

Price Discovery in the Presence of Boundedly Rational Agents

Keiber, Karl Ludwig

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Price Discovery in the Presence of Boundedly Rational
Agents*

Karl Ludwig Keiber[†]

For Peer Review Only

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[†]Address: WHU – Otto Beisheim School of Management, Burgplatz 2, D-56179 Vallendar, Germany.
Phone: +49-261-6509-422. Fax: +49-261-6509-409. E-mail: kkeiber@whu.edu.

Price Discovery in the Presence of Boundedly Rational Agents

Abstract

In this paper we propose a sequential model of security trading which, compared to existing models, is extended along the notions of Simon (1955), Rubinstein (1998), and Odean (1999) by adding boundedly rational traders. Our results indicate that both momentum and mean-reversion in asset prices can be attributed to the presence of agents who are subject to systematic errors in the process of forecasting the liquidation value of a risky security. The length of the momentum period is inversely related to both the amount of information-based trading in the market and the rate at which asset specific information is learned by boundedly rational agents. Furthermore, the model allows explicitly to establish a link between the component of the bid-ask spread that can be explained by bounded rationality and both momentum and reversal.

JEL classification: D4, D8, G1.

1 Introduction

Currently, the academic community witnesses an ongoing debate among financial economists whether models of financial markets should take into account the actual behavior of economic agents. The case for this approach is e.g. made by Becker (1993, p. 402) who points out that his “... *work may have sometimes assumed too much rationality.*” or by Thaler (1991, p. xxi) who claims that “... *it is time to recognize that there can be too much of a good thing, even rationality.*” In the same vein, Thaler (1999, p. 16) hopes that “... *in their enlightenment, economists will routinely incorporate as much ‘behavior’ into their models as they observe in the real world. After all to do otherwise would be irrational.*” However, the alternative is to stick to the well known rational man paradigm, which we are not only familiar with, but also has served successfully as workhorse for many generations of financial economists. Fama (1998) and Rubinstein (2001) argue in favor of the latter approach.

From the facts that bounded rationality was already stressed by Simon (1955) in the mid fifties and that the experimental evidence for bounded rationality was well documented by other academic disciplines such as e.g. psychology before, it is astonishing that this discussion in financial economics did not start earlier.¹ One reason for this delay might have been the lack of empirical evidence against the predictions of financial markets’ models. But, with the increased availability of financial markets’ data, it took little time until the first objections against the models based on full rationality emerged, and the body of literature documenting financial markets’ anomalies began to grow. Since then financial economists enriched their profession by creating models which account for the boundedly rational behavior of economic agents.²

According to this tendency in financial economics, we propose a sequential model of security trading that assumes bounded rationality on the part of the agents present in the market. The sequential structure of the trading process allows to inspect the implications of bounded rationality for the dynamics of various market characteristics, such as the transaction price and the bid–ask spread.

A simulation study of the sequential trading mechanism provides us with various insights. We find that the specification of our model allows to explain both momentum i.e. short–run positive autocorrelation, and mean–reversion i.e. long–run negative autocorrelation in asset price changes. These effects can hardly be explained within models assuming full rationality

¹See Conlisk (1996) and Rabin (1998) for a review of the evidence for bounded rationality of economic agents or Kahneman, Slovic and Tversky (1982) for an overview of the psychological biases economic agents are subject to.

²We will briefly work through the major recent approaches in section 2.

on the part of the agents. Obviously, the results on autocorrelation are similar to those reported in Barberis, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), and Hong and Stein (1999). Unlike Barberis, Shleifer and Vishny (1998) and Daniel, Hirshleifer and Subrahmanyam (1998) we do not perform our analysis within a representative agent framework. Rather, we are building on agents' heterogeneity as do Hong and Stein (1999).

In addition, the numerical comparative static results from the simulation study provide an ex-post rationalization of Hong, Lim and Stein's (2000) empirical findings. We illustrate that the momentum effect vanishes the faster the more information-based trading activity is present in the market. This result corresponds to the findings of these authors that momentum strategies work not so well for stocks that are covered by a large number of analysts. Moreover, a second related effect concerning momentum is identified. We find that the faster the fundamental value of an asset is learned from available information the shorter is the momentum period. This is reminiscent of Hong, Lim and Stein's (2000) finding that momentum strategies work better for stocks of small firms, as long as one agrees that information about small firms is more ambiguous and therefore learned more slowly, whereas information about large firms is salient and consequently easier to learn.

Furthermore, by building on a sequential trade model in the spirit of Glosten and Milgrom (1985) we are able to derive results on the bid-ask spread explicitly. These results naturally cannot be obtained by relying on a single price model as do Barberis, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998) and Hong and Stein (1999). In this respect our model allows to study an additional facet of financial markets. We are able to determine the adverse selection component of the bid-ask spread which can be explained by bounded rationality. As becomes evident from the simulation study this component almost vanishes at the end of the momentum period and increases again during the reversal period. Although Daniel, Hirshleifer and Subrahmanyam (1998) analyze a single price model, they conjecture that the adverse selection component of the bid-ask spread is to some extent related to both momentum and reversal period which we confirm in the present paper.

The paper proceeds as follows. Section 2 reviews briefly the related theoretical literature on security price patterns. We present our sequential trade model and derive straightforward implications of our setup in section 3. The dynamics of the proposed model are illustrated and the major results of our analysis are derived in section 4. Section 5 concludes.

2 Related literature

A large number of papers study sequential trade models in the spirit of Glosten and Milgrom (1985). Thus, a review of these papers is far beyond the scope of this paper.³ Here, we focus on those papers that derive results similar to ours and that share the same philosophy of assuming less than full rationality on the part of the agents. The approaches of Barberis, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), and Hong and Stein (1999) all belong to this category. The basic idea behind these behavioral finance models is to consistently generate explanations for well documented stylized facts in financial markets — such as momentum and reversal in asset returns — which conventional models usually fail to explain. The approaches of Barberis, Shleifer and Vishny (1998) and Daniel, Hirshleifer and Subrahmanyam (1998) are grounded on psychological biases and analyze a representative agent framework, whereas Hong and Stein (1999) focus on the interaction of heterogenous agents. In the following we will review quickly these three models.

In the model of Barberis, Shleifer and Vishny (1998) a representative agent believes the earnings shocks of a stock either to follow a trend or to be mean-reverting, while the true earnings shocks process is a random walk. Conditionally on the observed earnings shock she updates her belief about which of these two regimes is generating the next observation. Whenever she observes two consecutive earnings shocks of the same sign she increases the probability of the trending regime. If instead she observes two consecutive earnings shocks of different signs she increases the probability of the mean-reverting regime. The trending regime and the mean-reverting regime are argued to capture the psychological biases *representativeness* and *conservatism*, respectively.⁴ It is important to note that she has the wrong information generation process for the earnings shocks in mind. She thinks the earnings shocks to be generated by a mix of a trending process and a mean-reverting process, but not by a random walk. However, it is also of importance that she does not change her mental structure of the earnings shocks' generation process during time. This means

³For instance, the papers by Easley and O'Hara (1987), Diamond and Verrecchia (1987), Easley and O'Hara (1991), Easley and O'Hara (1992), Easley, Kiefer and O'Hara (1997) and Easley, O'Hara and Srinivas (1998) all feature sequential trade models.

⁴According to the representativeness bias the agent thinks that the last observation is representative for the next observation. Therefore, the agent assigns an earnings shock to be followed by another earnings shock of the same sign a high probability. This in turn makes him producing a trend.

If the agent is subject to conservatism she judges the actual observation to be an outlier and thinks conservatively that the next earnings shock is more likely to have the opposite sign. This generates mean-reversion in earnings shocks. Consequently, according to conservatism she assigns the event that the next earnings shock has the same sign a low probability.

that she does not figure out that her earnings shocks' generation model in mind does not fit the observations. This in turn implies that the probability she assigns to observing positive or negative earnings shocks does not match the true probabilities under the random walk. Therefore, the Bayesian inference process of the representative agent who learns from observed earnings shocks is biased. Applying these biased probabilities to the forecast of future earnings shocks, she consequently arrives at a biased estimate of the price of the stock. Finally, Barberis, Shleifer and Vishny (1998) demonstrate which parameter combinations of the representative agent's Bayesian inference problem allow the price to underreact and/or to overreact. The crucial ingredients to their model are (a.) the two Markov models, which lead to the representative agent's wrong predictions, and (b.) the random sequences of earnings shocks where random subsequences of the same sign bias the representative agent in favor of the trending regime, and two consecutive earnings shocks with different signs bias her in favor of the mean-reverting regime. This means that she reacts to patterns that are purely random. These patterns in earnings shocks are taken to be more informative than they really are.

Daniel, Hirshleifer and Subrahmanyam's (1998) model builds both on the psychological bias *overconfidence* and *biased self-attribution*. The representative agent observes a private noisy signal about a risky asset's final payoff. Thereafter, public noisy signals about the uncertain payoff become available one by one. The noise components of the public signals are assumed to be identically distributed, independent of each other, and independent of the private signal's noise. Overconfidence is captured by the underassessment of the private signal's variance. The public signals however are processed by the representative agent in an unbiased manner. The biased self-attribution affects pricing only in the dynamic context, but in an asymmetric way. Given that a public signal confirms her private signal, where confirmation loosely means that it points in the same direction as her private signal, the biased self-attribution increases her overconfidence. Thus, confirming public information exacerbates the underassessment of the private signal's variance. This in turn implies that the weight she puts on her private signal in the Bayesian inference increases. In the case the public signal is disconfirming, her overconfidence is reduced. Since the reduction of overconfidence is less than the increase, an asymmetry is introduced. Therefore, one confirming signal must be offset by more than one disconfirming signal in order to reduce the overconfidence and to arrive at a rational assessment of the asset's price finally. Naturally, as more public signals are available their average becomes an unbiased estimate of the asset's final payoff. Furthermore, the variance of the public signal's average converges to zero. This in turn increases the weight she puts on this average in the Bayesian inference pro-

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cess. In the end she is not overconfident anymore and the expected price of the risky asset converges to the true value. In a simulation study Daniel, Hirshleifer and Subrahmanyam (1998) demonstrate that these mechanics produce price patterns that exhibit short-run momentum and long-run reversal. However, the basic ingredients of their approach in order to derive these results are (a.) a biased Bayesian inference due to overconfidence, (b.) a mistaken private signal initially and (c.) a sequence of disconfirming public signals that imply via biased self-attribution both the variation and the reduction of the representative agent's overconfidence.

Another approach which, in contrast to the two models already discussed, is not motivated by psychological biases is that proposed by Hong and Stein (1999). They present a model that builds on the interaction of two groups of boundedly rational agents — newswatchers and momentum traders. *Bounded rationality* is introduced by the assumption that each group is only able to process a subset of available information. Heterogeneity comes from the assumption that the two groups process different subsets of information. On the part of the newswatchers bounded rationality is interpreted as the inability to extract information from prices in addition to private information. Because private information is assumed to consist of a set of signals which diffuse gradually across newswatchers, each subgroup of newswatchers has access to a different set of private information. In a setting with diverse private information the price aggregates diverse information and is informative itself. Therefore, not conditioning on prices is in fact less than fully rational. On the other hand, bounded rationality on the part of momentum traders is introduced by allowing them to condition only on past information in a rather simple fashion. They derive their asset demand conditionally on a single historical price change. The gradual information diffusion in addition to the inability of the newswatchers to condition on prices prevents the price from being pushed to the true value immediately. The price therefore underreacts to private information. However, the price trends to the true value. This trend is realized by the momentum traders who compensate part of the underreaction, and push the price up additionally thereby increasing the trend of prices. The underreaction caused by newswatchers nourishes the momentum traders who, by strengthening the trend, foster even more trend-chasing in subsequent periods. In the end these mechanics of momentum trading make the price overreact. In further trading rounds the overreaction is corrected and the expected price converges to the true value. Initially, the underreaction produces momentum in prices, and the correction of the overreaction in later trading rounds produces mean-reversion of the prices. It is worth summarizing the integral elements of Hong and Stein's (1999) approach, too. These are (a.) the gradual diffusion of private information together with (b.)

the inability of the newswatchers to condition on prices as well as (c.) the trend-chasing behavior of the momentum traders.

The models discussed above are paralleled by a large number of studies which provide the corresponding empirical evidence on security price patterns. Excellent overviews of these empirical studies can be found for instance in section two of Barberis, Shleifer and Vishny (1998), the appendix of Daniel, Hirshleifer and Subrahmanyam (1998), and section one of Hong and Stein (1999), as well as section two of Fama (1998).

Before we continue it is worth pointing out the existence of a further strand of the literature which also addresses the issue of security price patterns. In contrast to the above behavioral finance models which rely on biases in belief formation or limited availability of information, that strand of the literature explains security price patterns based on microeconomic principles, the temporal mismatch of supply and demand, in particular. More precisely, Caginalp and Ermentrout (1990), Caginalp and Balenovich (1994), and Caginalp and Balenovich (1999) describe the dynamics of securities prices by an ordinary differential equation.⁵ Caginalp and Balenovich (1994) reconcile this approach with data from both experimental asset markets and real-world financial markets, and demonstrate that it fits observable security price patterns in both environments. At the very heart of this approach are shifts of supply and demand stemming from two sources, namely, (i.) trends in the security price and (ii.) fundamental misvaluation which both affect the demand for and the supply of the security. The former effects trend-based trading whereas the latter generates value-based trading.

3 Model

This section proposes a sequential model of security trading which is in the spirit of Glosten and Milgrom (1985), but incorporates ideas of Simon (1955), Rubinstein (1998), as well as Odean (1999) and shares modeling features with the behavioral models discussed in section 2. First, we introduce the setup of the model. We describe how agents in our model behave, and which information is available to them. Afterwards, we discuss the implications of our setup.

⁵In Caginalp and Balenovich (1994) and Caginalp and Balenovich (1999) it is discussed how this ordinary differential equation can be obtained from some higher-order system of ordinary differential equations. Furthermore, it is demonstrated how to obtain stochastic counterparts of the(se) ordinary differential equation(s) and how to accommodate cash injections and withdrawals.

3.1 Setup

We analyze the market for a risky security with uncertain future liquidation value \tilde{v} . Before trading starts nature releases either the information ϕ_H or ϕ_L about the future liquidation value of the risky security. The shares of the risky security are exchanged through competitive risk neutral profit maximizing market makers. The risk free rate of return equals zero. Thus, the agents' time preferences do not affect the analysis. The market makers are assumed to be symmetric within all relevant characteristics allowing us to study the model from a single market maker's perspective. A transaction happens when a market order of unit size is cleared by some market maker.⁶ Each order is submitted by either an information-based trader or a liquidity trader. The orders are submitted consecutively. The traders are assumed to constitute a trader continuum. Due to trader anonymity, the market maker does not know to which group the trader submitting an order belongs. In order to rule out strategic behavior on the part of the traders, each trader is assumed to trade once only.

The fraction of the liquidity traders is assumed to be $1 - \alpha$ where $0 < \alpha < 1$. As usual, we assume the liquidity traders to trade solely for liquidity reasons such as life-cycle needs as well as for hedging purposes, but not for speculation. We further assume that the group of liquidity traders splits in two subgroups — buyers and sellers — with equal probability mass. Thus, the liquidity traders are assumed to submit buy orders and sell orders equally likely.

Consequently, the fraction of the information-based traders amounts to α . The information-based traders are assumed to be risk-neutral profit maximizing agents. Hence, by excluding risk aversion we solely concentrate on information effects. The information-based traders are assumed to have access to the information which is released by nature before trading starts. Therefore, the released information is private to them. Once the information-based traders observe the released information they forecast the liquidation value of the risky security. It is exactly in this forecast that the traders' cognitive capability and bounded rationality come into play. We assume that prior to transaction n only the fraction $1 - \varepsilon_n$ of the information-based traders comes up with a rational forecast. Here, rationality means that conditionally on ϕ_H they forecast the future liquidation value to be H . If ϕ_L is released, they rationally forecast L to be the risky security's future liquidation value. Formally, $E[\tilde{v}|\phi_H] = H$ and $E[\tilde{v}|\phi_L] = L$, where $L < H$ giving us the intuition of ϕ_H and ϕ_L representing good news and bad news, respectively. The remaining fraction ε_n of

⁶As the market makers are assumed to be symmetric, it is not relevant which market maker is clearing the order.

information-based traders is assumed to be boundedly rational. Those traders end up with mistaken forecasts like $E_{br}[\tilde{v}|\phi_H] = L$ and $E_{br}[\tilde{v}|\phi_L] = H$ where the subscript indicates that the forecast is subject to bounded rationality. Conditionally on the individual forecast, the information-based traders submit corresponding orders.⁷

However, we account for learning in our approach. Generally, we assume that the fraction of boundedly rational agents is reduced at some learning rate $0 < g < 1$. One might think of g to depend on the quality of the private information where a larger value of g comes along with salient information which can be learned easily, and smaller values of g apply to ambiguous information which is harder to evaluate. Hence, the fraction of boundedly rational agents present among information-based traders prior to transaction n is given by $\varepsilon_n = \varepsilon \cdot (1 - g)^n$, where ε denotes the fraction of boundedly rational traders present in the market before trading starts.⁸

Before we continue with the description of the market makers' behavior some comments have to be made about the setup described so far. We deviate with our setup from existing sequential trade models, which assume that being informed coincides with being rational. Therefore, we term those traders having access to information *information-based* traders, in contrast to informed traders or insiders. In our model we separate the availability of information and the agents' cognitive capabilities. In this respect we conform to Simon (1955, p. 99) who claims "... to replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, ...". Rubinstein (1998, p. 3) also supports our approach by noting that "... decision makers are not equally capable of analyzing a situation even when the information available to all of them is the same." Additionally, Odean (1999, p. 1282) reports findings which corroborate the view that "... investors receive useful information but are systematically biased in their interpretation of that information; that is the investors hold mistaken beliefs about the mean, instead of (or in addition to) the precision of the distribution of their information. If they believe they are correctly interpreting information that they misinterpret, they may choose to buy or sell securities which they would not have otherwise bought or sold." Moreover, Harris and Raviv (1993, p. 474) point out that "... people often share common information yet disagree as to the meaning of this infor-

⁷One can think of the information-based traders all being boundedly rational with error probability ε_n , alternatively. I am grateful to an anonymous referee for this interpretation of the setup.

⁸Allowing for learning on the part of the information-based traders explicitly captures the idea that in the end economic agents act fully rationally — an argument which is favored by proponents of as-if rationality. However, as usual in market microstructure, this paper studies the intermediate effects of the presence of bounded rationality on the one hand and the ignorance of bounded rationality on the other hand.

tion.” Furthermore, our approach might also be viewed to be complementary to Admati and Pfleiderer (1988) who divide the group of liquidity traders by different preferences concerning the timing of trades, whereas we divide the group of information-based traders by cognitive capabilities or by different levels of rationality, respectively.

After having introduced how the traders behave, we turn to the description of the market makers’ behavior. The market makers are obliged to supply liquidity to the traders. Therefore, the market makers have to continuously quote a bid price b_n and an ask price a_n at which an incoming order is cleared in transaction n . Thus, the transaction price p_n in transaction n is either a_n or b_n , depending on the submitted order being a buy order b or a sell order s , respectively. The market makers’ uncertainty arises from the fact that they do not know which information is released by nature, initially. Hence, the market makers are at an informational disadvantage compared to the information-based traders. However, the market makers presume both information events to occur with equal probability. That is, prior to the first transaction they assign the occurrence of the information ϕ_H the probability $\delta = 0.5$ and, consequently, the occurrence of the information ϕ_L the probability $1 - \delta$. With respect to the meaning of the released information for the future liquidation value the market makers are not subject to any bias. That is, the market makers would forecast $E[\tilde{v}|\phi_H] = H$ and $E[\tilde{v}|\phi_L] = L$ as do the rational information-based traders.

As the market makers do not observe the information which is released about the future liquidation value of the security before trading starts, they are subject to an adverse selection problem. The adverse selection problem is induced by the information-based traders who extract an informational rent on their informational advantage at the expense of the market making industry. Since the market makers are risk-neutral and competitive they implement a regret-free price policy. That is, prior to transaction n the market makers quote the ask price

$$\begin{aligned} a_n &= E[\tilde{v}|\tilde{o}_n = b] \\ &= E[\tilde{v}|\phi_H] \cdot \delta_n(b) + E[\tilde{v}|\phi_L] \cdot (1 - \delta_n(b)) \\ &= H \cdot \delta_n(b) + L \cdot (1 - \delta_n(b)) \\ &= \delta_n(b) \cdot (H - L) + L \end{aligned} \tag{1}$$

and the bid price

$$\begin{aligned} b_n &= E[\tilde{v}|\tilde{o}_n = s] \\ &= \delta_n(s) \cdot (H - L) + L, \end{aligned} \tag{2}$$

where \tilde{o}_n denotes the upcoming order.⁹ If the transaction n is triggered by a buy order (sell order), that is, if $\tilde{o}_n = b$ ($\tilde{o}_n = s$), then the transaction n is cleared at the ask price a_n (bid price b_n). The prices a_n and b_n at which the upcoming order \tilde{o}_n is cleared equal the market makers' expectation of the liquidation value conditional on the event $\tilde{o}_n = b$ or $\tilde{o}_n = s$, respectively. The calculation of these conditional expectations of the future liquidation value applies the conditional or posterior probabilities $\delta_n(b)$ and $\delta_n(s)$ which the market makers assign the information release ϕ_H conditionally on having to clear a buy order $\tilde{o}_n = b$ or a sell order $\tilde{o}_n = s$ in transaction n , respectively. By being calculated according to the posterior probabilities $\delta_n(\cdot)$, the prices a_n and b_n account for the information which the market makers can extract from the upcoming order about the initial information release. Therefore, the prices are referred to as regret-free. As the equations (1) and (2) show, the determination of the market makers' quote boils down to specifying the formation of the posterior belief $\delta_n(\cdot)$. The technical aspects of the market makers' belief formation are discussed in detail in section 3.2. In the following we describe the broad principle underlying their inference problem.

So far the description of the market makers' behavior coincides with the standard sequential trade models. Additionally, the market makers are supposed to be ignorant of both the fraction ε of boundedly rational information-based traders and the learning rate g . Consequently, they do not know the exact fraction ε_n prior to transaction n . Put differently, the market makers do not know the true order arrival process. Even if the market makers know that boundedly rational traders exist among the information-based traders, they are assumed to act *as if* all information-based traders were rational.¹⁰ Since the boundedly rational traders always buy when selling is the rational choice and vice versa, transactions with boundedly rational traders are systematically producing profits for the market makers. Under competition these profits are handed over to the group of traders as a whole by offering better quotes than in the absence of boundedly rational traders. Otherwise, the zero expected profits condition would not be met. Now, suppose the market makers overestimate the fraction ε_n of the boundedly rational traders. In this case they overestimate the profits from transaction with the boundedly rational traders, offer too good prices, and finally are

⁹As prior to transaction n the market makers do not know which order will be submitted next, the upcoming order is denoted by the random variable \tilde{o}_n .

¹⁰The market makers therefore behave as they have learned about economic agents from economics textbooks. Namely, that economic agents in the end get things right and act as if they were rational. For most economists this as-if rationality argument makes bothering with bounded rationality issues superfluous. This paper highlights the consequences if the market makers are not concerned about less than full rationality on the part of some traders.

left with negative profits on average. This would imply the market makers to stop supplying transaction services. Consequently, if the market makers do not know the fraction of boundedly rational agents the only way for them to avoid negative expected profits for sure is to assume $\varepsilon = \varepsilon_n = 0$ for all transactions.¹¹

The market makers' behavior requires some comments as did that of the information-based agents. The knowledge of the exact fraction of boundedly rational traders would imply a kind of super-rationality on the part of the market makers.¹² Consequently, if we intend to discuss the implications of less than fully rational behavior, it is less useful, if not counterproductive, to put super-rationality to the analysis. As will become clear from the analysis in section 4, super-rationality on the part of the market makers works as error correcting mechanism. Therefore, super-rationality on the part of the market makers would give them the role of mistake-identifiers in the sense of Rubinstein (1998) who noticed that "... , in real life, explicit 'mistake-identifiers' rarely exist." Hence, Aumann's (1997) and Rubinstein's (1998) notions corroborate the assumption that the market makers act as if all information-based traders were rational.

To summarize, we revisit the major ingredients of our approach. Once more, we point out where our approach shares modeling features with the models reviewed in section 2. The three key assumptions underlying our approach are:

- The boundedly rational traders come up with the wrong forecast of the risky security's liquidation value but submit the corresponding orders to the market makers i.e. they submit a buy order or sell order if they forecast the liquidation value to be H or L , respectively.¹³
- The boundedly rational agents learn as trading proceeds at the learning rate g .
- The market makers act as if all information-based traders were rational.

¹¹Note that this behavior of the market makers violates the zero expected profits condition, as long as there are at least some boundedly rational traders present in the market. But, it is the only way to prevent — with probability one — the market makers from running the risk of realizing negative expected profits. Obviously, there should be competition among the market makers concerning the actual fraction ε_n . However, that market makers are not always prone to quote competitively was reported e.g. by the Christie and Schultz (1994) study.

¹²The term super-rationality is borrowed from Aumann (1997, p. 8) who points out that "... You must be super-rational in order to deal with my irrationalities."

¹³Note that bounded rationality does not mean the complete absence of rationality as conditionally on the boundedly rational traders' forecast the submitted orders are rational. Thus, there is no inconsistent behavior of the boundedly rational traders as concerns order submission and forecasting.

The first assumption makes the order flow to provide wrong signals to the market makers in much the same way as Daniel, Hirshleifer and Subrahmanyam (1998) supply the representative agent in their simulation study with a wrong private signal initially. The second ingredient is reminiscent of the gradual information diffusion which plays a crucial role in Hong and Stein's (1999) model. Another way to think about this point is that the proportion of wrong signals in the order flow is reduced during time, just as the weight of the wrong private signal in the Bayesian inference of the representative agent in Daniel, Hirshleifer and Subrahmanyam (1998) becomes smaller. The last of the three modeling features implies that the market makers have the wrong information generation process i.e. the wrong order flow model in mind, like the representative agent in Barberis, Shleifer and Vishny (1998) who relies on the wrong model of the process generating earnings shocks.

3.2 Implications

In this section we establish some propositions that are straightforward implications of the model's setup. All proofs are moved to the appendix A. The case of super-rationality on the part of the market makers is analyzed in parallel to the case where the market makers act as if all information-based traders were rational. Thus, the case of super-rationality serves as benchmark.

Let us begin with the case of super-rational market makers which know exactly the fraction ε_n of boundedly rational traders present in the market prior to transaction n . According to super-rationality the market makers' belief evolves as given in proposition 1.

Proposition 1 *If the market makers are super-rational thus knowing exactly the fraction ε_n prior to transaction n their posterior belief is given by*

$$\delta_n^{\text{sr}}(b) = \frac{(1 + \alpha \cdot (1 - 2\varepsilon_n)) \cdot \delta_{n-1}^{\text{sr}}(o_{n-1})}{(1 + \alpha \cdot (1 - 2\varepsilon_n)) \cdot \delta_{n-1}^{\text{sr}}(o_{n-1}) + (1 - \alpha \cdot (1 - 2\varepsilon_n)) \cdot (1 - \delta_{n-1}^{\text{sr}}(o_{n-1}))}$$

and

$$\delta_n^{\text{sr}}(s) = \frac{(1 - \alpha \cdot (1 - 2\varepsilon_n)) \cdot \delta_{n-1}^{\text{sr}}(o_{n-1})}{(1 - \alpha \cdot (1 - 2\varepsilon_n)) \cdot \delta_{n-1}^{\text{sr}}(o_{n-1}) + (1 + \alpha \cdot (1 - 2\varepsilon_n)) \cdot (1 - \delta_{n-1}^{\text{sr}}(o_{n-1}))}$$

where the superscript sr indicates that the market makers' belief is based on super-rationality and o_{n-1} denotes the order which was cleared in transaction $n - 1$. The recursion for $\delta_n^{\text{sr}}(\cdot)$ is given by (5) and (6) in appendix A.1.

Having established proposition 1 it is straightforward to introduce in proposition 2 the evolution of the market makers' biased belief if they ignore the presence of boundedly rational traders in the market.

Proposition 2 *If the market makers act as if all information-based agents were rational their posterior belief is given by*

$$\delta_n^b(b) = \frac{(1 + \alpha) \cdot \delta_{n-1}^b(o_{n-1})}{(1 + \alpha) \cdot \delta_{n-1}^b(o_{n-1}) + (1 - \alpha) \cdot (1 - \delta_{n-1}^b(o_{n-1}))}$$

and

$$\delta_n^b(s) = \frac{(1 - \alpha) \cdot \delta_{n-1}^b(o_{n-1})}{(1 - \alpha) \cdot \delta_{n-1}^b(o_{n-1}) + (1 + \alpha) \cdot (1 - \delta_{n-1}^b(o_{n-1}))}$$

where the superscript b indicates that the market makers' belief is biased in the sense that they assume the absence of boundedly rational traders and o_{n-1} denotes the order which was cleared in transaction $n - 1$. The recursion for $\delta_n^b(\cdot)$ is given by (5) and (6) in appendix A.1.

Comparison of the propositions 1 and 2 yields the observation that the market makers' behavior of ignoring the boundedly rational traders implies a deviation of their posterior belief from that posterior belief which they would derive otherwise under super-rationality. The effects of this ignorance bias are quantified in proposition 3.

Proposition 3 *Assume the market makers have the posterior belief $\delta_{n-1}(o_{n-1})$ irrespective of their behavior. If the market makers ignore the presence of boundedly rational traders in the market then their posterior belief is biased compared to the case of super-rationality. It holds*

$$\delta_n^b(b) > \delta_n^{sr}(b) \quad \text{and} \quad \delta_n^b(s) < \delta_n^{sr}(s).$$

Proposition 3 reveals that acting as if all information-based traders were rational induces a bias in the market makers' posterior belief. By ignoring boundedly rational traders the market makers' posterior belief is biased upwards if the upcoming order is a buy order ($\tilde{o}_n = b$) or downwards if the next order is a sell order ($\tilde{o}_n = s$) compared to a super-rational assessment of the upcoming order's informational content. Basically, the overestimation of the likelihood that an order comes from a rational information-based trader implies a more pronounced adjustment of the market makers' posterior belief to the upcoming order.

To gain the intuition of proposition 3 imagine for instance that only information based-traders are present in the market. If the market makers ignore boundedly rational agents the first buy order (sell order) submitted to the market makers is judged to be perfectly informative for the fact that the information ϕ_H (ϕ_L) was released initially. If at least some information-based traders are boundedly rational then super-rational market makers would take into account that the first order might have been submitted by a boundedly rational

trader. Consequently, super-rational market makers would arrive at a posterior belief which puts less than unit weight on either ϕ_H or ϕ_L .

The market makers' price policy is represented by equations (1) and (2). The application of the posterior belief as given in proposition 2 immediately allows to conclude from proposition 3 that both the ask price and the bid price deviate from the prices which the market makers would quote under super-rationality.¹⁴ Hence, it is straightforward to study the impact of the market makers' biased Bayesian inference from the order flow on the quoted bid-ask spread. Therefore, we apply the posterior beliefs according to propositions 1 and 2 to the quoted prices (1) and (2). We obtain corollary 1.

Corollary 1 *Assume the market makers have the posterior belief $\delta_{n-1}(o_{n-1})$ irrespective of their behavior. Prior to transaction n , if the market makers act as if all information-based traders were rational then the quoted spread s_n^b is greater than the spread s_n^{sr} which the market makers would quote under super-rationality. Formally, $s_n^b - s_n^{sr} > 0$.*

Corollary 1 reports that the market makers' ignorance of the presence of boundedly rational agents yields a quoted bid-ask spread which exceeds the bid-ask spread they would quote if they were super-rational. Consequently, if the bid-ask spread under super-rationality allows them to earn zero expected profits, ignoring the presence of boundedly rational traders leads to a bid-ask spread that overcompensates the market making industry for the actual adverse selection problem. Under super-rationality the market makers are aware of the fact that they are systematically making profits in transactions with boundedly rational agents. As these profits offset the losses they incur in transactions with rational information-based traders, the super-rational market makers improve the terms of trade for liquidity traders by reducing the bid-ask spread.¹⁵ However, if the market makers ignore boundedly rational traders they charge a bid-ask spread which is too large. Hence, the ignorant market makers' earn positive expected profits. The simulation study in section 4 demonstrates that the positive expected profits which stem from ignoring the boundedly rational traders come along with negative externalities for the market as a whole.

Before we turn to studying the dynamics of our approach we point out another observation. If the market makers act as if all information-based traders were rational, then they behave like the market makers in Glosten and Milgrom's (1985) model. In the presence of

¹⁴Note that since all orders are cleared at the quoted prices, the transaction prices are biased too. The dynamics of the transaction prices are studied in section 4.

¹⁵Of course, under super-rationality the bid-ask spread becomes negative for $\varepsilon_n > 0.5$. However, the case of super-rationality simply serves as benchmark. If the presence of boundedly rational traders is ignored — which is the case we study — the bid-ask spread is strictly non-negative.

boundedly rational traders, this behavior implies a bid–ask spread wider than necessary in order to compensate the market makers for the losses they incur in transactions with rational information–based traders because the market makers systematically gain in transactions with boundedly rational traders. Consequently, the difference of the biased bid–ask spread and the bid–ask spread under super–rationality — that is $s_n^b - s_n^{sr}$ — comes from ignoring the boundedly rational traders in the market and quantifies the overcompensation of the ignorant market makers.

4 Dynamics of the model

Compared to approaches which achieve the final outcome in a single trading round the natural advantage of a sequential trade model is that it allows for studying the dynamics of the model characteristics. In particular, the sequential approach allows for the inspection of the price discovery process. We perform a simulation study in order to analyze the price discovery process from the dynamics of the transaction prices as well as the dynamics of other market characteristics, such as the component of the bid–ask spread which accounts for the overcompensation of the market makers.

We study the benchmark case of super–rational market makers and the case in which the market makers ignore the presence of boundedly rational traders in parallel. The simulation study proceeds according to the following steps:

STEP 1 (Information release) Choose the released information to be ϕ_L .

STEP 2 (Sampling the trader continuum) Draw uniformly distributed index $i_n \in [0, 1] \subset \mathbb{R}$. If $i_n \in [0, \frac{1-\alpha}{2})$ then $\tilde{o}_n = b$ (liquidity buyer). If $i_n \in [\frac{1-\alpha}{2}, 1-\alpha)$ then $\tilde{o}_n = s$ (liquidity seller). If $i_n \in [1-\alpha, 1-\alpha\varepsilon_n)$ then $\tilde{o}_n = s$ (rational seller) else $\tilde{o}_n = b$ (boundedly rational buyer).

STEP 3 (Transaction) If $\tilde{o}_n = b$ then $p_n^b = a_n^b$ and $p_n^{sr} = a_n^{sr}$ else $p_n^b = b_n^b$ and $p_n^{sr} = b_n^{sr}$.

By step 1 we restrict our simulation study to a market situation in which bad news about the future liquidation value is released before trading starts. In step 2 the order which is cleared in transaction n is determined from sampling the trader continuum.¹⁶ In step 3 the transaction price p_n of transaction n is either the bid price b_n or the ask price a_n . The prices

¹⁶Restricting on the release of good news in step 1 implies that the rational traders would submit a buy order and the boundedly rational traders would submit a sell order in step 2. However, the liquidity traders' trading intentions are unaffected by step 1.

in step 3 are obtained from applying the beliefs as given in the propositions 1 and 2 to the equations (1) and (2).

A simulation run iterates the steps 2 and 3 for $n = 1, \dots, 400$. That is, 400 transactions in a row are randomly generated throughout one simulation run. A simulation takes 5000 independent simulation runs. By performing the simulation with various parameter combinations for α and g the study provides us with a series of numerical comparative static results. All simulations are performed with $H = 1$, $L = 0$, $\delta = 0.5$, and $\varepsilon = 1$. The figures 1–3 in the appendix B all report averages from 5000 independent simulation runs. The outcomes which obtain in a single simulation run are depicted in figure 4. Each simulation run yields a series of 401 prices, namely, 400 transaction prices and the unconditional price of the risky security prior to the first transaction. Hence, each simulation run provides us with 400 transaction price changes. The calculation of the autocorrelation coefficient of those 400 changes at maximal lag $i = 150$ uses 250 transaction price changes. Therefore, we display transaction prices up to transaction $n = 250$ and the unconditional price of the risky security prior to the first transaction. We report average autocorrelation coefficients of transaction price changes at all lags $i = 1, \dots, 150$.

The transaction prices represented by the solid lines the panels (a) and (b) of figure 1 show that the market makers' behavior of ignoring boundedly rational traders prevents the transaction prices from converging monotonically to the true liquidation value on average. Thus, our model — as do Daniel, Hirshleifer and Subrahmanyam's (1998) model, and Hong and Stein's (1999) approach — produces a hump shaped pattern in transaction prices. During a first period the transaction prices depart from the true liquidation value before we observe a reversal to the true liquidation value. If the market makers' had perfect knowledge about the fraction ε_n of boundedly rational traders present in the market prior to transaction n , the transaction prices converge monotonically to the true liquidation value as is indicated by the dotted lines. From this perspective the super-rational market makers' truly could be referred to as mistake-identifiers. However, the flat segments of the dotted lines about $n = 35$ in panel 1(a) and both about $n = 15$ and $n = 35$ in panel 1(b) need some comments. For these transactions the fraction ε_n of boundedly rational traders is about fifty percent; cf. figure 5. In those transactions the order flow from the information-based traders is uninformative. Consequently, the market makers do not adjust their quoted prices on average.

Further inspection of panel 1(a) shows that although the deviation from the true liquidation value is more pronounced for a larger value of α , the transaction prices converge even faster to the true liquidation value than for a smaller value of α . This result obtains since

ceteris paribus the fraction of the order flow which is information-based is higher for larger values of α . This in turn implies a stronger Bayesian inference of both wrong and correct order flow information on the part of the ignorant market makers. Finally, we therefore observe a faster convergence to both the wrong and the true liquidation value for larger values of α .

The panel 1(b) reports that the higher is the learning rate of the boundedly rational agents the less pronounced is the departure from the true liquidation value and the faster is the convergence of the transaction prices to the true liquidation value. Hence, the faster the boundedly rational learn to forecast the liquidation value correctly the faster the wrong order flow information vanishes. Alternatively, a higher learning rate implies that the market makers' biased inference becomes less pronounced since the actual order arrival process converges faster to the order flow model which the market makers have in mind.

The panels (a) and (b) of figure 2 depict average autocorrelation coefficients of transaction price changes for the lags 1 to 150 for different combinations of simulation parameters. The message of these autocorrelation patterns is that the ignorance of boundedly rational traders allows to explain short-term momentum and long-term reversal. Put differently, if the market makers act as if all information-based traders were rational, then the transaction price changes are positively correlated for short lags and negatively correlated for long lags.

The panel 2(a) reports that the extent of both positive and negative autocorrelation is more pronounced the higher is the fraction of information-based trading. Thus, the larger deviation from the true liquidation value and the sharper reversal of the transaction prices coming along with a high value of α imply a more pronounced pattern of autocorrelation. The numerical results verify that the first negative autocorrelation coefficient occurs at lag 12 for $\alpha_1 = 0.5$ (dotted line) and at lag 17 for $\alpha_2 = 0.25$ (solid line). This provides us with the intuition that the momentum period is shorter the more information-based trading takes place in the market and vice versa.

The panel 2(b) delivers the observation that the faster the boundedly rational agents learn to forecast the liquidation value correctly the less pronounced is the momentum but the more pronounced becomes the reversal. Visually, the dotted line lies below the solid line which depicts the average autocorrelation coefficients resulting from a lower learning rate. As we have pointed out previously, a higher learning rate implies both a smaller deviation from the true liquidation value and a sharper reversal. Our numerical results underpin these observations. For $g_2 = 0.05$ the first negative autocorrelation can be reported at lag 12 whereas for $g_1 = 0.02$ the first negative autocorrelation is obtained at lag 17. This buys us the intuition that a higher learning rate comes along with a shorter momentum period and

vice versa.

Before we continue with the discussion of our simulation, we reconcile our finding on the positive average autocorrelation of the transaction price changes at short lags with the empirical evidence of Hong, Lim and Stein's (2000) study whose purpose is to verify Hong and Stein's (1999) model of gradual information diffusion. Hong, Lim and Stein (2000, p. 266) test the hypothesis that "... *stocks with slower information diffusion should exhibit more pronounced momentum*" by checking the profitability of momentum strategies sorting the sample stocks by size and analyst coverage.¹⁷ Their results show that (i.) the momentum strategy works best for small stocks with low analyst coverage and (ii.) that the momentum period for stocks with low coverage lasts by more than factor two longer than for stocks with high coverage.¹⁸ This evidence turns out to be persistent in a series of robustness checks.

Using the notion of Hong, Lim and Stein (2000, p. 266) that "... *information about small firms gets out more slowly*" and interpreting our learning rate g as proxy for the speed of information diffusion yields a match between their results and ours. In the case of slowly diffusing information the graphs in panel 2(b) show that (i.) the momentum that is the extent of positive autocorrelation is more pronounced and (ii.) the momentum period is longer compared to information diffusion at a higher rate. Hence, our numerical comparative static result with respect to the learning rate fits their empirical evidence, namely, that small stocks whose information is less salient and much harder to learn exhibit stronger momentum. Furthermore, let us reinterpret the extent of information-based trading α in our model as proxy for analyst coverage since more analysts spread more firm-specific information which is used by more investors ultimately. Given this reinterpretation the panel 2(a) reports that the length of the momentum period increases the less trading is information-based or the lower is analyst coverage, respectively. Hence, our numerical comparative static result again matches Hong, Lim and Stein's (2000) findings.

One might criticize the link to Hong, Lim and Stein's (2000) findings as our model applies to a single security whereas they study portfolios of securities. However, our numerical comparative statics rely on the average of 5000 independent simulation runs, and the profitability of the momentum strategies is studied for equally weighted portfolios. Thus, both procedures cancel out characteristics of individual simulation runs or individual securities, respectively. This makes the two studies comparable to each other.

Thus far the simulation study has demonstrated that the market makers' ignorance of

¹⁷The table II in Hong, Lim and Stein (2000) reports the systematic dependency of analyst coverage on size. Thus, the proxy for analyst coverage is not the raw number of analysts following a stock but the residual analyst coverage which is systematically corrected for size.

¹⁸Cf. table V and panel A of figure 2 in Hong, Lim and Stein (2000).

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boundedly rational traders serves as explanation for stylized facts about the predictability of asset prices as do the models discussed in section 2. Since we rely on a sequential trade model which allows modeling the bid–ask spread explicitly we are able to shed light on Daniel, Hirshleifer and Subrahmanyam’s (1998) conjecture about the relation between the adverse selection component of the bid–ask spread and both momentum and reversal. Note that Daniel, Hirshleifer and Subrahmanyam (1998) are not able to derive a corresponding result as they perform their analysis in a single price framework.¹⁹

The panel (a) of figure 3 shows that the component of the biased bid–ask spread which the market makers quote in excess of the bid–ask spread they would quote under super–rationality is always positive and converges to zero when the price discovery process is completed. During the momentum period this component becomes smaller whereas during the reversal period it increases again. In the reversal period this component peaks when the market makers uncertainty about the true liquidation value reaches a maximum. The peak is observed around the transactions $n = 80$ and $n = 40$ for $g_1 = 0.02$ and $g_2 = 0.05$, respectively. When the transaction prices return to the true liquidation value the adverse selection problem, which almost vanished in the momentum period, regains severity during the reversal period. At the end of the reversal period the adverse selection problem vanishes completely, and the additional component of the bid–ask spread converges to zero. Hence, the simulation study establishes a relation between the size of the bid–ask spread’s component which is induced by the market makers’ ignorance of boundedly rational traders and both momentum and reversal. Furthermore, the additional component of the bid–ask spread is the smaller the faster the boundedly rational traders learn. This can be grasped from the dotted line in the panel 3(a) lying below the solid line. Thus, the slower the boundedly rational traders learn the higher is the additional compensation of the market makers from ignoring bounded rationality.

The results which are reported cleanly for the average characteristics of 5000 simulation runs in the figures 1–3 can to some extent also be grasped from the figure 4 which displays the transaction prices, the additional component of the bid–ask spread, and the lagged autocorrelation coefficients of transaction price changes for a single simulation run. The solid line in panel 4(a) exhibits the hump–shaped pattern of the transactions prices if the market makers ignore boundedly rational agents. It reports the deviation from and the reversal to the liquidation value of the risky security. The panel 4(b) reports that the additional component of the bid–ask spread vanishes during the momentum period ($n = 1, \dots, 40$) and

¹⁹The same is true for the papers by Barberis, Shleifer and Vishny (1998) and Hong and Stein (1999). Hence, the present paper truly contributes to the body of literature.

reappears during the reversal period ($n = 60, \dots, 100$). Finally, the panel 4(c) reports that the lagged autocorrelation coefficients peak to positive values ($\rho_i > 0.2$) for short lags $i = 1, \dots, 50$ and to negative values ($\rho_i < -0.4$) for long lags $i = 50, \dots, 150$. The irregularities of the graphs in the figure 4 are due to the stochastic nature of the simulation run. Naturally, these characteristics display smoother in the figures 1–3 as those graphs report average values.

To summarize, the simulation study delivers negative externalities which are induced by the boundedly rational traders and the ignorant market makers. On the one hand, when the boundedly rational traders dominate the order flow which is true for $\varepsilon_n > 0.5$ they impose a negative externality on the rational information-based traders since the value of the rational traders' positions evolve in a way that is disadvantageous to the rational traders initially. Note that the rational information-based traders sell the risky security while its price is pushed upwards in the momentum period. On the other hand, also the market makers effect a negative externality by ignoring the presence of boundedly rational traders. Due to the biased Bayesian inference from the order flow the market makers prevent a faster price discovery process and impose additional transaction costs on the liquidity traders by quoting too large a bid–ask spread. Therefore, the positive expected profits which the market makers earn by ignoring boundedly rational agents must be traded off against (i.) the departure of transaction prices from the true liquidation value and (ii.) the delayed price discovery process. Both effects impose a negative externality on the market as a whole.

5 Conclusion

In this paper we have proposed a sequential model of security trading which compared to existing models is extended in the spirit of Simon (1955), Rubinstein (1998) and Odean (1999). In this respect the present paper is complementary to Odean (1998) who focuses on the analysis of deviations from traders' full rationality in both rational expectations models and strategic trader models. Our approach builds on trader heterogeneity with respect to both access to information and information processing capabilities. We assume that information-based traders are subject to bounded rationality but are learning to improve their information processing capabilities through time whereas the market makers always act as if all information-based traders were rational. The market makers' behavior can simply be described by increasing prices if a buy orders is to be cleared and by decreasing prices in case of a submitted sell order. The driving forces behind our model are that (a.) boundedly rational traders add mistaken signals to the order flow, (b.) the boundedly rational traders

learn to get things right through time, and (c.) the market makers by ignoring the boundedly rational traders misinterpret the signals provided by the order flow. From the setup we have seen that our approach is in line with recent related behavioral finance models.

Concerning our setup, one might criticize that all boundedly rational agents end up with the same forecast error. But, in this respect, the same critique obviously applies to all representative agent models in which all agents are assumed to be subject to the same bias or mental error, too. A second objection raised against our approach could be that we did not model explicitly how boundedly rational traders come up with their wrong forecasts. This may be correct, but in this paper we intended to focus on the aggregate effects of the interaction of different trader groups on the price discovery process and not on the individual trader's decision problem. The assumption that the boundedly rational traders learn at an exogenously specified learning rate could be subject to criticism too. Of course, specifying the learning process endogenously would be possible but at the cost of building a more complex model. However, one must see clearly that an endogenous learning process would not generate more results beyond those already discussed. Even more important it would not yield additional economic intuition. One must trade off this potential criticism against the strength of our approach to generate a variety of hypotheses on information-based trading and the quality of information from extending the standard model parsimoniously. Similarly, one might criticize that the traders which are assumed to be boundedly rational initially become fully rational in the end. Basically, the same problem is present in Daniel, Hirshleifer and Subrahmanyam's (1998) model. Their representative agent is not overconfident anymore with respect to the quality of private information, ultimately. Thus, a similar criticism applies, obviously. Anyway, at the very heart of the present paper is the suggestion of an alternative mechanism — ignorance of boundedly rational traders — in order to explain stylized facts in financial markets. Besides, the strength of the present paper is to derive results on a market characteristic — the bid-ask spread — which cannot be obtained in related models.

We have illustrated by means of a simulation study that our model consistently explains both momentum and reversal in security prices that is the departure from the true liquidation value in a first trading period and the convergence to the true liquidation value in a second trading period. Specifically, the proposed approach is able to generate positive autocorrelation of security price changes for short lags and negative autocorrelation for long lags. Furthermore, we provided an ex-post rationalization for the empirical findings of Hong, Lim and Stein (2000). Moreover, we are able to confirm Daniel, Hirshleifer and Subrahmanyam's (1998) conjecture concerning the relation between the adverse selection component of the

bid–ask spread and both momentum and reversal in security prices. Our observation that the adverse selection component is smaller during the transition from momentum to reversal can be tested empirically. This is probably the most clear cut avenue for future empirical research.

Ignoring bounded rationality among traders on the part of the market makers implies negative externalities for the market as a whole and the liquidity traders in particular, but not for those who generate to some extent these externalities, namely the market makers. They even profit on average from ignoring bounded rationality. The market as whole suffers from a temporary departure of the transaction prices from the true liquidation value and from the delayed price discovery process. The liquidity traders are subject to an additional adverse selection component. As it is clear from the above analysis the crucial parameter of our approach is the fraction of boundedly rational traders ε_n present in the market prior to the transaction n . The market makers' imperfect knowledge of this parameter induces the negative externalities.

The model of Daniel, Hirshleifer and Subrahmanyam (1998) and our approach have in common that the price setting agents — the representative agent in their model and the market makers in our model — are subject to some learning or inference bias. They must have some wrong model of the relevant processes underlying the financial markets in mind. But, in order to generate the hump shaped pattern accompanied by short–run momentum and long–run reversal in transaction prices, there has to be added some variation to the model in addition to this learning bias. In our model, this is achieved by varying the quality of the order flow as direct consequence of reducing the fraction of boundedly rational agents. In Daniel, Hirshleifer and Subrahmanyam's (1998) model it is the varying overconfidence from the self–attribution bias together with the exogenous public signals that point on average in the opposite direction compared to the private signal obtained by the representative agent initially. Anyway, it must be stressed that if financial economists like to explain variation which they observe in financial markets they somehow have to add variation to their models.

Finally, the diversity of approaches generated by financial economists in behavioral finance in order to explain security price patterns and to cope with predictability issues is probably an indicator of Aumann's (1997, p. 3) notion that “... *there is no unified theory of bounded rationality*.” This quotation and the insights that both economic agents are subject to bounded rationality and bounded rationality affects financial markets nourish the expectation that behavioral finance will be a vivid research discipline in the near future.

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A Proofs

A.1 Proof of proposition 1

Proof. Under super-rationality the market makers know the composition of the order flow exactly. Therefore, we have for transaction n

$$P(\tilde{o}_n = b | \phi_H) = \alpha \cdot (1 - \varepsilon_n) + \frac{1 - \alpha}{2} \quad \text{and} \quad (3a)$$

$$P(\tilde{o}_n = b | \phi_L) = \alpha \cdot \varepsilon_n + \frac{1 - \alpha}{2} \quad (3b)$$

as well as

$$P(\tilde{o}_n = s | \phi_H) = \alpha \cdot \varepsilon_n + \frac{1 - \alpha}{2} \quad \text{and} \quad (4a)$$

$$P(\tilde{o}_n = s | \phi_L) = \alpha \cdot (1 - \varepsilon_n) + \frac{1 - \alpha}{2}. \quad (4b)$$

Note that the probabilities of observing a buy order or a sell order depend on the released information (ϕ_H, ϕ_L) as well as on the transaction (n) . Therefore, the market makers' inference from the order flow is path-dependent and has to account for all orders prior to transaction n .

For $n \geq 1$ let $o_n \in \{b, s\}$ denote any realization of the upcoming order \tilde{o}_n . Define $\delta_n(o_n) \equiv P(\phi_H | o_n)$ as the posterior probability which the market makers assign the release of good news conditionally on clearing the order o_n in transaction n . Hence,

$$\begin{aligned} \delta_n : o_n &\mapsto \delta_n(o_n) \\ \{b, s\} &\rightarrow (0, 1) \subset \mathbb{R} \end{aligned}$$

is a function which maps the order triggering the transaction n to a posterior probability, where

$$\delta_n(o_n) = \begin{cases} \frac{P(\tilde{o}_n = b | \phi_H) \cdot \delta_{n-1}(o_{n-1})}{P(\tilde{o}_n = b | \phi_H) \cdot \delta_{n-1}(o_{n-1}) + P(\tilde{o}_n = b | \phi_L) \cdot (1 - \delta_{n-1}(o_{n-1}))} & \text{if } o_n = b \\ \frac{P(\tilde{o}_n = s | \phi_H) \cdot \delta_{n-1}(o_{n-1})}{P(\tilde{o}_n = s | \phi_H) \cdot \delta_{n-1}(o_{n-1}) + P(\tilde{o}_n = s | \phi_L) \cdot (1 - \delta_{n-1}(o_{n-1}))} & \text{if } o_n = s \end{cases} \quad (5)$$

with

$$\delta_0(o_0) = \delta. \quad (6)$$

Note that, first, (5) is obtained by Bayes law where $\delta_{n-1}(o_{n-1})$ is used as prior belief and, second, (5) defines the posterior belief $\delta_n(o_n)$ recursively with termination by (6). Hence, the posterior belief $\delta_n(o_n)$ accounts for all orders prior to transaction n . Plugging (3a), (3b), (4a), and (4b) into (5) yields the proposition, ultimately. This completes the proof. \square

A.2 Proof of proposition 2

Proof. If the market makers act as if all information-based traders were rational the probabilities underlying the order flow are

$$P(\tilde{o}_n = b|\phi_H) = \alpha + \frac{1-\alpha}{2} \quad \text{and} \quad (7a)$$

$$P(\tilde{o}_n = b|\phi_L) = \frac{1-\alpha}{2} \quad (7b)$$

as well as

$$P(\tilde{o}_n = s|\phi_H) = \frac{1-\alpha}{2} \quad \text{and} \quad (8a)$$

$$P(\tilde{o}_n = s|\phi_L) = \alpha + \frac{1-\alpha}{2}. \quad (8b)$$

Now, plugging (7a), (7b), (8a), and (8b) into (5) yields the proposition. This completes the proof. \square

A.3 Proof of proposition 3

Proof. We assume $\delta_{n-1}(o_{n-1}) \equiv \delta_{n-1}^b(o_{n-1}) = \delta_{n-1}^{sr}(o_{n-1})$. Straightforward calculation shows $\delta_n^b(b) - \delta_n^{sr}(b) > 0 \Leftrightarrow 2 \cdot (1 - \delta_{n-1}(o_{n-1})) > 0$ giving us the first part of the proposition since $0 < \delta_{n-1}(o_{n-1}) < 1$. Similarly, $\delta_n^b(s) - \delta_n^{sr}(s) < 0 \Leftrightarrow 2 \cdot (\delta_{n-1}(o_{n-1}) - 1) < 0$ which is true since $0 < \delta_{n-1}(o_{n-1}) < 1$. This completes the proof. \square

A.4 Proof of corollary 1

Proof. We assume $\delta_{n-1}(o_{n-1}) \equiv \delta_{n-1}^b(o_{n-1}) = \delta_{n-1}^{sr}(o_{n-1})$. The bid price and the ask price for transaction n are given in equations (1) and (2). According to proposition 2 the biased bid-ask spread results as

$$s_n^b = (H - L) \cdot (\delta_n^b(b) - \delta_n^b(s)) \quad (9)$$

whereas the bid-ask spread of super-rational market makers according to proposition 1 is

$$s_n^{sr} = (H - L) \cdot (\delta_n^{sr}(b) - \delta_n^{sr}(s)). \quad (10)$$

The difference of both bid-ask spreads

$$s_n^b - s_n^{sr} = (H - L) \cdot (\delta_n^b(b) - \delta_n^{sr}(b) - (\delta_n^b(s) - \delta_n^{sr}(s))) > 0 \quad (11)$$

by proposition 3. This completes the proof. \square

B Figures

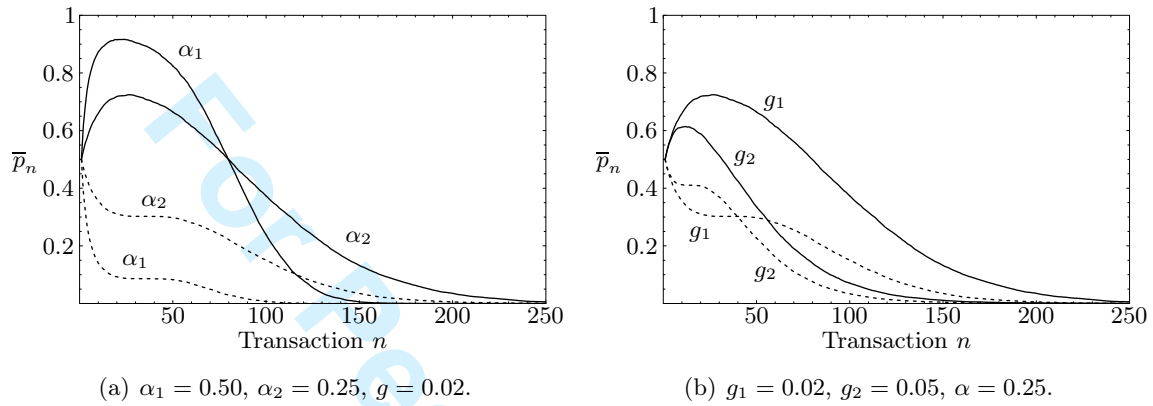


Figure 1: Panels (a) and (b) show average transaction prices \bar{p}_n . The solid (dotted) lines obtain if the market makers ignore boundedly rational agents (are super-rational).

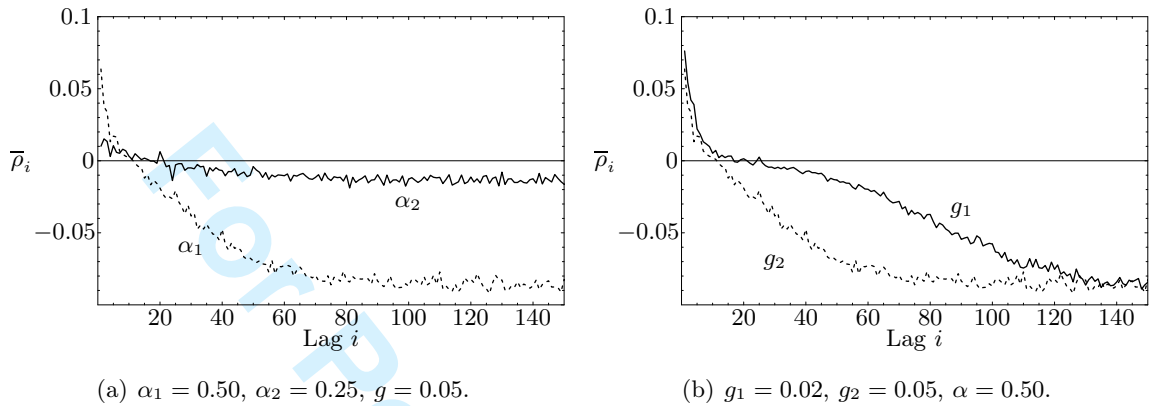


Figure 2: Panels (a) and (b) show average autocorrelation coefficients of transaction price changes $\bar{\rho}_i$ at lags $i = 1, \dots, 150$ if the market makers ignore boundedly rational agents.

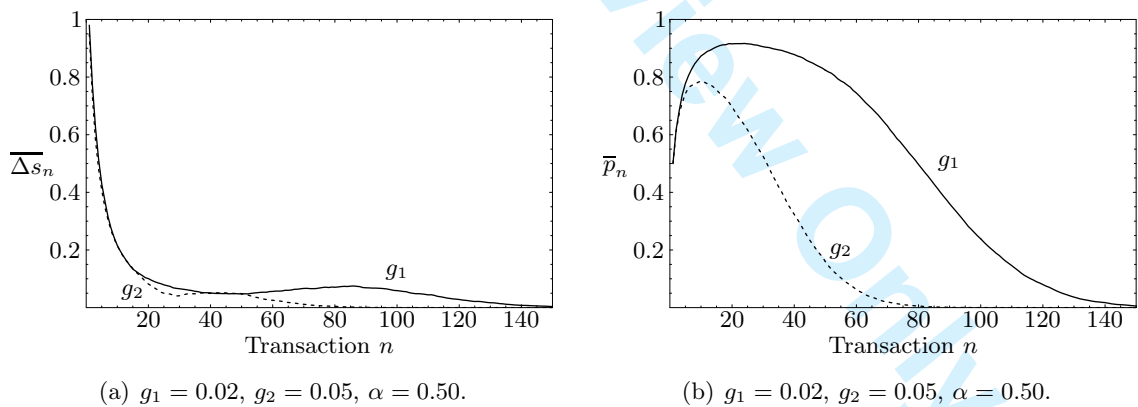


Figure 3: Panel (a) shows the average additional bid-ask spread $\Delta s_n = s_n^b - s_n^{sr}$ if the market makers ignore boundedly rational agents. Panel (b) depicts the corresponding average transaction prices.

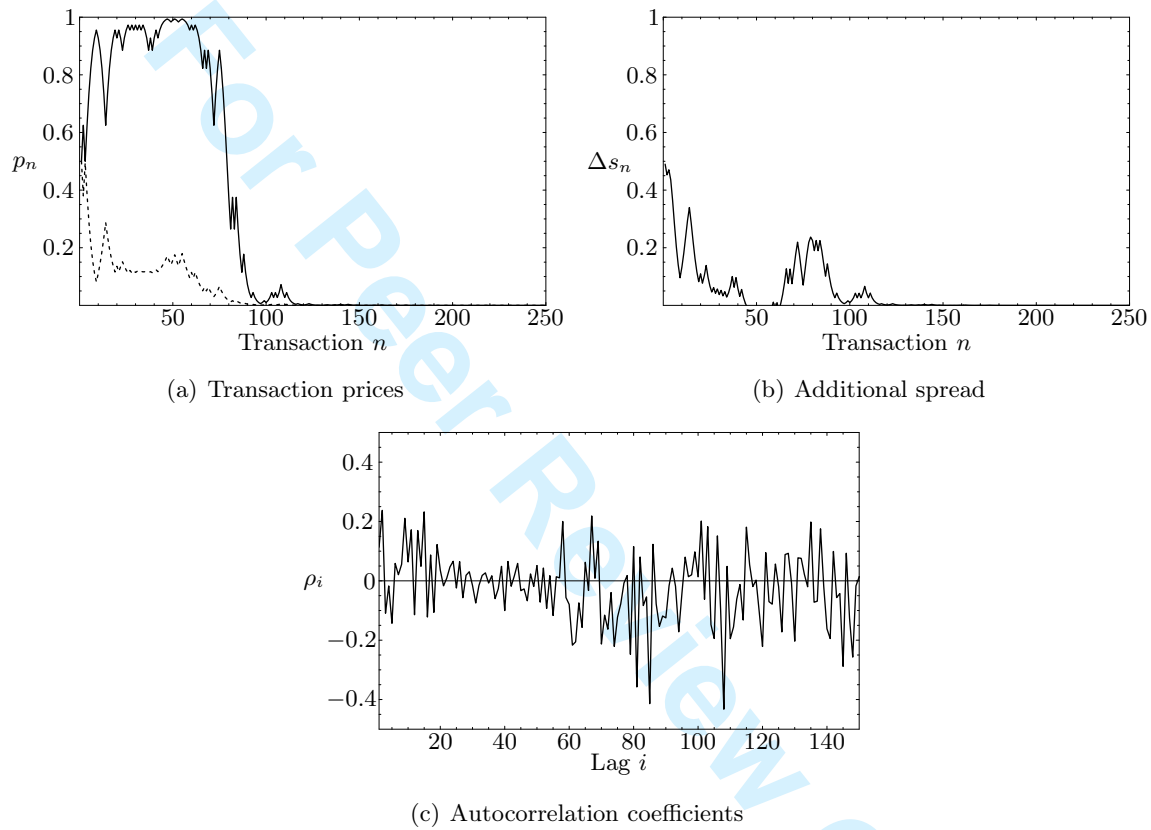


Figure 4: Panels (a), (b), and (c) exemplify the transaction prices p_n , the additional bid-ask spread $\Delta s_n = s_n^b - s_n^{sr}$, and the autocorrelation coefficients of lagged transaction price changes ρ_i for a single simulation run, respectively. The solid lines (dotted line) obtain(s) if the market makers ignore boundedly rational agents (are super-rational). For this simulation run, $\alpha = 0.25$ and $g = 0.02$.

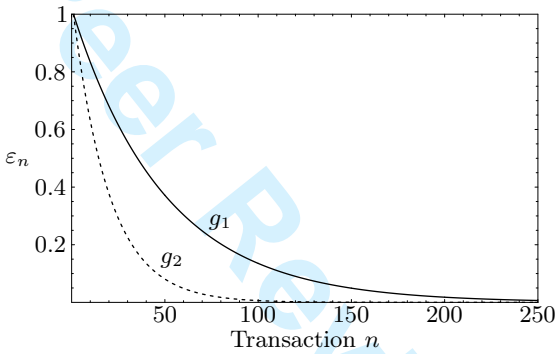
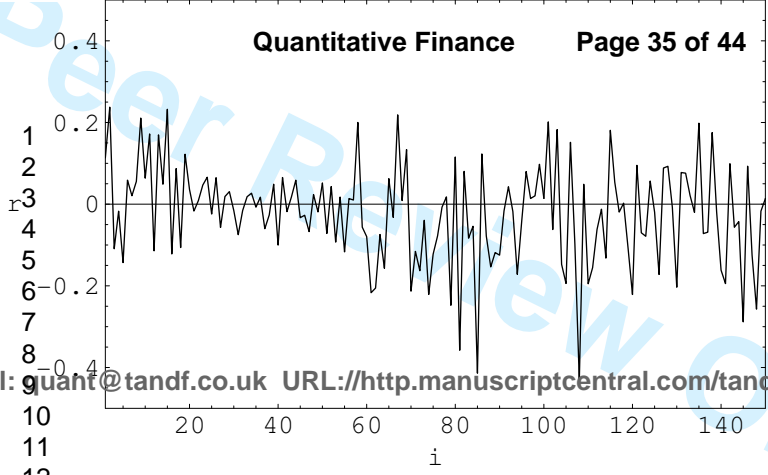
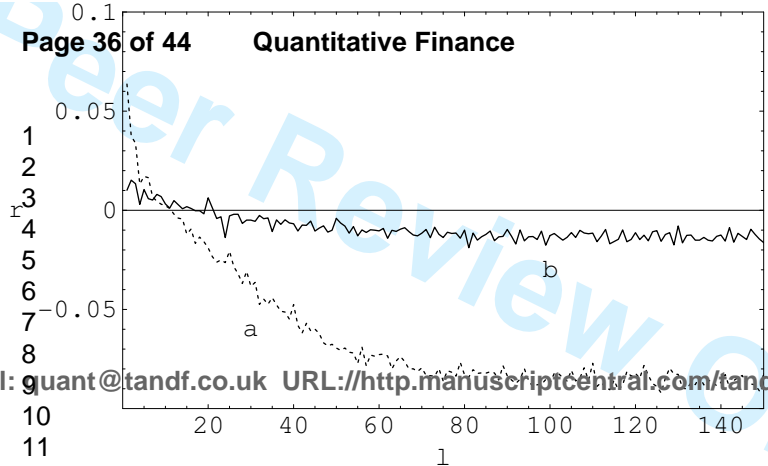
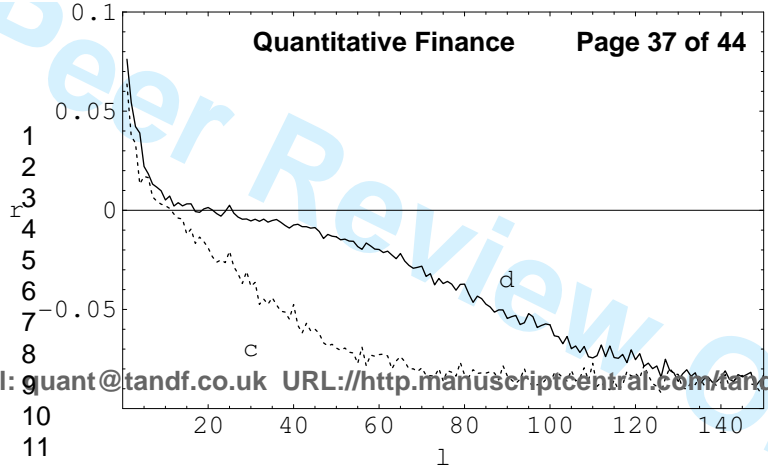
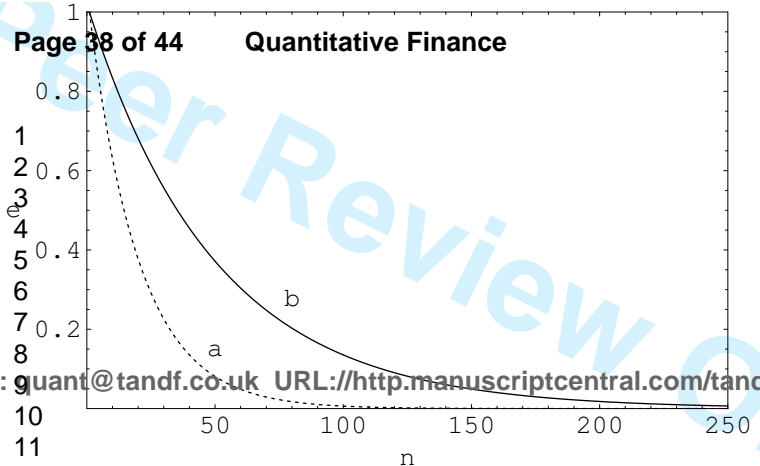


Figure 5: The graph shows the fraction ε_n of boundedly rational agents among the information-based traders. Here, $g_1 = 0.02$ and $g_2 = 0.05$.









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