Basic concepts and new methods of time series analysis in Historical Social Research

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Historical Social Research, Vol. 14 — 1989 — No. 1, 3-24

Basic Concepts and New Methods of Time Series Analysis in Historical Social Research

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Abstract: The main purpose of this lecture is the presentation of important concepts and tools of time series analysis in a nontechnical way. Therefore only elementary mathematical operations will be used. Proofs and a lot of details are ommitted completely. In a first part *»Time Series Models«* fundamentals and modelling of time series are discussed. The second part *»Filtering of lime Series«* is dedicated to filter-problems in time series analysis. The two parts are independent of each other.

TIME SERIES MODELS

I. Fundamentals

1. Introduction

A time series is a set of time-ordered observations of a process. Each observation should be an interval level measurement of the process and the time separating successive observations should be constant. Thus, by this definition, a time series is a discrete data set. For example, Figure 1 shows the plotted time series »Gross Fixed Capital Formation in the United Kingdom 1830-1979«, an annual series. In practice we can observe quarterly, monthly, weekly and even daily time series (like stock prices).

2. Some Objectives of Time Series Analysis

The social scientist is interested in making inferences about the process underlying a given time series. This implies that we consider a given time

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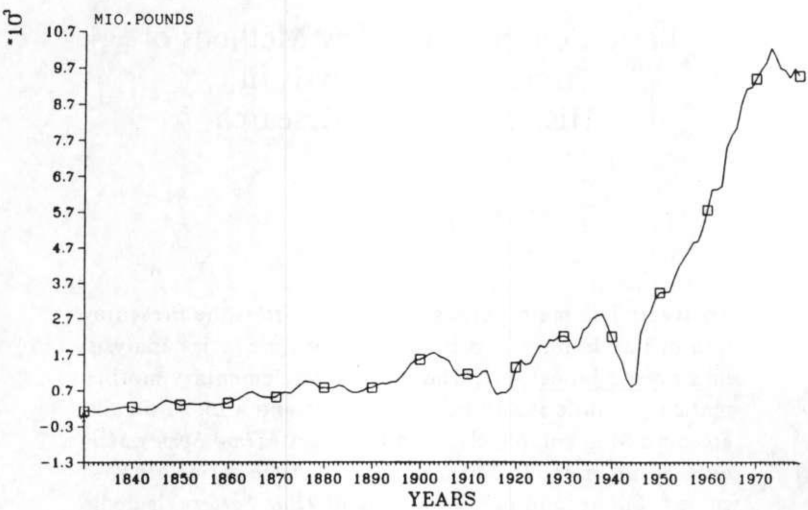


Fig. 1

series as a realization of an unknown process. If we succeed in identifying this unknown process, we can hope to »predict the future« of a social process. Furthermore, if we consider not only one time series but several series (a so-called »multivariate« instead of an »univariate« approach), we can hope to analyze dependencies between different social processes. This possibility seems to be especially promising for historical research.

Both in the univariate and in the multivariate approach an identification of the unknown underlying process is only possible if we succeed in »modelling« properly a given time series. For this to achieve, it is necessary to know some basic facts about stochastic processes which will be presented in the following.

II. Modelling of Time Series

1. Stochastic Processes

Simply spoken, a stochastic process is a sequence of random variables which are in general dependent (or correlated). In the special case where the random variables are uncorrelated, the process is called a »whitenoise«process. For such a process, the realization at a certain time point t_i

bears no informations about the next realization in t_{j+1}. Such a process is unpredictable. In general however, the correlation-structure of a stochastic process can be used for prediction.

Now, there are different types of stochastic processes. An important class of processes are the so-called »stationary« processes. They can be characterized by the fact that their mean and variance are constant for all time points. Besides that their correlation-structure is »homogeneous«, which means that the correlation of two variables at different time points does only depend on the length of the time-interval between the two time-points but not on the location of the two time-points on the time scale.

For »nonstationary«-processes one of these properties does not hold. As we mentioned above, we will consider a given time series as a realization of a stochastic process. The series of Figure 1 can certainly not considered to be a realization of a stationary process, since obviously both the mean and variance are not constant over time. This series contains a »trend«.

However, with the series plotted in Figure 2 we would have no problems to consider it as a realization of a stationary process.

There are certain »basic« stationary processes which prove to be very useful tools in building complex processes. There are the »autoregressive«- and the »movingaverage«processes. The simplest autoregressive process is given by the equation

$$(1) \qquad X_t = \alpha_1 X_{t-1} + u_t , \qquad t = 1,2, \ldots$$

where u_t denotes a white-noise-process. In order to be stationary the parameter a has to be smaller than one in absolute value. This process is called an autoregressive process of order 1 (in short: AR(1)). In general, an autoregressive process of order p (AR(p)) is given by

$$(2) \qquad X_t = \alpha_1 X_{t-1} + \alpha X_{t-2} + \ldots + \alpha_p X_{t-p} + u_t , \quad t = 1, 2, \ldots$$

The parameters $\alpha_1, \alpha_2, ..., \alpha_p$ have to obey certain conditions to guarantee the stationarity of AR(p). They cannot be mentioned here.
The simplest moving-average-process is defined by the equation

$$(3) \qquad X_t = u_t + \beta u_{t-1} , t = 1, 2, \ldots$$

This process is called a moving-average-process of order 1 (in short: MA(1)). An MA(l)process is stationary for arbitrary values of the para-

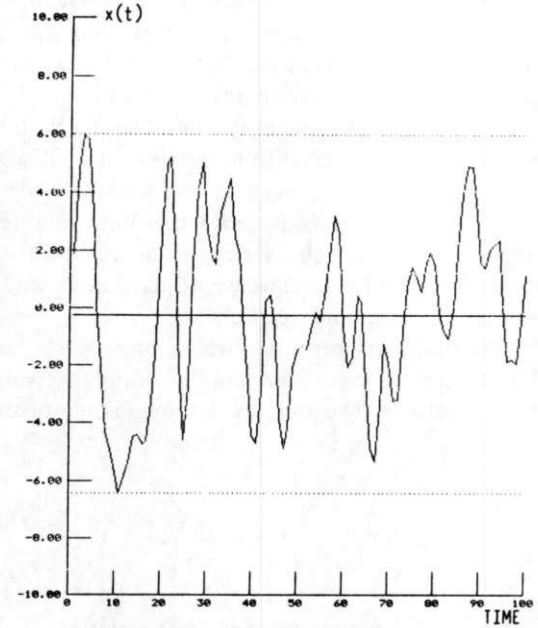


Fig. 2

meter β.

In general, a moving-average process of order q (MA(q)) is given by

$$(4) \qquad X_t = u_t + \beta_1 u_{t-1} + \beta_2 u_{t-2} + \ldots + \beta_q u_{t-q}, \qquad t = 1, 2, \ldots$$

If we would try to modelling a series by using either an AR- or a MA-process we frequently would see that the required order for a »good« model would be very high. This, however, would be an unsatisfactory state of affairs, since the precision of the estimation of the unknown parameters depends (given a limited set of data) on the number of parameters to be estimated. Therefore, we should look for models which contain as less parameters as possible. This is called the »principle of parsimony«. Now, it can be shown that a good strategy to realize this principle consists in combining the two »pure« approaches which leads to the so-called ARMA(p,q)processes:

$$(5) \quad X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} = u_t + \beta_1 u_{t-1} + \beta_2 u_{t-2} + \ldots + \beta_q u_{t-q}, \; t = 1, 2, \ldots$$

In practise, in many cases the orders $p \leq 2$, $q \leq 2$ prove to be sufficient.

2. ARIMA-Processes

Since practical time series frequently cannot considered to be realizations of stationary processes, it seems to be a poor strategy to use stationary ARMA(p,q)processes as modelling tools. Fortunately, there exists a simple device which transforms non-stationary series to stationarity - at least for most series encountering in practise. This transformation consists in differencing a series. The first difference is defined by

$$(6) \qquad Y_t =: X_t - X_{t-1}$$

If Y_t should not be stationary, we could try a second difference

$$(7) \qquad Z_t =: Y_t - Y_{t-1} = X_t - 2\,X_{t-1} + X_{t-2}$$

and so on. In most cases no higher than second differences are necessary to achieve stationarity.

If we are able to model a series after differencing we call this an ARIMA(p,d,q)-process, where d = 1, 2, ... denotes the degree of differencing.

Now the modelling of a time series by an ARIMA(p,d,q)-process in general requires the following three steps:

1) Identification of the process-type (what are the proper values for p, d, q?
2) Estimation of the unknown parameters.
3) Diagnostic checking of the residuals.

In step 1) the order of the process has to be determined. This is a relatively complicated and delicate matter which is solved mainly by using and interpreting different (estimated) correlation-functions. Details cannot be discussed here. Until recently, a lot of practical experience was necessary to do a good identification-job. But fortunately, more and more software is now available with an automatic identification-procedure. In step 2) the unknown parameters of the model proposed in step 1) are estimated. This requires complicated numerical procedures which, however, need not bother the model-builder, since they work fully automatically in all available software. The final step 3) checks the adequacy of a proposed model by analyzing the residuals which are defined as difference between the given series and the series predicted by the identified model. Necessary for adequacy of a model is that the residuals can be considered as a realization of a white-noise process. There exists different statistics for testing this hypothesis which cannot be considered here. If a model passes step 3) then it can be used for instance for forecasting. If not, then one would have to go back to step 1) again and try a different (hopefully better) modelspecification. So it might be necessary to iterate the three steps. For details the interested reader is referred to (1) and (2).

3. Some Practical Results

If we try to model the time series of Figure 1 we find as the best model the ARIMA(0, 1, 4)- or IMA(1, 4)process:

$$X_t - X_{t-1} = u_t + 0.47\, u_{t-1} + 0.15\, u_{t-2} + 0.2\, u_{t-3} + 0.19\, u_{t-4}. \tag{8}$$

Figure 3 shows the »long wave« which is hidden in this series. (The extraction of this signal using digital filters is discussed in Part Two *»Filtering of Time Series«).*

The best model here is the ARIMA(2,0,1> or ARMA(2,l)process:

$$X_t - 1.889X_{t-1} + 0.943X_{t-2} = u_t + 0.453u_{t-1} \tag{9}$$

We can use this process for forecasting. The forecasting-equations in this case are

$$\hat{X}_t\,(1) = 1.889X_t - 0.943X_{t-1}$$

$$\hat{X}_t\,(2) = 1.889\hat{X}_t\,(1) - 0.943\,X_t$$

and

$$\hat{X}_t\,(m) = 1.889\hat{X}_t\,(m-1) - 0.943\hat{X}_t\,(m-2), \quad m < 2$$

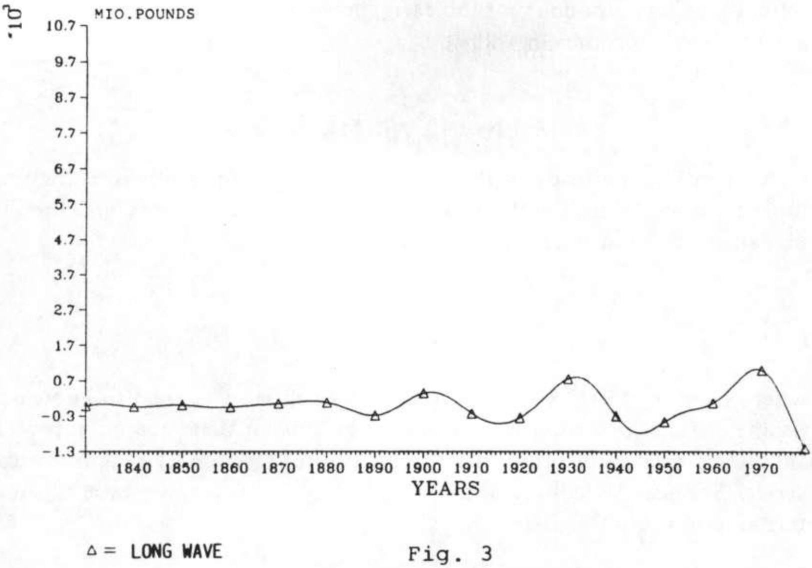


Fig. 3

where $\hat{X}_t(m)$ denotes the forecasted value for the lead time m.

In the next table we find both the forecasted values for 1980, 1981, ..., 1985 and the »true« values for these years. The latter ones are the result of updating the original series »Gross fixed Capital Formations

Table 1

Year	Forecasted Values	»True« Values
1980	-12.4	-9.0
1981	-13.0	-11.5
1982	-13.0	-13.2
1983	-12.2	-13.9
1984	-10.9	-13.6
1985	-9.0	-12.4

Comparing the forecasted values with the »true« ones, it is seen that they show the correct tendency: there is a »turning-point« (where the downswing stops) both in the »true« data and in the forecastings. The latter date this point one year too early, however. Besides that, the minima are very close together in value.

4. Multivariate ARIMA-Processes

As mentioned already in the beginning, it is also possible to modelling more than one series simultaneously. Here, one series can be considered to be dependent on n independent series, say:

$$Y_t = f(X_{1t}, X_{2t}, \ldots X_{nt}) + N_t \tag{11}$$

where N_t is an ARIMA-noise component. Such models are also known as »transfer-function« models. It seems to be evident that such an approach should be especially promising for analyzing dependencies of historical series. Because of lack of space it is not possible to give even a short introduction into this field.

FILTERING OF TIME SERIES

1. Introduction

Filter-methods may be very important in investigating historical phenomena. For instance, the search for »long swings« is a famous and well-known subject which keeps busy historians and economists since more than fifty years and in which an extensive use of filters is made. Although the purpose of my lecture is not the discussion of the existence (or non-existence) of »long swings«, I would like to start with a problem which is also important in the context of »long swings« and which gives a good illustration of why filtering is necessary and what are the main differences between traditional and modern approaches in this field.

For the sake of clarity I shall use firstly a simulated time series. Let us suppose, we generate a time series in the following manner. The series is supposed to be additively composed of three components (see Figure 4):

$$\begin{aligned} (12) \qquad & T_t = 0.001\, t^3 - 0.1\, t^2 + 0.5\, t + 100 \\ & C_t = 200 \cdot \sin 0.06\, \pi\, t \\ & R_t \sim N(0,1) \quad t = 1, 2, \ldots, 200 \end{aligned}$$

Historical Social Research, Vol. 14 — 1989 — No. 1, 3-24

that is

$$X_t = T_t + C_t + R_t, \quad t = 1, \ldots 200.$$

Let us further suppose that the time index t means years, so that our time series comprises 200 years, a time span not unusual for historical time series. Of course, the first component is the so-called long-term or secular trend which gives us the direction of the general development of the series in the long range. The second component (C_t) is a cyclical one which evolves with exact regularity with a period of about 33 years. Finally, the third component (R_t) is of a different nature than the other ones. It is a so-called random-component, that means, this component does not follow any regular pattern whatsoever. Its behaviour is unpredictable. This characterization involves that R_t contains no useful information for the historian. Therefore, R_t is »white-noise«. We assume for R_t a normal distribution with zero expectation and unit variance for all time-points. The following graph shows the series X_t and the components T_t and C_t.

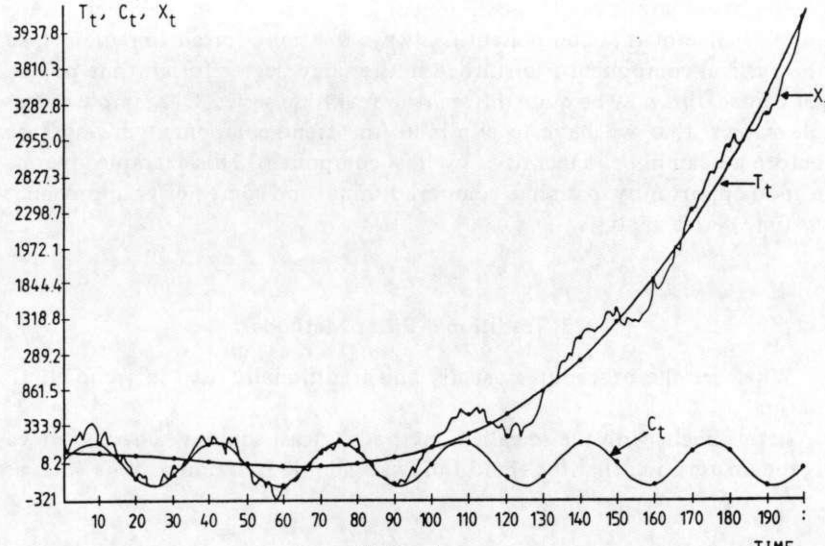


Fig. 4

Now, let us forget that we know the components of the time series and let us consider X_t to be a real time series. If X_t is a real series, we might be interested in the following questions:

- Does X_t contain a cyclical component and if yes, how regularly does it evolve in time?
- How does the trend develop in time?

Obviously, the first question seems to be the more difficult one: we have to extract a signal - namely the cyclical component C_t - which is contaminated by a second signal and by 'noise'. Of course, in our simple example, the trend is the more disturbing fact. This might be different in a different series. Now, how could we find out, if the series X_t contains a cyclical component at all?

The first and easiest method would be a simple visual inspection of the given series. In our example it is not hard to see that the series does indeed contain a cyclical component whose pattern is obviously very regular at least until t = 90. After that time point the pattern seems to loose its regularity (the maxima and minima are shifted) and the amplitude of the cyclical component seems to diminish. Thus we would conclude that there is a cyclical component which however is not stable in the whole series and which disappears towards the end of the series. Of course, this conclusion would be wrong as we know.

It is obvious that the trend makes the visual method an insufficient one. We see also that the random-component is not a severe problem here, since the variance of that component is always the same for all time-points, so the cyclical component is disturbed to the same degree for all time points. Of course, this may be quite different in real time series. Our simple example shows, that we have to eliminate the trend-component in any case before we can hope to identify a cyclical component. This example gives us a good opportunity to discuss some traditional and some newer approaches in time series analysis.

2. Traditional Filter-Methods

What are the procedures usually and traditionally used in trend-eliminating?

Let us begin with the so-called 'method of least-squares'. The basic idea is the assumption that the trend follows a simple function of time such as

$$T_t = a + bt \tag{13}$$

or

$$T_t = a + bt + ct^2$$

or in general

$$T_t = a + bt + ct^2 + dt^3 + \ldots + mt^n,$$

whose coefficients are unknown. According to the 'leastsquares'-principle, they are estimated in such a way that the differences between the values of the time series and the trend-values - squared and summed over all time-points - take a minimum. In the simplest case of a linear trend we minimize therefore the expression

$$(14) \qquad \sum_t [X_t - (\hat{a} + \hat{b}t)]^2 = \min_{\hat{a},\hat{b}}.$$

with respect to $\hat{a}$ and $\hat{b}$, where the 'hats' denote least-squares estimates. This approach is also called a 'regression'. After having estimated the trend-coefficients, a subtraction of the estimated trend from the given series results in a detrended series.

This brief nontechnical outline of least-squares shows us the crucial points of this method. Obviously, the most delicate point is the choice of the 'correct' trend-function. What degree is to be chosen? Is a linear function sufficient or should a higher degree be chosen? It is clear that an incorrect choice of the functional form - a misspecification - has severe consequences on the resulting detrended series. For instance, if we would choose a quadratic function instead of a cubic one, the detrended series would never be similar to a regular sinusoid with constant amplitude. Unfortunately, in general, we have no solid arguments which would allow us to prefer a special functional form for the trend in real time series. Neither economic theory nor history will give us reliable hints. So our choice will be more or less arbitrarily. This holds also of course for further results which depend on the detrended series. It shall be mentioned here that there exist special methods in time series analysis which may be useful in determining the 'adequate' degree of a trend-polynomial. However, they depend on certain assumptions which may not be fulfilled in reality. Maybe the most familiar is the so-called 'VariateDifferenceMethod'. It depends on a simple fact:

Let f(t) be a polynomial of degree n. Then the difference

$$\Delta f(t) := f(t) - f(t-1)$$

is a polynomial of degree not higher that n - 1. For instance, let

$$f(t) = a + bt + ct^2, \quad n = 2$$

Then

$$\Delta f(t) = a + bt + ct^2 - [a + b(t-1) + c(t-1)^2]$$
$$= 2ct + b - c$$

and

$$\Delta^2 f(t): = \Delta f(t) - \Delta f(t-1)$$
$$= 2c = \text{const.},$$

that is, the n-th difference is a constant. This gives us a practical procedure for determining the degree of a polynomial: by differencing a series successively we finally get a series of constants. The number of differences we need until we get constants is the degree of the trend-polynomial. If the series contains a noise-component we will not get exactly a series of constants, but a series which oscillates around zero. There exists significance-tests which are useful to decide if the differenced series can be considered indeed as a pure noise-process. Details are not important here.

However, this method depends on the assumption that a series does not contain a cyclical component. If a cyclical component is present, we can difference as often as we like, we never will get a constant or random-series. For instance, let us consider the cyclical component in our simulated series after differencing once:

(15) $$\Delta C_t = C_t - C_{t-1}$$
$$= 200 \sin 0.06\pi\, t - 200 \sin 0.06\, \pi\, (t-1)$$
$$= 200\, (1 - \cos 0.06\, \pi)\, \sin 0.06\, \pi\, t$$
$$+ 200 \sin 0.06\, \pi \cdot \cos 0.06\, \pi\, t.$$

If we perform more differences, the differenced cyclical component becomes a very complicated trigonometric expression which is not equal to a constant. So in our simulated series, the third difference is not a constant (or oscillating randomly around zero if we include the noise-component) although the trend is in fact a polynomial of the third degree. Thus, this method is of restricted value only. Further problems of the variate-differencemethod cannot be discussed here.

Let us consider some more details and problems of least-squares. Obviously, a further crucial point is the validity of the basic time series model, that is the additivity of the components. A subtraction of the estimated trend makes only sense when the postulated additivity holds. But the components might just as well be combined in a multiplicative manner. Or a mixed model might be adequate. Since there are no dead-sure methods to discriminate between the different approaches, a further source of possible specification-errors has to be taken into account.

There exist some more problems with the least-squares-approach which shall not be discussed here.

The second approach used in time series analysis for estimating a trend are the so-called 'moving-averages'. A moving-average is defined by:

$$(16)\quad Y_t := \frac{1}{2q+1} \sum_{s=-q}^{+q} X_{t-s}\,,\ t = q+1, \ldots, N-q$$

where 2q + 1 denotes the length of the average and N the length of the time series. In general, it is preferable to choose an average of odd length, because the averaged values can be assigned exactly to the given time points. With even length they would have to be assigned between time points.

The above formula shows that by the averaging-procedure q values are lost at both ends of the series. For example, if the filter-length is 7, then we would loose 3 values at each end of the series. Figure 5 shows a series where the trend is estimated by a 7-term and by a 13-term moving-average. Obviously, the longer the moving-average is the smoother a trend results. Since moving-averages have smoothingproperties, they are often called »smoothers«.

One of the problems with moving-averages is the question: how many terms should be included? As our example shows, this is indeed a crucial problem, since the smoothness of the resulting trend depends on this decision and of course the pattern of the detrended series also. This problem is of the same importance as the choice of the degree of the polynomial in the least-squares-approach. However, 1 would like to defer the discussion of this point until some more aspects of time series analysis are exposed.

Moving-averages as introduced above can be considered as special filters. A linear filter can simply be described as a transformation of a time series according to

$$Y_t := \sum_{s=-q}^{r} a_s X_{t-s}\,,\ t = s+1, \ldots, N-r$$

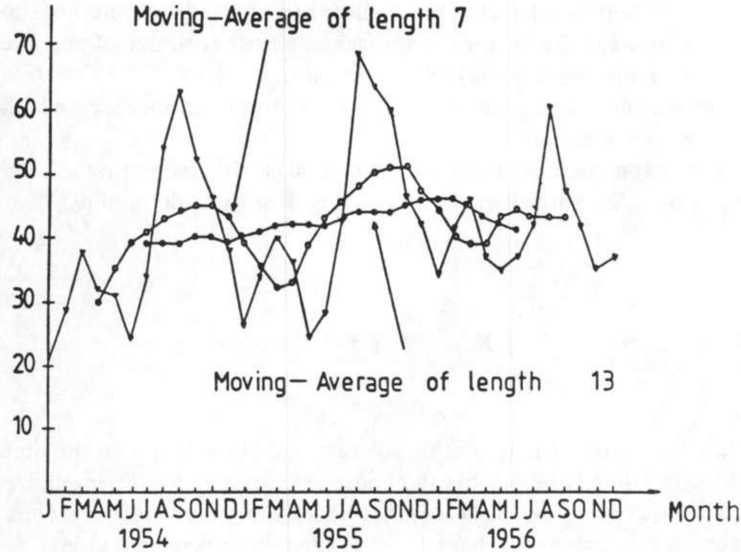


Fig. 5

with

X_t: = filter-input (given time series)

Y_t: = filter-output (filtered series)

a_s: = filter-weights.

Obviously, in the case of our moving-averages we have

$$a_s = \frac{1}{2q+1}$$

$$r = q,$$

that is, they are filters with constant coefficients and they are symmetrical. This last property implies that there is no phase-shift between filter-input and filter-output. In a nonsymmetrical filter or moving-average there is a phase-shift, which means that the filter-output is delayed against the given time series. This might be an undesirable feature of a filter.

Filters are important tools of modern time series analysis. Before we enter into a discussion of these concepts, I would like to make some final comments on the traditional approaches. Moving averages seem to be more flexible than least-squares. The assumption that a trend, for example, evolves according to one and the same function for all time-points seems to be a very rigid one, especially when the series is very long. Fortunately, least-squares-approaches can be modified in a way which makes them as flexible as moving averages. The modification is based on the assumption that the polynomial chosen does not hold for the complete series, but only for relatively small time-intervals. These intervals are overlapping and moving over the series. Therefore, this modification is called 'moving regression'. For an example, let us assume that a polynomial of the second degree is to be fitted to the first five values of a series by least squares. Then the coefficients of the polynomial are determined by minimizing the expression:

$$\sum_t [X_t - (\hat{a} + \hat{b}t + \hat{c}t^2)]^2.$$

As solution we get for the first trend value at t = 3

$$Y_3 = \frac{1}{35}(-3X_1 + 12X_2 + 17X_3 + 12X_4 - 3X_5)$$

$$Y_t = \frac{1}{35}(-3X_{t-2} + 12X_{t-1} + 17X_t + 12X_{t+1} - 3X_{t+2}).$$

$$Y_3 = \frac{1}{35}(-3X_1 + 12X_2 + 17X_3 + 12X_4 - 3X_5)$$

or in

$$Y_t = \frac{1}{35}(-3X_{t-2} + 12X_{t-1} + 17X_t + 12X_{t+1} - 3X_{t+2}).$$

The filter-weights are (-3/35, 12/35, 17/35, 12/35, -3/35). Note that they sum to unity. The first case with equal weights results by fitting a linear function.

Although least-squares can be made more flexible by using moving regression, the basic problem, namely the choice of the functional form, remains the same. Additionally, there is the problem of the length of the moving regression. This is the price to be paid for the higher flexibility.

3. Newer Filter-Methods

The modern approaches to be discussed here are methods which are based on filter-theory. We have learned already that moving-averages can also be considered as filters, although more elementary ones. Modern filter-theory can be characterized by the fact that no ad-hoc-solutions (like simple moving averages) are accepted, but filters are designed in such a way, that they accomplish prespecified criteria. These criteria are chosen according to the problems to be solved or the aims of the analysis.

A characteristic feature of modern filter-theory is the fact that filter-design can be made in the *frequency-domain* and not only in the time domain. This possibility offers a lot of advantages which will become evident when we consider details and examples. [See (4) and (5)].

The transition from the time domain to the frequency domain is done by mathematical operations which are known as Fourier transforms. However, I do not want to enter into this relatively complicated matter. Rather, I prefer to explain the essentials in a nontechnical way.

The thinking in frequencies seems to be complicated and unfamiliar. However, it is easily to understand, if we look at well-understood concepts in a way which is a little bit different than we might be used to it.

Let us begin with a monthly economic time series. Very often, economic series show a seasonal pattern, for example, the series of unemployment. The seasonal component of a series can roughly be characterized by its 'periodic' nature: it repeats every twelve months. Of course, the seasonal pattern is almost never of an exact regularity in reality, since the amplitude of the seasonal component is usually changing more or less. Therefore, we should characterize a seasonal component as a 'quasi-periodic' movement with a period of twelve months.

Now, instead of saying: 'the period of the seasonal component is 12 months' we might just as well say: 'in a time unit (which is a month here) 1/12 of the seasonal component has passed'. Or shorter: 'the frequency of the seasonal component is 1/12'. Next, let us consider the trend of a series. The trend can be characterized by the fact that it is a component which never repeats itself. That is, its periodicity is infinite or its frequency is zero. Frequencies which are between zero and 1/12 are called 'low frequencies' and frequencies beyond 1/12 are called 'high frequencies' (in monthly series).

Visually, low-frequency components correspond to smooth and slow movements in the time domain, whereas high-frequency components correspond to rough and fast movements in the time domain.

Instead of considering only two frequencies we might ask if a series contains maybe some more frequencies. Now, it may be proven mathematically, that there are in fact an infinity of frequencies. More exactly, it can be shown that a series in generally can be considered as being composed of a continuum of frequencies. The lowest frequency is zero and the highest is 0.5 (with discrete series).

If we design a filter in the frequency domain, we have the possibility to eliminate or preserve frequencies according to the aims of our analysis. For instance, if a series is to be detrended, the filter has to eliminate the zero-frequency. All other frequencies have to be preserved. A filter of this kind is called a *high pass* filter. If the trend is to be estimated, all low frequencies below the seasonal frequency should be left unaltered, whereas the high frequencies should be eliminated. Such a filter is called a *low-pass* filter.

The behaviour of a filter in the frequency-domain can be described completely by a function which is usually called *'transfer junction'*. This function gives us two informations: the first one shows us, if and how certain frequencies or frequency-bands are attenuated resp. eliminated or amplified. The second one shows, if there is a phase-shift between filter-input and filter-output. The transfer function of a filter is a complex valued function in general. Its absolute value gives us the first information. Therefore, this function is called the 'amplitude-function' of a filter.

Now, let us return to our problem of detrending a time series. As mentioned before, for solving this problem, a high-pass filter is needed. The amplitude function of such a filter has the shape shown in Figure 6.

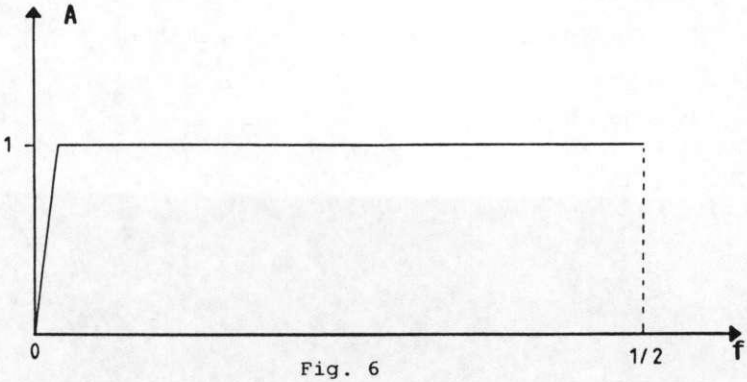


Fig. 6

A filter with such an amplitude function eliminates practically only the zero-frequency and leaves untouched all other frequencies.

If th is amplitude function is to be realized, how can we find the corresponding filter-equation? The solution of this problem is indeed difficult. It is not possible to discuss it here in detail. Only results can be given. It can be shown that the realization requires a set of three filter equations:

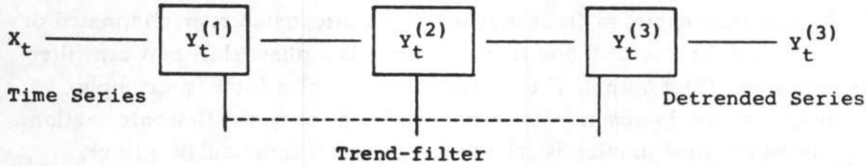
$$Y_t^{(1)} = X_t - 2\,X_{t-1} + X_{t-2} + 1.980098\;Y^{(1)}_{t-1} - 0.980487\;Y^{(1)}_{t-2}$$

(17) $$Y_t^{(1)} = X_t - 2\,X_{t-1} + X_{t-2} + 1.980098\;Y^{(1)}_{t-1} - 0.980487\;Y^{(1)}_{t-2}$$

$$Y_t^{(2)} = Y_t^{(1)} - 2\;Y^{(1)}_{t-1} + Y^{(1)}_{t-2} + 1.960530\;X^{(2)}_{t-1} - 0.960919\;Y^{(2)}_{t-2}$$

$$Y_t^{(3)} = 0.961057Y^{(2)}_{t} - 1.922114\;Y^{(2)}_{t-1} + 0.961057Y^{(2)}_{t-2} + 1.980098\;Y^{(3)}_{t-1} - 0.980487\;Y^{(3)}_{t-2}. \quad \text{[See (4)]}$$

The filter-output of the first filter ($Y_t^{(1)}$) is the filter-input of the second filter and its output ($Y_t^{(2)}$) is the input of the third filter. $Y_t^{(3)}$ finally, is the detrended series. The next diagram illustrates the complete filtering-process:



To demonstrate the efficiency of this filter, let us first consider a simulated series which is additively composed of the two components:

$$T_t = 0.001\, t^3 - 0.1\, t^2 + 0.5\, t + 100$$

$$C_t = 100\, (\sin 2\pi/60\, t + \sin 2\pi/40\, t + \sin 2\pi/20\, t),$$

that is

$$X_t = T_t + C_t.$$

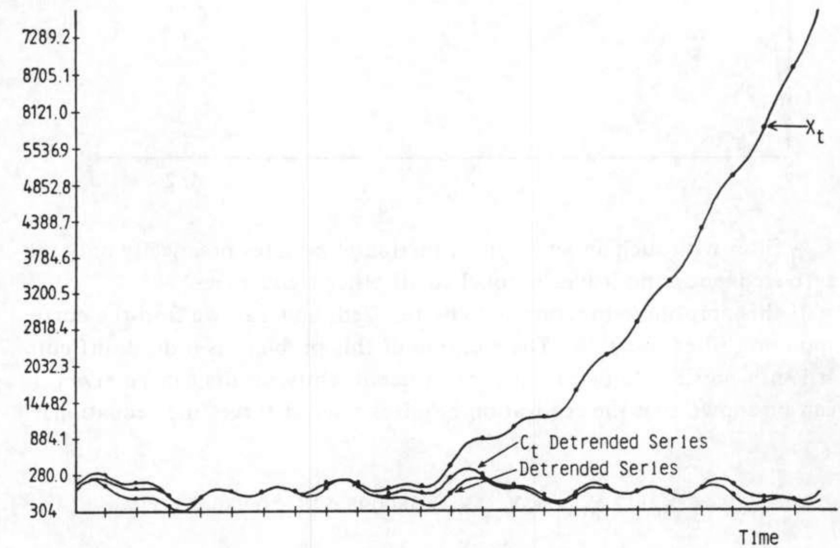


Fig. 7

The cyclical component is a superposition of three sinusoids with the periods 60, 40 and 20 time units (or with the frequencies 1/60, 1/40 and 1/20).

For the sake of simplicity, the series does not contain a random component. Figure 7 shows the series and the result of the detrending.

Obviously, C_t and the detrended series differ only marginally. Note, that no assumption has to be made about the functional form of the trend whatsoever.

Now, let us consider some real series (see Figures 8, 9 and 10). The series are English series relating to the production of consumption goods, investment goods and the production on tin. [See (3)J. Fortunately, the filter-design approach discussed above does not exhaust all possibilities of modern filter design. For certain types of filters like low-pass or band-pass-filters (which are very important in practice), such a design may have the drawback of a remarkable phase-shift which may be disturbing in analyzing certain problems.

This phase-shift can be avoided if we use a different approach. Its essential feature is the fact that filtering is done in the frequency-domain and not in the time-domain as in the first approach. This can be achieved by a transformation of the data into the frequency-domain which can be done by using a so-called »Discrete-Fourier-Transform (DFT). I shall not discuss details here [See (5)]. Rather, let us finally consider an example. For illustration we use the series Turnover of Rye in Cologne 1531 -1659'. For estimating the trend a low-pass filter was chosen which preserves all oscillations with a period longer than 60 years. To estimate a trend-free series, a high-pass filter was used which is complementary to the low-pass. Finally, for analyzing any possible long cycles, a band-pass filter was taken which preserves all cycles between 20 and 60 years. The amplitude functions of these filters are shown in Figure 11. The series and the results of the filtering are shown in Figure 12.

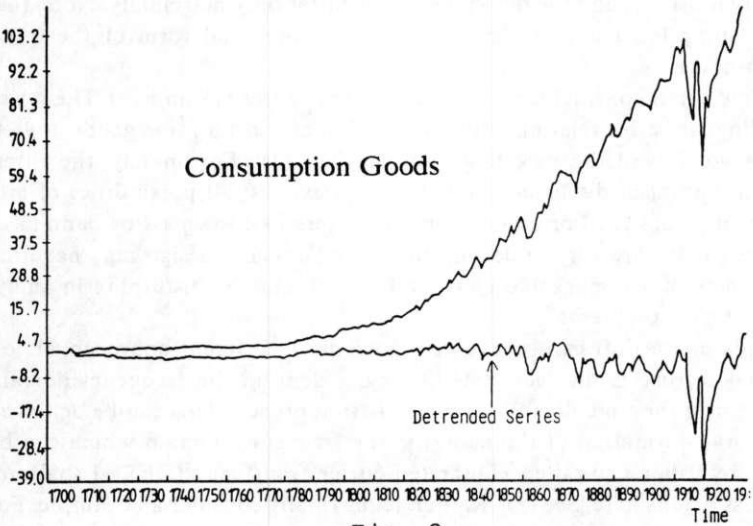

Consumption Goods
103.2
92.2
81.3
70.4
59.4
48.5
37.5
28.8
15.7
4.7
-8.2
-17.4
-28.4
-39.0
Detrended Series
1700 1710 1720 1730 1740 1750 1760 1770 1780 1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 19:
Time

Fig. 8


Investment Goods
106.9
95.0
89.2
71.3
59.5
47.8
35.7
23.9
12.0
0.1
-11.2
-23.8
-35.4
-47.3
Detrended Series
1700 1710 1720 1730 1740 1750 1760 1770 1780 1790 1800 1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 19:
Time

Fig. 9

Historical Social Research, Vol. 14 — 1989 — No. 1, 3-24

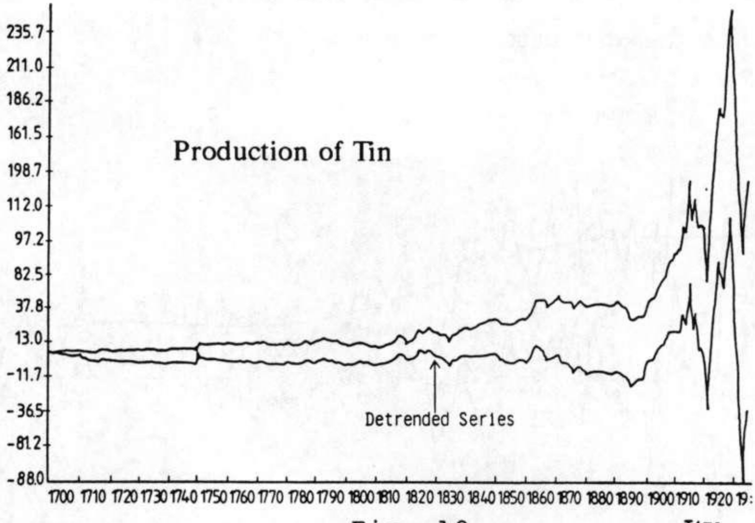


Fig. 10

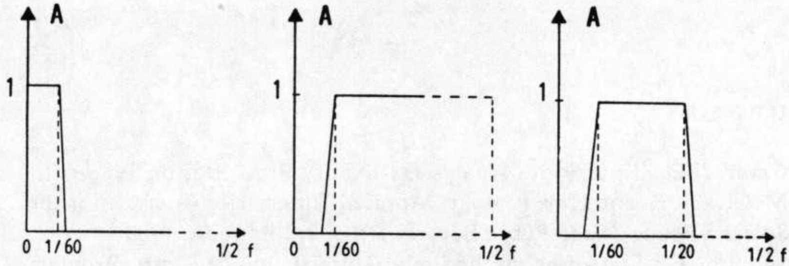


Fig. 11

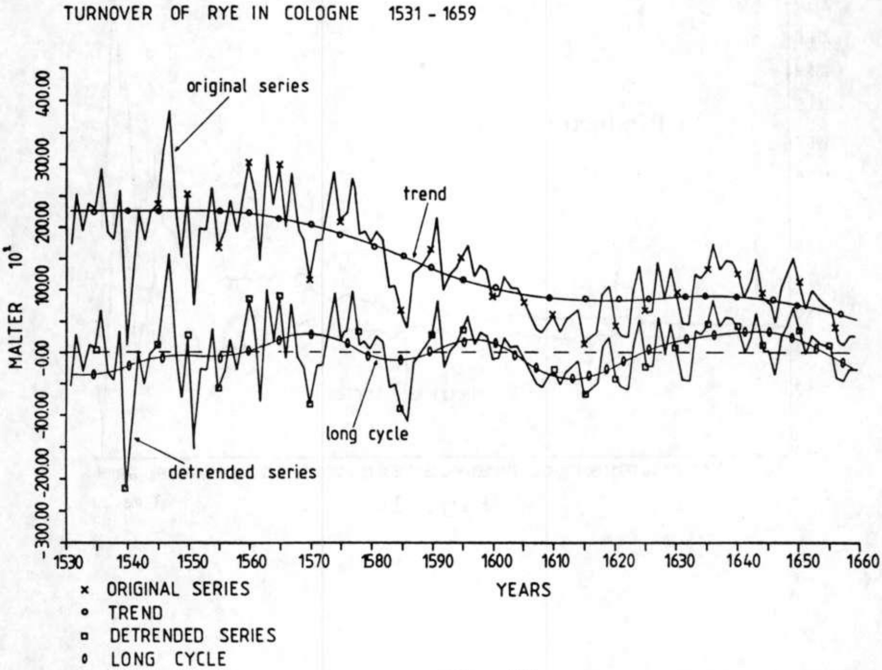


Fig. 12

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