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ABSTRACT

Identification of Network Externalities in Markets for Non-Durables

by Michal Grajek[1]

This paper introduces a structural econometric model of consumer demand for non-durable goods, which exhibits network externalities. The structural model allows us to identify the parameters, which determine the strength of the externalities in the underlying economic model from the empirical estimation results. The estimates of these parameters can then be employed to test the economic significance of the externalities and the compatibility of networks. The identifying assumption that drives our results is that consumers care about the lagged instead of the current network size. We argue that it does not necessarily bound their rationality. To complete our structural model, we provide an example of functional specification that yields a simple linear stochastic model of demand. Using this functional specification, we identify all structural parameters of the model. In the end, the estimation and the stochastic structure of the resulting econometric model are discussed.

Keywords: Structural Econometric Model, Network Externalities, Innovation Diffusion

JEL Classification: C51, D12

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ZUSAMMENFASSUNG

Identifikation der Netzwerkeffekten in den Märkten für nicht-dauerhafte Güter

Der vorliegende Beitrag stellt ein strukturelles ökonometrisches Modell der Konsumnachfrage für nicht-dauerhafte Güter mit externen Netzwerkeffekten vor. Das strukturelle Modell lässt uns die Parameter von Netzwerkeffekten im zugrunde liegenden ökonomischen Modell empirisch zu identifizieren. Die Schätzer der Strukturparameter könnten für das Testen der Netzwerkkompatibilität und der ökonomischen Signifikanz der Netzwerkeffekte verwendet werden. Für die Identifikation nehmen wir an, dass die Konsumenten die Netzwerksgröße verzögert wahrnehmen. Wir argumentieren, dass diese Annahme nicht notwendigerweise mit irrationalem Verhalten gleichzusetzen ist. Um das strukturelle Modell zu vollständigen, geben wir eine funktionale Spezifikation, aus der ein lineares stochastisches Nachfragemodell folgt. Unter Verwendung dieser Spezifikation sind alle Strukturparameter von dem Modell identifiziert. Zum Schluss diskutieren wir die Schätzung und die stochastische Struktur des sich ergebenden ökonometrischen Modells.

1. Introduction

This paper introduces a structural econometric model of consumer demand for non-durable goods or services exhibiting network externalities. Its main contribution is that it allows us to identify structural parameters that determine the extent of externalities in the underlying economic model from the empirical estimation results. The structural parameters' estimates can be employed in turn to test the economic significance of the externalities and the compatibility of networks. The structure that we derive is mainly suited to deal with direct network externalities, e.g. as in telecommunication services. However, one could also think of it as of a reduced form arising from indirect network externalities. E.g. in the case of experience goods, the installed base of consumers could matter, if they transmit information about quality of the good.

Generally, positive network externalities mean that utility, which users derive from consumption of a given good or service, increases with the number of other users.[1] The modern economic literature usually distinguishes two major types: direct and indirect network externalities (see e.g. Katz and Shapiro 1985, 1994; Economides and White 1994; Economides 1996). The first one is related to physical networks, e.g. supported by telecommunication technologies like telephone, telegraph, facsimile or e-mail. Clearly, the utility, which consumers derive from using any of these technologies, depends on the number of other users. The most obvious reason for a positive dependence is that a larger network allows consumers to satisfy more communication needs. The other reason might be the bandwagon effect, which arises because conspicuous consumption gives rise to a conformist behavior as argued by Leibenstein (1950). Blonski (2002) considers another explanation in the context of telecommunication market with competing networks. In his model, it is cheaper for consumers to call within their network, since network suppliers charge access fees for the calls from outside into their networks. As a consequence, consumers benefit from a larger network, because it implies a lower monthly bill, hence endogenous network externalities arise. A negative dependence between network size and utility, which consumers derive from network good, might be justified by congestion or by non-conformism of consumers.

In turn, in a typical virtual network the externality is indirect and comes from the hardware/software paradigm (see Katz and Shapiro, 1985). It applies when a good consists of two complementary components: hardware, which is durable, and software, which exhibits

[1] Throughout the paper, the term network externalities is used interchangeably with the term network effects. The difference between the two is that in addition to network effects, network externalities imply also a market failure (see Liebowitz and Margolis, 1994). In the context of our paper this might depend on interpretation of the economic model.

supply-side economies of scale. The number of users of a given hardware is relevant since it determines the size of the market for software and influences positively its variety and quality, hence enhancing the utility from using that hardware. This way of reasoning may be applied to computer operating systems, credit cards, video recorders, phonograph equipment etc.

The main difficulty, which our structural econometric model has to overcome, is the multiplicity of equilibria, a common result in theoretical studies of markets featured by network externality. In a one-shot (or static) setting, multiple equilibria are due to coordination problems (see Farrell and Klemperer, 2001, pp. 47-50). The simplest example, with one pure network good may be found in Economides and Himmelberg (1995).[2] They show that consumers' expectations of no network good provision as well as positive levels of the network good sales at a given non-negative price may actually be self-fulfilling equilibrium outcomes.

It is also a common wisdom that network externalities could give rise to some sort of S-shaped diffusion of network good's sales over time. Multiple steady states in dynamic models of demand with network externalities are analogous to multiple static equilibria. Switching from the low steady state to the high one can be seen as network diffusion (see Cabral, 1990). Since there are infinitely many diffusion paths, which are supported by fulfilled consumers' expectations, the question of interest is when and how fast the diffusion occurs. Cabral (1990) addresses this question in a perfectly competitive setting with one network good. As an equilibrium selection rule, he introduces lagged instead of expected network size into consumers' willingness-to-pay function. By doing so, he obtains a unique network diffusion path. This even holds when the lag length is infinitely small, in which case, consumers are claimed to be rational. A drawback of the model is that the infinitely small lag causes at the same time a discontinuous jump in the equilibrium network diffusion path. In other words, the "rational" diffusion process is infinitely fast. Being aware of this counterfactual feature, Cabral (1990) argues that the discontinuous diffusion path in his model can be treated as an approximation to the empirically observed S-shaped diffusion. In fact, assuming small but non-zero perception lag yields such result.

Economides and Himmelberg (1995) propose another solution in the context of perfectly competitive market. The unique network diffusion path in their model results from the assumption that supply of the network good is finitely elastic in the sense that the marginal cost function depends positively on the derivative of network size with respect to time. In other words, the change of network size is costly and these additional (over marginal)

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[2] However, seminal works are Rohlfs (1974) and Katz and Shapiro (1985)

costs are passed through to consumers. In this model consumers form expectations and the expectations are fulfilled along the equilibrium network diffusion path. Moreover, the assumption about finitely elastic supply resolves the discontinuity problem discussed above. This is because by construction of the marginal cost function, a discontinuous diffusion would imply an infinitely high price.

In this paper we take the former approach. Following Cabral (1990), we assume that consumers care about the lagged network size in their decision about buying the network good (joining the network). This approach is very appealing from an empirical perspective. First, as we will show, it allows us to identify the structural parameters from the estimation results. Economides and Himmelberg (1995) fail to prove that one can do this in their model. Second, the use of lagged dependent variable is common in econometric practice.

The limitation of the approach we follow is that consumers are rational only when the lag length tends to zero, at least in our simple setup.[3] If the lag becomes larger, as it is the case with empirical data time series, consumers do not consider that during the diffusion process the network grows in current period. Another implication of the model is that empirical magnitude of network effect depends to some extent on lag length, hence on data frequency. This is because the stronger network effects and more frequent updating of the network size both speed up the diffusion.

The model, which we derive, is also closely related to the marketing literature on diffusion of innovations. In seminal work of Bass (1969), a structural econometric model of new product diffusion is developed, which is driven solely by the diffusion of awareness of this product. The striking feature of the original Bass' (1969) model is that price does not influence the diffusion. The marketing scientist recognized that puzzle and developed many extended models with the price incorporated (e.g. Horsky, 1990; Jain and Rao, 1990; Bass, Krishnan and Jain ,1994). Our structural econometric model is not an extension of the seminal Bass' (1969) model. We use the theory of network externalities instead of imperfect information in order to facilitate diffusion. However, the equation to estimate in our example coincides with the equation proposed in Bass (1969), except the additional price variable, which is in there in our model.

So far, there is little empirical work that measures the consumption network externality. Some works follow the reduced form approach looking for an empirical evidence for network effects. Greenstein (1993) conducts the first research in that stream. He shows

[3] E.g. in the case of experience goods the lag might be justified, because the network externalities arise through information transmition, which takes time. Also in the case of direct network externalities, one might argue that the lag results from costly updating of information about the network size.

that compatibility with the installed base matters in the choice of the mainframe computer system. Gandal (1994, 1995) estimates hedonic price equations for spreadsheets and data base management systems and finds that consumers are willing to pay significant premium for software supporting a common file compatibility standard. This result is in line with the hypothesis that the software markets exhibit network externalities. Similar findings report Brynjolfsson and Kemerer (1996). Additionally, they find that a product's installed base increases the price of spreadsheets. Gandal, Greenstein and Salant (1999) show the two-way positive feedback between different components in competing microcomputer systems by means of vector autoregressive (VAR) analysis. In this way, they prove empirically the indirect network externality hypothesis. Dranove and Gandal (2000) study the effect of DIVX preannouncement in the DVD market. They find that the preannouncement indeed slowed down the adoption of DVD technology as predicted by the theory of competition with network externalities.

Economides and Himmelberg (1995) conduct the pioneering study that goes in the direction of structural econometric modeling. In the economic part of their paper they derive a dynamic model of a perfectly competitive market with consumption network externalities. The possible multiplicity of equilibrium network diffusion paths under fulfilled consumers' expectations is solved by the assumption of finitely elastic supply, as described above. In the empirical part they estimate the demand for facsimiles in the U.S. over 1978-1991. The assumption that facilitates the estimation is that expected network size is a linear function of the past network size. Fulfilled expectations would then lead to a constant growth rate of the U.S. fax network, which is counterfactual and breaks the consistency of that structural model. Another structural econometric work concerning network externalities includes Gandal, Kende, and Rob (2000) for the CD industry and Rysmann (2002) for the Yellow Pages market. These authors concentrate however on the indirect network effect and estimate two interrelated demand equations, for software and hardware. In this way, they model the complementarities between software and hardware in full instead of putting the network size into the consumers' utility function as in the direct network externality case.

The paper is organized as follows. Section 2 introduces the economic model, which yields the structure for the empirical investigation. Section 3 gives an example of the functional specification that leads to a simple stochastic model and discusses the identification and interpretation of the structural parameters. Section 4 concludes.

2. The Economic Model

2.1. Willingness-to-Pay Function

The demand model we use is a partial equilibrium, discrete choice, dynamic model. The good being considered is non-durable, *ex ante* homogenous[4] and subject to network externalities. We refer to the good supplied by different firms as brands. A consumer's willingness to pay for a given brand is influenced by her type and by the network size of that brand. We refer to the network as a set of subscribers and to the purchase of the non-durable network good as subscribing to the network.

Denote by i ($i = 1, 2, ..., I$) the brand of the homogenous good and assume that there is a measure one of infinitely living consumers, each demanding at most one unit of the good. Consumer v's preference for brand i at time t is represented by the instantaneous willingness-to-pay function $u(v,x_i(t-\delta))$, where v is the individual preference parameter, $x_i(t-\delta)$ is the lagged network size of brand i and the perception lag δ in an arbitrary number. Formally, we assume that the individual preference parameter v is distributed over the interval [0,1] according to the cumulative density function $F(v)$, and that $u(v,x_i(t-\delta))$ is strictly increasing and continuous in v. By construction, the parameter v establishes a rank ordering of the consumers according to their willingness to pay. We assume that the ranking is invariant with respect to changes in $x_i(t-\delta)$. As a matter of convention, the higher v is, the larger is the benefit of using each network. Network externalities are captured by the dependence of each consumer's willingness to pay on the network size $x_i(t-\delta)$.

Introduction of the lagged network size $x_i(t-\delta)$ into the willingness-to-pay function is crucial to our model. As pointed out by Cabral (1990), it is an equilibrium selection device that gives us the unique diffusion path of each network i. However, there appears a natural concern about consumer's rationality in this setting. Cabral (1990) argues that if the lag δ is infinitely small the consumers are rational. This is because their subscription decisions are identical to the ones done by forward-looking consumers. However, the construction of willingness-to-pay function does not allow them to coordinate in order to switch from the low steady state to the high one, when both are feasible. Section 2.5 explains these findings in detail.

From an empirical perspective, the lagged network size in the willingness-to-pay function corresponds to the lagged dependent variable in the estimated equation. Obviously, the lagged dependent variable is easier to work with than with some unobserved expectations of the consumers. The cost of this approach is that we have to give up the rationality of the consumers with respect to the network size in most of the cases. This is because the minimum lag length is naturally defined by the frequency of the data and is usually large. Consequently the approximation of the rational consumers' behavior is poor.

2.2. Subscription Demand

Each brand i has its network constituting of a set of subscribers. If the brands are incompatible, each makes up its own network so $x_i(t-\delta) = y_i(t-\delta)$, where $y_i(t-\delta)$ stands for the normalized sales of brand i (the number of subscribers to brand i). However, if the brands are perfectly compatible then the network is common, which is given by total sales of all brands $x_i(t-\delta) = \sum_{j=1}^{I} y_j(t-\delta)$. By homogeneity, brands with identical network size (in particular compatible brands) are perceived by consumers as perfect substitutes.

In a more general setting, partial compatibility may prevail. In this case, the network size of a brand is a weighted sum of its own and all other subscribers. Under symmetry assumption, we could write it as $x_i(t-\delta) = y_i(t-\delta) + w\sum_{j \neq i} y_j(t-\delta)$, where $w \in [0,1]$ measures the degree of compatibility. $w = 1$ and $w = 0$ correspond to the perfect compatibility and perfect incompatibility respectively and the interior values of w indicate a partial compatibility.

In each instance of time, consumer v decides to buy one of the brands or to stay out of the market in order to maximize her net utility

(1) $u(v,x_i(t-\delta)) - p_i(t)$

If (1) is negative for all brands, than she will not join any of them. This "static" decision rule in our dynamic model is appropriate, as we focus on non-durable goods. In the context of telecommunication service this would mean that consumers could initiate or relinquish their subscription costlessly.

The consumer for whom (1) equals zero is indifferent between subscribing to and staying out of a given network. Denote $v_{i,t}^* = v^*(x_i(t-\delta),p_i(t))$ the type of the indifferent consumer with respect to brand i in time t. $v_{i,t}^*$ can be obtained from

(2) $u(v_{i,t}^*,x_i(t-\delta)) = p_i(t)$.

The brand i for which $v_{i,t}^*$ is the lowest is the most attractive brand for all subscribers in time t. Define

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[4] By *ex ante* homogeneity we mean that different brands of the good are perceived as intrinsically equal. However, the difference in their valuation is possible *ex post*, when they have different network sizes.

(3) $v_{L,t}^{*} = \min_{i}\{v_{1,t}^{*}, v_{2,t}^{*}, \ldots, v_{I,t}^{*}\}.$

By construction, all consumers with higher preference parameter than $v_{L,t}^{*}$ buy the good. If $v_{i,t}^{*}$ is equal among some brands, then the subscribers choose among them with equal probability. Define

(4) $$H_i(\mathbf{v_t}^{*}) \equiv \begin{cases} \dfrac{1 - F(v_{L,t}^{*})}{I_{L,t}} & \text{if} \quad v_{i,t}^{*} = v_{L,t}^{*} \\ 0 & \text{otherwise} \end{cases},$$

where $\mathbf{v_t}^{*} = (v_{1,t}^{*}, v_{2,t}^{*}, \ldots, v_{I,t}^{*})$ is a vector of the indifferent types with respect to brand i in time t, $I_{L,t}$ is the number of brands for which $v_{i,t}^{*} = v_{L,t}^{*}$ and F is the distribution function of v. H_i equals the number of the consumers willing to buy brand i in time t. Now, the state equations, which describe the evolution of each brand's sales over time, are given by

(5) $y_i(t) = H_i(\mathbf{v_t}^{*})$.

In the steady-state (given that all prices stay constant) we expect that none of the consumers can increase her utility by changing the subscription decision, so each brand's sales stay constant over time

(6) $y_i(t) = y_i(t-\delta)$.

It is worth noting that the steady-state equilibrium demand of the above model coincides with the standard static model equilibrium with fulfilled consumers' expectations (see Rohlfs, 1974 and Economides and Himmelberg, 1995). Moreover, the process of achieving equilibrium, which we have described formally, is in line with the logic presented by Rohlfs (1974).

2.3. Switching Costs

The above model of demand with network externalities is probably the most obvious extension of the Cabral (1990) single brand model. It possesses however some unnatural features. One of them is a particular symmetry. In each instance of time, every active firm has an equal number of subscribers $y_i(t)$, which stays in contrast to the observation that real firms' market shares exhibit persistent differences.

The other feature corresponds to the Bertrand's paradox. If one firm undercuts the others just a little bit it wins immediately the whole market. This creates a strong incentive to undercut and results in fierce price competition. Moreover, with incompatibility, it is extremely difficult to recoup market shares once a firm lost its customers. This is because without installed base it needs to offer far more attractive price then the rival, which just won the whole market. In that case the Bertrand's paradox is even stronger and the market outcome is extremely tippy.

Switching costs offer a solution to the problems mentioned above and are particularly relevant to network markets. In fact, network externalities and switching costs are closely related to each other (see Farrell and Klemperer, 2001).

Suppose, switching costs are high enough, such that having bought one brand, consumers will never find it optimal to switch to another one later on. The type of switching costs we have in mind can be observed in mobile telecommunication markets. That is, consumers have to pay a penalty for premature cancellation of a long-term contract. As a security option, they have however the right to relinquish the subscription without any penalty when the firm raises the price. In other words, once the price goes up the switching costs are gone.

The introduction of switching costs of this kind changes the subscription demand described in the previous section to the extent that only the unattached consumers can feed the diffusion of the networks. So, we can rewrite (4) as

$$(7) \qquad H_i'(\mathbf{v}_t^*, \mathbf{v}_{t-\delta}^*) \equiv \begin{cases} \dfrac{F(v_{L,t-\delta}^*) - F(v_{L,t}^*)}{I_{L,t}} & \text{if} \quad v_{i,t}^* = v_{L,t}^* \\ 0 & \text{otherwise} \end{cases}.$$

Now, H_i' equals the number of the *new* consumers willing to buy the brand i in time t. Accordingly, the state equations are given by

$$(8) \qquad y_i(t) = H_i'(\mathbf{v}_t^*, \mathbf{v}_{t-\delta}^*) + y_i(t-\delta).$$

This demand specification allows for persisting differences in market shares of different brands. In particular, the incumbent's installed base of consumers constitutes a persisting competitive advantage over the entrant.

Together with switching costs we introduce new issues concerning the pricing by the firms. First of all, remember that the specification in (7) remains valid until the prices go up. Otherwise, we are back to the set-up without switching costs as in (4). The switching costs will change also the price setting itself. We discuss that to some extent in the next section.

2.4. Supply of the Network Good

To complete the economic model of the market we would need to model how the prices are determined. Since the paper focuses on the demand side of the market and, in particular, on identification of the network effects, we do not introduce a structure for the supply side. Instead, we discuss some possibilities of extending the economic model to contain the explicit pricing relation as well.

In the simplest case without switching costs, we could plausibly assume that fierce price competition drives prices down to marginal costs. If firms are symmetric regarding their production technology, prices of all brands will be equal and their changes over time will reflect some technological progress and/or economies of scale. As a consequence, market outcomes will be completely symmetric. If the firms start their activity simultaneously with zero network size their brands will remain equally attractive for the consumers. In other words, expression (1) will be equal for all i. No firm will drop out of the market ($I_{L,t} = I$) and their networks will grow (or decline) equally fast. This set of assumptions facilitates static, marginal-costs pricing relation, which is well established in the empirical Industrial Organization literature (see Bresnahan, 1989).

Once we introduce switching costs, a space for strategic pricing emerges. Indeed, switching costs in our set up tend to reduce competition and give firms the opportunity to earn some mark-ups.[5] In that case, firms face a trade-off. On the one hand, they want to keep prices high in order to exploit the installed base of consumers. On the other hand, they want to lower prices to attract new subscribers, i.e. to enhance the installed base in the future. Static, marginal-costs pricing is no longer appropriate for modeling this kind of pricing behavior, as it does not take account of this trade-off. Instead, state-space games, in which actions taken in one period shift payoffs in subsequent periods, could be utilized with installed bases of firms as natural state variables (see Basar and Olsder, 1999). Examples of empirical dynamic pricing models within this framework have been developed in the learning-by-doing literature (e.g. Jarmin, 1994). In these models, cumulative past sales benefit firms in that they lower production costs, what gives rise to a similar trade-off as with network externalities and switching costs.

From an econometric perspective, we do not necessarily need structure for supply relation to be able to correctly estimate the network externalities parameters. Endogeneity problems regarding the price variable can be resolved by instrumental variable technique.

2.5. Dynamics of the Network Good Adoption

To get the intuition of dynamics of the network good adoption, a graphical analysis is useful. For simplicity of the presentation we abstract from switching costs and assume the price to be equal across brands, as with perfect competition, and constant over time. We

[5] See Klemperer (1995) for an overview of switching costs impact on competition and prices.

present the common (compatible) network dynamics, so the subscripts i are omitted throughout this section. In other words, there is one network and one competitive price in this exercise. Because of the symmetry on the supply side, the evolution of the subscriber sets of particular brands is proportional to the common network evolution. To further simplify matters we assume also that the cumulative density function $F(v)$ and the willingness-to-pay function $u(v, x(t\text{-}\delta))$ are continuously differentiable in all arguments.

Detailed mathematical treatment of the equilibrium network size path in such model can be found in Cabral (1990). The author proves there that if networks externalities are strong and the lag length δ tends to zero the equilibrium adoption path is unique and discontinuous.

The equilibrium adoption path is described by equation (5). It says, that the function H maps the network size from time $t\text{-}\delta$ to t. Given the assumptions of this section, (5) simplifies to

(9) $x(t) = H(v_t^*) \equiv 1 - F(v_t^*)$

To gain intuition on how network externalities and price affect diffusion we calculate the derivatives of H with respect to the lagged network size $x(t\text{-}\delta)$ and price p in the appendix. Since H maps the network size from time $t\text{-}\delta$ to t, it is convenient to think of it as of a function of the lagged network size $x(t\text{-}\delta)$. Examination of (A.11) in the appendix leads to the following lemma

> **Lemma 1**: Whenever the solution to equation (2) exists, i.e. v_t^* is defined, and the density $f(v_t^*)$ is strictly positive, the extent of network externalities measured by $\frac{\partial u(v_t^*, x(t-\delta))}{\partial x(t-\delta)}$ determine the slope of the function H in the $x(t\text{-}\delta)$ domain, such that
>
> (i) H is non-decreasing if and only if network externalities are non-negative,
>
> (ii) the slope of H equals zero if there are no network externalities, and
>
> (iii) the slope of H is larger if network externalities are stronger, other things being equal.

Figure 1 illustrates dynamics of the network good adoption. In its upper part we draw H as the function of the lagged network size $x(t\text{-}\delta)$. Lemma 1 formalizes the link between the extent of network externalities and the slope of H, which in turn determines the dynamics of diffusion. The lower part of figure 1 shows the steady-state equilibria of the network size for each price p, denoted $D(p)$. As it has been already mentioned, it coincides with the analogous static model equilibria with fulfilled consumers' expectations. Dynamic model allows, however, to discriminate among multiple steady-state equilibria. Suppose for the moment that the market price is p^* such as in the figure 1. Then according to the state equation (9) the network size will evolve in the way the upper part of the figure 1 indicates. If it starts at some size smaller than x_1 it will eventually reach x_0, if the starting network size is bigger than x_1 it will end up in x_2. If the network size for some reason equals exactly x_1, it will stay there, but any arbitrarily small shock will lead to equilibrium at x_0 or x_2. Therefore we can conclude that x_0 and x_2 are stable steady states, while x_1 is unstable. To apply this way of reasoning to any price p consider the following lemma

> **Lemma 2**: Whenever the solution to equation (2) exists, i.e. v_t^* is defined, and the density $f(v_t^*)$ is strictly positive, changes in price p determine the shifts of the function H in the $x(t-\delta)$ domain, such that $H(v^*(x(t-\delta),p_1)>H(v^*(x(t-\delta),p_2)$ for every $x(t-\delta)$ if $p_1<p_2$.

Lemma 2 follows directly from examination of (A.7) in the appendix. It says that lowering the price shifts the function H up, although it does not need to be a parallel shift. Drawing steady states for each price yields the steady-state demand $D(p)$. We can conclude that

> **Theorem 1**: downward-sloping parts of the steady-state demand $D(p)$ consist of stable equilibria, while the upward-sloping parts are unstable, i.e. consist of critical-mass points.

Now, consider a case when price changes over time instead of being a constant. To see how the common network evolves let the price $p(t)$ be a continuous and decreasing function of time and let $p(0)>p_h$ (as in figure 1) and $x(p(0))$ be the unique steady-state network size given $p(0)$. As time passes and the price falls, the network size follows the lower steady-state size. Eventually the price reaches p_l and just after that the network size jumps discontinuously to the higher steady-state and grows further on along it. Formally, this diffusion pattern has been shown to be correct for infinitely small lag δ in Cabral (1990).

If the perception lag is strictly positive, the consumers are myopic with respect to the network size. They do not recognize that the network is going to grow in the current period. As a consequence, the equilibrium network size does not follow exactly but rather tends to the steady-state size. There is no discontinuous jump in the network diffusion either. Instead, the diffusion pattern takes an S-shape.

The dynamic perspective helps to understand the equilibrium selection rule assumed implicitly in our model by the lag structure. It does not allow for coordination among the consumers. Note, that it would be Pareto optimal to jump to the larger steady-state network size before price falls under p_l. However, this would require the coordination of the consumers' subscription decision in order to reach at least the critical mass.

Another insight drawn from the analysis is a sort of substitutability between the extent of network externalities and the lag length, which is described by the following theorem

Theorem 2: Network externalities, which extent is measured by $\frac{\partial u(v_t*, x(t-\delta))}{\partial x(t-\delta)}$ and the perception lag of length δ are substitutes in the sense that both strenghtening of network externalities and shortening of the lag length speed up adoption of the network good.

The arrows in the upper part of the figure 1 indicate the change of the network size from time *t-δ* to time *t*. The length of these arrows reflects the speed of the network size growth (or decline). Now, strengthening the network effects, which implies according to lemma 1 a larger slope of the function *H*, and lowering the lag length (say to *δ/2*, so between time *t-δ* and *t* there are two "updates" of the network size) one can achieve the same network size growth. In other words, a large extent of network externalities together with a large perception lag may result in the same network diffusion speed as a small extent of externalities and a small lag. One should keep in mind this substitutability when interpreting empirical results. On the other hand, however, manipulating the lag does not influence the steady-state equilibria (the fixed points of the function *H*), while strengthening of network externalities does. This observation will to be important for the empirical identification of the network effects' strength.

3. The Stochastic Model

3.1. Functional Specification

The next step towards the structural econometric model is to specify the functional forms in the underlying economic model. This section proposes an example of such specification. It has been chosen because of two reasons. First, the specification yields the demand relation as a simple linear equation (in parameters), which is convenient to work with empirically. Second, the demand relation nests the well-established Bass' (1969) diffusion model.

Assume the consumers' willingness-to-pay function to be

(10) $u(v, x_i(t-\delta)) = av + bx_i(t-\delta) + cx_i^2(t-\delta),$

where *a, b* and *c* are parameters. As before, $x_i(.)$ denotes the network size and *v* the consumer type. This specification implies that a network of size zero has no other than intrinsic value, which is proportional to the consumer preference parameter *v*. Network size enters additively into the utility function, which means that consumers are homogenous in their valuation of the network. The square function of the network size catches its non-linear influence on the willingness to pay, e.g. diminishing positive marginal network effect, which is usually assumed in the literature.

The distribution of the individual preference parameters (consumers' types) *v* is assumed to be uniform on the support [0,1], hence $F(v) = v$ on that support. This distributional assumption corresponds to the linear demand function when the network size is fixed. As pointed out by Economides and Himmelberg (1995), the distribution of types is an a priori assumption, on which the identification of network effects in data critically depends. In that sense the uniform distribution of types is not very fortunate, because it attaches significant proportion of the diffusion S-shape to the network effect arbitrarily. However, we can modify the interpretation of the network externalities parameters slightly in order to incorporate some of the distributional effects. The section 3.4 discusses that issue in more details.

Given all the functional assumptions, we can calculate the index of the indifferent consumer with respect to each brand *i* from the equation (2)

(11) $$v^*(x_i(t-\delta), p_i(t)) = \frac{1}{a}p_i(t) - \frac{b}{a}x_i(t-\delta) - \frac{c}{a}x_i^2(t-\delta).$$

The subscription demand of each brand at time *t* (the state equations) can be obtained from the equations (3)-(5)

(12) $$y_i(t) = \frac{1}{I_{L,t}}\left(1 - \frac{1}{a}p_i(t) + \frac{b}{a}x_i(t-\delta) + \frac{c}{a}x_i^2(t-\delta)\right).$$

To get single demand equation (12) instead of a switching regime, we assume, that without switching costs the competitive pressure drives the prices down to the marginal costs, so the firms that survive in the market are endowed with the same production technology and set equal prices. This demand relation might be also relevant for the single brand market ($I_{L,t} = 1$). The price then would be of course different from marginal cost.

In the economic model we assumed that there is a measure one of consumers in the market. To be consistent with data we enhance the market to m consumers and call it the market potential[6]. To get actual network size values instead of normalized ones multiply both sides of (12) by m

$$(13)\quad Y_i(t)=\frac{1}{I_{L,t}}\left(m-\frac{m}{a}p_i(t)+\frac{b}{a}X_i(t-\delta)+\frac{c}{am}X_i^{\,2}(t-\delta)\right),$$

where $Y_i(t) = my_i(t)$ and $X_i(t\text{-}\delta) = mx_i(t\text{-}\delta)$.

When the switching costs are consider we can still use the indifferent consumer indexes (11), but the subscription demand (actual, not normalized) obtained now from (3), (7) and (8) becomes

$$(14)\quad \Delta Y_i(t)=\frac{1}{I_{L,t}}\left(-\frac{m}{a}\Delta p_i(t)+\frac{b}{a}\Delta X_i(t-\delta)+\frac{c}{am}\Delta X_i^{\,2}(t-\delta)\right),$$

where $\Delta Y_i(t) = Y_i(t) - Y_i(t\text{-}\delta)$, $\Delta X_i(t\text{-}\delta) = X_i(t\text{-}\delta) - X_i(t\text{-}2\delta)$ and $\Delta X_i^2(t\text{-}\delta) = X_i^2(t\text{-}\delta) - X_i^2(t\text{-}2\delta)$. Again, to simplify matters we assume that the firms keep equal hedonic (i.e. adjusted for the network size) prices all the time. In other words, they compete for the new subscribers continuously. In principle, it would be also possible that they price low and high interchangeably. So that there would be periods over which one firm attracts the new subscribers and the other extracts a rent from the installed base and periods over which the roles are reversed. Actually, such consecutive pricing pattern is found in Farrell and Shapiro (1988), but it hinges rather on particular assumptions of their model[7].

3.2. Identification

Since data is in discrete time, we need the analogues of (13) and (14) for estimation purposes. Additionally, we let some stochastic noise enter the equations. This yields

$$(15)\quad Y_{i,t}=\frac{1}{I_{L,t}}\left(\alpha+\beta p_{i,t}+\gamma_1 X_{i,t-1}+\gamma_2 X_{i,t-1}^2\right)+\xi_{i,t} \quad \text{and}$$

$$(16)\quad \Delta Y_{i,t}=\frac{1}{I_{L,t}}\left(\beta\Delta p_{i,t}+\gamma_1\Delta X_{i,t-1}+\gamma_2\Delta X_{i,t-1}^2\right)+\zeta_{i,t}$$

respectively, where $Y_{i,t}$ is the discrete analogue of $Y_i(t) = my_i(t)$, which is the number of brand i's customers in time t and $X_{i,t\text{-}1}$ is the analogue of $X_i(t\text{-}\delta)$, the lagged network size of brand i. $\xi_{i,t}$ and $\zeta_{i,t}$ stand for the error terms and reflect the stochastic noise in the data. Because of the

[6] The market potential differs in our formulation from the market potential in Bass (1969) in that it does not depend on price.

[7] See the discussion in Klemperer (1995).

lagged dependent variables the stochastic structure of these equations might be quite complex. We introduce it in more details in the section 3.3.

All four structural parameters *a, b, c*, and *m* in (13) ($I_{L,t}$ is observable from the market structure) are uniquely identified from the estimates of (15). Simple algebra yields the scaling parameter $a = -\alpha/\beta$, the network externalities parameters $b = -\alpha\gamma_1/\beta$ and $c = -\alpha^2\gamma_2/\beta$, as well as the market potential $m = \alpha$.

In the case with switching costs we need some more manipulations. One cannot recover all the structural parameters from the estimates of (16) directly. This is because in contrast to (15), the equation (16) is expressed in terms of differences. By differentiating we loose the constant term, so there are only three parameter estimates with four structural parameters to identify. To solve this problem we need to write the sales equation in terms of levels, which yields

$$(17)\quad Y_{i,t} = \alpha_i E_t + \frac{1}{I_{L,t}}\left(\alpha + \beta p_{i,t} + \gamma_1 X_{i,t-1} + \gamma_2 X_{i,t-1}^2\right) + \psi_{i,t}$$

where α_i is a firm specific constant. E_t is a dummy variable indicating new entry, which is equal to zero in the periods prior to entry and one otherwise. This result is formally derived in the appendix for a general case, i.e. without any functional assumptions. The intuition for this is as follows. Remember, that we have assumed equal hedonic prices among firms each period. Given this assumption, it follows from our economic structure that all active firms attract equal number of new subscribers each period. The only possible source of sustaining differences in total sales is a new entry. Because of the switching costs, the installed base of the incumbent constitutes the competitive advantage over the entrants. This advantage (or disadvantage in case of the entrants) is summarized by the firm specific constants α_i. The formulation in (17) indicates that the entry happened only once. It is straightforward to extend it to multiple entries.

Now, we are able to identify all four structural parameters under switching costs as well. The same formulas as before (without switching costs) applied to the estimates of (17) yield the desired results. Note, that the interpretation of the parameters in (17) is the same as in (15). Indeed, as it is shown in the appendix, the sum of firm-specific constants equals zero. Switching costs do not influence the market potential or the network effects in our model. They simply allow for persisting asymmetries among firms expressed by nonzero firm specific constants α_i.

Equation (17) is convenient also because it nests the two regimes, with and without switching costs. In particular, when the firm specific constants α_i are zero, the sales of the firms are equal, and the equation (17) boils down to (15), which describes the evolution of sales under no switching costs. However, to get the simple sales equation under switching costs we assumed previously that the prices did not rise. Since we do not need that additional assumption under no switching costs, the validity of the equation (15) is slightly less restrained. That is why we decided to keep the two cases separately.

Now, let us turn to the question of compatibility of the brands. Our structure allows us to investigate the compatibility, that is to check to what extent the network externality operates at the industry level. To test the hypothesis of compatibility empirically, we let

(18) $X_{i,t\text{-}1} = Y_{i,t\text{-}1} + wY_{j,t\text{-}1}$,

where $Y_{j,t-1} = \sum_{k \neq i} Y_{k,t-1}$ is the sum of all other brands' customers in time t-1 and $w \in [0,1]$ measures the degree of compatibility, as described in the section 2.2. Then (17) becomes

(19) $$Y_{i,t} = \alpha_i E_t + \frac{1}{I_{L,t}}\left(\alpha + \beta p_{i,t} + \gamma_1 Y_{i,t-1} + \gamma_{11} Y_{j,t-1} + \gamma_2 Y_{i,t-1}^2 + \gamma_{21} Y_{i,t-1} Y_{j,t-1} + \gamma_{22} Y_{j,t-1}^2\right) + \psi_{i,t}\,.$$

The identification of the structural parameters a, b, c, and m remains unchanged, since the estimates of α, β, γ_1, and γ_2 are still available there in (19). The new structural parameter w is however overidentified, because there are three new parameters γ_{11}, γ_{21} and γ_{22} in the equation (19). It can be recovered from $w = \gamma_{11}/\gamma_1$, from $w = \gamma_{21}/2\gamma_2$, and from $w^2 = \gamma_{22}/\gamma_2$. It follows that when the externality operates at the firm level only (incompatible networks, $w = 0$) we expect the estimates of γ_{11}, γ_{21} and γ_{22} to be zero. In the polar case, when the externality operates at the industry level (fully compatible networks, $w = 1$), we expect $\gamma_{11} = \gamma_1$, $\gamma_{22} = \gamma_2$ and $\gamma_{21} = 2\gamma_2$. All the intermediate cases with partial compatibility can be easily obtained from the three equalities as well.

The overidentification of the structural compatibility parameter w gives a scope for a specification test. All three equalities identifying w must hold, otherwise there is something wrong with our model. Either the estimates are not correct, or data reject the structure.

On the other hand, we could use the overidentification to introduce parameter restrictions and to save on degrees of freedom. For example, recover w from $w = \gamma_{11}/\gamma_1$ and impose $\gamma_{21} = 2\gamma_{11}\gamma_2/\gamma_1$ and $\gamma_{22} = (\gamma_{11}/\gamma_1)^2\gamma_2$. So we could estimate five instead of seven parameters in (19).

Last, as mentioned at the beginning of this section our structure corresponds to the information diffusion models widely studied in the marketing science. In particular, equation (19) nests the original diffusion equation proposed by Bass (1969) for the single product case. When we consider single brand diffusion, (19) simplifies to the original Bass model if $\beta = 0$ (i.e. price does not matter for the network diffusion).

3.3. Stochastic Structure

The final step in the structural econometric model is to introduce the stochastic structure. So far, we have not imposed any assumptions on the error terms in (15), (16), (17), and (19). We have not proposed any estimation technique either. This is because it may be far less trivial than the simple, linear in parameters functional form of these equations suggests.

To illustrate the potential econometric pitfalls let us consider the market with network externalities operating at the industry level (fully compatible networks) and two competing brands. The equation (19) becomes then

$$(20)\quad Y_{i,t} = \frac{1}{2}\left(\alpha + \beta p_{i,t} + \gamma_1 \left(Y_{i,t-1} + Y_{j,t-1}\right) + \gamma_2 \left(Y_{i,t-1} + Y_{j,t-1}\right)^2\right) + \psi_{i,t}\,.$$

The most obvious way the stochastic noise can enter the empirical relation is a measurement error in the dependent variable. This makes sense in our model, since the price is usually easily observable by an econometrician, while the network size might be not. Suppose, that we observe the brand sales with some noise $Y_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t}$ is some i.i.d. measurement error. In the estimation we put the observation with noise into our equation (20) creating the error term of the form

$$(21)\quad \psi_{i,t} = -\varepsilon_{i,t} + \frac{\gamma_1}{2}\left(\varepsilon_{i,t-1} + \varepsilon_{j,t-1}\right) + \frac{\gamma_2}{2}\left(\varepsilon_{i,t-1} + \varepsilon_{j,t-1}\right)^2 + \gamma_2 \left(Y_{i,t-1} + Y_{j,t-1}\right)\left(\varepsilon_{i,t-1} + \varepsilon_{j,t-1}\right),$$

which is clearly not an i.i.d. error. The error structure (21) received some attention in the time series econometrics[8]. The second and third term on the RHS of (21) indicate a multivariate nonlinear moving average. The fourth term points to a multivariate bilinear process. To correctly estimate the structural parameters of this model one needs to take care for the error generating process that is consistent with the assumed structure. A good news is that our economic model gives rise not only to the equations, but to a particular error structure in the econometric model as well. As a consequence, we do not need to rely solely on the statistical procedures to choose the appropriate error structure.

3.4. Interpretation of the Identified Structural Parameters

Interpretation of the identified structural network externalities parameters *b* and *c* directly is difficult because of two reasons. First, as indicated already in the section 3.1, the empirical identification of the network effects relies heavily on the functional assumptions. In particular, the distribution of types plays a key role. Another assumption that influences the

[8] See Granger and Teräsvirta (1993) for a nice overview of the nonlinear time series analysis.

estimates of the network effects is the consumer perception lag δ that we impose by choosing data frequency. Second, even if we have statistically significant and correct estimates of b and c we still miss some threshold, which tells us which values of the parameters correspond to the economically significant network effects.

Going back to the first problem, our empirical estimates of the network effects can be biased because of the functional assumptions. In particular the uniform distribution of types is likely to bias the network estimates upward, i.e. to attach significant proportion of the diffusion S-shape to the network effect arbitrarily. The natural assumption is that the distribution of types mimics the distribution of consumer income, which is usually log-normal[9]. The section 2.5 on the dynamics of network growth helps to understand how any bell shaped distribution of types contributes to the S-shape of the diffusion curve.

In the case of the perception lag the direction of bias is less clear. In general, we can expect that imposing larger (smaller) lag than the actual one creates an upward (downward) bias. But, the question, how large the actual lag is, remains open. The common sense just tells us that using monthly data (hence the one month perception lag) is more appropriate than using yearly data. In the section 2.5 we formalized the intuitive relationship between the lag length and the strength of network effects. We noted also that the steady-state equilibria are not affected by the lag manipulations.

Being aware of the possible bias in our estimates, how can we infer the economic significance of the identified network effects parameters? We propose to calculate the steady-state inverse demand functions from (19) replacing all the parameters with their empirical estimates and imposing the steady-state conditions (6). All the important economic phenomena driven by the network effects, like multiple equilibria and critical mass of adopters, apply to the case with upward sloping demand. Therefore, the existence of the upward sloping part in the empirical steady-state demand function indicates strong network externalities.

The empirical steady-state demand function seems also more robust to the improper functional assumptions than the identified structural parameters themselves. First, the steady-state equilibria are not affected by the lag manipulations. And second, the intuition suggests that attaching some distribution-of-types effects to the network effects should not change dramatically the shape of the steady-state demand function. Therefore, it seems unlikely that we obtain an upward sloping part of the demand function in the estimation, while in fact it is all along downward sloping in the given market.

Some additional information about the source of the network effect in the market under consideration can be obtained from the estimate of the compatibility parameter w. For example, the ability to satisfy more communication needs with the bigger consumers' pool may give rise to direct network externalities, which operate at the industry level (compatible networks) in telecommunication markets. Whereas endogenous externalities as in Blonski (2002), which are created by firms charging an access fee for the calls from outside into their networks, operate at the firm level (incompatible brands).

Last, the estimate of the market potential parameter m may serve as another specification test. Usually, we have a rough guess of the total number of consumers, which could potentially subscribe to a network. Outstanding values of m signal problems with the estimates.

4. Conclusions

This paper introduces a structural econometric model of consumer demand for non-durable goods exhibiting network externalities. Its main contribution is that it allows us to recover the structural parameters responsible for the externalities. The structural parameters' estimates can be employed in turn to test the economic significance of the externalities and the compatibility of networks.

The identifying assumption that drives our results is that the consumers care about the lagged instead of the current network size in their subscription decision. As Cabral (1990) we argue that when the lag is infinitely small this behavior is rational. It does not allow only for coordination among the consumers.

However, an empirical implementation of the model leads naturally (because of the data frequency) to a bigger than infinitely small lag. Then, if we interpret the economic models in terms of direct network externality, consumers are myopic with respect to the network size. In other words, they do not recognize that during the diffusion process the network grows in current period. Instead, they use the previous period network size in their purchase (subscription) decision. The "myopic" diffusion is slower and smoother (does not exhibit discontinuous jump) than "rational" diffusion.

On the other hand, the installed base of users could matter for the reasons other than the direct network externalities. For example, in the experience good case the installed base could transmit the information about the quality of the good enhancing the willingness to pay for it. In this case the lag in the network size has a natural explanation, since the transmission of

[9] Economides and Himmelberg (1995) study the distribution of consumer income in depth to obtain more

information takes time. This reinterpretation of the network effect preserves the rationality of consumers.

Our structure gives also important insights for interpretation of the empirical results. First, as mentioned already in Economides and Himmelberg (1995) the distribution of types is an important a priori assumption that the identification of network effects in data relies on. Second, the lag induced naturally by data frequency influences the empirically identified network effects too. The strength of the network effects and the lag length are in a sense substitutes in generating the diffusion S-shape.

We provide an example of functional specification that yields a simple linear stochastic model of demand. This demand model nests the original Bass' (1969) model of innovation diffusion. Using the economic structure and the functional specification we are able to identify all structural parameters of the model. Interpreting the parameters correctly, we can still investigate the economic significance of the identified network effects.

Last, but not least, we brought stochastic structure into the model. Introducing a measurement error, as the most obvious source of stochastic noise in data, results in a non-trivial error structure in our econometric model. To correctly estimate the structural parameters of the model one needs to take care for the error structure.

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reliable estimates of network effects.

Appendix

A.1. Derivatives of the function H with respect to x(t-δ) and p

First, note that v_t* is an implicit function of *x(t-δ)* and *p*, what under simplifying assumptions in the section 2.5 is described by

(A.1) $u(v_t^*, x(t-\delta)) = p.$

To calculate the derivative of *H* with respect to the lagged network size *x(t-δ)* we first apply the chain rule to the definition of *H* given in (9). We obtain

(A.2) $$\frac{\partial}{\partial x(t-\delta)} H(v_t^*) = -\frac{\partial F(v_t^*)}{\partial v_t^*} \cdot \frac{\partial v_t^*}{\partial x(t-\delta)}$$

The first term on the RHS of (A.2) is just the density of *v* at v_t*. To calculate the second term note that the total derivative of $u(v_t^*, x(t-\delta))$ with respect to *x(t- δ)* must stay constant in order to satisfy equation (A.1). This holds for

(A.3) $$\frac{\partial u(v_t^*, x(t-\delta))}{\partial x(t-\delta)} = -\frac{\partial u(v_t^*, x(t-\delta)}{\partial v_t^*} \cdot \frac{\partial v_t^*}{\partial x(t-\delta)}$$

Solving (A.3) for $\frac{\partial v_t^*}{\partial x(t-\delta)}$ and substituting that into (A.2) yields the desired result

(A.4) $$\frac{\partial}{\partial x(t-\delta)} H(v_t^*) = f(v_t^*) \cdot \left(\frac{\partial u(v_t^*, x(t-\delta)}{\partial v_t^*} \right)^{-1} \cdot \frac{\partial u(v_t^*, x(t-\delta))}{\partial x(t-\delta)},$$

where *f* is a density function of *v*.

Analogously, to calculate the derivative of *H* with respect to the price *p* we first apply the chain rule to obtain

(A.5) $$\frac{\partial}{\partial p} H(v_t^*) = -\frac{\partial F(v_t^*)}{\partial v_t^*} \cdot \frac{\partial v_t^*}{\partial p}.$$

Then we note that

(A.6) $$\frac{\partial u(v_t^*, x(t-\delta)}{\partial v_t^*} \cdot \frac{\partial v_t^*}{\partial p} = 1,$$

and substitute to get

(A.7) $$\frac{\partial}{\partial p} H(v_t^*) = -f(v_t^*) \cdot \left(\frac{\partial u(v_t^*, x(t-\delta))}{\partial v_t^*} \right)^{-1}.$$

A.2. State equations with Firm-Specific Constants

To see how we can nest the two regimes (with and without switching costs) in a single set of the state equations rewrite (8) using the definitions (4) and (7) to

(A.8) $y_i(t) = H_i(\boldsymbol{v_t}^*) + y_i(t\text{-}\delta) - H_i(\boldsymbol{v_{t-\delta}}^*)\dfrac{I_{L,t-\delta}}{I_{L,t}}$.

Remember that we assume equal hedonic prices among firms. You can think of (A.8) as of a decomposition of the total sales of bran i in time t under switching costs. The first term on the RHS of (A.8) gives the total sales of brand i (number of subscribers) if there were no switching costs. The second and the third term adds and subtracts the installed base of brand i respectively in a way that is sensitive to the number of active firms in the market. To see how this can lead to persistent asymmetries among firms expand the recursive equation (A.8) to

$$\text{(A.9)}\quad y_i(T) = H_i(\boldsymbol{v}_T^*) + H_i(\boldsymbol{v}_{T\text{-}\delta}^*) - H_i(\boldsymbol{v}_{T\text{-}\delta}^*)\frac{I_{L,T-\delta}}{I_{L,T}} + H_i(\boldsymbol{v}_{T\text{-}2\delta}^*) - H_i(\boldsymbol{v}_{T\text{-}2\delta}^*)\frac{I_{L,T-2\delta}}{I_{L,T-\delta}} + \ldots + y_i(0)$$

$$- H_i(\boldsymbol{v}_0^*)\frac{I_{L,0}}{I_{L,\delta}},$$

where $t = 0$ indicates the time when the market starts up so there are no sales at that time and $T > 0$.

Suppose, there is constant number of firms active in the market such that $I_{L,t} = I_L$ for $t \in (0,T)$. Then the last two terms on the RHS of (A.9) equal zero, because every firm is active from the very beginning of the market, and all the middle terms cancel out. In this case (A.9) simplifies to (5), i.e. the state equations with and without switching costs are the same.

Now suppose, there was an entry into the market in $t = E$, and $0 < E < T$. This means that $I_{L,t}$ rises discontinuously in $t = E$ and stays at the higher level afterwards. The sales equations of the incumbents do not simplify to (5) any longer. They become instead

$$\text{(A.10)}\quad y_i^{inc}(T) = H_i(\boldsymbol{v}_T^*) + \int_E^{E+\delta} [H_i(\boldsymbol{v_{t-\delta}}^*) - H_i(\boldsymbol{v_{t-\delta}}^*)\frac{I_{L,t-\delta}}{I_{L,t}}]dt,$$

for $T \geq E+\delta$. The integral in (A.10) is positive. It is also invariant with respect to any events in $T > E+\delta$ and can be trated therefor as a firm-specific constant in the post-entry period.

In contrast, the expansion of the recursive equation (A.8) does not go back to $t = 0$ for the entrants. Their history starts at $t = E$ and the sales can be described by

$$\text{(A.11)}\quad y_i^{ent}(T) = H_i(\boldsymbol{v}_T^*) + \int_E^{E+\delta} [-H_i(\boldsymbol{v_{t-\delta}}^*)\frac{I_{L,t-\delta}}{I_{L,t}}]dt,$$

for $T \geq E+\delta$. To see this result, refer to (A.8) and note that $y_i^{ent}(t\text{-}\delta) = 0$ for $t \in (E, E+\delta)$. The integral in (A.11) plays analogous role for the entrants as the integral in (A.10) for incumbents, but it is negative. Therefore, we can conclude that the incumbents have a constant (in terms of the difference in the total sales) competitive advantage over the entrants.

Moreover one can show that the the fixed effects caused by entry sum up to zero. To see that the number of incumbents as *A* and the number of entrants as *B*. The sum of the effects is then

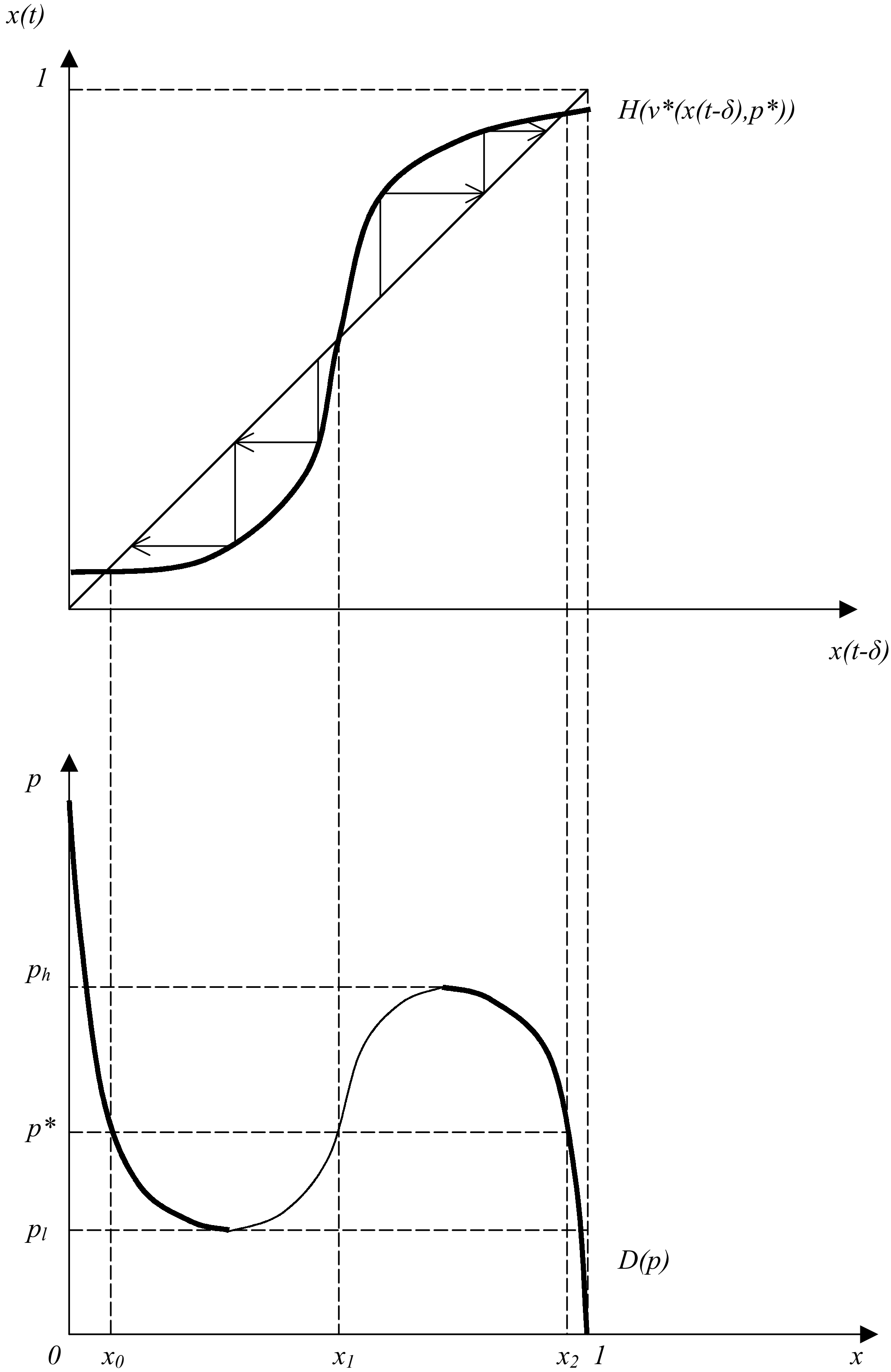
$$A \int_{E}^{E+\delta} [H_i(\mathbf{v}_{t\text{-}\delta}^*) - H_i(\mathbf{v}_{t\text{-}\delta}^*) \frac{I_{L,t-\delta}}{I_{L,t}}]dt + B \int_{E}^{E+\delta} [-H_i(\mathbf{v}_{t\text{-}\delta}^*) \frac{I_{L,t-\delta}}{I_{L,t}}]dt =$$

$$\text{(A.12)} = A \int_{E}^{E+\delta} [H_i(\mathbf{v}_{t\text{-}\delta}^*) - H_i(\mathbf{v}_{t\text{-}\delta}^*) \frac{A}{A+B}]dt + B \int_{E}^{E+\delta} [-H_i(\mathbf{v}_{t\text{-}\delta}^*) \frac{A}{A+B}]dt =$$

$$= \left(A - \frac{A^2}{A+B} - \frac{AB}{A+B} \right) \int_{E}^{E+\delta} H_i(\mathbf{v}_{t\text{-}\delta}^*)\, dt = 0.$$

One could also investigate the effects of exit in the analogous manner. Since in our economic structure there is no reason for a firm to leave the market we skip this discussion.

Figure 1. Stable vs. unstable equilibria.



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