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Nonlinearity and Structural Breaks in Irish PPP Relationships: An Application of Random Field Regression*

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Abstract

Using nominal and real exchange rates for Ireland relative to Germany and the UK from 1975 to 2003, this paper explores likely sources of nonlinearity in purchasing power parity (PPP) relationships and difficulties in employing an $I(1)/I(0)$ econometric framework. Tests for fractional integration and nonlinearity, including random field regression-based procedures, are applied. Results reveal shortcomings in the standard cointegration and smooth transition autoregression approaches to modelling, and point to multiple structural changes models. Such a model for the case of Ireland and Germany suggests that PPP holds not only in the long run but also in the medium to short term.

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1. Introduction

Purchasing power parity (PPP) continues to be a major subject of applied economic research. Results of empirical studies, however, have been very heterogeneous (see, for example, Taylor and Taylor, 2004). From broad acceptance in the 1970s to firm rejection in the 1980s, PPP has generally been cautiously accepted more recently (Taylor, 2006). These developments are, in part, due to contemporaneous developments in econometric theory. Another important factor throughout this period has been the changing monetary landscape, from Bretton-Woods to the European Monetary System (EMS) and eventual European Monetary Union (EMU).

Early investigations of PPP usually took one of two approaches, examining either the co-movement of price indices or the behaviour of the real exchange rate, with a particular emphasis on the long run (see, for example, Sarno and Taylor, 2002). The perceived difficulties with these approaches, which frequently employed cointegration techniques, were generally attributed to the low power of unit root tests. Efforts to overcome these difficulties focused on obtaining long-span data series, using alternative testing procedures and panel data approaches (see, for example, Papell, 2006).

However, two new approaches have grown in importance, focusing on the persistence in the real exchange rate and the possibility of nonlinearity. Persistence may be due to aggregation bias in the data and nonlinearity may arise from asymmetric adjustment to PPP (Rogoff, 1996). Several studies have placed PPP in the fractional (co)integration framework or used long memory models (see, for example, Villeneuve and Handa, 2006). The most commonly used nonlinear technique has been smooth transition autoregression (Schnatz, 2006). Although this approach may be appealing theoretically, it tests the null of linearity against just one nonlinear specification, thereby disregarding any other form of nonlinearity; a more general approach may be better. Also, these approaches have usually been considered in isolation, although it is clear from the econometrics literature that nonstationarity, be it fractional or otherwise, and nonlinearity are closely related.

This paper aims to model the nominal and real exchange rates for Ireland relative to Germany and the United Kingdom (UK) from 1975 to 2003, with a particular emphasis on persistence and nonlinearity. Adopting an approach similar to Johansen and Juselius (1992), the paper initially explores PPP in a cointegration framework. The possibilities of both persistent deviation from PPP and nonlinearity are then considered. Two approaches, which have yet to be employed in this area and which have the potential to overcome some of the difficulties encountered in previous studies, are introduced. The first, the fractional augmented Dickey-Fuller test, examines the hypothesis of fractional integration against that of integer integration, and may help distinguish between stationary, nonstationary and long memory processes. The second, random field regression, offers a new approach to testing for nonlinearity and specifying nonlinear models. Importantly, this technique assumes no prior knowledge of the likely form of nonlinearity.

The structure of the paper is as follows. Section 2 provides relevant background material, sketching the theory of PPP, the results of previous studies using Irish data and a brief history of important monetary developments. Section 3 describes the data and precise methodology used, and presents and discusses the results. Finally, Section 4 concludes.

2. Purchasing Power Parity

A simple statement of the purchasing power parity hypothesis is that national price levels should be equal when expressed in a common currency. If s_t is the logarithm of the nominal exchange rate (expressed as units of foreign currency per unit of domestic currency), p_t and p_t^* are the logarithms of the domestic and foreign price levels, respectively, and q_t is the logarithm of the real exchange rate in period $t = 1,2,\ldots,T$, then for all t,

$$q_t = s_t + p_t - p_t^* . \qquad (1)$$

It follows that q_t must be stationary for long-run PPP to hold. If the mean of q_t, $E(q_t)$, is zero, PPP is absolute, whereas if $E(q_t) \neq 0$, PPP is relative. Most empirical studies of PPP have either been concerned with testing whether q_t has a mean reversion tendency over time or whether s_t, p_t and p_t^* move together over time.[2]

This latter work has generally been concerned with models whose simplest form is

$$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \varepsilon_t, \tag{2}$$

where ε_t is white noise. Early studies were concerned with whether the estimated values of the parameters of various versions of (2) were as predicted (see, for example, MacDonald and Taylor, 1992). As awareness of time series dynamics increased, the issue changed to whether (2) is a cointegrating regression. Wright (1994) takes such an approach with Irish data, using the Johansen (1988) approach to cointegration.

The emphasis subsequently shifted to considering directly the behaviour of $\{q_t\}_{t=1}^{T}$, the sequence of real exchange rate values. Within the $I(1)/I(0)$ framework, most initial studies failed to reject the hypothesis that real exchange rates were $I(1)$ for periods of flexible exchange rates, which implies a lack of mean reversion and undermines the PPP hypothesis. The explanation often given for this non-rejection is the recognised low power of traditional unit root tests, such as the standard Dickey-Fuller (1981) test. To overcome this problem, two general approaches were adopted. First, the construction and use of long series of exchange rate data and more powerful asymptotic tests (see, for example, Taylor, 2002). Secondly, the estimation of the half-life of the mean reversion of the real exchange rate, using panel data (Cashin and McDermott, 2004). There is, though, another possibility that is receiving increasing attention, and this is described in the following subsection.

2.1 Nonlinearity and purchasing power parity

Among the various alternative approaches to modelling PPP relationships that have been put forward, much recent interest has focused on nonlinearity. Taylor (2006) details three of the most commonly cited sources of potential nonlinearity in PPP. The first relates to the underlying assumption that transport costs, tariffs and other barriers to trade are negligible or non-existent. If this assumption is false, these costs may cause frictions in the markets for goods and services.

[2] Taylor (1995) provides an excellent survey of the literature.

Such frictions can lead to so-called 'bands of inaction', within which it is unprofitable to arbitrage the deviations from the law of one price, causing discontinuities in the relationship.

The second source of nonlinearity in PPP, originally proposed by Kilian and Taylor (2003), may arise from the interaction of heterogeneous agents in the foreign exchange market. When the exchange rate is close to its PPP equilibrium level, agents would hold a diverse range of views regarding its (mis)alignment; but as the exchange rate deviates further from its equilibrium level, views regarding future movements converge.

The third possible source of nonlinearity, proposed by Sarno and Taylor (2001) and Taylor (2004), relates to official intervention in the foreign exchange market. If misalignments in the equilibrium level of exchange rates are viewed as being due to problems of co-ordination between traders and monetary authorities, official intervention may be required to correct the misalignment. This view is supported empirically by Taylor (2004, 2005) and more recently by Reitz and Taylor (2008).

The persistence of deviations from PPP has been a source of much study. While these deviations may result from nonlinearities such as those induced by the factors just described, there is a further possibility. Persistent deviations from PPP may be due to long memory processes generating the data and these in turn may arise from data aggregation (Granger, 1980). Taylor (2006) discusses the role of aggregation bias in the PPP 'puzzle', but fails to make the link between the aggregation of data and fractional integration. Imbs *et al.* (2005) find that this bias may be more significant for data which excludes the non-traded sector, but that the bias may be overcome by using nonlinear models.

2.2 The Irish experience

Testing PPP for Ireland has produced varying results. In some cases, PPP could not be accepted, whereas in others it could not be rejected. Bradley (1977) found evidence in favour of short-run and long-run PPP, using pre-EMS data for Ireland and the UK. Thom (1989) also found some support for PPP using data for Ireland relative to Germany and the United States. However, Callan and Fitzgerald (1989) rejected PPP for Irish, German and UK data.

While rejection of PPP was common, particularly when data from the EMS period was used, non-rejection seemed most common when either alternative price indices were used or other variables were included in the model. For instance, Wright (1994) considered interest rate differentials, along with the variables in (2), while others have distinguished between prices in the traded and non-traded sectors. In an effort to explore the long-run PPP relationship, this study uses data from 1975 to 2003. This period, however, saw the inception of the EMS and EMU. It is important, therefore, to note the events relating to monetary integration in this period.

Ireland joined the EMS at its outset in 1979, as did Germany; the UK did not. During the early years of EMS, the Irish currency depreciated against the basket of European currencies of EMS participants, known as the European Currency Unit (ECU), as the Deutsche-Mark was re-valued in 1979, 1981 and 1982. The Irish pound continued to depreciate against the Deutsche-Mark until 1985 but remained stable within the EMS until its re-alignment in August 1986, when it was devalued by 8 per cent relative to the ECU. This devaluation was brought about by a loss of competitiveness vis-à-vis the UK, due to movements in the Deutsche-Mark/Sterling exchange rate.

From 1987 to 1992, the Irish pound was stable against the Deutsche-Mark. This period was notable, as the UK joined the EMS in 1989 and Germany re-unified in 1990. These events were followed by a period of sustained pressure on the Irish pound within the EMS, culminating in another devaluation in January 1993. This followed Sterling's devaluation in September 1992 and ultimate exit from the system shortly after. This period of ‘crises’ for the EMS resulted in a widening of the currency fluctuation bands. The penultimate step towards monetary union was taken in 1996, in the form of the new exchange rate mechanism. Both Thom (1989) and Honohan and Leddin (2006), however, have argued that these re-alignments should not be viewed as shocks, but rather as corrective adjustments, which are not necessarily inconsistent with PPP. This view coincides with that of Taylor (2005) regarding official intervention in the foreign exchange market, and suggests that this may be a likely cause of nonlinearity in the PPP relationship.

3. Methodology, Results and Discussion

The model used throughout this analysis follows Johansen and Juselius (1992) and Wright (1994). The specification is

$$s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^* + \alpha_3 i_t + \alpha_4 i_t^* + \varepsilon_t, \quad (3)$$

where, in addition to the variables defined in Section 2, i_t and i_t^* are the domestic and foreign short-term interest rates.[3] The real exchange rate series, $\{q_t\}_{t=1}^{T}$, is constructed using (1). Wholesale price indices are used in preference to consumer price indices as they offer a better approximation to price developments in the traded sector. The data are quarterly for the period 1975 Q1 to 2003 Q3, a total of 115 observations, and are displayed in Figs 1 and 2. These observations span several monetary regimes and crises, as described above. Wright (1994) used the shorter period from 1981 to 1992 to avoid these regime changes.

3.1 Univariate analysis

To put the long memory and random field analysis into context, standard unit root testing was conducted. The strategy of Dolado *et al.* (1990), to determine whether the augmented Dickey-Fuller (ADF) regressions have significant constants or trends, was adopted. These results generally seem to suggest that most series are $I(1)$.[4]

The issue of fractional integration was then investigated. The approach to applying the fractional ADF (FADF) test suggested by Dolado *et al.* (2002), is to obtain a consistent parametric estimate of the order of integration, d, and apply the FADF test for this value. The 'over-differenced' *ARFIMA* model, which uses the first-differences of the observations on a variable, was estimated to avoid problems associated with drift. Two parametric estimates of d were calculated, namely, the exact maximum likelihood (EML) estimate and a nonlinear least squares (NLS) estimate. The nonparametric estimate of d from the logperiodogram method

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[3] The short-term (3-month) interest rates were obtained from EcoWin; the remainder of the series were provided by Jonathan H. Wright. The data are available on request from the authors.

[4] Results are omitted for compactness but are available in the working paper version of this paper as Tables 1, 2, and 3, from http://www.ecb.int/pub/pdf/scpwps/ecbwp823.pdf. Tests conducted included the ADF, KPSS and Ng and Perron procedures, along with HEGY tests for seasonal unit roots.

(GPH) and the semiparametric estimate from the Gaussian method (GSP), were also calculated.[5] The estimates of d were then used in the FADF test, with the modified Akaike information criterion (MAIC) being used to set the lag length for the test.

Table 1 gives the results of the simple fractional integration analysis and Table 2 presents the results of the FADF test. For each series, estimates of d are given in Table 1, together with their estimated standard errors. As the FADF test is only meaningful if $d \leq 1$, when the probabilities to be applied to the test statistics are the standard normal ones, it is only reported in Table 2 for relevant cases. The results are interesting and seem to imply that the only series that is likely to be unambiguously fractionally integrated is the Irish interest rate. While all the estimates of d for the nominal exchange rate between Ireland and the UK are less than one, the FADF test fails to reject the null hypothesis of a unit root. For all other series, the estimates of d gave conflicting values, although a unit root is suggested in the Ireland/UK real exchange rate. The FADF test only gave strong evidence of fractional integration in the case of the Ireland/Germany nominal and real exchange rates when the GPH and GSP estimates of d were used.

The correlograms shown in Figs 3 and 4 appear to support the fractionality of the Irish, German and UK interest rates, and also the Ireland/UK exchange rate; they suggest unit roots for the other series. Thus the results of the FADF test are broadly in line with conclusions that might be drawn from inspection of correlograms, but point estimates of d suggest a somewhat higher incidence of fractionality.

3.2 Cointegration analysis

Traditional cointegration analysis was then applied to model (3). Firstly, the Engle and Granger (1987) two-step procedure was used, with the lagged residuals from the levels regression serving as the error-correction term. Then the Johansen *VAR* approach was applied. The effect of applying the Johansen (2002) small-sample bias correction was also investigated.

By treating the variables as $I(1)$ and applying the standard Engle-Granger (AEG) analysis, cointegration of the nominal exchange rate, price levels and interest rates is overwhelmingly rejected for both the Ireland/UK and the Ireland/Germany data. These results, shown in Table 3,

[5] Estimates were computed using the *ARFIMA* package for Ox; see Doornik and Ooms (1999).

are confirmed by the findings of cointegrating regression Durbin-Watson (CRDW) tests.[6] The results of trying to estimate parsimonious error-correction models, using the first lag of the residuals from the corresponding levels model as the error-correction term in each of the two cases, confirm the conclusion about the lack of cointegration. The error-correction mechanism (ECM) test also rejects cointegration in all cases.

The results from the Johansen procedure are reported in Table 4. They show evidence of one cointegrating vector in the Ireland/Germany case, when interest rates are excluded from the equation. Importantly, this result is overturned by the trace test when Johansen's small-sample correction to that test is applied. However, when interest rates are included, one cointegrating vector is suggested whether or not the small-sample correction is used. For the Ireland/UK relationship, the finding of one cointegrating vector in the specification without interest rates is also overturned by the adjusted trace test. In contrast, two vectors are suggested when the interest rates are included.

Taken together, the results so far are rather mixed and indicate that there is little evidence of cointegration in a traditional PPP setting, but that the introduction of interest rates appears to be important. Overall, as in previous studies, this attempt to place the PPP analysis of Irish data in a cointegration framework is not entirely satisfactory. We therefore turn to the results from the alternative nonlinear methodologies.

3.3 Nonlinearity tests

For the causal models, the RESET test, using quadratic as well as linear terms, and random field-based tests were applied.[7] Also, for an autoregressive model involving q_t, the now standard smooth transition autoregression (STAR) tests for nonlinearity were used. In all tests, the null hypothesis is that the model/series is linear. For the RESET test, both the F and LR variants are given. For the STAR test, an F version is used. The Akaike information criterion suggested a lag length of three for the STAR test in the case of the Ireland/Germany exchange rate and a lag

[6] For the complete results, see tables 5 to 9 in the working paper: http://www.ecb.int/pub/pdf/scpwps/ecbwp823.pdf.
[7] Details of the random field-based tests can be found in Hamilton (2001) and Dahl and González-Rivera (2003).

length of two for the Ireland/UK case. The Schwarz information criterion suggested a lag length of one in both cases. Table 5 gives the results.

As can be seen from the upper section of Table 5, which relates to the nominal exchange rate, the RESET test and the four random field-based tests emphatically reject linearity at the 5 per cent significance level in the case of the Ireland/Germany model. For the Ireland/UK model, however, there is a marked contrast between the findings from the two test approaches, with the RESET test failing to reject linearity but all of the random field tests strongly rejecting it.

The lower panel of Table 5 contains similar, though opposite findings for the real exchange rate. The RESET test, STAR tests and random field-based tests all suggest that the assumption of linearity is adequate for the Ireland/UK real exchange rate; but whereas the random field tests overwhelmingly support linearity of the Ireland/Germany rate, the STAR test based on the use of three lags gives some indications of nonlinearity and the RESET test rejects linearity very strongly. It is difficult to explain these conflicting outcomes, especially in the absence of information on the relative power of the different types of test. Given these results, the remainder of the paper concentrates on modelling the nominal exchange rate.

3.4 Random field regressions[8]

Random field regressions were estimated for the nominal exchange rates using the re-specification

$$s_t = \alpha_0 + \mathbf{x}_t'\boldsymbol{\alpha} + \lambda m(\mathbf{g} \circ \mathbf{x}_t) + \varepsilon_t = \mu(\mathbf{x}_t) + \varepsilon_t, \quad (4)$$

where $\mathbf{x}_t = \begin{bmatrix} p_t & p_t^* & i_t & i_t^* \end{bmatrix}'$, $\boldsymbol{\alpha} = \left[\alpha_j\right]$ and $\mathbf{g} = \left[g_j\right]$ are 4-vectors of parameters, λ is a scalar parameter, $m(\cdot)$ is a realisation of a stochastic process called a random field, $\circ$ denotes element-by-element multiplication, and all other symbols are as previously defined. The scalars λ and g_j, $j = 1,2,3,4$, characterise the relationship between $m(\cdot)$ and the conditional expectation function $\mu(\mathbf{x}_t)$. Specifically, λ is a measure of the overall 'weight' of the process $m(\cdot)$ in the conditional expectation, while the magnitudes of the g_j indicate the degree of

[8] A detailed description of random field regression can be found in Hamilton (2001) and Bond *et al.* (2005).

nonlinearity due to their associated variables. To carry out the estimation, the GAUSS code provided by Hamilton (2001) was adapted to apply the algorithm-switching approach to the numerical optimisation suggested by Bond *et al.* (2005).[9]

The results of the random field regressions are given in Table 6. Given that the bulk of the results in Table 5 suggest that the linear equation (3) used in the earlier analysis of PPP is not an appropriate specification, these results for the nonlinear random field models are of considerable interest. In the case of both country pairings, the standard model and the augmented model exhibit nonlinearity with respect to the two price variables, the price coefficients in the nonlinear component of the models being highly significant. However, in the augmented Ireland/Germany model, the German interest rate is nonlinearly significant, while in the Ireland/UK model it is the Irish interest rate that appears to have a significantly nonlinear influence on the nominal exchange rate.

Most striking, perhaps, is the fact that when nonlinearity is modelled by means of a random field, the coefficients on the domestic and foreign prices in the specifications with and without interest rates, are not statistically significantly different from their -1 and 1 values under PPP theory. This finding contrasts with the findings in the earlier Irish studies by, for example, Thom (1989) and Wright (1994), both of whom report cointegrating vectors, corresponding to the vector of variables s_t, p_t and p_t^*, that are markedly different from (1, -1, 1).

To infer a suitable nonlinear model, a method suggested by Bond *et al.* (2008) was used. This exploits the fact that the random field regression consists of two components: a linear and a nonlinear one. In the context of PPP, these two components can be viewed as a linear long-run approximation to the PPP relation over the sample period and a nonlinear dynamic or deviation component. The procedure was applied to the Irish/German data.[10] An estimate of the linear term was plotted as the 'fitted' term along with the actual dependent variable against time. This is shown in Fig. 5, together with the re-scaled difference between the two plots. Examining this difference or 'residual', several breaks are apparent, particularly around 1978, 1986, and 1996. To infer the form of nonlinearity that may account for these breaks, the residuals were plotted

[9] Hamilton's (2001) GAUSS code is available at http://weber.ucsd.edu/~jhamilto/.
[10] For this analysis, the data sample was truncated to exclude the period of fixed exchange rates under EMU.

against the three significantly nonlinear variables, respectively.[11] Evidence of regime changes is suggested by these plots, which indicate shifts corresponding approximately to 1978, 1986, 1990 and 1996. These break dates are very much in line with monetary developments affecting the Irish nominal exchange rate. The year 1978 saw the end of the peg to Sterling and the start of the EMS the following year; the Irish currency was devalued in 1986; and in 1989-1990 the UK joined the EMS and Germany re-unified. The final break in 1996 may relate to the introduction of the new exchange rate mechanism around that time, in preparation for EMU.

3.5 Multiple structural changes models

In view of these findings, break-date tests and time-varying parameter estimation, following Bai and Perron (1998, 2003), were used.[12] This multiple structural changes model approach is based on the regression

$$s_t = \mathbf{y}_t'\boldsymbol{\beta} + \mathbf{z}_t'\boldsymbol{\delta}_k + \varepsilon_t, \quad t = T_{k-1}+1,\ldots,T_k, \;\; k = 1,\ldots,n+1, \tag{5}$$

where $\mathbf{y}_t = \begin{bmatrix} i_t & i_t^* \end{bmatrix}'$ and its associated coefficient vector, $\boldsymbol{\beta}$, is not subject to change, $\mathbf{z}_t = \begin{bmatrix} 1 & p_t & p_t^* \end{bmatrix}'$ and its associated coefficient vector, $\boldsymbol{\delta}_k$, is subject to change, $T_0 = 0$, n is the number of break points, and all other symbols are as previously defined. Estimation of the model includes the appropriate number of break points and their timing.

Table 7 shows the results of this approach for Ireland/Germany, excluding intercepts. Four significant breaks are identified at 1978 Q2, 1986 Q2, 1990 Q3 and 1995 Q3. The $\sup F_T(l)$, $\sup F_T(l+1\,|\,l)$, *UDmax* and *WDmax* tests are all significant at the 5 per cent level for four breaks. Fig. 6 shows a plot over time of actual versus fitted s_t. The plot is based on estimates from the time-varying parameter model and is much improved compared with that seen in Fig. 5. Even more noteworthy are the coefficients reported in Table 7. In three out of five regimes, the coefficients for p_t and p_t^* are not statistically significantly different from -1 and 1, the values

[11] While not reported here, these plots are available from the authors on request.

[12] The GAUSS code to implement these techniques is available from http://people.bu.edu/perron/code.html.

predicted by PPP theory. For the second regime, coefficients of -0.725 and 0.813 are statistically significantly different from -1 and 1, respectively, yet remain plausible in magnitude. It is only for the fourth regime that the parameter estimates deviate substantially from theory, at approximately ±2. This regime is for the period 1990 Q3 to 1995 Q3. There is some limited evidence of a further break at 1993, but this was not found using the Bai and Perron approach.[13] Recall also that this period can be characterised as one of crisis for the EMS, and this may go some way to explaining this result. Nevertheless, these findings do not detract greatly from the overall results, which suggest that PPP does in fact hold for Ireland, in both the medium and long run.[14]

4. Summary and Conclusion

This paper has modelled the nominal exchange rates for Ireland relative to Germany and the UK from 1975 to 2003. It has used new approaches, not previously applied in this area, and has shown that PPP can be effectively modelled for those bilateral exchange rates by using random field regression and, in particular, multiple structural changes models.

Unit root tests found that most series could be characterised as nonstationary but the fractional augmented Dickey-Fuller test found little evidence of fractionality. Initial attempts to model the nominal exchange rate used the Engle-Granger and Johansen approaches. These illustrated the difficulties inherent in placing the study of PPP in the standard $I(1)/I(0)$ framework, which are implicit in the very mixed results of previous Irish studies.

Nonlinearity was then tested using a range of methods. Random field-based tests strongly indicated nonlinearity of the nominal exchange rate, while STAR-based tests were much more ambiguous, frequently failing to reject linearity. However, little if any nonlinearity was found in the real exchange rate data. This, taken with the evidence of the FADF tests, suggested that modelling the real exchange rate as a long memory or nonlinear process was not warranted.

Given the findings of nonlinearity in the nominal exchange rate, random field regressions were estimated. These produced striking results: the estimated coefficients of the linear

[13] The Irish currency devalued relative to the ECU in 1993.
[14] A similar approach was undertaken for the UK, the results of which are available in Bond *et al.* (2007).

component of the model were not significantly different from those expected under PPP and both price indices were found to be nonlinearly significant in each case. It was clear from graphical analysis following the random field regression that a series of breaks occurred in the data, which coincide accurately with monetary developments in the economies in question, and this suggested that a multiple structural changes model may be appropriate. Such a model was estimated and break dates were tested. Interestingly, this approach indicated very similar breaks to those found previously, and these were highly statistically significant. The estimated coefficients from these models were also very close to those theoretically predicted by PPP in the case of Ireland/Germany. The good fit achieved by this model is also noteworthy.

These results provide strong evidence for nonlinearity in the PPP relationship for these data, resulting from monetary developments. This supports the view that shocks relating to official intervention in the foreign exchange market may result in nonlinearity, but that when such shocks are modelled, the PPP relationship is linear. This certainly appears to be the case for the Ireland/Germany data, as PPP holds even in some of the short periods between structural changes. It remains to be seen whether similar findings to these apply to other currencies. Likewise, the interesting, though complex issue concerning the relationship between persistence and structural change is left for future research.

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Appendix A.1: Tables

Table 1: Fractional Integration Analysis.

Variables	EML	NLS	GPH	GSP
Irish Price Level	1.46 (0.04)	1.50 (0.07)	1.01 (0.11)	0.89 (0.07)
Irish Interest Rate	0.79 (0.10)	0.78 (0.10)	0.97 (0.10)	0.80 (0.06)
Ire./Ger. Nom. Exch. Rate	1.49 (0.14)	1.89 (0.10)	0.94 (0.11)	0.82 (0.07)
German Price Level	1.46 (0.05)	1.57 (0.09)	1.02 (0.11)	0.92 (0.07)
German Interest Rate	0.69 (0.24)	0.65 (0.23)	1.12 (0.11)	1.03 (0.07)
Ire./Ger. Real Exch. Rate	1.41 (0.08)	1.48 (0.08)	0.98 (0.11)	0.85 (0.07)
Ire./UK Nom. Exch. Rate	0.95 (0.09)	0.95 (0.09)	0.88 (0.11)	0.91 (0.07)
UK Price Level	1.48 (0.02)	1.55 (0.06)	0.99 (0.11)	0.87 (0.07)
UK Interest Rate	1.07 (0.09)	1.08 (0.10)	1.00 (0.11)	0.94 (0.07)
Ire./UK Real Exch. Rate	1.07 (0.09)	1.08 (0.09)	1.15 (0.11)	0.97 (0.07)

Note: standard errors in parentheses.

Table 2: Fractional Augmented Dickey-Fuller Tests.

Variables	EML	NLS	GPH	GSP
Irish Price Level	–	–	–	4.50
Irish Interest Rate	-3.22	-3.21	-3.35	-3.23
Ire./Ger. Nom. Exch. Rate	–	–	-5.48	-5.51
German Price Level	–	–	–	2.89
German Interest Rate	-1.49	-1.48 [a]	–	–
Ire./Ger. Real Exch. Rate	–	–	-5.05	-5.12
Ire./UK Nom. Exch. Rate	-1.60	-1.60	-1.61	-1.60
UK Price Level	–	–	5.03	4.69
UK Interest Rate	–	–	–	-2.53
Ire./UK Real Exch. Rate	–	–	–	-1.09

a Trend and constant not included.
– Indicates FADF test not applicable.

Table 3: *I(1)/I(0)* Levels Regression and Error Correction Analysis.

Test	Ireland & Germany		Ireland & United Kingdom	
	Excl. int. rate	Incl. int. rate	Excl. int. rate	Incl. int. rate
AEG	-2.475 [-3.817]	-2.835 [-4.540]	-2.653 [-3.817]	-2.728 [-4.540]
CRDW	0.186 [0.48]	0.245 [0.68]	0.239 [0.48]	0.250 [0.68]
ECM Test	-0.108 [-3.244]	-0.107 [-3.787]	-0.133 [-3.244]	-0.124 [-3.787]

Note: critical values in square brackets.

Table 4: Johansen Cointegrating Rank (Trace) Test.

Hypothesis		Test Statistic	0.05 Critical value	Modified 0.05 Critical Value
		Ireland / Germany excluding interest rates[a]		
$r = 0$	$r \geq 1$	39.203	34.870	45.68
$r \leq 1$	$r \geq 2$	13.347	20.180	–
		Including interest rates[b]		
$r = 0$	$r \geq 1$	111.587	87.170	93.328
$r \leq 1$	$r \geq 2$	57.298	63.000	–
		Ireland / UK excluding interest rates[b]		
$r = 0$	$r \geq 1$	57.532	42.340	70.030
$r \leq 1$	$r \geq 2$	21.695	25.770	–
		Including interest rates[b]		
$r = 0$	$r \geq 1$	127.997	87.170	85.427
$r \leq 1$	$r \geq 2$	77.194	63.000	61.740
$r \leq 2$	$r \geq 3$	41.665	42.340	–

a Cointegration with restricted intercepts and no trends in the *VAR*.
b Cointegration with unrestricted intercepts and restricted trends in the *VAR*.

Table 5: Nonlinearity Tests – Causal Models.

Test	Test Statistic	*p*-value	Bootstrap *p*-value	Test Statistic	*p*-value	Bootstrap *p*-value
	Ireland & Germany			Ireland & United Kingdom		
			Nominal Exchange Rates			
RESET						
			Excluding interest rates			
F	35.040	0.000		0.948	0.431	
LR	77.646	0.000		3.969	0.414	
			Including interest rates			
F	24.474	0.000		0.882	0.477	
LR	60.085	0.000		3.765	0.439	
Random Field						
			Excluding interest rates			
$\lambda_H^E(\mathbf{g})$	575.388	0.000	0.001	648.928	0.000	0.001
λ_{OP}^A	324.321	0.000	0.001	151.160	0.000	0.001
$\lambda_{OP}^E(\mathbf{g})$	233.907	0.000	0.001	233.152	0.000	0.001
g_{OP}	11.380	0.044	0.001	104.661	0.000	0.001
			Including interest rates			
$\lambda_H^E(\mathbf{g})$	179.66	0.000	0.001	205.475	0.000	0.001
λ_{OP}^A	224.382	0.000	0.001	545.731	0.000	0.001
$\lambda_{OP}^E(\mathbf{g})$	180.758	0.000	0.001	161.323	0.000	0.001
g_{OP}	156.695	0.000	0.001	211.304	0.000	0.001
			Real Exchange Rates			
RESET						
F	8.136	0.000		1.043	0.376	
LR	23.606	0.000		3.969	0.349	
STAR				lag length 1		
F		0.236			0.576	
F4		0.379			0.952	
F3		0.121			0.169	
F2		0.303			0.764	
		lag length 3			lag length 2	
F		0.010			0.207	
F4		0.054			0.108	
F3		0.010			0.236	
F2		0.039			0.591	
Random Field						
$\lambda_H^E(\mathbf{g})$	2.410	0.121	0.058	0.187	0.665	0.653
λ_{OP}^A	4.481	0.923	0.369	6.721	0.751	0.394
$\lambda_{OP}^E(\mathbf{g})$	0.035	0.852	0.922	1.056	0.304	0.562
g_{OP}	4.551	0.871	0.367	2.847	0.970	0.458

The subscripts and superscripts on λ indicate the precise nature of the LM tests; see Dahl and González-Rivera (2003).

Table 6: Random Field Analysis – Ireland, Germany & UK.

Model Component	Ireland & Germany		Ireland & United Kingdom	
Linear				
c	0.332	0.769	1.176	0.907
	(1.488)	(1.121)	(0.751)	(0.213)
p_t	-0.896	-0.836	-1.439	-1.093
	(0.191)	(0.152)	(0.308)	(0.239)
p_t^*	0.892	0.724	1.164	0.882
	(0.502)	(0.390)	(0.320)	(0.218)
i_t		-0.0004		0.009
		(0.002)		(0.004)
i_t^*		0.007		-0.009
		(0.005)		(0.004)
Nonlinear				
σ	0.019	0.010	0.021	0.009
	(0.002)	(0.004)	(0.003)	(0.004)
ζ	3.987	5.859	9.572	8.148
	(0.817)	(2.551)	(2.109)	(4.368)
p_t	4.265	4.609	0.480	2.777
	(0.375)	(1.103)	(0.116)	(1.214)
p_t^*	11.068	16.971	-1.864	10.454
	(0.733)	(3.021)	(0.044)	(1.846)
i_t		-0.032		0.118
		(0.023)		(0.039)
i_t^*		-0.146		-2.26 E-7
		(0.052)		(0.040)

Note: standard errors in parentheses. ζ is defined as $\zeta = \lambda/\sigma$, where σ is the standard deviation of the white noise disturbance ε_t.

Table 7: Multiple Structural Changes Model Estimation: Ireland-Germany.

Coefficients	Variables	Estimate	Standard Error	p-value
$\hat{\boldsymbol{\delta}}_1$	p_t	-1.034	0.059	0.000
	p_t^*	1.077	0.051	0.000
$\hat{\boldsymbol{\delta}}_2$	p_t	-0.725	0.043	0.000
	p_t^*	0.813	0.042	0.000
$\hat{\boldsymbol{\delta}}_3$	p_t	-0.787	0.386	0.045
	p_t^*	0.849	0.385	0.030
$\hat{\boldsymbol{\delta}}_4$	p_t	-1.961	0.311	0.000
	p_t^*	1.999	0.312	0.000
$\hat{\boldsymbol{\delta}}_5$	p_t	-0.843	0.499	0.094
	p_t^*	0.894	0.499	0.077
	i_t	-0.003	0.002	0.070
	i_t^*	0.008	0.002	0.000
R^2		0.985		
Adjusted R^2		0.983		
$F(12, 85)$		468.237		0.000

Estimated Break Dates and Confidence Intervals[a]

$\hat{T}_1$	1978 Q2	1978 Q1 – 1981 Q2
$\hat{T}_2$	1986 Q2	1986 Q1 – 1986 Q3
$\hat{T}_3$	1990 Q3	1990 Q2 – 1990 Q4
$\hat{T}_4$	1995 Q3	1994 Q2 – 1996 Q2

Break Tests

$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$\sup F_T(4)$
90.144	99.056	160.258	110.216
[11.470]	[9.750]	[8.360]	[7.190]

$\sup F_T(2\|1)$	$\sup F_T(3\|2)$	$\sup F_T(4\|3)$
96.265	12.233	19.191
[11.470]	[12.950]	[14.030]

UDmax	*WDmax*
160.58	219.875
[11.700]	[12.810]

a The 95 per cent confidence interval for break date.
Note: 5 per cent critical values in parentheses.

Appendix A.2: Figures

Fig. 1: Irish, German and UK Exchange and Interest Rates.

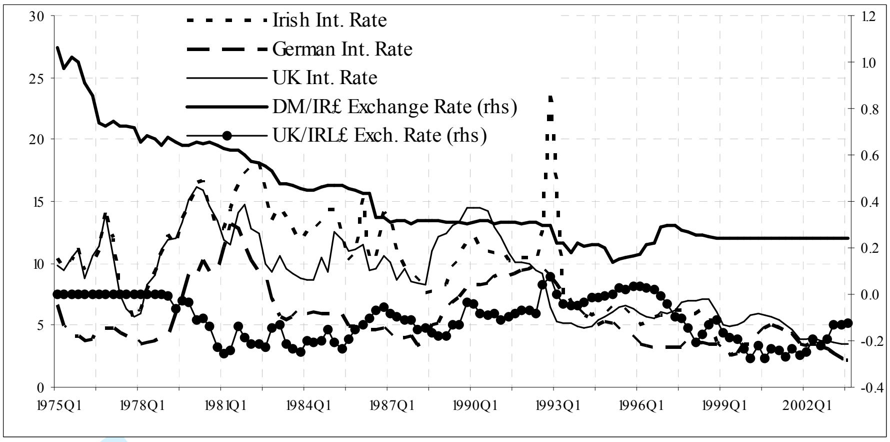

Fig. 2: Irish, German and UK Price Levels.

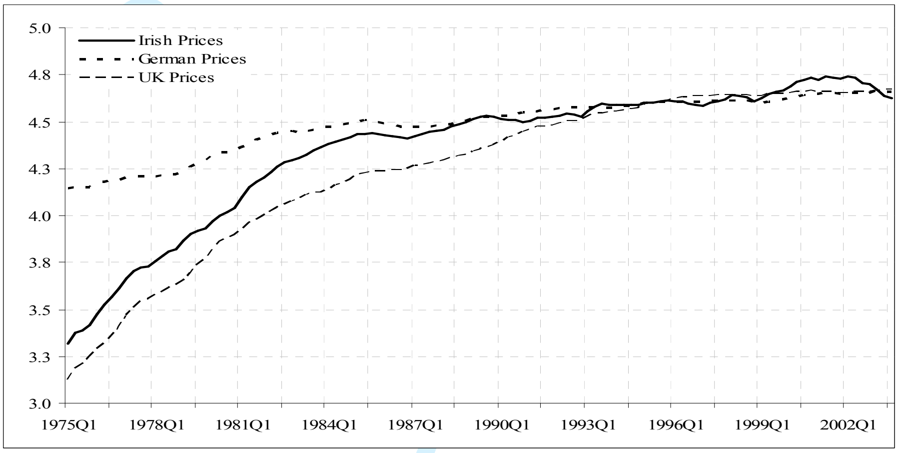

Fig. 3: Correlograms – Irish and German Series.

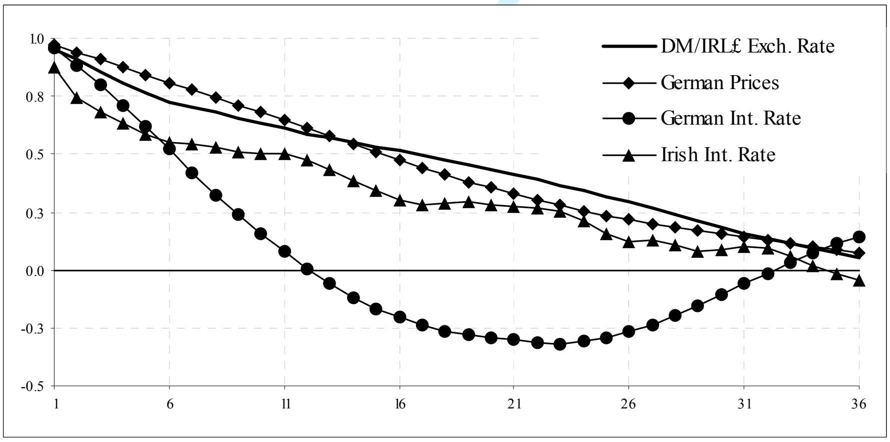

Fig. 4: Correlograms – Irish and UK Series.

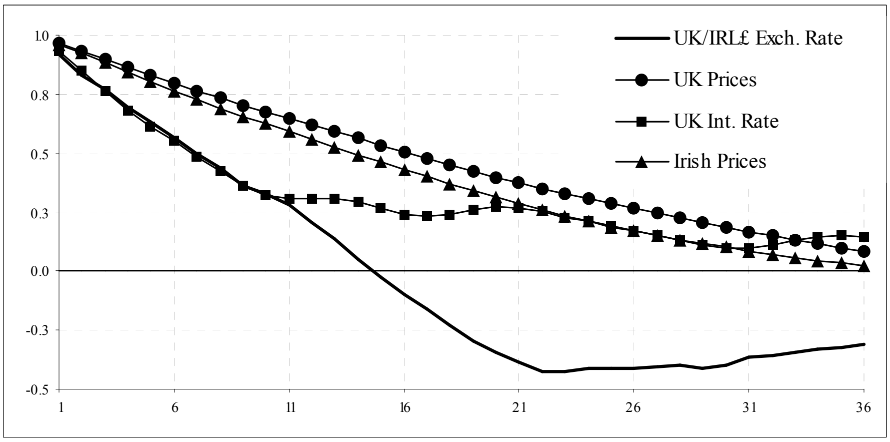

Fig. 5: Ireland/Germany – actual versus fitted based on random field regression.

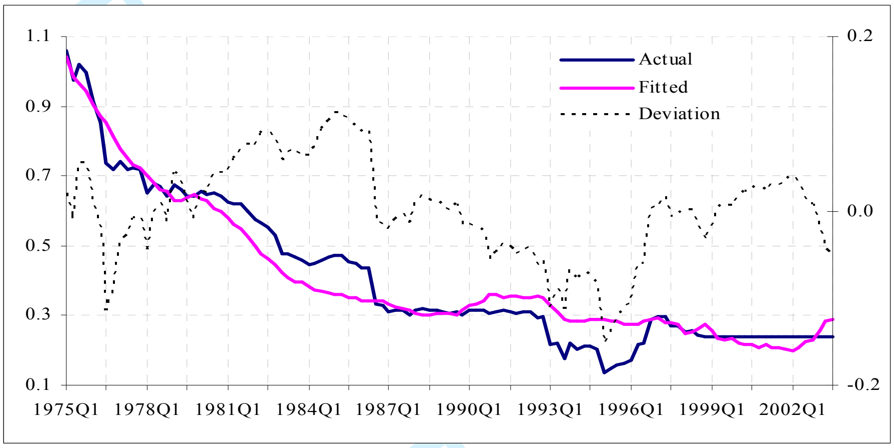

Fig. 6: Ireland/Germany – actual versus fitted based on structural changes model.