 'presupposes' that i be a rational economic agent – trying to obtain those things he or she mostly prefers – and this in turn is just another form of stating that i maximizes its utility. We conclude that U is PEE-theoretical (in the weaker sense).

U's theoreticity implies that of E. For members of E can be determined only by means of the theory: we have to determine whether q satisfies the axioms in order to determine whether q belongs to E or not. But in order to demonstrate that q satisfies the axioms we must presuppose U and therefore PEE.

V. HOW TO USE THE FORMALISM

The considerations of Sec. IV give rise to a discussion of the actual use of PEE's models made by economists. For if our claim of U's PEE-theoreticity is correct some widely accepted uses of PEE are submitted to considerable constraints.

The first use of PEE's models is this. One presupposes the utility functions as given and uses the axioms in order to calculate or to predict the quantities of commodities which will be exchanged provided the system will attain a state of equilibrium. That is, starting from a given U economists predict the individual demands and offers to be expected if the system comes into equilibrium. These predictions essentially use (T3) of Sec. III. For (T3) says, given U, p and suitable conditions of 'well-behavedness' of U an equilibrium distribution is uniquely determined (by the f in T3)) by the prices.

Another way of using PEE is to presuppose U and to calculate or predict equilibrium prices from given distributions. That is, on the assumption that U is known and has special properties, and that q is given and the system will attain equilibrium economists calculate the prices that will obtain in equilibrium. This use is possible because of (T2) which determines price-ratios in terms of utilities.

Both these uses are mentioned in a typical quotation from [6], p. 126: 'theoretical analysis contains data, variables, and behaviour assumptions that allow the determination of specific values for the variables once the data are known'. And

> The data for the determination of a general multimarket equilibrium are the utility... functions of all consumers and their initial... endowments of... commodities. The variables are the prices of all... commodities and the quantities purchased and sold by each consumer... The behaviour assumptions require utility... maximization with the condition that every market be cleared. ([6], p. 127).

Finally PEE can be used to determine utility functions. That is, assuming q^0 and p to be given one tries to find utility functions which – when added to q^0 and p – yield a non-empty set of equilibrium distributions. A fourth use of the formalism differs from the last one only in checking whether the equilibrium distributions predicted by PEE in fact coincide with those actually being reached after exchanges. All these four uses are compatible with our formulation which therefore can be said to be 'neutral to actual usage'.

It is illuminating to compare these uses to the use of the axioms of, say, classical mechanics. These axioms in a crude way can be rephrased by a differential equation

$$(+) \qquad m\ddot{s} = f(s)$$

where s is the position function (representing a particle's path), and m and f denote mass and force, respectively. Physicists often use this equation by presupposing m and f and solving it in order to obtain 'admissible' position functions. This corresponds to the first use of PEE. Second, they also try to find out m by presupposing s and f as given – which corresponds to PEE's second use. And third, they start with s and try to see whether m and f can be found satisfying $(+)$ above. This corresponds to the third and fourth use of PEE.

Now the fact of U's being PEE-theoretical (or, similarly, of m and f being theoretical in mechanics) imposes restrictions on the first two uses: to use PEE in these two ways does not yield empirical claims. In other words: starting from given utilities is not compatible with claiming to arrive by using PEE at empirical statements or empirical predictions. Or still more sharply: those uses of PEE have only intra-theoretical value comparable to the value of mathematical proofs in mathematics.

This is a strong offence but as soon as one accepts the PEE-theoreticity of U one is forced to surrender. The argument runs like this. The derivation of prices or quantities from given utilities by means of PEE can yield empirical statements or predictions only if PEE is applied to a concrete system and if it is possible to test that this system is a model of PEE. But to test whether some system is a model of PEE it is necessary to determine the values of the functions occurring in PEE, especially of U. Now determination of the values of U presupposes – because of the theoreticity criterion – that the process of determination already is a model of PEE. If one wants to test whether this process of determination indeed is a model of PEE one is in the same situation as at the beginning. That is, the attempt to test PEE's axioms in concrete systems ends in circularity. In the two uses of PEE mentioned above – where U is assumed to be given – this amounts to the following. Statements arrived at in this way can be called empirical only if it can be empirically tested that U in fact has the form it is assumed to have. But the test of U immediately leads to a circle of the kind just described. This is a special instance of what Stegmüller has called the problem of theoretical terms in his [9].

The only uses of PEE which are free from this difficulty are those in which q^0 and p are 'observed' and U is 'calculated' in a way to fit the axioms. This can indeed lead to testable or empirical statements for 'observing' q^0 and p does not imply circularities of the above kind. But what do these empirical statements look like? Roughly, they say that for given q^0 and p there *exists* – there 'can be found' – a utility function such that q^0, p and U satisfy the axioms. This certainly is not the typical form of what was thought to be an empirical statement. However, if U is PEE-theoretical then only statements of this form can be called empirical, provided we do not want 'empiricity' to include 'circularity'.

More precisely, the only use of PEE that *yields an empirical claim* is the following. We consider those structures obtained from potential models by omitting the theoretical terms U and E. They are called partial potential models and defined as follows.

D4 y is a *partial potential model of PEE* ($y \in M_{pp}$) iff there exist $\mathscr{F}$, G, $\tilde{q}$, q^0 and p such that

(1) $y = \langle \mathscr{F}, G, \tilde{q}, q^0, p \rangle$

(2) requirements (2)–(5) of (D1) are satisfied

Now a class of concrete economical systems is specified: the class of PEE's intended applications, denoted by I. (The problems connected with I will be discussed in the following Section.) Let us for the moment assume that there is a class of intended applications of PEE. There are no difficulties in further assuming that these intended applications can be described as partial potential models. So we may assume that there is a set $I \subseteq M_{pp}$ of intended applications. Now we can use PEE in order to formulate an empirical claim as follows: for each intended application y there exists a utility function U such that U together with y satisfies the axioms and yields a non-empty set of equilibrium distributions. A claim of this form can be called 'empirical' because intuitively its truth depends on how the intended applications look. We can think of intended applications for which suitable utility functions exist, and we can think of intended applications – with the qualifications to follow in Sec. VI – for which there is no utility function satisfying PEE's axioms.

VI. IS PEE 'PURE' OR 'EMPIRICAL'?

Even if we exclude those uses of PEE which start from utility functions, and concentrate on the use of PEE as a means of finding the 'right' utilities there still is the question of empiricity. The question 'Is PEE an empirical theory?' or 'Does PEE have empirical content?' cannot be settled by a straightforward "yes" in the face of the lack of – or very slow progress in the formulation of – true predictions. Although empirical claims of the form sketched at the end of Section V are free from intrinsic difficulties there are reasons for doubt. Before discussing these let us precisely reformulate the empirical claim.

D5 (a) If $x = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E \rangle \in M_p$ then $r(x) := \langle \mathscr{F}, G, \tilde{q}, q^0, p \rangle$

(b) $\bar{r}(M) := \{y \in M_{pp} / \exists x \in M(r(x) = y)\}$

(c) The empirical claim of PEE formulated with M and I is that: $I \subseteq \bar{r}(M)$

r just cuts off theoretical terms from models of PEE. The empirical claim then says that the intended applications can be augmented by theoretical terms such that one obtains a set of models. Conversely, if we call $r(x)$ the 'reduct' of x then (D5-c) says that every intended application is a reduct of some *model.*

Now the first reason to doubt PEE's empiricity comes from the following theorem.

T5 For every $Y \subseteq M_{pp}$: $Y \subseteq \overline{r}(M)$

Proof: We have to show that for $y = \langle \mathscr{F}, G, \tilde{q}, q^0, p \rangle \in M_{pp}$ there exist U and E such that $x = \langle \mathscr{F}, G, \tilde{q}\ q^0, p, U, E \rangle \in M$. We take $E = \{q^0\}$, then $E \neq \emptyset$ and $E \subseteq B_x$. So all we have to do is to find a function U such that (D1–6) and (D3–5) are satisfied. This can be done by defining functions $U_i : \mathbb{R}^m \to \mathbb{R}$ such that (D1–6) holds and each U_i has an *absolute* maximum at point $\langle q^0(i, 1), \ldots, q^0(i, m) \rangle$. Such functions are found easily. For instance, we can take the well known

$$U_i(\alpha_1, \ldots, \alpha_m) := \frac{1}{\sqrt{(2\pi)^m}}\, e^{-\frac{1}{2}\sum_{k \leq m}(\alpha_k - q^0(i,k))^2}$$

(T5) shows that the empirical claim formulated with M and I is true no matter how I looks. If we take an arbitrary economical system and describe it as a partial potential model we are always sure to find utility functions which satisfy PEE's axioms. In this sense the empirical claim formulated with I and M is trivial or empirically empty.

It is not convincing, however, to use this fact as an argument against PEE's empiricity. For we have similar situations in well-established physical theories – as for instance classical mechanics and classical equilibrium thermodynamics – which nobody would hesitate to call empirical. The general situation of mature empirical theories seems to be that empirical claims formulated with the very basic axioms only turn out to be empty in the way made precise by (T5). In the case of physical theories this triviality is eliminated by adducing to the basic axioms two further kinds of requirements: constraints and special laws. Constraints are requirements 'across' different models while special laws are additional requirements to hold in models which already satisfy the basic axioms.

Concerning constraints in economics we cannot refer to explicit statements of economists because there are none. Intuitively, constraints say how things behave if they are 'transported' from one application (or model) to a different one. Usually one expects that 'instrinsic' properties do not change from system to system. The classical example stems from physics. There, the mass of a body is an intrinsic property which does not change, at least in the classical theory, when the body is transported to a different application. Constraints are necessary only if we want to take serious the idea of a theory having many different 'local' applications – and not only one big universal application.

The only reasonable candidate in PEE is an equality constraint for the individual utility functions U_i defined by $U_i(\alpha_1, \ldots, \alpha_m) = \mathrm{U}(i, \alpha_1, \ldots, \alpha_m)$. An equality constraint requires that something – namely U_i in the present case – remains equal in different systems. It seems natural to think of utility as something intrinsic to the economic agent. If an individual is given a certain bundle of commodities the utility depends on the quantities of these commodities and on nothing else. It does not change, for instance, when the individual emigrates to another economical system (country). This, it might be argued, contradicts to what actually happens because it is well known that people's tastes change when people change their surroundings. There are two aspects of this objection. First, change is something we cannot express in the present vocabulary. What can we do to deal with such phenomena if we do not want to wait until sociologists provide a successful theory of how people's tastes change? Are we to give up the theory or will the theory still be of some value even if we exclude such phenomena? This is of course the question of whether the 'ceteris paribus' condition is acceptable. The reader will not expect a decisive answer to this question. The second aspect of the objection refers to the concept of time implicitly used when speaking of change. One might argue that PEE is static or very 'local' with respect to time. So changes which evidently take a lot of time are not relevant for the theory. But this reveals a wrong conception of the role of constraints. Even in static theories constraints cover dynamical aspects. Two different applications in which the same individual occurs may be separated by quite a big interval of time. Indeed, it will always take time for a person to travel from one application (place) to another.

We will assume for the moment that ceteris can be paribus and formulate an equality constraint for the functions U_i.

D6 C is the *constraint for PEE* iff for all $\mathfrak{X}$:
$\mathfrak{X} \in C$ iff $\mathfrak{X} \subseteq M_p$ and
$\forall x, x' \in \mathfrak{X} \forall i \forall q: x = \langle \mathcal{F}, G, \tilde{q}, q^0, p, U, E \rangle \wedge x' = \langle \mathcal{F}', G', \tilde{\tilde{q}}, q^{0\prime}, p', U', E' \rangle \wedge i \in \mathcal{F} \cap \mathcal{F}' \wedge q \in Z_x \cap Z_{x'} \rightarrow U(i, q(i, 1), \ldots, q(i, m)) = U'(i, q(i, 1), \ldots, q(i, m))$.

The elements $\mathfrak{X}$ of C can be imagined as combinations of models a combination being characterized by the fact that the utilities of a person i occurring in different models at the same time are the same in all those models. Such combinations of models come up either successively in time or if different models are considered which are sub-systems of each other, e.g. a town and a country.

Using this constraint the empirical claim of PEE defined in (D5) can be sharpened to an empirical claim with respect to I, M and C.

D7 (a) $A(M, C) := \{Y / Y \subseteq M_{pp} \wedge Y \subseteq \bar{r}(M) \wedge \exists X(X \in C \wedge \bar{r}(X) = Y)\}$

(b) the empirical claim of PEE with respect to I, M and C is that $I \in A(M, C)$

(b) says that I consists of reducts of models and, in addition, I is a combination of reducts such that the theoretical augmentations form a combination satisfying the constraint. For this stronger claim we cannot prove a theorem analogous to (T5). On the contrary, the following seems to be true.

T6 Not for every $Y \subseteq M_{pp}$: $Y \subseteq A(M, C)$

Although we have no formal proof of (T6) we are rather convinced of its being true. Intuitively, if Y is infinite and even non-denumerable a construction similar to that in the proof of (T5) does not seem to work any longer.

So the first reason to doubt PEE's empiricity, namely the triviality of the empirical claim formulated with M and I can be devaluated by assuming the ceteris paribus condition to make sense in PEE. If this assumption is taken for granted then the non-triviality of the empirical claim $I \in A(M, C)$ will depend on the special form of I.

Here there is a second difficulty for PEE's empiricity. For there is doubt whether PEE in fact has something like a set of intended applications. The general method of fixing *I* is to list some 'paradigm' elements of *I* – e.g. the city of Nürnberg on a market day in the 15th century – and to say that all other elements of *I* be sufficiently similar to those paradigm elements. All elements of *I* must of course describe real systems. But in economics we cannot point out a single real, concrete system which is commonly accepted by economists to be a standard example of PEE. There are those who will say that since there are no accepted intended applications PEE is no empirical theory at all but rather a 'pure' theory. A pure theory is a theory in which intended applications and empirical claims do not matter. Pure theories are not constructed in order to formulate empirical claims, they are constructed for other reasons. In pure theories also there is no need to draw a distinction between theoretical and non-theoretical terms which is relevant only for the formulation of empirical claims. By looking on what economists actually do one feels strongly inclined to agree that they, in fact, construct pure theories. Studies in the historical development of economical theories (e.g. (5)) enforce this view.

But on the other hand – and here we leave the position of descriptive philosophy of science – by reflecting on why this is so one is lead to entertain the idea that an empirical form of PEE at least should be possible. There are three reasons why PEE has no intended applications. First, the description of economical systems is very complex. Although the picture of our medieval village gives an impression of simplicity the situation is still much more complex than, for instance, in models of physics. Even the most simple models will contain a number of kinds of commodities and persons, and hence a considerable number of quantities of commodities. Complexitity becomes really relevant if we think of present day systems with their huge numbers of commodities and persons. Second, even if there were simple models in history the relevant data are not recorded and cannot be fully reconstructed from the historical material available. Third, and most important, it seems impossible to reproduce even the most simple economical system in a way coming close to the original.

These three reasons up to now have been sufficient to suppress a development of intended applications. But it seems not too fictitious to think of a state of affairs in which these difficulties can be overcome to a certain extent. With the help of computers very complex data structures can be recorded and stored for a long time. And if complexity is no longer the main difficulty we can think of the possibility to recognize similar 'initial conditions' in different systems although we might not be able to 'reproduce' any system.

VII. SPECIALIZATIONS

We conclude by giving some examples of specializations of the basic core of PEE described so far. Specializations consist essentially of special laws which are added to those of (D3). This yields another way to come to stronger empirical claims – if empiricity is possible at all – or at least to stronger theoretical models. Before describing these specializations let us recall what might be called the basic core of PEE.

D8 (a) K is the *basic core* of PEE iff $K = \langle M_p, M, M_{pp}, C\rangle$
(b) T is the *basic theory-element* of PEE iff $T = \langle K, I\rangle$

We speak of a 'theory-element' because PEE consists of more than just T. Roughly, PEE consists of a net of elements of the same form as T such that T provides a basis for the whole net. If PEE is regarded as a pure theory then (D8-b) is not relevant.

Specializations of K and T can be defined quite generally as follows.

D9 (a) K' is a *core-specialization* of K ($K' \sigma_c K$) iff there exist M' and C' such that
(1) $K' = \langle M_p, M', M_{pp}, C'\rangle$
(2) $M' \subseteq M$
(3) $C' \subseteq C$
(b) T', is a *specialization* of T ($T' \sigma T$) iff there exist K' and I' such that
(1) $T' = \langle K', I'\rangle$
(2) $K' \sigma_c K$
(3) $I' \subseteq I$
(c) if $T_1 \sigma T$ and $T_2 \sigma T$ then T_1 is a specialization of T_2 ($T_1 \sigma T_2$) iff $T_1 = \langle\langle M_p, M_1, M_{pp}, C_1\rangle, I_1\rangle$ and $T_2 = \langle\langle M_p, M_2, M_{pp}, C_2\rangle, I_2\rangle$ and $M_1 \subseteq M_2 \wedge C_1 \subseteq C_2 \wedge I_1 \subseteq I_2$

Specializations of K and T consist just of restrictions of the set of models M, and eventually of suitable restrictions of the constraints and intended applications. In (D9-c) the notion of specialization is extended to arbitrary 'theory-elements' of the form of T which are specializations of T.

A first specialization is given by the special law expressing that in equilibrium all markets are cleared. We add this law as a further axiom to those of (D3) thereby obtaining a subset M_1 of M.

*D*10 (a) x is a model of PEE *with all markets cleared* ($x \in M_1$) iff

(1) $x = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E \rangle \in M$

(2) $\forall g \in G \, \forall q \in E \, (\sum_{i \in \mathscr{F}} q(i, g) = \sum_{i \in \mathscr{F}} q^0(i, g))$

(b) $K_1 := \langle M_p, M_1, M_{pp}, C \rangle$ and $T_1 := \langle K_1, I_1 \rangle$ where I_1 is a suitable set of intended applications for K_1

In models of M_1 the total amount of commodity g as initially distributed among the individuals $\sum_{i \in \mathscr{F}} q^0(i, g)$ is the same after exchange for all commodities. Thus everything offered has found a demand.

A second specialization is given by requiring utilities to increase with increasing consumption of commodities.

*D*11 (a) x is a model of PEE *with increasing utilities* ($x \in M_2$) iff

(1) $x = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E \rangle \in M$

(2) $\forall i \in \mathscr{F} \forall q, q' \in Z_x$: $(\forall g \in G(q(i, g) < q'(i, g))) \rightarrow U(i, q(i, 1), \ldots, q(i, m)) < U(i, q'(i, 1), \ldots, q'(i, m))$

(b) $K_2 := \langle M_p, M_2, M_{pp}, C \rangle$ and $T_2 := \langle K_2, I_2 \rangle$ where I_2 is a suitable set of intended applications for K_2

If person i's bundle of commodities is increased from $\langle q(i, 1), \ldots, q(i, m) \rangle$ to $\langle q'(i, 1), \ldots, q'(i, m) \rangle$ by increase of all quantities then utility will increase, too. This is a special law which will not be satisfied in all of PEE's applications. For instance, by consuming ever more ice cream utility does not necessarily increase. A further specialization consists of the law of decreasing marginal utility.

*D*12 (a) x is a model of PEE *with decreasing marginal utilities* ($x \in M_3$) iff

(1) $x = \langle \mathscr{F}, G, \tilde{q}, q^0, p, U, E \rangle \in M$

(2) $x \in M_2$

(3) $\forall i \in \mathscr{F} \forall q, q' \in Z_x: (\forall g \in G(q(i, g) < q'(i, g))) \to \forall j \in G(D_j U(i, q(i, 1), \ldots, q(i, m)) < D_j U(i, q'(i, 1), \ldots, q'(i, m)))$

(b) $K_3 := \langle M_p, M_3, M_{pp}, C \rangle$ and $T_3 := \langle K_3, I_3 \rangle$ where I_3 is a set of intended applications for K_3

$D_j U$ denotes the partial derivative of U with respect to its $j + 1$-th argument. (2) requires U to increase with increasing quantities but (3) says that this increase becomes ever smaller. If the original bundle of commodities becomes bigger and bigger then an increase of this bundle will yield smaller and smaller increases of utilities. For a person with average salary 1000 \$ in addition will yield a considerable increase of utility. A person owing already 50 000 000 \$ will hardly consider 1000 \$ to affect his or her utility.

A final specialization requires U to have a very special mathematical form already mentioned in Sec. IV: the form of a so called Stone-Geary function.

*D*13 (a) x is a model of PEE *with Stone-Geary function* ($x \in M_4$) iff

(1) $x = \langle \mathscr{F}, G, \hat{q}, q^0, p, U, E \rangle \in M$

(2) $\forall i \in \mathscr{F} \exists \beta_{i1}, \ldots, \beta_{im}, \delta_{i1}, \ldots, \delta_{im} \forall \alpha_1, \ldots, \alpha_m \; U(i, \alpha_1, \ldots, \alpha_m) = \sum_{j \leq m} \beta_{ij} \log (\alpha_j - \delta_{ij})$

(b) $K_4 := \langle M_p, M_4, M_{pp}, C \rangle$ and $T_4 := \langle K_4, I_4 \rangle$ where I_4 is a set of intended applications for K_4

In Stone-Geary functions U can be determined as follows. First, for a single commodity g the utility depends on the logarithm of the quantity $\alpha_g - \delta_g$. So utility increases with increasing quantity and the marginal utility decreases. δ_g denotes a minimal amount of commodity g which is necessary for person i to survive. Only if i has at least quantity δ_g of commodity g its utility is defined, if i has less the δ_g of g then i will not participate in the economic system: i will be dead. This implies a little formal difficulty with U's domain which here is neglected. Second, each such utility is multiplied with a 'weight' β_g expressing the 'importance' of commodity g with respect to the other commodities. Finally, all these utilities are summed up in order to yield the total utility.

The following theorem is obvious.

T7 (a) for $i = 1, \ldots, 4$: $K_i \sigma_c K$ and $T_i \sigma T$
(b) $T_4 \sigma T_3$ and $T_3 \sigma T_2$, especially: $K_4 \sigma_c K_3$ and $K_3 \sigma_c K_2$

We thus have a small 'theory-net' of specializatoons of PEE's basic theory-element. A theory-net in this special context can be defined as follows.

*D*14 X is a *theory-net* over T iff there exist N and σ such that
(1) $X = \langle N, \sigma \rangle$
(2) N is a finite set of specializations of T
(3) $\sigma \subseteq N \times N$ is the specialization relation on N defined in (D9-b) and (D9-c)
(4) $T \in N$
(5) for all $T' \in N$: $T' \sigma T$

It is clear that, in general, empirical claims formulated with specializations can be logically stronger than those formulated with the basic element. In physics, for instance, there are many specializations with non-trivial empirical claims. A last reason for doubt of PEE's being an empirical theory now consists of the observation that the situation in PEE is different from that in physics with respect to specializations. Whereas commonly accepted physical theories derive their empiricity from the existence of specializations with non-trivial empirical claims in PEE even the specializations are empirically trivial. For instance, we have the following theorem.

T8 For all $Y \subseteq M_{pp}$: $Y \subseteq \bar{r}(M_4)$

Proof: For each $y \in M_{pp}$ we have to find U and E such that $\langle y, U, E \rangle \in M_4$. This can be done by choosing $E = \{q^0\}$ and defining a Stone-Geary utility function U such that all U_i are identical and touch the hyperplane B_x exactly in point $\langle q^0(i, 1), \ldots, q^0(i, m) \rangle$. If q^0 is a point on the boundary of Z_x the domain of U has to be shifted suitably into the negative numbers. The calculation of the coefficients $\beta_1, \ldots, \beta_m, \delta_1, \ldots, \delta_m$ is a matter of routine.

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NOTES

[1] [7], p. 253. The authors there are dealing with classical mechanics. But the situation in economics is the same.

[2] m is just the number of kinds of commodities and uniquely given by G. We could have been more pedantic and have chosen m as a primitive instead of G. But G is chosen to suggest that kinds of commodities are entities different from natural numbers.

[3] Of course there may be cases where we exclude a certain person from being treated as economic agent (although being present in the economic system). For term t to be *not* T-theoretical, however, it suffices to find at least one independent method of measurement. So t can be T-non-theoretical although there may be quite a number of methods of determination for t which presuppose T.

[4] I am indebted to E. Händler at this point. Also I want to thank B. Hamminga and M. Küttner for helpful remarks on an earlier draft. The most simple method of determination for p is of course to observe that all individual prices are equal. But this method does not always work.

[5] This condition is satisfied if, for instance, det $((D_j D_\kappa U_i)_{j,\kappa \leq m}) \neq 0$ at point $\langle p(1), \ldots, p(m), q(i, 1), \ldots, q(i, m)\rangle$.

[6] Such existence theorems belong to the most subtle results of modern economics. The first theorem of this kind is due to Debreu.

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