

## Stochastic dynamic programming in life course analysis: basic concepts and a simple model

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"Bildung", die Erstberufe "freie Berufe" und "Klerus" und bei den Berufen des Vaters ebenfalls "freie Berufe" und "Klerus" lokalisiert. D.h., bleibt die Berufsstabilität und die Berufsvererbung außerhalb der Betrachtung, so gaben diejenigen, die 1848/49 hauptberuflich in der Bildung tätig waren, als Erstberuf sowie als Beruf des Vaters überdurchschnittlich oft "Klerus" und "freie Berufe" an. Ähnliches gilt für den Hauptberuf "Justiz" (rechter oberer Quadrant); die Angehörigen dieses Berufes kamen, abgesehen von "Justiz", überdurchschnittlich oft aus der "Verwaltung" bzw. sie waren als "Advokaten" tätig.

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## 2. Stochastic Dynamic Programming in Life Course Analysis: Basic Concepts and A simple model

*Ulrich Mueller*

For causal analysis in demography, individual longitudinal data are indispensable. Life table and transition rate methods, however, allow the analysis of single events only, they do not capture the character of the human life course as a adaptive sequence of transitions. Using concepts from evolutionary life history research, a new approach to analyzing whole life courses is presented: from measuring trade-offs between life course traits identifying optimal life courses with dynamic stochastic programming, and modeling the effect of covariates as determining deviations from the optimal sequence.

Stochastic Dynamic Programming is the tool of choice for the problem of optimizing the overall outcome of a sequence of decisions when the optimal choice of later steps depends on earlier steps (Mangel and Clark 1988; Puterman 1994).

Fundamental to Stochastic Dynamic Programming is the principle of optimality:

*"From any point on an optimal trajectory, the remaining trajectory is optimal for the corresponding problem initiated at that point"* (Luenberger 1979, 419).

The trick in this seemingly trivial statement is that, if the trajectory is optimal, the principle works backwards as forwards. Assume the trajectory is a sequence of discrete moves, with discrete outcomes. We consider a possible outcome of the process at terminal time T. Once we

know that the trajectory leading to this outcome is optimal, we can determine the optimal last move at time  $T-1$ , which has brought the trajectory from its position at time  $T-1$  to its position at time  $T$ . Next, from the penultimate position of the process at time  $T-1$ , we can determine its position at time  $T-2$ , and so forth determine the optimal trajectory all the way backwards until the beginning of the process at time  $t=0$ .

The application to the problem of an optimal sequence of reproduction decisions would be like this: Consider an individual in a stationary population whose total remaining reproductive potential  $V$  at age  $t+1$  is

$$V_{t+1} = V_t + G_t - P_t \quad (1)$$

the total reproductive potential  $V$  at age  $t$  plus some current income  $G_t$  minus current parental investment  $P_t$  at  $t$ ,  $G_t$  being a non increasing function of  $P_t$ .  $G_t$  is reaching its maximum, if there is no parental investment in this period: there exists a trade-off between  $P_t$  and  $G_t$ .

Assuming that all individuals die at terminal age  $T$ , the optimal allocation at this age is to divert all remaining resources into current investment. From that the optimal allocation at time  $T-1$  can be calculated, from that the one at  $T-2$ , and so forth down to age  $0$ . Thus, for any assumed reproductive investment in the last time period of the individual's life, the sequence of allocation decisions can be derived which maximizes lifetime fitness.

Up to now, most applications of stochastic dynamic programming to life history analysis have been for animals: hunting behavior of lions (Mangel and Clark 1988); migration by salmon (Levy 1987, quoted in Puterman 1994); foraging of small birds in winter (Houston et al. 1988); the dawn chorus of birds in spring (McNamara et al. 1987); parental allocation and clutch size (Mangel 1987; Mangel and Clark 1988); mate desertion in hawks (Kelly and Kennedy 1993); foraging in many species (references in Clark 1991). Any application to human reproductive strategies could easily make use of many well documented trade-offs between costs and returns of parental investment.

We want to determine the optimal number and optimal timing of births which under certain environment conditions maximize expected lifetime fitness. As an illustration, we are describing a very simple example of a life course problem.

From maturity on, in a population, a female with an ever-present, faithful husband, or, alternatively, an abundant supply of males may live uniformly seven years during which she may reproduce. She may give birth once per year but may decide not to. Beyond what she needs for her own subsistence, she has a fixed income of one resource unit per year. As long as she has no children, she saves this amount. Once she has children, the current income is split up evenly among them. Her savings (the resources she had saved before reproduction) will be split up among her children after her death. This assumption may reflect the investment she has made in her own rank which in many species is known to be transferred to children and, thus, to increase reproductive success of children beyond current maternal investment.

In order to keep things simple, let children uniformly start reproduction after the death of their mother, notwithstanding their own age at this moment.

In consistence with the fitness criterion from stochastic population theory as sketched above, her goal is to maximize the survival of her lineage, or to minimize the probability that no descendants of her survive. Simulation can support the intuitive notion that in sexually reproducing species, the vast majority of all lineage extinction events takes place in the first generation, therefore we will take into account only lineage survival probabilities into this first generation (Mueller 1992).

Let there be two built in-trade-offs with respect to number of children.

1) Mortality of children shall go up with number of children already present as an effect of the decreasing share of current income per child. Empirical evidence for that assumption can be found in the studies of Anderson (1990) and Haukioja et al. (1989). By comparing infant mortality of twins and singletons these studies circumvent the problem that realized number of children may be a reaction to expected infant mortality, as well as the problem that subsequent children are born at increasing age of mothers with increasing risk of maternal and infant mortality.

2) Chances of adult children of not remaining childless shall go up with increasing amount of resources each child inherits from parents. This inheritance in turn is the bigger the longer the parents have postponed reproduction and the smaller the number of their children is. Positive correlations between number of children and percentage of children remaining childless were found in the families of US American professional soldiers (noncommissioned and commissioned officers) (Mueller 1992), in the families of West German and East German physicians (Mueller unpublished), and in the families of German female university professors (Schmid 1994).

In both tradeoffs, we model the desired effect not as a proportional function of the share of current income or of inheritance but a logarithmic function, taking into account that from an early age on, life chances of children depend on parental investment only with decreasing marginal effect.

Finally, we will consider two situations. In the *no-care-necessary* situation, children can be on their own and are able to start reproduction already in the first year they have entered alive. In the *care-necessary* situation, children need two full years of maternal care, before they can start reproduction: the first year they have entered alive, and the following year. In this situation, a child, whose mother does not live through these two years, will never reproduce.

For any stochastic dynamic optimization model, essential components are (Mangel and Clark 1988, 215-233; Puterman 1994, 17-25):

- a state variable  $X_t$ ;
- a set of constraints on  $X_t$ , defining a state space;
- a set of actions  $i$ , which can be used with probability  $b_i$ , such that  $\sum b_i = 1$ ;
- a state dynamics;
- an optimization criterion.

Applied to our model, we have

- 1) the state variable  $X_t$ , the expected number of children at time  $t$ ;
- 2) a discrete time structure, consisting of right-side-open intervals  $[t, t+1)$  of equal length, with a start period  $[t_0, t_1)$  and with a terminal period, denoted as  $[T, T]$ . We will think of time periods as years. The beginning of year  $[t, t+1)$  is denoted as  $t$ . Where the context is unambiguous,  $t$  may also denote the year  $[t, t+1)$ . The state variable is measured at the beginning

of each year, reproduction and mortality occur during the year, the female dies at the end of the year  $T, T]$ ;

3) the constraints  $0 \leq X_t, 0 \leq (X_t - X_{t-1}) \leq 1, 0 = X_t$  for  $t = t_0$ :

the expected number of children alive at the beginning of the process is zero, thereafter a non-negative number not exceeding the number of years passed, since there can be only one birth per year;

4) the set of actions: Here simply the binary decision to give birth to a child in year;  $[t, t+1)$  with probability  $b_t$ , or not, with probability  $(1-b_t)$ ;

5) the state dynamics given by:

$$X_{t+1} = \{ X_t(1 - m_t(m, X_t) + 1) b_t + X_t(1 - m_t(m, X_t))(1 - b_t) \} \tag{2}$$

The number of children  $X_{t+1}$  at time  $t+1$  equals the number of children  $X_t$  at time  $t$ , diminished by the mortality  $m_t(m, X_t)$  during the year  $[t, t+1)$ , which is a function of the number of children present at  $t$ , and some base line mortality  $m$ , an environment parameter, the one a child is subject to if there are no other children present and it gets the whole of the parental investment. In order to model the first trade-off, we want  $m_t$  to be an increasing function of number of children present, but with decreasing increment, and propose:

$$m_t := m^{(1/(1+\ln(X_t)))} \tag{3}$$

Neither age of children nor age of mother shall have an effect on child mortality, which for simplicity shall work only on children alive at the beginning of the year  $[t, t+1)$ .

6) The optimality criterion is the lifetime fitness function  $\varphi_T(X_T, R_T)$ , the function which maps number of children alive at the beginning of the terminal year  $[T, T]$  and the resources  $R_T$  which they will receive after the death of their mother at the end of  $[T, T]$ , into the expected lineage survival probability = lifetime fitness.

In the model, we assume that  $R_T = t^*$  with  $b_t^* > b_{t-1} = b_{t-2} = \dots = b_0$ , since the mother can save one resource unit each year before she starts reproducing.

Appropriately, lifetime fitness is measured at the end of  $[T, T]$ . The more resources going to a child, the smaller the probability  $c$  that this child will remain childless (not finding a mate, being sterile, or die before onset of reproduction). As in the case of infant mortality  $m$ , in order to model the second trade-off, we want the probability  $c$  be an increasing function of number of resources going to a child, but with decreasing increment, and propose:

$$\varphi_T(X_T, R_T) := 1 - c(\gamma, R_T)^{X_T} \quad \text{and} \quad c(\gamma, R_T) := \gamma / (1 + \ln(1 + R_T / X_T)) \tag{4}$$

The lineage survival probability, according to our definition - see above - is the complement to the probability that all children  $X_T$  will remain childless. This is the individual chances of childlessness among these children, dependent on some base line childlessness  $\gamma$  and the number of resources per capita  $R_T / X_T$ , raised to the power  $X_T$ .

Now we can determine the optimal trajectory, that is the optimal timing of reproduction decisions  $b_0, b_1, \dots, b_{T-1}, b_T$ .

The general procedure is:

1) For each possible value of the state variable  $X_T$ , the expected number of children at terminal time  $T$ , we determine the optimal trajectory, which maximizes the terminal fitness function  $\varphi$  for that  $X_T$ . We may call such a trajectory a locally optimal trajectory for  $X_T$ . There may be more than one locally optimal trajectory for a given  $X_T$ .

2) Then we select from that set of locally optimal trajectories for all values of  $X_T$  which are possible under the constraints of the state space, the one trajectory which leads to the highest value of the terminal fitness function  $\varphi$  over the whole state space. We may call this trajectory the globally optimal trajectory for all  $X_T$ . Again, there may be more than one globally optimal trajectory for all  $X_T$ , several distinct sequences of reproduction decisions may be equally globally optimal.

While the second half of the task does not pose specific problems, the first one requires stochastic dynamic programming, a working backwards in the state dynamics, as defined in eq. (5).

We introduce a new function

$$F(x, t, T) := \max E\{\varphi(X_T) \mid X_t = x\} \quad (5)$$

$F$  is the function, which at the end of  $[T, T]$  maximizes expected lifetime fitness of the state variable  $X_T$ , under the condition that the state variable  $X$  at time  $t$  equals  $x$ . In order to find a general method of calculating  $F$ , we write

$$F(x, T, T) := \max E\{\varphi((X_T(1-m_T) + 1) b_T) + \varphi((X_T(1-m_T))(1 - b_T))\} \quad (6)$$

The right hand side of this equation gives the expected value of the expected lifetime fitness at the end of the terminal year for any  $X_T$  depending on

- the mortality in  $T$ , itself a function of  $X_T$ ,
- the fitness function  $\varphi$ , which also is a function of  $X_T$  and  $R_T$ , the latter at least being bound by  $X_T$ ;
- and the action "reproduction", taken with probability  $b_T$  and avoided with probability  $(1 - b_T)$ .

Finding  $F(x, T, T)$  means determining the value of  $b_T$ , which maximizes the right hand side of eq.(6).

After we have found  $F(x, T, T)$ , we want to find  $F(x, T-1, T)$ .

$$\text{Let } x'_t := [(X_{t-1})(1-m_{t-1}) + 1]$$

$$\text{and } x''_t := [(X_{t-1})(1-m_{t-1})]$$

Then, by the total probability formula, we have

$$\begin{aligned}
 F(x, T-1, T) &= \max \{F(x, T, T) \mid X_{t-1} = x\} \\
 &= \{F(x', T, T)b_{T-1} + F(x'', T, T)(1 - b_{T-1})\}
 \end{aligned}
 \tag{7}$$

Again, we solve this equation by an appropriate choice of  $b_{T-1}$ , which together with  $b_T$  gives  $F(x, T-1, T)$ . Applying the same algorithm, we obtain  $F(x, T-2, T)$ , and all the way backwards to  $F(x, t_0, T)$  for any given  $X_{t_0}$ .

Since we want to keep this model as simple as possible, we set child mortality equal at all ages. First we consider the *no-care-necessary* situation when no child reproduces as long as the parents are alive, but starts to do so as soon as they are dead, never mind how old the child is. We also disregard adult mortality among children.

For the simple model presented here, inspection of eq.(7) shows that, in the *no-care-necessary* situation  $b_T$  must equal one, and so must  $b_{T-1}$ , and so forth, down to some  $t$ , such that the optimal trajectory = sequence of reproduction decisions has the following general form:

$$b_0 = b_1 = \dots = b_{t-1} = 0, \quad \text{and} \quad b_t = \dots = b_{T-1} = b_T = 1$$

Once reproduction has begun, it is not advisable to take a break.

Imagine that the female has achieved the optimal number of births, the one which maximizes survival probability of her lineage. In this situation the best last move for the female must have been to place this last birth in the last year of her life, because any other choice would unnecessarily expose the last child to additional years with the risk to die before commencing reproduction.

The best move before this last best move again is to give birth to a child the year immediately before and so on backwards. Put it in forward perspective: as soon as the female has started reproduction, she should get a child every year until death.

Thus, if the optimal number of births is 1,2,3 ... , we know the optimal timing of births. In order to find the number of births, which actually maximizes lineage survival chances, we first calculate the expected number of children alive at the death of their parents and then the lineage survival probability from this number and from the size of the maternal inheritance (her mother's savings) bequeathed to them.

Next we may wish to consider the *care-necessary* situation: no child may be able to survive the first two years without the ongoing support of the mother. That means, the mother should give birth to her last child not immediately before her death. But from that time backwards, by force of the same argument already applied, the female, from the onset of her reproduction on, should have a baby each year. And again, in order to find the optimal number of births, we first calculate the expected number of children alive at the death of their mother and then the lineage survival probability from this number and from the size of the inheritance left to them.

In the *care-necessary* situation  $b_T$  must be zero, and so must  $b_{T-1}$ , but from there on  $b_{T-2} = b_{T-3} \dots = 1$ . Here the optimal trajectory = sequence of reproduction decisions has the following general form:

$$b_0 = b_1 = \dots = b_{t-1} = 0, \quad b_t = \dots = b_{T-3} = b_{T-2} = 1 \text{ and } b_{T-1} = b_T = 0$$

In the last step, the maximal fitness value of the various locally optimal trajectories has to be found which identifies the globally optimal trajectory.

We consider three types of environments,

- one "rough" with child mortality at .15 and childlessness chances at .40,
- one "moderate" with child mortality at .10 per year and childlessness chances at .30,
- one "rich" with child mortality at .05 per year and a .20 chance of an adult child to remain childless.

Figures 1-6 show for both situations (*no-care-necessary* and *care-necessary*) and for all three environments (rough, moderate, and rich) the optimal life course in terms of number and timing of births.

In a rough environment, in the *no-care-necessary* situation, if children can be on their own immediately after birth, the mother should wait three years, have four children and then die immediately after the birth of her last child. In the *care-necessary* situation, if children need two years before they can be on their own, the female should wait three years, have two children without a break, and then care for these children during her last two years.

In a moderate environment, in the *no-care-necessary* situation, the mother should wait three years, have four children and then die immediately after the birth of her last child. In the *care-necessary* situation, the female should wait two years, have three children without a break, and then care for these children during her last two years.

In a rich environment, in the *no-care-necessary* situation, the mother should wait two years after maturity, have five children and then die immediately after the birth of her last child. In the *care-necessary* situation, the female should wait one year, have four children without a break, and then care for these children during her last two years.

Figure 1: Rough environment, without care

Figure 2: Rough environment, with care

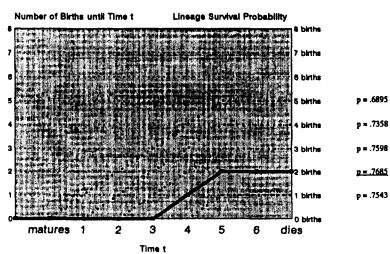
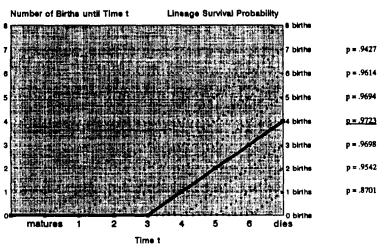




Figure 3: Moderate environment, without care

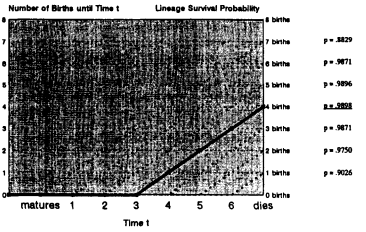


Figure 4: Moderate environment, with care

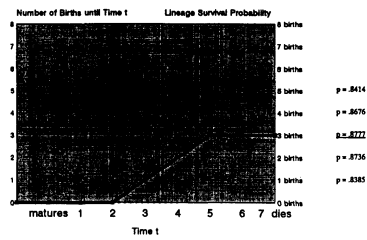


Figure 5: Rich environment, without care

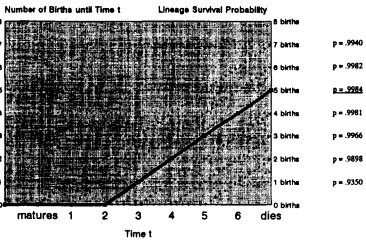
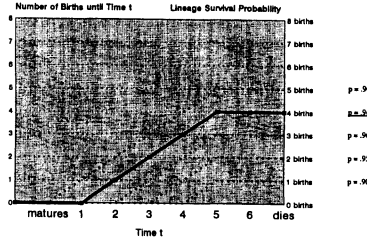


Figure 6: Rich environment, with care



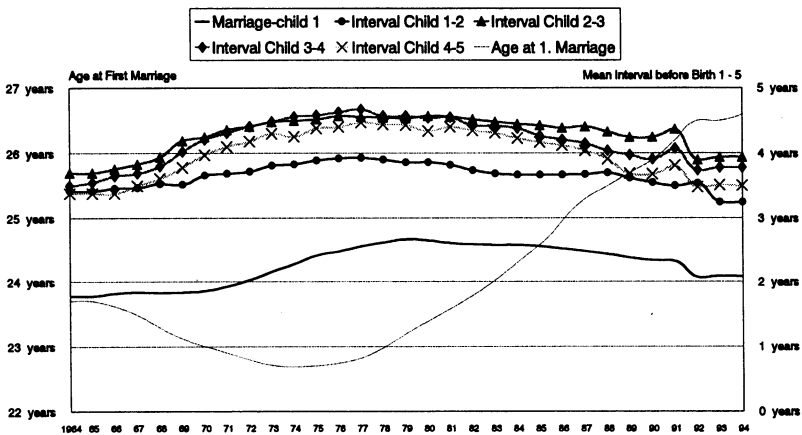
Further model refinements, in order to make it more realistic, are easily conceivable. We could vary child mortality by age of child. We could allow children start reproducing with their parents still alive, we could make the time of inheritance more flexible (allow for dowries) and so on. The next step would then be fitting the model to real data in order to see how close the distribution of empirical life courses in a population matches the optimum predicted by the model. For this, the two tradeoff functions probably would have to be re-specified.

On the other hand, very simple models can have considerable predictive power. For example, the relatively inflexible spacing of the optimal number of children which was not included in the assumptions of the model, fairly well reflects the empirical findings. In subsistence societies like the hunter-gatherers of the Kalahari, the optimal spacing is about 4 years (Blurton Jones 1986, 1987). In modern societies, made possible by modern medicine and improved nutrition, optimal spacing, can be somewhat shorter (Yamaguchi and Ferguson 1995): but apparently not less than three years.

But also in modern societies, with a much lower number of births per women, leaving much more freedom of choice for the timing of births, adaptation to environment fluctuations occurs by varying the onset of reproduction and - possibly - the total number of children born, but not by varying the spacing between births. Figure 6 shows the intervals between marriage and birth of

first child, between first and second child and so forth, on the one side, the age at first marriage for women in West Germany 1964-94. Age at first marriage varied between 22.7 and 26.9, more than 4 years. The crude birth rate varied between 17.4 and 9.9 per 1000. The spacing between births, however, was remarkably stable: changes of one year most. There seems to be a large evolutionary premium on having the optimal number of births in the shortest possible period in life, once reproduction has begun. In estimating the effects of covariates on the extent of eventual deviations, some techniques analogous to semiparametric transition rate models may be useful. The general procedure could be as follows:

Figure 7: Age at First Marriage and Intervals before Birth 1 - 5  
German Women 1964-1994 / Legitimate Births only



Mean age at first marriage and intervals before birth 1-5, German Women 1964-1994 (Federal Statistical Office, own computations)

- 1) Determine trade-off functions from empirical data;
- 2) Derive the optimal transition probabilities, and, hence, the optimal life course in this environment;
- 3) Link the variation in the covariates with the variation in the observed transition probabilities. Standard linking models from life event analysis (proportional and linear hazard models; accelerated failure time models) may possibly be too simple. Imagine that one deviation may be a condensation in time (have your first child later, your last one earlier), the other deviation a stretching of events (have your first child earlier, your last one later). Better might be just determining the degree of suboptimality of deviations, and identifying the contribution of single transitions to this differential. This could be done analogous to determining the elasticity

- of the intrinsic growth rate with respect to changes in age specific vital rates. It has been shown also for industrial societies that changes in the optimal age class have the greatest impact on the overall growth rate, as predicted (Caswell 1989, 133).
- 4) If there are several fitness maxima, measuring the degree of suboptimality may require additional assumptions. Whether stable polymorphism as compared to varying degrees of suboptimality of life courses in human populations is a frequent situation, must be decided empirically.

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