Bargaining theory and the analysis of Belgian multy-party-government formation during the interwar period

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Bargaining Theory and the Analysis of Belgian Multy-Party Government Formation during the Interwar Period

*Erik Buyst, Luc Lauwers, Patrick Uytterhoeven**

Abstract: The distribution of minister portfolios in government coalitions is the result of a complex process of bargaining. In that way power relations among political parties are reflected in the distribution of cabinet posts. In this paper the predictions of game theoretical concepts (bargaining set, e*-core) and the Gamson hypothesis are compared with the actual portfolio distribution. In most Belgian interwar governments the relatively small liberal party was able to obtain a share of ministeries substantially larger than their share of parliamentary seats in coalition. Game theoretical models proved superior in capturing this effect. They also accentuate the disruptive effect of the 1936 elections. The success of extreme parties changed the simple bargaining structure of the 1919-1936 period into a complicated one. Moreover, the 1936 game is as far removed as possible from an Pareto optimal solution, which explains the labarious formation of short-lived governments.

1. Introduction

In many Western democraties no single political party is able to obtain an absolute majority in parliament. It is therefore necessary, or at least desirable, to form a coalition between two or more parties in order to acquire a governing majority. A minority government is usually not a preferred option as it continuously faces the risk of losing a motion of no-confidence in the legislative assembly.

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Coalition formation, and more particularly the distribution of cabinet posts among coalition partners, is the result of a complex process of bargaining. Some historians emphasize the personalities of individual politicians and the power relations among them in explaining the outcomes of negotiation. Other researchers reject this personalistic point of view and maintain that the number of portfolios obtained by a coalition member is directly proportional to the percentage share of that party in the seats occupied by the coalition (Gamson hypothesis). The latter opinion has in its turn been widely criticized by game theorists. They argue that the proportion of seats is a poor proxy for power relations. It is indeed not exceptional that a party acquiring more seats in a legislature actually loses its capacity to influence the outcome of a vote, and vica versa.

In this paper the distribution of cabinet posts is considered as a game where every party tries to control as many portfolios as possible, taking into account the different political weight of the positions on offer. The use of game-theoretical concepts has several advantages. First, it takes account of the criticism levelled against the Gamson hypothesis. Secondly, it dissociates our analysis from occasional events, and therefore provides a general framework to evaluate their influence.

As a test case we deal with multi-party government formation in Belgium during the interwar period. In a first section we give a concise overview of Belgian politics between the wars. Next, the game-theoretical framework is presented. Finally, we compare the predictions of several models (Gamson hypothesis, bargaining set, and e*-core) with the actual portfolio distribution.

2. Belgian politics between the wars: a brief overview

Belgium is an interesting case to test bargaining models. Table 1 shows that during the interwar period no single political party was ever able to obtain an absolute majority in parliament. As a result, all but one of the twenty-four interwar governments were coalitions between two or three parties.

Another feature of interest is that extreme left- and right-wing parties (on the one side Communists and on the other Rexists and Flemish Nationalists) had considerable electoral success in the second half of the 1930s. Until 1936 extreme parties were barely represented in Parliament. Catholics and Socialists each controlled about 40 percent of the seats. The conservative liberal party was much smaller in size and occupied on average 15 percent of the seats. The 1936 elections disrupted this traditional election pattern as the share of seats obtained by extreme parties jumped from 6 to 23 percent. The Catholics suffered an especially crushing defeat (1). How was the changed political landscape reflected in the distribution of cabinet posts?

Table 1: The composition of the Belgian House of Representatives (2)

	CAT 1	LIB 2	SOC 3	FLN 4	COM 5	REX 6	OTH 7	TOTAL
1919	73	34	70	5			4	186
1921	80	33	68	4			1	186
1925	78	23	78	6	2			187
1929	76	28	70	10	1		2	187
1932	79	24	73	8	3			187
1936	63	23	70	16	9	21		202
1939	73	33	64	17	9	4	2	202

CAT : Catholics REX : Rexists
SOC : Socialists COM : Communists
LIB : Liberals OTH : Others
FLN : Flemish Nationalists. Since 1933 they were united in the VNV (»Vlaams Nationaal Verbond«)

Source: Luykx 1977 , pp 426-7.

Table 2 shows that out of the twenty-four interwar governments, fifteen were two-party coalitions of Catholics and Liberals, or Catholics and Socialists. The former were conservative, while the latter, only two in number, were more leftish.

In exceptional circumstances the two-party formula was replaced by the so-called governments of national union, coalitions of all three center parties. This happened immediately after the First World War as Catholics, Socialists and Liberals shared the responsability for rebuilding the economy and changing the constitution. In 1926 the same coalition was constituted to solve the severe financial and monetary crisis that necessitated a substantial devaluation of the Belgian franc. Nine years later, the failure of deflationary policies to counter the Great Depression made a new devaluation inevitable and the three democratic parties joined together to undertake the operation (3). The success of the extreme parties in the 1936 elections compelled the traditional parties to cooperate for another two years. In September 1939, finally, the outbreak of World War II once again led Catholics, Socialists and Liberals to form a three-party government. In total no less than eight governments of national union were formed to face the numerous crises that threatened Belgium during the interwar period.

Table 2 : Government coalitions in Belgium during the interwar period

Elections 16 November 1919		
1. L. Delacroix	(2.12.1919- 3.11.1920)	CAT, LIB, SOC
2. H. Carton de Wiart I	(20.11.1920 - 24.10.1921)	CAT, LIB, SOC
3. H. Carton de Wiart II	(24.10.1921 - 20.11.1921)	CAT, LIB
Elections 22 November 1921		
4. G. Theunis I	(16.12.1921 - 5.07.1923)	CAT, LIB
5. G. Theunis II	(5.07.1923 - 27.02.1924)	CAT, LIB
6. G. Theunis III	(10.03.1924 - 5.04.1925)	CAT, LIB
Elections 5 April 1925		
7. A. Vande Vyvere	(13.05.1925 - 22.05.1926)	CAT
8. P. Poullet- E. Vandervelde	(17.06.1925 - 8.05.1926)	CAT, SOC
9. H. Jaspar I	(20.05.1926 - 21.11.1927)	CAT, LIB, SOC
10. H. Jaspar II	(22.11.1927 - 26.05.1929)	CAT, LIB
Elections 26 May 1929		
11. H. Jaspar III	(26.05.1929 - 21.05.1931)	CAT, LIB
12. J. Renkin I	(5.06.1931 - 23.05.1932)	CAT, LIB
13. J. Renkin II	(23.05.1932 - 18.10.1932)	CAT, LIB
14. Ch. de Broqueville I	(22.10.1932	
Elections 27 November 1932		
	- 13.12.1932)	CAT, LIB
15. Ch. de Broqueville II	(17.12.1932- 6.06.1934)	CAT, LIB
16. Ch. de Broqueville III	(12.06.1934 - 13.11.1934)	CAT, LIB
17. G. Theunis IV	(20.11.1934 - 19.03.1935)	CAT, LIB
18. P. Van Zeeland I	(25.03.1935	
Elections 24 May 1936		
	- 26.05.1936)	CAT, LIB, SOC
19. P. Van Zeeland II	(13.06.1936 - 25.10.1937)	CAT, LIB, SOC
20. P.E. Janson	(23.11.1937 - 12.05.1938)	CAT, LIB, SOC
21. P.H. Spaak	(15.05.1938 - 9.02.1939)	CAT, LIB, SOC
22. H. Pierlot I	(21.02.1939 - 27.02.1939)	CAT, SOC
Elections 2 April 1939		
23. H. Pierlot II	(18.04.1939 - 3.09.1939)	CAT, LIB
24. H. Pierlot III	(3.09.1939 - 10.05.1940)	CAT, LIB, SOC

Source: Luykx 1977 , pp 413-8.

3. Game-theoretic preliminaries

In this section the notions of bargaining set and ε^*-core are presented. Let *C* denote a government coalition and x a relative utility (portfolio) distribution. Then x is said to be in the bargaining set if, for each party p in C, a threat by p to leave the coalition in order to gain more portfolios in another winning coalition can be countered by some party of C.

The concept of the ε-core is based upon the notion of ε-blocking. Let C and x be as above and let *C* be the set of players which are not in C. Assume that *C* can convince some players in *C* to leave *C* by offering this group of players a surplus of ε above their joint utilities in x. Then *C* is said to ε-block *C.* The e-core is defined as the set of distribution vectors which cannot be e-blocked. Distribution vectors which cannot be O-blokked coincide with the Pareto optimal points. In the framework of coalition forming this 0-core is empty. The smallest ε for which the ε-core is a nonempty set is denoted by ε^*. In this way, the concept of ε^*-core answers the question of how close the game is to having a Pareto optimal distribution.

3.1 Introduction and notation

Here we simply state some definitions concerning cooperative n-person games (Friedman 1986 , pp 184-208).

Definition 1: An (essential) n-person game G is played by a set of n players. This set is labeled $N = \{1,2,.., n\}$ with players numbered 1, 2,.., n. Subsets $C \subset N$ are called coalitions. Denote by | C | the number of players in a coalition C. Let 2^N be the power set of *N.* Then, the game G can be presented by its characteristic function, which is a real valued function y: 2^N -> IR, that satisfies

(i) $\gamma(\phi) = 0,$

(ii) $\gamma(S \cup T) \geq \gamma(S) + \gamma(T)$, **provided** $S \cap T = \phi$ **(superadditivity),**

(iii) $\gamma(N) > \Sigma_{i=1}^{n} \gamma(\{i\})$.

Example 1: A game G for which the images of its characteristic function belong to the set {0,1} is called a simple game. Such a game is uniquely presented by the set W [G] = $\gamma^{-1}(\{1\})$. The set W [G] is interpreted as the set of the winning coalitions. If in a simple game G, player 1 belongs to every winning coalition, player 1 is called a veto player.

Assume that each player i is equipped with a natural number v_i, that is called the weight of player i. Let q be a number which satisfies $1/2\ (v_x + v_2 + .. + v_n) < q \leq (y_x + v_2 + .. + v_n)$. The number q is called the quota. The simple game G for which the characteristic function is given by $\gamma(\mathbf{A}) = 1$ if and only if $\Sigma_{j \varepsilon A} v_j \geq q$, is called a weighted voting game. It is denoted by G = $[q \mid v_1, v_2, .., v_n]$. If G is a weighted voting game and if *C* is a winning coalition for which every proper subcoalition is not winning, then *C* is said to be a minimal winning coalition. The set of all minimal winning coalitions is denoted by MW[G }

In our application the players will be interpreted as political parties. We take the number of seats in the House of Representatives of the party i to be the weight v_i. The quota q is given by

$$q = (v_1 + v_2 + .. + v_n) / 2.$$

It is the threshold to make a coalition winning. The interpretation is clear: if a coalition *C* wants to pass a bill in the House of Representatives, the bill is accepted when $C \varepsilon$ W[G 1 i.e. when the total number of seats $\Sigma_{j \varepsilon C} v_j$ of the coalition is greater than the quota q. Clearly, political parties are assumed to act homogeneously. Thus fractionalization within a party is not considered, although this often occurs in a historical context.

Assume that an n-person game G is played. The players will have to divide the total utility $\gamma(N)$. The division of this total utility is, of course, subject to certain constraints.

Definition 2: An *imputation* for the game G is a vector

$$x = (x_1, x_2, .., x_n)$$

satisfying

(i) $\Sigma_{i=1}^{n} x_i = \gamma(N)$ (group rationality),

(ii) $x_i \geq \gamma(\{i\})$ for all $i \varepsilon N$ (individual rationality).

Let 5 be a coalition and **x** and **y** be two imputations, then **y** *dominates* **x** *through S* if

(i) $y_i > x_i$ for all $i \varepsilon S$,

(ii) $\Sigma_{j \varepsilon S} y_j \leq \gamma(S)$.

* And, **y** *dominates* **x** if **y** *dominates* **x** through some coalition *S.*

As a first attempt to solve an n-person game one defines the *core.*

Definition 3: The core of a game is the set of imputations **x** which are not dominated, or equivalently, the core is the set of imputations **x** which satisfy $\Sigma_{i \varepsilon S} x_i \geq \gamma(S)$ for all $S \varepsilon 2^N$ (Friedman [1986], p 191).

Unfortunately, the core of a simple game without veto players appears to be empty. Indeed, suppose that **x** is an imputation such that $x_1 > 0$. Then y = $(0, x_2 + \varepsilon, x_3 + \varepsilon, .., x_n + \varepsilon)$ with $\varepsilon = x_1/(n-1)$ dominates x through *S* = {2, 3, .., n}. Moreover $\Sigma_{j \varepsilon S} y_j = 1$ and $\gamma(S) = 1$, since 1 is not a veto player. Thus, y is an imputation which dominates x. Hence, other concepts have to be considered.

3.2 ε-core

The notion of core can be weakened by looking for imputations x which satisfy $\Sigma_{i \varepsilon S}\, x_i \geqq \gamma(S) - \varepsilon$ for all $S \in 2^N$, where e is some fixed (positive) real number. The set of such imputations is called the ***ε-core*** (Friedman [1986], p 192).

The notion of domination can be strengthened to ε-*domination*, i.e. y ε-dominates x if there is some coalition S such that

(i) $y_i > x_i$ for all $i \in S$,
(ii) $\Sigma_{j \varepsilon S}\, (y_j - x_j) > \varepsilon$,
(iii) $\Sigma_{j \varepsilon S}\, y_j \leqq \gamma(S)$.

Then, the ε-core coincides with the set of imputations which are not ε-dominated. Indeed, suppose that x is e-dominated by y through a coalition S, then $\Sigma_{j \varepsilon S}\, x_i < \Sigma_{i \varepsilon S}\, y_i - \varepsilon \leqq \gamma(S) - \varepsilon$, and x does not belong to the ε-core. Conversely, suppose that the imputation x does not belong to the e-core. Then there exists some nonempty coalition S such that $\Sigma_{i \varepsilon S}\, x_i = s < \gamma(S) - \varepsilon$.

And x is e-dominated through S by the imputation

$y_i = x_i + \delta$ for $i \in S$,
$y_j = \gamma(\{j\}) + \delta'$ for $j \notin S$,
where $\delta = (\gamma(S) - s) / |S| > \varepsilon / |S| > 0$
and $\delta' = [\gamma(N) - \gamma(S) - \Sigma_{j \varepsilon N\text{-}S}\, \gamma(\{j\})] / |N - S| \geqq 0$.

There exists a real number ε for which the ε-core is a nonempty set. Indeed, let $\varepsilon \geqq \gamma(N)$ and the corresponding ε-core is equal to the set of all imputations. Friedman suggests to look for the smallest ε^* for which the ε^*-core is not empty (4).

Lemma 1: Let G be an n-person game with γ its characteristic function and assume that the core is empty. The smallest ε^* for which the corresponding ε^*-core is not empty is found by solving the equation

$$E(\varepsilon) = 0,$$

where $E(\varepsilon)$ is defined as the solution of the linear programming problem

minimize $f = \Sigma_{i=1}^{n}\, x_i - \gamma(N)$,

subject to the following inequalities

(*) $\Sigma_{i \varepsilon S}\, x_i \geqq \gamma(S) - \varepsilon$ for all coalitions $S \in 2^N - \{N\}$,

$$x_j \geqq \gamma(\{j\}) \text{ for } j \in N.$$

Proof: The constraint set D (ε) of vectors $\mathbf{x} \in (\mathbb{R}^+)^n$ which satisfies the system (*) of inequalities is obtained by an intersection of several half-spaces and is a convex polyhedron. Since the objective function f is linear, the minimum will be found at one of the vertices of D (ε). Hence, a vector $\mathbf{x}^*$, for which f is minimal will satisfy the following system

(i) $\Sigma_{i \in S}\, x_i^* = \gamma(S) - \varepsilon$, for $S \in V \subset 2^N$

(ii) $\Sigma_{i \in S}\, x_i^* > \gamma(S) - \varepsilon$, for $S \in 2^N - V\ \{N\}$

(iii) $x_j^* = \gamma(x_j^*)$, for $j \in V \subseteq N$,

(iv) $x_j^* > \gamma(x_j^*)$, for $j \in N - V$,

where the nonempty set (i), (iii) of equalities defines the vertex (or the edge which contains the vertex) for which f is minimal. If $\delta < \varepsilon$, then $D(\delta)$ C $D(\varepsilon)$ and the boundary of the set $D(\delta)$ is disjoint with the boundary of the set $D(\varepsilon)$. Conclude that $E\{\delta) >$ $E\{\varepsilon)$, or, E is a monotonically decreasing function. Note that since the core is empty £(0)> 0, and that $E(\gamma(N))$ < 0. Since all operations are continuous, the function E(ε) is continuous and has a unique zero ε^*.

When E(ε) > 0, then $\mathbf{x}^*$ is not an imputation and the corresponding ε-core is empty. And, if E(ε) = 0, then $\mathbf{x}^*$ is an imputation and the corresponding ε-core contains $\mathbf{x}^*$. Conclude the lemma.

Note that for an n-person weighted voting game G the set of constraining inequalities can be restricted to

$\Sigma_S\, x_j \geqq 1 - \varepsilon$, where S runs over MW[G J

If, in addition, there is no veto player ε^* satisfies $e = (|D|-1)/|D| \geqq \varepsilon^*$, where D is a minimal winning coalition for which \D | is minimal. Indeed, let x be the imputation defined by $x\text{-}_j = 1/|D|$ for $j \in D$ and $x_k = 0$ otherwhise. Clearly, x is not e-dominated.

3.3 Bargaining set

$(x,\tau) = (x_1, \ldots, x_2;\ T_1, \ldots, T_m)$

the discussion that

where $\tau = \{T_1, \ldots, T_m\}$ is a partition of the set N, litional definitions

$x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ satisfies

(i) $\Sigma_{Tk}\ x_i = \gamma(T_k)$ k= 1, . . ., m,

(ii) $x_j \geqq \gamma(\{j\})$ for all $j \in N$.

a pair

$(x,\tau) = (x_1, \ldots, x_2;\ T_1, \ldots, T_m)$

where $\tau = \{T_1, \ldots, T_m\}$ is a partition of the set N,

$x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ satisfies

(i) $\Sigma_{Tk}\ x_i = \gamma(T_k)$ k= 1, . . ., m,

(ii) $x_j \geqq \gamma(\{j\})$ for all $j \in N$.

Axiom (ii) is called the individual rationality.

Definition 5: Let (x, τ) be a payoff configuration. Let *K, L* be two nonempty and disjoint subsets of $T \varepsilon \tau$. Then, the payoff configuration (y,υ), with $K \subseteq U \varepsilon \upsilon$, is an objection of *K* against *L* concerning (x τ) if

(i) $U \cap L = \phi$,
(ii) $y_i > x_i$ for all i ε *K*,
(iii) $y_j \geq x_i$ for all j ε *U*.

This payoff configuration (y,υ) can be countered by *L* through a *counterobjection* (z, V), with $L \subseteq V \varepsilon V$. The payoff configuration (z, V) has to satisfy

(i) the set *K* is not a subset of *V*,

(ii) $z_i > x_i$ for all i ε *V*,

(iii) $z_j \geq x_j$ for all j ε $(U \cap V)$.

Definition 6: The bargaining set B is the set of the payoff configurations (x, τ) such that whenever $i \varepsilon T \varepsilon \tau, L \subseteq T - \{i\}$ and i has an objection against *L* , then *L* can counter the objection.

3.4 Explicit calculations

Example 2: The 1936 elections

There are seven players involved. Player 7 has weight equal to zero and is excluded in the calculations. According to Table 1, the game is denoted by [102 I 63,23,70,16, 9,21]. The set MW[G] contains the following coalitions: {1,2,41 (1,2,6), {1,3}, {1,4,5,61 {2,3,41 {2,3,51 {2, 3, 6} and {3,4, 6}. Since the coalition A = {1, 2, 3}between Catholics, Liberals and Socialists was the actual government, we only consider payoff configurations (x, τ) with $x_4 = x_5 = x_6 = 0$ and A e τ.

The bargaining set B

Suppose player 1 has an objection against player 2 through a configuration $(X j \varepsilon, 0, 1 - X - \varepsilon, 0,0,0)$, where e is small and positive. Since player 3 is contained in all winning coalitions which contain player 2 and avoid player 1, this is the best way for player 1 to object to player 2. Player 2 can counter through the configuration $(0, x_2, 1 - x_1 - \varepsilon, \delta, 0,0)$ provided $1 - x_1 - \varepsilon + x_2 < 1$ or $x_2 \leq x_1$ Analogously, $x_2 \geq x_3$ (player 3 objects to player 2).

Consider an objection $(0,0, x_3 + \varepsilon, \delta, 0, \delta)$, $2\delta + x_3 + \delta = 1$, of player 3 against 1. Player 1 can counter through $(x_1,x_2,0,x_3,0,0)$ if $x_3 \geq 8$ or $x_3 \geq 1/3$. Analogously, $x_3 \leq (1+x)/2$ (player 1 objects to player 3). Player 2 objects to player 3 through the coalition {1,2,61 Two situations occur. First, if $x_3 \geq$ **Xp** he objects through (δ,x_2+ δ,0,0,0, δ) with 2 δ + x_2+ ε = 1. Player 3 can counter if 1-$x_3 \geq \delta$ or if $x_3 \leq$ (x_2+l)/2. Secondly, if $x_3 < x_1$ then player 3 can counter any objection of player 2.

Thirdly, if $x_3 \leq x_1$ then player 2 can object to player 1 through the configuration (0,x_2+ e, δ,0,0, δ), 2 δ + x_2+ ε = 1. And if $x_i \leq$ (x_2+l)/2 then player 1 can counter the objection.

Finally, consider the condition induced by an objection of player 1 to {2, 3}through (x_1+ ε, 0,0, δ, δ, δ) with δ = (1-x_1,- ε)/3. {2,3} can counter through (0, x_2, x_3, x_1, 0, 0) provided $x_1 \geq$ 1/4.

The ε*-core

Following Lemma 1, we have to solve the linear programming problem (*). Then, E(ε) = 1-2 ε. Hence, ε* = 1/2 and the ε*-core consists of the point E* = (1/2, 0, 1/2). Note that ε* has the largest value possible.

Example 3: The 1939 elections

The game **G** is denoted by [102 | 73, 33, 64, 17, 9, 4, 2]. The set **MW[G]** consists of the following sets: {1,2), {1,3), {1,4,5,6), {2,3,4}, {2, 3,5}and {2, 3, 6,71

Let A = {1, 2, 3). Configurations (x_1, x_2, x_3, 0,.., 0 ; *T)* with A ε *T* belong to the bargaining set B provided x_2 = x_3 and 1/3 $\leq$ **Xj** $\leq$ 3/5. The ε*-core consists of the set {(1/2, x_2, x_3) | x_2 + x_3 = 1/2} and ε* = 1/2. In case B = {1,2}ε 7, the configuration *(x_v \-x_v* 0,.., 0 ; *T)* belongs to the bargaining set *B* provided 1/2 $\leq$ $x_1 \leq$ 2/3. The case {1,3} ε *T* is treated analogously.

4. An application to the interwar period

Before examining what model explains most adequately about how the power relations among political parties are translated into the distribution of minister portfolios, it is necessary to deal with some data problems.

4.1 Preparation of the data

First we have to make a distinction between the number of cabinet members on the one hand, and the number of portfolios on the other. During the interwar period it was a common practise, especially among catholic ministers, to accumulate several portfolios. Mr. Ch. du Bus de Warnaffe, for instance, combined Transportation and Post, Telegraph and Telephone in the Theunis II government. Therefore we construct two different datasets CM and PORT.

CM = {(CM_1, ..., CM_7)$_t$ }, with CM_i the number of cabinet members of party i,

PORT = $\{(PORT_1, \ldots PORT_7)_t\}$ with $PORT_i$ the number of portfolios of party i.

The parameter t runs over the successive governments.
In CMj, for instance, Mr. Ch. du Bus de Warnaffe is included only once while he is counted twice in $PORT_1$.

Secondly, we have to bear in mind that not all portfolios share the same political weight. Hence distinction is made between major and minor departments (see Table 3). Moreover, in the course of time some departments gained and others lost political weight. In the early twenties, for example, the competence of the Ministry of Economic Affairs was largely limited to war debt management. During the Great Depression, however, it became one of the key departments. In the course of time some other departments were split and new ones were created.

Table 3: List of ministries according to their political weight

A : List of major departments	
Prime Minister,	
Defence,	
Economic Affairs	(from 1932),
Education,	
Finance,	
Foreign Affairs,	
Industry and Labour (5)	(until 1932),
Interior,	
Justice,	
Railways, P.T.T. (6)	(until 1926).
B : List of minor departments	
Agriculture,	
Colonies (7),	
Economic Affairs	(until 1932),
Industry, Labour and Social Affairs	(from 1932),
National Information	(created in 1939),
Provisioning and Absorption of Unemployment	(created in 1939),
Public Health	(created in 1935),
Public Works,	
Transportation, P.T.T.(8)	(from 1926).

With the help of Table 3 we refine our data. For each of the two datasets (CM and PORT) a set is defined counting only the major portfolios:

CMMA = $\{(CMMA_1, .., CMMA_7)_t\}$, with $CMMA_i$ the number of cabinet members belonging to party i and occupying major departments,

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$PORTMA = \{ (PORTMA_1, \ldots, PORTMA_7)_t \}$, with $PORTMA_i$ the number of major portfolios of party i.

The parameter t runs over the successive governments.
The four datasets CM, CMMA, PORT, PORTMA include all cabinet reshuffles within a government's term (9).

To make matters even more complicated each party had a different preference ordering over the various cabinet posts. The Department of Agriculture, for instance, was more important to catholic politicians than to Liberals or Socialists. In our analysis this element is not taken into account.

Many interwar governments included some so-called technicians, ministers who had not been elected to parliament. The centre-left Poulet-Vandervelde cabinet, for instance, included two liberaloriented technicians to make the coalition acceptable to the conservative wing of the catholic party and the Belgian establishment in general. Only technicians with clear political affiliations have been included in our datasets.

Finally, the catholic minority government formed by Vande Vyvere is excluded from our analysis. The framework applied is not suited to deal with minority governments. This is not a serious problem since the Vande Vyvere cabinet survived for just one week.

4.2 The 1919-1936 period

Throughout the 1919-1936 period the bargaining set *B* gives a unique solution, equal to the ε^*-core solution. The bargaining concept and the ε^*-core predict that each center party entering a government coalition receives an equal reward: one half in a two-party coalition and one third in a coalition of national union.

Two conclusions can be drawn: first, ignoring ideological preferences, each party has the same probability of entering a government. Consequently, the bargaining power of the liberal party is equal to that of the Socialists or Catholics, despite its considerably smaller representation in the Belgian House of Representatives. Second, election gains or losses have no influence on power relations.

4.3 The 1936-1939 period

The simple bargaining structure of the 1919-1936 period is completely disrupted by the electoral success of some extreme parties in 1936. Negotiations to form a government were thwarted by this evolution. In one month eight successive politicians were appointed to bring about a government, but none succeeded. Finally, the government Van Zeeland II was formed. The different predictions and the actual outcome are denoted in

Figure 1 (Appendix), which represents the simplex $\{(x_1,x_2,x_3) \mid x_1,x_2,x_3 \geq 0$ and $x_1 + x_2 + x_3 = 1)$. The shaded area is the bargaining set *B*. The ε^*-core is denoted by the point **E***.

5. An empirical test

In this section we investigate whether the observed payoff and the predicted payoff are proportional on a linear base. This hypothesis can be phrased as $S = a + bP$, where *S* is the actual payoff according to one of the datasets (CM, CMMA, PORT, PORTMA) and P is a predictor (Gamson predictor, the ε^*-core and the bargaining set *B).* The relation is tested by linear least squares methods. In case $a = 0$ and $b = 1$, the predicted and the actual distributions coincide. Otherwhise, e.g., $b < 1$ and $a > 0$, the straight lines .S = P and $S = a + bP$ have an intersection point T. On the left side of this point T the actual payoffs are underestimated and on the right side they are overestimated. For $b > 1$ and $a < 0$ the converse holds.

The results of the linear regression of *S* on P are reported in Table 4 (Standard errors are given between brackets). In all estimates the intersection point *T* has abscis equal to 0.43.

Hence the relatively small liberal party is underestimated by all concepts. The slope of the bargaining predictor is close to one, while the slopes of the Gamson and the ε^*-core predictor are considerably smaller than one. This result is not surprising. Indeed, the Gamson hypothesis only takes into account the weight of the parties, and does not consider the way in which players learn how to use their weight. On the other hand, the ε^*-core intends to underreward players which are not needed to make the coalition a winning one. This was the case in 1936, when the government was composed of Catholics, Liberals and Socialists, although a majority could have been obtained by a coalition of the catholic and socialist parties.

Table 4: Results of regression between the observed payoff *S* and the predicted payoff P

	intercept (a)	slope (b)	R^2
S refers to CM			
Gamson	.2383	.4468	.6087
	(.0181)	(.0384)	
B	.0376	.9132	.5756
	(.0369)	(.0841)	
ε*-core	.1532	.6464	.5250
	(.0294)	(.0639)	
S refers to CMMA			
Gamson	.3064	.2822	.2008
	(.0285)	(.0604)	
B	.0612	.8531	.4154
	(.0477)	(.1085)	
ε*-core	.1920	.5504	.3147
	(.0388)	(.0871)	
S refers to PORT			
Gamson	.1809	.5763	.7372
	(.0176)	(.0373)	
B	.0091	.9748	.4746
	(.0483)	(.1099)	
ε*-core	.1411	.6696	.4077
	(.0386)	(.0865)	
S refers to PORTMA			
Gamson	.2692	.3692	.3595
	(.0249)	(.0528)	
B	.0435	.8943	.4776
	(.0441)	(.1063)	
ε*-core	.1841	.5687	.3516
	(.0369)	(.0828)	

Our analysis has some limitations and drawbacks. First, the political parties are considered as the central actors of the game. This approach neglects the considerable influence of the King on government formation during the interwar period. For example, after the 1921 elections King Albert appointed Mr. Theunis to form a government, despite the fact that Mr. Theunis was a rather unknown politician even in his own party. His conservative policy was strongly resisted by the labour oriented wing of the catholic party. In such a situation the party is no longer the principal element in the game and the prime minister himself has to make his government acceptable to his own rank and file. The influence of political parties on government formation increased throughout the interwar period and during the thirties government policies were highly dependent on their approval.

Secondly, the catholic party was not a party in the modern sense. Rather it was a loose collection of different catholic tendencies. These groups frequently disagreed and sometimes voted against each other in Parliament. In the analysis above it has been assumed that each party acted as a homogeneous block.

6. Conclusion

In this paper the distribution of minister portfolios in Belgian government coalitions during the interwar period was investigated. Bargaining set theory appears to be an adequate tool to describe this process. It enabled us to disentangle the process of government formation from occasional events (e.g., the personalities of the politicians involved), and to explain the distribution of ministerial portfolios, even if the actual distribution is counter intuitive at first sight.

In most interwar governments the small liberal party was able to obtain a share of ministries substantially larger than their share of parliamentary seats in the coalition. The larger catholic party was usually disadvantaged in comparison with its share of seats in the coalition. The game-theoretic concepts (the bargaining set B and the ε^*-core) proved superior to the popular Gamson hypothesis in capturing this effect.

Our analysis also accentuates the significance of the 1936 elections. Throughout the 1919-1936 period the bargaining structure is very simple. After the 1936 elections, however, it became fairly complicated because of the success of extreme parties. Moreover, the 1936 game is as far removed as possible from a Pareto optimal solution. This dissatisfaction is reflected in the succession of short-lived governments.

Notes

The authors wish to thank Prof. Dr. H. van der Wee and Dr. P. Solar for their helpful comments.

(1) The electoral success of extreme parties in 1936 is mainly explained by the inability of successive governments to tackle the severe socio-economic difficulties related to the Great Depression. In addition, the traditional parties were weakened by financial scandals, corruption and the quarrels between French and Dutch speakers. It fuelled the general opinion that a parliamentary system was no longer up to the

task of managing a modern society. Rex, for instance, wanted to impose an authorian regime modelled on Italian fascism. For more details, see Luykx 1977, pp 348-62.

(2) We focus on the election results for the House of Representatives since not all members of the Belgian Senate are directly elected.

(3) For more information about the monetary crises of 1926 and 1935, see Van der Wee and Tavernier 1975 , pp 79-212 and pp 257-289.

(4) Apparently the ε^* Friedman refers to, is not the smallest one for which the ε^*-core is not empty. The n-person weighted voting game [n-1 I n-2, 1, .., 1] is a counterexample. The ε^* according to Friedman [1986], p 192 is equal to (n-2)/(n-1). However the (n-2)/(2n-3)-core is also a nonempty set.

(5) Since the government de Broqueville the importance of the Department of Industry was declining and from the government Theunis onwards its task were largely taken over by Economic Affairs.

(6) Its significance declined considerably since the creation of the *National Railway Company* in 1926.

(7) The colonies itself were of course important as an economic resource, but economic affairs were dominated by the *Société Générale de Belgique.*

(8) From 19 October 1929 Transportation and P.T.T. became two separate departments.

(9) For a complete list, see Luykx 1977 , pp 414-8.

(10) Until 20 January 1939, Mr. P. Heymans. was Minister of Agriculture and Economic Affairs. He was succeeded by Mr. d'Aspremont Lynden for Agriculture and by Mr. Barnich for Economic Affairs.

References

Buyst E., Lauwers L. and Uytterhoeven P., »A game theoretical approach to the results of parliamentary elections in Belgium between the wars«, Social Science History 13:3 (1989), pp 237-254.

Friedman J.W., Game Theory with Applications to Economics, New York-Oxford, Oxford University Press, 1986.

Gamson W.A., »A Theory of Coalition Formation«, The American Sociological Review 26 (1961), pp 373-382.

Gerard E., De Katholieke Partij in crisis. Partijpolitick leven in Belgie (1918-1940) [The Catholic Party in Crisis], Leuven, Kritak, 1985.

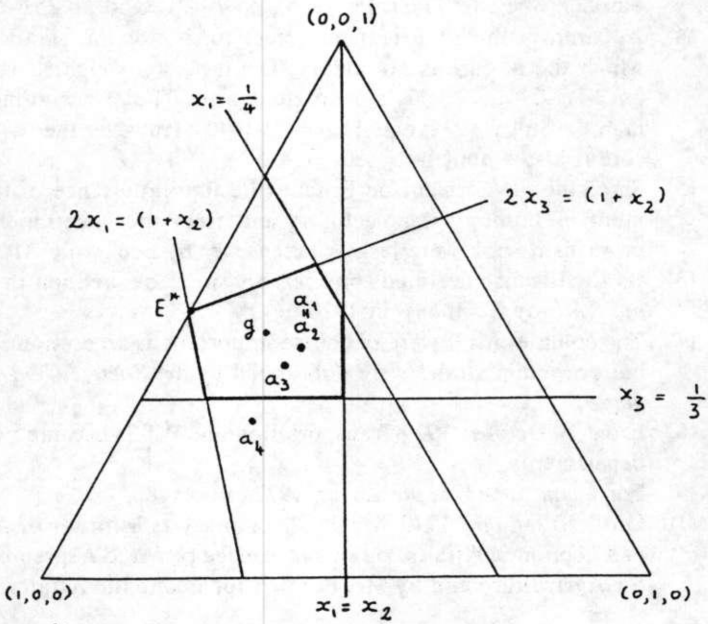
Luykx T., Politieke geschiedenis van Belgifc* [Political History of Belgium], Brüssel-Amsterdam, Elsevier, 1977.

Van der Wee H. and Tavernier K., La Ban que nationale de Belgique et lliistoire mon taire entre les deux guerres mondiales, Brussels, 1975.

Appendix

Figure 1

The 1936 elections



g: Gamson predictor.
a_1: actual distribution Van Zeeland II (13.06.1936-25.10.1937).
a_2: actual distribution P.E. Janson (23.11.1937-12.05.1938).
a_3: actual distribution P.H. Spaak (15.05.1938-20.01.1939).
a_4: actual distribution P.H. Spaak (21.01.1939- 9.02.1939)[10].