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Journal:	*Applied Economics*
Manuscript ID:	APE-05-0609.R1
Journal Selection:	Applied Economics
JEL Code:	C10 - General < C1 - Econometric and Statistical Methods: General < C - Mathematical and Quantitative Methods, C20 - General < C2 - Econometric Methods: Single Equation Models < C - Mathematical and Quantitative Methods
Keywords:	ARIMA, UNIT ROOT, POWER, MONTE CARLO SIMULATIONS, CRITICAL VALUES

FULL TITLE

TESTING FOR A UNIT ROOT UNDER THE ALTERNATIVE HYPOTHESIS OF ARIMA (0,2,1)

By

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ABSTRACT

Showing a dual relationship between ARIMA (0,2,1) with parameter θ=−1 and the random walk, a new alternative hypothesis in the form of ARIMA (0,2,1) is established in this paper for evaluating unit root tests. The power of four methods of testing for a unit root is investigated under the new alternative, using Monte Carlo simulations. The first method testing θ=–1 in second differences and using a new set of critical values suggested by the two authors in finite samples, is the most appropriate from the integration order point of view. The other three methods refer to tests based on t and Φ statistics introduced by Dickey & Fuller, as well as, the non-parametric Phillips-Perron test. Additionally, for cases where for the first method a low power is met, we studied the validity of prediction interval for a future value of ARIMA (0,2,1) with θ close but greater of –1, using the prediction equation and the error variance of the random walk. Keeping the forecasting horizon short, the coverage of the interval ranged at expected levels, but its average half-length ranged up to four times more than its true value.

Running Head: Unit root – ARIMA(0,2,1)

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Acknowlegdements: Thanks are due to an anonymous referee for the very helpful and constructive comments. Any remaining errors are solely the authors' responsibility.

1. INTRODUCTION

Dickey and Fuller (1976) in their pioneer work, considered the random walk model with drift as a special case of the Gaussian AR(1), $y_t = \mu + \phi y_{t-1} + \varepsilon_t$, with $\phi = 1$ and $\varepsilon_t \sim$ i.i.d. $N(0, \sigma_\varepsilon^2)$. Taking first differences in $y_t = \mu + y_{t-1} + \varepsilon_t$ gives a Gaussian white noise. Over-differencing the random walk model, we would expect again a Gaussian white noise with mean $E(\Delta^2 y_t) = E(\varepsilon_t - \varepsilon_{t-1}) = 0$, and variance $V(\Delta^2 y_t) = E(\varepsilon_t^2) + E(\varepsilon_{t-1}^2) = 2\sigma_\varepsilon^2$. Empirical results from Monte-Carlo simulations, although they support the outcome of first differences, they do not agree with a white noise in second differences.

Figure 1 displays the sample autocorrelation (ACF) and partial autocorrelation (PACF) functions of first and second differences of a typical realisation of size 250 observations from the random walk with $\mu = 2$, $y_o = 0$, and $\sigma_\varepsilon = 1$. Details about the adopted random number generator can be found in Kevork (1990). For first differences, the plots confirm a white noise process. On the contrary, the sample ACF and PACF of second differences, combined with a stationary time series plot, indicate an MA(1) process, as a significant negative autocorrelation occurs at lag 1, and the partial autocorrelations decay exponentially to zero.

Figure 1 about here

The findings of Monte-Carlo simulations are shown analytically by first rewriting the second differences of the random walk model as $\Delta^2 y_t = \mu - \Delta y_{t-1} + \varepsilon_t$, second, replacing y_{t-1} in the right hand side with $\mu + y_{t-2} + \varepsilon_t$, and finally, after cancellations, writing $\Delta^2 y_t = \varepsilon_t + (-1)\varepsilon_{t-1}$. Thus, overdifferencing a random walk model with drift leads to ARIMA (0,2,1) with parameter $\theta = -1$. But also the random walk model can be considered as a special case of ARIMA (0,2,1) with $\theta = -1$. Rewriting ARIMA (0,2,1) as $\Delta y_t = \Delta y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$, and relating Δy_t's to current and past ε_t's terms we take the following set of recursive equations:

$$
\begin{aligned}
\Delta y_t - \Delta y_{t-1} &= \varepsilon_t + \theta\varepsilon_{t-1} \\
\Delta y_{t-1} - \Delta y_{t-2} &= \varepsilon_{t-1} + \theta\varepsilon_{t-2} \\
&\;\;... \\
\Delta y_2 - \Delta y_1 &= \varepsilon_2 + \theta\varepsilon_1 \\
\Delta y_1 &= \mu + \varepsilon_1
\end{aligned}
\qquad (1)
$$

Adding the terms in each side of (1), we eliminate all past values of Δy_t, and get

$$\Delta y_t = \mu + (1+\theta)(\varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3} + \ldots + \varepsilon_1) + \varepsilon_t \tag{2}$$

Setting $\theta = -1$ in (2), we take eventually the random walk model with drift.

To investigate further the behaviour of ARIMA (0,2,1) for θ greater but close to –1, we generated 1000 replications of 600 observations each, from the population model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$, under different values of θ, with $y_o = 0$, $y_1 = 2 + \varepsilon_t$, and $\varepsilon_t \sim$ i.i.d. $N(0,1)$. Using the same streams of values generated from the standard normal for $\theta = -1$, the previous specification lead to the same realisations with those produced by the random walk model with drift $y_t = 2 + y_{t-1} + \varepsilon_t$ with $y_o = 0$. Figures 2a up to 2c display the time series plots of first differences, as well as, the sample ACF of first and second differences, for a typical realisation of 600 observations. As θ is approaching to -1^+, the time series plots of first differences are changing gradually from an obvious non-stationary pattern to a stationary one. The corresponding plots of ACF and PACF have the representative pattern of a non-stationary process for θ greater but quite far away from –1, but for θ close to –1, they indicate a white noise process. Finally, the graphs in figure 2c indicate an MA(1) in second differences, even when θ is not close to –1.

Figure 2a-c about here

The behaviour of ARIMA (0,2,1) for θ close to –1 leads us to consider it as the alternative hypothesis for evaluating and comparing the power of unit root test methods. Four such general methods are considered in this paper. The first method tests in second differences the null hypothesis H_o: $\theta = -1$, against the alternative H_o: $\theta > -1$, using a new set of critical values, which Halkos and Kevork generated for testing a unit root in finite samples from an MA(1). The second method is based on testing the statistical significance of the estimated coefficient of the lagged dependent variable in the right hand side of the "intercept" and the "trend-and-intercept" models, using Mac-Kinnon critical values. The third method includes common regression F-tests introduced by Dickey and Fuller, which are known as Φ tests. Finally the last method concerns the non-parametric Phillips-Perron (PP) test, which is applied after estimation of the "intercept" model".

The previous discussion makes the structure of the paper to be as follows: In the next section we review the relevant literature on over-differencing empirical non-stationary series, and validity of unit root tests. In section 3, we revise the theoretical background for exploring the existence of a random walk by testing for a unit root in ARIMA (0,2,1). The power of such a test is also presented for different combinations of sample sizes and values of θ. In section 4, we estimate the power of t, Φ, and PP tests on ARIMA (0,2,1). The method, based on testing for a unit root in ARIMA (0,2,1), attains low power for small samples and values of θ close to –1. For such cases, in section 5, we explore the consequences of a wrong decision, namely to accept a random walk model for making predictions for future values of the true process. Finally, section 6 summarises the main findings of this research.

2. LITERATURE REVIEW

The literature review focuses on two basic issues: over-differencing empirical non-stationary series and validity of unit root tests. Specifically, in empirical research applications of Box-Jenkins ARIMA (p,d,q) models for making valid predictions, we have to identify correctly the proper ARIMA model, which governs the behaviour of the empirical time series (hereafter TS). For a non-stationary time series before identifying the parameter p and q we must identify the times the series should be differenced.

The number of times that the TS under consideration must be differenced is determined intuitively by using the autocorrelation or/and partial autocorrelation functions of the differenced series. Model identification is complicated especially if the TS under consideration is seasonal or periodic. For non-seasonal TS, manual identification may be achieved by using the autocorrelation or/and partial autocorrelation functions, the extended autocorrelation function and the smallest canonical correlation table (Tsay and Tiao, 1984, 1985, Box and Jenkins, 1970, Box *et a.,* 1994, Pankratz, 1991). The above methods seem to be ineffective in seasonal TS. In this case the identification may be performed using a filtering method (Liu, 1989, 1999, Liu and Hudak, 1992). This method is effective for automatic identification of ARIMA models for both seasonal and non-seasonal TS.

Koreisha and Pukkila (1993) presented two methods for determining the degree of differencing in order to achieve stationarity in the data. Using simulation of different model structures, they confirmed the results. Hall (1989) proposes a test for unit root relying on an instrumental variable (IV) estimator, which was applied in the case where the series is generated by an ARIMA(0,1,q) process. Pantula and Hall (1991) extended Hall's framework to the case of a series generated by an ARIMA(p,1,q) model. To obtain the asymptotic distributions, they assumed that either p or q was known, and using simulations they provide evidence that the finite-sample distributions of their test statistics were well approximated by the Dickey-Fuller distributions even in the case of over-specifying p and q.

Reilly (1980) and Reynolds *et al.* (1995) develope automatic methods for identifying ARIMA models for TS. The method developed by Reynolds *et al.* (1995) employs a neural network approach and is restricted to non-seasonal TS, while the method developed by Reilly (1980) works properly for non-seasonal TS but it is less effective in the case of seasonal TS.

The above-mentioned methods require the existence of long TS, which are used for model development and validation before we proceed to parameter estimation and predictions. The ARIMA approach for TS predictive model development is justified in both theoretical and statistical grounds. But Makridakis *et al.* (1983) claim the complexity of these models has been an obstacle for their adoption as a forecasting tool in organisations. The one-step ahead forecast for an ARIMA (0,1,1) model is equivalent to forecasting using an exponential smoothing method, when the smoothing constant leads to minimum mean square error forecast (Abraham and Ledolter, 1983).

A unit root in the moving average polynomial can be interpreted in various ways depending on the modeling application. Testing for a unit root in the moving average polynomial is equivalent to test that the series is over-differenced (Brockwell and Davis, 2002). A difficulty with the null hypothesis H_0: $\theta=1$ is that estimating a moving average model with a unit root is an irregular problem. The asymptotic distribution of the maximum likelihood estimator of a non-invertible MA parameter is unknown but there is a positive probability that a local maximum is attained by the likelihood function at a point of a unit root (Anderson and Takemura, 1986, Tanaka and Satchell, 1989). This implies that the development of LR and Wald tests is "intractable". Lagrange multiplier tests can be obtained, as they require the estimation of the model under the null hypothesis. Ahtola and Tiao (1984) prove it in the case of an MA(1) model with a zero mean value, while Tanaka (1990) obtains a general score-type test for the MA unit root hypothesis. Phillips (1987) and Phillips and Perron (1988) extends this work on autoregressive unit root tests.

Similarly Saikkonen and Luukkonen (1993) derive two tests for the MA unit root hypothesis. In the case of serially uncorrelated errors these can be motivated by local optimality arguments. Halkos and Kevork (2005c), using the exact maximum likelihood estimator of θ from the MA(1), and a certain simulation strategy, estimate appropriate percentiles, together with their standard errors, offering a new set of critical values for testing in finite samples $H_o: \theta = -1$, against $H_1: \theta > -1$. In this way, appropriate regions for rejecting the null or being in uncertainty are defined.

A large literature has been recently developed for analyzing TS regression with difference stationary processes. Dickey (1976) and Dickey and Fuller (1976, 1981) in their seminal papers examine the OLS estimation when the innovations in the unit root process are i.i.d. Phillips (1987) extends these results to a more general setting for the innovation process in such a way as to allow both time dependence and heterogeneity. Phillips and Perron (1988) explore data generating mechanisms with drift and trend. Phillips (1990) and Chan and Tran (1989) have explored the estimation of the autoregressive parameter and tested for a unit root when the random walk process has errors, which obey to a stable law. Phillips (1990) generalises this case using a semi-parametric modification of the usual t-ratio.

Leybourne and Newbold (1999) using simple theoretical calculations, confirm simulation evidences that probabilities of rejecting the null hypothesis of the Dickey Fuller and the Phillips-Perron tests differ substantially when the true generating process is the stationary second order autoregression. Halkos and Kevork (2005b) using certain estimates from Monte-Carlo simulations and considering the random walk as the true model, derived the probability the prediction interval to include any future value y_{T+s} of AR(1).

Ahn *et al.* (2001) analyse both asymptotically and in finite sample the properties of some unit root test, when the errors obey to a stable law. They consider a number of test statistics (such as the Dickey Fuller and the Lagrange Multiplier), when the data generating process is a driftless random walk and the regression model matches exactly the data generation process. Gallegari *et al.* (2003), in a similar analysis, characterize as limited both the behavior of OLS estimators of regression coefficients and the DF tests under the data generating processes usually encountered in the unit root literature (random walk with and without drift and the associated regression models with constant term, without deterministic component and with constant and time trend terms). They also investigate the consequences of the 'local to finite' variance analysis assessing that the size distortion of the DF test as the departure from the standard finite variance set up tends to decrease as the sample size tends to infinite.

Dickey and Fuller (1979) based their analysis on the asymptotic properties of the OLS estimator. Important variations of the DF tests are their extensions to other estimation methods such as Maximum Likelihood (Shin and Lee, 2000, Skin and Fuller, 1998), the generalised least squares detrending under a fixed local alternative (Elliott *et al.*, 1996, Xiao and Phillips, 1998, Hwang and Schmidt, 1996) and the weighted symmetric estimator (Park and Fuller, 1995, Fuller, 1996). Hassler and Wolters (1994) claim that the Augmented Dickey Fuller (hereafter ADF) compared to fractional alternatives loses considerable power when augmented terms are added. On the other hand, Krämer (1998) finds that ADF is consistent if the order of autoregression does not tend to infinity too fast. Bisaglia and Procidano (2002), using Monte Carlo simulations, clarify this contradiction and find that the ADF bootstrap works in general better than the ADF even if the power of the test is quite low, especially if the data generating process is a non-stationary fractional integrated one.

On the contrary, Halkos and Kevork (2005a) evaluated simple versions of the Dickey-Fuller test under the null hypothesis of a random walk model or an alternative non-stationary mean reverted process. Through Monte Carlo simulations they show that, apart from few cases, testing the existence of a unit root, using both McKinnon critical values and an F test, recommended by Pindyck and Rubinfeld, they obtain actual type I error and power very close to their nominal levels.

Finally, a number of researchers have developed tests for a single structural break with unknown break points in various dynamic models (Andrews, 1993; Perron and Vogelsang, 1992; Sen, 2004; Blot and Serranito, 2006). In most cases, these tests were either designed to test for a structural change in regression coefficients with stationary series or for a unit root against a stationary alternative with an unknown single break point. The applications of these tests were extremely successful in analysing breaking points in variables like real exchange rates, real GNP and other integrated processes (Banerjee *et al.*, 1992; Perron and Vogelsang, 1992; Zivot and Andrews, 1992; Charemza *et al.*, 2005; Harvey and Mills, 2005).

3. TESTING FOR A UNIT ROOT IN ARIMA (0,2,1)

For the non-invertible MA(1), $y_t - \mu = \varepsilon_t + \theta\varepsilon_{t-1}$, given a sample of size T, the *ln* of the exact likelihood function is given by

$$\ln\left(L\left(\theta, \sigma_\varepsilon^2\right)\right) = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln\sigma_\varepsilon^2 - \frac{1}{2}\sum_{t=1}^{T}\psi_{tt} - \frac{1}{2\sigma_\varepsilon^2}\sum_{t=1}^{T}\frac{\tilde{y}_t^2}{\psi_{tt}} \tag{3}$$

where

$$\psi_{tt} = \begin{cases} \dfrac{1-\theta^{2(t-1)}}{1-\theta^{2t}} & , \quad \theta \neq \pm 1 \\[2ex] \dfrac{t+1}{t} & , \quad \theta = \pm 1 \end{cases}$$

$$\tilde{y}_t = \begin{cases} y_t - \mu - \theta\dfrac{1-\theta^{2(t-1)}}{1-\theta^{2t}}\tilde{y}_{t-1} & , \quad \theta \neq \pm 1 \\[2ex] y_t - \mu - \theta\dfrac{t-1}{t}\tilde{y}_{t-1} & , \quad \theta = \pm 1 \end{cases}$$

and $\tilde{y}_1 = y_1 - \mu$. Using an appropriate simulation strategy after maximising (3), Halkos and Kevork (2005c) offer a new set of critical values for testing H_o: $\theta = -1$, against H_1: $\theta > -1$, in samples of size T = 25, 50, 100, 250, 500. The case of an estimate of θ less than –1 does not contradict the alternative hypothesis, as (3) is maximised at $\theta = \tilde{\theta} = \tilde{\theta}^{-1}$ (Hamilton, 1994). So, whenever $\tilde{\theta}$ is less than –1, the test is applied by taking the reciprocal of $\tilde{\theta}$.

In the introductory section we illustrated the dual relationship between the random walk model and ARIMA (0,2,1). That is, overdifferencing the random walk model leads to ARIMA (0,2,1) with parameter $\theta = -1$, as well as, the random walk model can be considered as a special case of ARIMA (0,2,1) with $\theta = -1$. This means that applying the test H_o: $\theta = -1$, against H_1: $\theta > -1$, to second differences of a time series realisation is equivalent of testing in the same series for the existence of a unit root. The only problem, which occurs in such a case is that taking second differences, the actual sample size is reduced from T to T-2. For this reason, we applied a linear interpolation to the initially suggested critical values of Halkos and Kevork, resulting to those presented in Table 1. The critical values of table 1 can be used for testing a unit root in ARIMA (0,2,1) at nominal sample sizes T=25, 50, 100, 250 and 500, or actual ones T =

23, 48, 98, 248, and 498, leading to rejection of H_o when $\hat{\theta} > \Theta_\alpha^U$, and to uncertainty whenever $\Theta_\alpha^L < \hat{\theta} < \Theta_\alpha^U$.

To estimate the power of the unit root test in ARIMA (0,2,1) using the critical values of table 1, for each value of $\theta =$ -0.90, -0.95, -0.98, we generated 1000 replications of size 600 observations from model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$. The initial conditions, as well as, comments about the validity of the random number generator have been already discussed in the introductory section. In each replication, and for nominal T=25, 50, 100, 250, 500, after subtracting the mean of second differences, we fit an MA(1) without a constant term to second differences in each replication, by maximising (3). The maximisation process was performed in E-VIEWS.

In each replication, to update θ and σ_ε^2 in successive iterations, we used the Marquard first derivative method, where first derivatives were evaluated analytically. The computation of $\ln\left(L\left(\theta, \sigma_\varepsilon^2\right)\right)$ was repeated until the improvement between iterations was less than 0.001. The Marquardt algorithm modifies the Gauss-Newton by adding a correction matrix to the Hessian approximation. This ridge correction copes with numerical problems in case the outer product is near singular, improving the convergence rate. Estimation using the analytical evaluation of first derivatives was preferred as it involves fewer function evaluations, and therefore is faster than evaluating the derivatives analytically.

As starting values for θ and σ_ε^2, we used those provided by E-VIEWS, using the method of backcasting (Box and Jenkins; 1976). EVIEWS backcasts MA terms by computing the unconditional residuals and using the backward recursion to compute backcast values. To start this recursion the values for the innovations beyond the estimation sample are set to zero and a forward recursion is used for estimating the values of the innovations using the backcasted values of the innovations and the actual residuals. The sum of squared residuals is minimized, after having being formed as a function of θ and relying on the fitted values of the lagged innovations. The backcast step, forward recursion and the minimization of SSR are repeated till the convergence of estimates of θ.

The standard errors of the estimated θ and σ_ε^2 were computed from the Gauss-Newton Hessian. Gauss-Newton follows Newton-Raphson replacing the negative of the Hessian using for each observation the approximation of the sum of the outer product of the gradient vectors and its contribution to the objective function. In this way we have just to evaluate the first derivatives, and the outer product is necessarily positive and semi-definite. But in case that it is away from the maximum, we may approximate poorly the overall shape of the function. This implies the need for more iterations for convergence.

Table 1 about here

For each combination of T and θ, table 2 displays the percentages of replications where the null hypothesis of a unit root in ARIMA (0,2,1) either is rejected or we are uncertain for making a decision. The estimated probabilities are reported for nominal level of significance, α = 0.10, where the maximum power is attained between the three traditional levels of significance, 1%, 5% and 10%. It is obvious that for $\theta = -0.90$ and $\theta = -0.95$ acceptable power can be attained when the sample size is greater than 250 and 500 observations respectively. For $\theta = -0.98$, the power remains at very low levels, even with a sample of size 500. Further, for all the examined cases, the probability of being in uncertainty ranges below 4%. The cases where low power is observed will be further discussed in section 5, where we shall investigate, from the forecasting point of view, the consequences of not rejecting the null hypothesis on the validity of prediction intervals generated using the random walk model.

Table 2 about here

4. POWER OF UNIT ROOT TESTS ON ARIMA (0,2,1)

In this section, we evaluate traditional unit root tests on ARIMA (0,2,1), using the 1000 replications already generated from model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$, for θ = -0.90, -0.95, -0.98. In the following lines, we present first the theoretical properties of each test separately, and the way of implementing it in each replication from ARIMA (0,2,1).

The first two tests are carried out by estimating using ordinary least squares (OLS) the "intercept" model

$$\Delta y_t = \mu + \gamma y_{t-1} + \sum_{j=1}^{P} \phi_j \Delta y_{t-j} + \varepsilon_t \qquad (4)$$

The first test is based on the t_{INT} statistic of the estimated parameter γ. Whenever t_{INT} is less than the corresponding Mac-Kinnon critical value, we reject H_o: γ=0, in favour of H_1: γ<0. The second test, known as Φ_1, concerns the null hypothesis H_o: μ=0, γ=0, and is applied by estimating first (4), and then its restricted form under H_o. The null hypothesis is rejected whenever the calculated ratio $(T-\kappa-P+1)(ESS_R-ESS_{UR})/(2ESS_{UR})$ is greater than the corresponding critical value of table IV in Dickey and Fuller (1981). ESS_{UR} and ESS_R are respectively the sum of squared residuals in the unrestricted and restricted regressions, while κ is the number of estimated parameters in the unrestricted model.

The next three tests assume that the time movement of the series, y_t, is described by the following "trend and intercept" model

$$\Delta y_t = \mu + \beta t + \gamma y_{t-1} + \sum_{j=1}^{P} \phi_j \Delta y_{t-j} + \varepsilon_t \tag{5}$$

The third test is based again on the t_{TREND} statistic of the estimated γ. The unit root null hypothesis, H_o: γ=0, is rejected in favour of the alternative H_1: γ<0, whenever t_{TREND} is less than the corresponding Mac-Kinnon critical value. Furthermore, the next two tests are the common regression "F-tests" (as Φ_1 was), known as Φ_2 and Φ_3. For Φ_2, the null hypothesis H_o: μ=0, β=0, γ=0, is rejected when the ratio $(T-\kappa-P+1)(ESS_R-ESS_{UR})/(3ESS_{UR})$ is greater than the DF critical value of table V. In a similar manner, for Φ_3, when the ratio defined in Φ_1 test is greater than the DF critical value of table VI, the null hypothesis H_o: μ=0, γ=0, is rejected. The latter test is referred by Pindyck and Rubinfeld (1988) as an F-ratio test.

Regarding the three Φ tests, adding lagged difference terms of y_t to the right-hand side of (4) and (5), the sample size is reduced. For this reason, before the implementation of the Φ tests, an appropriate linear interpolation was applied again to the corresponding DF critical values. Additionally to this, adding Δy_{t-j} terms to the right-hand side of (4) and (5), not rejecting the unit root null hypothesis in Φ tests leads to ARIMA (P,1,0). On the contrary, not including such lagged difference terms of the dependent variable in the testing models, the rejection of the null leads to the rejection of the random walk model.

The final test is the non-parametric Phillips-Perron (PP), which assumes the following model

$$\Delta y_t = \mu + \gamma y_{t-1} + \varepsilon_t \tag{6}$$

To test the random walk null hypothesis H_o: γ=0 against H_1: γ<0, a correction to the t-statistic estimated from an OLS regression on (6) is applied. The corrected t-statistic is given by

$$t_{PP} = \frac{t_{\gamma}\sqrt{\hat{\gamma}_o}}{\omega} - \frac{T(\omega^2 - \hat{\gamma}_o)s_{\gamma}}{2\omega\hat{\sigma}_{\varepsilon}}$$

where t_{γ} and s_{γ} are respectively the t-statistic and the standard error from applying OLS regression to (6), $\hat{\sigma}_{\varepsilon}$ the standard error of the regression, $\omega^2 = \hat{\gamma}_o + 2\sum_{j=1}^{q}[1 - j/(q+1)]\hat{\gamma}_j$, $\hat{\gamma}_j = \sum_{t=j+1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_{t-j} \Big/ T$, and q the truncation lag defined as the largest integer not exceeding $4(0.01T)^{2/9}$. In this test, the null hypothesis is rejected whenever t_{PP} is less than the corresponding Mac-Kinnon critical values.

Tables 3 up to 7 present the results from the application of all the previous tests on ARIMA (0,2,1), for different sample sizes T=25, 50, 100, 250, 500, as θ is approaching to -1^+. Considering t_{INT}, t_{TREND}, and PP tests, the first interesting remark is a non-negligible probability to estimate a positive γ, something which contradicts the nature of the alternative hypothesis, $\gamma<0$, in these tests (see table 3). For t_{INT}, and PP, and for every combination of T and θ, this probability ranges close to 0.50, while no significant differences are observed between the two tests. Besides, adding lagged difference terms of the dependent variable to the right hand side of (4), no considerable changes are observed, while increasing the sample size these probabilities rise slightly in both tests. On the other hand, t_{TREND} statistic displays lower probabilities of a positive γ compared to the other two tests. Furthermore, the probability related to t_{TREND} statistic is rather low either for small samples or for θ quite close to -1. Increasing the sample size, this probability is getting larger and larger, approaching θ=1, the probability declines, as well as, adding Δy_{t-j} terms to the right hand side of (5), the probability declines again.

For the previous three tests, tables 4 displays their "conditional" power, at nominal level of significance 10%. The term conditional implies that the power has been estimated as a percentage of replications where the null hypothesis is rejected, from the subset of replications, where a negative γ-estimate is obtained. For the three tests, the estimated conditional power increases as T is getting larger, but decreases as θ is approaching -1^+. However, the most important feature, regarding t_{INT} and t_{TREND} tests, is that for large T (e.g T≥100), when θ is not very close to -1, adding more Δy_{t-j} terms in (4) and (5), trying to correct autocorrelation in the error term, we lead the tests to lower power. Comparing now the performance between the three tests, which one of the three attains the highest power depends upon the sample size and how close θ is to –1. For example, for small samples (e.g. T≤50) and θ not close to –1, PP attains the highest power, while for T=500, t_{INT} succeeds the highest power, even when $\theta = -0.98$.

Tables 5, 6 and 7 present the power (again at nominal level of significance 10%) of the Φ tests, for which the problem of a negative γ-estimate is not met. For this reason, the power was estimated with reference to the total number of 1000 replications for each case. For P=0, the power of Φ_3 statistic is higher than the power of Φ_2 and Φ_3. Besides, the power of Φ_3 test, (a) declines as θ takes values closer to –1, (b) increases by drawing a larger sample, and (c) has a downward trend for θ not very close to –1, by adding more Δy_{t-j} terms in the right-hand side of (5). Regarding now Φ_1 and Φ_2 tests, adding lagged difference terms of the dependent variable to the right hand side of (4) and (5), we observe extremely high powers. Especially, for θ not so close to –1, the maximum power is attained in most of the cases at P=1. But we should not forget that for models (4) and (5) with P≥1, rejecting Ho in Φ_1 and Φ_2 leads not only to reject the null hypothesis but also the hypothesis that the population model is integrated of order 2.

Tables 3-7 about here

5. FORECASTING ARIMA (0,2,1) USING THE RANDOM WALK MODEL

In table 2, testing a unit root on ARIMA (0,2,1), there are cases (combinations of T and θ) where the power remains at low levels. In such cases, therefore, it is very likely to accept that the process of generating the data is the random walk model, when in fact this is not true. The consequences of such a wrong decision are investigated in the current section, by evaluating the validity of the prediction interval for a future value $y_T(\ell)$ of ARIMA (0,2,1), using, however, the random walk prediction equation and error variance for the $\ell-$period forecast, computed as

$\hat{y}_T(\ell) = y_T + \ell\hat{\mu}$, and $V\left(y_T(\ell) - \hat{y}_T(\ell)\right)^2 = \ell\hat{\sigma}_v^2$ respectively, where

$$\hat{\mu} = \frac{\sum_{t=2}^{T} \Delta y_t}{T-1} = \frac{y_T - y_1}{T-1}$$

$$\hat{\sigma}_v^2 = \frac{\sum_{t=2}^{T} \left(\Delta y_t - \hat{\mu}\right)^2}{T-2}$$

are the corresponding OLS estimates obtained after fitting the model $\Delta y_t = \mu + v_t$ to the available realisation.

The first criterion of evaluation is the coverage of the prediction interval

$$y_T + \ell\hat{\mu} - 2\hat{\sigma}_v\sqrt{\ell} \le y_T(\ell) \le y_T + \ell\hat{\mu} + 2\hat{\sigma}_v\sqrt{\ell} \quad (7)$$

namely, the percentage the interval to include the true value, with reference all those replications where the random walk null hypothesis cannot be rejected. Table 8 presents the coverage, but only for those combinations of T and θ of table 2, where the method based on testing the unit root on ARIMA (0,2,1) gave low power. The general remark is that keeping ℓ close to T, accordingly to the values of T and θ, coverage is acceptable, as it is over 90%. On the other hand, increasing ℓ, coverage declines reaching low levels, especially using very large samples.

The second criterion concerns the precision of prediction interval (7), expressed as the ratio of its average half length (computed as the mean of half length of prediction intervals constructed from those replications where the random walk null hypothesis cannot be rejected) over the true value of the half length. It is known for ARIMA models that, given information up to time T, the conditional distribution of $y_T(\ell)$ is normal with mean $\hat{y}_T(\ell)$ and variance $\sigma_\varepsilon^2\left(1+\sum_{j=1}^{\ell-1}\psi_j^2\right)$. For ARIMA (0,2,1), the ψ weights are computed recursively from $\psi_j = j(1+\theta)+1$. Using the last relationship, the variance of the $\ell-$period forecast error for ARIMA (0,2,1) is given by

$$E(y_T(\ell) - \hat{y}_T(\ell))^2 = \ell\sigma_\varepsilon^2\left\{1 + (1+\theta)^2\frac{(\ell-1)(2\ell-1)}{6} + (\ell-1)(1+\theta)\right\} \quad (8)$$

Thus the true value of the half-length, in the context of our simulation experiments, is computed by multiplying (8) by 2, replacing $\sigma_\varepsilon^2 = 1$, and using the appropriate value for θ. Table 9 displays this ratio, again at those cases of table 2, where we meet low power. The ratio although is above one in all cases, indicating a smaller precision than it should be expected, it reduces by increasing ℓ, resulting however in smaller coverage. Summarising, therefore, using the random walk model to predict future values of ARIMA (0,2,1) when θ is close to –1, for ℓ close to T, coverage is ranged at satisfactory levels, but we take as a penalty a lower precision.

Tables 8-9 about here

6. SUMMARY AND CONCLUSIONS

In this paper we show that taking second differences in the random walk model generates an ARIMA (0,2,1) with parameter $\theta = -1$. The reverse argument also holds, as a random walk model can be also considered as a special case of ARIMA (0,2,1) with $\theta = -1$. Using this dual relationship between a random walk model and an ARIMA (0,2,1), we illustrate that the behaviour of the latter model in levels and first differences, when θ is close but greater to –1, resembles the behaviour of either the random walk model with drift or the behaviour of the stationary AR(1) with autoregressive coefficient close to unity [Halkos & Kevork(2005b)]. The last finding leads us to suggest a new alternative hypothesis, in the form of replications from ARIMA (0,2,1) with θ close but greater to –1, to estimate and to compare the power of alternative methods of unit root tests, using Monte Carlo simulations.

Four such general methods are considered in the current research. The first is based on testing H_o: $\theta = -1$ to second differences, rejecting the random walk null hypothesis using a set of critical values suggested by Halkos & Kevork (2005c) for finite samples. The second method refers to tests based on the t-statistic of the estimated coefficient of the lagged dependent variable in the right hand side of the "intercept" and the "trend-and-intercept" models. The third method includes tests based on the known Φ statistics, which are calculated by estimating the unrestricted and the restricted forms of the "intercept model" (Φ_1 test) and the trend-and-intercept model (Φ_2 and Φ_3 tests). Finally the last method concerns the non-parametric Phillips-Perron (PP) test, which is applied after again the estimation of the "intercept" model, without the presence of lagged difference terms of the dependent variable in the right hand side.

Considering the first method, for any θ greater than –0.95, a sample over 250 observations generates acceptable power, while for θ very close to –1, unfortunately, only an enormously large sample might lead to satisfactory levels of power. Regarding now the two tests based on t-statistics together with the PP test, which one of the three attains the highest power depends upon the sample size, and how close θ is to –1. Finally, comparing the power of Φ tests without including lagged difference terms of the dependent variable to the right hand side of the corresponding models, the Φ_3 statistic performs better than the other two. However, with the inclusion of lagged difference terms, the Φ_1 and Φ_2 statistics produce extremely high powers, rejecting not only the unit root null hypothesis but also the hypothesis that the population model is ARIMA (0,2,1). Two other interesting findings should be also reported for some of the tests under investigation. The first concerns the t and PP test statistics, where a positive value might occur, something that contradicts the nature of the alternative hypothesis. Second, for the t and Φ_3 tests, adding more and more lagged difference terms of the dependent variable to the right hand side of the testing models, in an effort to eliminate autocorrelation in the error term, unfortunately the power shows a downward trend.

Up to now, the most known disadvantage of unit root tests has been the extremely low power when the parameter(s) of the model representing the alternative hypothesis take values close to limiting situations. On the other hand, this paper brings forward an additional problem concerning the selection of the appropriate unit root test, even when in the alternative hypothesis model its parameter(s) are not taking values close to a certain limit. For example, the "trend and intercept model" with an autoregressive coefficient not very close to unity cannot be easily differentiated and identified from an ARIMA (0,2,1) with θ close but greater to –1. This happens because in finite samples, both models display similar patterns in the plots of levels and in the sample autocorrelation and partial autocorrelation functions of first differences. And for the "trend and intercept model", Halkos & Kevork (2005a) showed that the Φ_3 statistic should be preferred, while this paper suggests for ARIMA (0,2,1) the null hypothesis H_o: $\theta = -1$ being tested to second differences of the series.

We believe that the research on unit root tests, apart from developing new methods, should also take a new direction on exploring the consequences of reaching a wrong conclusion due to the low power of a specific unit root test. This is something that we also explore in the current paper. More specifically, regarding the first method, in cases where it attains low power, we study the validity of the prediction interval for a future value of ARIMA (0,2,1), when, however, we use the prediction equation and the error variance for the $\ell-$period forecast of the random walk. Besides, the validity is explored only to those replications where the random walk hypothesis cannot be rejected. The results indeed are not disappointed, looking at the estimated probability (coverage) the prediction interval to include a future value of ARIMA (0,2,1) $\ell-$periods ahead. Keeping ℓ low, the prediction interval with half-width 2 times the square root of the random walk error variance attains coverage more than 90%. For such cases, however, the average half-length of the prediction interval might reach to be four times more than its true value.

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Table 1: Critical values for a unit root test in ARIMA (0,2,1)

T	$\Theta^{L}_{0.10}$	$\Theta^{U}_{0.10}$	$\Theta^{L}_{0.05}$	$\Theta^{U}_{0.05}$	$\Theta^{L}_{0.01}$	$\Theta^{U}_{0.01}$
25	-0,7358	-0,7243	-0,6775	-0,6578	-0,4675	-0,3685
50	-0,8287	-0,8222	-0,7980	-0,7891	-0,7087	-0,6692
100	-0,8933	-0,8897	-0,8765	-0,8720	-0,8387	-0,8248
250	-0,9422	-0,9403	-0,9340	-0,9316	-0,9176	-0,9127
500	-0,9635	-0,9625	-0,9587	-0,9573	-0,9493	-0,9464

Table 2: Probability of rejecting a unit root in ARIMA(0,2,1); $\alpha = 0.10$

	Sample size									
Θ	T=25		T=50		T=100		T=250		T=500	
	Uncertainty	Reject	Uncertainty	Reject	Uncertainty	Reject	Uncertainty	Reject	Uncertainty	Reject
-0.90	0,025	0,160	0,024	0,197	0,037	0,367	0,013	0,872	0,000	0,994
-0.95	0,029	0,143	0,024	0,149	0,024	0,192	0,036	0,406	0,019	0,808
-0.98	0,024	0,141	0,026	0,131	0,017	0,158	0,022	0,204	0,025	0,277

Table 3: Probability of a positive t_{INT}, t_{TREND}, and PP

		t_{INT}						t_{TREND}						PP
T	θ	P=0	P=1	P=2	P=3	P=4	P=5	P=0	P=1	P=2	P=3	P=4	P=5	
25	-0.90	0,451	0,447	0,457				0,019	0,029	0,036				0,456
	-0.95	0,430	0,440	0,451				0,012	0,016	0,029				0,437
	-0,98	0,426	0,434	0,452				0,007	0,013	0,027				0,442
50	-0.90	0,471	0,477	0,476				0,054	0,050	0,057				0,474
	-0.95	0,461	0,460	0,459				0,015	0,012	0,019				0,463
	-0,98	0,434	0,446	0,460				0,006	0,005	0,006				0,442
100	-0.90	0,498	0,500	0,496	0,493			0,174	0,154	0,144	0,133			0,496
	-0.95	0,497	0,491	0,495	0,500			0,059	0,049	0,047	0,048			0,497
	-0,98	0,483	0,483	0,483	0,481			0,004	0,008	0,006	0,008			0,485
250	-0.90	0,493	0,495	0,494	0,489	0,489		0,398	0,368	0,348	0,321	0,299		0,493
	-0.95	0,474	0,476	0,473	0,481	0,481		0,251	0,233	0,223	0,219	0,201		0,473
	-0,98	0,463	0,466	0,469	0,465	0,467		0,051	0,048	0,044	0,044	0,048		0,464
500	-0.90	0,563	0,563	0,558	0,555	0,546	0,538	0,467	0,446	0,416	0,397	0,368	0,344	0,561
	-0.95	0,505	0,504	0,504	0,503	0,505	0,504	0,383	0,375	0,367	0,352	0,342	0,325	0,505
	-0,98	0,511	0,508	0,504	0,508	0,507	0,507	0,184	0,178	0,173	0,173	0,167	0,162	0,511

Table 4: Conditional power of t_{INT}, t_{TREND}, and PP test statistics

		t_{INT}						t_{TREND}						PP
T	θ	P=0	P=1	P=2	P=3	P=4	P=5	P=0	P=1	P=2	P=3	P=4	P=5	
25	-0.90	0,071	0,058	0,057				0,077	0,104	0,080				0,097
	-0.95	0,046	0,030	0,044				0,083	0,102	0,080				0,066
	-0,98	0,030	0,027	0,038				0,099	0,109	0,083				0,050
50	-0.90	0,168	0,140	0,124				0,072	0,072	0,072				0,192
	-0.95	0,050	0,050	0,039				0,085	0,098	0,092				0,073
	-0,98	0,012	0,023	0,020				0,100	0,108	0,096				0,034
100	-0.90	0,458	0,388	0,347	0,292			0,054	0,037	0,040	0,043			0,405
	-0.95	0,187	0,175	0,141	0,138			0,055	0,054	0,060	0,059			0,209
	-0,98	0,029	0,033	0,033	0,029			0,089	0,096	0,078	0,080			0,039
250	-0.90	0,724	0,671	0,617	0,569	0,524		0,223	0,177	0,147	0,125	0,097		0,659
	-0.95	0,532	0,515	0,488	0,489	0,464		0,072	0,072	0,051	0,047	0,041		0,518
	-0,98	0,194	0,200	0,185	0,170	0,173		0,054	0,043	0,042	0,046	0,050		0,209
500	-0.90	0,828	0,778	0,719	0,667	0,606	0,545	0,484	0,394	0,320	0,264	0,217	0,175	0,752
	-0.95	0,770	0,746	0,718	0,704	0,669	0,645	0,271	0,234	0,202	0,188	0,173	0,151	0,731
	-0,98	0,444	0,431	0,417	0,407	0,400	0,396	0,067	0,062	0,060	0,052	0,056	0,055	0,431

Table 5: Power of Dickey-Fuller Φ_1 statistic

T	θ	P=0	P=1	P=2	P=3	P=4	P=5
25	-0.90	0,042	0,964	0,740			
	-0.95	0,017	0,972	0,766			
	-0,98	0,011	0,972	0,771			
50	-0.90	0,136	0,999	0,986			
	-0.95	0,036	1	0,994			
	-0,98	0,012	1	0,996			
100	-0.90	0,404	0,997	0,995	0,992		
	-0.95	0,134	1	1	1		
	-0,98	0,016	1	1	1		
250	-0.90	0,719	0,981	0,976	0,967	0,956	
	-0.95	0,502	1	1	1	1	
	-0,98	0,138	1	1	1	1	
500	-0.90	0,843	0,971	0,959	0,943	0,930	0,915
	-0.95	0,745	0,999	0,998	0,998	0,997	0,997
	-0,98	0,391	1	1	1	1	1

Table 6: Power of Dickey-Fuller Φ_2 statistic

T	θ	P=0	P=1	P=2	P=3	P=4	P=5
25	-0.90	0,068	0,935	0,703			
	-0.95	0,059	0,956	0,744			
	-0,98	0,055	0,966	0,760			
50	-0.90	0,149	0,999	0,975			
	-0.95	0,073	1	0,991			
	-0,98	0,057	1	0,991			
100	-0.90	0,364	0,998	0,997	0,990		
	-0.95	0,130	1	1	1		
	-0,98	0,060	1	1	1		
250	-0.90	0,712	1	0,999	0,997	0,989	
	-0.95	0,469	1	1	1	1	
	-0,98	0,128	1	1	1	1	
500	-0.90	0,850	0,996	0,993	0,989	0,981	0,968
	-0.95	0,733	1	1	1	1	1
	-0,98	0,377	1	1	1	1	1

Table 7: Power of Dickey-Fuller Φ_3 statistic

T	θ	P=0	P=1	P=2	P=3	P=4	P=5
25	-0.90	0,111	0,127	0,094			
	-0.95	0,092	0,123	0,084			
	-0,98	0,098	0,118	0,086			
50	-0.90	0,191	0,162	0,147			
	-0.95	0,118	0,111	0,113			
	-0,98	0,095	0,107	0,101			
100	-0.90	0,420	0,354	0,305	0,274		
	-0.95	0,176	0,150	0,138	0,133		
	-0,98	0,104	0,099	0,095	0,089		
250	-0.90	0,725	0,683	0,635	0,598	0,553	
	-0.95	0,516	0,487	0,461	0,445	0,417	
	-0,98	0,173	0,163	0,166	0,158	0,144	
500	-0.90	0,861	0,819	0,779	0,723	0,677	0,632
	-0.95	0,747	0,724	0,706	0,676	0,643	0,623
	-0,98	0,418	0,411	0,408	0,395	0,385	0,371

Table 8: Coverage of prediction intervals

		ℓ – period forecast								
T	Θ	1	2	3	4	5	10	15	20	40
25	-0,90	0,934	0,921	0,893	0,872	0,866				
	-0,95	0,931	0,925	0,915	0,899	0,891				
	-0,98	0,935	0,934	0,927	0,904	0,899				
50	-0,90	0,941	0,920	0,869	0,860	0,855	0,729			
	-0,95	0,944	0,949	0,907	0,925	0,917	0,877			
	-0,98	0,951	0,953	0,929	0,941	0,944	0,929			
100	-0,90	0,919	0,909	0,861	0,834	0,805	0,681	0,601		
	-0,95	0,936	0,941	0,930	0,912	0,893	0,858	0,807		
	-0,98	0,941	0,950	0,943	0,939	0,941	0,932	0,920		
250	-0,95	0,950	0,930	0,892	0,884	0,869	0,756	0,667	0,616	
	-0,98	0,956	0,960	0,948	0,944	0,928	0,899	0,884	0,862	
500	-0,98	0,947	0,933	0,918	0,923	0,920	0,871	0,834	0,794	0,672

Table 9: Ratio of the estimated average half-length of the prediction interval over its true one

		LAG								
T	θ	1	2	3	4	5	10	15	20	40
25	-0,90	2,37	2,26	2,15	2,05	1,96				
	-0,95	2,36	2,30	2,24	2,19	2,14				
	-0,98	2,36	2,34	2,32	2,29	2,27				
50	-0,90	2,53	2,41	2,29	2,19	2,09	1,71			
	-0,95	2,37	2,31	2,25	2,20	2,15	1,92			
	-0,98	2,34	2,32	2,29	2,27	2,25	2,14			
100	-0,90	3,44	3,28	3,12	2,98	2,85	2,33	1,96		
	-0,95	2,53	2,47	2,41	2,35	2,30	2,05	1,85		
	-0,98	2,40	2,38	2,35	2,33	2,31	2,20	2,10		
250	-0,95	3,67	3,58	3,49	3,41	3,33	2,97	2,68	2,44	
	-0,98	2,57	2,55	2,52	2,50	2,47	2,36	2,25	2,15	
500	-0,98	2,89	2,86	2,83	2,80	2,77	2,64	2,52	2,41	2,05

Figure 1: Sample autocorrelation and partial autocorrelation functions of first and second differences from the random walk model $y_t = 2 + y_{t-1} + \varepsilon_t$

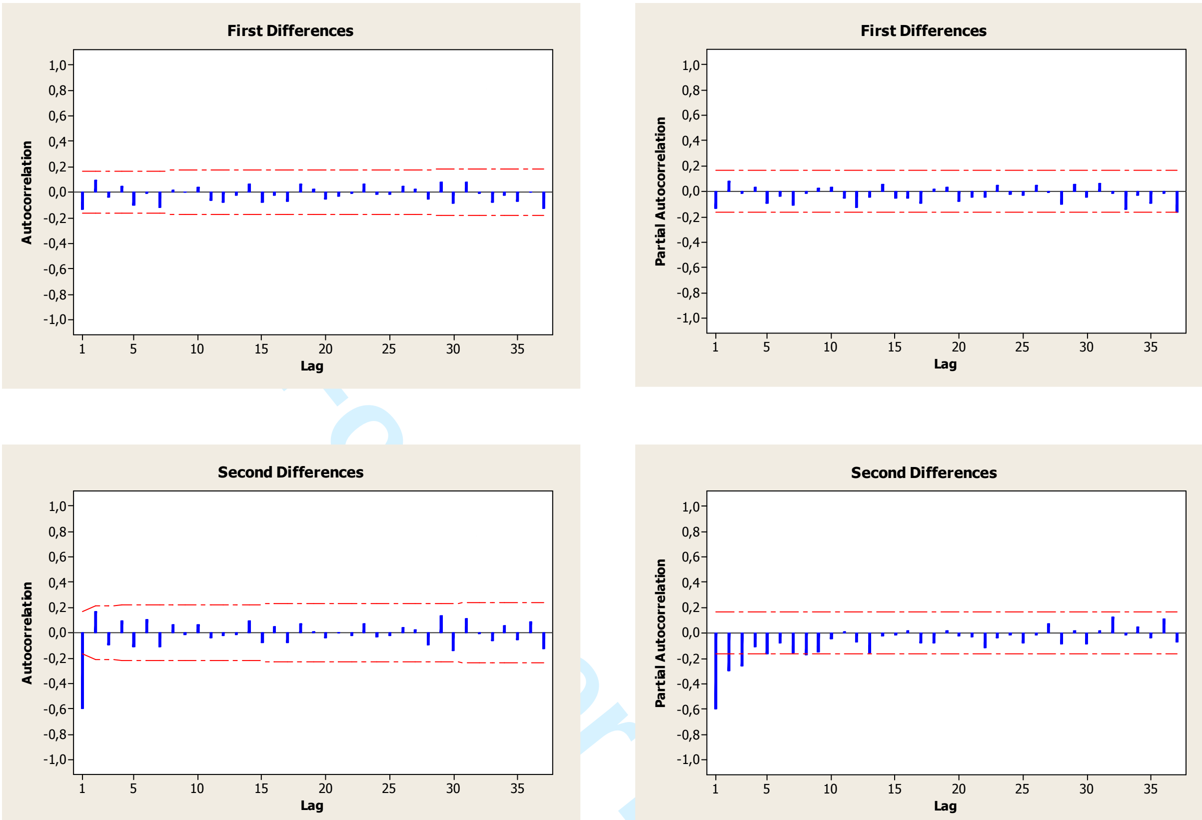
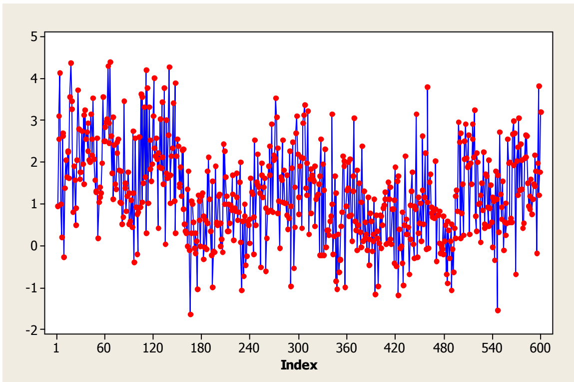
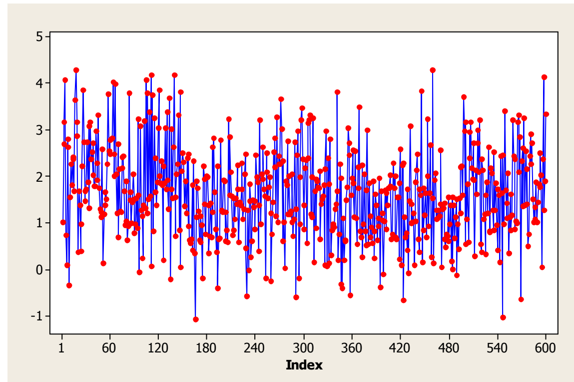


Figure 2a: Time series plots of first differences for model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$

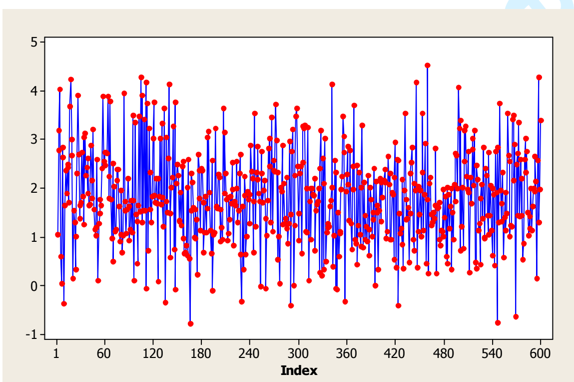
$\theta = -0.80$



$\theta = -0.90$



$\theta = -0.95$



$\theta = -0.98$

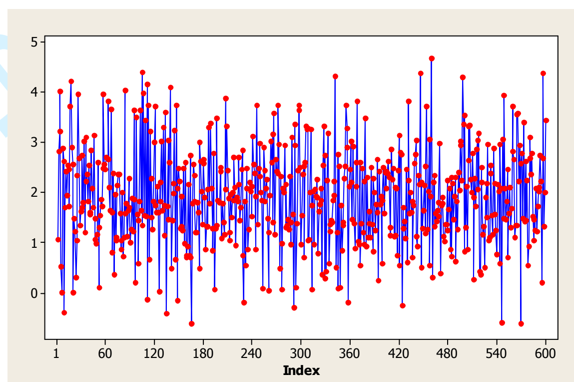
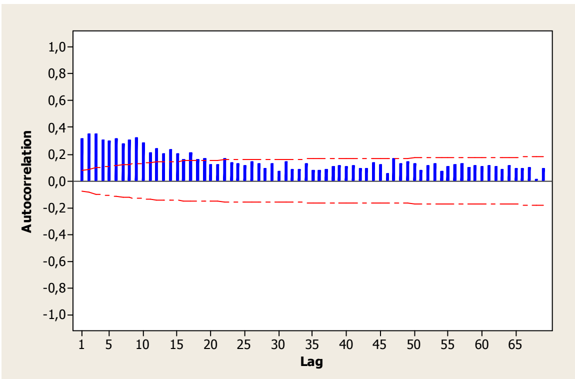
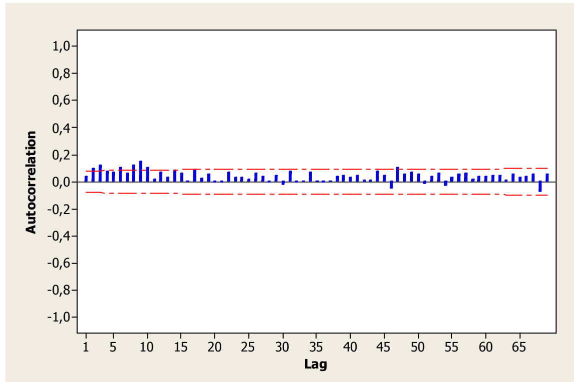


Figure 2b: Sample ACF of first differences for model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$

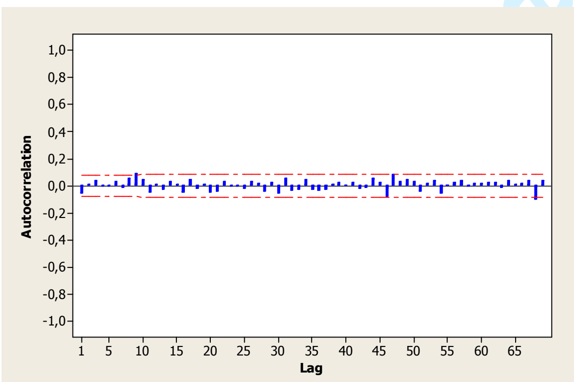
$\theta = -0.80$



$\theta = -0.90$



$\theta = -0.95$



$\theta = -0.98$

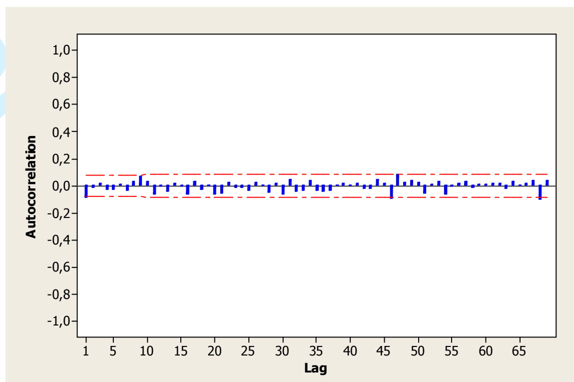
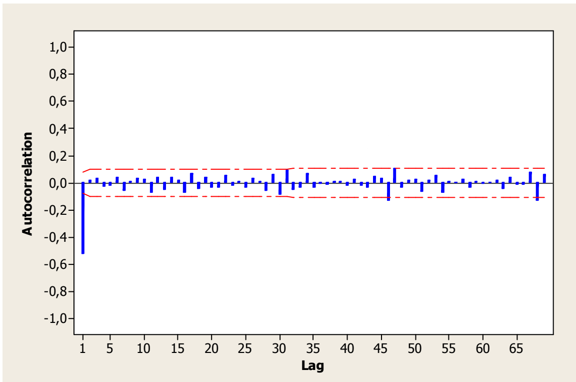
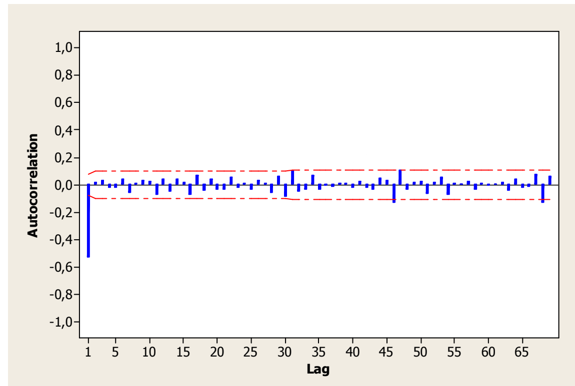


Figure 2c: Sample ACF of second differences for model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$

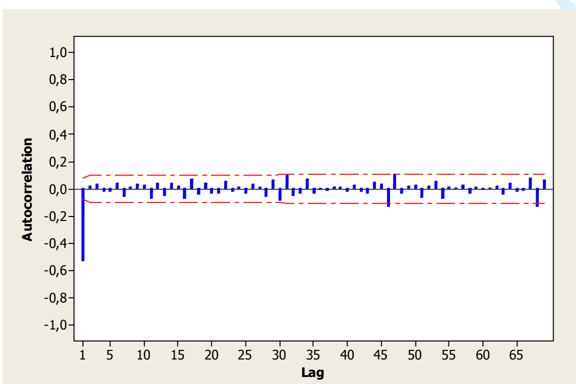
$\theta = -0.80$



$\theta = -0.90$



$\theta = -0.95$



$\theta = -0.98$