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Abstract. Several attempts have been made to reduce the impact of estimation errors on the optimal portfolio composition. On the one hand, improved estimators of the necessary moments have been developed and on the other hand, heuristic methods have been generated to enhance the portfolio performance, for instance the "resampled efficiency" of Michaud (1998). We compare the out-of-sample performance of traditional Mean-Variance optimization by Markowitz (1952) with Michaud's resampled efficiency in a comprehensive simulation study for a large number of relevant estimators appearing in the literature. In this context we consider different estimation periods as well as unconstrained and constrained portfolio optimization problems. The main finding of our simu-lation study concerning the optimization approach is that Markowitz outperforms Mi-chaud on average. Further-more, the estimation strategy of Frost/Savarino (1988) proves to work excellent in all analyzed situations.

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1. INTRODUCTION

Portfolio selection according to Markowitz (1952) is based on the assumption that investment decisions only depend on the expectation value μ_P and the variance σ_P^2 of the total portfolio return. Against this background, the portfolio optimization procedure requires the knowledge of the following input parameters: μ_i as the expected return of asset i, σ_i as the standard deviation of the return of asset i, and ρ_{ij} as the correlation between the returns of the assets i and j (i, j = 1, …, N). These parameters are assumed to be known within the procedure although the investors have to estimate them.

Naturally, estimation strategies entail estimation errors which in turn affect the solution of the portfolio selection problem often resulting in extreme portfolio weights, an unbalanced asset allocation, and a lack of diversification. Moreover, the composition of optimal portfolios is very sensitive to changes in expected returns, variances and covariances. Especially estimation errors in expected returns have a strong impact on portfolio allocation (Chopra and Ziemba, 1993). Against this background, a practically reasonable approach is to determine the sensitivity of the results to the different input parameters (Best and Grauer, 1991) and to focus estimation efforts for the parameters with greatest sensitivity. Beyond that, several attempts have been made to reduce the impact of estimation errors and to improve portfolio performance. On the one hand, advancements in estimation or portfolio optimization techniques have been built up, e.g. Bayesian approaches or the Black-Litterman model (e.g. Jorion, 1985; Garlappi *et al.*, 2007; Black and Litterman, 1992; Da Silva *et al.*, 2009), that should lead to reasonable portfolio compositions. On the other hand, heuristic methods have been developed to achieve this aim, for instance upper-bound constraints on portfolio weights (Frost and Savarino, 1988; Eichhorn *et al.*, 1998) or the concept of Michaud (1998) as well as Michaud and Michaud (2008a). Michaud's "resampled efficiency" is based on a resampling of portfo-

lio returns to reflect the uncertainty in the return process and has been widely discussed in the literature (e.g. see Scherer, 2002, and the literature introduced in the following paragraph).

In order to analyze the performance of the approach of Michaud, some studies deal with the comparison between traditional mean-variance (MV) optimization by Markowitz and the resampled efficiency by Michaud. The results are ambiguous: in a capital market study by Fletcher and Hillier (2001), Michaud's procedure outperforms the approach of Markowitz, but the improvements are not statistically significant. In a simulation study of Michaud and Michaud (2008b), resampled efficiency leads to the best outcomes. Markowitz and Usmen (2003) also find strong evidence for a better performance of the resampled efficiency compared to a Bayesian estimator using a diffused prior within a simulation study. However, the results of Harvey *et al.* (2008) and Scherer (2006) are completely different. First, Harvey *et al.* did a rematch of the simulation game of Markowitz and Usmen (2003) with a more sophisticated prior distribution and a more appropriate algorithm. In this setting, they obtain rather balanced results between the resampled efficiency and the optimization of Markowitz using their Bayesian estimator. Moreover, in a second competition, Markowitz always wins against Michaud. Scherer (2006) uses a James-Stein prior instead of a diffuse prior since evidence has shown the usefulness of this prior. In this simulation study, the Bayesian player almost always shows a significantly better performance compared to the resampled efficiency. However, comparing resampled efficiency and Markowitz optimization using classical historical estimators, Michaud's procedure performs slightly better but the difference is frequently not significant.

Furthermore, there are several studies that concentrate on the impact of different estimation techniques on portfolio optimization with Markowitz. In a study of Kempf and Memmel (2003), several estimation strategies are tested. They detect the approach of

Ledoit and Wolf (2003) to be the approach with the best results. Duchin and Levy (2009) find that the naïve 1/N-rule outperforms the classical Markowitz optimization for small portfolios but for large portfolios the Markowitz's strategy is superior. In a capital market study, DeMiguel *et al.* (2009) analyze several optimization strategies and conclude that no considered strategy beats the 1/N-rule persistently.

However, even if there are some studies comparing the performance of Markowitz and Michaud, each of these studies concentrates on a specific setting, which hardly leads to general recommendations. First, some studies analyze the performance with an out-of-sample capital market study (cf. Fletcher and Hillier, 2001), which has the advantage of realistic characteristics of stock returns, but the results highly depend on the development of the stock market in the certain time period. As noted by Michaud (2003), for a specific time series a good strategy can perform badly and a bad strategy can deliver a good performance. For this reason, some authors apply a simulation study instead of a capital market study (e.g. Markowitz and Usmen, 2003; Scherer, 2006; Harvey *et al.*, 2008; Michaud and Michaud, 2008b). Since such studies, however, are based on a very small set of "true" input parameters, e.g. Michaud and Michaud (2008b) based their findings on only one true parameter set, the same aspect ought to be criticized. Therefore, we perform a simulation study with 100 true parameter sets, which are generated to have similar characteristics as capital market data.

Second, none of the simulation studies mentioned above allows for riskless borrowing and lending. Even though this opportunity does not have to be implemented, in real-world applications it is mostly possible to combine risk-free and risky investments, which generally lead to different compositions of the risky part of the portfolio. Against this background, we additionally consider the possibility of riskless borrowing and lending.

Third, only Fletcher and Hillier (2001) test the Markowitz and Michaud optimization with different estimators while all other examinations are confined to the comparison of the classical MV-approach, Michaud's resampled efficiency, and the MV-approach with a Bayesian estimator. Since different estimators can be applied to both optimization approaches, we provide an extensive comparison of a number of relevant estimators appearing in the literature for Markowitz and Michaud. Furthermore, we estimate both the expected returns and the variance-covariance matrix since both variables are unknown in practical implementations, which is in contrast to Scherer (2006), who assumes the variance-covariance matrix to be known.

Fourth, most studies rely on a time series of 216 months (Markowitz and Usmen, 2003; Harvey *et al.*, 2008; Scherer, 2006), although a shorter estimation period is used frequently in financial research and practical applications.[1] Thus, we consider seven estimation periods ranging from a short period of 24 months to a long horizon of 216 months to account for the effects of different period lengths.

Fifth, almost all studies compare Markowitz and Michaud in the long-only constrained case (Markowitz and Usmen, 2003; Harvey *et al.*, 2008; Michaud and Michaud, 2008b). We also allow for unconstrained optimizations as Michaud's resampled efficiency intends to reduce the impact of estimation errors, which should have the greatest effect in the unconstrained case. Furthermore, this case is of practical interest since e.g. hedge funds are usually not restricted to long-only positions.

The main findings of our simulation study concerning the optimization approach are as follows: In the case with long-only constraints in the absence of riskless borrowing and lending opportunities, the results of Markowitz versus Michaud are rather balanced, which, in essence, confirms the results in the literature. However, under consid-

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[1] See Chopra *et al.* (1993), Scherer (2002), or DeMiguel *et al.* (2009).

eration of riskless lending and borrowing, the optimization procedure of Markowitz performs significantly better than Michaud in all relevant situations regardless of whether or not long-only constraints are taken into account. Furthermore, we find that the results are very sensitive to the length of the estimation horizon and can give advice for different initial situations of investors. Moreover, we show in which situations a constraint leads to improved results. Finally, we find that the estimation strategy of Frost/Savarino works excellent in all analyzed situations. This largely confirms Harvey *et al.* (2008) and Scherer (2006), and extends the results for unconstrained optimization problems as well as for varying lengths of observation periods.

The remainder of the article is organized as follows: We start with a short description of the Markowitz MV-approach and Michaud's resampled efficiency. Before explaining the database and the methodology of our simulation study, the underlying estimators are specified. Afterwards, the results are presented and finally our findings are summarized.

2. MARKOWITZ MV OPTIMIZATION

We consider the portfolio selection problem of an investor who can allocate his or her wealth to N risky assets and one riskless asset. In the framework of Markowitz under the hypothesis of multivariate normal distributed returns, the investor maximizes the following preference function in X:

$$\phi = X'\mu - (\lambda / 2) \cdot X'\Sigma X , \tag{1}$$

where $X = (x_1, \ldots, x_N)'$ represents the vector of portfolio shares of the N risky assets, μ is the vector of expected excess returns, Σ the variance-covariance matrix and λ the risk aversion coefficient.[2] Consequently, the difference $1 - x_1 - \ldots - x_N$ is invested in the

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[2] Assuming multivariate normal distributed returns, the preference function results as the certainty equivalent return from a utility function with constant absolute risk aversion.

riskless asset. Since the investor does not know the true parameters μ and Σ of the return distribution, he or she has to estimate them. The estimators $\hat{\mu}$ and $\hat{\Sigma}$ are inserted in (1) for μ and Σ and the optimization procedure is accomplished.

3. MICHAUD'S RESAMPLED EFFICIENCY

The basic concept of Michaud's resampled efficiency comprises (a) a generation of a sequence of returns, which are statistically equivalent to the actual time series of returns, through a Monte Carlo simulation, (b) the subsequent determination of portfolio weights for every resample, and finally (c) the averaging over the obtained portfolio weights to obtain the optimal portfolio weights according to Michaud. The aim of resampled efficiency is to minimize the impact of estimation risk on the portfolio composition, to get a more balanced asset allocation, and to improve the portfolio performance compared to Markowitz.

   The specific steps of the procedure are listed below:[3]

1) Estimate the input parameters $\hat{\mu}$ and $\hat{\Sigma}$.

2) Resample from the inputs of 1) by taking T draws from a multivariate normal distribution $N(\hat{\mu}, \hat{\Sigma})$ and estimate new input parameters $\hat{\hat{\mu}}$ and $\hat{\hat{\Sigma}}$.

3) Identify the optimal portfolio composition $\hat{\hat{X}}$ by maximizing equation (1) with the new estimators $\hat{\hat{\mu}}$ and $\hat{\hat{\Sigma}}$.

4) Repeat steps 2) and 3) 500 times.

5) Calculate the average portfolio weight vector $\hat{X}$ from the 500 different optimal weight vectors and chose $\hat{X}$ as the optimum.

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[3] The procedure of Michaud is modified slightly to take into account the insertion of the riskless asset and omitted long-only constraints.

Notice that the optimization in step 3) is not only conducted with long-only constraints but also for the unconstrained case although Michaud emphasizes the restriction of portfolio weights between 0 and 1. We advance the view that one has to compare the two procedures of Markowitz and Michaud with the same constraints. Otherwise the comparison is not admissible as the potential reduction of estimation risk could be driven by the constraints instead of the procedure itself.

4. ESTIMATION STRATEGIES

We apply six widespread estimation strategies for the determination of μ and Σ, which are composed of the different estimators we present in the following.[4] The standard estimators for μ and Σ are the maximum likelihood estimators $\hat{\mu}_{hist}$ and $\hat{\Sigma}_{hist}$, which are computed on the basis of an excess return series of length T:

$$\hat{\mu}_{hist} = \frac{1}{T} \cdot \sum_{t=1}^{T} r^{(t)} , \tag{2}$$

$$\hat{\Sigma}_{hist} = \frac{1}{T} \cdot \sum_{t=1}^{T} \left(r^{(t)} - \hat{\mu}_{hist} \right) \cdot \left(r^{(t)} - \hat{\mu}_{hist} \right)' , \tag{3}$$

where $r^{(t)}$ is the N-vector of excess returns in t. According to Turner and Hensel (1993), mean stock returns in the 1980s were statistically indistinguishable. If we, moreover, consider the strong sensitivity of changes in expected returns to portfolio weights, it might be consequent to set all expected excess returns equal to the average excess return of the considered stocks.[5] The so called grand mean $\hat{\mu}_0$ can be determined in the following way:

$$\hat{\mu}_0 = \frac{1}{T} \cdot \sum_{t=1}^{T} \sum_{j=1}^{N} r_j^{(t)} \cdot 1_N , \tag{4}$$

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[4] We adhere to the estimation strategies presented in Kempf and Memmel (2003).

[5] This procedure could be problematic if the data contain assets from different asset classes. However, our simulation study comprehends only stock market equivalent data.

where 1_N represents a N-vector with ones: $1_N = (1,\dots,1)'$. If this concept is carried forward to presumed equal values of correlations and variances of stock returns, variance-covariance matrix $\hat{\Sigma}_0$ results.

As mentioned before, Bayesian and James/Stein (JS) estimators have been developed to mitigate the estimation risk for better portfolio performance.[6] James and Stein (1961) show that the historical mean $\hat{\mu}_{hist}$ for N greater than two is inadmissible for a quadratic loss function and suggest the following estimator:

$$\hat{\mu}_{JS} = a \cdot \hat{\mu}_{hist} + (1-a) \cdot \hat{\mu}_0, \tag{5}$$

where a is a parameter which must be specified from the time series:

$$a = 1 - \frac{N-2}{T-N+2} \cdot \frac{1}{(\hat{\mu}_{hist} - \hat{\mu}_0)' \hat{\Sigma}_{hist}^{-1} (\hat{\mu}_{hist} - \hat{\mu}_0)}. \tag{6}$$

The closer the single historical expected returns are located around the grand mean, the more $\hat{\mu}_{JS}$ is shrunken towards the grand mean. Ledoit and Wolf (2003) construct a shrinkage estimator of the variance-covariance matrix:

$$\hat{\Sigma}_{LW} = b \cdot \hat{\Sigma}_{hist} + (1-b) \cdot \hat{\Sigma}_0, \tag{7}$$

where b can be calculated in the following way:

$$b = \frac{\mathrm{tr}\left(\left(\Sigma_0 - \hat{\Sigma}_{hist}\right)^2\right)}{\mathrm{tr}\left(\left(\Sigma_0 - \hat{\Sigma}_{hist}\right)^2\right) + (1/T) \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} (\hat{\sigma}_{hist,ij}^2 + \hat{\sigma}_{hist,i}^2 \cdot \hat{\sigma}_{hist,j}^2)}, \tag{8}$$

with $\mathrm{tr}(A)$ representing the trace of matrix A.

Table 1 illustrates the combinations of these six introduced estimators. Application of the maximum-likelihood estimators $\hat{\mu}_{hist}$ and $\hat{\Sigma}_{hist}$ leads to the widely used classical historical estimation strategy, denoted with (C). If we replace $\hat{\mu}_{hist}$ by the grand mean

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[6] James/Stein estimators can be seen as empirical Bayes estimators where the input parameters are taken from the time series of returns; see Gruber (1995).

$\hat{\mu}_0$ for all stocks, the solution of the classical approach complies with the minimum variance portfolio (MV). In contrast, substitution of $\hat{\mu}_{hist}$ by the James/Stein estimator $\hat{\mu}_{JS}$ results in a composition which was presented by Jorion (1985) (J). The equally weighted portfolio (EW) evolves from imposing the same distribution parameters for all stocks, thereby no stock is preferred. This corresponds to the naïve 1/N-rule for the risky assets. Ledoit and Wolf (2003) (LW) assume equal expectation values for all stocks and analyze their shrinkage estimator $\hat{\Sigma}_{LW}$ for the variance-covariance matrix. Both shrinkage estimators $\hat{\mu}_{JS}$ and $\hat{\Sigma}_{LW}$ are implemented in the approach of Frost and Savarino (1986) (FS).[7]

Table 1 about here

5. SIMULATION STUDY

5.1 Setting

In order to test the performance of the portfolio optimization techniques of Markowitz and Michaud with different estimation strategies, we perform a two-step simulation study. In the first step, "true" parameters μ and Σ are generated. In the second step, we draw realizations of excess returns for a fixed observation period from a multivariate normal distribution with parameters μ and Σ. On the basis of these drawn excess returns, we estimate the parameters μ and Σ using the above introduced estimation strategies and apply the optimization techniques under consideration. Thus, the second step corresponds to real-life application, where the input parameters μ and Σ have to be estimated. Solely a "referee" knows the true parameters due to simulation step 1. Hence, we are

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[7] The variance-covariance estimator of Frost and Savarino (1986) is not exactly the same as of Ledoit and Wolf (2003) since a third summand is added in the denominator of $\hat{b}$. Nevertheless, we implemented the estimator of Ledoit and Wolf (2003) in both cases, which is in line with Kempf and Memmel (2003).

able to evaluate the performance of different approaches with respect to the true result-ing preference values. The explained procedure is outlined in Figure 1.

Figure 1 about here

1) The choice of the true parameters μ and Σ is based on 216 monthly returns of 24 Euro Stoxx 50 stocks between May 1991 and April 2009.[8] The grand mean of the excess returns is $\mu_0 = 0.64\%$ per month, the averaged standard deviation of these stocks is $\sigma_0 = 9.43\%$ per month, and the averaged correlation is $\rho_0 = 36.41\%$. Using the latter values, we construct a variance-covariance matrix Σ_0 with the elements $\sigma_0^{\,2}$ on the di-agonal and $\sigma_0^{\,2} \cdot \rho_0$ for the off-diagonal entries. In order to achieve results that do not rely on one specific parameter setting, we use these values as a basis to generate 100 true variance-covariance matrices $\Sigma_1, \ldots, \Sigma_{100}$ for 10 stocks from a Wishart distribution with $\nu = 26$ degrees of freedom:

$$\Sigma_s \sim W_{10}\left(\frac{\Sigma_0}{\nu}, \nu\right), \text{ with } s = 1, \ldots, 100. \tag{9}$$

The corresponding expected returns $\mu_1, \ldots, \mu_{100}$ are drawn from a multivariate normal distribution

$$\mu_s \mid \Sigma_s \sim N_{10}\left(\mu_0, \frac{1}{\tau}\Sigma_s\right), \text{ with } s = 1, \ldots, 100 \tag{10}$$

with homogeneity parameter $\tau = 13$. The choice of ν and τ leads to a good match be-tween simulated and real capital market data.[9]

2) For each of these true parameter settings, we generate 100 "observable" time se-ries, each consisting of T monthly returns. In order to test the influence of different ob-

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[8] We used all stocks that are listed in the Euro Stoxx 50 at May 22th 2009 for which historical stock pric-es are available for the complete observation period.

[9] We minimize the squared difference between the variance of drawn parameters and the parameter va-riance computed from subsamples of the empirical data and obtain optimal parameter realizations of $\nu = 26.3895$ and $\tau = 13.2790$.

servation periods on the results, we choose T = 24, 36, 60, 84, 120, 168, 216 months (2–18 years). These returns are drawn from a multivariate normal distribution:

$$r_{s,i,t} \sim N_{10}\left(\mu_s, \Sigma_s\right), \text{ with } i = 1,...,100 \text{ and } t = 1,...,T \qquad (11)$$

with s,i indicating the observable time series i of the true parameter combination s.[10] Thus, for 100 true parameter settings we have in total 10,000 observable time series. The assumption of normally distributed returns could be problematic, especially in the aftermath of the subprime crisis. Therefore, we tested the return history (216 months) of the above-mentioned Euro Stoxx 50 stocks for the multivariate normal distribution. Only for three stocks the normality hypothesis had to be rejected at the 5 percent level using the KS-Test.[11] Furthermore, to account for the financial crisis, we split the sample period into two subsamples: One sample ends at June 14th 2007 and the other one starts at this time, where June 14th designates the starting point of the subprime crisis.[12]0 For none of the 24 stocks the hypothesis of normality can be rejected at the 5 percent level for the two subsamples. Hence, based on monthly data, the assumption of normally distributed returns proves to be rather unproblematic.

3) As a next step, we estimate the expected return vector $\hat{\mu}_{s,i}$ and the variance-covariance matrix $\hat{\Sigma}_{s,i}$ for each of these 10,000 time series. For this purpose, we implement the six estimation strategies of the previous section: the classical historical estimation, application of the grand mean for all stocks (resulting in the minimum variance portfolio), imposing the same distribution parameters for all stocks (resulting in the

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[10] The generation of our simulation data is similar to Simaan (1997) and Frost and Savarino (1986), who also generate multivariate normally distributed stock returns in their examinations of estimation risk.

[11] These stocks are Fortis (Belgium), Ing Groep (Netherlands), and Unicredit (Italy).

[12] The Bank of International Settlements used this date as the starting point of the subprime crisis; cf. Bank of International Settlements (2008).

equally weighted portfolio), and the estimation strategies of Ledoit/Wolf, Jorion, and Frost/Savarino.

4) Subsequently, the Markowitz portfolio optimization as well as Michaud's re-sampled efficiency are applied for each estimated parameter combination $\hat{\mu}_{s,i}$, $\hat{\Sigma}_{s,i}$. These optimization strategies are realized under consideration of riskless borrowing and lending without long-only constraints ("no constraints" case) or with long-only constraints (i.e. $0 \leq X \leq 1$, so called "long-only constraints" case). These both cases are compared with the case with present long-only constraints in the absence of riskless borrowing opportunities (i.e. $0 \leq X \leq 1$ and $x_1+\ldots+x_N=1$, so-called "no borrowing and lending" case), which is usually analyzed in the literature. As a result, we get "optimal" portfolio weights $\hat{X}_{s,i}$ for each approach and estimated parameter combination. To have a benchmark, we also compute the true optimal portfolio weights X_s^*, which would result if the true parameters were applied.

5) In order to assess and to compare the performance of all approaches, we measure the corresponding preference values $\phi_{s,i}$. As we also know the true parameters μ_s and Σ_s, we are able to determine the out-of-sample or rather the true preference values that result by application of the different approaches (Markowitz vs. Michaud in combination with the different estimation strategies):

$$\phi_{s,i} = \hat{X}_{s,i}'\cdot\mu_s - \frac{\lambda}{2}\cdot\hat{X}_{s,i}'\cdot\Sigma_s\cdot\hat{X}_{s,i}. \tag{12}$$

The better the portfolio optimization strategy in combination with the estimation of the input parameters, the closer the resulting preference level is to the true optimum, calculated as

$$\phi_s = X_s^{*'}\cdot\mu_s - \frac{\lambda}{2}\cdot X_s^{*'}\cdot\Sigma_s\cdot X_s^*. \tag{13}$$

A comparison of the preference levels of all 10,000 parameter combinations indicates which approach performs best. For this reason the following measure

$$\Phi_1^{(A,B)} = \frac{\sum_{s=1}^{100} \sum_{i=1}^{100} I\left(\phi_{s,i}^{(\text{Approach A})} > \phi_{s,i}^{(\text{Approach B})}\right)}{10,000} \tag{14}$$

serves as a performance index with $I(\cdot)$ defining the indicator function, which equals one if the event is true and otherwise equals zero. Thus, $\Phi_1^{(A,B)}$ indicates the percentage of simulations where approach A performs better than approach B. Furthermore, a comparison of the accumulated preference levels for all 100 true parameters s leads to a corresponding index

$$\Phi_2^{(A,B)} = \frac{\sum_{s=1}^{100} I\left(\sum_{i=1}^{100} \phi_{s,i}^{(\text{Approach A})} > \sum_{i=1}^{100} \phi_{s,i}^{(\text{Approach B})}\right)}{100}. \tag{15}$$

In addition, we introduce

$$\varphi^{(A)} = \frac{\sum_{s=1}^{100} \sum_{i=1}^{100} \phi_{s,i}^{(\text{Approach A})}}{10,000} \tag{16}$$

as the overall average of the preference values for approach A. The latter measure can be used to rank all approaches.

5.2 Results

First, we present the results of Markowitz' and Michaud's optimization for an observation period of 60 months, since this estimation period is of widespread use in practice.[13] We start with the "no constraints" case and the entries of Table 2 characterize the proportion of the 10,000 trials the row-strategy wins against the column-strategy (index $\Phi_1^{(\text{row,column})}$ according to (14)). Table 3 contains the corresponding comparison of the

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[13] Cf. DeMiguel *et al.* (2009).

accumulated preference levels regarding the 100 true parameters (index $\Phi_2^{(\text{row,column})}$ according to (15)).[14]

Table 2 about here
Table 3 about here

Within the Markowitz optimization, we find that the estimation strategy of Frost/Savarino outperforms the other strategies with higher preference values in $\Phi_1^{(\text{Frost/Savarino,other})} \geq 67\%$ of 10,000 scenarios. In addition, the outperformance ($\phi_{s,i}^{(\text{Frost/Savarino})} > \phi_{s,i}^{(\text{other})}$) is statistically significant at the 1 percent level.[15] After aggregating the results for each of the 100 true parameters, the result is even more evident with $\Phi_2^{(\text{Frost/Savarino,other})} \geq 96\%$ wins. The second best performing approach is Jorion, the classical historical estimator is on rank 3. The other approaches show a rather poor performance. Thus, the loss of information resulting from using the grand mean instead of the individual means outweighs the benefit from pooling the data, which reduces the estimation error.

The results within Michaud's resampling technique are similar, as the estimation strategy of Frost/Savarino followed by the approach of Jorion outperforms the other strategies. Only for the classical historical approach the results vary considerably as it does not show a convincing performance. The grey areas of the tables compare the optimizations of Markowitz and Michaud. Dark-grey areas compare one strategy combined with the optimization of Markowitz and the same strategy combined with the optimization of Michaud. Having a look at these results, we detect that the standard optimization procedure of Markowitz does a better job than Michaud for each pair of identical estimation techniques, which is in each case significant at the 1 percent level.

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[14] A risk aversion coefficient of $\lambda = 2$ was used to generate the results in the tables and charts.

[15] Significance is tested with the Wilcoxon signed-rank test.

If the same analyses are carried out for the optimizations with long-only constraints, most results remain equal (see Table 4 and 5), e.g. the estimation approaches of Frost/Savarino and Jorion outperform the other approaches and the standard optimization of Markowitz dominates the procedure of Michaud. Interestingly, the classical historical estimator seems to benefit most from the long-only constraints and evidently outperforms the remaining estimators within the approaches of Markowitz and Michaud. It appears as if this is a consequence of eliminated outliers, which only occur if short sales are allowed.

Table 4 about here
Table 5 about here

The results for the "no borrowing and lending" case with $0 \leq X \leq 1$, which is also the

original implementation of Michaud, are shown in Table 6 and 7. Within the optimization of Markowitz, the estimation approaches of Frost/Savarino and Jorion still lead to the best results. However, the classical historical estimator performs almost as good as these estimators. In this setting, the comparison of Markowitz and Michaud leads to ambiguous outcomes. The equally weighted as well as the minimum variance portfolio perform slightly better in the optimization procedure of Michaud, but the optimization of Markowitz wins for the other estimators, including the classical historical estimator, which leads to the best results within Michaud's procedure.

Table 6 about here
Table 7 about here

After dealing with an observation period of 60 months, we demonstrate the impact of a varying time horizon. In Figure 2 and 3 the respective results in the "no constraints" case are visualized. The exhibited average preference values are the overall averages resulting from the 10,000 optimizations (measure $\varphi^{(A)}$ according to (16)). In addition to the considered strategies, the figures contain a strategy "Max", which stands for the

knowledge of the true parameters and consequently characterizes the average of the maximum achievable preference values. As expected, it can be found that the average preference values are strictly increasing with longer observation periods as the estimation error for both parameters is decreasing.[16] However, the degree of this behavior is very different for the approaches: For very short observation periods, the equally weighted and Ledoit/Wolf estimation strategy lead to relatively high preference values. By contrast, if the observation period becomes longer, at the latest for the case of 84 months, the approaches where the return of every stock is estimated individually perform better. Although the approach of Frost/Savarino performs slightly poorer than the classical historical estimator and the approach of Jorion for very long observation periods, it is the approach with the best overall performance as it works very well for short observation periods, too. This is true for both the optimization procedure of Markowitz and the procedure of Michaud.

Figure 2 about here
Figure 3 about here

Figure 4 and 5 show the corresponding results in the "long-only constraints" case. As expected, the maximum achievable average preference value "Max" is significantly lower but the constraint leads to a cut-off of very low preference levels since extreme weights are avoided. This results in better outcomes for short observation periods to a large extent. Estimators which rely on individual parameters for expected returns instead of grand means benefit most from this characteristic as the usage of grand means results in more balanced weights whether or not constraints are present. In comparison, the approach of Michaud cannot benefit from these constraints to the same degree but the overall performance is very similar to Markowitz.

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[16] Since we assume stationary returns, the estimation error converges to zero if the observation period goes to infinity.

In Figure 6 and 7 the "no borrowing and lending" case is presented. The stronger constraints lead to an additional reduction of the maximum preference level but also to a cut-off of low preference levels. In this setting, the length of the observation period has a rather low impact on the preference value. This shows that estimation errors play a minor role if this constraint is implemented. A comparison of Figure 6 and 7 confirms our finding that both optimization procedures lead to similar results when the portfolio weights are constrained to $0 \leq X \leq 1$.

Next, we compare the performance of the Markowitz and Michaud optimization in our 3 settings (1: no constraints, 2: long-only constraints, 3: no borrowing and lending) for the classical historical estimator. As can be seen in Figure 8, it is advisable to implement a long-only constraint or even the $0 \leq X \leq 1$ constraint for both the Markowitz and Michaud optimization if the chosen observation period is rather short, e.g. smaller than 60 months. However, if the observation period is longer, the average preference value is considerably higher in the unconstrained case. Furthermore, in setting 1 and 2 the average preference loss of Michaud in comparison to Markowitz is quite high for small observation samples, whereas the preference values are similar for long estimation periods. In the third setting, the results of Markowitz and Michaud are almost identically for all presented estimation periods. This supports previous studies which show that for an observation period of 216 months both procedures lead to almost identical results if setting 2 or 3 is implemented. However, our analyzes show that in all situations where estimation errors are most relevant, that is for the unconstrained setting and the long-

only constraints in combination with short observation periods, Markowitz clearly outperforms Michaud.

Figure 8 about here

Finally, we analyze the best performing approach, the estimation strategy of Frost/Savarino. This strategy not only performs very good for long observation periods but also outperforms all other approaches for short observation periods, regardless of whether constraints are considered or not. Again, we find an advantage of the standard Markowitz optimization (see Figure 9). In contrast to the classical historical estimator, the curves "no constraints", "long-only constraints", and "no borrowing and lending" do not intersect. The preference level for the unconstrained case is higher for all considered observation periods, even if the difference is lower for small samples. Thus, in most situations it seems advisable to implement the Frost/Savarino estimation strategy without constraints using the Markowitz optimization.

Figure 9 about here

We conducted several robustness checks to ascertain meaningful results. First, we tested several common risk aversion coefficients (λ = 1, 1.5, 2, 2.5, 3), and 150 instead of 100 observable time series per true parameter. All results presented here were robust to these modifications. Second, we applied all analyses to a larger data set consisting of 20 instead of 10 stocks, which mainly influences the complexity of the variance-covariance matrix. In the unconstrained case, we detect the maximum average preference value to be significantly higher compared to the universe of 10 stocks, which is a consequence of the improved diversification possibilities. Consequently, the average preference values for estimation strategies which apply individual means are higher for long observation periods. For short observation periods the opposite is true as the negative impact of the more complex variance-covariance matrix – a higher estimation error – becomes appar-

ent. In this case, we find that it is recommendable to rely on strategies using the shrinkage estimator $\hat{\Sigma}_{LW}$ for the variance-covariance matrix. Again, the estimation strategy of Frost/Savarino shows a very stable performance and Markowitz outperforms Michaud. The results using long-only constraints correspond to the results for 10 stocks to a large extent except the fact that the higher estimation error leads to reduced average preference values for small observation periods.

6. CONCLUSION

In the following our findings are summarized:

1. The optimization procedure of Markowitz possesses in nearly every case a superior performance compared to Michaud. This is especially true in situations where estimation errors are most serious, notably the unconstrained and long-only constrained settings with short observation periods.

2. For short observation periods strategies using the grand mean perform quite well. Moreover, the effect of implementing constraints is rather small for these estimators. However, for longer periods the estimation of individual expected returns pays off.

3. Similarly, for strategies estimating individual expected returns, applying long-only constraints or weights between 0 and 1 is reasonable when the estimation is based on short observation periods. However, for longer observation periods, the inclusion of constraints involves considerable reduced preference values.

4. Altogether, the estimation strategy of Frost/Savarino has shown the best overall performance (no constraints/long-only constraints/no riskless borrowing and lending; Markowitz/Michaud; short/long observation period). For this strategy, it is profitable (if possible) not to insert long-only constraints irrespective of the estimation period, as the average preference value for the unconstrained case is larger than for the constrained

cases even for short observation periods. For longer time series of returns, the advantage of the unconstrained case increases.

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Table 1

Composition of the Implemented Estimation Strategies

Estimator	$\hat{\mu}_{hist}$	$\hat{\mu}_0$	$\hat{\mu}_{JS}$
$\hat{\Sigma}_{hist}$	Classic (C)	Minimum Variance Portfolio (MV)	Jorion (J)
$\hat{\Sigma}_0$		Equally Weighted Portfolio (EW)	
$\hat{\Sigma}_{LW}$		Ledoit/Wolf (LW)	Frost/Savarino (FS)

Table 2

Performance Index Φ_1 to Compare Different Approaches ("no constraints" case with T=60)

			Markowitz						Michaud				
	↓ wins against →	C	EW	MV	LW	J	FS	C	EW	MV	LW	J	FS
Markowitz	C	-	63.9% ***	73.8% ***	63.5% ***	10.1% ***	20.8% ***	99.4% ***	64.1% ***	78.2% ***	63.9% ***	49.1% ***	31.0% ***
	EW	36.1% ***	-	70.9% ***	46.5% ***	23.0% ***	5.7% ***	64.3% ***	60.1% ***	76.0% ***	50.3%	35.7% ***	7.8% ***
	MV	26.2% ***	29.1% ***	-	26.5% ***	13.8% ***	2.6% ***	52.6% ***	29.2% ***	85.2% ***	28.6% ***	24.5% ***	3.6% ***
	LW	36.5% ***	53.5% ***	73.5% ***	-	23.6% ***	6.2% ***	64.4% ***	55.0% ***	78.0% ***	54.8% ***	36.1% ***	8.5% ***
	J	89.9% ***	77.0% ***	86.2% ***	76.4% ***	-	32.4% ***	97.4% ***	77.2% ***	88.7% ***	77.1% ***	89.1% ***	44.2% ***
	FS	79.2% ***	94.3% ***	97.4% ***	93.8% ***	67.6% ***	-	93.9% ***	94.5% ***	97.9% ***	94.3% ***	78.8% ***	80.8% ***
Michaud	C	0.6% ***	35.7% ***	47.4% ***	35.6% ***	2.6% ***	6.1% ***	-	35.7% ***	53.0% **	35.6% ***	8.4% ***	11.7% ***
	EW	35.9% ***	39.9% ***	70.8% ***	45.1% ***	22.8% ***	5.5% ***	64.3% ***	-	76.1% ***	43.2% ***	35.6% ***	7.5% ***
	MV	21.8% ***	24.0% ***	14.9% ***	22.0% ***	11.4% ***	2.1% ***	47.0% **	23.9% ***	-	23.6% ***	18.9% ***	2.8% ***
	LW	36.2% ***	49.7%	71.4% ***	45.2% ***	22.9% ***	5.7% ***	64.4% ***	56.9% ***	76.4% ***	-	35.7% ***	7.8% ***
	J	50.9% ***	64.3% ***	75.5% ***	63.9% ***	10.9% ***	21.2% ***	91.6% ***	64.5% ***	81.1% ***	64.3% ***	-	31.9% ***
	FS	69.0% ***	92.2% ***	96.4% ***	91.6% ***	55.8% ***	19.2% ***	88.3% ***	92.5% ***	97.2% ***	92.2% ***	68.1% ***	-

*   Significant at the 10 percent level.

**   Significant at the 5 percent level.

*** Significant at the 1 percent level.

Table 3

Performance Index Φ_2 to Compare Different Approaches ("no constraints" case with T=60)

		Markowitz						Michaud					
	↓ wins against →	C	EW	MV	LW	J	FS	C	EW	MV	LW	J	FS
Markowitz	C	-	63%	89%	63%	0%	0%	100%	63%	94%	62%	59%	3%
	EW	37%	-	87%	42%	13%	2%	99%	100%	92%	64%	42%	3%
	MV	11%	13%	-	11%	1%	0%	84%	13%	100%	13%	8%	0%
	LW	37%	58%	89%	-	14%	4%	99%	63%	95%	64%	42%	3%
	J	100%	87%	99%	86%	-	3%	100%	87%	99%	87%	100%	11%
	FS	100%	98%	100%	96%	97%	-	100%	98%	100%	98%	100%	95%
Michaud	C	0%	1%	16%	1%	0%	0%	-	1%	33%	1%	0%	0%
	EW	37%	0%	87%	37%	13%	2%	99%	-	92%	38%	42%	2%
	MV	6%	8%	0%	5%	1%	0%	67%	8%	-	8%	2%	0%
	LW	38%	36%	87%	36%	13%	2%	99%	62%	92%	-	42%	2%
	J	41%	58%	92%	58%	0%	0%	100%	58%	98%	58%	-	3%
	FS	97%	97%	100%	97%	89%	5%	100%	98%	100%	98%	97%	-

Table 4

Performance Index Φ_1 to Compare Different Approaches ("long-only constraints" case with T=60)

		Markowitz						Michaud					
	↓ wins against →	C	EW	MV	LW	J	FS	C	EW	MV	LW	J	FS
Markowitz	C	-	80.5% ***	83.4% ***	79.7% ***	36.6% ***	43.4% ***	85.7% ***	82.6% ***	86.4% ***	82.3% ***	68.4% ***	60.7% ***
	EW	19.5% ***	-	39.0% ***	26.9% ***	11.1% ***	7.1% ***	27.6% ***	61.4% ***	64.6% ***	52.7% ***	20.4% ***	13.2% ***
	MV	16.6% ***	61.0% ***	-	22.2% ***	8.3% ***	6.1% ***	23.5% ***	48.6% ***	70.4% ***	47.8% ***	15.9% ***	12.1% ***
	LW	20.3% ***	73.1% ***	77.8% ***	-	11.8% ***	7.6% ***	28.4% ***	61.4% ***	67.4% ***	60.9% ***	21.1% ***	14.6% ***
	J	63.4% ***	89.0% ***	91.7% ***	88.2% ***	-	46.1% ***	77.9% ***	86.6% ***	90.1% ***	86.3% ***	81.8% ***	66.6% ***
	FS	56.6% ***	92.9% ***	93.9% ***	92.4% ***	53.9% ***	-	71.7% ***	90.5% ***	91.6% ***	90.2% ***	72.4% ***	80.2% ***
Michaud	C	14.3% ***	72.4% ***	76.5% ***	71.6% ***	22.1% ***	28.4% ***	-	73.2% ***	79.5% ***	72.8% ***	37.5% ***	40.6% ***
	EW	17.4% ***	38.6% ***	51.5% ***	38.6% ***	13.4% ***	9.5% ***	26.8% ***	-	58.5% ***	39.8% ***	19.4% ***	11.8% ***
	MV	13.6% ***	35.4% ***	29.6% ***	32.6% ***	9.9% ***	8.4% ***	20.5% ***	41.5% ***	-	34.6% ***	12.4% ***	10.4% ***
	LW	17.7% ***	47.4% ***	52.2% ***	39.1% ***	13.7% ***	9.8% ***	27.2% ***	60.2% ***	65.4% ***	-	19.6% ***	12.2% ***
	J	31.6% ***	79.7% ***	84.1% ***	78.9% ***	18.2% ***	27.6% ***	62.5% ***	80.6% ***	87.6% ***	80.4% ***	-	47.3% ***
	FS	39.3% ***	86.8% ***	87.9% ***	85.5% ***	33.4% ***	19.8% ***	59.4% ***	88.2% ***	89.6% ***	87.8% ***	52.7% ***	-

*   Significant at the 10 percent level.

**   Significant at the 5 percent level.

*** Significant at the 1 percent level.

Table 5

Performance Index Φ_2 to Compare Different Approaches ("long-only constraints" case with T=60)

	↓ wins against →	Markowitz						Michaud					
		C	EW	MV	LW	J	FS	C	EW	MV	LW	J	FS
Markowitz	C	-	82%	86%	83%	11%	15%	100%	83%	86%	82%	76%	41%
	EW	18%	-	62%	36%	15%	9%	29%	83%	90%	65%	20%	9%
	MV	14%	38%	-	14%	11%	9%	21%	51%	97%	48%	12%	10%
	LW	17%	64%	86%	-	15%	9%	30%	77%	93%	76%	20%	12%
	J	89%	85%	89%	85%	-	27%	100%	89%	92%	87%	97%	61%
	FS	85%	91%	91%	91%	73%	-	100%	94%	93%	94%	96%	96%
Michaud	C	0%	71%	79%	70%	0%	0%	-	72%	84%	72%	13%	14%
	EW	17%	17%	49%	23%	11%	6%	28%	-	80%	34%	18%	8%
	MV	14%	10%	3%	7%	8%	7%	16%	20%	-	16%	10%	8%
	LW	18%	35%	52%	24%	13%	6%	28%	66%	84%	-	18%	8%
	J	24%	80%	88%	80%	3%	4%	87%	82%	90%	82%	-	22%
	FS	59%	91%	90%	88%	39%	4%	86%	92%	92%	92%	78%	-

Table 6

Performance Index Φ_1 to Compare Different Approaches ("no borrowing and lending" case with T=60)

	↓ wins against →	Markowitz						Michaud					
		C	EW	MV	LW	J	FS	C	EW	MV	LW	J	FS
Markowitz	**C**	-	99.5% ***	98.0% ***	99.0% ***	40.2% ***	43.5% ***	72.0% ***	99.5% ***	98.2% ***	99.5% ***	78.1% ***	78.7% ***
	EW	0.5% ***	-	51.3% *	47.2% ***	0.4% ***	0.6% ***	0.1% ***	47.3% *	51.0%	45.3% ***	0.3% ***	0.2% ***
	MV	2.0% ***	48.7% *	-	45.8% ***	1.7% ***	1.9% ***	1.3% ***	48.7% *	44.8% ***	47.8% ***	1.3% ***	2.6% ***
	LW	1.0% ***	52.8% ***	54.2% ***	-	0.8% ***	1.0% ***	0.4% ***	52.8% ***	53.9% ***	52.2% ***	0.3% ***	1.0% ***
	J	59.8% ***	99.6% ***	98.3% ***	99.2% ***	-	43.2% ***	74.0% ***	99.6% ***	98.5% ***	99.6% ***	80.0% ***	80.0% ***
	FS	56.5% ***	99.5% ***	98.1% ***	99.0% ***	56.8% ***	-	72.6% ***	99.5% ***	98.2% ***	99.4% ***	78.0% ***	80.8% ***
Michaud	**C**	28.0% ***	99.9% ***	98.7% ***	99.6% ***	26.0% ***	27.4% ***	-	99.9% ***	98.9% ***	99.9% ***	83.9% ***	80.7% ***
	EW	0.5% ***	52.7% *	51.3% *	47.2% ***	0.4% ***	0.6% ***	0.1% ***	-	51.0% ***	45.3% ***	0.3% ***	0.2% ***
	MV	1.8% ***	49.0%	55.2% ***	46.1% ***	1.5% ***	1.9% ***	1.2% ***	49.0%	-	48.3% ***	1.2% ***	2.4% ***
	LW	0.5% ***	54.7% ***	52.2% ***	47.8% ***	0.4% ***	0.6% ***	0.1% ***	54.7% ***	51.7% **	-	0.3% ***	0.3% ***
	J	21.9% ***	99.7% ***	98.7% ***	99.7% ***	20.0% ***	22.0% ***	16.2% ***	99.7% ***	98.8% ***	99.7% ***	-	58.9% ***
	FS	21.3% ***	99.8% ***	97.4% ***	99.0% ***	20.0% ***	19.2% ***	19.3% ***	99.8% ***	97.7% ***	99.7% ***	41.2% ***	-

*   Significant at the 10 percent level.

**   Significant at the 5 percent level.

**   Significant at the 1 percent level.

27

Table 7

Performance Index Φ_2 to Compare Different Approaches ("no borrowing and lending" case with T=60)

	↓ wins against →	Markowitz						Michaud					
		C	EW	MV	LW	J	FS	C	EW	MV	LW	J	FS
Markowitz	C	-	100%	98%	100%	53%	49%	86%	100%	99%	100%	92%	95%
	EW	0%	-	49%	48%	0%	0%	0%	46%	48%	47%	0%	0%
	MV	2%	51%	-	46%	2%	2%	2%	51%	39%	49%	1%	2%
	LW	0%	52%	54%	-	0%	0%	0%	52%	55%	52%	0%	1%
	J	47%	100%	98%	100%	-	51%	89%	100%	99%	100%	94%	95%
	FS	51%	100%	98%	100%	49%	-	80%	100%	98%	100%	92%	96%
Michaud	C	14%	100%	98%	100%	11%	20%	-	100%	99%	100%	98%	92%
	EW	0%	54%	49%	48%	0%	0%	0%	-	48%	47%	0%	0%
	MV	1%	52%	61%	45%	1%	2%	1%	52%	-	51%	1%	2%
	LW	0%	53%	51%	48%	0%	0%	0%	53%	49%	-	0%	0%
	J	8%	100%	99%	100%	6%	8%	2%	100%	99%	100%	-	60%
	FS	5%	100%	98%	99%	5%	4%	8%	100%	98%	100%	40%	-

Figure 1

Procedure of the Simulation Study

(1) 100 „true" parameter combinations

(2) 100x100 „observable" time series of returns
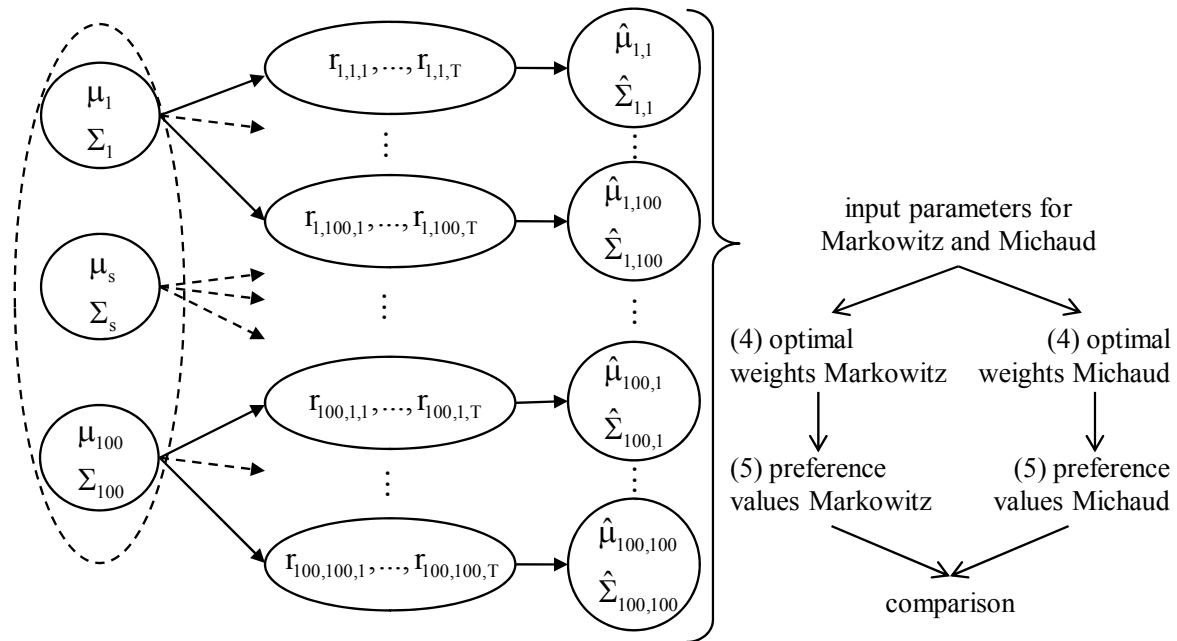
(3) parameter estimation for each time series

Figure 2

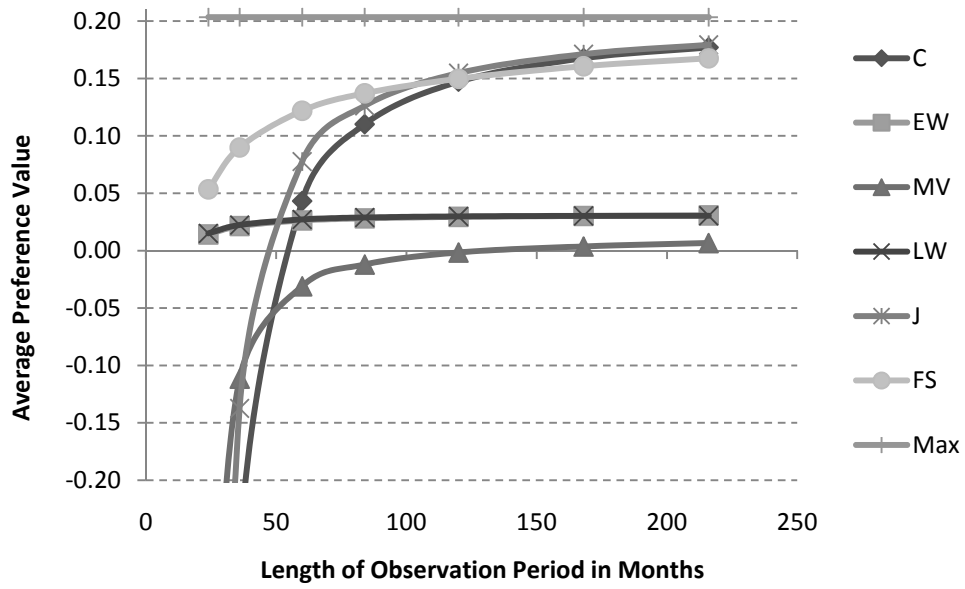Average Preference Values φ for Markowitz ("no constraints" case)

Figure 3

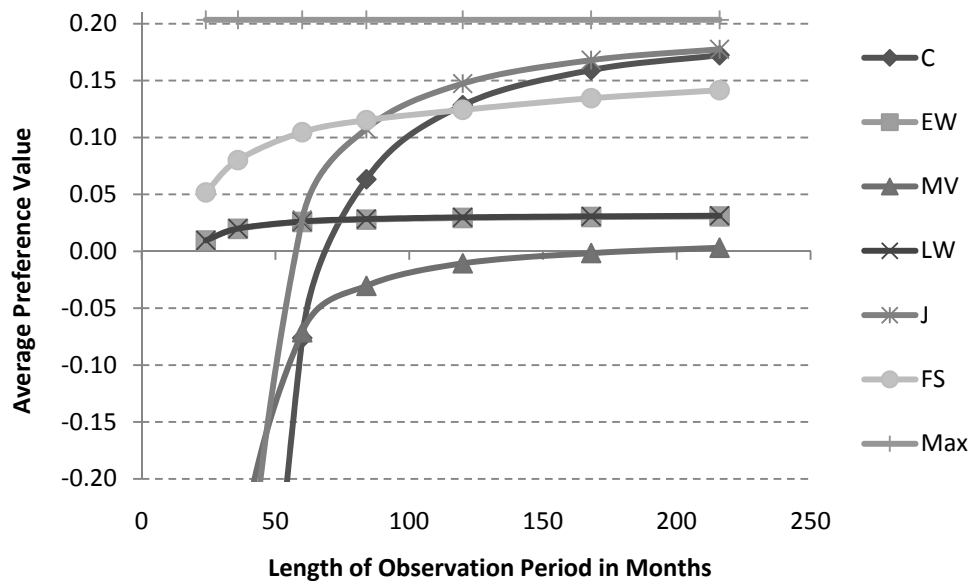Average Preference Values φ for Michaud ("no constraints" case)

Figure 4

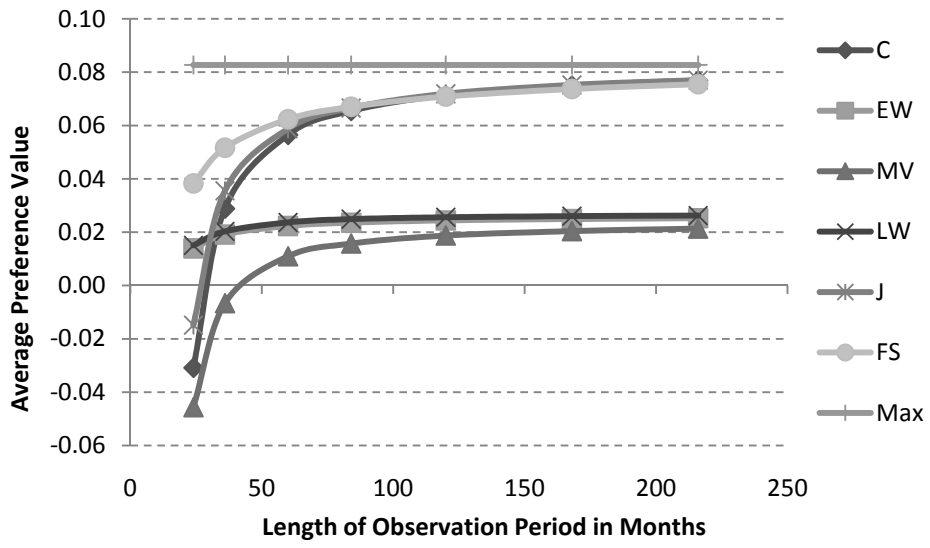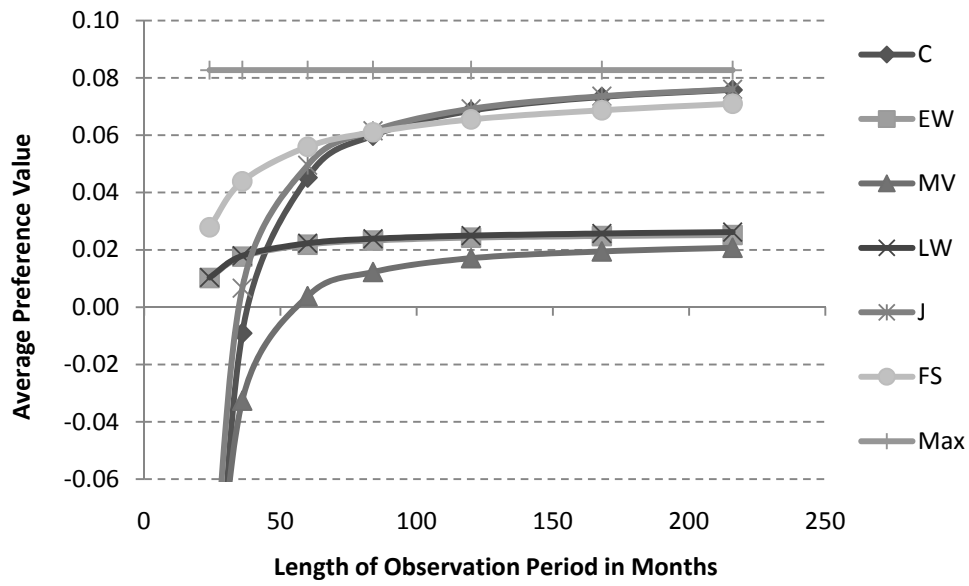Average Preference Values φ for Markowitz ("long-only constraints" case)

Figure 5

Average Preference Values φ for Michaud ("long-only constraints" case)

Figure 6

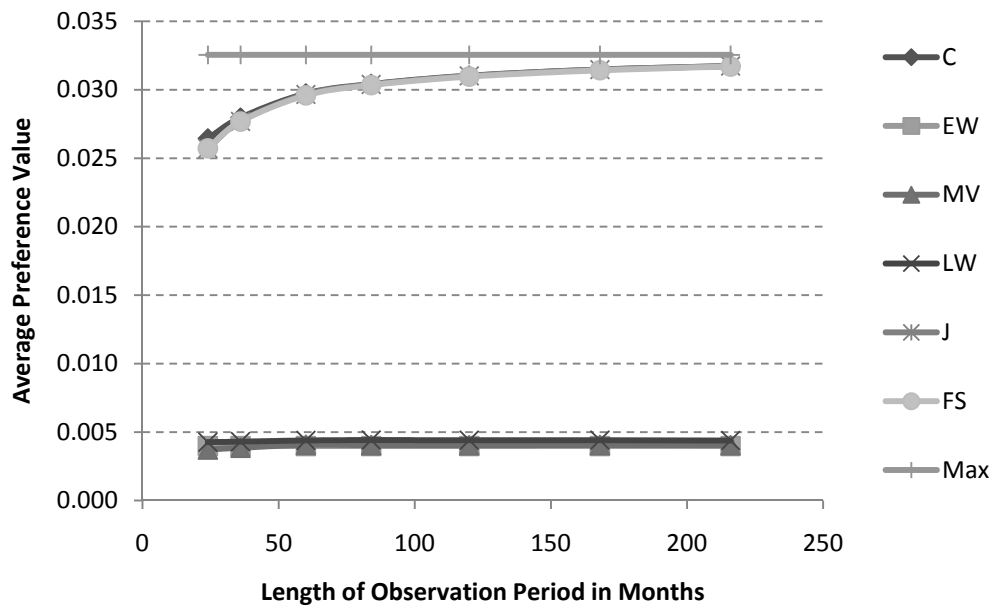Average Preference Values φ for Markowitz ("no borrowing and lending" case)

Figure 7

Average Preference Values φ for Michaud ("no borrowing and lending" case)
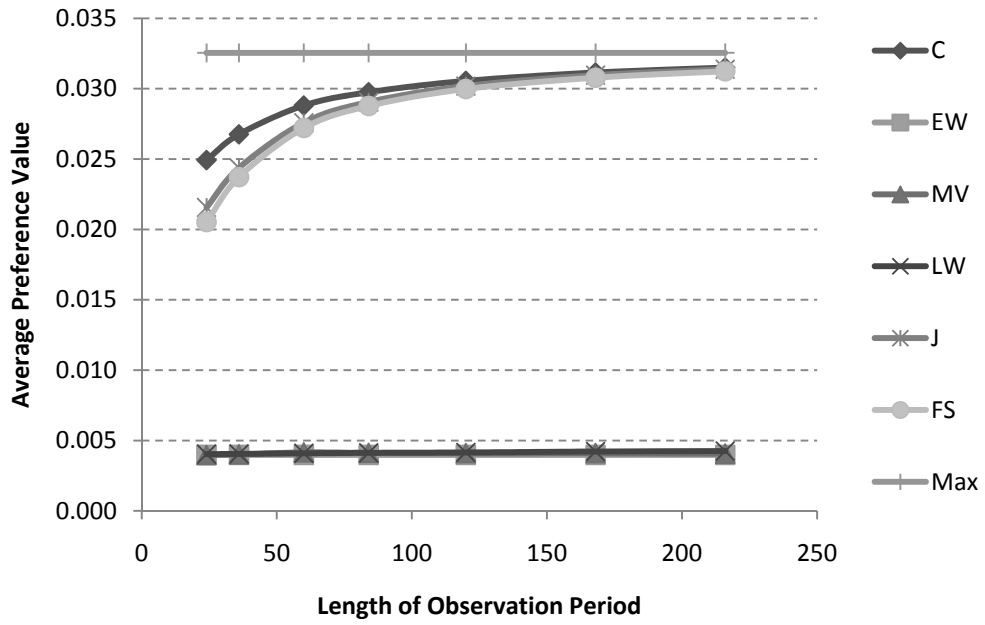
Figure 8

Average Preference Values φ (Classical Historical Estimation Strategy)
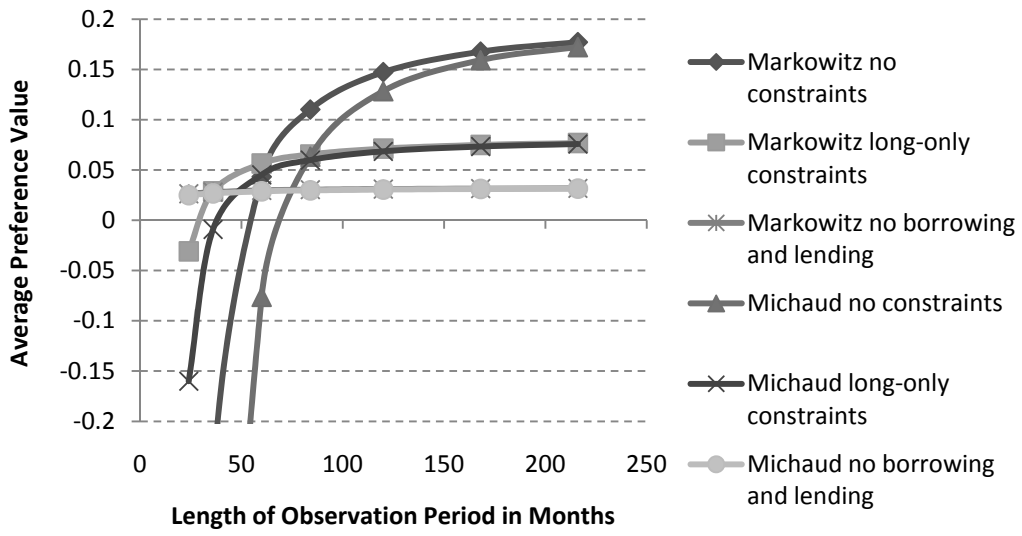
Figure 9

Average Preference Values φ (Estimation Strategy of Frost/Savarino)