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Computer Modeling of Historical Processes by Means of Fractal Geometry

Dmitry Zhukov & Sergey Lyamin *

Abstract: »Computermodellierung von historischen Prozessen mittels fraktaler Geometrie«. This article is dedicated to application of theory and methodology of fractal geometry in historical research. The article represents the concrete historic issue mathematical model, specifically: the dynamics of the conscience and social environment modernization. On the basis of this model a computer program, which generates fractal images of attractors, attractor basins, and phase transformations of the social systems studied subject to user-entered numerical indicators of certain factors, has been developed. The article represents the principal approaches to the qualitative interpretation of the fractal images obtained.

Keywords: fractal modeling, computer modeling, fractal geometry, Benoit Mandelbrot, modernization, traditional society.

1. General Principles of Fractal Modeling

The theory and methodology of fractal geometry have been settled in social and humanitarian spheres of knowledge for about thirty years. However, in most historical studies the matter concerns not concrete fractal models but only the assertion of self-similarity of different levels of the social systems considered and (or) a certain cycling of tendencies. Nevertheless, we suppose fractal methodology allows to produce the concrete mathematical models of historical phenomena and processes as well as to perform computer experiments imitating these phenomena and processes.

Fractal geometry appeared in 1977 after the publication of “Fractals: Form, Chance, and Dimension” by Benoit Mandelbrot (Mandelbrot 1977) (revised edition – “The Fractal Geometry of Nature” (Mandelbrot 1982)). In the simplest case a fractal is a special type of geometrical figure, and “fractal” is a characteristic of a structure, phenomenon or process possessing properties of a

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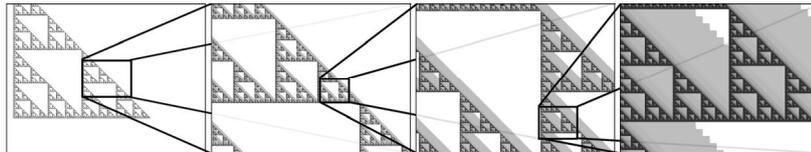
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fractal. The definition of a fractal given by B. Mandelbrot himself is as follows: “Fractal is called a structure consisting of parts which are in a certain sense similar to the whole”. In other words, one of the properties of fractals is self-similarity. This means that a small part of a fractal contains information about the whole fractal.

Thus, it is a so-called scale invariance that is characteristic of a fractal. “No matter on what scale we consider a fractal we always see the same or anyhow something similar” (Zhikov 1996, 109) (see Figure 1).

Figure 1: Scale Invariance of Fractal



The easiness of making the fractals similar to real objects enables fractal geometry to be a convenient means for modeling the reality. The logical question arises, how broad the fractal modeling application field is, and how large the number of fractal-like structures in nature is. Benoit Mandelbrot replies categorically: it is the fractal manner of self-organization that is typical for nature.

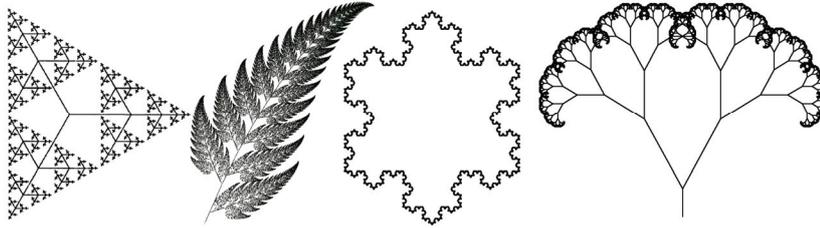
Indeed, one can see fractals in the borders of clouds and coastlines, in turbulent flows, in the cracks of rocks, in ice-ferns and in snowflakes, in roots, leaves and branches of plants, in tissues and organs of a living being, including a human, in the dynamics of price changes at stock exchange, in recurring social processes, in the organization of political and social structures, etc. Owing to the fact that fractals are represented in nature, the methods of fractal geometry have penetrated and are penetrating into different (if not into all) branches of science (Gleick 1987).

To imagine the variety of fractals we will use the generally accepted classification of them (Morozov 1999). According to the method of formation fractals are usually subdivided into geometric and algebraical ones.

Geometrical fractals are the most obvious. They are produced with the help of a broken line or a surface called a generator. The generator repeats itself over scales (see Figure 2).

Algebraical fractals evolve as a result of certain mathematical operations. Numerical results of these operations are regarded as coordinates of points which are set in the coordinate plane. These points make a figure – fractal. It came as a surprise for the researchers that it was possible to generate complex nontrivial fractal structures by means of simple mathematical algorithms.

Figure 2: Geometrical Fractals



Algebraical fractals are used, among other things, in the studies of dynamic systems. Non-linear dynamic systems can exist in several stable states. The state in which the system is after some time depends, among other things, on its reference state. That is why every stable state (attractor) possesses a certain field of reference states starting from which the system will necessarily get into the considered final state (this attractor).

The researchers bring forward a river basin as a metaphor for such phenomena. The attractor of the system is a river mouth here. The initial conditions are springs. Wherever in the basin the springs are located, the water coming out of them will certainly be at the mouth. There is a watershed between the basins of different rivers. The mouth of what river the water of a certain spring gets depends on its location in regard to the watershed. The characteristics of reference states and attractors of a system can be expressed in numbers; these values can be taken for the coordinates of points which form a certain figure in the coordinate plane. It turns out that the images of attractors as well as images of total reference states of these attractors (“catchment” basins) look like fractals in many cases.

The constructing of algebraical fractals allows to model processes in the phase space. The phase space can be represented as a coordinate plane in which the values of a certain system parameter are plotted along every axis. If we aim at studying of two parameters of the system, we can set the point in the plane the coordinates of which correspond to the parameter values. The change of parameters through time (that is the evolution of the system) is marked as the point motion trajectory in the phase space.

The phase space is a convenient instrument for studying attractors. Attractors possess the most important quality – stability. The simplest attractors can be depicted as fixed points and closed curves. Such attractors describe the behavior of the systems which have reached a stable state or repeat themselves continuously. We can also indicate the initial conditions of the system in the phase space – the point it starts from. Every attractor of the system (there can be several of them) has its own domain of initial conditions in the phase space.

When constructing an algebraical fractal a point is plotted on the graph not when it satisfies the equation but when it generates a certain type of behavior.

For example, a starting point attractor can lie in certain specified visible bounds of the phase space, and can tend to infinity without control.

The point trajectory in the phase space is a numerical series (that is a series of coordinates) calculated in the course of iterations of this or that mathematical formula. The iteration process in this case is a folded insertion of the result obtained with this formula before. When constructing algebraical fractals formulas with complex numbers are used. A complex number consisting of two parts can be represented as a point in the plane of complex numbers. If we insert the initial number into the formula, we will calculate the second number which in its turn will be used to calculate the third number and so on. Thus, we obtain the point motion trajectory and can study the attractor of this trajectory.

Let us consider the plane of complex numbers as the phase space and apply some kind of a grid with a certain size of mesh or pitch to it. We will test the points at the nodes of this grid one by one, that is: we will insert them as the initial values into the iterated formula and find the attractor of every point out, i.e. where every point will move to after a considerable number of iterations. We can study attractors and their basins in that way.

So, having described mathematically the interaction of several system factors one can predict the results of its development with high probability. Of course, it is done only with the help of a computer experiment. Having made the fractal model of an object and carrying out a computer experiment with a fractal we can reveal and predict the behavior of its real prototype exactly enough.

2. Mathematical Model of Mental Fractal

In the course of our studies a mathematical model which describes the processes of modernization of urban social sphere and city people conscience in Russia in the second half of XX century (on the materials of the city of Tambov) has been worked out.

Let us take the plane of complex numbers. The continuity of real and imaginary parts of the complex number can be likened to the continuity of individual conscience and the environment in which it (the conscience) is placed. Therefore, let us assume that k (is plotted along the y -axis) expresses a certain characteristic of individual conscience and d (is plotted along the x -axis) expresses a certain characteristic of the environment. We will fill up with certain content the expression "a certain characteristic": d is the degree of environment modernization, k is the degree of conscience modernization.

At that, the value equal to -1 and 1 (both for k and d) is the boundary which divides between the traditional environment-conscience and the modernized one. Thus, all values in the ranges $-1 \leq k \leq 1$ и $-1 \leq d \leq 1$ attest to the domination of modernized features. The closer the values of k and d to 0 , the higher traditionalism is.

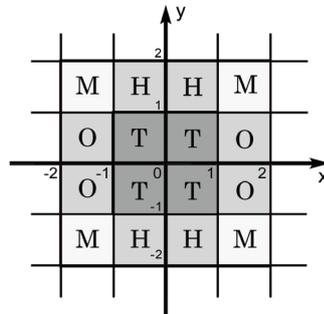
For example, a person with the characteristic of conscience $k = 1,5$ is more traditional than a person with the characteristic of conscience $k = -1,7$, though both of them bear the modernized conscience as compared to a person with $k = 0,5$.

It is necessary to specify here that since the scale of traditional conscience-environment along the x -axis and along the y -axis is in the range $[-1; 1]$, then, correspondently, the scale of the modernized conscience-environment along the x -axis and the y -axis must be within numerically equal interval $(1; 2]$, $[-2; -1)$. It is necessary to accept as a condition that $k > 2$ and $k < -2$ as well as $d > 2$ and $d < -2$ do not exist. In other words, if one or both characteristics of environment-conscience are outside the fixed range $[-2; 2]$ then such environment-conscience characteristics do not exist at all or they are not integrated into the system of functional relations and the historical context; and owing to that it is possible and necessary not to take such characteristics into account in the model.

So, the points of this complex plane are the conscience placed into the environment and at the same time the environment surrounding the conscience. Hence, every type of environment-conscience has two coordinates. All possible types are set in the square with coordinates of the diagonal $(-2; -2)$, $(2; 2)$. At that, one can differentiate several domains in this square.

The T-domain – people with traditional conscience in the traditional environment. The M-domain – people with modernized conscience in the modernized environment. The O-domain (“migrant in city”) – all people with traditional conscience in the modernized environment. The N-domain (“narodnik in village”) – all people with modernized conscience in the traditional environment (see Figure 3).

Figure 3: Domains of the Complex Plane Studied



We will further denote the individual type of environment-conscience by a complex number Z . Our task is to express the dynamics of environment-conscience mathematically. For that it is necessary to analyze and formalize the factors which influence the dynamics of Z_n . One of such factors is the influence of the immediate environment on a person – people with similar environment-

conscience characteristics. With a reasonable degree of accuracy this factor can be denoted as Z_n . In everyday life a person as a rule has to do with his like. Especially it is typical for Russia in the time of Great Reforms when the social structure had distinct corporate features.

The effect of the change of the environment and human conscience under the influence of closest (identical) environment can be expressed by the formula:

$$Z_{n+1} = Z_n^2$$

Here indices n and $n+1$ express different states of one and the same environment-conscience type – before and after the interaction with his like environments-consciences. For example, Z_1 type after the interaction with Z_1^2 acquires new characteristics Z_2 . At next stage the type interacts again with its like – with those which possess characteristics Z_2 as well as it itself. The result of such interaction is $Z_3 = Z_2^2$. Finally we obtain the continuity of points in the complex plane. This continuity will reflect the dynamics of the environment-conscience change under the influence of one of the factors, namely under the influence of interaction with its like.

But a person interacts not only with his like. He interacts with environments-consciences which are different from the environment-conscience of his own.

Meanwhile, the formula $Z_{n+1} = Z_n^2$ reflects the interaction of environments-consciences in a maximum stable, that is a hypothetic society – in the society which is not intermixed and is unaffected from the outside. This is the society where every person interacts with his like in his small group. In order to correct this effect it is necessary to multiply Z_n^2 by the anomalous diffusion coefficient (A) which expresses the result of breach of social structure absolute stability.

The fact is that in after-reform Russia a certain removal of social strata occurred what had been caused first of all by the processes of urbanization and migration of agrarian population to city. This led to the traditionalization of environment-conscience of urban population on the whole. To reflect this factor we introduce the anomalous diffusion coefficient (A).

A is a real number in the range $[0;2]$ where all values $A > 1$ reflect the modernization of environments-consciences following the destabilization of social structure, and all values $A < 1$ express the traditionalization of environments-consciences following the destabilization. When multiplying A by Z_n^2 the result ensued becomes more traditional if $0 < A < 1$ as the real number is multiplied by both parts of the complex one. At the same time if $1 < A < 2$, then when multiplying A by Z_n^2 the d_{zn} and k_{zn} modulus grows. This means that the expression on the whole acquires less traditional value.

The indicator for determining of A can be the intensity of migration flows from the countryside to city. It is such a migration that led to the traditionalization of environment-conscience of urban population on the whole –

“peasantization” of cities. So, A will be within the range from 0 to 1 in this very model.

At that, if the number of migrants was 100% of the total number of urban population, then $A = 0$, and if the number of migrants was 0%, then $A=1$. These values determine the scale for measuring A . However, the variant of modernizing migration is possible. In that case A will be within the range from 1 to 2.

Thus, we have the formula:

$$Z_{n+1} = Z_n^2 A$$

Finally, we should take into account such a factor as the influence of outside forces towards environment-conscience. In after-reform Russia there was a factor which had changed radically the environment of people’s staying – it was the state. The modernization role of the state and its influence on the Russian society are well known.

One more factor influencing the individual conscience was public conscience which was mainly traditionalizing. In our mathematical model the influence of outside forces – C (complex number) – acts as the biunity of d_c (state) and k_c (public conscience).

The interaction of all factors specified on the elementary level in the short-term outlook has been studied well (Zhukov 2007), so we can make a formula which describes their interaction taking into consideration the geometric meaning of operations with complex numbers. In other words, we know how and in which direction the position of the point in the complex plane (in phase space) will change under the influence of this or that factor. We will calculate the value of this influence on the basis of studying the each factor indicators.

The final formula for iteration is as follows:

$$Z_{n+1} = Z_n^2 A + C \quad (1)$$

C is added in this formula. The question arises: why addition but not multiplication? The fact is that at multiplication the point in the complex plane changes its position depending not only on a multiplier but on its starting value. At addition all points to which a number is added change their position over an equal distance in the same direction. If we admit that the difference between the initial position of the point and the result of number C influence depends only on number C , and this difference is equal for different points, then we talk about addition. This corresponds to the qualitative sense of influence of public conscience and the state, as under their influence all environments-consciences moved in one definite direction.

It is necessary to introduce an amendment here. Number C specifies the direction and the length of movement of the point. But this movement must be symmetric concerning all coordinates axes.

Otherwise, if all points move in one direction (e.g. from the bottom upwards or from left to right), then it will result in the increase of modulus d_{zn} and k_{zn} for

positive numbers and in the decrease of modulus d_{zn} and k_{zn} for negative numbers, even if the matter concerns the modernization influence of the state.

We introduce the following rule to eliminate this effect and obtain the essential symmetry.

- Let us represent the expression $Z_n^2 A$ by a complex number F .
- Direction (positive or negative) influence of the factor

At the modernization influence of the state:

- If d_f is a negative number, then d_c is also a negative number.
- If d_f is a positive number, then d_c is also a positive number.

When these conditions are fulfilled, the addition of d_c results in increasing of the modulus of real part of initial number, and the point will tend to the domains M and O (see Fig. 3). This is the modernization of environment.

At the traditionalization influence of conscience:

- If k_f is a negative number, then k_c is a positive number.
- If k_f is a positive number, then k_c is a negative number.

When these conditions are fulfilled, the addition of k_c results in decreasing of the modulus of imaginary part of initial number, and the point will tend to the domains T and O (see Fig. 3). This is the traditionalization of conscience.

At the traditionalization influence of the state:

- If d_f is a negative number, then d_c is a positive number.
- If d_f is a positive number, then d_c is a negative number.
- At the modernization influence of public conscience:
- If k_f is a negative number, then k_c is a negative number.
- If k_f is a positive number, then k_c is a positive number.

Thus, C is a rather “peculiar” parameter, as the moduli of its parts are constant but signs can change. Let us pay attention to the specific of value C quality. The point is that modernization and traditionalism in C are expressed by the signs ahead of the parts of C , and the moduli express the degree of quality determined by signs not depending if these numbers are larger or smaller than number 1. In other words, for example $d_{zn} = 0,5$ means the traditionalism of environment whereas $d_c = 0,5$ means the modernization influence of the state when d_f is positive. This condition should be taken into consideration when the scale for d_c and k_c is determined. It is obvious that their values cannot exceed 2 in modulus.

Determining the signs ahead of the parts of C we can specify the direction (modernizing or traditionalizing character) of outside influence, and determining the moduli of the parts of C we can specify the intensity of outside influence.

It remains to add that the qualitative sense of all numbers except C is irrelative to their signs. Therefore, if we take positive values of A , it is necessary to remember that it is rightful as well as the use of negative values of A with the same modulus. We can take a negative A in some cases and a positive one in

other cases but we use only a positive A (or only a negative A) to make calculation convenient.

Introducing new and new values into formula 1 on the principle of folded connection (that is making iterations) we can observe the moving of environment-conscience in the complex plane different domains of which have a different qualitative sense. At that, under the influence of factors represented in the iterated formula the initial conditions of existence of environment-conscience (Z_1) generate a certain final state of environment-conscience, Z_{300} for example.

Using iterated formula 1 and having set a concrete value of coefficients beforehand we will test all points of the chosen section of complex plane which are taken with a certain step of grid. So we can discover the attractors (and their basins) of all possible types of environments-consciences.

All factors except A are expressed in complex numbers in our mathematical model. The results of mathematical operations with complex numbers are not always similar to the results of operations with “usual” real numbers. Therefore at iteration of formula 1 certain, at first site paradoxical effects appear. We consider this circumstance as one of the strongest sides of the model because it allows to take into account non-linear effects of human mentality and microenvironment. This results in the refusal from the logic of “single-valued and proportionate causes and effects” and allows to take into consideration the situations when a butterfly’s flap of wing causes a typhoon, or when the road to hell is paved with good intentions.

It is necessary to remember that the influence of a certain factor on the considered system is a non-linear process. So, changes occurring in the system are not always proportionate to the direction and value of the factors acting. The system can both respond to the acting factors (resist or transform) or remain neutral (not react) or “split” (different subsystems can respond or not respond to the acting factors differently).

3. Description of the Program “Mental Fractal”

On the basis of the developed mathematical model a programmer Julia Movchko has made the program “Mental Fractal” which generates the fractal images of attractors, attractor basins and phase transfers of the systems studied depending on the numerical parameters of certain factors which are input by a user (see Figure 4).

The program in compliance with the specified step of grid tests the total values of complex plane points in the square with diagonal coordinates $(2;2)$; $(-2;-2)$ or specified by a user. In other words, the value of every point is introduced into the iterated formula as the initial value of Z_1 . Then the program makes iterations the number of which is specified by a user and analyzes the final result, i.e. for example Z_{300} coordinates, if the user has demanded to make 300

iterations. The experience accumulated at the studies of algebraical fractals shows that 300 iterations are quite enough for the approximate determination of movement attractor of the point, i.e. the system's behavior type.

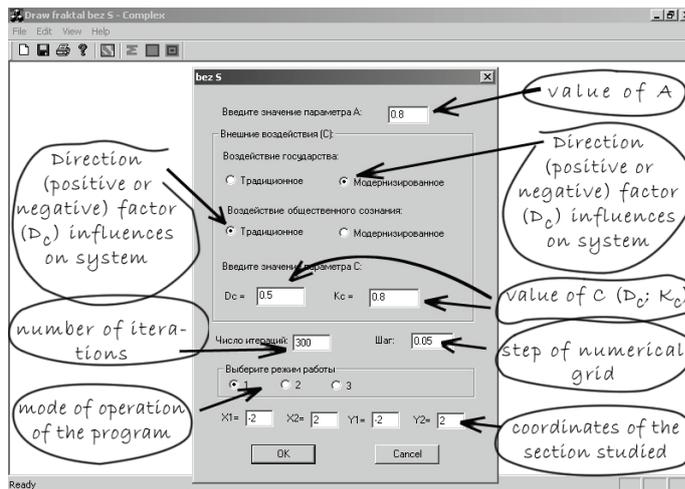
If after a certain number of iterations the final point is outside the section bounded by the square with the diagonal vertexes $(2;2)$ and $(-2;-2)$ in the complex plane, then the starting point is colored pink. This is the basin of attractors which tend to infinity.

If after a certain number of iterations the final point is within the section bounded by the square with the diagonal vertexes $(2;2)$ and $(-2;-2)$ in the complex plane, then the starting point is of other colors.

Thus, we will be able to discover which types of environments-consciences have attractor inside the viewed section of complex plane and can be integrated into the concrete society. The iteration process expresses here social interaction, and the final point gives a rough idea of attractor.

The pink-colored points denote such types of environment-conscience which cannot be inscribed in the society or exist at the parameter values given at all. So we nominally call the pink-colored section the "Giordano Bruno Section".

Figure 4: Options Box of the Program "Mental Fractal"



A user can introduce the values of A and C , the number of iterations, the step of numerical grid of the section studied, the coordinates of the section studied, as well as the mode of operation of the program. The first and the third modes allow to display the basins (starting points) of attractors lying in the range $x \in [-2; 2], y \in [-2; 2]$. At that, at the third mode the basins are colored in

different colors depending in what section of complex plane the attractors of the basins lie.

The black color of the basin indicates that its attractors tend to zero; the white (or pink) color of the basin, as it was already mentioned, means that its attractors lie at infinity; the red color of the basin indicates that the attractors lie in the O-domain; the yellow color of the basin indicates that the attractors lie in the N-domain; the green color of the basin indicates that the attractors lie in the T-domain; the blue color of the basin indicates that the attractors lie in the M-domain.

The second mode of “Mental Fractal” allows to display the image of attractors’ position, of course except those lying at infinity and at zero.

4. Determination of the Parameters (Factors) Values

4.1 Parameter A

As the object of research we have determined the state of environment and mental characteristics of Tambov as well as the factors influencing on them as of 1872. This year belongs to the period of modernization growth when the effects of Great Reforms including those of the Municipal Reform became apparent. The Municipal Reform had a significant influence on the modernization processes in provincial cities of Russia and, among others, in Tambov.

Parameter A is determined as anomalous diffusion coefficient which expresses the result of absolute instability of the city social structure. This coefficient in the considered concrete historical situation shows the number of migrants from rural areas which brought to cities the traditional social practice and mentality.

A is a real number in the interval $[0;2]$. All the values $A>1$ reflect the modernization of environments-consciences as a result of social structure destabilization, and all the values $A<1$ express the traditionalization of environments-consciences. Our task is to calculate the percentage of migrants in the city as of 1872. However it is necessary to take into account not only the number of newly arrived migrants but also those who had settled in the city before the date under research, as they also had a significant traditionalizing influence on the city environment. In this connection a calculation on the basis of migration flows seem problematic, especially taking into account the statistics drawbacks as of 1872. Nevertheless we know the number of people attached to the peasantry who lived on the territory of Tambov at different time. Undoubtedly, not all “peasants by class” were migrants in the first generation but all of them bore the traditional mentality and social practices.

If in 1861 peasants made 22,1% of the Tambov population, then in 1897 this number increased up to 49,7%. On the basis of these numbers we will make the smoothing of the tendency of peasants’ change in number and determine their

part in the city population as of 1872. We get 30,5%. Then we convert these data into the scale assumed by us. We will get $A=0,695$.

4.2 Parameter C ($d_c ; k_c$)

To determine the parameter “influence of the state” (d_c) there has been used an indicator based on the ratio of cases of modernizing and traditionalizing nature which passed through the Regional Administration Office for the period of 1872¹. At that the cases initiated not only in 1872 but also those continuing in this period of time were taken into account.

Let us remind that to measure this parameter in accordance with the conditions of the model the following scale has been assumed. The nature of the state’s influence (traditionalizing or modernizing) is expressed by the signs ahead of the parameter value. The value modulus can vary from 0 to 2. The modulus of the parameter value expresses the intensity of effect but not the direction (nature). In other words, if the impact of the state is 0,2, then under certain conditions this is the evidence of an insignificant modernizing impact but not of a strong traditionalizing pressure.

So, our task is to determine the portion of cases of a modernizing and a traditionalizing nature among the amount of cases in the Governor’s Office. The impact of the state could be multidirectional, but the impacts of one nature fully or partially compensated for the impact of another nature. Therefore, the difference between the portions of modernizing and traditionalizing cases will show the intensity and direction of the state’s total influence on the society (at the local level).

We have got the following data in the course of calculation (see Table 1).

Table 1: The Calculation of the Indicator “Impact of the State” Value

Total number of cases ($a=b+c+r$)	129	100%
Number of cases of a neutral nature (b)	91	70,50%
Number of cases of a modernizing nature (c)	27	21%
Number of cases of a traditionalizing nature (r)	11	8,50%
The intensity of the state’s modernizing influence ($((100(c-r))/a)$)		12,50%

So, the modernizing influence of the state made 12,5% of the total maximum possible pressure of the state on the society. If we convert these data into the scale from 0 to 2, then 100% should be equal to 2 (both values are considered to be maximum possible influence). Therefore, the value of the parameter “impact of the state” is 0,25.

¹ Source: The State Archives of the Tambov Region (SATR). Fund No. 4. The Tambov Regional Administration Office.

In a similar way we can calculate the value of the parameter “influence of the public conscience”. In this case we defined the concept as the active influence of mentality on an individual. The instruments of such an active influence are social inspections and evaluations that are the instruments of forming and maintaining of certain views, thinking and behavior principles accepted, shared and approved by a significant part of the society.

We have obtained the inspections and evaluations significant for an individual from the texts of the Municipal Duma members’ speeches². The choice of source was determined by the fact that, firstly, the Municipal Duma members were authoritative representatives of the city society and, secondly, they could not help reflecting in their public speeches those requirements to a personality, its behavior, etc. which prevailed in the society. Both evaluations and inspections can be qualified by this criterion as traditional, modernizing or neutral.

In spite of a rather modernizing nature of the Municipal Duma as an institution, we came across a considerable inertia of the traditional thinking of the Municipal Duma members if the matter concerned private, personal evaluations of different phenomena of the city life. Such situation seems paradoxical only at first sight. In reality the traditional norms of conscience did not disappear in the process of forming the modernized institutions but accustomed themselves to them and accustomed these modernized institutions “to themselves”. The modernizing nature of the municipality’s activity was combined with the traditional nature of the city people conscience.

The scale for measuring the considered parameter is determined in the same way as the one for the parameter “impact of the state”.

Let us calculate the portions of traditional and modernizing evaluations and inspections in their total amount. Then we calculate the difference of the portions and therefore determine the degree which differently directed impacts compensate each other to. We assume the total number of evaluations and inspections as 100% (or as 2) as this number shows the maximum possible intensity of public conscience’s influence according to this indicator; and thus we calculate the intensity and direction of the parameter’s total influence (see Table 2).

Table 2: The Calculation of the Indicator “Influence of Public Conscience”

Total number of evaluations and inspections ($a=b+c+r$)	156	100%
Number of evaluations and inspections neutral by nature (b)	48	30,8%
Number of evaluations and inspections modernizing by nature (c)	11	7%
Number of evaluations and inspections traditional by nature (r)	97	62,2%
Intensity of public conscience’ traditionalizing influence $((100(r-c))/a)$		55,2%

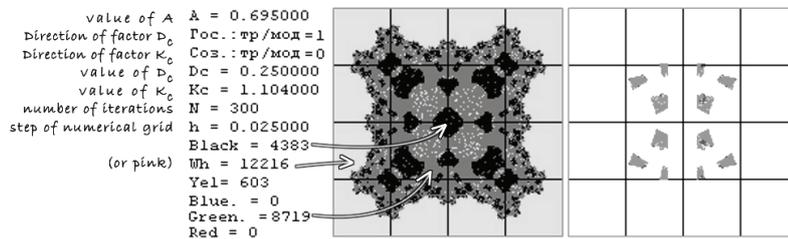
² Source: The Minute Books of the Tambov Municipal Duma for 1872.

So, the traditionalizing impact of public conscience made 55,2% of the total maximum possible impact of the public conscience on an individual. If we convert these data into the scale from 0 to 2, the value of the studied parameter is 1,104.

5. Determination of the Object Domain

The fractal structure obtained on the basis of the input parameters represents a set of basins and attractors of all possible variants for combination of different modernization degrees of conscience and environment (see Figure 5).

Figure 5: The General Results of “Mental Fraktal” Work with Parameters for 1872 (Basins and Attractors). The Tambovskiy Krest (Tambov Cross) Fractal



However our purpose is to research into the concrete historical problem, that is: the society of a certain city at a definite period. In other words, of all possible types of environments-consciences we should choose the ones which really existed in this city for the period of time under research. This sample will be represented as a certain sector in the complex plane in accordance with the conditions of the model.

It is absolutely obvious that the city is populated with people who due to different reasons have different characteristics of the environment-conscience by the degree of modernization. Consequently, the target sector will be represented neither as a point nor a line. We shall call the target sector as the research object domain.

How can we find the exact coordinates of the object domain? Obviously, it is necessary to measure the value of modernization of the environment and modernization of conscience of the society under research. In other words, we should formalize the concrete historical material. Besides, we need to get not one but a range of values for the characteristics of both environment and conscience.

We should choose indicators to measure the specified characteristics. The choice of indicators is connected with the determination of the concepts “city environment” and “city people conscience”. When we talk about the city envi-

ronment as applied to Tambov of 1870-s we mean the social structures interacting within one administrative-territorial formation. We do not consider the city people conscience as the conscience of representatives of different social groups or cultural-anthropological types, but as the conscience of people associated with the city-wide social and political life. This condition is the criterion of differentiating city people as an integrated mental community from non-city people who even if could live in the city did not participate in its social and political processes.

Such definitions of environment and conscience urge us to employ as concrete historic material the facts of social and political history of the city, which have been most representatively reflected in the documents of the city authorities formed after the Municipal Reform of 1870. Undoubtedly, only a small part of the city people possessed elective rights (somewhat over 8%), however the local authorities were outside-of-class, missioned to consolidate the urban community. In practice it meant that the local authorities were occupied with the things which concerned not only those who voted but all city people. That is why the indicators which can be revealed and measured by the local authorities' documents reflect correctly the modernization level of both the urban environment in the whole and the city people conscience.

As indicators must be easily formalized it is convenient to choose those of them which concerned expenditures and revenues (expressed in definite sums) as well as the indicators which allow to make the statistical calculation of these or those actions of the city people and (or) local authorities. To calculate the indicators only those revenues, expenditures and actions of the subjects are chosen which have either traditional or modernized nature.

After calculating the ratios of the modernized and traditional elements within each indicator (earnings, expenditures, and actions) we can determine the numerical ratio of modernized and traditional elements within the indicator. If we assume 0% per 100% as extreme values of the scale and, respectively, 100% per 0% as values of modernization and traditional expenditures, of earnings and actions, then we will be able to calculate the exact numerical expression of modernization level as to the given indicator.

For example, let us take as a modernized environment indicator the following: the ratio of the agricultural income and commercial-and-industrial activities income within the municipal budget. It is not typical for people of a modernized (industrial) city to do farming. So, the part of income from agriculture should be minimal in the revenue structure of an industrial city. In other words, the maximum modernized urban environment presupposes the following correlation: 0% of the municipal budget earnings from agriculture per 100% of earnings from commercial-and-industrial activities. After analyzing the structure of the municipal budget revenues we can determine and express in concrete numbers the really existed ratio of modernized and traditional city revenues; this will be one of the indicators.

A natural question arises: what are the criteria of relating a certain income, expenditure and action to the category of modernized or traditional ones? It is necessary to emphasize here that the calculation of the indicators values is based, in addition to the quantitative analysis, on the qualitative interpretation of their individual elements. The theory of modernization has a large number of tools to conduct such qualitative analyses – the theory has clear standards which determine a modernized or traditional nature of a wide range of social phenomena. It is especially important for our research that these modernized standards spread not only to objective phenomena but also to subjective, mental ones.

In many cases the same indicators are applied to the calculation of both environment and conscience characteristics. It is not surprising because the environment and conscience are closely connected and are often shown in the same phenomena.

A few words are about the sources on the basis of which the indicators were calculated. These are the minute-books of the Tambov Municipal Duma for 1872. They reflect, apart from the debates, the city people's appeals, the initiatives and reports of the Board, resolutions of the Municipal Duma, etc. The work with the budget indicators calculated according to "The Income and Expenditure Statement of the Town of Tambov"³ was based on the following observations. We took into account the plans (budget) for 1873 as, firstly, this budget was formed in 1872 and, secondly, it is obvious, that this budget was formed proceeding from the calculation of the actual income and expenditures of 1872.

The methods chosen presuppose the total (not sample) study of all modernized and traditional manifestations of the town life for the specified period reflected in the representative sources. Otherwise the correlation obtained could have a casual nature. All the minute-books of the Municipal Duma for 1872 have been studied as well as the budget documents of Tambov for the specified period.

We confined ourselves to 21 indicators for determination of the modernization degree of the city environment and the city people conscience for 1872 (see Tables 3 and 4).

After the calculation of indicators the task is to determine the sought ranges of values for the characterizing conscience and environment on the basis of the indicators values. In other words, we should determine the limits of the actual conscience and the environment characteristics for Tambov citizens for 1872. We have preferred using one of the simplest and most effective statistical methods – the evaluation of the average mean of absolute values of the data points deviations from the average (average deviation). The average deviation

³ Source: Income and Expenditure Statement of the Town of Tambov for 1873.

is the measure of dispersion of the data set. If we subtract the average deviation from the arithmetical mean value in the series of indicators values and then add it, then we get the sought range for the determination of the research object domain as a result of these two operations (see Table 5).

Table 3: Indicators of Mentality Modernization Level of the Population of Tambov (1872)

Indicator	Indicator Value
Ratio of the cases of a traditional and modernized nature considered by the Municipal Duma (MD) ⁴ *	0,928
Ratio of the resolutions of traditional and modernized nature carried by the MD*	1,31
Ratio of city people's appeals to the MD of traditional and modernized nature	0,696
The ratio of the cases related to arrears forgiveness and unforgiveness in the scope of events referring to arrears, discussed by the MD	1,467
Ratio of the cases of the MD extraordinary spendings of traditional and modernized nature	0,5
Ratio of the MD refusal from extraordinary spendings of a traditional and modernized nature	1
Ratio of public-service employees holding their posts and those who submitted their resignation by traditional motives	0,8
The ratio (in the total number of the MD cases in dispute with the Regional Administration Office) of the cases, in relation where to there have been filed the MD complaints to the Senate, per the cases left to the Regional Administration Office discretion	0,857
Ratio of the facts of return and non-return of public funds, obtained by the city people illegally, to the Municipal Treasury in the total number of the amounts discovered	0,5
Ratio of the municipal budget expenses of a modernized and traditional nature*	1,43
Ratio in the municipal budget of the expenditures for police and education*	0,573
Ratio of amounts by new expenditure items of a modernized and traditional nature*	1,326

Table 4: Indicators of the Tambov Environment Modernization Level (1872)

Indicator	Indicator Value
Ratio of the number of cases of a traditional and modernized nature considered by the MD*	0,928
Ratio of the number resolutions of a traditional and modernized nature, carried by the MD*	1,31

⁴ The symbol «*» marks the indicators which concern both the definition of characteristics of conscience and characteristics of environment.

<i>Table 4 continued...</i>	
Ratio of the amounts of the MD extraordinary expenses of a traditional and modernized nature	1,506
Ratio of the amounts of the MD refusals from extraordinary expenses of a traditional and modernized nature	0,886
Ratio of the amounts returned and not returned to the Municipal Treasury	0,421
Ratio of the city's income from commercial-and- industrial activity and income from agriculture	1,647
Ratio of the municipal budget expenditures of a traditional and modernized nature*	1,43
Ratio of the expenditures for police and education in the municipal budget*	0,573
Ratio of the amounts by new expenditure items of a traditional and modernized nature*	1,326

Table 5: Calculation of the Object Domain Values

The arithmetic mean of the conscience modernization indicators values	0,95
The average of absolute values of the data points deviations from the average	0,29
The minimal value of the object domain (along the axis "conscience characteristic")	0,65
The maximal value of the object domain (along the axis "conscience characteristic")	1,25
The arithmetic mean of the environment modernization indicators values	1,11
The average of absolute values of the data points deviations from the average	0,37
The minimal value of the object domain (along the axis "environment characteristic")	0,75
The maximal value of the object domain (along the axis "environment characteristic")	1,48

6. Difficulties in Convertation: Approaches to the Interpretation of Modelling Results

The modelling results represent a set of a great number of effects which reflect a great variety of historical situations. However, our objective is a qualitative interpretation of the results of the model functioning within the range of the object domain interesting for us and with the specified, historically real values of the factors (parameters).

Practically, the process of modelling consists in converting the historical discourse into the mathematical one at first, and then, vice versa, – in converting the formalized results of the program's work into the language of qualitative concepts historians have accustomed to. So, to fulfill the task assigned we should elaborate certain notions of the following: which historical phenomena are behind one or another form of the final modelling data. In other words, we

need a glossary of basic (or at least certain) notions which connect a mathematical model with a historical discourse. To make such a glossary we consider some typical phenomena appearing in the model in the course of computer experiments undertaken.

However, at first it is necessary to specify that the phenomena studied do not exhaust the whole variety of modelling results as all the combinations of factors and parameters (even if the variation interval of the input values is moderately large) can amount to hundreds of thousands.

It is also necessary to pay attention that we research into the basins having variable attractors of development of the environment-conscience together with the analysis of location of the attractors themselves.

1) At the values of the parameters being maximal and minimal (or close to maximal and minimal), in the total area of complex plane, the results of modelling demonstrate the high determinacy of development perspectives of conscience and environment in attractors located at infinity or at zero. Such a phenomenon points to the perspectives of transformation of the environment-conscience at maximal and minimal values into ultra-traditional or ultra-modernized qualitative states which can not be analyzed within the limits of the given mathematical model.

The parameter values being minimal, the figure of the basin usually represents a fractal close to the sphere contour, the inner part of which is inclined to the absolutely traditional total and the outer part is drawn to the absolutely modernized one. In this case the society (as well as the parameters values being maximal when a fractal image is absent) in accordance with the conditions of the model is not integrated, too polarized to function normally. The spherical form of the fractal is also easily explained: low values of the factors lead to a high-level determinacy of attractors by the initial conditions (that is the initial points data). In other words, the initial states points located around zero have attractors at zero; all the remaining points – at infinity.

2) The growth of factors values leads to diversification and to society complication: various areas being originally well organized are revealed inside the fractals of the basin. This means that different groups of population have different development perspectives of mentality and the social environment around them. In whole, this is not the evidence that the society is disintegrated, qualitatively transformed, as diversification originally causes the complementarity of social elements and then disintegration.

The appearing of differently colored areas of the basins in the plane of attractors corresponds to the phenomenon which we called the first explosion of attractors. The thing is that originally situated near zero (in the T-area) the usual four attractors break into several pieces moving, with factors increasing, in different directions, specifically: and in the direction of the boundaries, and moving away from each other. This indicates that certain

social communities maintain this or that degree of common “social fate” and evolve in a consolidated manner in the varying degree.

- 3) It is worthy of note that the attractors’ explosions are phase changes, when an insignificant change of factors values leads to the qualitative transformation of the system.
- 4) The second explosion of attractors takes place when the values of factors further increase and represents the start of destruction of the existing social institutions and social practices and thinking norms associated with them, as the attractors consolidated in the varying degree and forming splinters of the first explosion, break to separate points. In this case each point of the basins plane has its own attractor or represents a set of a very small number of them, what means that people of different types of the environment conscience lose complementarity.

In the plane of basins the second explosion of attractors is accompanied by the emergence of chaos in the fractal structure: at first the accurate structure of differently colored areas of basins’ fractal is disrupted; then the scattered areas of tending to zero and to infinity appear; and, at last, the fractal gradually disappears, what is also the evidence of disintegration of the previously existing society with the peculiar characteristics of the environment-conscience. There is no doubt that it is the disintegration which takes place only within the limits of the model considered and nothing more than that. The model here does not take into account the factors holding the society and contributing to the change from fast disintegration to gradual transformation.

In this connection it is especially important to emphasize the following circumstance. The attraction of new factors apart from those appearing in the model allows to discover those ones which interfere with social cataclysms, revolutions, and degradation of society. So, the mathematical modelling allows to compare different factors by analyzing their different combinations. On the basis of this analysis we can reveal historical phenomena and facts contributing to the consolidation or erosion of the existing concrete historical society, and even to determine the comparative degree of their influence on the processes of consolidation and erosion.

- 5) At certain combinations of the factors values there are periods of stability when even a significant change in the values does not lead to the qualitative change of the system. Quite the contrary, at some other values the system undergoes the state of phase change when a small change of factors leads to the fundamental change of the system, usually in the form of the first or second explosion of attractors. Such effects correspond to the ideas about the complexity of historical process, about possible great consequences of the influence of small reasons, as well as about the possible existence of long-lasting, determined and predictable processes in the historical reality.

- 6) The larger numerical value the factors have (the stronger their influence on the system is), the more complex the picture of basins is (the more complex the social organism is), the shorter stability intervals are, and the more probable a phase change is. (This observation does not concern maximal or close to maximal values of factors). If the factors which influence the society are very large, any misbalance of the factors is fraught with serious consequences. Quite the contrary, the external influence being small, the colossal internal inertia of the society suppresses outer impulses arising as a result of balance shift of outer acting factors.

The environments-conscience development diversity which generates the complexity and effectiveness of the society leads gradually to the accumulation of disfunctionality, like a human being's maturing ultimately becomes ageing.

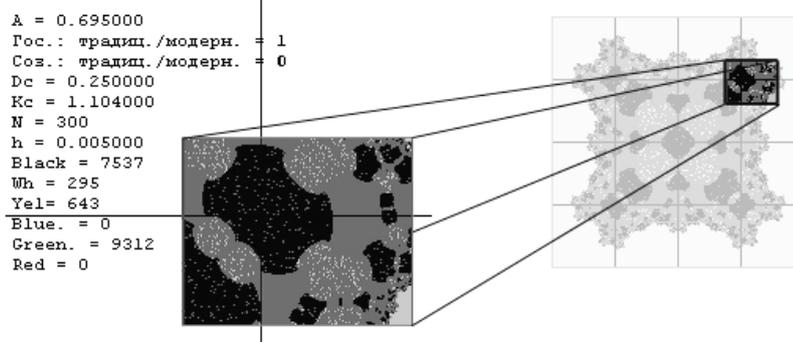
- 7) The diversity of society in the interval between the first and the second explosions of attractors reduces the influence of attractors of individual environment-conscience social evolution on the initial conditions. Even an insignificant change of the initial conditions of an individual's evolution in a society with a complex organization leads to the change of attractors. And, even a more insignificant change of initial conditions is required to change the attractor in the state of chaos. However, in the first case we deal only with the multi-variety of social perspectives in a society with a complex structure, and in the second case we deal with the disintegration of the society, and its regeneration which is connected with the destruction of existing social structures and social practices and thinking norms typical for them, when the environment-conscience evolution of a certain type does not already occur synchronously and complementarily with similar evolution of other environments-conscience types.

7. Interpretation of Modelling Results in Statics

- 1) The analysis of the object domain when the values of factors (the model's parameters) are real shows that the urban environment in the whole and the types of conscience typical for it had a traditional nature (see Figure 6).

We can add that the Tambov urban community had no chances to evolve to the attractor in the M-domain in a consolidated manner within this model. With the considered values of parameters the attractors are in the other domains – in the T-domain (the larger part) and in the N-domain (the smaller part).

Figure 6: The Object Domain with Parameters of 1872



So, with the factors really existed and taken into account in the model, the Tambov community could not successfully implement a modernized project in the perspective. That is why the response to the state's modernized pressure did not take the form of a gradual drift to a bright modernized future. In other words, the state's modernized pressure did not eliminate the influence of the traditionalizing factors – the flow of migrants to the urban environment and the traditionalizing influence of public conscience.

- 2) A significant part of starting conditions in the plane of basins is colored black. This indicates that these areas are close to zero, tending to overtraditional state. And the “black areas” are relatively consolidated and low-scattered: they form fractal structures, poorly dissected in the centre and much more dissected peripherally. Being of such fractal nature, the “black areas” pass through all the segments of the object domain without dissolving in the green (“T”) basin. Actually, this is the evidence of the presence in the urban environment of the environment-conscience type, which does not fit well into traditional type of environment-conscience predominating in the city. We can suppose that we deal with local, closed and self-reproducing social phenomena – be it a community of migrants, residents of agricultural settlements, etc. – which have been considered even by relatively traditional city people contemporaries as elements foreign to urban environment – “muzhichki!”.

And yet inside the “black areas” there are rather rare green points scattered to attest to certain (although insignificant) infiltration of more modernized (whilst not completely modernized) norms of thinking and social practices.

- 3) The most insignificant areally and at the same time the most homogeneous and consolidated basin is the “pink area” (types of the conscious environments the attractors of which are at infinity). In any society there are people whose conscience and social experience are so innovative that they are not compatible with the predominating apprehension.

- 4) In the “green area” of the traditional urban population there are yellow points scattered, i.e. initial conditions with the attractor in the N-domain (“narodnik in village”, that is to say, the modernized conscience in the traditional environment). It is worthy of note that the “yellow centers” are not consolidated and not numerous, but they are the evidence of successful modernization of conscience of a certain part of social environment.

8. Interpretation of Modelling Results in Dynamics

So, we have considered the static profile of the object domain structure when the parameters values are fixed and real. To analyze more thoroughly the meaning of this structure it is necessary to place it into the context of other profiles with the combinations of the parameters values close to real. To do that we studied the profiles in which A , d_c and k_c take the values $\pm 0,2$ of real figures. The results of the comparative study of the profiles obtained are given in Figures 7, 8, 9 and 10.

Figure 7: The Dynamics of the “Mental Fractal” Work Results at Changing of Certain Parameters ($d_c ; k_c$) (Basins)

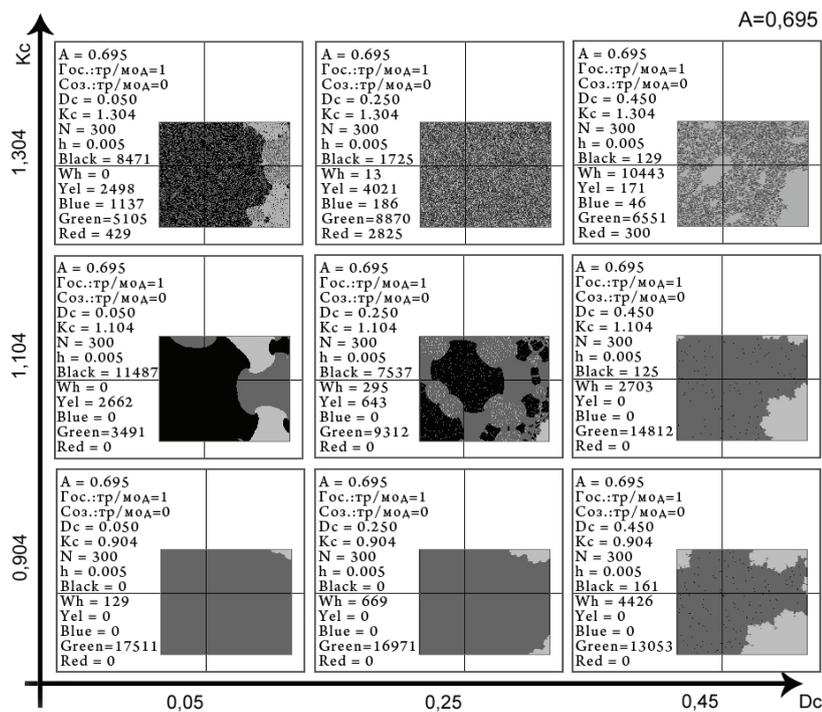
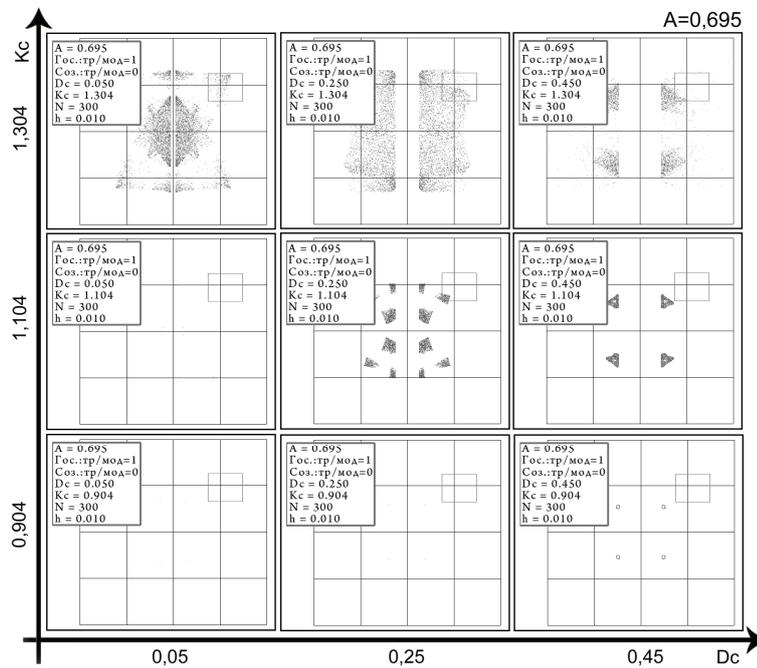


Figure 8: The Dynamics of the “Mental Fractal” Work Results at Changing of Certain Parameters (d_c ; k_c) (Attractors)



- On the basis of the results received the following observations can be made.
- 1) When the modernizing influence of the state increases, the two other parameters of the model being fixed, the dialectical consolidation of attractors takes place; that is the attractors gather into relatively compact clusters after the phase of a certain spread (see Figure 7). In other words, the community is finally unified under the influence of the state – the number of variants of social perspectives is reduced. The modernizing pressure of the state caused the destabilization of the existing social organism and then its consolidation in a new qualitative state. At that, the modernizing pressure contributed to scattering of both black and yellow zones which are relatively opposite by their characteristics. So, the state contributed to not only the consolidation of the society but its consolidation at the level which was historically possible, that is corresponding to the modernizing potential of the community. When the state’s modernizing pressure increases the archaic ultra-traditional strata and the strata which belong to the type “narodnik in village” are mixed into a relatively homogeneous community traditional by its characteristics but without perspectives of archaization. If we look at the picture of attractors (see Figure 7), we can discover that in the considered perspective

the attractors are located closer to the modernized zone than to zero. The reduction of individuals of “narodnik in village” type can be connected both with the persecution of the revolutionarily inclined city people by the state and with the satisfaction of their modernized ambitions by the state’s reform policy. The plane of attractors with real parameters (see Figure 8) can be considered as the beginning or ending of attractors’ consolidation phase depending on whether the matter concerns the increase or reduction of modernizing pressure. Anyhow, the introduction of real parameters gives results close to phase change (the explosion of attractors). This is the evidence that the year of 1872 chosen for the demonstration of the model’s capabilities was included into the chronological confines dividing between different historical phases.

- 2) Let us trace back the dynamics of traditionalizing influence of the public conscience, the other two parameters actual values being fixed (see Figure 9). The increase of traditionalizing influence causes the explosion of attractors. At that, after the first explosion a relatively homogeneous society is diversified and at the next stage is disintegrated – the second explosion of attractors occurs. After the second explosion of attractors the inner structure of basins’ fractals disappears in the chaotic mixture of all colors zones. For the first time the type “migrant in town” (traditional conscience in the modernized environment) and M-type (modernized conscience in the modernized environment) appear. The appearance of such types should not be considered as the logic result of modernization but as an unhealthy reaction to traditionalization.

The society in its previous forms is atomized: the homogeneousness turns to the homogeneous heterogeneity. These are the starting conditions for forming a society of a new type which can be described as the society marginalization. Let us remind that the mixing of the basins of different colors describes the situation when social perspectives of an individual can rapidly and radically change as a result of extremely insignificant change in its existence conditions. (A point can easily transfer from one basin to another if they are too split.) The disintegration of homogeneous color basins can be interpreted as the marginalization of the society.

If the upper strata are unable to increase the power of modernizing pressure and reactionary character of public conscience increase, then the modernizing needs of the society appear in pathologic forms: contrary to but not within the limits of the existing social institutions. At that, the second explosion of attractors and disintegration of the society in its previous forms takes place depending on the following: the more A values are, the less k_c values are (that is the less migrants from village are in city). It is quite explainable as the high level of migration from village which means that is the strengthening of traditionalism of the urban way of life reduces the modernizing needs of the society until migrants turn to proletariat.

Figure 9: The Dynamics of the “Mental Fractal” Work Results at Changing of Certain Parameters ($A ; k_c$) (Basins)

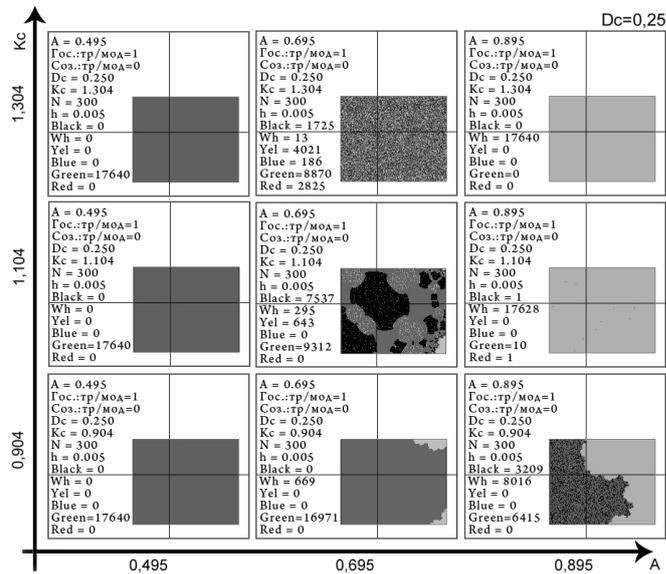
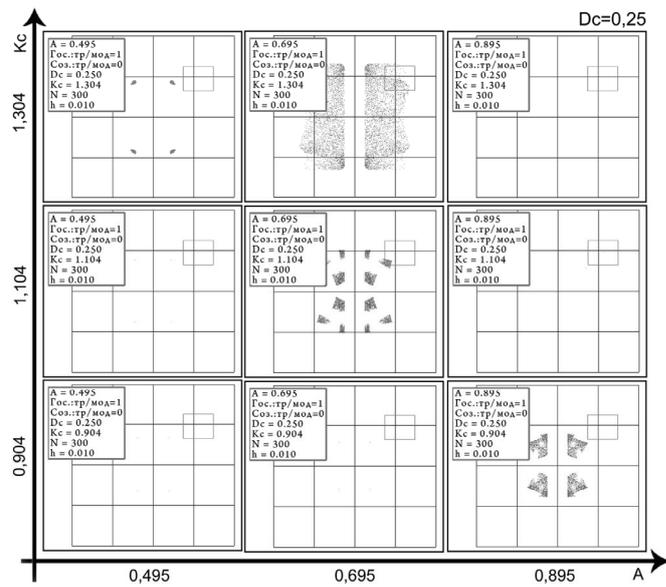


Figure 10: The Dynamics of the “Mental Fractal” Work Results at Changing of Certain Parameters ($A ; k_c$) (Attractors)



9. Can History Study the Future?

In the presented model of “Mental Fractal” there is a strict mathematical procedure between our historical and sociological concepts of modernization-traditionalization counteraction and the image which graphically expresses this counteraction. The qualitative analysis of the situation is transformed to a mathematical model, and due to that a rather accurate metaphor of the situation which corresponds to the qualitative data is produced.

This means that the model is really a rather accurate approach to the historically existing social system. That is why the experiment with the model specified seems to be heuristically correct, specifically: the change of the original parameters and the analysis of new results. We cannot experiment with the historical material – “there happened to be what happened to be, and we do not know, how could that happen otherwise, and whether it could happen the other way at all”. As for a mathematical model we can make computer experiments considering other variants of development under different initial conditions.

The model is a kind of matrix of the different factors interaction. The parameters of these factors with some restrictions can be preset reasoning from the modern state of social systems.

It is necessary to make a significant reservation. Models do not represent the exact reflection of the reality but functional (and functioning) generalization of several factors – the generalization which in such a form can be used in the generalizations of a higher level. That is why a model must not be and cannot be the exact copy of the reality. We do not judge about the whole reality in the infinite manifold of its interconnections by a model but only about its several segments abstracted and formalized in numerical values and interconnected by a certain mathematical apparatus.

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