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Unit root testing against an ST-MTAR alternative: Finite-sample properties and an application to the UK housing market

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ABSTRACT

A class of smooth transition momentum-threshold autoregressive (ST-MTAR) tests is proposed to allow testing of the unit root hypothesis against an alternative of asymmetric adjustment about a smooth non-linear trend. Monte Carlo simulation is employed to derive finite-sample critical values for the proposed test and illustrate its attractive power properties against a range of stationary alternatives. The empirical relevance of the ST-MTAR test is highlighted via an application to aggregate house price data for the UK. Interestingly, house prices are found to exhibit structural change characterised a fitted logistic smooth transition process, with the newly proposed ST-MTAR test providing the most significant results of the alternative smooth transition unit root tests available.

Keywords:
Unit roots; Momentum-threshold autoregression; Smooth transition; Structural change; UK house prices.

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1 Introduction

Following the seminal study of Perron (1989), a large literature has emerged considering the issue of testing the unit root hypothesis in the presence of structural change. In response to Perron's finding that the Dickey-Fuller (1979) (DF) test can exhibit low power when applied to series which are stationary about a deterministic component subject to structural change, a number of authors have examined the issue of unit root testing in the presence of regime shifts. While some authors have followed Perron's seminal study and assumed structural change to be abrupt or instantaneous (see, *inter alia*, Banerjee *et al.* 1992; Zivot and Andrews 1992), Leybourne, Newbold and Vougas (1998) (LNV) suggest an alternative approach which allows for gradual adjustment. Using the logistic smooth transition function, LNV permit testing of the unit root hypothesis against an alternative of stationarity about a non-linear trend which allows for gradual adjustment between two regimes. The resulting smooth transition (ST) unit root tests have an obvious appeal, particularly in the analysis of economics data which frequently exhibit changing patterns of behaviour in their evolution. In more recent research, Sollis (2004) has extended this approach to incorporate the possibility of asymmetric adjustment about a non-linear trend specified by a logisitic smooth transition. Drawing upon the methods of Enders and Granger (1998), Sollis (2004) employs threshold autoregression (TAR) to develop a ST-TAR unit root test. In the present paper, the research of Sollis (2004) is itself extended in two ways. First, an alternative method of capturing asymmetric adjustment is proposed based upon momentum-threshold autoregression (MTAR). The extension of smooth transition unit root tests to consider MTAR adjustment has an obvious appeal as the use of MTAR rather than TAR adjustment has been found to result in higher power when extending the Dickey-Fuller (1979) unit root test and Engle-Granger (1986) cointegration test (see Enders and Granger (1998) and Enders and Siklos (2001) respectively). Using Monte Carlo simulation, the present paper derives finite-sample critical values of the resulting ST-MTAR tests and examines their power properties in the presence of stationary alternatives. In addition, the properties of the ST-TAR and ST-MTAR tests are compared under the alternative TAR and MTAR adjustment schemes. That is, ST-TAR (ST-MTAR) tests are examined in the context of misspecification when the data generation process is actually MTAR (TAR). Following this approach, the results of Cook (2003a) for Enders-Granger asymmetric unit root tests show the MTAR models to outperform TAR models even in the presence of TAR adjustment. These findings provide further justification for analysis of MTAR adjustment in the present context. Second, the analysis is extended to provide asymmetric versions of all existing specifications of the ST unit root test. While Sollis (2004) considered ST-TAR testing based upon the Models A, B and C of LNV, a further testing specification denoted as Model D by Vougas (2005) which has since been proposed, was not considered. In the present paper, all four specifications are extended to provide alternative ST-MTAR tests which allow for the inclusion of alternative deterministic terms and breaks. To illustrate the empirical relevance of the ST-MTAR testing procedure, an application to aggregate UK house price data is presented. Over recent years a large literature has emerged examining UK house prices. A feature of this literature is the inference that the UK aggregate house price series is a unit root process (see, *inter alia*, Cook 2003b, Meen 1999, Peterson *et al.* 2002). The application of smooth transition unit root tests overturns this inference, with the ST-MTAR unit root test found to provide the most conclusive results of the alternative tests considered.

This paper proceeds as follows. In section [2], the ST unit root tests of LNV are presented along with the ST-TAR tests of Sollis (2004) and the newly proposed ST-MTAR tests developed herein. Section [3] presents critical values for the ST-MTAR tests and an analysis of the empirical powers of the ST, ST-TAR and ST-MTAR tests in the presence of both TAR and MTAR adjustment. An empirical application of the tests to aggregate house price data for the UK is provided in section [4], with section [5] concluding.

2 Smooth transition unit root tests

To allow the unit root hypothesis to be tested against an alternative of structural change in the form of gradual rather than abrupt adjustment, LNV employ the deterministic logistic smooth transition $S_t(\gamma,\tau)$ which is defined as:

$$S_t(\gamma,\tau) = [1+\exp\{-\gamma(t-\tau T)\}]^{-1} \qquad \gamma>0 \qquad t=1,...,T \tag{1}$$

where T is the sample size, τ is the parameter determining the fraction of the sample at which the transition occurs, while γ determines the speed of transition. LNV propose three smooth transition unit root tests based upon the following models denoted as A, B and C:

$$\text{Model A} \quad : \quad y_t = \alpha_1 + \alpha_2 S_t(\gamma,\tau) + u_{at} \tag{2}$$

$$\text{Model B} \quad : \quad y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma,\tau) + u_{bt} \tag{3}$$

$$\text{Model C} \quad : \quad y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma,\tau) + \beta_2 t S_t(\gamma,\tau) + u_{ct} \tag{4}$$

where u_{it} are zero mean I(0) error processes. A further testing equation, denoted as Model D, is provided in subsequent analysis by Vougas (2005):

$$\text{Model D:} \;\; y_t = \alpha_1 + \beta_2 t S_t(\gamma, \tau) + u_{dt} \tag{5}$$

where u_{dt} is a zero mean I(0) error process. The four proposed tests therefore differ according to deterministic terms included and form of break considered. To test for the presence of a unit root, the null hypothesis of a unit root or unit root with drift is tested against an alternative given by Model A, B, C or D as appropriate:

$$\begin{aligned} H_0 &: \;\; y_t = \mu_t, \quad \mu_t = \mu_{t-1} + \varepsilon_t \\ H_1 &: \;\; (2), (3), (4) \text{ or } (5) \end{aligned}$$

$$\begin{aligned} H_0 &: \;\; y_t = \mu_t, \quad \mu_t = \kappa + \mu_{t-1} + \varepsilon_t \\ H_1 &: \;\; (3), (4) \text{ or } (5) \end{aligned}$$

where ε_t is an error term. The alternative models therefore permit various gradual changes in either intercept and/or trend, with a fixed intercept (and trend in the cases of Models B and C) also included. To implement the ST test, a two-step approach is followed.[1] In the first step, Models A, B, C and D are estimated using a non-linear least squares (NLS) algorithm with the resulting the residual processes ($\widehat{u}_{it}$, $i = a, b, c, d$) stored. These processes are given as:

$$\text{Model A} \;:\; \widehat{u}_{at} = y_t - \widehat{\alpha}_1 - \widehat{\alpha}_2 S_t(\widehat{\gamma}, \widehat{\tau}) \tag{6}$$

$$\text{Model B} \;:\; \widehat{u}_{bt} = y_t - \widehat{\alpha}_1 - \widehat{\beta}_1 t - \widehat{\alpha}_2 S_t(\widehat{\gamma}, \widehat{\tau}) \tag{7}$$

$$\text{Model C} \;:\; \widehat{u}_{ct} = y_t - \widehat{\alpha}_1 - \widehat{\beta}_1 t + \widehat{\alpha}_2 S_t(\widehat{\gamma}, \widehat{\tau}) - \widehat{\beta}_2 t S_t(\widehat{\gamma}, \widehat{\tau}) \tag{8}$$

$$\text{Model D} \;:\; \widehat{u}_{dt} = y_t - \widehat{\alpha}_1 - \widehat{\beta}_2 t S_t(\widehat{\gamma}, \widehat{\tau}) \tag{9}$$

[1] In the interest of brevity calculation of the smooth transition unit root tests is only outlined here. Further details can be obtained from reference to LNV and Vougas (2005).

In the second step, an augmented DF test is performed using the t-ratio of ψ_i from the following regression:

$$\Delta \hat{u}_{it} = \psi_i \hat{u}_{it-1} + \sum_{j=1}^{p_i} \phi_{ij} \Delta \hat{u}_{it-j} + \varepsilon_{it} \qquad i = a, b, c, d \tag{10}$$

The test statistics for testing the unit hypothesis $\psi_i = 0$ in (10) are denoted as s_α, $s_{\alpha(\beta)}$, $s_{\alpha\beta}$, and s_β for Models A to D respectively, with (2), (3), (4) or (5) used as appropriate. Recently, Sollis (2004) has extended the ST unit root tests given by Models A, B and C $\left(s_\alpha, s_{\alpha(\beta)}, s_{\alpha\beta}\right)$ to allow for asymmetric adjustment about the non-linear trend specified by the fitted smooth transition. To do this, Sollis employs threshold autoregression with a Heaviside indicator function (I_t) defined as below:

$$I_t = \begin{cases} 1 & \text{if } \hat{u}_{it-1} \geq 0 \\ 0 & \text{if } \hat{u}_{it-1} < 0 \end{cases} \tag{11}$$

This indicator function is then employed to extend (10) as follows:

$$\Delta \hat{u}_t = I_t \rho_1 \hat{u}_{t-1} + (1 - I_t)\, \rho_2 \hat{u}_{t-1} + \sum_{j=1}^{p} \psi_j \Delta \hat{u}_{t-j} + \eta_t \tag{12}$$

Asymmetric adjustment is therefore permitted as two adjustment parameters (ρ_i) are now present in (12), as compared to a single parameter (ρ) and speed of adjustment in (10). The unit root null hypothesis is then tested via either via the joint hypothesis H$_0$: $\rho_1 = \rho_2 = 0$ or the more significant t-statistic of H$_0$: $\rho_1 = 0$ or H$_0$: $\rho_2 = 0$. To test the unit root hypothesis against the different alternative hypotheses, Sollis (2004) combines (11) and (12) with (2), (3) or (4) respectively for Models A, B and C. The resulting test F and t statistics are denoted as F_α and ts_α for Model A, $F_{\alpha(\beta)}$ and $ts_{\alpha(\beta)}$ for Model B and $F_{\alpha\beta}$ and $ts_{\alpha\beta}$ for Model C. However, while this TAR-based extension to permit asymmetry is to be welcomed, it is suggested here that an MTAR-based indicator function can be employed also to derive a range of alternative asymmetric ST tests. As stated above, this extension has obvious appeal given the power advantage of MTAR tests relative to TAR tests in the context of unit root and cointegration analysis noted previously in the literature (see Enders and Granger 1998; Enders and Siklos 2001). Under MTAR adjustment, the relevant indicator function is then defined as:

$$I_t = \begin{cases} 1 & \text{if } \Delta y_{t-1} \geq 0 \\ 0 & \text{if } \Delta y_{t-1} < 0 \end{cases} \tag{13}$$

The resulting ST-MTAR tests therefore combine (12) and (13) with either (2), (3), (4) or (5) depending upon whether the smooth transition is given by Model A, B, C or D. As with the ST-TAR tests, the unit root hypothesis is tested via the joint significance of the adjustment parameters $\{\rho_i\}$ or the individually most significant parameter. Following earlier notation, the resulting F and t statistics are denoted as F^*_α and ts^*_α for Model A, $F^*_{\alpha(\beta)}$ and $ts^*_{\alpha(\beta)}$ for Model B, $F^*_{\alpha\beta}$ and $ts^*_{\alpha\beta}$ for Model C and F^*_β and ts^*_β for Model D.[2]

3 Critical values and empirical power analyses

3.1 Finite-sample critical values of the ST-MTAR test

To generate critical values for the newly proposed ST-MTAR tests, the following data generation process (DGP) is employed:

$$y_t = y_{t-1} + \eta_t \qquad t = 1, ..., T \tag{14}$$

$$\eta_t \sim i.i.d.\ \mathsf{N}(0,1) \tag{15}$$

All experiments are performed over 10,000 replications using GAUSS, with the error series $\{\eta_t\}$ generated via the RNDNS procedure with $y_0 = 0$. The resulting critical values for the alternative ST-MTAR tests are reported in Table One for a range of sample sizes ($T = 50, 100, 250, 500$) and levels of significance ($10\%, 5\%, 1\%$).

Table One about here

3.2 Empirical power analyses

To examine the empirical powers of the ST, ST-TAR and ST-MTAR unit root tests, Monte Carlo simulation experimentation is undertaken. The experimental design employed is based upon Sollis

[2] Vougas (2005) provides a detailed discussion of alternative approaches to the NLS estimation required for smooth transition unit root tests, with close attention paid to the impact of differing optimisation algorithms upon resulting critical values. In this paper, critical values for the ST-MTAR tests are generated using the superior NLP® constrained optimiser of the GAUSS subroutine FANPAC®. This optimiser combines the Broyden, Fletcher, Goldfarb and Shanno (BFGS) algorithms utilised by LNV, with the Newton-Raphson algorithm. This superior optimiser is utilised for all of the tests employed. Also, an initial grid search is employed for both τ and γ, thereby fully endogenising structural change.

(2004), with its basic structure given as below:

$$y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + v_t \tag{16}$$

$$\Delta\widehat{v}_t = I_t\rho_1\widehat{v}_{t-1} + (1 - I_t)\,\rho_2\widehat{v}_{t-1} + \eta_t \tag{17}$$

$$v_0 = 0 \quad \eta_t \sim \text{NID}\,(0, 1) \tag{18}$$

The basis of this DGP is therefore provided by the Model A specification given above. To ensure a stationary asymmetric DGP with structural change, the following design parameters are employed: $\rho_1 = -0.1, -0.3, -0.9$; $\rho_2 = -0.1, -0.3, -0.9$; $\alpha_1 = 1$; $\alpha_2 = 2, 5, 10$; $\gamma = 0.5, 5$; $\tau = 0.5$. With the exception of the inclusion of an additional break size ($\alpha_2 = 5$), these values follow Sollis (2004) and allow varying degrees of asymmetry and stationary to be considered in conjunction with a range of breaks of differing sizes occurring at different times. All possible combinations of the above parameters are considered subject to $\rho_1 \neq \rho_2$ to ensure asymmetric adjustment about the underlying smooth transition. However, in contrast to Sollis (2004) where asymmetric adjustment of a TAR form alone is considered, both TAR and MTAR adjustment are considered here. Therefore two sets of experiments are performed. For the first set of experiments, I_t is specified as in (11) allowing the properties of the alternative tests to be examined under a DGP where stationary TAR asymmetric adjustment occurs around a trend exhibiting smooth transition. For the second set of experiments, I_t is given by (13) to permit a similar analysis where asymmetric adjustment is of an MTAR form. Given the above design, the alternative TAR and MTAR tests are based upon the use of Model A with the powers of the s_α, ts_α, Fs_α, ts^*_α and F^*_α tests calculated at the 5% and 10% nominal levels of significance. Under the ST-TAR DGP, ts_α and Fs_α represent correctly specified tests, while under the ST-MTAR DGP they are clearly mis-specified. Similarly, ts^*_α and F^*_α are correctly specified under the ST-MTAR DGP but are mis-specified under the ST-TAR DGP. The experimental design therefore permits a standard analysis of the tests under the assumption of correct specification, while also allowing a form of mis-specification analysis where an investigator employs the incorrect form of asymmetric test. All experiments are conducted over 2,000 replications for a representative sample size of 100 observations.

The power results for the ST-TAR DGP are presented in Table Two with the results for the ST-MTAR DGP provided in Table Three. While empirical powers of the tests can be seen to vary across the considered designs, the following points can be made to summarise the results. Under a TAR adjustment scheme, it can be seen that for low levels of asymmetry (and consequently stationarity), the original LNV test can exhibit greater power than the TAR or MTAR tests. This is an intuitive finding as the asymmetric tests involve the estimation of additional parameters and the presence of a reasonable degree of asymmetry may be required to offset this and allow greater power to be observed. Considering the F and t forms of the asymmetric tests, it can be seen that for both TAR and MTAR adjustment, the former is a more powerful form than the latter. Comparing the relative powers of the TAR and MTAR tests, the TAR tests are generally more powerful than the MTAR tests. However, this property does not hold for all experimental designs and the difference in power where it does occur is often very small. This is perhaps a surprising finding as the MTAR tests are mis-specified under the current DGP and therefore might be expected to be dominated by the TAR tests. Turning to the results for the ST-MTAR DGP, the most striking feature of the results is superiority of the MTAR tests. Throughout, it is either the F_α^* or ts_α^* test which is the most powerful of all the tests considered. In contrast, the TAR tests which are now mis-specified under this DGP, possess similar power to the LNV test when applied in F-form, but less power when applied t-form. To illustrate the relative powers of the tests, consider the first design where $\{\rho_1, \rho_2, \alpha_2, \gamma\} = \{-0.1, -0.3, 2, 0.5\}$. For this case, the ts_α^* test has a power advantage of 4%, 34%, 36% and 57% relative to the F_α^*, F_α, s_α and ts_α tests respectively. Therefore, while the F_α^* or ts_α^* tests are similar in terms of power, they are both substantially more powerful than their rivals.

Tables Two and Three about here

4 Examining the order of integration of UK aggregate house prices

To illustrate the empirical relevance of the proposed ST-MTAR test, the order of integration of aggregate house prices for the UK is examined. The house price data considered are seasonally adjusted, quarterly observations on house prices over the period 1973(4) to 2005(1).[3] The series is analysed in its natural logarithmic form. Before considering the smooth transition based unit root tests detailed above, the unit root hypothesis is tested using commonly employed unit root tests. The two tests applied are the augmented Dickey-Fuller (1979) (ADF) test and the higher powered GLS-based Dickey-Fuller test of Elliott *et al.* (1996). These resulting unit root test statistics are denoted as τ_τ and τ_τ^{gls} respectively. From inspection of Figure One it can be seen that UK house prices display strong trending behaviour. As a result of this, an intercept and linear trend are included as deterministic terms when employing the τ_τ and τ_τ^{gls} tests. The degree of augmentation of the tests is determined via the Akaike Information Criterion following initial consideration of a

[3]The series are mixed-adjusted observations on all properties drawn from the Nationwide Building Society.

maximum lag length given by $int\left[12\left(T/100\right)\right]^{0.25}$. Justification of this upper bound is provided by Hayashi (2000). The resulting calculated test statistics are reported in Table One. Comparison of the reported values to the appropriate critical values as provided by, *inter alia,* Fuller (1996) and Elliott *et al.* (1996), shows that the unit root hypothesis cannot be rejected at even the 10% level of significance. However, from closer inspection of Figure One it is apparent that despite a clear upward trend, UK house price have not risen continuously over the sample period. In particular, the well documented recessionary period of the early to mid 1990s is apparent. This change in behaviour and its following gradual or steady recovery would appear to be well suited to modelling by a smooth transition process. In light of this, it is perhaps unsurprising that the above τ_τ and τ_τ^{gls} tests with their common underlying assumption of a maintained linear trend are unable to reject the unit root hypothesis. To incorporate the structural change depicted in the house price series under investigation, the smooth transition $s_{\alpha\beta}$, $F_{\alpha\beta}$, $ts_{\alpha\beta}$, $F_{\alpha\beta}^*$ and $ts_{\alpha\beta}^*$ test statistics are calculated. These tests are performed using equations (1), (4) and (10) for $s_{\alpha\beta}$, (1), (4), (11) and (12) for $F_{\alpha\beta}$ and $ts_{\alpha\beta}$, and (1), (4), (12) and (13) for $F_{\alpha\beta}^*$ and $ts_{\alpha\beta}^*$. All of the smooth transition tests are therefore based upon Model C given the noted trending nature of the series, with the degree of augmentation determined as for the τ_τ and τ_τ^{gls} tests.

Before considering the results for the smooth transition unit root tests, Figure Two presents the UK house price series along with the fitted smooth transition. It can be seen that the smooth transition process closely fits the recessionary and recovery periods observed in the 1990s. In Table Four, the calculated $s_{\alpha\beta}$, $F_{\alpha\beta}$, $ts_{\alpha\beta}$, $F_{\alpha\beta}^*$ and $ts_{\alpha\beta}^*$ test statistics are provided. Considering the original LNV $s_{\alpha\beta}$ test, it can be seen that incorporation of structural change via a smooth transition allows the unit root null to be rejected at the 5% level of significance. To examine whether the detected reversion to an underlying smooth transition attractor is of an asymmetric nature, the results for the ST-TAR and ST-MTAR tests can be considered. The results for the ST-TAR tests of Sollis (2004) show that the unit root is again rejected, this time at the 10% level of significance for $F_{\alpha\beta}$, and the 5% level of significance for $ts_{\alpha\beta}$. The results for the ST-MTAR are more significant, with the unit root hypothesis rejected at the 5% level of significance for $F_{\alpha\beta}^*$, and the 1% level of significance for $ts_{\alpha\beta}^*$. Therefore, while all of the smooth transition tests are able to reject the null of a unit root, the most significant rejection (at the 1% level) results from application of the newly proposed ST-MTAR test. To investigate further the relative performances of the tests, the AIC for each of the smooth transition tests is reported in Table Four. These results provide additional support for the ST-MTAR test, with this specification delivering the minimum AIC. Considering the estimated asymmetric adjustment parameters of the ST-TAR and

ST-MTAR tests, it can be seen that two very similar values are obtained under the former test with $(\rho_1, \rho_2) = (-0.150, -0.156)$, while the estimated values of the two adjustment parameters under MTAR adjustment differ with $(\rho_1, \rho_2) = (-0.186, -0.094)$. This latter finding indicates that the speed of adjustment is much faster for positive changes in the lagged residual in (13), than for negative changes.

Figures One and Two about here

Table Four about here

5 Conclusion

In this paper, recent developments in the testing of the unit root hypothesis have been extended with a class of ST-MTAR tests proposed and examined. Via simulation analysis, the ST-MTAR tests have been shown to possess some advantages relative to the existing ST-TAR tests of Sollis (2004). However, the ST-MTAR tests have not been developed with the intention of encompassing or dominating the ST-TAR tests, but are instead viewed as an alternative which might prove more appropriate in the presence of a differing asymmetric adjustment scheme. To illustrate the empirical relevance of the newly proposed tests, an empirical application to aggregate UK house prices was undertaken. In contrast to previous results in the literature which have concluded house prices in the UK to be I(1), application of ST, ST-TAR and ST-MTAR tests resulted rejection of the unit root null in the direction of asymmetric stationarity about a smooth transition. In particular, it was found that the presence of a unit root could be rejected beyond the 1% level of significance using the ST-MTAR test which was seen to provide the most significant results of all tests considered.

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Table One: Critical values for the ST-MTAR unit root test

T		ts^*_α	F^*_α	$ts^*_{\alpha(\beta)}$	$F^*_{\alpha(\beta)}$	$ts^*_{\alpha\beta}$	$F^*_{\alpha\beta}$	ts^*_β	F^*_β
50	10%	−3.587	8.620	−4.075	11.553	−4.302	13.037	−3.465	7.948
	5%	−3.901	10.063	−4.387	13.197	−4.630	14.937	−3.808	9.434
	1%	−4.538	13.269	−5.062	16.792	−5.338	19.008	−4.436	12.924
100	10%	−3.562	8.335	−3.966	10.754	−4.162	12.091	−3.417	7.713
	5%	−3.858	9.653	−4.250	12.177	−4.468	13.663	−3.754	9.029
	1%	−4.475	12.917	−4.871	15.400	−5.098	16.994	−4.387	12.223
250	10%	−3.486	8.077	−3.894	10.315	−4.116	11.670	−3.364	7.416
	5%	−3.788	9.329	−4.174	11.524	−4.383	13.057	−3.675	8.699
	1%	−4.328	12.018	−4.713	14.180	−4.935	16.154	−4.224	11.323
500	10%	−3.474	7.935	−3.875	10.138	−4.065	11.400	−3.341	7.282
	5%	−3.751	9.022	−4.158	11.541	−4.353	12.721	−3.628	8.450
	1%	−4.309	11.611	−4.684	14.176	−4.892	15.507	−4.208	11.111

* *Notes*: The tabulated figures represent finite-sample critical values for the ST-MTAR unit root test using Models A, B, C and D for a range of sample sizes and levels of significance.

Table Two: Empirical powers in the presence of a stationary ST-TAR process

				s_α		ts_α		F_α		ts^*_α		F^*_α	
ρ_1	ρ_2	α_2	γ	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%
−0.1	−0.3	2	0.5	0.427	0.266	0.371	0.222	0.425	0.269	0.309	0.187	0.412	0.256
−0.1	−0.9	2	0.5	0.726	0.589	0.752	0.620	0.747	0.615	0.669	0.531	0.753	0.615
−0.3	−0.1	2	0.5	0.424	0.265	0.387	0.248	0.420	0.267	0.319	0.195	0.417	0.261
−0.9	−0.1	2	0.5	0.705	0.527	0.746	0.627	0.741	0.578	0.658	0.511	0.736	0.586
−0.1	−0.3	5	0.5	0.418	0.245	0.358	0.214	0.415	0.251	0.307	0.177	0.400	0.248
−0.1	−0.9	5	0.5	0.721	0.565	0.751	0.628	0.747	0.601	0.665	0.521	0.748	0.608
−0.3	−0.1	5	0.5	0.410	0.245	0.373	0.236	0.404	0.252	0.312	0.194	0.409	0.255
−0.9	−0.1	5	0.5	0.695	0.522	0.748	0.621	0.729	0.569	0.650	0.516	0.729	0.575
−0.1	−0.3	10	0.5	0.397	0.246	0.342	0.215	0.392	0.247	0.302	0.178	0.382	0.247
−0.1	−0.9	10	0.5	0.702	0.522	0.730	0.608	0.725	0.567	0.611	0.492	0.710	0.568
−0.3	−0.1	10	0.5	0.375	0.225	0.348	0.203	0.377	0.226	0.298	0.180	0.384	0.238
−0.9	−0.1	10	0.5	0.687	0.526	0.746	0.627	0.724	0.576	0.632	0.493	0.712	0.576
−0.1	−0.3	2	5	0.419	0.248	0.357	0.222	0.414	0.256	0.303	0.178	0.399	0.247
−0.1	−0.9	2	5	0.704	0.554	0.724	0.598	0.732	0.588	0.639	0.509	0.728	0.588
−0.3	−0.1	2	5	0.404	0.233	0.369	0.227	0.403	0.238	0.313	0.187	0.407	0.247
−0.9	−0.1	2	5	0.693	0.549	0.749	0.625	0.732	0.590	0.665	0.508	0.731	0.583
−0.1	−0.3	5	5	0.375	0.223	0.336	0.198	0.374	0.230	0.285	0.171	0.369	0.226
−0.1	−0.9	5	5	0.677	0.495	0.725	0.581	0.714	0.539	0.597	0.469	0.686	0.540
−0.3	−0.1	5	5	0.350	0.203	0.321	0.179	0.351	0.204	0.280	0.165	0.361	0.216
−0.9	−0.1	5	5	0.660	0.491	0.738	0.597	0.698	0.547	0.618	0.465	0.686	0.540
−0.1	−0.3	10	5	0.307	0.175	0.286	0.171	0.309	0.177	0.259	0.153	0.315	0.181
−0.1	−0.9	10	5	0.623	0.453	0.710	0.575	0.667	0.517	0.597	0.467	0.667	0.515
−0.3	−0.1	10	5	0.309	0.178	0.282	0.163	0.309	0.186	0.237	0.140	0.311	0.182
−0.9	−0.1	10	5	0.615	0.456	0.703	0.563	0.661	0.504	0.595	0.456	0.650	0.514

Notes: The tabulated figures represent empirical power of the alternative smooth transition unit root tests under a ST-TAR data generation process.

Table Three: Empirical powers in the presence of a stationary ST-MTAR process*

				s_α		ts_α		F_α		ts^*_α		F^*_α	
ρ_1	ρ_2	α_2	γ	10%	5%	10%	5%	10%	5%	10%	5%	10%	5%
−0.1	−0.3	2	0.5	0.534	0.340	0.465	0.295	0.532	0.345	0.621	0.464	0.643	0.447
−0.1	−0.9	2	0.5	0.997	0.986	0.995	0.969	0.999	0.989	1.000	1.000	1.000	1.000
−0.3	−0.1	2	0.5	0.541	0.350	0.492	0.324	0.545	0.361	0.626	0.465	0.646	0.457
−0.9	−0.1	2	0.5	0.998	0.983	0.994	0.978	0.999	0.990	1.000	1.000	1.000	1.000
−0.1	−0.3	5	0.5	0.526	0.329	0.453	0.279	0.523	0.334	0.614	0.460	0.635	0.442
−0.1	−0.9	5	0.5	0.996	0.987	0.995	0.971	0.999	0.991	1.000	1.000	1.000	1.000
−0.3	−0.1	5	0.5	0.527	0.340	0.463	0.302	0.528	0.344	0.620	0.451	0.638	0.454
−0.9	−0.1	5	0.5	0.999	0.986	0.996	0.982	1.000	0.992	1.000	1.000	1.000	1.000
−0.1	−0.3	10	0.5	0.504	0.328	0.440	0.287	0.500	0.335	0.594	0.446	0.617	0.444
−0.1	−0.9	10	0.5	0.995	0.984	0.994	0.981	0.997	0.988	1.000	1.000	1.000	1.000
−0.3	−0.1	10	0.5	0.502	0.316	0.455	0.293	0.502	0.322	0.597	0.454	0.615	0.441
−0.9	−0.1	10	0.5	0.997	0.988	0.995	0.983	0.999	0.993	1.000	1.000	1.000	1.000
−0.1	−0.3	2	5	0.519	0.327	0.445	0.277	0.517	0.342	0.596	0.422	0.609	0.422
−0.1	−0.9	2	5	0.995	0.979	0.991	0.974	0.997	0.984	1.000	1.000	1.000	1.000
−0.3	−0.1	2	5	0.525	0.332	0.466	0.313	0.533	0.337	0.590	0.436	0.627	0.442
−0.9	−0.1	2	5	0.997	0.987	0.996	0.982	1.000	0.993	1.000	1.000	1.000	1.000
−0.1	−0.3	5	5	0.477	0.302	0.409	0.263	0.469	0.299	0.580	0.416	0.585	0.405
−0.1	−0.9	5	5	0.994	0.981	0.994	0.979	0.996	0.988	1.000	1.000	1.000	1.000
−0.3	−0.1	5	5	0.470	0.283	0.421	0.258	0.474	0.290	0.573	0.421	0.586	0.404
−0.9	−0.1	5	5	0.996	0.982	0.993	0.979	0.999	0.991	1.000	1.000	1.000	1.000
−0.1	−0.3	10	5	0.430	0.248	0.381	0.232	0.432	0.256	0.604	0.429	0.575	0.379
−0.1	−0.9	10	5	0.993	0.976	0.992	0.971	0.997	0.984	1.000	1.000	1.000	1.000
−0.3	−0.1	10	5	0.425	0.255	0.374	0.239	0.424	0.261	0.572	0.411	0.566	0.378
−0.9	−0.1	10	5	0.997	0.984	0.993	0.980	0.998	0.991	1.000	1.000	1.000	1.000

Notes: The tabulated figures represent empirical power of the alternative smooth transition unit root tests under a ST-MTAR data generation process.

Table Four: Linear and smooth transition unit root tests for UK house prices

Test	Calculated test statistic		AIC	(ρ_1, ρ_2)
ADF	$\tau_\tau : -2.207$			
GLS-DF	$\tau_\tau^{gls} : -1.737$			
LNV	$s_{\alpha\beta} : -5.184^{**}$		-8.175	$-$
ST-TAR	$F_{\alpha,\beta} : 13.328^{*}$	$ts_{\alpha,\beta} : -4.223^{**}$	-8.159	$-0.150, -0.156$
ST-MTAR	$F^*_{\alpha,\beta} : 15.304^{**}$	$ts^*_{\alpha,\beta} : -5.401^{***}$	-8.187	$-0.186, -0.094$

Notes: The tabulated figures represent empirical output resulting from the application of linear and smooth transition unit root test statistics for UK aggregate house price data. All smooth transition unit root tests are performed using the Model C testing specification. A single, double or triple asterisk denotes rejection at the 10%, 5% or 1% levels of significance respectively.

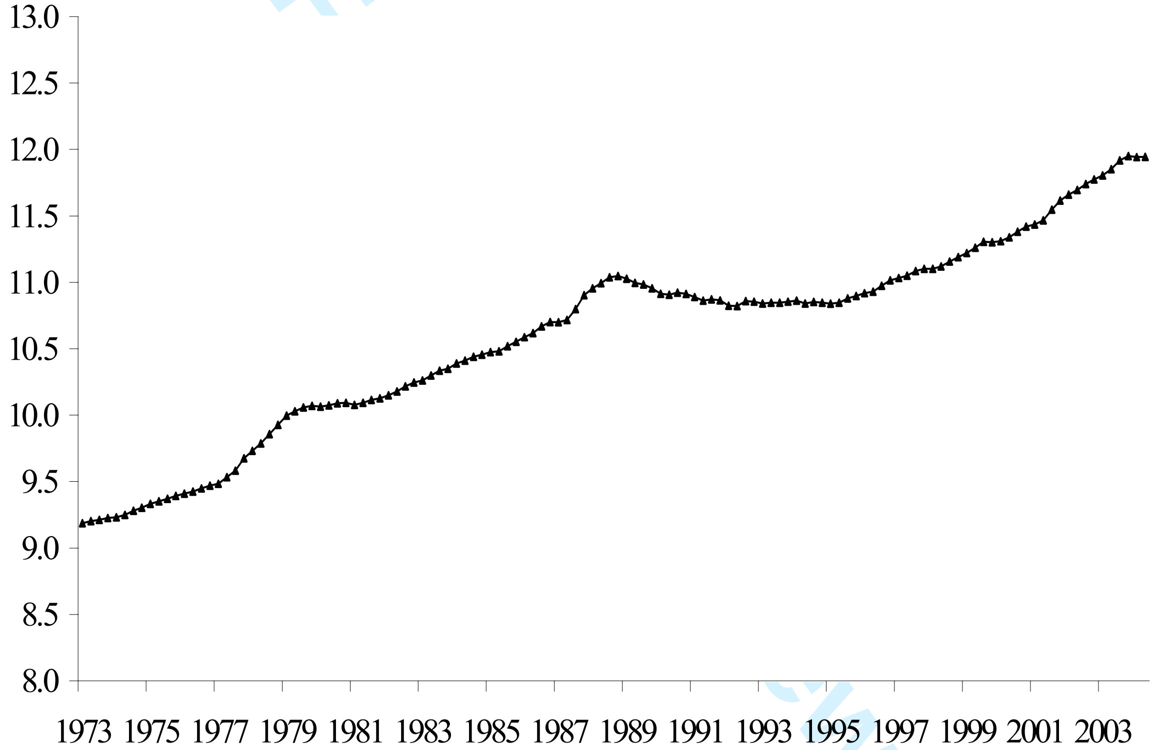

Figure 1: Aggregate house prices in the UK, 1973-2005.

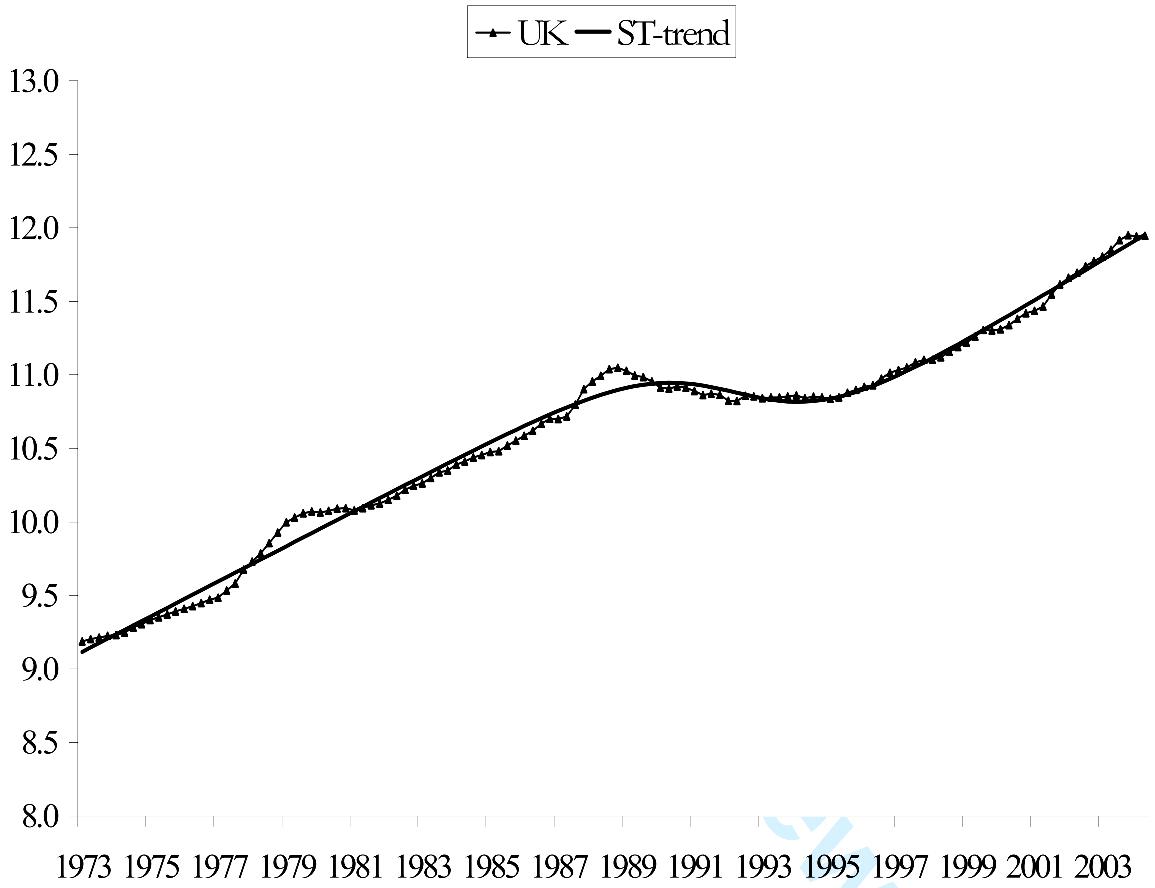

Figure 2: Aggregate house prices in the UK and fitted logistic smooth transition function.